Instructor's Manual

By

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For

Microeconomics

Eighth Edition

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Contents

PART ONE

Introduction: Markets and Prices

Chapter 1
Preliminaries

■ Teaching Notes

Chapter 1 covers basic concepts students first saw in their introductory course but could bear some repeating. Since most students will not have read this chapter before the first class, it is a good time to get them talking about some of the concepts presented. You might start by asking for a definition of economics. Make sure to emphasize scarcity and trade-offs. Remind students that the objective of economics is to explain observed phenomena and predict behavior of consumers and firms as economic conditions change. Ask about the differences (and similarities) between microeconomics and macroeconomics and the difference between positive and normative analysis. Review the concept of a market and the role prices play in allocating resources. Discussions of economic theories and models may be a bit abstract at this point in the course, but you can lay the groundwork for a deeper discussion that might take place when you cover consumer behavior in Chapter 3.

Section 1.3 considers real and nominal prices. Given the reliance on dollar prices in the economy, students must understand the difference between real and nominal prices and how to compute real prices. Most students know about the Consumer Price Index, so you might also mention other price indexes such as the Producer Price Index and the Personal Consumption Expenditures (PCE) Price Index, which is the Fed's preferred inflation measure.[1] It is very useful to go over some numerical examples using goods that are in the news and/or that students often purchase such as gasoline, food, textbooks, and a college education.[2]

In general, the first class is a good time to pique student interest in the course. It is also a good time to tell students that they need to work hard to learn how to do economic analysis, and that memorization alone will not get them through the course. Students must learn to think like economists, so encourage them to work lots of problems. Also encourage them to draw graphs neatly and large enough to make them easy to interpret. It always amazes me to see the tiny, poorly drawn graphs some students produce. It is no wonder their answers are often incorrect. You might even suggest they bring a small ruler and colored pencils to class as they can draw good diagrams.

■ Questions for Review

1. **It is often said that a good theory is one that can be refuted by an empirical, data-oriented study. Explain why a theory that cannot be evaluated empirically is not a good theory.**

   A theory is useful only if it succeeds in explaining and predicting the phenomena it was intended to explain. If a theory cannot be evaluated or tested by comparing its predictions to known facts and data, then we have no idea whether the theory is valid. If we cannot validate the theory, we cannot have any confidence in its predictions, and it is of little use.

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[1] The *CPI* and *PPI* are reported by the Bureau of Labor Statistics (www.bls.gov). The PCE Price Index is compiled by the Bureau of Economic Analysis in the Commerce Department (www.bea.gov).
[2] The College Board collects data on college tuition (www.collegeboard.com).

2. Which of the following two statements involves positive economic analysis and which normative? How do the two kinds of analysis differ?

a. Gasoline rationing (allocating to each individual a maximum amount of gasoline that can be purchased each year) is poor social policy because it interferes with the workings of the competitive market system.

Positive economic analysis is concerned with explaining *what is* and predicting *what will be*. Normative economic analysis describes *what ought to be*. Statement (a) is primarily normative because it makes the normative assertion (i.e., a value judgment) that gasoline rationing is "poor social policy." There is also a positive element to statement (a), because it claims that gasoline rationing "interferes with the workings of the competitive market system." This is a prediction that a constraint placed on demand will change the market equilibrium.

b. Gasoline rationing is a policy under which more people are made worse off than are made better off.

Statement (b) is positive because it predicts how gasoline rationing affects people without making a value judgment about the desirability of the rationing policy.

3. Suppose the price of regular-octane gasoline were 20 cents per gallon higher in New Jersey than in Oklahoma. Do you think there would be an opportunity for arbitrage (i.e., that firms could buy gas in Oklahoma and then sell it at a profit in New Jersey)? Why or why not?

Oklahoma and New Jersey represent separate geographic markets for gasoline because of high transportation costs. There would be an opportunity for arbitrage if transportation costs were less than 20 cents per gallon. Then arbitrageurs could make a profit by purchasing gasoline in Oklahoma, paying to transport it to New Jersey and selling it in New Jersey. If the transportation costs were 20 cents or higher, however, no arbitrage would take place.

4. In Example 1.3, what economic forces explain why the real price of eggs has fallen while the real price of a college education has increased? How have these changes affected consumer choices?

The price and quantity of goods (e.g., eggs) and services (e.g., a college education) are determined by the interaction of supply and demand. The real price of eggs fell from 1970 to 2010 because of either a reduction in demand (e.g., consumers switched to lower-cholesterol food), an increase in supply due perhaps to a reduction in production costs (e.g., improvements in egg production technology), or both. In response, the price of eggs relative to other foods decreased. The real price of a college education rose because of either an increase in demand (e.g., the perceived value of a college education increased, population increased, etc.), a decrease in supply due to an increase in the cost of education (e.g., increase in faculty and staff salaries), or both.

5. Suppose that the Japanese yen rises against the U.S. dollar—that is, it will take more dollars to buy a given amount of Japanese yen. Explain why this increase simultaneously increases the real price of Japanese cars for U.S. consumers and lowers the real price of U.S. automobiles for Japanese consumers.

As the value of the yen grows relative to the dollar, it takes more dollars to purchase a yen, and it takes fewer yen to purchase a dollar. Assume that the costs of production for both Japanese and U.S. automobiles remain unchanged. Then using the new exchange rate, the purchase of a Japanese automobile priced in yen requires more dollars, so for U.S. consumers the real price of Japanese cars in dollars increases. Similarly, the purchase of a U.S. automobile priced in dollars requires fewer yen, and thus for Japanese consumers the real price of a U.S. automobile in yen decreases.

6. **The price of long-distance telephone service fell from 40 cents per minute in 1996 to 22 cents per minute in 1999, a 45% (18 cents/40 cents) decrease. The Consumer Price Index increased by 10% over this period. What happened to the real price of telephone service?**

   Let the *CPI* for 1996 equal 100 and the *CPI* for 1999 equal 110, which reflects a 10% increase in the overall price level. Now let's find the real price of telephone service (in 1996 dollars) in each year. The real price in 1996 is 40 cents. To find the real price in 1999, divide CPI_{1996} by CPI_{1999} and multiply the result by the nominal price in 1999. The result is $(100/110) \times 22 = 20$ cents. The real price therefore fell from 40 to 20 cents, a 50% decline.

■ Exercises

1. **Decide whether each of the following statements is true or false and explain why:**

   a. **Fast food chains like McDonald's, Burger King, and Wendy's operate all over the United States. Therefore the market for fast food is a national market.**

   This statement is false. People generally buy fast food locally and do not travel large distances across the United States just to buy a cheaper fast food meal. Because there is little potential for arbitrage between fast food restaurants that are located some distance from each other, there are likely to be multiple fast food markets across the country.

   b. **People generally buy clothing in the city in which they live. Therefore there is a clothing market in, say, Atlanta that is distinct from the clothing market in Los Angeles.**

   This statement is false. Although consumers are unlikely to travel across the country to buy clothing, they can purchase many items online. In this way, clothing retailers in different cities compete with each other and with online stores such as L.L. Bean. Also, suppliers can easily move clothing from one part of the country to another. Thus, if clothing is more expensive in Atlanta than Los Angeles, clothing companies can shift supplies to Atlanta, which would reduce the price in Atlanta. Occasionally, there may be a market for a specific clothing item in a faraway market that results in a great opportunity for arbitrage, such as the market for blue jeans in the old Soviet Union.

   c. **Some consumers strongly prefer Pepsi and some strongly prefer Coke. Therefore there is no single market for colas.**

   This statement is false. Although some people have strong preferences for a particular brand of cola, the different brands are similar enough that they constitute one market. There are consumers who do not have strong preferences for one type of cola, and there are consumers who may have a preference, but who will also be influenced by price. Given these possibilities, the price of cola drinks will not tend to differ by very much, particularly for Coke and Pepsi.

2. **The following table shows the average retail price of butter and the Consumer Price Index from 1980 to 2010, scaled so that the *CPI* = 100 in 1980.**

	1980	1990	2000	2010
CPI	100	158.56	208.98	218.06
Retail price of butter (salted, grade AA, per lb.)	$1.88	$1.99	$2.52	$2.88

a. Calculate the real price of butter in 1980 dollars. Has the real price increased/decreased/ stayed the same from 1980 to 2000? From 1980 to 2010?

Real price of butter in year $t = \frac{CPI_{1980}}{CPI_t} \times$ nominal price of butter in year t.

	1980	1990	2000	2010
Real price of butter (1980 $)	$1.88	$1.26	$1.21	$1.32

The real price of butter decreased from $1.88 in 1980 to $1.21 in 2000, and it increased from $1.88 in 1980 to $1.32 in 2010, although it did increase between 2000 and 2010.

b. What is the percentage change in the real price (1980 dollars) from 1980 to 2000? From 1980 to 2010?

Real price decreased by $0.67 (1.88 − 1.21 = 0.67) between 1980 and 2000. The percentage change in real price from 1980 to 2000 was therefore (–0.67/1.88) × 100% = –35.6%. The decrease was $0.56 between 1980 and 2000 which, in percentage terms, is (–0.56/1.88) × 100% = –29.8%.

c. Convert the *CPI* into 1990 = 100 and determine the real price of butter in 1990 dollars.

To convert the *CPI* into 1990 = 100, divide the *CPI* for each year by the *CPI* for 1990 and multiply that result by 100. Use the formula from part a and the new *CPI* numbers below to find the real price of milk in 1990 dollars.

	1980	1990	2000	2010
New *CPI*	63.07	100	131.80	137.53
Real price of butter (1990 $)	$2.98	$1.99	$1.91	$2.09

d. What is the percentage change in the real price (1990 dollars) from 1980 to 2000? Compare this with your answer in (b). What do you notice? Explain.

Real price decreased by $1.07 (2.98 − 1.91 = 1.07). The percentage change in real price from 1980 to 2000 was therefore (−1.07/2.98) × 100% = –35.9%. This answer is the same (except for rounding error) as in part b. It does not matter which year is chosen as the base year when calculating percentage changes in real prices.

3. At the time this book went to print, the minimum wage was $7.25. To find the current value of the *CPI*, go to http://www.bls.gov/*CPI*/home.htm. Click on "*CPI* Tables," which is found on the left side of the web page. Then, click on "Table Containing History of *CPI*-U U.S. All Items Indexes and Annual Percent Changes from 1913 to Present." This will give you the *CPI* from 1913 to the present.

a. With these values, calculate the current real minimum wage in 1990 dollars.

The last year of data available when these answers were prepared was 2010. Thus, all calculations are as of 2010. You should update these values for the current year.

Real minimum wage in 2010 $= \frac{CPI_{1990}}{CPI_{2010}} \times$ minimum wage in 2010 $= \frac{130.7}{218.056} \times \$7.25 = \$4.35$.

So, as of 2010, the real minimum wage in 1990 dollars was $4.35.

b. Stated in real 1990 dollars, what is the percentage change in the real minimum wage from 1985 to the present?

The minimum wage in 1985 was \$3.35. You can get a complete listing of historical minimum wage rates from the Department of Labor, Wage and Hour Division at http://www.dol.gov/whd/minwage/chart.htm.

Real minimum wage in 1985 $= \dfrac{CPI_{1990}}{CPI_{1985}} \times \$3.35 = \dfrac{130.7}{107.6} \times \$3.35 = \$4.07.$

The real minimum wage therefore increased slightly from \$4.07 in 1985 to \$4.35 in 2010 (all in 1990 dollars). This was an increase of $\$4.35 - 4.07 = \$0.28$, so the percentage change was $(0.28/4.07) \times 100\% = 6.88\%$.

Chapter 2
The Basics of Supply and Demand

■ Teaching Notes

This chapter reviews the basics of supply and demand that students should be familiar with from their introductory economics courses. You may choose to spend more or less time on this chapter depending on how much review your students require. Chapter 2 departs from the standard treatment of supply and demand basics found in most other intermediate microeconomics textbooks by discussing many real-world markets (copper, office space in New York City, wheat, gasoline, natural gas, coffee, and others) and teaching students how to analyze these markets with the tools of supply and demand. The real-world applications are intended to show students the relevance of supply and demand analysis, and you may find it helpful to refer to these examples during class.

One of the most common problems students have in supply/demand analysis is confusion between a *movement along a supply or demand curve* and a *shift in the curve*. You should stress the *ceteris paribus* assumption, and explain that all variables except price are held constant along a supply or demand curve. So movements along the demand curve occur *only with changes in price*. When one of the omitted factors changes, the entire supply or demand curve shifts. You might find it useful to make up a simple linear demand function with quantity demanded on the left and the good's price, a competing good's price and income on the right. This gives you a chance to discuss substitutes and complements and also normal and inferior goods. Plug in values for the competing good's price and income and plot the demand curve. Then change, say, the other good's price and plot the demand curve again to show that it shifts. This demonstration helps students understand that the other variables are actually in the demand function and are merely lumped into the intercept term when we draw a demand curve. The same, of course, applies to supply curves as well.

It is important to make the distinction between quantity demanded as a function of price, $Q_D = D(P)$, and the inverse demand function, $P = D^{-1}(Q_D)$, where price is a function of the quantity demanded. Since we plot price on the vertical axis, the inverse demand function is very useful. You can demonstrate this if you use an example as suggested above and plot the resulting demand curves. And, of course, there are "regular" and inverse supply curves as well.

Students also can have difficulties understanding how a market adjusts to a new equilibrium. They often think that the supply and/or demand curves shift as part of the equilibrium process. For example, suppose demand increases. Students typically recognize that price must increase, but some go on to say that supply will also have to increase to satisfy the increased level of demand. This may be a case of confusing an increase in quantity supplied with an increase in supply, but I have seen many students draw a shift in supply, so I try to get this cleared up as soon as possible.

The concept of elasticity, introduced in Section 2.4, is another source of problems. It is important to stress the fact that any elasticity is the ratio of two percentages. So, for example, if a firm's product has a price elasticity of demand of −2, the firm can determine that a 5% increase in price will result in a 10% drop in sales. Use lots of concrete examples to convince students that firms and governments can make important use of elasticity information. A common source of confusion is the negative value for the price elasticity of demand. We often talk about it as if it were a positive number. The book is careful in referring to the "magnitude" of the price elasticity, by which it means the absolute value of the price elasticity, but students may not pick this up on their own. I warn students that I will speak of price elasticities as if they were positive numbers and will say that a good whose elasticity is −2 is more elastic (or greater) than one whose elasticity is −1, even though the mathematically inclined may cringe.

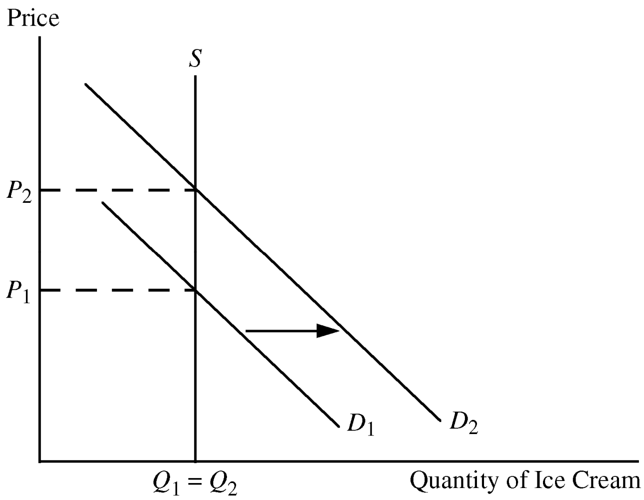
Section 2.6 brings a lot of this material together because elasticities are used to derive demand and supply curves, market equilibria are computed, curves are shifted, and new equilibria are determined. This shows students how we can estimate the quantitative (not just the qualitative) effects of, say, a disruption in oil supply as in Example 2.9. Unfortunately, this section takes some time to cover, especially if your students' algebra is rusty. You'll have to decide whether the benefits outweigh the costs.

Price controls are introduced in Section 2.7. Students usually don't realize the full effects of price controls. They think only of the initial effect on prices without realizing that shortages or surpluses are created, so this is an important topic. However, the coverage here is quite brief. Chapter 9 examines the effects of price controls and other forms of government intervention in much greater detail, so you may want to defer this topic until then.

■ Questions for Review

1. **Suppose that unusually hot weather causes the demand curve for ice cream to shift to the right. Why will the price of ice cream rise to a new market-clearing level?**

   Suppose the supply of ice cream is completely inelastic in the short run, so the supply curve is vertical as shown below. The initial equilibrium is at price P_1. The unusually hot weather causes the demand curve for ice cream to shift from D_1 to D_2, creating short-run excess demand (i.e., a temporary shortage) at the current price. Consumers will bid against each other for the ice cream, putting upward pressure on the price, and ice cream sellers will react by raising price. The price of ice cream will rise until the quantity demanded and the quantity supplied are equal, which occurs at price P_2.



2. Use supply and demand curves to illustrate how each of the following events would affect the price of butter and the quantity of butter bought and sold:

a. An increase in the price of margarine.

Butter and margarine are substitute goods for most people. Therefore, an increase in the price of margarine will cause people to increase their consumption of butter, thereby shifting the demand curve for butter out from D_1 to D_2 in Figure 2.2.a. This shift in demand causes the equilibrium price of butter to rise from P_1 to P_2 and the equilibrium quantity to increase from Q_1 to Q_2.

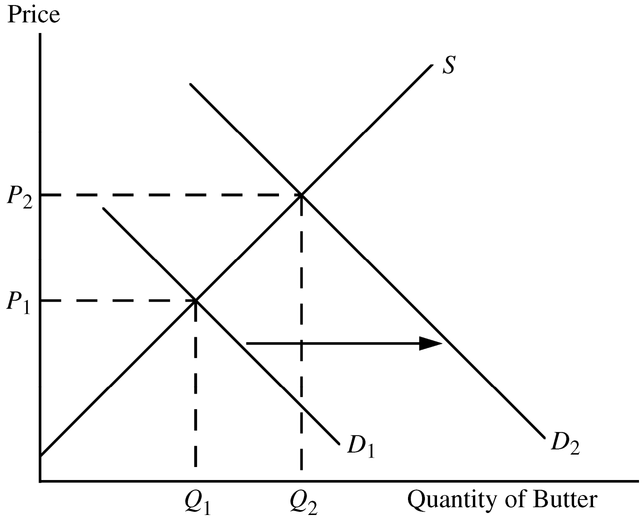


Figure 2.2.a

b. An increase in the price of milk.

Milk is the main ingredient in butter. An increase in the price of milk increases the cost of producing butter, which reduces the supply of butter. The supply curve for butter shifts from S_1 to S_2 in Figure 2.2.b, resulting in a higher equilibrium price, P_2 and a lower equilibrium quantity, Q_2, for butter.

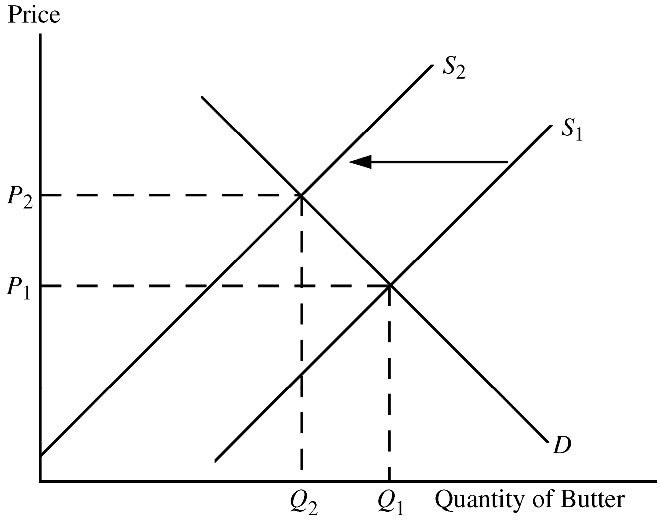


Figure 2.2.b

Note: Butter is in fact made from the fat that is skimmed from milk; thus butter and milk are joint products, and this complicates things. If you take account of this relationship, your answer might change, but it depends on why the price of milk increased. If the increase were caused by an increase in the demand for milk, the equilibrium quantity of milk supplied would increase. With more milk being produced, there would be more milk fat available to make butter, and the price of milk fat would fall. This would shift the supply curve for butter to the right, resulting in a drop in the price of butter and an increase in the quantity of butter supplied.

c. A decrease in average income levels.

Assuming that butter is a normal good, a decrease in average income will cause the demand curve for butter to decrease (i.e., shift from D_1 to D_2). This will result in a decline in the equilibrium price from P_1 to P_2, and a decline in the equilibrium quantity from Q_1 to Q_2. See Figure 2.2.c.

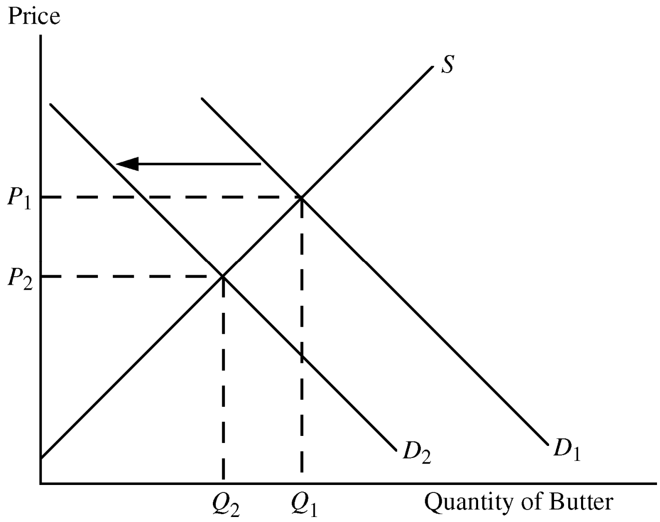


Figure 2.2.c

3. If a 3% increase in the price of corn flakes causes a 6% decline in the quantity demanded, what is the elasticity of demand?

The elasticity of demand is the percentage change in the quantity demanded divided by the percentage change in the price. The elasticity of demand for corn flakes is therefore

$$E_P^D = \frac{\%\Delta Q}{\%\Delta P} = \frac{-6}{+3} = -2.$$

4. Explain the difference between a shift in the supply curve and a movement along the supply curve.

A movement along the supply curve occurs when the price of the good changes. A shift of the supply curve is caused by a change in something other than the good's price that results in a change in the quantity supplied at the current price. Some examples are a change in the price of an input, a change in technology that reduces the cost of production, and an increase in the number of firms supplying the product.

5. Explain why for many goods, the long-run price elasticity of supply is larger than the short-run elasticity.

The price elasticity of supply is the percentage change in the quantity supplied divided by the percentage change in price. In the short run, an increase in price induces firms to produce more by using their facilities more hours per week, paying workers to work overtime and hiring new workers. Nevertheless, there is a limit to how much firms can produce because they face capacity constraints in the short run. In the long run, however, firms can expand capacity by building new plants and hiring new permanent workers. Also, new firms can enter the market and add their output to total supply. Hence a greater change in quantity supplied is possible in the long run, and thus the price elasticity of supply is larger in the long run than in the short run.

6. Why do long-run elasticities of demand differ from short-run elasticities? Consider two goods: paper towels and televisions. Which is a durable good? Would you expect the price elasticity of demand for paper towels to be larger in the short run or in the long run? Why? What about the price elasticity of demand for televisions?

Long-run and short-run elasticities differ based on how rapidly consumers respond to price changes and how many substitutes are available. If the price of paper towels, a non-durable good, were to increase, consumers might react only minimally in the short run because it takes time for people to change their consumption habits. In the long run, however, consumers might learn to use other products such as sponges or kitchen towels instead of paper towels. Thus, the price elasticity would be larger in the long run than in the short run. In contrast, the quantity demanded of durable goods, such as televisions, might change dramatically in the short run. For example, the initial result of a price increase for televisions would cause consumers to delay purchases because they could keep on using their current TVs longer. Eventually consumers would replace their televisions as they wore out or became obsolete. Therefore, we expect the demand for durables to be more elastic in the short run than in the long run.

7. Are the following statements true or false? Explain your answers.

a. The elasticity of demand is the same as the slope of the demand curve.

False. Elasticity of demand is the percentage change in quantity demanded divided by the percentage change in the price of the product. In contrast, the slope of the demand curve is the change in quantity demanded (in units) divided by the change in price (typically in dollars). The difference is that elasticity uses percentage changes while the slope is based on changes in the number of units and number of dollars.

b. The cross-price elasticity will always be positive.

False. The cross-price elasticity measures the percentage change in the quantity demanded of one good due to a 1% change in the price of another good. This elasticity will be positive for substitutes (an increase in the price of hot dogs is likely to cause an increase in the quantity demanded of hamburgers) and negative for complements (an increase in the price of hot dogs is likely to cause a decrease in the quantity demanded of hot dog buns).

c. The supply of apartments is more inelastic in the short run than the long run.

True. In the short run it is difficult to change the supply of apartments in response to a change in price. Increasing the supply requires constructing new apartment buildings, which can take a year or more. Therefore, the elasticity of supply is more inelastic in the short run than in the long run.

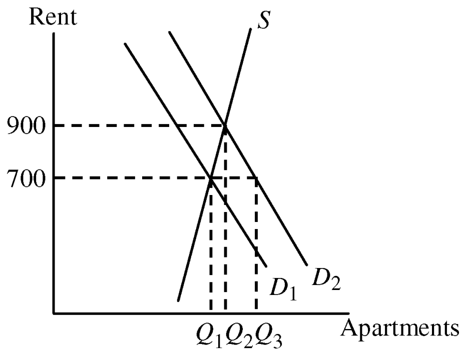
8. Suppose the government regulates the prices of beef and chicken and sets them below their market-clearing levels. Explain why shortages of these goods will develop and what factors will determine the sizes of the shortages. What will happen to the price of pork? Explain briefly.

If the price of a commodity is set below its market-clearing level, the quantity that firms are willing to supply is less than the quantity that consumers wish to purchase. The extent of the resulting shortage depends on the elasticities of demand and supply as well as the amount by which the regulated price is set below the market-clearing price. For instance, if both supply and demand are elastic, the shortage is larger than if both are inelastic, and if the regulated price is substantially below the market-clearing price, the shortage is larger than if the regulated price is only slightly below the market-clearing price. Factors such as the willingness of consumers to eat less meat and the ability of farmers to reduce the size of their herds/flocks will determine the relevant elasticities. Customers whose demands for beef and chicken are not met because of the shortages will want to purchase substitutes like pork. This increases the demand for pork (i.e., shifts demand to the right), which results in a higher price for pork.

9. The city council of a small college town decides to regulate rents in order to reduce student living expenses. Suppose the average annual market-clearing rent for a two-bedroom apartment had been \$700 per month and that rents were expected to increase to \$900 within a year. The city council limits rents to their current \$700-per-month level.

a. Draw a supply and demand graph to illustrate what will happen to the rental price of an apartment after the imposition of rent controls.

Initially demand is D_1 and supply is S, so the equilibrium rent is \$700 and Q_1 apartments are rented. Without regulation, demand was expected to increase to D_2, which would have raised rent to \$900 and resulted in Q_2 apartment rentals. Under the city council regulation, however, the rental price stays at the old equilibrium level of \$700 per month. After demand increases to D_2, only Q_1 apartments will be supplied while Q_3 will be demanded. There will be a shortage of $Q_3 - Q_1$ apartments.

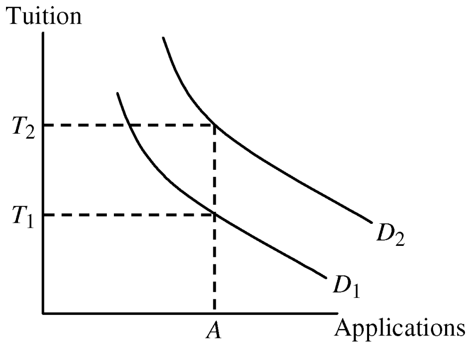


a. Do you think this policy will benefit all students? Why or why not?

No. It will benefit those students who get an apartment, although these students may find that the cost of searching for an apartment is higher given the shortage of apartments. Those students who do not get an apartment may face higher costs as a result of having to live outside the college town. Their rent may be higher and their transportation costs will be higher, so they will be worse off as a result of the policy.

10. In a discussion of tuition rates, a university official argues that the demand for admission is completely price inelastic. As evidence, she notes that while the university has doubled its tuition (in real terms) over the past 15 years, neither the number nor quality of students applying has decreased. Would you accept this argument? Explain briefly. (*Hint*: The official makes an assertion about the demand for admission, but does she actually observe a demand curve? What else could be going on?)

I would not accept this argument. The university official assumes that demand has remained stable (i.e., the demand curve has not shifted) over the 15-year period. This seems very unlikely. Demand for college educations has increased over the years for many reasons—real incomes have increased, population has increased, the perceived value of a college degree has increased, etc. What has probably happened is that tuition doubled from T_1 to T_2, but demand also increased from D_1 to D_2 over the 15 years, and the two effects have offset each other. The result is that the quantity (and quality) of applications has remained steady at A. The demand curve is not perfectly inelastic as the official asserts.



11. Suppose the demand curve for a product is given by $Q = 10 - 2P + P_s$, where P is the price of the product and P_s is the price of a substitute good. The price of the substitute good is \$2.00.

a. Suppose P = \$1.00. What is the price elasticity of demand? What is the cross-price elasticity of demand?

Find quantity demanded when P = \$1.00 and P_s = \$2.00. $Q = 10 - 2(1) + 2 = 10$.

$$\text{Price elasticity of demand} = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{1}{10}(-2) = -\frac{2}{10} = -0.2.$$

$$\text{Cross-price elasticity of demand} = \frac{P_s}{Q}\frac{\Delta Q}{\Delta P_s} = \frac{2}{10}(1) = 0.2.$$

b. Suppose the price of the good, P, goes to \$2.00. Now what is the price elasticity of demand? What is the cross-price elasticity of demand?

When P = \$2.00, $Q = 10 - 2(2) + 2 = 8$.

$$\text{Price elasticity of demand} = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{2}{8}(-2) = -\frac{4}{8} = -0.5.$$

$$\text{Cross-price elasticity of demand} = \frac{P_s}{Q}\frac{\Delta Q}{\Delta P_s} = \frac{2}{8}(1) = 0.25.$$

12. Suppose that rather than the declining demand assumed in Example 2.8, a decrease in the cost of copper production causes the supply curve to shift to the right by 40%. How will the price of copper change?

If the supply curve shifts to the right by 40% then the new quantity supplied will be 140% of the old quantity supplied at every price. The new supply curve is therefore the old supply curve multiplied by 1.4.

$Q_S' = 1.4(-9 + 9P) = -12.6 + 12.6P$. To find the new equilibrium price of copper, set the new supply equal to demand. Thus, $-12.6 + 12.6P = 27 - 3P$. Solving for price results in $P = \$2.54$ per pound for the new equilibrium price. The price decreased by 46 cents per pound, from \$3.00 to \$2.54, a drop of about 15.3%.

13. Suppose the demand for natural gas is perfectly inelastic. What would be the effect, if any, of natural gas price controls?

If the demand for natural gas is perfectly inelastic, the demand curve is vertical. Consumers will demand the same quantity regardless of price. In this case, price controls will have no effect on the quantity demanded, but they will still cause a shortage if the supply curve is upward sloping and the regulated price is set below the market-clearing price, because suppliers will produce less natural gas than consumers wish to purchase.

■ Exercises

1. Suppose the demand curve for a product is given by $Q = 300 - 2P + 4I$, where I is average income measured in thousands of dollars. The supply curve is $Q = 3P - 50$.

a. If $I = 25$, find the market-clearing price and quantity for the product.

Given $I = 25$, the demand curve becomes $Q = 300 - 2P + 4(25)$, or $Q = 400 - 2P$. Set demand equal to supply and solve for P and then Q:

$$400 - 2P = 3P - 50$$
$$P = 90$$
$$Q = 400 - 2(90) = 220.$$

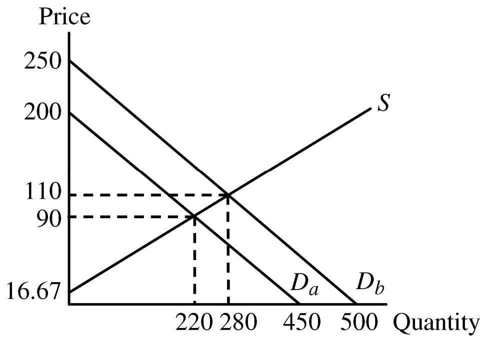
b. If $I = 50$, find the market-clearing price and quantity for the product.

Given $I = 50$, the demand curve becomes $Q = 300 - 2P + 4(50)$, or $Q = 500 - 2P$. Setting demand equal to supply, solve for P and then Q:

$$500 - 2P = 3P - 50$$
$$P = 110$$
$$Q = 500 - 2(110) = 280.$$

c. Draw a graph to illustrate your answers.

It is easier to draw the demand and supply curves if you first solve for the inverse demand and supply functions, i.e., solve the functions for P. Demand in part a is $P = 200 - 0.5Q$ and supply is $P = 16.67 + 0.333Q$. These are shown on the graph as D_a and S. Equilibrium price and quantity are found at the intersection of these demand and supply curves. When the income level increases in part b, the demand curve shifts up and to the right. Inverse demand is $P = 250 - 0.5Q$ and is labeled D_b. The intersection of the new demand curve and original supply curve is the new equilibrium point.



2. Consider a competitive market for which the quantities demanded and supplied (per year) at various prices are given as follows:

Price (Dollars)	Demand (Millions)	Supply (Millions)
60	22	14
80	20	16
100	18	18
120	16	20

a. Calculate the price elasticity of demand when the price is \$80 and when the price is \$100.

$$E_D = \frac{\frac{\Delta Q_D}{Q_D}}{\frac{\Delta P}{P}} = \frac{P}{Q_D}\frac{\Delta Q_D}{\Delta P}.$$

With each price increase of \$20, the quantity demanded decreases by 2 million. Therefore,

$$\left(\frac{\Delta Q_D}{\Delta P}\right) = \frac{-2}{20} = -0.1.$$

At $P = 80$, quantity demanded is 20 million and thus

$$E_D = \left(\frac{80}{20}\right)(-0.1) = -0.40.$$

Similarly, at $P = 100$, quantity demanded equals 18 million and

$$E_D = \left(\frac{100}{18}\right)(-0.1) = -0.56.$$

b. Calculate the price elasticity of supply when the price is $80 and when the price is $100.

$$E_S = \frac{\frac{\Delta Q_S}{Q_S}}{\frac{\Delta P}{P}} = \frac{P}{Q_S}\frac{\Delta Q_S}{\Delta P}.$$

With each price increase of $20, quantity supplied increases by 2 million. Therefore,

$$\left(\frac{\Delta Q_S}{\Delta P}\right) = \frac{2}{20} = 0.1.$$

At $P = 80$, quantity supplied is 16 million and

$$E_S = \left(\frac{80}{16}\right)(0.1) = 0.5.$$

Similarly, at $P = 100$, quantity supplied equals 18 million and

$$E_S\left(\frac{100}{18}\right)(0.1) = 0.56.$$

c. What are the equilibrium price and quantity?

The equilibrium price is the price at which the quantity supplied equals the quantity demanded. Using the table, the equilibrium price is $P^* = \$100$ and the equilibrium quantity is $Q^* = 18$ million.

d. Suppose the government sets a price ceiling of $80. Will there be a shortage, and if so, how large will it be?

With a price ceiling of $80, price cannot be above $80, so the market cannot reach its equilibrium price of $100. At $80, consumers would like to buy 20 million, but producers will supply only 16 million. This will result in a shortage of 4 million units.

3. Refer to Example 2.5 (page 37) on the market for wheat. In 1998, the total demand for U.S. wheat was $Q = 3244 - 283P$ and the domestic supply was $Q_S = 1944 + 207P$. At the end of 1998, both Brazil and Indonesia opened their wheat markets to U.S. farmers. Suppose that these new markets add 200 million bushels to U.S. wheat demand. What will be the free-market price of wheat and what quantity will be produced and sold by U.S. farmers?

If Brazil and Indonesia add 200 million bushels of wheat to U.S. wheat demand, the new demand curve will be $Q + 200$, or

$$Q_D = (3244 - 283P) + 200 = 3444 - 283P.$$

Equate supply and the new demand to find the new equilibrium price.

$$1944 + 207P = 3444 - 283P, \text{ or}$$

$$490P = 1500, \text{ and thus } P = \$3.06 \text{ per bushel.}$$

To find the equilibrium quantity, substitute the price into either the supply or demand equation. Using demand,

$$Q_D = 3444 - 283(3.06) = 2578 \text{ million bushels.}$$

4. A vegetable fiber is traded in a competitive world market, and the world price is $9 per pound. Unlimited quantities are available for import into the United States at this price. The U.S. domestic supply and demand for various price levels are shown as follows:

Price	U.S. Supply (Million Lbs.)	U.S. Demand (Million Lbs.)
3	2	34
6	4	28
9	6	22
12	8	16
15	10	10
18	12	4

a. What is the equation for demand? What is the equation for supply?

The equation for demand is of the form $Q = a - bP$. First find the slope, which is $\frac{\Delta Q}{\Delta P} = \frac{-6}{3} = -2 = -b$. You can figure this out by noticing that every time price increases by 3, quantity demanded falls by 6 million pounds. Demand is now $Q = a - 2P$. To find a, plug in any of the price and quantity demanded points from the table. For example: $Q = 34 = a - 2(3)$ so that $a = 40$ and demand is therefore $Q = 40 - 2P$.

The equation for supply is of the form $Q = c + dP$. First find the slope, which is $\frac{\Delta Q}{\Delta P} = \frac{2}{3} = d$. You can figure this out by noticing that every time price increases by 3, quantity supplied increases by 2 million pounds. Supply is now $Q = c + \frac{2}{3}P$. To find c, plug in any of the price and quantity supplied points from the table. For example: $Q = 2 = c + \frac{2}{3}(3)$ so that $c = 0$ and supply is $Q = \frac{2}{3}P$.

b. At a price of $9, what is the price elasticity of demand? What is it at a price of $12?

Elasticity of demand at $P = 9$ is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{9}{22}(-2) = \frac{-18}{22} = -0.82$.

Elasticity of demand at $P = 12$ is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{12}{16}(-2) = \frac{-24}{16} = -1.5$.

c. What is the price elasticity of supply at $9? At $12?

Elasticity of supply at $P = 9$ is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{9}{6}\left(\frac{2}{3}\right) = \frac{18}{18} = 1.0$.

Elasticity of supply at $P = 12$ is $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{12}{8}\left(\frac{2}{3}\right) = \frac{24}{24} = 1.0$.

d. In a free market, what will be the U.S. price and level of fiber imports?

With no restrictions on trade, the price in the United States will be the same as the world price, so $P = \$9$. At this price, the domestic supply is 6 million lbs., while the domestic demand is 22 million lbs. Imports make up the difference and are 16 million lbs.

5. Much of the demand for U.S. agricultural output has come from other countries. In 1998, the total demand for wheat was $Q = 3244 - 283P$. Of this, total domestic demand was $Q_D = 1700 - 107P$, and domestic supply was $Q_S = 1944 + 207P$. Suppose the export demand for wheat falls by 40%.

a. U.S. farmers are concerned about this drop in export demand. What happens to the free-market price of wheat in the United States? Do farmers have much reason to worry?

Before the drop in export demand, the market equilibrium price is found by setting total demand equal to domestic supply:

$$3244 - 283P = 1944 + 207P, \text{ or}$$

$$P = \$2.65.$$

Export demand is the difference between total demand and domestic demand: $Q = 3244 - 283P$ minus $Q_D = 1700 - 107P$. So export demand is originally $Q_e = 1544 - 176P$. After the 40% drop, export demand is only 60% of the original export demand. The new export demand is therefore, $Q'_e = 0.6Q_e = 0.6(1544 - 176P) = 926.4 - 105.6P$. Graphically, export demand has pivoted inward as illustrated in the figure below.

The new total demand becomes

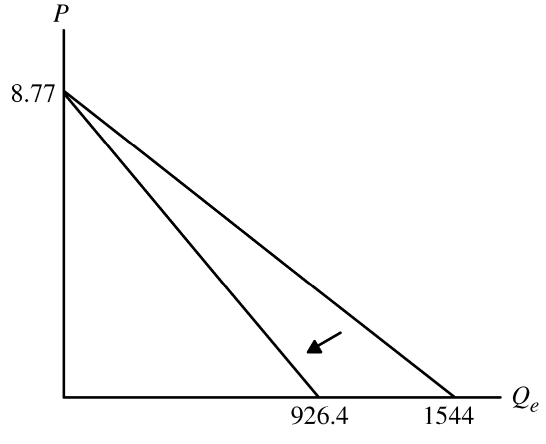
$$Q' = Q_D + Q'_e = (1700 - 107P) + (926.4 - 105.6P) = 2626.4 - 212.6P.$$

Equating total supply and the new total demand,

$$1944 + 207P = 2626.4 - 212.6P, \text{ or}$$

$$P = \$1.63,$$

which is a significant drop from the original market-clearing price of \$2.65 per bushel. At this price, the market-clearing quantity is about $Q = 2281$ million bushels. Total revenue has decreased from about \$6609 million to \$3718 million, so farmers have a lot to worry about.



b. Now suppose the U.S. government wants to buy enough wheat to raise the price to $3.50 per bushel. With the drop in export demand, how much wheat would the government have to buy? How much would this cost the government?

With a price of $3.50, the market is not in equilibrium. Quantity demanded and supplied are

$$Q' = 2626.4 - 212.6(3.50) = 1882.3, \text{ and}$$

$$Q_S = 1944 + 207(3.50) = 2668.5.$$

Excess supply is therefore 2668.5 − 1882.3 = 786.2 million bushels. The government must purchase this amount to support a price of $3.50, and will have to spend $3.50(786.2 million) = $2751.7 million.

6. The rent control agency of New York City has found that aggregate demand is $Q_D = 160 - 8P$. Quantity is measured in tens of thousands of apartments. Price, the average monthly rental rate, is measured in hundreds of dollars. The agency also noted that the increase in *Q* at lower *P* results from more three-person families coming into the city from Long Island and demanding apartments. The city's board of realtors acknowledges that this is a good demand estimate and has shown that supply is $Q_S = 70 + 7P$.

a. If both the agency and the board are right about demand and supply, what is the free-market price? What is the change in city population if the agency sets a maximum average monthly rent of $300 and all those who cannot find an apartment leave the city?

Set supply equal to demand to find the free-market price for apartments:

$$160 - 8P = 70 + 7P, \text{ or } P = 6,$$

which means the rental price is $600 since price is measured in hundreds of dollars. Substituting the equilibrium price into either the demand or supply equation to determine the equilibrium quantity:

$$Q_D = 160 - 8(6) = 112$$

and

$$Q_S = 70 + 7(6) = 112.$$

The quantity of apartments rented is 1,120,000 since *Q* is measured in tens of thousands of apartments. If the rent control agency sets the rental rate at $300, the quantity supplied would be 910,000 ($Q_S = 70 + 7(3) = 91$), a decrease of 210,000 apartments from the free-market equilibrium. Assuming three people per family per apartment, this would imply a loss in city population of 630,000 people. ***Note***: At the $300 rental rate, the demand for apartments is 1,360,000 units, and the resulting shortage is 450,000 units (1,360,000 − 910,000). However, excess demand (the shortage) and lower quantity demanded are not the same concept. The shortage of 450,000 units is the difference between the number of apartments demanded at the new lower price (including the number demanded by new people who would have moved into the city), and the number supplied at the lower price. But these new people will not actually move into the city because the apartments are not available. Therefore, the city population will fall by 630,000, which is due to the drop in the number of apartments available from 1,120,000 (the old equilibrium value) to 910,000.

b. Suppose the agency bows to the wishes of the board and sets a rental of $900 per month on all apartments to allow landlords a "fair" rate of return. If 50% of any long-run increases in apartment offerings come from new construction, how many apartments are constructed?

At a rental rate of $900, the demand for apartments would be 160 − 8(9) = 88, or 880,000 units, which is 240,000 fewer apartments than the original free-market equilibrium number of 1,120,000. Therefore, no new apartments would be constructed.

7. In 2010, Americans smoked 315 billion cigarettes, or 15.75 billion packs of cigarettes. The average retail price (including taxes) was about $5.00 per pack. Statistical studies have shown that the price elasticity of demand is –0.4, and the price elasticity of supply is 0.5.

a. Using this information, derive linear demand and supply curves for the cigarette market.

Let the demand curve be of the form $Q = a - bP$ and the supply curve be of the form $Q = c + dP$, where a, b, c, and d are positive constants. To begin, recall the formula for the price elasticity of demand

$$E_P^D = \frac{P}{Q}\frac{\Delta Q}{\Delta P}.$$

We know the demand elasticity is –0.4, $P = 5$, and $Q = 15.75$, which means we can solve for the slope, $-b$, which is $\Delta Q/\Delta P$ in the above formula.

$$-0.4 = \frac{5}{15.75}\frac{\Delta Q}{\Delta P}$$

$$\frac{\Delta Q}{\Delta P} = -0.4\left(\frac{15.75}{5}\right) = -1.26 = -b.$$

To find the constant a, substitute for Q, P, and b in the demand function to get $15.75 = a - 1.26(5)$, so $a = 22.05$. The equation for demand is therefore $Q = 22.05 - 1.26P$. To find the supply curve, recall the formula for the elasticity of supply and follow the same method as above:

$$E_P^S = \frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

$$0.5 = \frac{5}{15.75}\frac{\Delta Q}{\Delta P}$$

$$\frac{\Delta Q}{\Delta P} = 0.5\left(\frac{15.75}{5}\right) = 1.575 = d.$$

To find the constant c, substitute for Q, P, and d in the supply function to get $15.75 = c + 1.575(5)$ and $c = 7.875$. The equation for supply is therefore $Q = 7.875 + 1.575P$.

b. In 1998, Americans smoked 23.5 billion packs cigarettes, and the retail price was about $2.00 per pack. The decline in cigarette consumption from 1998 to 2010 was due in part to greater public awareness of the health hazards from smoking, but was also due in part to the increase in price. Suppose that the *entire decline* was due to the increase in price. What could you deduce from that about the price elasticity of demand?

Calculate the arc elasticity of demand since we have a range of prices rather than a single price. The arc elasticity formula is

$$E_P = \frac{\Delta Q}{\Delta P}\frac{\overline{P}}{\overline{Q}}$$

where $\bar{P}$ and $\bar{Q}$ are average price and quantity, respectively. The change in quantity was 15.75 −23.5 = −7.75, and the change in price was 5 − 2 = 3. The average price was (2 + 5)/2 = 3.50, and the average quantity was (23.5 + 15.75)/2 = 19.625. Therefore, the price elasticity of demand, assuming that the *entire decline* in quantity was due solely to the price increase, was

$$E_P = \frac{\Delta Q}{\Delta P}\frac{\bar{P}}{\bar{Q}} = \frac{-7.75}{3}\frac{3.50}{19.625} = -0.46.$$

8. In Example 2.8 we examined the effect of a 20% decline in copper demand on the price of copper, using the linear supply and demand curves developed in Section 2.6. Suppose the long-run price elasticity of copper demand were –0.75 instead of –0.5.

a. Assuming, as before, that the equilibrium price and quantity are P^* = $3 per pound and Q^* = 18 million metric tons per year, derive the linear demand curve consistent with the smaller elasticity.

Following the method outlined in Section 2.6, solve for a and b in the demand equation $Q_D = a - bP$. Because $-b$ is the slope, we can use $-b$ rather than $\Delta Q/\Delta P$ in the elasticity formula. Therefore, $E_D = -b\left(\frac{P^*}{Q^*}\right)$. Here $E_D = -0.75$ (the long-run price elasticity), $P^* = 3$ and $Q^* = 18$. Solving for b,

$$-0.75 = -b\left(\frac{3}{18}\right), \text{ or } b = 0.75(6) = 4.5.$$

To find the intercept, we substitute for b, Q_D (= Q^*), and P (= P^*) in the demand equation:

$$18 = a - 4.5(3), \text{ or } a = 31.5.$$

The linear demand equation is therefore

$$Q_D = 31.5 - 4.5P.$$

b. Using this demand curve, recalculate the effect of a 20% decline in copper demand on the price of copper.

The new demand is 20% below the original (using our convention that quantity demanded is reduced by 20% at every price); therefore, multiply demand by 0.8 because the new demand is 80% of the original demand:

$$Q'_D = (0.8)(31.5 - 4.5P) = 25.2 - 3.6P.$$

Equating this to supply,

$$25.2 - 3.6P = -9 + 9P, \text{ so}$$
$$P = \$2.71.$$

With the 20% decline in demand, the price of copper falls from $3.00 to $2.71 per pound. The decrease in demand therefore leads to a drop in price of 29 cents per pound, a 9.7% decline.

9. In Example 2.8 (page 52), we discussed the recent increase in world demand for copper, due in part to China's rising consumption.

a. Using the original elasticities of demand and supply (i.e., $E_S = 1.5$ and $E_D = -0.5$), calculate the effect of a 20% *increase* in copper demand on the price of copper.

The original demand is $Q = 27 - 3P$ and supply is $Q = -9 + 9P$ as shown on page 51. The 20% increase in demand means that the new demand is 120% of the original demand, so the new demand is $Q'_D = 1.2Q$. $Q'_D = (1.2)(27 - 3P) = 32.4 - 3.6P$. The new equilibrium is where Q'_D equals the original supply:

$$32.4 - 3.6P = -9 + 9P.$$

The new equilibrium price is $P^* = \$3.29$ per pound. An increase in demand of 20%, therefore, entails an increase in price of 29 cents per pound, or 9.7%.

b. Now calculate the effect of this increase in demand on the equilibrium quantity, Q^*.

Using the new price of \$3.29 in the supply curve, the new equilibrium quantity is $Q^* = -9 + 9(3.29) = 20.61$ million metric tons per year, an increase of 2.61 million metric tons (mmt) per year. Except for rounding, you get the same result by plugging the new price of \$3.29 into the new demand curve. So an increase in demand of 20% entails an increase in quantity of 2.61 mmt per year, or 14.5%.

c. As we discussed in Example 2.8, the U.S. production of copper declined between 2000 and 2003. Calculate the effect on the equilibrium price and quantity of *both* a 20% increase in copper demand (as you just did in part a) *and* of a 20% decline in copper supply.

The new supply of copper falls (shifts to the left) to 80% of the original, so $Q'_S = 0.8Q = (0.8)(-9 + 9P) = -7.2 + 7.2P$. The new equilibrium is where $Q'_D = Q'_S$.

$$32.4 - 3.6P = -7.2 + 7.2P$$

The new equilibrium price is $P^* = \$3.67$ per pound. Plugging this price into the new supply equation, the new equilibrium quantity is $Q^* = -7.2 + 7.2(3.67) = 19.22$ million metric tons per year. Except for rounding, you get the same result if you substitute the new price into the new demand equation. The combined effect of a 20% increase in demand and a 20% decrease in supply is that price increases by 67 cents per pound, or 22%, and quantity increases by 1.22 mmt per year, or 6.8%, compared to the original equilibrium.

10. Example 2.9 (page 54) analyzes the world oil market. Using the data given in that example:

a. Show that the short-run demand and competitive supply curves are indeed given by

$$\mathbf{D = 33.6 - 0.020P}$$

$$\mathbf{S_c = 18.05 + 0.012P.}$$

The competitive (non-OPEC) quantity supplied is $S_c = Q^* = 19$. The general form for the linear competitive supply equation is $S_C = c + dP$. We can write the short-run supply elasticity as $E_S = d(P^*/Q^*)$. Since $E_S = 0.05$, $P^* = \$80$, and $Q^* = 19$, $0.05 = d(80/19)$. Hence $d = 0.011875$. Substituting for d, S_c, and P in the supply equation, $c = 18.05$, and the short-run competitive supply equation is $S_c = 18.05 + 0.012P$.

Similarly, world demand is $D = a - bP$, and the short-run demand elasticity is $E_D = -b(P^*/Q^*)$, where Q^* is total world demand of 32. Therefore, $-0.05 = -b(80/32)$, and $b = 0.020$. Substituting $b = 0.02$, $D = 32$, and $P = 80$ in the demand equation gives $32 = a - 0.02(80)$, so that $a = 33.6$. Hence the short-run world demand equation is $D = 33.6 - 0.020P$.

b. Show that the long-run demand and competitive supply curves are indeed given by

$$D = 41.6 - 0.120P$$

$$S_C = 13.3 + 0.071P.$$

Do the same calculations as above but now using the long-run elasticities, $E_S = 0.30$ and $E_D = -0.30$: $E_S = d(P^*/Q^*)$ and $E_D = -b(P^*/Q^*)$, implying $0.30 = d(80/19)$ and $-0.30 = -b(80/32)$. So $d = 0.07125$ and $b = 0.12$.

Next solve for c and a: $S_c = c + dP$ and $D = a - bP$, implying $19 = c + 0.07125(80)$ and $32 = a - 0.12(80)$. So $c = 13.3$ and $a = 41.6$.

c. In Example 2.9 we examined the impact on price of a disruption of oil from Saudi Arabia. Suppose that instead of a decline in supply, OPEC production *increases* by 2 billion barrels per year (bb/yr) because the Saudis open large new oil fields. Calculate the effect of this increase in production on the supply of oil in both the short run and the long run.

OPEC's supply increases from 13 bb/yr to 15 bb/yr as a result. Add 15 bb/yr to the short-run and long-run competitive supply equations. The new total supply equations are:

Short-run: $S_T' = 15 + S_c = 15 + 18.05 + 0.012P = 33.05 + 0.012P$, and

Long-run: $S_T'' = 15 + S_c = 15 + 13.3 + 0.071P = 28.3 + 0.071P$.

These are equated with short-run and long-run demand, so that:

$33.05 + 0.012P = 33.6 - 0.020P$, implying that $P = \$17.19$ in the short run, and

$28.3 + 0.071P = 41.6 - 0.120P$, implying that $P = \$69.63$ in the long run.

In the short run, total supply is $33.05 + 0.012(17.19) = 33.26$ bb/yr. In the long run, total supply remains virtually the same at $28.3 + 0.071(69.63) = 33.24$ bb/yr. Compared to current total supply of 32 bb/yr, supply increases by about 1.25 bb/yr.

11. Refer to Example 2.10 (page 59), which analyzes the effects of price controls on natural gas.

a. Using the data in the example, show that the following supply and demand curves describe the market for natural gas in 2005–2007:

$$\textbf{Supply: } Q = 15.90 + 0.72P_G + 0.05P_O$$

$$\textbf{Demand: } Q = 0.02 - 1.8P_G + 0.69P_O$$

Also, verify that if the price of oil is \$50, these curves imply a free-market price of \$6.40 for natural gas.

To solve this problem, apply the analysis of Section 2.6 using the definition of cross-price elasticity of demand given in Section 2.4. For example, the cross-price elasticity of demand for natural gas with respect to the price of oil is:

$$E_{GO} = \left(\frac{\Delta Q_G}{\Delta P_O}\right)\left(\frac{P_O}{Q_G}\right).$$

$\left(\frac{\Delta Q_G}{\Delta P_O}\right)$ is the change in the quantity of natural gas demanded because of a small change in the price of oil, and for linear demand equations, it is constant. If we represent demand as $Q_G = a - bP_G + eP_O$ (notice that income is held constant), then $\left(\frac{\Delta Q_G}{\Delta P_O}\right) = e$. Substituting this into the cross-price elasticity, $E_{GO} = e\left(\frac{P_O^*}{Q_G^*}\right)$, where P_O^* and Q_G^* are the equilibrium price and quantity. We know that $P_o^* = \$50$ and $Q_G^* = 23$ trillion cubic feet (Tcf). Solving for e,

$$1.5 = e\left(\frac{50}{23}\right), \text{ or } e = 0.69.$$

Similarly, representing the supply equation as $Q_G = c + dP_G + gP_O$, the cross-price elasticity of supply is $g\left(\frac{P_O^*}{Q_G^*}\right)$, which we know to be 0.1. Solving for g, $0.1 = g\left(\frac{50}{23}\right)$, or $g = 0.5$ rounded to one decimal place.

We know that $E_S = 0.2$, $P_G* = 6.40$, and $Q* = 23$. Therefore, $0.2 = d\left(\frac{6.40}{23}\right)$, or $d = 0.72$. Also, $E_D = -0.5$, so $-0.5 = -b\left(\frac{6.40}{23}\right)$, and thus $b = 1.8$.

By substituting these values for d, g, b, and e into our linear supply and demand equations, we may solve for c and a:

$23 = c + 0.72(6.40) + 0.05(50)$, so $c = 15.9$, and

$23 = a - 1.8(6.40) + 0.69(50)$, so that $a = 0.02$.

Therefore, the supply and demand curves for natural gas are as given. If the price of oil is \$50, these curves imply a free-market price of \$6.40 for natural gas as shown below. Substitute the price of oil in the supply and demand equations. Then set supply equal to demand and solve for the price of gas.

$$15.9 + 0.72P_G + 0.05(50) = 0.02 - 1.8P_G + 0.69(50)$$

$$18.4 + 0.72P_G = 34.52 - 1.8P_G$$

$$P_G = \$6.40.$$

b. Suppose the regulated price of gas were \$4.50 per thousand cubic feet instead of \$3.00. How much excess demand would there have been?

With a regulated price of \$4.50 for natural gas and the price of oil equal to \$50 per barrel,

$$\text{Demand: } Q_D = 0.02 - 1.8(4.50) + 0.69(50) = 26.4, \text{ and}$$

$$\text{Supply: } Q_S = 15.9 + 0.72(4.50) + 0.05(50) = 21.6$$

With a demand of 26.4 Tcf and a supply of 21.6 Tcf, there would be an excess demand (i.e., a shortage) of 4.8 Tcf.

c. Suppose that the market for natural gas remained unregulated. If the price of oil had increased from $50 to $100, what would have happened to the free-market price of natural gas?

In this case

$$\text{Demand: } Q_D = 0.02 - 1.8P_G + 0.69(100) = 69.02 - 1.8P_G, \text{ and}$$

$$\text{Supply: } Q_S = 15.9 + 0.72P_G + 0.05(100) = 20.9 + 0.72P_G.$$

Equating supply and demand and solving for the equilibrium price,

$$20.9 + 0.72P_G = 69.02 - 1.8P_G, \text{ or } P_G = \$19.10.$$

The free-market price of natural gas would have almost tripled from $6.40 to $19.10.

12. The table below shows the retail price and sales for instant coffee and roasted coffee for two years.

Year	Retail Price of Instant Coffee ($/Lb)	Sales of Instant Coffee (Million Lbs)	Retail Price of Roasted Coffee ($/Lb)	Sales of Roasted Coffee (Million Lbs)
Year 1	10.35	75	4.11	820
Year 2	10.48	70	3.76	850

a. Using these data alone, estimate the short-run price elasticity of demand for roasted coffee. Derive a linear demand curve for roasted coffee.

To find elasticity, first estimate the slope of the demand curve:

$$\frac{\Delta Q}{\Delta P} = \frac{820 - 850}{4.11 - 3.76} = \frac{-30}{0.35} = -85.7$$

Given the slope, we can now estimate elasticity using the price and quantity data from the above table. Assuming the demand curve is linear, the elasticity will differ the two years because price and quantity are different. We can calculate the elasticities at both points and also find the arc elasticity at the average point between the two years:

$$E_P^1 = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{4.11}{820}(-85.7) = -0.043$$

$$E_P^2 = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{3.76}{850}(-85.7) = -0.038$$

$$E_P^{ARC} = \frac{\overline{P}}{\overline{Q}}\frac{\Delta Q}{\Delta P} = \frac{3.935}{835}(-85.7) = -0.040.$$

To derive the demand curve for roasted coffee, $Q = a - bP$, note that the slope of the demand curve is $-85.7 = -b$. To find the coefficient a, use either of the data points from the table above so that $820 = a - 85.7(4.11)$ or $850 = a - 85.7(3.76)$. In either case, $a = 1172.2$. The equation for the demand curve is therefore

$$Q = 1172.2 - 85.7P.$$

b. Now estimate the short-run price elasticity of demand for instant coffee. Derive a linear demand curve for instant coffee.

To find elasticity, first estimate the slope of the demand curve:

$$\frac{\Delta Q}{\Delta P} = \frac{75-70}{10.35-10.48} = \frac{5}{-0.13} = -38.5$$

Given the slope, we can now estimate elasticity using the price and quantity data from the above table. Assuming demand is of the form $Q = a - bP$, the elasticity will differ in the two years because price and quantity are different. The elasticities at both points and at the average point between the two years are:

$$E_P^1 = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{10.35}{75}(-38.5) = -5.31$$

$$E_P^2 = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{10.48}{70}(-38.5) = -5.76$$

$$E_P^{ARC} = \frac{\overline{P}}{\overline{Q}}\frac{\Delta Q}{\Delta P} = \frac{10.415}{72.5}(-38.5) = -5.53.$$

To derive the demand curve for instant coffee, note that the slope of the demand curve is $-38.5 = -b$. To find the coefficient a, use either of the data points from the table above so that $a = 75 + 38.5(10.35) = 473.5$ or $a = 70 + 38.5(10.48) = 473.5$. The equation for the demand curve is therefore

$$Q = 473.5 - 38.5P.$$

c. Which coffee has the higher short-run price elasticity of demand? Why do you think this is the case?

Instant coffee is significantly more elastic than roasted coffee. In fact, the demand for roasted coffee is inelastic and the demand for instant coffee is highly elastic. Roasted coffee may have an inelastic demand in the short run because many people think of coffee as a necessary good. Changes in the price of roasted coffee will not drastically affect the quantity demanded because people want their roasted coffee. Many people, on the other hand, may view instant coffee as a convenient, though imperfect and somewhat inferior, substitute for roasted coffee. So if the price of instant coffee rises, the quantity demanded will fall by a large percentage because many people will decide to switch to roasted coffee instead of paying more for a lower quality substitute.

PART TWO

Producers, Consumers, and Competitive Markets

Chapter 3
Consumer Behavior

■ Teaching Notes

Now we step back from supply and demand analysis to gain a deeper understanding of what lies behind the supply and demand curves. It will help students understand where the course is heading if you explain that this chapter builds the foundation for deriving demand curves in Chapter 4, and that you will do the same for supply curves later in the course (beginning in Chapter 6).

It is important to explain that economists approach behavior somewhat differently than, say, psychologists. We take preferences to be a given and don't question how they came to be. Psychologists, on the other hand, are interested in how preferences are formed and how and why they change, among other things. Economists usually assume consumers are rational, while psychologists explore alternative explanations for behavior. It is very useful to describe what we mean by rational, because that term is often misunderstood. We mean that people have goals, and they make decisions that will enable them to achieve those goals. Rational does not mean that the goals are somehow rational or appropriate, nor does it mean that people do what others might think is right or best for them. Economists generally assume that consumers want to maximize their happiness or satisfaction (i.e., utility), and as long as consumers are making decisions that achieve that goal, they are being rational. If someone who absolutely loves fast cars buys an expensive Porsche and consequently lives in a dump, wears worn-out clothes and eats poorly, he is being completely rational if that is what makes him most happy.

Students sometimes think that economists view people as being self-centered and concerned only with themselves. This is not necessarily the case. A consumer's utility can depend on other consumers' purchases or well-being in either a positive or negative way. I had a colleague once who taught Sunday school and said, somewhat jokingly, that it was all just a matter of interdependent utility functions. You can come back to this issue when covering the material on network externalities in Section 4.5 of Chapter 4 if you wish.

Many students find the consumer behavior material to be highly theoretical and not very "realistic." I like to use Milton Friedman's billiards player example to illustrate that theories do not have to be realistic to be useful.[1] In his famous essay, "The Methodology of Positive Economics," Friedman argues that economic theories should be judged by how well they predict and not by the descriptive realism of their assumptions. He suggests that if we wanted to predict how a skilled billiards player would play a particular shot, we could assume that the player knows the laws of physics and can do the calculations in his head to determine how to strike the cue ball. This theory would probably give very good predictions even though the billiards player knows nothing about physics, because he has learned how to make the shot through long practice. Likewise, consumers have learned what makes them happy through experience, so even though the assumption that consumers maximize utility subject to a budget constraint is pretty unrealistic, the theory predicts behavior well and is quite useful.

It is possible to discuss consumer choice without going into extensive detail on utility theory, relying instead on preference relationships and indifference curves. However, if you plan to discuss uncertainty in Chapter 5, you should cover marginal utility (Section 3.5). Even if you cover utility theory only briefly, make sure students are comfortable with the term utility because it appears frequently in Chapter 4. Also, emphasize that a consumer's utility for a product does not depend on the price of the product. Students sometimes

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[1] Milton Friedman, "The Methodology of Positive Economics," in *Essays in Positive Economics*, University of Chicago Press, 1953.

say, for example, that they would prefer a huge pickup truck with dual rear wheels to a small convertible, even though they would be much happier driving around in the convertible. The reason they give is that the pickup costs a lot more, so they could sell the pickup, buy the convertible and have money left over to purchase other things. This confuses preference with choice.

When introducing indifference curves, stress that physical quantities are represented on the two axes. After discussing supply and demand, students may think that price should be on the vertical axis. To illustrate indifference curves, pick an initial bundle on the graph and ask which other bundles are likely to be more preferred and less preferred to the initial bundle. This will divide the commodity space into four quadrants as in Figure 3.1, and it is then easier for students to figure out the set of bundles between which the consumer is indifferent. It is helpful to present examples with different types of goods (and bads) and see if the class can figure out how to draw the indifference curves.

The concept of utility follows naturally from the discussion of indifference curves. Emphasize that it is the ranking that is important and not the utility number, and point out that if we can graph an indifference curve we can certainly find an equation to represent it, so utility functions aren't so far-fetched. Finally, what is most important is the rate at which consumers are willing to exchange goods (the marginal rate of substitution), and this is based on the relative satisfaction that they derive from each good at any particular time.

The marginal rate of substitution, *MRS*, can be confusing. Some students confuse the *MRS* with the ratio of the two quantities. If this is the case, point out that the slope is equal to the ratio of the rise, ΔY, and the run, ΔX. This ratio is equal to the ratio of the intercepts of a line just tangent to the indifference curve. As we move along a convex indifference curve, these intercepts and the *MRS* change. Another problem is the terminology "of *X* for *Y*." This is confusing because we are not substituting "*X* for *Y*," but *Y* for one unit of *X*. You may want to present a variety of examples in class to explain this important concept.

Budget lines are easier to understand than indifference curves for most students, so you need not spend as much time on them. Be certain to point out that the two intercepts represent the number of units of each good the consumer could purchase if the consumer spent all of his or her income on that good. Also be sure to go over some numerical examples illustrating how the budget line shifts with changes in prices and income.

If you want to cover the utility maximization model mathematically, the Appendix to Chapter 4 lays out the Lagrangian method for solving constrained optimization problems and applies it to the maximization of utility subject to a budget constraint. This appendix also shows how demand curves are derived and discusses the Slutsky equation.

■ Questions for Review

1. **What are the four basic assumptions about individual preferences? Explain the significance or meaning of each.**

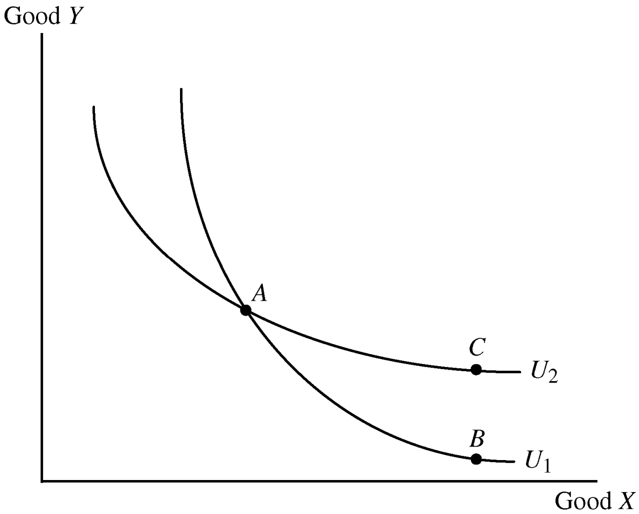
   (1) Preferences are complete: this means that the consumer is able to compare and rank all possible baskets of goods and services. (2) Preferences are transitive: this means that preferences are consistent, in the sense that if bundle *A* is preferred to bundle *B* and bundle *B* is preferred to bundle *C*, then bundle *A* is preferred to bundle *C*. (3) More is preferred to less: this means that all goods are desirable, and that the consumer always prefers to have more of each good. (4) Diminishing marginal rate of substitution: this means that indifference curves are convex, and that the slope of the indifference curve increases (becomes less negative) as we move down along the curve. As a consumer moves down along her indifference curve she is willing to give up fewer units of the good on the vertical axis in exchange for one more unit of the good on the horizontal axis. This assumption also means that balanced market baskets are generally preferred to baskets that have a lot of one good and very little of the other good.

2. Can a set of indifference curves be upward sloping? If so, what would this tell you about the two goods?

A set of indifference curves can be upward sloping if we violate assumption number three: more is preferred to less. When a set of indifference curves is upward sloping, it means one of the goods is a "bad" so that the consumer prefers less of that good rather than more. The positive slope means that the consumer will accept more of the bad only if he also receives more of the other good in return. As we move up along the indifference curve the consumer has more of the good he likes, and also more of the good he does not like.

3. Explain why two indifference curves cannot intersect.

The figure below shows two indifference curves intersecting at point *A*. We know from the definition of an indifference curve that the consumer has the same level of utility for every bundle of goods that lies on the given curve. In this case, the consumer is indifferent between bundles *A* and *B* because they both lie on indifference curve U_1. Similarly, the consumer is indifferent between bundles *A* and *C* because they both lie on indifference curve U_2. By the transitivity of preferences this consumer should also be indifferent between *C* and *B*. However, we see from the graph that *C* lies above *B*, so *C* must be preferred to *B* because *C* contains more of Good *Y* and the same amount of Good *X* as does *B*, and more is preferred to less. But this violates transitivity, so indifference curves must not intersect.



4. Jon is always willing to trade one can of Coke for one can of Sprite, or one can of Sprite for one can of Coke.

a. What can you say about Jon's marginal rate of substitution?

Jon's marginal rate of substitution can be defined as the number of cans of Coke he would be willing to give up in exchange for a can of Sprite. Since he is always willing to trade one for one, his *MRS* is equal to 1.

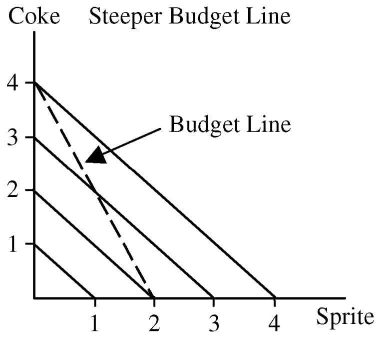
b. Draw a set of indifference curves for Jon.

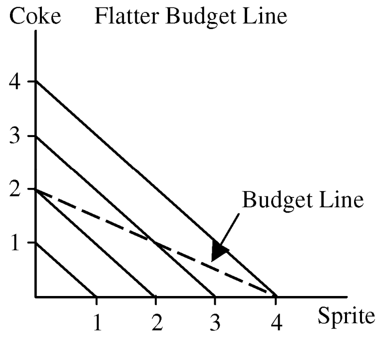
Since Jon is always willing to trade one can of Coke for one can of Sprite, his indifference curves are linear with a slope of –1. See the diagrams below part c.

c. Draw two budget lines with different slopes and illustrate the satisfaction-maximizing choice. What conclusion can you draw?

Jon's indifference curves are linear with a slope of −1. Jon's budget line is also linear, and will have a slope that reflects the ratio of the two prices. If Jon's budget line is steeper than his indifference curves, he will choose to consume only the good on the vertical axis. If Jon's budget line is flatter than his indifference curves, he will choose to consume only the good on the horizontal axis. Jon will always choose a corner solution where he buys only the less expensive good, unless his budget line has the same slope as his indifference curves. In this case any combination of Sprite and Coke that uses up his entire income will maximize Jon's satisfaction.

The diagrams below show cases where Jon's budget line is steeper than his indifference curves and where it is flatter. Jon's indifference curves are linear with slopes of −1, and four indifference curves are shown in each diagram as solid lines. Jon's budget is $4.00. In the diagram on the left, Coke costs $1.00 and Sprite costs $2.00, so Jon can afford 4 Cokes (if he spends his entire budget on Coke) or 2 Sprites (if he spends his budget on Sprite). His budget line is the dashed line. The highest indifference curve he can reach is the one furthest to the right. He can reach that level of utility by purchasing 4 Cokes and no Sprites. In the diagram on the right, the price of Coke is $2.00 and the price of Sprite is $1.00. Jon's budget line is now flatter than his indifference curves, and his optimal bundle is the corner solution with 4 Sprites and no Cokes.





5. What happens to the marginal rate of substitution as you move along a convex indifference curve? A linear indifference curve?

The *MRS* measures how much of a good you are willing to give up in exchange for one more unit of the other good, keeping utility constant. The *MRS* diminishes along a convex indifference curve. This occurs because as you move down along the indifference curve, you are willing to give up less and less of the good on the vertical axis in exchange for one more unit of the good on the horizontal axis. The *MRS* is also the negative of the slope of the indifference curve, which decreases (becomes closer to zero) as you move down along the indifference curve. The *MRS* is constant along a linear indifference curve because the slope does not change. The consumer is always willing to trade the same number of units of one good in exchange for the other.

6. Explain why an *MRS* between two goods must equal the ratio of the price of the goods for the consumer to achieve maximum satisfaction.

The *MRS* describes the rate at which the consumer is willing to trade off one good for another to maintain the same level of satisfaction. The ratio of prices describes the trade-off that the consumer is able to make between the same two goods in the market. The tangency of the indifference curve with the budget line represents the point at which the trade-offs are equal and consumer satisfaction is maximized. If the *MRS* between two goods is not equal to the ratio of prices, then the consumer could trade one good for another at market prices to obtain higher levels of satisfaction. For example, if the slope of the budget line (the ratio of the prices) is −4, the consumer can trade 4 units of *Y* (the good on the vertical axis) for one unit of *X* (the good on the horizontal axis). If the *MRS* at the current bundle is 6, then the consumer is willing to trade 6 units of *Y* for one unit of *X*. Since the two slopes are not equal the consumer is not maximizing her satisfaction. The consumer is willing to trade 6 but only has to trade 4, so she should make the trade. This trading continues until the highest level of satisfaction is achieved. As trades are made, the *MRS* will change and eventually become equal to the price ratio.

7. **Describe the indifference curves associated with two goods that are perfect substitutes. What if they are perfect complements?**

   Two goods are perfect substitutes if the *MRS* of one for the other is a constant number. In this case, the slopes of the indifference curves are constant, and the indifference curves are therefore linear. If two goods are perfect complements, the indifference curves are L-shaped. In this case the consumer wants to consume the two goods in a fixed proportion, say one unit of good 1 for every one unit of good 2. If she has more of one good than the other, she does not get any extra satisfaction from the additional units of the first good.

8. **What is the difference between ordinal utility and cardinal utility? Explain why the assumption of cardinal utility is not needed in order to rank consumer choices.**

   Ordinal utility implies an ordering among alternatives without regard for intensity of preference. For example, if the consumer's first choice is preferred to his second choice, then utility from the first choice will be higher than utility from the second choice. How much higher is not important. An ordinal utility function generates a ranking of bundles and no meaning is given to the magnitude of the utility number itself. Cardinal utility implies that the intensity of preferences may be quantified, and that the utility number itself has meaning. An ordinal ranking is all that is needed to rank consumer choices. It is not necessary to know how intensely a consumer prefers basket *A* over basket *B*; it is enough to know that *A* is preferred to *B*.

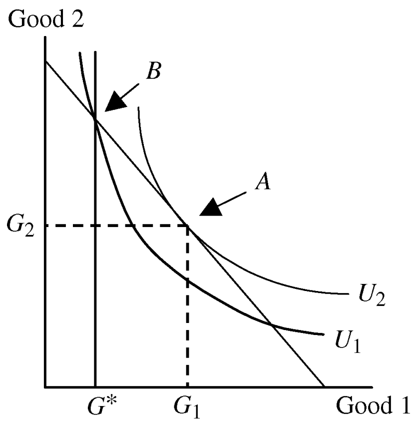
9. **Upon merging with the West German economy, East German consumers indicated a preference for Mercedes-Benz automobiles over Volkswagens. However, when they converted their savings into deutsche marks, they flocked to Volkswagen dealerships. How can you explain this apparent paradox?**

   There is no paradox. Preferences do not involve prices, and East German consumers preferred Mercedes based solely on product characteristics. However, Mercedes prices are considerably higher than Volkswagen prices. So, even though East German consumers preferred a Mercedes to a Volkswagen, they either could not afford a Mercedes or they preferred a bundle of other goods plus a Volkswagen to a Mercedes alone. While the marginal utility of consuming a Mercedes exceeded the marginal utility of consuming a Volkswagen, East German consumers considered the marginal utility per dollar for each good and, for most of them, the marginal utility per dollar was higher for Volkswagens. As a result, they flocked to Volkswagen dealerships to buy VWs.

10. **Draw a budget line and then draw an indifference curve to illustrate the satisfaction-maximizing choice associated with two products. Use your graph to answer the following questions.**

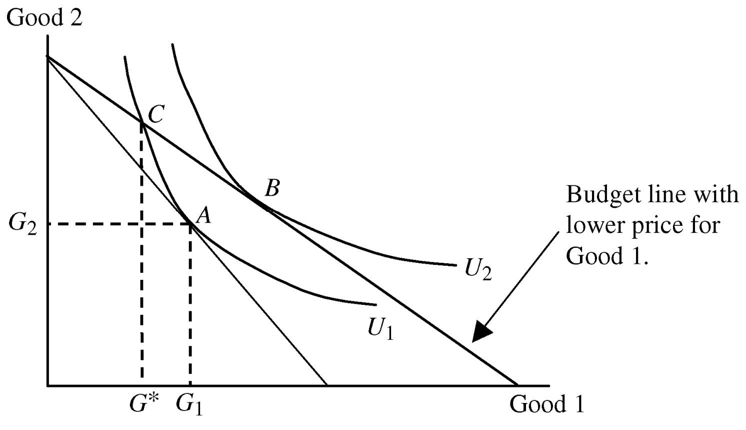
    a. **Suppose that one of the products is rationed. Explain why the consumer is likely to be worse off.**

       When goods are not rationed, the consumer is able to choose the satisfaction-maximizing bundle where the slope of the budget line is equal to the slope of the indifference curve, or the price ratio is equal to the *MRS*. This is point *A* in the diagram below where the consumer buys G_1 of good 1 and G_2 of good 2 and achieves utility level U_2. If good 1 is now rationed at G^* the consumer will no longer be able to attain the utility maximizing point. He or she cannot purchase amounts of good 1 exceeding G^*. As a result, the consumer will have to purchase more of the other good instead. The highest utility level the consumer can achieve with rationing is U_1 at point B. This is not a point of tangency, and the consumer's utility is lower than at point A, so the consumer is worse off as a result of rationing.



b. Suppose that the price of one of the products is fixed at a level below the current price. As a result, the consumer is not able to purchase as much as she would like. Can you tell if the consumer is better off or worse off?

No, the consumer could be better off or worse off. When the price of one good is fixed at a level below the current (equilibrium) price, there will be a shortage of that good, and the good will be effectively rationed. In the diagram below, the price of good 1 has been reduced, and the consumer's budget line has rotated out to the right. The consumer would like to purchase bundle B, but the amount of good 1 is restricted because of a shortage. If the most the consumer can purchase is G^*, she will be exactly as well off as before, because she will be able to purchase bundle C on her original indifference curve. If there is more than G^* of good 1 available, the consumer will be better off, and if there is less than G^*, the consumer will be worse off.



11. Describe the equal marginal principle. Explain why this principle may not hold if increasing marginal utility is associated with the consumption of one or both goods.

The equal marginal principle states that to obtain maximum satisfaction the ratio of the marginal utility to price must be equal across all goods. In other words, utility maximization is achieved when the budget is allocated so that the marginal utility per dollar of expenditure (MU/P) is the same for each good. If the MU/P ratios are not equal, allocating more dollars to the good with the higher MU/P will increase utility. As more dollars are allocated to this good its marginal utility will decrease, which causes its MU/P to fall and ultimately equal that of the other goods.

If marginal utility is increasing, however, allocating more dollars to the good with the larger MU/P causes MU to *increase*, and that good's MU/P just keeps getting larger and larger. In this case, the consumer should spend all her income on this good, resulting in a corner solution. With a corner solution, the equal marginal principle *does not* hold.

12. The price of computers has fallen substantially over the past two decades. Use this drop in price to explain why the Consumer Price Index is likely to overstate substantially the cost-of-living index for individuals who use computers intensively.

The Consumer Price Index measures the cost of a basket of goods purchased by a typical consumer in the current year relative to the cost of the basket in the base year. Each good in the basket is assigned a weight, which reflects the importance of the good to the typical consumer, and the weights are kept fixed from year to year. One problem with fixing the weights is that consumers will shift their purchases from year to year to give more weight to goods whose prices have fallen, and less weight to goods whose prices have risen. The *CPI* will therefore give too much weight to goods whose prices have risen, and too little weight to goods whose prices have fallen. In addition, for non-typical individuals who use computers intensively, the fixed weight for computers in the basket will understate the importance of this good, and will hence understate the effect of the fall in the price of computers for these individuals. The *CPI* will overstate the rise in the cost of living for this type of individual.

13. Explain why the Paasche index will generally understate the ideal cost-of-living index.

The Paasche index measures the current cost of the current bundle of goods relative to the base year cost of the current bundle of goods. The Paasche index will understate the ideal cost-of-living index because it assumes the individual buys the current year bundle in the base period. In reality, at base year prices the consumer would have been able to attain the same level of utility at a lower cost by altering his or her consumption bundle in light of the base year prices. Since the base year cost is overstated, the denominator of the Paasche index will be too large and the index will be too low, or understated.

■ Exercises

1. In this chapter, consumer preferences for various commodities did not change during the analysis. In some situations, however, preferences do change as consumption occurs. Discuss why and how preferences might change over time with consumption of these two commodities:

a. Cigarettes

The assumption that preferences do not change is a reasonable one if choices are independent across time. It does not hold, however, when "habit-forming" or addictive behavior is involved, as in the case of cigarettes. The consumption of cigarettes in one period influences the consumer's preference for cigarettes in the next period: the consumer desires cigarettes more because he has become more addicted to them.

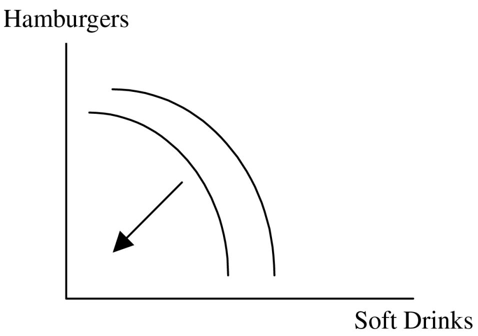
b. Dinner for the first time at a restaurant with a special cuisine

The first time you eat at a restaurant with a special cuisine can be an exciting new dining experience. This may make eating at the restaurant more desirable. But once you've eaten there, it isn't so exciting to do it again ("been there, done that"), and preference changes. On the other hand, some people prefer to eat at familiar places where they don't have to worry about new and unknown cuisine. For them, the first time at the restaurant would be less pleasant, but once they've eaten there and discovered they like the food, they would find further visits to the restaurant more desirable. In both cases, preferences change as consumption occurs.

2. Draw indifference curves that represent the following individuals' preferences for hamburgers and soft drinks. Indicate the direction in which the individuals' satisfaction (or utility) is increasing.

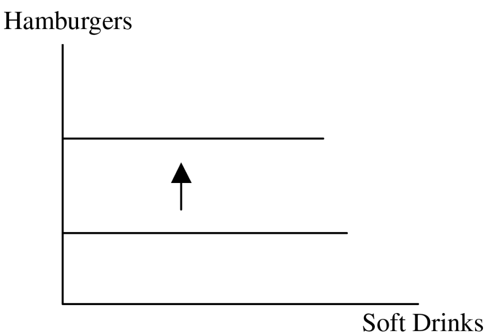
a. Joe has convex preferences and dislikes both hamburgers and soft drinks.

Since Joe dislikes both goods, he prefers less to more, and his satisfaction is increasing in the direction of the origin. Convexity of preferences implies his indifference curves will have the normal shape in that they are bowed towards the direction of increasing satisfaction. Convexity also implies that given any two bundles between which the Joe is indifferent, any linear combination of the two bundles will be in the preferred set, or will leave him at least as well off. This is true of the indifference curves shown in the diagram below.



b. Jane loves hamburgers and dislikes soft drinks. If she is served a soft drink, she will pour it down the drain rather than drink it.

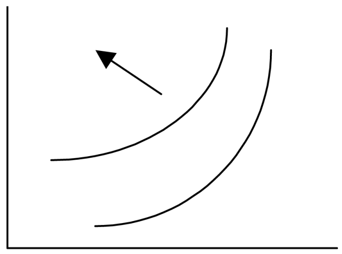
Since Jane can freely dispose of the soft drink if it is given to her, she considers it to be a neutral good. This means she does not care about soft drinks one way or the other. With hamburgers on the vertical axis, her indifference curves are horizontal lines. Her satisfaction increases in the upward direction.



c. Bob loves hamburgers and dislikes soft drinks. If he is served a soft drink, he will drink it to be polite.

Since Bob will drink the soft drink in order to be polite, it can be thought of as a "bad." When served another soft drink, he will require more hamburgers at the same time in order to keep his satisfaction constant. More soft drinks without more hamburgers will worsen his utility. More hamburgers and fewer soft drinks will increase his utility, so his satisfaction increases as we move upward and to the left.

Hamburgers

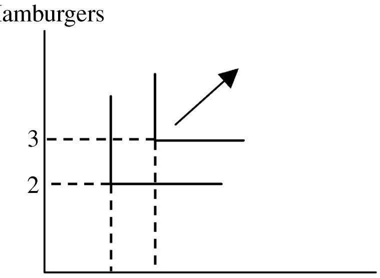


Soft Drinks

d. Molly loves hamburgers and soft drinks, but insists on consuming exactly one soft drink for every two hamburgers that she eats.

Molly wants to consume the two goods in a fixed proportion so her indifference curves are L-shaped. For a fixed amount of one good, she gets no extra satisfaction from having more of the other good. She will only increase her satisfaction if she has more of both goods.

Hamburgers

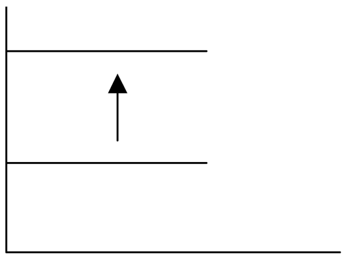


Soft Drinks

e. Bill likes hamburgers, but neither likes nor dislikes soft drinks.

Like Jane, Bill considers soft drinks to be a neutral good. Since he does not care about soft drinks one way or the other we can assume that no matter how many he has, his utility will be the same. His level of satisfaction depends entirely on how many hamburgers he has, so his satisfaction increases in the upward direction only.

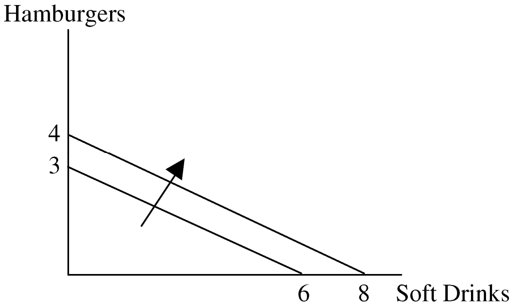
Hamburgers



Soft Drinks

f. Mary always gets twice as much satisfaction from an extra hamburger as she does from an extra soft drink.

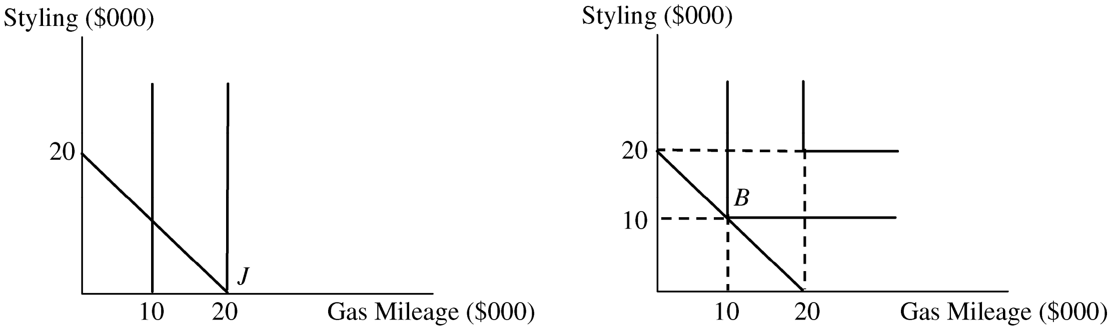
How much extra satisfaction Mary gains from an extra hamburger or soft drink tells us something about the marginal utilities of the two goods and about her *MRS*. If she always receives twice the satisfaction from an extra hamburger then her marginal utility from consuming an extra hamburger is twice her marginal utility from consuming an extra soft drink. Her *MRS*, with hamburgers on the vertical axis, is 1/2 because she will give up one hamburger only if she receives two soft drinks. Her indifference curves are straight lines with a slope of −1/2.



3. If Jane is currently willing to trade 4 movie tickets for 1 basketball ticket, then she must like basketball better than movies. True or false? Explain.

This statement is not necessarily true. If she is *always* willing to trade 4 movie tickets for 1 basketball ticket then yes, she likes basketball better because she will always gain the same satisfaction from 4 movie tickets as she does from 1 basketball ticket. However, it could be that she has convex preferences (diminishing *MRS*) and is at a bundle where she has a lot of movie tickets relative to basketball tickets. As she gives up movie tickets and acquires more basketball tickets, her *MRS* will fall. If *MRS* falls far enough she might get to the point where she would require, say, two basketball tickets to give up another movie ticket. It would not mean though that she liked basketball better, just that she had a lot of basketball tickets relative to movie tickets. Her willingness to give up a good depends on the quantity of each good in her current basket.

4. Janelle and Brian each plan to spend $20,000 on the styling and gas mileage features of a new car. They can each choose all styling, all gas mileage, or some combination of the two. Janelle does not care at all about styling and wants the best gas mileage possible. Brian likes both equally and wants to spend an equal amount on each. Using indifference curves and budget lines, illustrate the choice that each person will make.

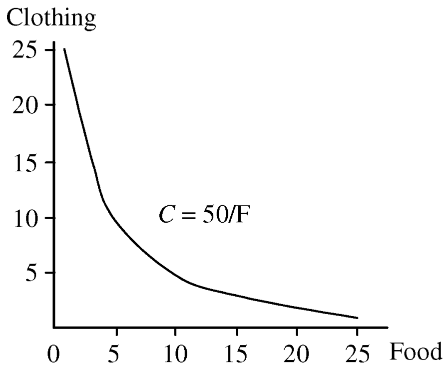


Plot thousands of dollars spent on styling on the vertical axis and thousands spent on gas mileage on the horizontal axis as shown above. Janelle, on the left, has indifference curves that are vertical. If the styling is there she will take it, but she otherwise does not care about it. As her indifference curves move over to the right, she gains more gas mileage and more satisfaction. She will spend all $20,000 on gas mileage at point *J*. Brian, on the right, has indifference curves that are L-shaped. He will not spend more on one feature than on the other feature. He will spend $10,000 on styling and $10,000 on gas mileage. His optimal bundle is at point *B*.

5. Suppose that Bridget and Erin spend their incomes on two goods, food (*F*) and clothing (*C*). Bridget's preferences are represented by the utility function $U(F,C) = 10FC$, while Erin's preferences are represented by the utility function $U(F,C) = 0.20F^2C^2$.

a. With food on the horizontal axis and clothing on the vertical axis, identify on a graph the set of points that give Bridget the same level of utility as the bundle (10,5). Do the same for Erin on a separate graph.

The bundle (10,5) contains 10 units of food and 5 of clothing. Bridget receives a utility of 10(10)(5) = 500 from this bundle. Thus, her indifference curve is represented by the equation $10FC = 500$ or $C = 50/F$. Some bundles on this indifference curve are (5,10), (10,5), (25,2), and (2,25). It is plotted in the diagram below. Erin receives a utility of $0.2(10^2)(5^2) = 500$ from the bundle (10,5). Her indifference curve is represented by the equation $0.2F^2C^2 = 500$, or $C = 50/F$. This is the same indifference curve as Bridget. Both indifference curves have the normal, convex shape.



b. On the same two graphs, identify the set of bundles that give Bridget and Erin the same level of utility as the bundle (15,8).

For each person, plug $F = 15$ and $C = 8$ into their respective utility functions. For Bridget, this gives her a utility of 1200, so her indifference curve is given by the equation $10FC = 1200$, or $C = 120/F$. Some bundles on this indifference curve are (12,10), (10,12), (3,40), and (40,3). The indifference curve will lie above and to the right of the curve diagrammed in part a. This bundle gives Erin a utility of 2880, so her indifference curve is given by the equation $0.2F^2C^2 = 2880$, or $C = 120/F$. This is the same indifference curve as Bridget.

c. Do you think Bridget and Erin have the same preferences or different preferences? Explain.

They have the same preferences because their indifference curves are identical. This means they will rank all bundles in the same order. Note that it is not necessary that they receive the same level of utility for each bundle to have the same set of preferences. All that is necessary is that they rank the bundles in the same order.

6. Suppose that Jones and Smith have each decided to allocate $1000 per year to an entertainment budget in the form of hockey games or rock concerts. They both like hockey games and rock concerts and will choose to consume positive quantities of both goods. However, they differ substantially in their preferences for these two forms of entertainment. Jones prefers hockey games to rock concerts, while Smith prefers rock concerts to hockey games.

a. Draw a set of indifference curves for Jones and a second set for Smith.

Given they each like both goods and they will each choose to consume positive quantities of both goods, we can assume their indifference curves have the normal convex shape. However since Jones has an overall preference for hockey and Smith has an overall preference for rock concerts, their two sets of indifference curves will have different slopes. Suppose that we place rock concerts on the vertical axis and hockey games on the horizontal axis, Jones will have a larger *MRS* than Smith. Jones is willing to give up more rock concerts in exchange for a hockey game since he prefers hockey games. Thus, indifference curves for Jones will be steeper than the indifference curves for Smith.

b. Using the concept of marginal rate of substitution, explain why the two sets of curves are different from each other.

At any combination of hockey games and rock concerts, Jones is willing to give up more rock concerts for an additional hockey game, whereas Smith is willing to give up fewer rock concerts for an additional hockey game. Since the *MRS* is a measure of how many of one good (rock concerts) an individual is willing to give up for an additional unit of the other good (hockey games), the *MRS*, and hence the slope of the indifference curves, will be different for the two individuals.

7. The price of DVDs (*D*) is $20 and the price of CDs (*C*) is $10. Philip has a budget of $100 to spend on the two goods. Suppose that he has already bought one DVD and one CD. In addition there are 3 more DVDs and 5 more CDs that he would really like to buy.

a. Given the above prices and income, draw his budget line on a graph with CDs on the horizontal axis.

His budget line is $P_D D + P_C C = I$, or $20D + 10C = 100$. If he spends his entire income on DVDs he can afford to buy 5. If he spends his entire income on CDs he can afford to buy 10. His budget line is linear with these two points as intercepts.

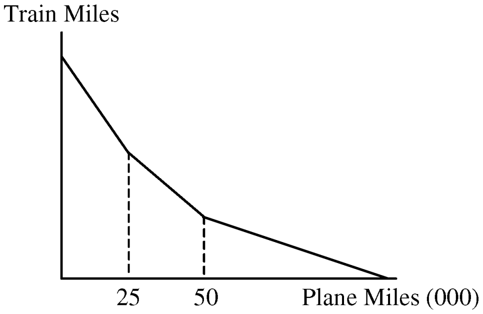
b. Considering what he has already purchased and what he still wants to purchase, identify the three different bundles of CDs and DVDs that he could choose. For this part of the question, assume that he cannot purchase fractional units.

He has already purchased one of each for a total of $30, so he has $70 left. Since he wants 3 more DVDs, he can buy these for $60 and spend his remaining $10 on 1 CD. This is the first bundle below. He could also choose to buy only 2 DVDs for $40 and spend the remaining $30 on 3 CDs. This is the second bundle. Finally, he could purchase 1 more DVD for $20 and spend the remaining $50 on the 5 CDs he would like. This is the final bundle shown in the table below.

Purchased Quantities		**Total Quantities**	
DVDs	**CDs**	**DVDs**	**CDs**
3	1	4	2
2	3	3	4
1	5	2	6

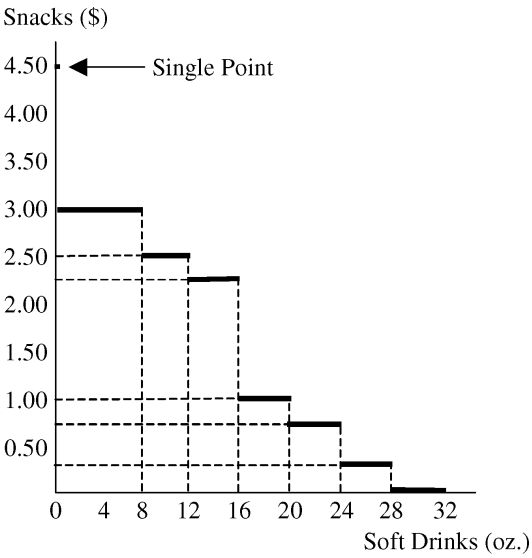
8. Anne has a job that requires her to travel three out of every four weeks. She has an annual travel budget and can travel either by train or by plane. The airline on which she typically flies has a frequent-traveler program that reduces the cost of her tickets according to the number of miles she has flown in a given year. When she reaches 25,000 miles, the airline will reduce the price of her tickets by 25% for the remainder of the year. When she reaches 50,000 miles, the airline will reduce the price by 50% for the remainder of the year. Graph Anne's budget line, with train miles on the vertical axis and plane miles on the horizontal axis.

The typical budget line is linear (with a constant slope) because prices do not change. In this case, the price of airline miles changes depending on how many miles Anne purchases. As the price changes, the slope of the budget line changes. Because there are three prices, there will be three slopes (and two kinks) to the budget line. Since the price falls as Anne flies more miles, her budget line will become flatter with each price change.



9. Debra usually buys a soft drink when she goes to a movie theater, where she has a choice of three sizes: the 8-ounce drink costs $1.50, the 12-ounce drink $2.00, and the 16-ounce drink $2.25. Describe the budget constraint that Debra faces when deciding how many ounces of the drink to purchase. (Assume that Debra can costlessly dispose of any of the soft drink that she does not want.)

First notice that as the size of the drink increases, the price per ounce decreases. So, for example, if Debra wants 16 ounces of soft drink, she should buy the 16-ounce size and not two 8-ounce size drinks. Also, if Debra wants 14 ounces, she should buy the 16-ounce drink and dispose of the last 2 ounces. The problem assumes she can do this without cost. As a result, Debra's budget constraint is a series of horizontal lines as shown in the diagram below.

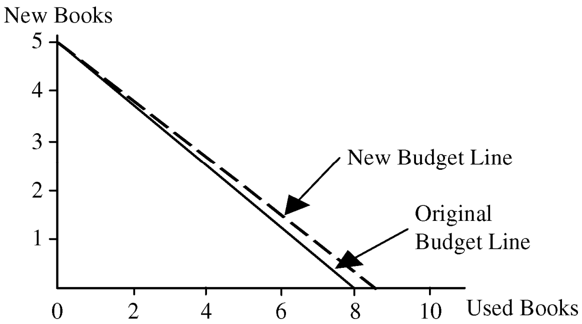


The diagram assumes Debra has a budget of $4.50 to spend on snacks and soft drinks at the movie. Dollars spent on snacks are plotted on the vertical axis and ounces of soft drinks on the horizontal. If Debra wants just an ounce or two of soft drink, she has to purchase the 8-ounce size, which costs $1.50. Thus, she would have $3.00 to spend on snacks. If Debra wants more than 16 ounces of soft drink, she has to purchase more than one drink, and we have to figure out the least-cost way for her to do that. If she wants, say, 20 ounces, she should purchase one 8-ounce and one 12-ounce drink. All of this must be considered in drawing her budget line.

10. Antonio buys five new college textbooks during his first year at school at a cost of $80 each. Used books cost only $50 each. When the bookstore announces that there will be a 10% increase in the price of new books and a 5% increase in the price of used books, Antonio's father offers him $40 extra.

a. What happens to Antonio's budget line? Illustrate the change with new books on the vertical axis.

In the first year Antonio spends $80 each on 5 new books for a total of $400. For the same amount of money he could have bought 8 used textbooks. His budget line is therefore $80N + 50U = 400$, where N is the number of new books and U is the number of used books. After the price change, new books cost $88, used books cost $52.50, and he has an income of $440. If he spends all of his income on new books, he can still afford to buy 5 new books, but he can now afford to buy 8.4 used books if he buys only used books. The new budget line is $88N + 52.50U = 440$. The line has rotated out to the right and become slightly flatter as shown in the diagram.



b. Is Antonio worse or better off after the price change? Explain.

The first year he bought 5 new books at a cost of $80 each, which is a corner solution. The new price of new books is $88 and the cost of 5 new books is now $440. The $40 extra income will cover the price increase. Antonio is definitely not worse off since he can still afford the same number of new books. He may in fact be better off if he decides to switch to some used books, although the slight shift in his budget line suggests that the new optimum will most likely be at the same corner solution as before.

11. Consumers in Georgia pay twice as much for avocados as they do for peaches. However, avocados and peaches are the same price in California. If consumers in both states maximize utility, will the marginal rate of substitution of peaches for avocados be the same for consumers in both states? If not, which will be higher?

The marginal rate of substitution of peaches for avocados is the maximum amount of avocados that a person is willing to give up to obtain one additional peach, or $MRS = -\dfrac{\Delta A}{\Delta P}$, where A is the number of avocados and P the number of peaches. When consumers maximize utility, they set their MRS equal to the price ratio, $\dfrac{p_P}{p_A}$, where p_P is the price of a peach and p_A is the price of an avocado. In Georgia, avocados cost twice as much as peaches, so the price ratio is ½, but in California, the prices are the same, so the price ratio is 1. Therefore, when consumers are maximizing utility (assuming they buy positive amounts of both goods), the marginal rates of substitution will not be the same for consumers in both states. Consumers in California will have an *MRS* that is twice as large as consumers in Georgia.

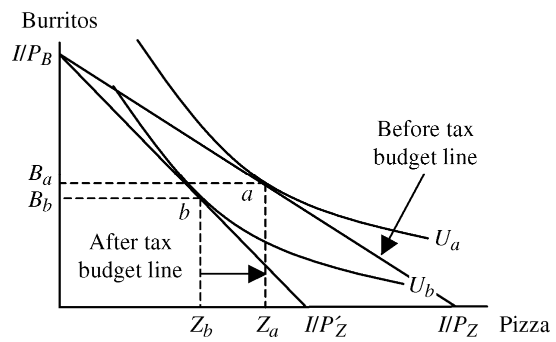
12. Ben allocates his lunch budget between two goods, pizza and burritos.

a. Illustrate Ben's optimal bundle on a graph with pizza on the horizontal axis.

In the diagram below, Ben's income is I, the price of pizza is P_Z, and the price of burritos is P_B. Ben's budget line is linear, and he consumes at the point where his indifference curve is tangent to his budget line at point a in the diagram. This places him on the highest possible indifference curve, which is labeled U_a. Ben buys Z_a pizza and B_a burritos.

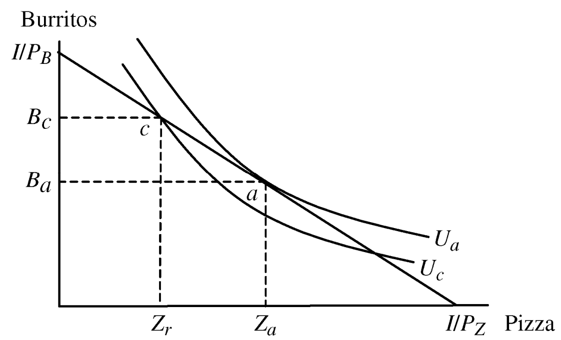
b. Suppose now that pizza is taxed, causing the price to increase by 20%. Illustrate Ben's new optimal bundle.

The price of pizza increases 20% because of the tax, and Ben's budget line pivots inward. The new price of pizza is $P'_Z = 1.2P_Z$. This shrinks the size of Ben's budget set, and he will no longer be able to afford his old bundle. His new optimal bundle is where the lower indifference curve U_b is tangent to his new budget line. Ben now consumes Z_b pizza and B_b burritos. ***Note*:** The diagram shows that Ben buys fewer burritos after the tax, but he could buy more if his indifference curves were drawn differently.



c. Suppose instead that pizza is rationed at a quantity less than Ben's desired quantity. Illustrate Ben's new optimal bundle.

Rationing the quantity of pizza that can be purchased will result in Ben not being able to choose his preferred bundle, *a*. The rationed amount of pizza is Z_r in the diagram. Ben will choose bundle *c* on the budget line that is above and to the left of his original bundle. He buys more burritos, B_c, and the rationed amount of pizza, Z_r. The new bundle gives him a lower level of utility, U_c.



13. Brenda wants to buy a new car and has a budget of $25,000. She has just found a magazine that assigns each car an index for styling and an index for gas mileage. Each index runs from 1 to 10, with 10 representing either the most styling or the best gas mileage. While looking at the list of cars, Brenda observes that on average, as the style index increases by one unit, the price of the car increases by $5000. She also observes that as the gas-mileage index rises by one unit, the price of the car increases by $2500.

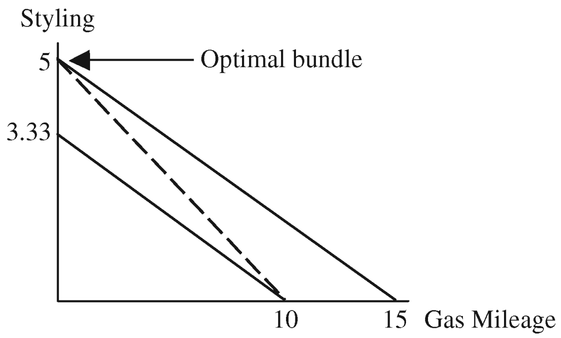
a. Illustrate the various combinations of style (*S*) and gas mileage (*G*) that Brenda could select with her $25,000 budget. Place gas mileage on the horizontal axis.

For every $5000 she spends on style the index rises by one, so the most she can achieve is a car with a style index of 5. For every $2500 she spends on gas mileage the index rises by one, so the most she can achieve is a car with a gas-mileage index of 10. The slope of her budget line is therefore −1/2 as shown by the dashed line in the diagram for part b.

b. Suppose Brenda's preferences are such that she always receives three times as much satisfaction from an extra unit of styling as she does from gas mileage. What type of car will Brenda choose?

If Brenda always receives three times as much satisfaction from an extra unit of styling as she does from an extra unit of gas mileage, then she is willing to trade one unit of styling for three units of gas mileage and still maintain the same level of satisfaction. Her indifference curves are straight lines with slopes of −1/3. Two are shown in the graph as solid lines. Since her *MRS* is a constant 1/3 and the slope of her budget line is −1/2, Brenda will choose all styling.

You can also compute the marginal utility per dollar for styling and gas mileage and note that the MU/P for styling is always greater, so there is a corner solution. Two indifference curves are shown on the graph as solid lines. The higher one starts with styling of 5 on the vertical axis. Moving down the indifference curve, Brenda gives up one unit of styling for every 3 additional units of gas mileage, so this indifference curve intersects the gas mileage axis at 15. The other indifference curve goes from 3.33 units of styling to 10 of gas mileage. Brenda reaches the highest indifference curve when she chooses all styling and no gas mileage.



c. Suppose that Brenda's marginal rate of substitution (of gas mileage for styling) is equal to *S*/(4*G*). What value of each index would she like to have in her car?

To find the optimal value of each index, set *MRS* equal to the price ratio of 1/2 and cross multiply to get $S = 2G$. Now substitute into the budget constraint, $5000S + 2500G = 25{,}000$, to get $5000(2G) + 2500G = 25{,}000$ or $12{,}500G = 25{,}000$. Therefore, $G = 2$ and $S = 4$.

d. Suppose that Brenda's marginal rate of substitution (of gas mileage for styling) is equal to (3*S*)/*G*. What value of each index would she like to have in her car?

Now set her new *MRS* equal to the price ratio of 1/2 and cross multiply to get $G = 6S$. Substitute into the budget constraint, $5000S + 2500G = 25{,}000$, to get $5000S + 2500(6S) = 25{,}000$. Solving, $G = 7.5$ and $S = 1.25$.

14. Connie has a monthly income of $200 that she allocates among two goods: meat and potatoes.

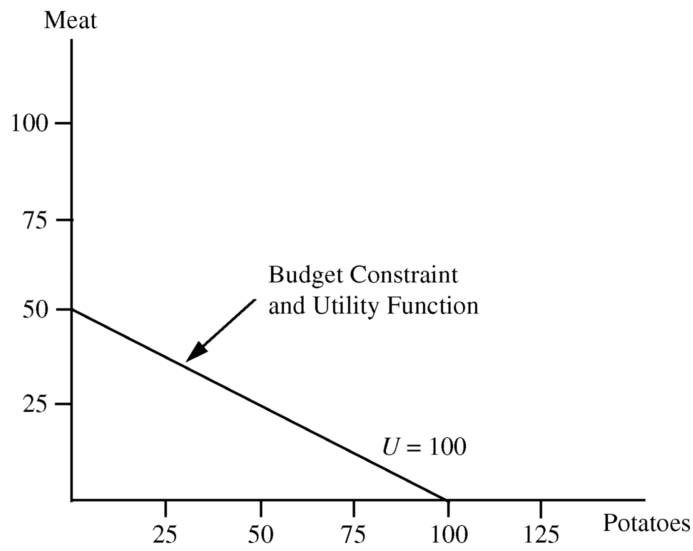
a. Suppose meat costs $4 per pound and potatoes $2 per pound. Draw her budget constraint.

Let M = meat and P = potatoes. Connie's budget constraint is

$$4M + 2P = 200, \text{ or}$$

$$M = 50 - 0.5P.$$

As shown in the graph below, with M on the vertical axis, the vertical intercept is 50 pounds of meat. The horizontal intercept may be found by setting $M = 0$ and solving for P. The horizontal intercept is therefore 100 pounds of potatoes.

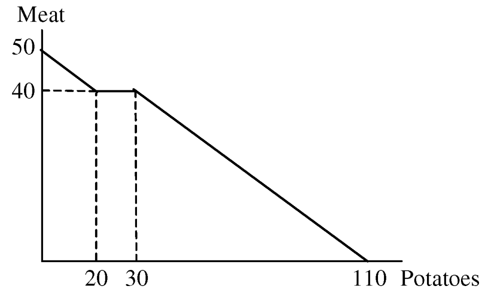


b. Suppose also that her utility function is given by the equation $U(M, P) = 2M + P$. What combination of meat and potatoes should she buy to maximize her utility? (*Hint*: Meat and potatoes are perfect substitutes.)

When the two goods are perfect substitutes, the indifference curves are linear. To find the slope of the indifference curve, choose a level of utility and find the equation for a representative indifference curve. Suppose $U = 50$, then $2M + P = 50$, or $M = 25 - 0.5P$. Therefore, Connie's budget line and her indifference curves have the same slope. This indifference curve lies below the one shown in the diagram above. Connie's utility is equal to 100 when she buys 50 pounds of meat and no potatoes or no meat and 100 pounds of potatoes. The indifference curve for $U = 100$ coincides with her budget constraint. Any combination of meat and potatoes along this line will provide her with maximum utility.

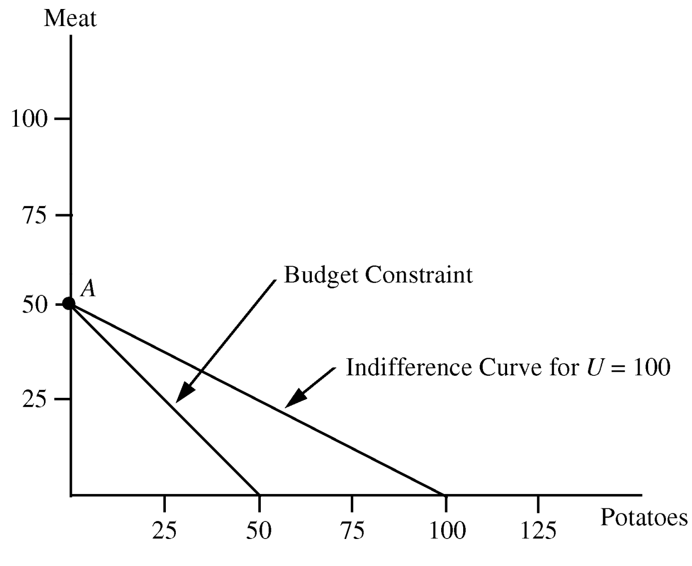
c. Connie's supermarket has a special promotion. If she buys 20 pounds of potatoes (at $2 per pound), she gets the next 10 pounds for free. This offer applies only to the first 20 pounds she buys. All potatoes in excess of the first 20 pounds (excluding bonus potatoes) are still $2 per pound. Draw her budget constraint.

With potatoes on the horizontal axis, Connie's budget constraint has a slope of −1/2 until Connie has purchased 20 pounds of potatoes. Then her budget line is flat from 20 to 30 pounds of potatoes, because the next 10 pounds of potatoes are free, and she does not have to give up any meat to get these extra potatoes. After 30 pounds of potatoes, the slope of her budget line becomes −1/2 again until it intercepts the potato axis at 110.



d. An outbreak of potato rot raises the price of potatoes to $4 per pound. The supermarket ends its promotion. What does her budget constraint look like now? What combination of meat and potatoes maximizes her utility?

With the price of potatoes at $4, Connie may buy either 50 pounds of meat or 50 pounds of potatoes, or any combination in between. See the diagram below. She maximizes utility at $U = 100$ at point A when she consumes 50 pounds of meat and no potatoes. *This is a corner solution.*



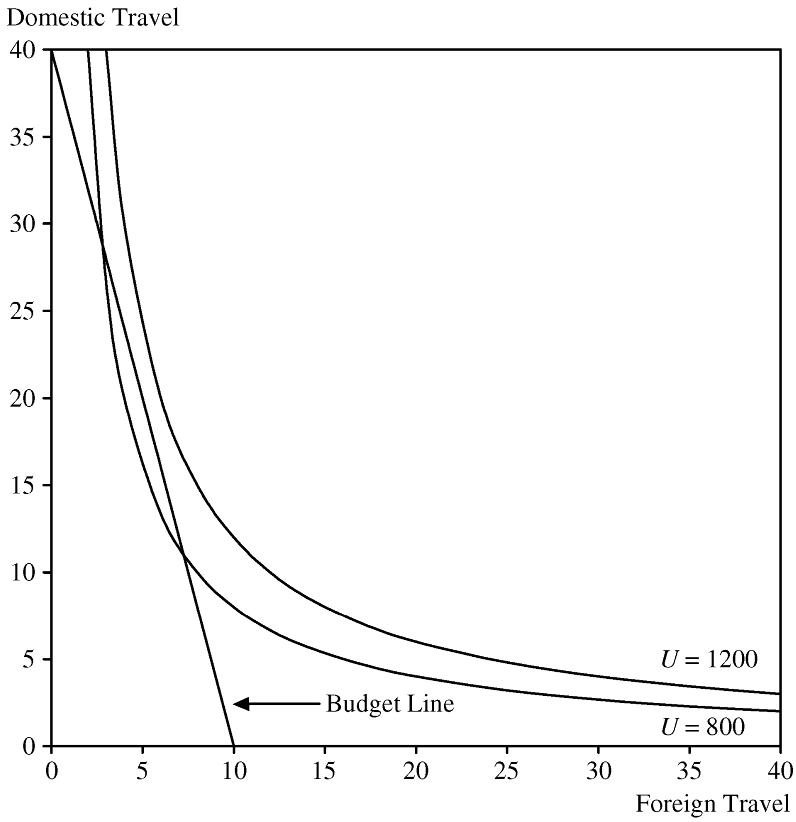
15. Jane receives utility from days spent traveling on vacation domestically (D) and days spent traveling on vacation in a foreign country (F), as given by the utility function $U(D,F) = 10DF$. In addition, the price of a day spent traveling domestically is \$100, the price of a day spent traveling in a foreign country is \$400, and Jane's annual travel budget is \$4000.

a. Illustrate the indifference curve associated with a utility of 800 and the indifference curve associated with a utility of 1200.

The indifference curve with a utility of 800 has the equation $10DF = 800$, or $D = 80/F$. Find combinations of D and F that satisfy this equation (such as $D = 8$ and $F = 10$) and plot the indifference curve, which is the lower of the two on the graph in part b. The indifference curve with a utility of 1200 has the equation $10DF = 1200$, or $D = 120/F$. Find combinations of D and F that satisfy this equation and plot the indifference curve, which is the upper curve on the graph.

b. Graph Jane's budget line on the same graph.

If Jane spends all of her budget on domestic travel she can afford 40 days. If she spends all of her budget on foreign travel she can afford 10 days. Her budget line is $100D + 400F = 4000$, or $D = 40 - 4F$. This straight line is plotted in the graph below.



c. Can Jane afford any of the bundles that give her a utility of 800? What about a utility of 1200?

Jane can afford some of the bundles that give her a utility of 800 because part of the $U = 800$ indifference curve lies below the budget line. She cannot afford any of the bundles that give her a utility of 1200 as this indifference curve lies entirely above the budget line.

d. Find Jane's utility maximizing choice of days spent traveling domestically and days spent in a foreign country.

The optimal bundle is where the ratio of prices is equal to the *MRS*, and Jane is spending her entire income. The ratio of prices is $\frac{P_F}{P_D} = 4$, and $MRS = \frac{MU_F}{MU_D} = \frac{10D}{10F} = \frac{D}{F}$. Setting these two equal and solving for D, we get $D = 4F$.

Substitute this into the budget constraint, $100D + 400F = 4000$, and solve for F. The optimal solution is $F = 5$ and $D = 20$. Utility is 1000 at the optimal bundle, which is on an indifference curve between the two drawn in the graph above.

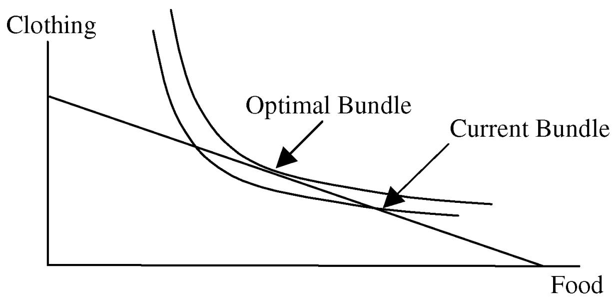
16. Julio receives utility from consuming food (*F*) and clothing (*C*) as given by the utility function $U(F, C) = FC$. In addition, the price of food is \$2 per unit, the price of clothing is \$10 per unit, and Julio's weekly income is \$50.

a. What is Julio's marginal rate of substitution of food for clothing when utility is maximized? Explain.

Plotting clothing on the vertical axis and food on the horizontal, as in the textbook, Julio's utility is maximized when his *MRS* (of food for clothing) equals P_F/P_C, the price ratio. The price ratio is $2/10 = 0.2$, so Julio's *MRS* will equal 0.2 when his utility is maximized.

b. Suppose instead that Julio is consuming a bundle with more food and less clothing than his utility maximizing bundle. Would his marginal rate of substitution of food for clothing be greater than or less than your answer in part a? Explain.

In absolute value terms, the slope of his indifference curve at this non-optimal bundle is less than the slope of his budget line, because the indifference curve is flatter than the budget line. He is willing to give up more food than he has to at market prices to obtain one more unit of clothing. His *MRS* is less than the answer in part a.



17. The utility that Meredith receives by consuming food *F* and clothing *C* is given by $U(F,C) = FC$. Suppose that Meredith's income in 1990 is \$1200 and that the prices of food and clothing are \$1 per unit for each. By 2000, however, the price of food has increased to \$2 and the price of clothing to \$3. Let 100 represent the cost of living index for 1990. Calculate the ideal and the Laspeyres cost-of-living index for Meredith for 2000. (*Hint*: Meredith will spend equal amounts on food and clothing with these preferences.)

First, we need to calculate F and C, which make up the bundle of food and clothing that maximizes Meredith's utility given 1990 prices and her income in 1990. Use the hint to simplify the problem: since she spends equal amounts on both goods, she must spend half her income on each. Therefore, $P_F F = P_C C = \$1200/2 = \$600$. Since $P_F = P_C = \$1$, F and C are both equal to 600 units, and Meredith's utility is $U = (600)(600) = 360{,}000$.

***Note*:** You can verify the hint mathematically as follows. The marginal utilities with this utility function are $MU_C = \Delta U/\Delta C = F$ and $MU_F = \Delta U/\Delta F = C$. To maximize utility, Meredith chooses a consumption bundle such that $MU_F/MU_C = P_F/P_C$, which yields $P_F F = P_C C$.

Laspeyres Index:

The Laspeyres index represents how much more Meredith would have to spend in 2000 versus 1990 if she consumed the same amounts of food and clothing in 2000 as she did in 1990. That is, the Laspeyres index (*LI*) for 2000 is given by:

$$LI = 100\,(I')/I,$$

where I' represents the amount Meredith would spend at 2000 prices consuming the same amount of food and clothing as in 1990. In 2000, 600 clothing and 600 food would cost \$3(600) + \$2(600) = \$3000.

Therefore, the Laspeyres cost-of-living index is:

$$LI = 100(\$3000/\$1200) = 250.$$

Ideal Index:

The ideal index represents how much Meredith would have to spend on food and clothing in 2000 (using 2000 prices) to get the same amount of utility as she had in 1990. That is, the ideal index (II) for 2000 is given by:

$$II = 100(I'')/I, \text{ where } I'' = P'_F F' + P'_C C' = 2F' + 3C',$$

where F' and C' are the amount of food and clothing that give Meredith the same utility as she had in 1990. F' and C' must also be such that Meredith spends the least amount of money at 2000 prices to attain the 1990 utility level.

The bundle (F',C') will be on the same indifference curve as (F,C) so $F'C' = FC = 360{,}000$ in utility, and $2F' = 3C'$ because Meredith spends the same amount on each good.

We now have two equations: $F'C' = 360{,}000$ and $2F' = 3C'$. Solving for F' yields:

$$F'[(2/3)F'] = 360{,}000 \text{ or } F' = \sqrt{[(3/2)360{,}000)]} = 734.85.$$

From this, we obtain C',

$$C' = (2/3)F' = (2/3)734.85 = 489.90.$$

In 2000, the bundle of 734.85 units of food and 489.90 units of clothing would cost 734.85(\$2) + 489.9(\$3) = \$2939.40, and Meredith would still get 360,000 in utility.

We can now calculate the ideal cost-of-living index:

$$II = 100(\$2939.40/\$1200) = 245.$$

This is slightly less than the Laspeyres Index of 250 and illustrates the fact that a Laspeyres type index tends to overstate inflation compared to the ideal cost-of-living index.

Chapter 4
Individual and Market Demand

■ Teaching Notes

Chapter 4 builds on the consumer choice model presented in Chapter 3. Students find this material very abstract and “unrealistic,” so it is important to convince them that there are good reasons for studying how consumers make purchasing decisions in some detail. Most importantly, we gain a deeper understanding of what lies behind demand curves and why, for example, demand curves almost always slope downward. The utility maximizing model is also crucial in determining the supply of labor in Chapter 14, general equilibrium in Chapter 16, and market failure in Chapter 18. So play up these applications when selling students on the importance of the material in this chapter.

Section 4.1 focuses on graphically deriving individual demand and Engel curves by changing price and income. Section 4.2 develops the income and substitution effects of a price change and explains the (perhaps mythical) Giffen good case where the demand curve slopes upward. Section 4.3 shows how to derive the market demand curve from individuals’ demand curves. The remaining sections cover consumer surplus, network externalities, and empirical demand estimation.

When discussing the derivation of demand, review how the budget line pivots around an intercept as price changes and how optimal quantities change as the budget line pivots. Most of the diagrams in the book analyze decreases in price, so you might want to go over an example in which price increases. This will come in handy if you later go over the effects of a gasoline tax in Example 4.2. Once students understand the effect of price changes on consumer choice, they can grasp the derivation of the price-consumption path and the individual demand curve. Remind students that the price a consumer is willing to pay is a measure of the marginal benefit of consuming another unit. This is important for understanding consumer surplus in Section 4.4.

Income and substitution effects are difficult for most students, so spend some extra time going over this material. Students frequently have trouble remembering which effect is which on the graph. Emphasize that the substitution effect explains the change in quantity demanded caused by the change in relative prices holding utility constant (a change in the slope of the budget line while staying on the original indifference curve), while the income effect explains the change in quantity demanded caused by a change in purchasing power (a shift of the budget line). Be sure to explain that the substitution effect is always negative (i.e., relative price and quantity are negatively related). On the other hand, the direction of the income effect depends on whether the product is a normal or inferior good. It is a good idea to cover both a price increase and a price decrease.

If students are having trouble understanding the income effect, you might want to give a few numerical examples of how purchasing power changes as the result of a price change. For example, suppose a consumer typically buys a can of soda every day for \$0.75 per can. If the price increases to \$1.00, the consumer’s purchasing power drops by \$0.25 per day, and she will have to spend less on other goods and/or buy less soda. Over a month this amounts to a reduction in real income of roughly $\$0.25 \times 30 = \$7.50$. You can show how the income effect can be large or small depending on how much the consumer spends on the product and how much the price changes.

When covering the aggregation of individual demand curves in Section 4.3, stress that this is equivalent to the horizontal summation of the individual demand curves because we want to add up the quantities demanded by all consumers at each price. To obtain the market demand curve, you must have demand written in the form $Q = f(P)$ as opposed to the inverse demand $P = f(Q)$. The concept of a kink in the market demand curve is often new to students. Emphasize that this is because not all consumers are in the market at all prices. With many consumers there can be many kinks. With thousands of consumers, however, we hardly notice the individual kinks, so it is reasonable to draw most market demand curves as smooth lines.

The concept of elasticity is reintroduced and further explored in Section 4.3. In particular, the relationship between elasticity and revenue (or total expenditures) is explained. Here is a way to help students remember this relationship. Think of price and revenue as being connected by a link of some sort. If the link is elastic, price and revenue move in opposite directions because the link between them stretches like a rubber band (for example, an increase in price leads to a decrease in revenue), but if the link is inelastic, price and revenue must move in the same direction because the link is inflexible.

Consumer surplus is introduced in Section 4.4. Emphasize that it measures the value a consumer places on a good in excess of the price the consumer pays for the good. It is the difference between what the consumer is willing to pay and what he or she actually has to pay for the good. We usually measure consumer surplus using a market demand curve, in which case we are finding the sum of all the individual consumer surpluses. Go over an example with a linear demand curve, and remind students that the area of a triangle is ½ × (base) × (height). Example 4.6 is a nice application of consumer surplus, but students have a hard time understanding the demand curve for reductions in air pollution, so expect to spend some time if you cover this example. The related topic of producer surplus is covered in Chapter 8, and both producer and consumer surplus are used extensively in Chapter 9 and later chapters.

Network externalities in Section 4.5 can be covered quickly, and they are pretty intuitive for most students. Figures 4.17 and 4.18 are a bit complicated, but you do not have to cover them to get the main points across. A nice example of a negative network externality that is covered only briefly in the text is congestion. Road congestion is something most students can relate to, and you could mention the comment about a popular New York City restaurant attributed to Yogi Berra that goes something like, “It’s gotten so crowded, nobody goes there anymore.”

The first part of Section 4.6, “The Statistical Approach to Demand Estimation,” is fairly straightforward. It is important for students to understand that demand curves really do exist and can be estimated. Many seem to think demand curves are figments of economists’ imaginations and that we draw them more or less randomly, so this part of the section is very useful. The second part, “The Form of the Demand Relationship,” is more complicated and difficult for students who do not understand logarithms.

The Appendix is intended for students with a background in calculus. It goes through the maximization of utility subject to a budget constraint using the Lagrange multiplier method. Demand curves are derived and many of the conditions developed in Chapter 3 are shown mathematically. There is a brief treatment of duality in consumer theory, and the mathematical form for the Slutsky equation is discussed but not derived mathematically.

■ Questions for Review

1. Explain the difference between each of the following terms:

a. a price consumption curve and a demand curve

The price consumption curve (*PCC*) shows the quantities of two goods a consumer will purchase as the price of one of the goods changes, while a demand curve shows the quantity of one good a consumer will purchase as the price of that good changes. The graph of the *PCC* plots the quantity of one good on the horizontal axis and the quantity of the other good on the vertical axis. The demand curve plots the quantity of the good on the horizontal axis and its price on the vertical axis.

b. an individual demand curve and a market demand curve

An individual demand curve plots the quantity demanded by one person at various prices. A market demand curve is the horizontal sum of all the individual demand curves. It plots the total quantity demanded by all consumers at various prices.

c. an Engel curve and a demand curve

An Engel curve shows the quantity of one good that will be purchased by a consumer at different income levels. The quantity of the good is plotted on the horizontal axis and the consumer's income is on the vertical axis. A demand curve is like an Engel curve except that it shows the quantity purchased at different prices instead of different income levels.

d. an income effect and a substitution effect

Both the substitution effect and income effect occur because of a change in the price of a good. The substitution effect is the change in the quantity demanded of the good due to the price change, holding the consumer's utility constant. The income effect is the change in the quantity demanded of the good due to the change in purchasing power brought about by the change in the good's price.

2. Suppose that an individual allocates his or her entire budget between two goods, food and clothing. Can both goods be inferior? Explain.

No, the goods cannot both be inferior; at least one must be a normal good. Here's why. If an individual consumes only food and clothing, then any increase in income must be spent on either food or clothing or both (recall, we assume there are no savings and more of any good is preferred to less, even if the good is an inferior good). If food is an inferior good, then as income increases, consumption of food falls. With constant prices, the extra income not spent on food must be spent on clothing. Therefore as income increases, more is spent on clothing, i.e., clothing is a normal good.

3. Explain whether the following statements are true or false:

a. The marginal rate of substitution diminishes as an individual moves downward along the demand curve.

True. The consumer maximizes his utility by choosing the bundle on his budget line where the price ratio is equal to the *MRS*. For goods 1 and 2, $P_1/P_2 = MRS$. As the price of good 1 falls, the consumer moves downward along the demand curve for good 1, and the price ratio (P_1/P_2) becomes smaller. Therefore, *MRS* must also become smaller, and thus *MRS* diminishes as an individual moves downward along the demand curve.

b. The level of utility increases as an individual moves downward along the demand curve.

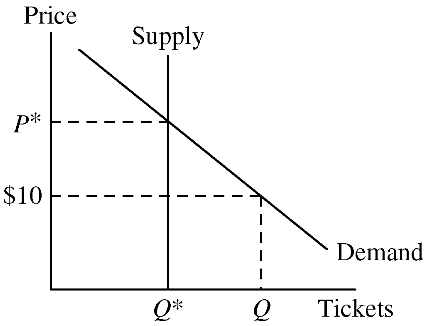
True. As the price of a good falls, the budget line pivots outward, and the consumer is able to move to a higher indifference curve.

c. Engel curves always slope upward.

False. If the good is an inferior good, quantity demanded decreases as income increases, and therefore the Engel curve slopes downward.

4. Tickets to a rock concert sell for $10. But at that price, the demand is substantially greater than the available number of tickets. Is the value or marginal benefit of an additional ticket greater than, less than, or equal to $10? How might you determine that value?

The diagram below shows this situation. At a price of $10, consumers want to purchase *Q* tickets, but only *Q** are available. Consumers would be willing to bid up the ticket price to *P**, where the quantity demanded equals the number of tickets available. Since utility-maximizing consumers are willing to pay more than $10, the marginal increase in satisfaction (i.e., the value or marginal benefit of an additional ticket) is greater than $10. One way to determine the value of an additional ticket would be to auction it off. Another possibility is to allow scalping. Since consumers are willing to pay an amount equal to the marginal benefit they derive from purchasing an additional ticket, the scalper's price equals that value.



5. Which of the following combinations of goods are complements and which are substitutes? Can they be either in different circumstances? Discuss.

a. a mathematics class and an economics class

If the math class and the economics class do not conflict in scheduling, then the classes could be either complements or substitutes. Math is important for understanding economics, and economics can motivate mathematics, so the classes could be complements. If the classes conflict or the student has room for only one in his schedule, they are substitutes.

b. tennis balls and a tennis racket

Tennis balls and a tennis racket are both needed to play tennis, thus they are complements.

c. steak and lobster

Foods can both complement and substitute for each other. Steak and lobster can be substitutes, as when they are listed as separate items on a menu. However, they can also function as complements because they are often served together.

d. a plane trip and a train trip to the same destination

Two modes of transportation between the same two points are substitutes for one another.

e. bacon and eggs

Bacon and eggs are often eaten together and are complementary goods in that case. However, in relation to something else, such as pancakes, bacon and eggs can function as substitutes.

6. Suppose that a consumer spends a fixed amount of income per month on the following pairs of goods:

a. tortilla chips and salsa

b. tortilla chips and potato chips

c. movie tickets and gourmet coffee

d. travel by bus and travel by subway

If the price of one of the goods increases, explain the effect on the quantity demanded of each of the goods. In each pair, which are likely to be complements and which are likely to be substitutes?

a. If the price of tortilla chips increases, the consumer will demand fewer tortilla chips. Since tortilla chips and salsa are complements, the demand for salsa will drop (the demand curve will shift to the left), and the consumer will demand less salsa.

b. If the price of tortilla chips increases, the consumer will demand fewer tortilla chips. Since tortilla chips and potato chips are substitutes, the demand for potato chips will increase (the demand curve will shift to the right), and the consumer will demand more potato chips.

c. The consumer will demand fewer movies if the price of tickets increases. You might think the demands for movies and gourmet coffee would be independent of each other. However, because the consumer spends a fixed amount on the two, the demand for coffee will depend on whether the consumer spends more or less of her fixed budget on movies after the price increase. If the consumer's demand elasticity for movie tickets is elastic, she will spend less on movies, and therefore more of her fixed income will be available to spend on coffee. In this case, her demand for coffee increases, and she buys more gourmet coffee. The goods are substitutes in this situation. If her demand for movies is inelastic, however, she will spend more on movies after the price increase, and therefore less on coffee. In this case, she will buy less of both goods in response to the price increase for movies, so the goods are complements. Finally, if her demand for movies is unit elastic, she will spend the same amount on movies and therefore will not change her spending on coffee. In this case, the goods are unrelated, and the demand curve for coffee is unchanged.

d. If the price of bus travel increases, the amount of bus travel demanded will fall, and the demand for subway rides will rise, because travel by bus and subway are typically substitutes. The demand curve for subway rides will shift to the right.

7. Which of the following events would cause a movement *along* the demand curve for U.S. produced clothing, and which would cause a *shift* in the demand curve?

a. the removal of quotas on the importation of foreign clothes

The removal of quotas will allow U.S. consumers to buy more foreign clothing. Because foreign produced goods are substitutes for domestically produced goods, the removal of quotas will result in a decrease in demand (a shift to the left) for U.S. produced clothes. There could be a smaller secondary effect also. When the quotas are removed, the total supply (foreign plus domestic) of clothing will increase, causing clothing prices to fall. The drop in clothing prices will lead consumers to buy more U.S. produced clothing, which is a movement along the demand curve.

b. an increase in the income of U.S. citizens

When income rises, expenditures on normal goods such as clothing increase, causing the demand curve to shift out to the right.

c. a cut in the industry's costs of producing domestic clothes that is passed on to the market in the form of lower prices

A cut in an industry's costs will shift the supply curve out. The equilibrium price will fall and quantity demanded will increase. This is a movement along the demand curve.

8. For which of the following goods is a price increase likely to lead to a substantial income (as well as substitution) effect?

a. salt

Small income effect, small substitution effect: The amount of income that is spent on salt is very small, so the income effect is small. Because there are few substitutes for salt, consumers will not readily substitute away from it, and the substitution effect is therefore small.

b. housing

Large income effect, small substitution effect: The amount of income spent on housing is relatively large for most consumers. If the price of housing rises, real income is reduced substantially, leading to a large income effect. However, there are no really close substitutes for housing, so the substitution effect is small.

c. theater tickets

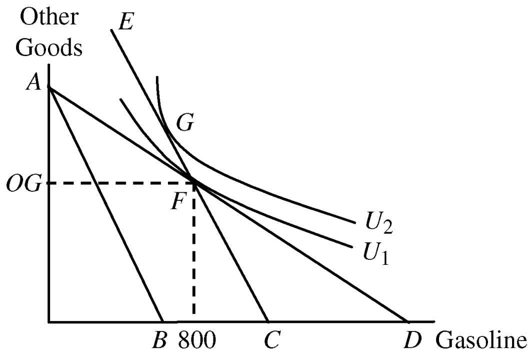
Small income effect, large substitution effect: The amount of income spent on theater tickets is usually relatively small, so the income effect is small. The substitution effect is large because there are many good substitutes such as movies, TV shows, bowling, dancing and other forms of entertainment.

d. food

Large income effect, virtually no substitution effect: As with housing, the amount of income spent on food is relatively large for most consumers, so the income effect is large. Although consumers can substitute out of particular foods, they cannot substitute out of food in general, so the substitution effect is essentially zero.

9. Suppose that the average household in a state consumes 800 gallons of gasoline per year. A 20-cent gasoline tax is introduced, coupled with a $160 annual tax rebate per household. Will the household be better or worse off under the new program?

If the household does not change its consumption of gasoline, it will be unaffected by the tax-rebate program, because the household pays ($0.20)(800) = $160 in taxes and receives $160 as an annual tax rebate. The two effects cancel each other out. However, the utility maximization model predicts that the household will not continue to purchase 800 gallons of gasoline but rather will reduce its gasoline consumption because of the substitution effect. As a result, it will be better off after the tax and rebate program. The diagram shows this situation. The original budget line is *AD*, and the household maximizes its utility at point F where the budget line is tangent to indifference curve U_1. At *F*, the household consumes 800 gallons of gasoline and *OG* of other goods. The 20-cent increase in price brought about by the tax pivots the budget line to *AB* (which is exaggerated to make the diagram clearer). Then the $160 rebate shifts the budget line out in a parallel fashion to *EC* where the household is again able to purchase its original bundle of goods containing 800 gallons of gasoline. However, the new budget line intersects indifference curve U_1 and is not tangent to it. Therefore, point *F* cannot be the new utility maximizing bundle of goods. The new budget line is tangent to a higher indifference curve, U_2 at point *G*. Point *G* is therefore the new utility maximizing bundle, and the household consumes less gasoline (because *G* is to the left of *F*) and is better off because it is on a higher indifference curve.



10. Which of the following three groups is likely to have the most, and which the least, price-elastic demand for membership in the Association of Business Economists?

a. students

The major differences among the groups are the level of income and commitment to a career in business economics. We know that demand will be more price-elastic (all else equal) if a good's consumption constitutes a large percentage of an individual's income, because the income effect will be large. Also demand is less elastic the more the good is seen as a necessity. For students, membership in the Association is likely to represent a larger percentage of income than for the other two groups, and students are less likely to see membership as critical for their success. Thus, their demand will be the most price-elastic.

b. junior executives

The level of income for junior executives will be larger than for students but smaller than for senior executives. They will see membership as important but perhaps not as important as for senior executives. Therefore, their demand will be less price-elastic than students but more elastic than senior executives.

c. senior executives

The high earnings among senior executives and the high importance they place on membership will result in the least elastic demand for membership.

11. Explain which of the following items in each pair is more price elastic.

a. The demand for a specific brand of toothpaste and the demand for toothpaste in general

The demand for a specific brand is more elastic because the consumer can easily switch to another brand if the price goes up. It is not so easy to switch to a different tooth brushing agent (baking soda?).

b. The demand for gasoline in the short run and the demand for gasoline in the long run

Demand in the long run is more elastic since consumers have more time to adjust to a change in price. For example, consumers can buy more fuel efficient vehicles, move closer to work or school, organize car pools, etc.

12. Explain the difference between a positive and a negative network externality and give an example of each.

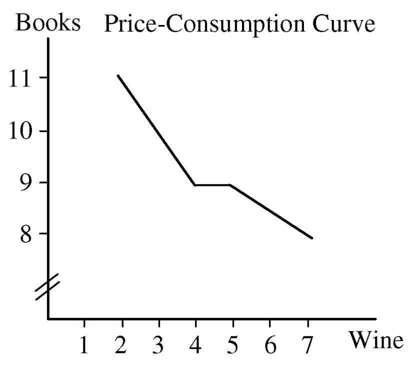
A positive network externality exists if one individual's demand increases in response to the purchase of the good by other consumers. Fads are an example of a positive network externality. For example, each individual's demand for baggy pants increases as more other individuals begin to wear baggy pants. This is also called a bandwagon effect. Another example of a positive network externality occurs with communications equipment such as telephones. A telephone is more desirable when there are a large number of other phone owners to whom one can talk. A negative network externality exists if the quantity demanded by one individual decreases in response to the purchase of the good by other consumers. In this case the individual prefers to be different from other individuals. As more people adopt a particular style or purchase a particular type of good, this individual will reduce his demand for the good. Goods like designer clothing can have negative network externalities, as some people would not want to wear the same clothes that many other people are wearing. This is also known as the snob effect. Another example of a negative network externality is road congestion. As more people use a road, the more congested it becomes, and the less valuable it is to each driver. Some people will drive on the road less often (i.e., demand less road services) when it becomes overly congested.

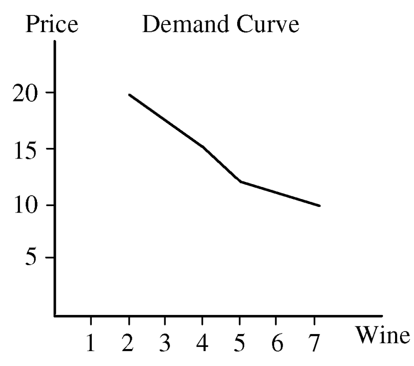
■ Exercises

1. An individual sets aside a certain amount of his income per month to spend on his two hobbies, collecting wine and collecting books. Given the information below, illustrate both the price-consumption curve associated with changes in the price of wine and the demand curve for wine.

Price Wine	Price Book	Quantity Wine	Quantity Book	Budget
$10	**$10**	**7**	**8**	**$150**
$12	**$10**	**5**	**9**	**$150**
$15	**$10**	**4**	**9**	**$150**
$20	**$10**	**2**	**11**	**$150**

The price-consumption curve connects each of the four optimal bundles given in the table, while the demand curve plots the optimal quantity of wine against the price of wine in each of the four cases. See the diagrams below.

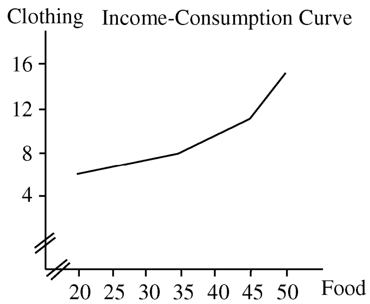


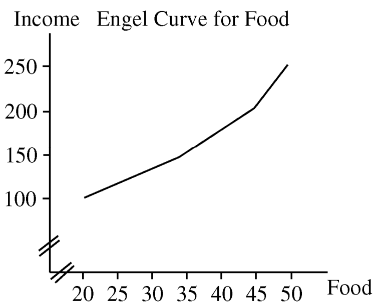


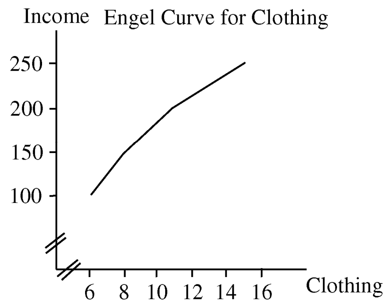
2. **An individual consumes two goods, clothing and food. Given the information below, illustrate both the income-consumption curve and the Engel curve for clothing and food.**

Price Clothing	Price Food	Quantity Clothing	Quantity Food	Income
$10	$2	6	20	$100
$10	$2	8	35	$150
$10	$2	11	45	$200
$10	$2	15	50	$250

The income-consumption curve connects each of the four optimal bundles given in the table above. As the individual's income increases, the budget line shifts out and the optimal bundles change. The Engel curve for each good illustrates the relationship between the quantity consumed and income (on the vertical axis). Both Engel curves are upward sloping, so both goods are normal.





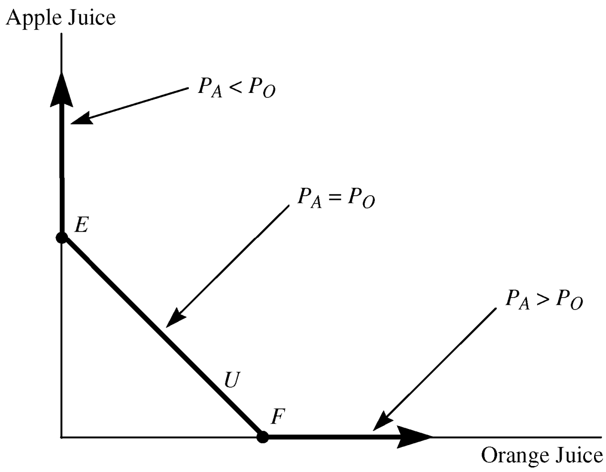


3. **Jane always gets twice as much utility from an extra ballet ticket as she does from an extra basketball ticket, regardless of how many tickets of either type she has. Draw Jane's income-consumption curve and her Engel curve for ballet tickets.**

Ballet tickets and basketball tickets are perfect substitutes for Jane. Therefore, she will consume either all ballet tickets or all basketball tickets, depending on the two prices. As long as ballet tickets are less than twice the price of basketball tickets, she will choose all ballet. If ballet tickets are more than twice the price of basketball tickets, she will choose all basketball. This can be determined by comparing the marginal utility per dollar for each type of ticket, where her marginal utility from another ballet ticket is 2 times her marginal utility from another basketball ticket regardless of the number of tickets she has. Her income-consumption curve will then lie along the axis of the good that she chooses. As income increases and the budget line shifts out, she will buy more of the chosen good and none of the other good. Her Engel curve for the good chosen is an upward-sloping straight line, with the number of tickets equal to her income divided by the price of the ticket. For the good not chosen, her Engel curve lies on the vertical (income) axis because she will never purchase any of those tickets regardless of how large her income becomes.

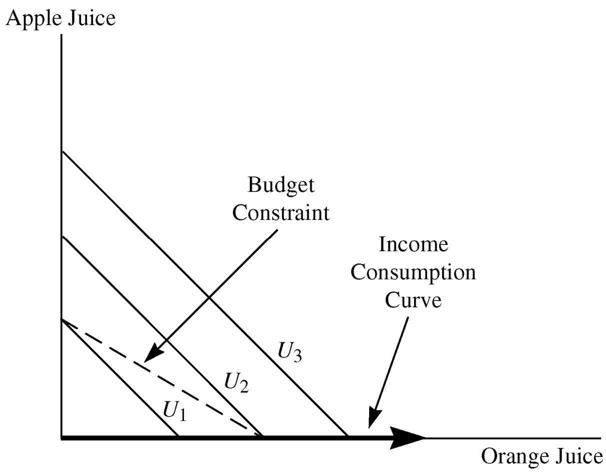
4. a. Orange juice and apple juice are known to be perfect substitutes. Draw the appropriate price-consumption curve (for a variable price of orange juice) and income-consumption curve.

We know that indifference curves for perfect substitutes are straight lines like the line *EF* in the price-consumption curve diagram below. In this case, the consumer always purchases the cheaper of the two goods (assuming a one-for-one tradeoff). If the price of orange juice is less than the price of apple juice, the consumer will purchase only orange juice and the price-consumption curve will lie along the orange juice axis of the graph (from point *F* to the right).



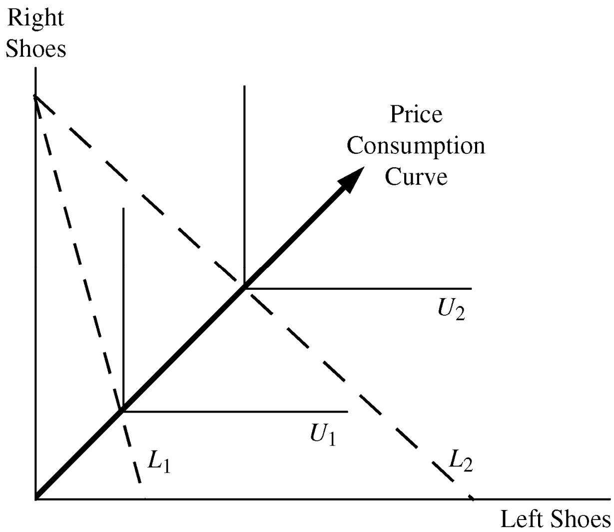
If apple juice is cheaper, the consumer will purchase only apple juice and the price-consumption curve will be on the apple juice axis (above point *E*). If the two goods have the same price, the consumer will be indifferent between the two; the price-consumption curve will coincide with the indifference curve (between *E* and *F*).

Assuming that the price of orange juice is less than the price of apple juice, the consumer will maximize her utility by consuming only orange juice. As income varies, only the amount of orange juice varies. Thus, the income-consumption curve will be along the orange juice axis as in the figure below. If apple juice were cheaper, the income-consumption curve would lie on the apple juice axis.

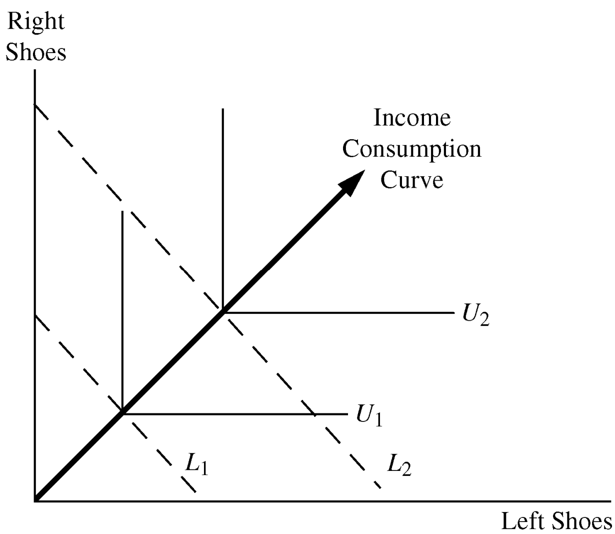


b. Left shoes and right shoes are perfect complements. Draw the appropriate price-consumption and income-consumption curves.

For perfect complements, such as right shoes and left shoes, the indifference curves are L-shaped. The point of utility maximization occurs when the budget constraints, L_1 and L_2 touch the kink of U_1 and U_2. See the following figure.



In the case of perfect complements, the income consumption curve is also a line through the corners of the L-shaped indifference curves. See the figure below.



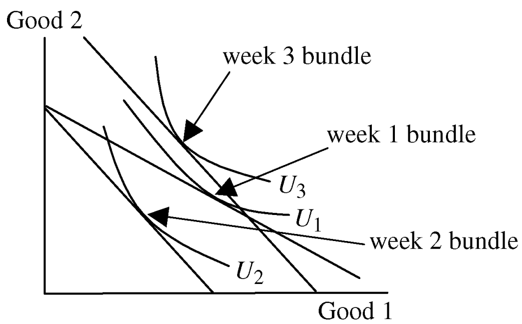
5. Each week, Bill, Mary, and Jane select the quantity of two goods, x_1 and x_2, that they will consume in order to maximize their respective utilities. They each spend their entire weekly income on these two goods.

a. Suppose you are given the following information about the choices that Bill makes over a three-week period:

	x_1	x_2	P_1	P_2	I
Week 1	**10**	**20**	**2**	**1**	**40**
Week 2	**7**	**19**	**3**	**1**	**40**
Week 3	**8**	**31**	**3**	**1**	**55**

Did Bill's utility increase or decrease between week 1 and week 2? Between week 1 and week 3? Explain using a graph to support your answer.

Bill's utility fell between weeks 1 and 2 because he consumed less of both goods in week 2. Between weeks 1 and 2 the price of good 1 rose and his income remained constant. The budget line pivoted inward and he moved from U_1 to a lower indifference curve, U_2, as shown in the diagram. Between week 1 and week 3 his utility rose. The increase in income more than compensated him for the rise in the price of good 1. Since the price of good 1 rose by $1, he would need an extra $10 to afford the same bundle of goods he chose in week 1. This can be found by multiplying week 1 quantities times week 2 prices. However, his income went up by $15, so his budget line shifted out beyond his week 1 bundle. Therefore, his original bundle lies within his new budget set as shown in the diagram, and his new week 3 bundle is on the higher indifference curve U_3.

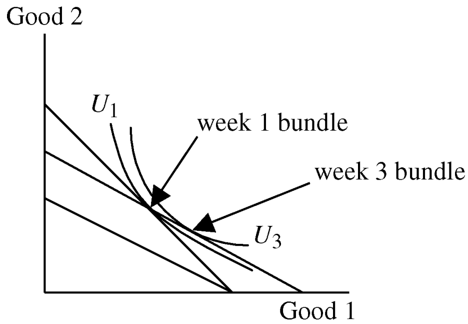


b. Now consider the following information about the choices that Mary makes:

	x_1	x_2	P_1	P_2	I
Week 1	**10**	**20**	**2**	**1**	**40**
Week 2	**6**	**14**	**2**	**2**	**40**
Week 3	**20**	**10**	**2**	**2**	**60**

Did Mary's utility increase or decrease between week 1 and week 3? Does Mary consider both goods to be normal goods? Explain.

Mary's utility went up. To afford the week 1 bundle at the new prices, she would need an extra $20, which is exactly what happened to her income. However, since she could have chosen the original bundle at the new prices and income but did not, she must have found a bundle that left her slightly better off. In the graph to the right, the week 1 bundle is at the point where the week 1 budget line is tangent to indifference curve U_1, which is also the intersection of the week 1 and week 3 budget lines. The week 3 bundle is somewhere on the week 3 budget line that lies above the week 1 indifference curve. This bundle will be on a higher indifference curve, U_3 in the graph, and hence Mary's utility increased. A good is normal if more is chosen when income increases. Good 1 is normal because Mary consumed more of it when her income increased (and prices remained constant) between weeks 2 and 3. Good 2 is not normal, however, because when Mary's income increased from week 2 to week 3 (holding prices the same), she consumed less of good 2. Thus good 2 is an inferior good for Mary.

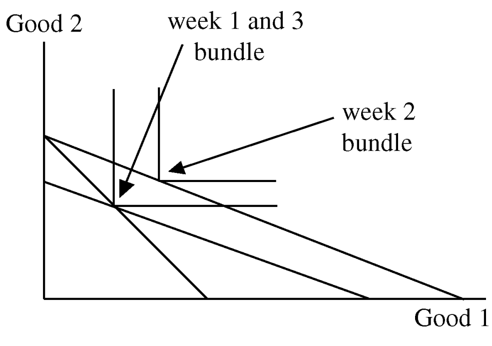


c. Finally, examine the following information about Jane's choices:

	x_1	x_2	P_1	P_2	I
Week 1	**12**	**24**	**2**	**1**	**48**
Week 2	**16**	**32**	**1**	**1**	**48**
Week 3	**12**	**24**	**1**	**1**	**36**

Draw a budget line-indifference curve graph that illustrates Jane's three chosen bundles. What can you say about Jane's preferences in this case? Identify the income and substitution effects that result from a change in the price of good x_1.

In week 2, the price of good 1 drops, Jane's budget line pivots outward, and she consumes more of both goods. In week 3 the prices remain at the new levels, but Jane's income is reduced. This leads to a parallel leftward shift of her budget line and causes Jane to consume less of both goods. Notice that Jane always consumes the two goods in a fixed 1:2 ratio. This means that Jane views the two goods as perfect complements, and her indifference curves are L-shaped. Intuitively, if the two goods are complements, there is no reason to substitute one for the other during a price change, because they have to be consumed in a set ratio. Thus the substitution effect is zero. When the price ratio changes and utility is kept at the same level (as happens between weeks 1 and 3), Jane chooses the same bundle (12, 24), so the substitution effect is zero.



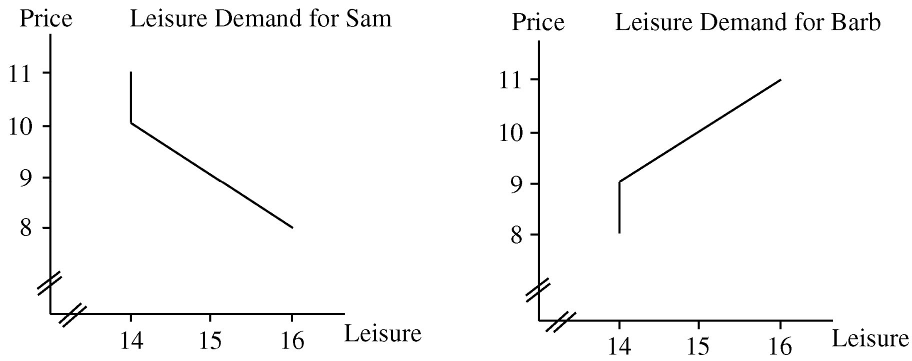
The income effect can be deduced from the changes between weeks 1 and 2 and also between weeks 2 and 3. Between weeks 2 and 3 the only change is the \$12 drop in income. This causes Jane to buy 4 fewer units of good 1 and 8 less units of good 2. Because prices did not change, this is purely an income effect. Between weeks 1 and 2, the price of good 1 decreased by \$1 and income remained the same. Since Jane bought 12 units of good 1 in week 1, the drop in price increased her purchasing power by (\$1)(12) = \$12. As a result of this \$12 increase in real income, Jane bought 4 more units of good 1 and 8 more of good 2. We know there is no substitution effect, so these changes are due solely to the income effect, which is the same (but in the opposite direction) as we observed between weeks 1 and 2.

6. Two individuals, Sam and Barb, derive utility from the hours of leisure (*L*) they consume and from the amount of goods (*G*) they consume. In order to maximize utility, they need to allocate the 24 hours in the day between leisure hours and work hours. Assume that all hours not spent working are leisure hours. The price of a good is equal to \$1 and the price of leisure is equal to the hourly wage. We observe the following information about the choices that the two individuals make:

Price of *G*	Price of *L*	Sam *L* (hours)	Barb *L* (hours)	Sam *G* (\$)	Barb *G* (\$)
1	8	16	14	64	80
1	9	15	14	81	90
1	10	14	15	100	90
1	11	14	16	110	88

Graphically illustrate Sam's leisure demand curve and Barb's leisure demand curve. Place price on the vertical axis and leisure on the horizontal axis. Given that they both maximize utility, how can you explain the difference in their leisure demand curves?

It is important to remember that less leisure implies more hours spent working. Sam's leisure demand curve is downward sloping. As the price of leisure (the wage) rises, he chooses to consume less leisure and thus spend more time working at a higher wage to buy more goods. Barb's leisure demand curve is upward sloping. As the price of leisure rises, she chooses to consume more leisure (and work less) since her working hours are generating more income per hour. See the leisure demand curves below.



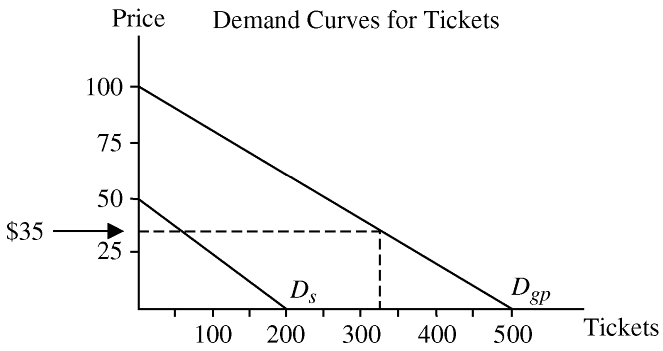
This difference in demand can be explained by examining the income and substitution effects for the two individuals. The substitution effect measures the effect of a change in the price of leisure, keeping utility constant (the budget line rotates along the current indifference curve). Since the substitution effect is always negative, a rise in the price of leisure will cause both individuals to consume less leisure. The income effect measures the effect of the change in purchasing power brought about by the change in the price of leisure. Here, when the price of leisure (the wage) rises, there is an increase in purchasing power (the new budget line shifts outward). Assuming both individuals consider leisure to be a normal good, the increase in purchasing power will increase demand for leisure. For Sam, the reduction in leisure demand caused by the substitution effect outweighs the increase in demand for leisure caused by the income effect, so his leisure demand curve slopes downward. For Barb, her income effect is larger than her substitution effect, so her leisure demand curve slopes upward.

7. The director of a theater company in a small college town is considering changing the way he prices tickets. He has hired an economic consulting firm to estimate the demand for tickets. The firm has classified people who go to the theater into two groups, and has come up with two demand functions. The demand curves for the general public (Q_{gp}) and students (Q_s) are given below:

$$Q_{gp} = 500 - 5P$$
$$Q_s = 200 - 4P$$

a. Graph the two demand curves on one graph, with *P* on the vertical axis and *Q* on the horizontal axis. If the current price of tickets is \$35, identify the quantity demanded by each group.

Both demand curves are downward sloping and linear. For the general public, D_{gp}, the vertical intercept is 100 and the horizontal intercept is 500. For the students, D_s, the vertical intercept is 50 and the horizontal intercept is 200. When the price is \$35, the general public demands $Q_{gp} = 500 - 5(35) = 325$ tickets and students demand $Q_s = 200 - 4(35) = 60$ tickets.



b. Find the price elasticity of demand for each group at the current price and quantity.

The elasticity for the general public is $\varepsilon_{gp} = \dfrac{P}{Q}\dfrac{\Delta Q}{\Delta P} = \dfrac{35}{325}(-5) = -0.54$, and the elasticity for students is $\varepsilon_S = \dfrac{P}{Q}\dfrac{\Delta Q}{\Delta P} = \dfrac{35}{60}(-4) = -2.33$. If the price of tickets increases by 10% then the general public will demand 5.4% fewer tickets and students will demand 23.3% fewer tickets.

c. Is the director maximizing the revenue he collects from ticket sales by charging \$35 for each ticket? Explain.

No, he is not maximizing revenue because neither of the calculated elasticities is equal to −1. The general public's demand is inelastic at the current price. Thus the director could increase the price for the general public, and the quantity demanded would fall by a smaller percentage, causing revenue to increase. Since the students' demand is elastic at the current price, the director could decrease the price students pay, and their quantity demanded would increase by a larger amount in percentage terms, causing revenue to increase.

d. What price should he charge each group if he wants to maximize revenue collected from ticket sales?

To figure this out, use the formula for elasticity, set it equal to −1, and solve for price and quantity. For the general public:

$$\begin{aligned}\varepsilon_{gp} &= \frac{-5P}{Q} = -1\\ 5P &= Q = 500 - 5P\\ P &= 50\\ Q &= 250.\end{aligned}$$

For the students:

$$\begin{aligned}\varepsilon_{s} &= \frac{-4P}{Q} = -1\\ 4P &= Q = 200 - 4P\\ P &= 25\\ Q &= 100.\end{aligned}$$

These prices generate a larger total revenue than the \$35 price. When price is \$35, revenue is $(35)(Q_{gp} + Q_s) = (35)(325 + 60) = \$13{,}475$. With the separate prices, revenue is $P_{gp}Q_{gp} + P_sQ_s = (50)(250) + (25)(100) = \$15{,}000$, which is an increase of \$1525, or 11.3%.

8. Judy has decided to allocate exactly \$500 to college textbooks every year, even though she knows that the prices are likely to increase by 5 to 10% per year and that she will be getting a substantial monetary gift from her grandparents next year. What is Judy's price elasticity of demand for textbooks? Income elasticity?

Judy will spend the same amount (\$500) on textbooks even when prices increase. We know that total revenue (i.e., total spending on a good) remains constant when price changes only if demand is unit elastic. Therefore Judy's price elasticity of demand for textbooks is −1. Her income elasticity must be zero because she does not plan to purchase more books even though she expects a large monetary gift (i.e., an increase in income).

9. The ACME Corporation determines that at current prices the demand for its computer chips has a price elasticity of −2 in the short run, while the price elasticity for its disk drives is −1.

a. If the corporation decides to raise the price of both products by 10%, what will happen to its sales? To its sales revenue?

We know the formula for the elasticity of demand is:

$$E_P = \frac{\%\Delta Q}{\%\Delta P}.$$

For computer chips, $E_P = -2$, so $-2 = \%\Delta Q/10$, and therefore $\%\Delta Q = -2(10) = -20$. Thus a 10% increase in price will reduce the quantity sold by 20%. For disk drives, $E_P = -1$, so a 10% increase in price will reduce sales by 10%.

Sales revenue will decrease for computer chips because demand is elastic and price has increased. We can estimate the change in revenue as follows. Revenue is equal to price times quantity sold. Let $TR_1 = P_1Q_1$ be revenue before the price change and $TR_2 = P_2Q_2$ be revenue after the price change. Therefore

$\Delta TR = P_2Q_2 - P_1Q_1$

$\Delta TR = (1.1P_1)(0.8Q_1) - P_1Q_1 = -0.12P_1Q_1$, or a 12% decline.

Sales revenue for disk drives will remain unchanged because demand elasticity is −1.

b. Can you tell from the available information which product will generate the most revenue? If yes, why? If not, what additional information do you need?

No. Although we know the elasticities of demand, we do not know the prices or quantities sold, so we cannot calculate the revenue for either product. We need to know the prices of chips and disk drives and how many of each ACME sells.

10. By observing an individual's behavior in the situations outlined below, determine the relevant income elasticities of demand for each good (i.e., whether it is normal or inferior). If you cannot determine the income elasticity, what additional information do you need?

a. Bill spends all his income on books and coffee. He finds $20 while rummaging through a used paperback bin at the bookstore. He immediately buys a new hardcover book of poetry.

Books are a normal good since his consumption of books increases with income. Coffee is a neutral good since consumption of coffee stayed the same when income increased.

b. Bill loses $10 he was going to use to buy a double espresso. He decides to sell his new book at a discount to a friend and use the money to buy coffee.

When Bill's income decreased by $10 he decided to own fewer books, so books are a normal good. Coffee appears to be a neutral good because Bill's purchase of the double espresso did not change as his income changed.

c. Being bohemian becomes the latest teen fad. As a result, coffee and book prices rise by 25%. Bill lowers his consumption of both goods by the same percentage.

Books and coffee are both normal goods because Bill's response to a decline in real income is to decrease consumption of both goods. In addition, the income elasticities for both goods are the same because Bill reduces consumption of both by the same percentage.

d. Bill drops out of art school and gets an M.B.A. instead. He stops reading books and drinking coffee. Now he reads *The Wall Street Journal* and drinks bottled mineral water.

His tastes have changed completely, and we do not know how he would respond to price and income changes. We need to observe how his consumption of the *WSJ* and bottled water change as his income changes.

11. Suppose the income elasticity of demand for food is 0.5 and the price elasticity of demand is –1.0. Suppose also that Felicia spends $10,000 a year on food, the price of food is $2, and that her income is $25,000.

a. If a sales tax on food caused the price of food to increase to $2.50, what would happen to her consumption of food? (*Hint*: Because a large price change is involved, you should assume that the price elasticity measures an arc elasticity, rather than a point elasticity.)

The arc elasticity formula is:

$$E_P = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{(P_1 + P_2)/2}{(Q_1 + Q_2)/2}\right).$$

We know that $E_P = -1$, $P_1 = 2$, $P_2 = 2.50$ (so $\Delta P = 0.50$), and $Q_1 = 5000$ units (because Felicia spends \$10,000 and each unit of food costs \$2). We also know that Q_2, the new quantity, is $Q_2 = Q_1 + \Delta Q$. Thus, if there is no change in income, we may solve for ΔQ:

$$-1 = \left(\frac{\Delta Q}{0.5}\right)\left(\frac{(2+2.5)/2}{(5000+(5000+\Delta Q))/2}\right).$$

By cross-multiplying and rearranging terms, we find that $\Delta Q = -1000$. This means that she decreases her consumption of food from 5000 to 4000 units. As a check, recall that total spending should remain the same because the price elasticity is -1. After the price change, Felicia spends (\$2.50)(4000) = \$10,000, which is the same as she spent before the price change.

b. Suppose that Felicia gets a tax rebate of \$2500 to ease the effect of the sales tax. What would her consumption of food be now?

A tax rebate of \$2500 is an income increase of \$2500. To calculate the response of demand to the tax rebate, use the definition of the arc elasticity of income.

$$E_I = \left(\frac{\Delta Q}{\Delta I}\right)\left(\frac{(I_1+I_2)/2}{(Q_1+Q_2)/2}\right).$$

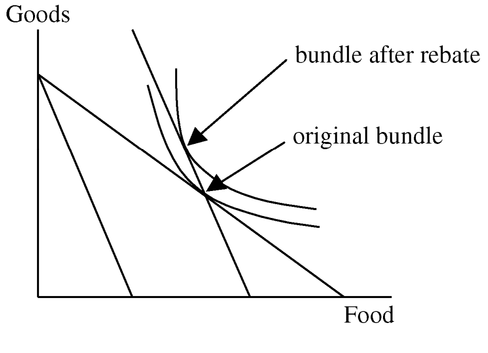
We know that $E_1 = 0.5$, $I_1 = 25{,}000$, $\Delta I = 2500$ (so $I_2 = 27{,}500$), and $Q_1 = 4000$ (from the answer to 11a). Assuming no change in price, we solve for ΔQ.

$$0.5 = \left(\frac{\Delta Q}{2500}\right)\left(\frac{(25{,}000+27{,}500)/2}{(4000+(4000+\Delta Q))/2}\right).$$

By cross-multiplying and rearranging terms, we find that $\Delta Q = 195$ (approximately). This means that she increases her consumption of food from 4000 to 4195 units.

c. Is she better or worse off when given a rebate equal to the sales tax payments? Draw a graph and explain.

Felicia is better off after the rebate. The amount of the rebate is enough to allow her to purchase her original bundle of food and other goods. Recall that originally she consumed 5000 units of food. When the price went up by fifty cents per unit, she needed an extra (5000)(\$0.50) = \$2500 to afford the same quantity of food without reducing the quantity of the other goods consumed. This is the exact amount of the rebate. However, she did not choose to return to her original bundle. We can therefore infer that she found a better bundle that gave her a higher level of utility. In the graph below, when the price of food increases, the budget line pivots inward. When the rebate is given, this new budget line shifts out to the right in a parallel fashion. The bundle after the rebate is on that part of the new budget line that was previously unaffordable, and that lies above the original indifference curve. It is on a higher indifference curve, so Felicia is better off after the rebate.



12. You run a small business and would like to predict what will happen to the quantity demanded for your product if you raise your price. While you do not know the exact demand curve for your product, you do know that in the first year you charged \$45 and sold 1200 units and that in the second year you charged \$30 and sold 1800 units.

a. If you plan to raise your price by 10%, what would be a reasonable estimate of what will happen to quantity demanded in percentage terms?

We must first find the price elasticity of demand. Because the price and quantity changes are large in percentage terms, it is best to use the arc elasticity measure. $E_P = (\Delta Q/\Delta P) \times$ (average P/average Q) = (600/−15) × (37.50/1500) = −1. With an elasticity of −1, a 10% increase in price will lead to a 10% decrease in quantity.

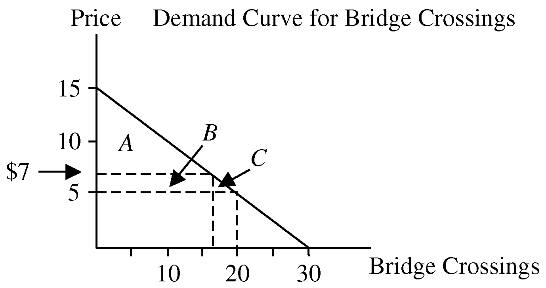
b. If you raise your price by 10%, will revenue increase or decrease?

When elasticity is −1, revenue will remain constant if price is increased.

13. Suppose you are in charge of a toll bridge that costs essentially nothing to operate. The demand for bridge crossings Q is given by $P = 15 - \frac{1}{2}Q$.

a. Draw the demand curve for bridge crossings.

The demand curve is linear and downward sloping. The vertical intercept is 15 and the horizontal intercept is 30.



b. How many people would cross the bridge if there were no toll?

At a price of zero, 0 = 15 − (1/2)Q, so Q = 30. The quantity demanded would be 30.

c. What is the loss of consumer surplus associated with a bridge toll of \$5?

If the toll is \$5 then the quantity demanded is 20. The lost consumer surplus is the difference between the consumer surplus when price is zero and the consumer surplus when price is \$5. When the toll is zero, consumer surplus is the entire area under the demand curve, which is (1/2)(30)(15) = 225. When P = 5, consumer surplus is area $A + B + C$ in the graph above. The base of this triangle is 20 and the height is 10, so consumer surplus = (1/2)(20)(10) = 100. The loss of consumer surplus is therefore \$225 − 100 = \$125.

d. The toll-bridge operator is considering an increase in the toll to \$7. At this higher price, how many people would cross the bridge? Would the toll-bridge revenue increase or decrease? What does your answer tell you about the elasticity of demand?

At a toll of \$7, the quantity demanded would be 16. The initial toll revenue was \$5(20) = \$100. The new toll revenue is \$7(16) = \$112, so revenue increases by \$12. Since the revenue goes up when the toll is increased, demand is inelastic (the 40% increase in price outweighs the 20% decline in quantity demanded).

e. Find the lost consumer surplus associated with the increase in the price of the toll from \$5 to \$7.

The lost consumer surplus is area $B + C$ in the graph above. Thus, the loss in consumer surplus is $(16) \times (7 - 5) + (1/2) \times (20 - 16) \times (7 - 5) = \$36$.

14. Vera has decided to upgrade the operating system on her new PC. She hears that the new Linux operating system is technologically superior to Windows and substantially lower in price. However, when she asks her friends, it turns out they all use PCs with Windows. They agree that Linux is more appealing but add that they see relatively few copies of Linux on sale at local stores. Vera chooses Windows. Can you explain her decision?

Vera is influenced by a positive network externality (not a bandwagon effect). When she hears that there are limited software choices that are compatible with Linux and that none of her friends use Linux, she decides to go with Windows. If she had not been interested in acquiring much software and did not think she would need to get advice from her friends, she might have purchased Linux.

15. Suppose that you are the consultant to an agricultural cooperative that is deciding whether members should cut their production of cotton in half next year. The cooperative wants your advice as to whether this action will increase members' revenues. Knowing that cotton (*C*) and soybeans (*S*) both compete for agricultural land in the South, you estimate the demand for cotton to be $C = 3.5 - 1.0P_c + 0.25P_s + 0.50I$, where P_c is the price of cotton, P_s the price of soybeans, and *I* income. Should you support or oppose the plan? Is there any additional information that would help you to provide a definitive answer?

If production of cotton is cut in half, then the price of cotton will increase, given that the equation above shows that demand is downward sloping (since the sign on P_c is negative). With price increasing and quantity demanded decreasing, revenue could go either way. It depends on whether demand is elastic or inelastic. If demand is elastic, a decrease in production and an increase in price would decrease revenue. If demand is inelastic, a decrease in production and an increase in price would increase revenue. You need a lot of information before you can give a definitive answer. First, you must know the current prices for cotton and soybeans plus the level of income; then you can calculate the quantity of cotton demanded, *C*. Next, you have to cut *C* in half and determine the effect that will have on the price of cotton, assuming that income and the price of soybeans are not affected (which is a big assumption). Then you can calculate the original revenue and the new revenue to see whether this action increases members' revenues or not.

Chapter 5
Uncertainty and Consumer Behavior

■ Teaching Notes

This is a useful chapter for business-oriented courses, particularly if you intend to cover the role of risk in capital markets in Chapter 15. It is also good background for Chapter 17 on asymmetric information. On the other hand, you will not able to cover everything in the book, so if there are other topics you wish to cover instead, you may skip this chapter without disrupting the overall flow of the text.

You might start by asking students how uncertainty affects the decisions made by consumers and firms. Consumers may not know their incomes for sure, they are uncertain about the quality of some of the goods they buy (so it is difficult to know the utility they will receive from purchasing those goods); business firms are uncertain about the demands for their products, the future costs of inputs, and the exchange rate for foreign currencies; investors don't know whether their investments will increase or decrease in value; etc. Then ask how people deal with these risks. For example, there are many types of insurance including auto, life, and unemployment; companies offer product warranties and refunds, farmers and other firms can hedge against price uncertainty in futures markets, businesses can hedge foreign exchange risks in forward markets, and investors can diversity.

If students have not previously been exposed to probability, expected value, and variance, the basics are covered in Section 5.1. However, this is a fast run-through even for those who have had a basic statistics course, so you may find Exercises 1 through 5 useful as they provide practice calculating expected value and variance. Many students think of risk as arising from the possibility of loss or injury; they do not consider that risk can also be due to uncertain gains. It is easy to construct simple examples where there are only gains to make the point that risk still exists. For example, with alternative 1 you get nothing if a coin comes up heads and $1000 if it comes up tails. With alternative 2, you get $300 for sure. Alternative 1 is clearly more risky. You can mention that variance (or standard deviation), which is often used as a measure of riskiness, takes into account both gains and losses.

Preferences toward risk depend on the decision maker's von Neumann-Morgenstern utility function, which is different from the utility functions in Chapters 3 and 4. Utility in this chapter has some cardinal properties and depends on the monetary payoff to the decision maker, whereas utility in earlier chapters was ordinal and depended on the amount of various goods consumed. To emphasize this difference, utility is denoted by a lower case *u* in this chapter.

There are a number of issues that trip up students in Chapter 5. Most importantly, students often confuse expected utility and the utility of the expected value. Give them a couple of examples to make sure they understand the difference. For instance, if $u(x)=\sqrt{x}$, the expected utility for alternative 1 in the example above is $Eu = 0.5u(0) + 0.5u(1000) = 0.5\sqrt{0} + 0.5\sqrt{1000} = 15.81$. On the other hand, the expected value for alternative 1 is $E(x) = 0.5(0) + 0.5(1000) = 500$, and the utility of the expected value is therefore $u[E(x)] = \sqrt{500} = 22.36$, which is quite different from the expected utility.

Students also have difficulty understanding Figure 5.4 that illustrates the risk premium. They do not understand why the point on the chord (point *F*) represents expected utility. You will need to explain this carefully. You should also make sure students understand that even risk-averse people take risks. Being risk averse does not mean avoiding all risks. Everyone takes risks when the possible rewards are greater than the costs. For instance, most people have parked illegally when they are in a hurry (or late for an exam). We all drive cars, risking accidents and injury, and many people buy stocks and bonds even though those investments may decrease in value. In fact, there really is no way to live and avoid all risks.

Even if your students have not fully understood the technical aspects of choice under uncertainty, they should easily comprehend Examples 5.1 and 5.2 (the latter example leads to Exercise 8). This is also true of the topics presented in Section 5.3, diversification, insurance, and value of information, and Examples 5.3 and 5.4. Also, you might mention the problems of adverse selection and moral hazard in insurance, to be discussed in Chapter 17.

Section 5.4 is more difficult and may be skipped or postponed until after the class has completed the discussion of risk and rates of return in Chapter 15. The last two sections discuss bubbles and behavioral economics. Most students find these sections quite interesting because they consider situations in which people do not behave as rationally as economists typically assume they do and because they cover a number of recent events and studies.

■ Questions for Review

1. **What does it mean to say that a person is *risk averse*? Why are some people likely to be risk averse while others are risk lovers?**

   A risk-averse person has a diminishing marginal utility of income and prefers a certain income to a gamble with the same expected income. A risk lover has an increasing marginal utility of income and prefers an uncertain income to a certain income when the expected value of the uncertain income equals the certain income. To some extent, a person's risk preferences are like preferences for different vegetables. They may be inborn or learned from parents or others, and we cannot easily say why some people are risk averse while others like taking risks. But there are some economic factors that can affect risk preferences. For example, a wealthy person is more likely to take risks than a moderately well-off person, because the wealthy person can better handle losses. Also, people are more likely to take risks when the stakes are low (like office pools around NCAA basketball time) than when stakes are high (like losing a house to fire).

2. **Why is the variance a better measure of variability than the range?**

   Range is the difference between the highest possible outcome and the lowest possible outcome. Range ignores all outcomes except the highest and lowest, and it does not consider how likely each outcome is. Variance, on the other hand, is based on all the outcomes and how likely they are to occur. Variance weights the difference of each outcome from the mean outcome by its probability, and thus is a more comprehensive measure of variability than the range.

3. **George has $5000 to invest in a mutual fund. The expected return on mutual fund *A* is 15% and the expected return on mutual fund *B* is 10%. Should George pick mutual fund *A* or fund *B*?**

   George's decision will depend not only on the expected return for each fund, but also on the variability of each fund's returns and on George's risk preferences. For example, if fund *A* has a higher standard deviation than fund *B*, and George is risk averse, then he may prefer fund *B* even though it has a lower expected return. If George is not particularly risk averse he may choose fund *A* even if its return is more variable.

4. What does it mean for consumers to maximize expected utility? Can you think of a case in which a person might *not* maximize expected utility?

To maximize expected utility means that the individual chooses the option that yields the highest average utility, where average utility is the probability-weighted sum of all utilities. This theory requires that the consumer knows each possible outcome that may occur and the probability of each outcome. Sometimes consumers either do not know all possible outcomes and the relevant probabilities, or they have difficulty evaluating low-probability, extreme-payoff events. In some cases, consumers cannot assign a utility level to these extreme-payoff events, such as when the payoff is the loss of the consumer's life. In cases like this, consumers may make choices based on other criteria such as risk avoidance.

5. Why do people often want to insure fully against uncertain situations even when the premium paid exceeds the expected value of the loss being insured against?

Risk averse people have declining marginal utility, and this means that the pain of a loss increases at an increasing rate as the size of the loss increases. As a result, they are willing to pay more than the expected value of the loss to insure against suffering the loss. For example, consider a homeowner who owns a house worth $200,000. Suppose there is a small 0.001 probability that the house will burn to the ground and be a total loss and a high probability of 0.999 that there will be no loss. The expected loss is 0.001(200,000) + 0.999(0) = $200. Many risk averse homeowners would be willing to pay a lot more than $200 (like $400 or $500) to buy insurance that will replace the house if it burns. They do this because the disutility of losing their $200,000 house is more than 1000 times larger than the disutility of paying the insurance premium.

6. Why is an insurance company likely to behave as if it were risk neutral even if its managers are risk-averse individuals?

A large insurance company sells hundreds of thousands of policies, and the company's managers know they will have to pay for losses incurred by some of their policyholders even though they do not know which particular policies will result in claims. Because of the law of large numbers, however, the company can estimate the total number of claims quite accurately. Therefore, it can make very precise estimates of the total amount it will have to pay in claims. This means the company faces very little risk overall and consequently behaves essentially as if it were risk neutral. Each manager, on the other hand, cannot diversify his or her own personal risks to the same extent, and thus each faces greater risk and behaves in a much more risk-averse manner.

7. When is it worth paying to obtain more information to reduce uncertainty?

It is worth paying for information if the information leads the consumer to make different choices than she would have made without the information, and the expected utility of the payoffs (deducting the cost of the information) is greater with the information than the expected utility of the payoffs received when making the best choices without knowing the information.

8. How does the diversification of an investor's portfolio avoid risk?

An investor reduces risk by investing in many assets whose returns are not highly correlated and, even better, some whose returns are negatively correlated. A mutual fund, for example, is a portfolio of stocks of many different companies. If the rate of return on each company's stock is not highly related to the rates of return earned on the other stocks in the portfolio, the portfolio will have a lower variance than any of the individual stocks. This occurs because low returns on some stocks tend to be offset by high returns on others. As the number of stocks in the portfolio increases, the portfolio's variance decreases. While there is less risk in a portfolio of stocks, risk cannot be completely avoided; there is still some market risk in holding a portfolio of stocks compared to a low-risk asset, such as a U.S. government bond.

9. Why do some investors put a large portion of their portfolios into risky assets, while others invest largely in risk-free alternatives? (*Hint*: Do the two investors receive exactly the same return on average? If so, why?)

Most investors are risk averse, but some are more risk averse than others. Investors who are highly risk averse will invest largely in risk-free alternatives while those who are less risk averse will put a larger portion of their portfolios into risky assets. Of course, because investors are risk averse, they will demand higher rates of return on investments that have higher levels of risk (i.e., higher variances). So investors who put larger amounts into risky assets expect to earn greater rates of return than those who invest primarily in risk-free assets.

10. What is an endowment effect? Give an example of such an effect.

An endowment effect exists if an individual places a higher value on an item that is in her possession as compared to the value she places on the same item when it is not in her possession. For example, some people might refuse to pay $5 for a simple coffee mug but would also refuse to sell the same mug for $5 if they already owned it or had just gotten it for free.

11. Jennifer is shopping and sees an attractive shirt. However, the price of $50 is more than she is willing to pay. A few weeks later, she finds the same shirt on sale for $25 and buys it. When a friend offers her $50 for the shirt, she refuses to sell it. Explain Jennifer's behavior.

To help explain Jennifer's behavior, we need to look at the reference point from which she is making the decision. In the first instance, she does not own the shirt so she is not willing to pay the $50 to buy the shirt. In the second instance, she will not accept $50 for the shirt from her friend because her reference point has changed. Once she owns the shirt, the value she attaches to it increases. Individuals often value goods more when they own them than when they do not. This is called the endowment effect.

■ Exercises

1. Consider a lottery with three possible outcomes:

- **$125 will be received with probability 0.2**
- **$100 will be received with probability 0.3**
- **$50 will be received with probability 0.5**

a. What is the expected value of the lottery?

The expected value, *EV*, of the lottery is equal to the sum of the returns weighted by their probabilities:

$$EV = (0.2)(\$125) + (0.3)(\$100) + (0.5)(\$50) = \$80.$$

b. What is the variance of the outcomes?

The variance, σ^2, is the sum of the squared deviations from the mean, $80, weighted by their probabilities:

$$\sigma^2 = (0.2)(125 - 80)^2 + (0.3)(100 - 80)^2 + (0.5)(50 - 80)^2 = \$975.$$

c. What would a risk-neutral person pay to play the lottery?

A risk-neutral person would pay the expected value of the lottery: $80.

2. Suppose you have invested in a new computer company whose profitability depends on two factors: (1) whether the U.S. Congress passes a tariff raising the cost of Japanese computers and (2) whether the U.S. economy grows slowly or quickly. What are the four mutually exclusive states of the world that you should be concerned about?

The four mutually exclusive states may be represented as:

	Congress passes tariff	**Congress does not pass tariff**
Slow growth rate	State 1: Slow growth with tariff	State 2: Slow growth without tariff
Fast growth rate	State 3: Fast growth with tariff	State 4: Fast growth without tariff

3. Richard is deciding whether to buy a state lottery ticket. Each ticket costs \$1, and the probability of winning payoffs is given as follows:

Probability	**Return**
0.50	**\$0.00**
0.25	**\$1.00**
0.20	**\$2.00**
0.05	**\$7.50**

a. What is the expected value of Richard's payoff if he buys a lottery ticket? What is the variance?

The expected value of the lottery is equal to the sum of the returns weighted by their probabilities:

$$EV = (0.5)(0) + (0.25)(\$1.00) + (0.2)(\$2.00) + (0.05)(\$7.50) = \$1.025$$

The variance is the sum of the squared deviations from the mean, \$1.025, weighted by their probabilities:

$$\sigma^2 = (0.5)(0 - 1.025)^2 + (0.25)(1 - 1.025)^2 + (0.2)(2 - 1.025)^2 + (0.05)(7.5 - 1.025)^2, \text{ or}$$

$$\sigma^2 = 2.812.$$

b. Richard's nickname is "No-Risk Rick" because he is an extremely risk-averse individual. Would he buy the ticket?

An extremely risk-averse individual would probably not buy the ticket. Even though the expected value is higher than the price of the ticket, \$1.025 > \$1.00, the difference is not enough to compensate Rick for the risk. For example, if his wealth is \$10 and he buys a \$1.00 ticket, he would have \$9.00, \$10.00, \$11.00, and \$16.50, respectively, under the four possible outcomes. If his utility function is $U = W^{0.5}$, where W is his wealth, then his expected utility is:

$$EU = (0.5)(9^{0.5}) + (0.25)(10^{0.5}) + (0.2)(11^{0.5}) + (0.05)(16.5^{0.5}) = 3.157.$$

This is less than 3.162, which is his utility if he does not buy the ticket ($U(10) = 10^{0.5} = 3.162$). Therefore, he would not buy the ticket.

c. Richard has been given 1000 lottery tickets. Discuss how you would determine the smallest amount for which he would be willing to sell all 1000 tickets.

With 1000 tickets, Richard's expected payoff is \$1025. He does not pay for the tickets, so he cannot lose money, but there is a wide range of possible payoffs he might receive ranging from

\$0 (in the extremely unlikely event that all 1000 tickets pay nothing) to \$7500 (in the even more unlikely case that all 1000 tickets pay the top prize of \$7.50), and virtually everything in between. Given this variability and Richard's high degree of risk aversion, we know that Richard would be willing to sell all the tickets for less (and perhaps considerably less) than the expected payoff of \$1025. More precisely, he would sell the tickets for \$1025 minus his risk premium. To find his selling price, we would first have to calculate his expected utility for the lottery winnings. This would be like point *F* in Figure 5.4 in the text, except that in Richard's case there are thousands of possible payoffs, not just two as in the figure. Using his expected utility value, we then would find the certain amount that gives him the same level of utility. This is like the \$16,000 income at point *C* in Figure 5.4. That certain amount is the smallest amount for which he would be willing to sell all 1000 lottery tickets.

d. In the long run, given the price of the lottery tickets and the probability/return table, what do you think the state would do about the lottery?

Given the price of the tickets, the sizes of the payoffs and the probabilities, the lottery is a money loser for the state. The state loses \$1.025 − 1.00 = \$0.025 (two and a half cents) on every ticket it sells. The state must raise the price of a ticket, reduce some of the payoffs, raise the probability of winning nothing, lower the probabilities of the positive payoffs, or some combination of the above.

4. Suppose an investor is concerned about a business choice in which there are three prospects—the probability and returns are given below:

Probability	Return
0.4	**\$100**
0.3	**30**
0.3	**–30**

What is the expected value of the uncertain investment? What is the variance?

The expected value of the return on this investment is

$$EV = (0.4)(100) + (0.3)(30) + (0.3)(-30) = \$40.$$

The variance is

$$\sigma^2 = (0.4)(100 - 40)^2 + (0.3)(30 - 40)^2 + (0.3)(-30 - 40)^2 = 2940.$$

5. You are an insurance agent who must write a policy for a new client named Sam. His company, Society for Creative Alternatives to Mayonnaise (SCAM), is working on a low-fat, low-cholesterol mayonnaise substitute for the sandwich-condiment industry. The sandwich industry will pay top dollar to the first inventor to patent such a mayonnaise substitute. Sam's SCAM seems like a very risky proposition to you. You have calculated his possible returns table as follows:

Probability	Return	Outcome
0.999	**–\$ 1,000,000**	**(he fails)**
0.001	**\$ 1,000,000,000**	**(he succeeds and sells his formula)**

a. What is the expected return of Sam's project? What is the variance?

The expected return, *ER*, of Sam's investment is

$$ER = (0.999)(-1{,}000{,}000) + (0.001)(1{,}000{,}000{,}000) = \$1000.$$

The variance is

$$\sigma^2 = (0.999)(-1{,}000{,}000 - 1000)^2 + (0.001)(1{,}000{,}000{,}000 - 1000)^2, \text{ or}$$

$$\sigma^2 = 1{,}000{,}998{,}999{,}000{,}000.$$

b. What is the most that Sam is willing to pay for insurance? Assume Sam is risk neutral.

Suppose the insurance guarantees that Sam will receive the expected return of $1000 with certainty regardless of the outcome of his SCAM project. Because Sam is risk neutral and because his expected return is the same as the guaranteed return with insurance, the insurance has no value to Sam. He is just as happy with the uncertain SCAM profits as with the certain outcome guaranteed by the insurance policy. So Sam will not pay anything for the insurance.

c. Suppose you found out that the Japanese are on the verge of introducing their own mayonnaise substitute next month. Sam does not know this and has just turned down your final offer of $1000 for the insurance. Assume that Sam tells you SCAM is only six months away from perfecting its mayonnaise substitute *and* that you know what you know about the Japanese. Would you raise or lower your policy premium on any subsequent proposal to Sam? Based on his information, would Sam accept?

The entry of the Japanese lowers Sam's probability of a high payoff. For example, assume that the probability of the billion-dollar payoff cut in half. Then the expected outcome is:

$$ER = (0.9995)(-\$1{,}000{,}000) + (0.0005)((\$1{,}000{,}000{,}000) = -\$499{,}500.$$

Therefore you should raise the policy premium substantially. But Sam, not knowing about the Japanese entry, will continue to refuse your offers to insure his losses.

6. Suppose that Natasha's utility function is given by $u(I) = \sqrt{10I}$, where I represents annual income in thousands of dollars.

a. Is Natasha risk loving, risk neutral, or risk averse? Explain.

Natasha is risk averse. To show this, assume that she has $10,000 and is offered a gamble of a $1000 gain with 50% probability and a $1000 loss with 50% probability. The utility of her current income of $10,000 is $u(10) = \sqrt{10(10)} = 10$. Her expected utility with the gamble is:

$$EU = (0.5)(\sqrt{10(11)}) + (0.5)(\sqrt{10(9)}) = 9.987 < 10.$$

She would avoid the gamble. If she were risk neutral, she would be indifferent between the $10,000 and the gamble, and if she were risk loving, she would prefer the gamble.

You can also see that she is risk averse by noting that the square root function increases at a decreasing rate (the second derivative is negative), implying diminishing marginal utility.

b. Suppose that Natasha is currently earning an income of $40,000 ($I = 40$) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a 0.6 probability of earning $44,000 and a 0.4 probability of earning $33,000. Should she take the new job?

The utility of her current salary is $\sqrt{10(40)} = 20$. The expected utility of the new job is

$$EU = (0.6)(\sqrt{10(44)}) + (0.4)(\sqrt{10(33)}) = 19.85,$$

which is less than 20. Therefore, she should not take the job. You can also determine that Natasha should reject the job by noting that the expected value of the new job is only $39,600, which is less than her current salary. Since she is risk averse, she should never accept a risky salary with a lower expected value than her current certain salary.

c. In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (*Hint*: What is the risk premium?)

This question assumes that Natasha takes the new job (for some unexplained reason). Her expected salary is 0.6(44,000) + 0.4(33,000) = \$39,600. The risk premium is the amount Natasha would be willing to pay so that she receives the expected salary for certain rather than the risky salary in her new job. In (b) we determined that her new job has an expected utility of 19.85. We need to find the certain salary that gives Natasha the same utility of 19.85, so we want to find I such that $u(I) = 19.85$. Using her utility function, we want to solve the following equation: $\sqrt{10I} = 19.85$. Squaring both sides, $10I = 394.0225$, and $I = 39.402$. So Natasha would be equally happy with a certain salary of \$39,402 or the uncertain salary with an expected value of \$39,600. Her risk premium is \$39,600 – 39,402 = \$198. Natasha would be willing to pay \$198 to guarantee her income would be \$39,600 for certain and eliminate the risk associated with her new job.

7. Suppose that two investments have the same three payoffs, but the probability associated with each payoff differs, as illustrated in the table below:

Payoff	Probability (Investment *A*)	Probability (Investment *B*)
\$300	**0.10**	**0.30**
\$250	**0.80**	**0.40**
\$200	**0.10**	**0.30**

a. Find the expected return and standard deviation of each investment.

The expected value of the return on investment *A* is

$$EV = (0.1)(300) + (0.8)(250) + (0.1)(200) = \$250.$$

The variance on investment *A* is

$$\sigma^2 = (0.1)(300 - 250)^2 + (0.8)(250 - 250)^2 + (0.1)(200 - 250)^2 = 500,$$

and the standard deviation on investment *A* is $\sigma = \sqrt{500} = \$22.36$.

The expected value of the return on investment *B* is

$$EV = (0.3)(300) + (0.4)(250) + (0.3)(200) = \$250.$$

The variance on investment *B* is

$$\sigma^2 = (0.3)(300 - 250)^2 + (0.4)(250 - 250)^2 + (0.3)(200 - 250)^2 = 1500,$$

and the standard deviation on investment *B* is $\sigma = \sqrt{1500} = \$38.73$.

b. Jill has the utility function $U = 5I$, where I denotes the payoff. Which investment will she choose?

Jill's expected utility from investment *A* is

$$EU = (0.1)(5 \times 300) + (0.8)(5 \times 250) + (0.1)(5 \times 200) = 1250.$$

Jill's expected utility from investment *B* is

$$EU = (0.3)(5 \times 300) + (0.4)(5 \times 250) + (0.3)(5 \times 200) = 1250.$$

Since both investments give Jill the same expected utility she will be indifferent between the two. Note that Jill is risk neutral, so she cares only about expected values. Since investments *A* and *B* have the same expected values, she is indifferent between them.

c. Ken has the utility function $U = 5\sqrt{I}$. Which investment will he choose?

Ken's expected utility from investment *A* is

$$EU = (0.1)(5)\sqrt{300} + (0.8)(5)\sqrt{250} + (0.1)(5)\sqrt{200} = 78.98.$$

Ken's expected utility from investment *B* is

$$EU = (0.3)(5)\sqrt{300} + (0.4)(5)\sqrt{250} + (0.3)(5)\sqrt{200} = 78.82.$$

Ken will choose investment *A* because it has a slightly higher expected utility. Notice that Ken is risk averse, and since the two investments have the same expected return, he prefers the investment with less variability.

d. Laura has the utility function $U = 5I^2$. Which investment will she choose?

Laura's expected utility from investment *A* is

$$EU = (0.1)(5 \times 300^2) + (0.8)(5 \times 250^2) + (0.1)(5 \times 200^2) = 315{,}000.$$

Laura's expected utility from investment *B* is

$$EU = (0.3)(5 \times 300^2) + (0.4)(5 \times 250^2) + (0.3)(5 \times 200^2) = 320{,}000.$$

Laura will choose investment *B* since it has a higher expected utility. Notice that Laura is a risk lover, and since the two investments have the same expected return, she prefers the investment with greater variability.

8. As the owner of a family farm whose wealth is \$250,000, you must choose between sitting this season out and investing last year's earnings (\$200,000) in a safe money market fund paying 5.0% or planting summer corn. Planting costs \$200,000, with a six-month time to harvest. If there is rain, planting summer corn will yield \$500,000 in revenues at harvest. If there is a drought, planting will yield \$50,000 in revenues. As a third choice, you can purchase AgriCorp drought-resistant summer corn at a cost of \$250,000 that will yield \$500,000 in revenues at harvest if there is rain, and \$350,000 in revenues if there is a drought. You are risk averse, and your preference for family wealth (W) is specified by the relationship $U(W) = \sqrt{W}$. The probability of a summer drought is 0.30, while the probability of summer rain is 0.70. Which of the three options should you choose? Explain.

Calculate the expected utility of wealth under the three options. Wealth is equal to the initial \$250,000 plus whatever is earned growing corn or investing in the safe financial asset. Expected utility under the safe option, allowing for the fact that your initial wealth is \$250,000, is:

$$E(U) = (250{,}000 + 200{,}000(1 + 0.05))^{0.5} = 678.23.$$

Expected utility with regular corn, again including your initial wealth, is:

$$E(U) = 0.7(250{,}000 + (500{,}000 - 200{,}000))^{0.5} + 0.3(250{,}000 + (50{,}000 - 200{,}000))^{0.5}$$
$$= 519.13 + 94.87 = 614.$$

Expected utility with drought-resistant corn is:

$$E(U) = 0.7(250{,}000 + (500{,}000 - 250{,}000))^{0.5} + 0.3(250{,}000 + (350{,}000 - 250{,}000))^{0.5}$$
$$= 494.975 + 177.482 = 672.46.$$

You should choose the option with the highest expected utility, which is the safe option of not planting corn.

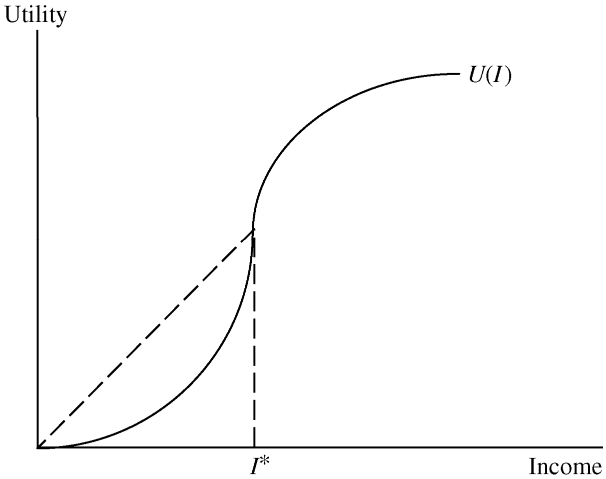
***Note*:** There is a subtle time issue in this problem. The returns from planting corn occur in 6 months while the money market fund pays 5%, which is presumably a yearly interest rate. To put everything on equal footing, we should compare the returns of all three alternatives over a 6-month period. In this case, the money market fund would earn about 2.5%, so its expected utility is:

$$E(U) = (250{,}000 + 200{,}000(1 + 0.025))^{0.5} = 674.54.$$

This is still the best of the three options, but by a smaller margin than before.

9. Draw a utility function over income *u*(*I*) that describes a man who is a risk lover when his income is low but risk averse when his income is high. Can you explain why such a utility function might reasonably describe a person's preferences?

The utility function will be *S*-shaped as illustrated below. Preferences might be like this for an individual who needs a certain level of income, I^*, in order to stay alive. An increase in income above I^* will have diminishing marginal utility. Below I^*, the individual will be a risk lover and will take unfavorable gambles in an effort to make large gains in income. Above I^*, the individual will purchase insurance against losses and below I^* will gamble.



10. A city is considering how much to spend to hire people to monitor its parking meters. The following information is available to the city manager:

- **Hiring each meter monitor costs $10,000 per year.**
- **With one monitoring person hired, the probability of a driver getting a ticket each time he or she parks illegally is equal to 0.25.**
- **With two monitors, the probability of getting a ticket is 0.5; with three monitors, the probability is 0.75; and with four, it's equal to 1.**
- **With two monitors hired, the current fine for overtime parking is $20.**

a. Assume first that all drivers are risk neutral. What parking fine would you levy, and how many meter monitors would you hire (1, 2, 3, or 4) to achieve the current level of deterrence against illegal parking at the minimum cost?

If drivers are risk neutral, their behavior is influenced only by their expected fine. With two meter monitors, the probability of detection is 0.5 and the fine is $20. So, the expected fine is (0.5)($20) + (0.5)(0) = $10. To maintain this expected fine, the city can hire one meter monitor and increase the fine to $40, or hire three meter monitors and decrease the fine to $13.33, or hire four meter monitors and decrease the fine to $10.

If the only cost to be minimized is the cost of hiring meter monitors at $10,000 per year, you (as the city manager) should minimize the number of meter monitors. Hire only one monitor and increase the fine to $40 to maintain the current level of deterrence.

b. Now assume that drivers are highly risk averse. How would your answer to (a) change?

If drivers are risk averse, they would want to avoid the possibility of paying parking fines even more than would risk-neutral drivers. Therefore, a fine of less than $40 should maintain the current level of deterrence.

c. (For discussion) What if drivers could insure themselves against the risk of parking fines? Would it make good public policy to permit such insurance?

Drivers engage in many forms of behavior to insure themselves against the risk of parking fines, such as checking the time often to be sure they have not parked overtime, parking blocks away from their destination in non-metered spots, or taking public transportation. If a private insurance firm offered insurance that paid the fine when a ticket was received, drivers would not worry about getting tickets. They would not seek out unmetered spots or take public transportation; they would park in metered spaces for as long as they wanted at zero personal cost. Having the insurance would lead drivers to get many more parking tickets. This is referred to as moral hazard and may cause the insurance market to collapse, but that's another story (see Section 17.3 in Chapter 17).

It probably would not make good public policy to permit such insurance. Parking is usually metered to encourage efficient use of scarce parking space. People with insurance would have no incentive to use public transportation, seek out-of-the-way parking locations, or economize on their use of metered spaces. This imposes a cost on others who are not able to find a place to park. If the parking fines are set to efficiently allocate the scarce amount of parking space available, then the availability of insurance will lead to an inefficient use of the parking space. In this case, it would not be good public policy to permit the insurance.

11. A moderately risk-averse investor has 50% of her portfolio invested in stocks and 50% in risk-free Treasury bills. Show how each of the following events will affect the investor's budget line and the proportion of stocks in her portfolio:

a. The standard deviation of the return on the stock market increases, but the expected return on the stock market remains the same.

From Section 5.4, the equation for the budget line is

$$R_p = \left[\frac{R_m - R_f}{\sigma_m}\right]\sigma_p + R_f,$$

where R_p is the expected return on the portfolio, R_m is the expected return from investing in the stock market, R_f is the risk-free return on Treasury bills, σ_m is the standard deviation of the return from investing in the stock market, and σ_p is the standard deviation of the return on the portfolio. The budget line is linear and shows the positive relationship between the return on the portfolio, R_p, and the standard deviation of the return on the portfolio, σ_p, as shown in Figure 5.6.

In this case σ_m, the standard deviation of the return on the stock market, increases. The slope of the budget line therefore decreases, and the budget line becomes flatter. The budget line's intercept stays the same because R_f does not change. Thus, at any given level of portfolio return, the portfolio now has a higher standard deviation. Since stocks have become riskier without a compensating increase in expected return, the proportion of stocks in the investor's portfolio will fall.

b. The expected return on the stock market increases, but the standard deviation of the stock market remains the same.

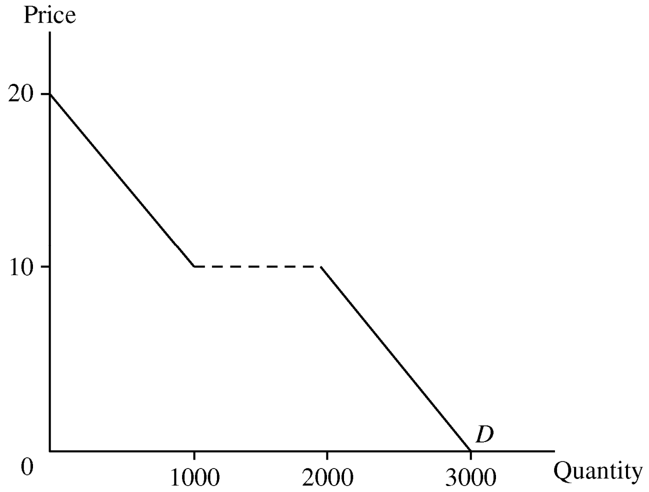
In this case, R_m, the expected return on the stock market, increases, so the slope of the budget line becomes steeper. At any given level of portfolio standard deviation, σ_p, there is now a higher expected return, R_p. Stocks have become relatively more attractive because investors now get greater expected returns with no increase in risk, and the proportion of stocks in the investor's portfolio will rise as a consequence.

c. The return on risk-free Treasury bills increases.

In this case there is an increase in R_f, which affects both the intercept and slope of the budget line. The budget line shifts up and becomes flatter as a result. The proportion of stocks in the portfolio could go either way. On one hand, Treasury bills now have a higher return and so are more attractive. On the other hand, the investor can now earn a higher return from each Treasury bill and so could hold fewer Treasury bills and still maintain the same level of risk-free return. In this second case, the investor may be willing to place more of her money in the stock market. It will depend on the particular preferences of the investor as well as the magnitude of the returns to the two asset classes. An analogy would be to consider what happens to savings when the interest rate increases. On the one hand, savings tend to increase because the return is higher, but on the other hand, spending may increase and savings decrease because a person can save less each period and still wind up with the same accumulation of savings at some future date.

12. Suppose there are two types of e-book consumers: 100 "standard" consumers with demand $Q = 20 - P$ and 100 "rule-of-thumb" consumers who buy 10 e-books only if the price is less than \$10. (Their demand curve is given by $Q = 10$ if $P < 10$ and $Q = 0$ if $P \geq 10$.) Draw the resulting total demand curve for e-books. How has the "rule-of-thumb" behavior affected the elasticity of total demand for e-books?

The demand for e-books is shown below. The upper part of the demand curve will look like the demands of the standard consumers. For example, when $P = \$20$, no e-books will be demanded, and when $P = \$10$, each standard consumer will demand 10 e-books and each rule-of-thumb consumer will demand zero e-books. Since there are 100 standard consumers, the total quantity demanded will be 1000. When the price drops to \$9.99, all 100 rule-of-thumb consumers will demand 10 e-books each, so the total quantity demanded will jump out to 2000 e-books. Finally, when price drops to zero, each standard consumer will demand 20 e-books, making the total number demanded 3000.



The equation for the upper part of the market demand curve is 100 times the standard demand equation, so it is $Q = 2000 - 100P$. Therefore, the slope of the demand curve is $\frac{\Delta Q}{\Delta P} = -100$. The elasticity of demand is $E_D = \frac{P}{Q}\frac{\Delta Q}{\Delta P}$, and at $P = \$10$, elasticity is therefore $E_D = \frac{10}{1000}(-100) = -1$.

When price drops to $P = \$9.99$ and the rule-of-thumb consumers jump into the market, elasticity becomes $E_D = \frac{9.99}{2000}(-100) = -0.50$. So elasticity drops considerably as a result of the behavior of the rule-of-thumb consumers. This makes sense, because the rule-of-thumb consumers have a perfectly inelastic demand once they enter the market, so they will lower the overall elasticity of demand for e-books at prices below \$10.

Chapter 6
Production

■ Teaching Notes

Chapters 3, 4, and 5 examined consumer behavior and demand. Now, in Chapter 6, we start looking more deeply at supply by studying production. Students often find the theory of supply easier to understand than consumer theory because it is less abstract, and the concepts are more familiar. It is helpful to emphasize the similarities between utility maximization and cost minimization—indifference curves and budget lines become isoquants and isocost lines. Once students have seen consumer theory, production theory usually is a bit easier.

While the concept of a production function is not difficult, the mathematical and graphical representation can sometimes be confusing. Numerical examples are very helpful. Be sure to point out that the production function tells us the *greatest* level of output for any given set of inputs. Thus, engineers have already determined the best production methods for any set of inputs, and all this is captured in the production function. Example 1 in Chapter 6, which discusses inefficiency in U.S. health care, covers this issue nicely. While technical efficiency is assumed throughout, you may want to discuss the importance of improving productivity and the concept of learning by doing, which is covered in Section 7.6 in Chapter 7. Examples 2 and 3 in Chapter 6 are good for highlighting this issue.

It is important to emphasize that the inputs used in production functions represent *flows* such as labor hours per week. Capital is measured in terms of capital services used during a period of time (e.g., machine hours per month) and not the number of units of capital. Capital flows are especially difficult for students to understand, but it is important to make the point here so that the discussion of input costs in Chapter 7 is easier for students to grasp.

Graphing the one-input production function in Section 6.2 leads naturally to a discussion of marginal product and diminishing marginal returns. Emphasize that diminishing returns exist because some factors are fixed by definition, and that diminishing returns does **not** mean negative returns. If you have not discussed marginal utility, now is the time to make sure that students know the difference between average and marginal. An example that captures students' attention is the relationship between average and marginal test scores. If their latest grade is greater than their average grade to date, it will increase their average.

Isoquants are defined and discussed in Section 6.3 of the chapter. Although the first few sentences in this section suggest that the one-input case corresponds to the short run while the two-input case occurs in the long run, you might want to point out that isoquants can also describe substitution among variable inputs in the short run. For example, skilled and unskilled labor, or labor and raw material can be substituted for each other in the short run. Rely on the students' understanding of indifference curves when discussing isoquants, and point out that, as with indifference curves, isoquants are a two-dimensional representation of a three-dimensional production function. A key concept in this section is the marginal rate of technical substitution, which is like the *MRS* in consumer theory.

Figure 6.5 is especially useful for demonstrating how diminishing marginal returns depend on the isoquant map. For example, if capital is held constant at 3 units, you can trace out the increase in output as labor increases and see that there are diminishing returns to labor.

Section 6.4 defines returns to scale, which has no counterpart in consumer theory because we do not care about the cardinal properties of utility functions. Be sure to explain the difference between diminishing returns to an input and decreasing returns to scale. Unfortunately, these terms sound very similar and frequently confuse students.

■ Questions for Review

1. **What is a production function? How does a long-run production function differ from a short-run production function?**

   A production function represents how inputs are transformed into outputs by a firm. In particular, a production function describes the *maximum* output that a firm can produce for each specified combination of inputs. In the short run, one or more factors of production cannot be changed, so a short-run production function tells us the maximum output that can be produced with different amounts of the variable inputs, holding fixed inputs constant. In the long-run production function, all inputs are variable.

2. **Why is the marginal product of labor likely to increase initially in the short run as more of the variable input is hired?**

   The marginal product of labor is likely to increase initially because when there are more workers, each is able to specialize in an aspect of the production process in which he or she is particularly skilled. For example, think of the typical fast food restaurant. If there is only one worker, he will need to prepare the burgers, fries, and sodas, as well as take the orders. Only so many customers can be served in an hour. With two or three workers, each is able to specialize, and the marginal product (number of customers served per hour) is likely to increase as we move from one to two to three workers. Eventually, there will be enough workers and there will be no more gains from specialization. At this point, the marginal product will begin to diminish.

3. **Why does production eventually experience diminishing marginal returns to labor in the short run?**

   The marginal product of labor will eventually diminish because there will be at least one fixed factor of production, such as capital. As more and more labor is used along with a fixed amount of capital, there is less and less capital for each worker to use, and the productivity of additional workers necessarily declines. Think for example of an office where there are only three computers. As more and more employees try to share the computers, the marginal product of each additional employee will diminish.

4. **You are an employer seeking to fill a vacant position on an assembly line. Are you more concerned with the average product of labor or the marginal product of labor for the last person hired? If you observe that your average product is just beginning to decline, should you hire any more workers? What does this situation imply about the marginal product of your last worker hired?**

   In filling a vacant position, you should be concerned with the marginal product of the last worker hired, because the marginal product measures the effect on output, or total product, of hiring another worker. This in turn determines the additional revenue generated by hiring another worker, which should then be compared to the cost of hiring the additional worker.

The point at which the average product begins to decline is the point where average product is equal to marginal product. As more workers are used beyond this point, both average product and marginal product decline. However, marginal product is still positive, so total product continues to increase. Thus, it may still be profitable to hire another worker.

5. **What is the difference between a production function and an isoquant?**

   A production function describes the maximum output that can be achieved with any given combination of inputs. An isoquant identifies all of the different combinations of inputs that can be used to produce one particular level of output.

6. **Faced with constantly changing conditions, why would a firm ever keep *any* factors fixed? What criteria determine whether a factor is fixed or variable?**

   Whether a factor is fixed or variable depends on the time horizon under consideration: all factors are fixed in the very short run while all factors are variable in the long run. As stated in the text, "All fixed inputs in the short run represent outcomes of previous long-run decisions based on estimates of what a firm could profitably produce and sell." Some factors are fixed in the short run, whether the firm likes it or not, simply because it takes time to adjust the levels of those inputs. For example, a lease on a building may legally bind the firm, some employees may have contracts that must be upheld, or construction of a new facility may take a year or more. Recall that the short run is not defined as a specific number of months or years but as that period of time during which some inputs cannot be changed for reasons such as those given above.

7. **Isoquants can be convex, linear, or L-shaped. What does each of these shapes tell you about the nature of the production function? What does each of these shapes tell you about the *MRTS*?**

   Convex isoquants indicate that some units of one input can be substituted for a unit of the other input while maintaining output at the same level. In this case, the *MRTS* is diminishing as we move down along the isoquant. This tells us that it becomes more and more difficult to substitute one input for the other while keeping output unchanged. Linear isoquants imply that the slope, or the *MRTS*, is constant. This means that the same number of units of one input can always be exchanged for a unit of the other input holding output constant. The inputs are perfect substitutes in this case. L-shaped isoquants imply that the inputs are perfect complements, and the firm is producing under a fixed proportions type of technology. In this case the firm cannot give up one input in exchange for the other and still maintain the same level of output. For example, the firm may require exactly 4 units of capital for each unit of labor, in which case one input cannot be substituted for the other.

8. **Can an isoquant ever slope upward? Explain.**

   No. An upward sloping isoquant would mean that if you increased both inputs output would stay the same. This would occur only if one of the inputs reduced output; sort of like a bad in consumer theory. As a general rule, if the firm has more of all inputs it can produce more output.

9. **Explain the term "marginal rate of technical substitution." What does a *MRTS* = 4 mean?**

   MRTS is the amount by which the quantity of one input can be reduced when the other input is increased by one unit, while maintaining the same level of output. If the *MRTS* is 4 then one input can be reduced by 4 units as the other is increased by one unit, and output will remain the same.

10. **Explain why the marginal rate of technical substitution is likely to diminish as more and more labor is substituted for capital.**

    As more and more labor is substituted for capital, it becomes increasingly difficult for labor to perform the jobs previously done by capital. Therefore, more units of labor will be required to replace each unit of capital, and the *MRTS* will diminish. For example, think of employing more and more farm labor while reducing the number of tractor hours used. At first you would stop using tractors for simpler tasks such as driving around the farm to examine and repair fences or to remove rocks and fallen tree limbs from fields. But eventually, as the number or labor hours increased and the number of tractor hours declined, you would have to plant and harvest your crops primarily by hand. This would take huge numbers of additional workers.

11. Is it possible to have diminishing returns to a single factor of production and constant returns to scale at the same time? Discuss.

Diminishing returns and returns to scale are completely different concepts, so it is quite possible to have both diminishing returns to, say, labor and constant returns to scale. Diminishing returns to a single factor occurs because all other inputs are fixed. Thus, as more and more of the variable factor is used, the additions to output eventually become smaller and smaller because there are no increases in the other factors. The concept of returns to scale, on the other hand, deals with the increase in output when *all* factors are increased by the same proportion. While each factor by itself exhibits diminishing returns, output may more than double, less than double, or exactly double when all the factors are doubled. The distinction again is that with returns to scale, all inputs are increased in the same proportion and no inputs are fixed. The production function in Exercise 10 is an example of a function with diminishing returns to each factor and constant returns to scale.

12. Can a firm have a production function that exhibits increasing returns to scale, constant returns to scale, and decreasing returns to scale as output increases? Discuss.

Many firms have production functions that exhibit first increasing, then constant, and ultimately decreasing returns to scale. At low levels of output, a proportional increase in all inputs may lead to a larger-than-proportional increase in output, because there are many ways to take advantage of greater specialization as the scale of operation increases. As the firm grows, the opportunities for specialization may diminish, and the firm operates at peak efficiency. If the firm wants to double its output, it must duplicate what it is already doing. So it must double all inputs in order to double its output, and thus there are constant returns to scale. At some level of production, the firm will be so large that when inputs are doubled, output will less than double, a situation that can arise from management diseconomies.

13. Suppose that output q is a function of a single input, labor (L). Describe the returns to scale associated with each of the following production functions:

a. $q = L/2$**.** Let q' be output when labor is doubled to $2L$. Then $q' = (2L)/2 = L$. Compare q' to q by dividing q' by q. This gives us $q'/q = L/(L/2) = 2$. Therefore when the amount of labor is doubled, output is also doubled. Hence there are constant returns to scale.

b. $q = L^2 + L$**.** Again, let q' be output when labor is doubled. $q' = (2L)^2 + 2L = 4L^2 + 2L$. Dividing by q yields $q'/q = (4L^2 + 2L)/(L^2 + L) > 2$. To see why this ratio is greater than two, note that it would be exactly two if q' were to equal $2L^2 + 2L$, but q' is larger than that, so the ratio is greater than two, indicating increasing returns to scale.

c. $q = \log(L)$**.** In this case, $q' = \log(2L) = \log(2) + \log(L)$, using the rules for logarithms. Then $q'/q = [\log(2) + \log(L)]/\log(L) = \log(2)/\log(L) + 1$. This expression is greater than, equal to or less than 2 when L is less than, equal to or greater than 2. So this production function exhibits increasing returns to scale when $L < 2$, constant returns to scale when $L = 2$, and decreasing returns to scale when $L > 2$.

■ Exercises

1. **The menu at Joe's coffee shop consists of a variety of coffee drinks, pastries, and sandwiches. The marginal product of an additional worker can be defined as the number of customers who can be served by that worker in a given time period. Joe has been employing one worker, but is considering hiring a second and a third. Explain why the marginal product of the second and third workers might be higher than the first. Why might you expect the marginal product of additional workers to diminish eventually?**

   The marginal product could well increase for the second and third workers because each would be able to specialize in a different task. If there is only one worker, that person has to take orders, prepare the food, serve the food, and do all the cleanup. With two or three workers, however, one could take orders and serve the food while the others do most of the coffee and food preparation and cleanup. Eventually, however, as more workers are employed, the marginal product would diminish because there would be a large number of people behind the counter and in the kitchen trying to serve more and more customers with a limited amount of equipment and a fixed building size.

2. **Suppose a chair manufacturer is producing in the short run (with its existing plant and equipment). The manufacturer has observed the following levels of production corresponding to different numbers of workers:**

Number of Workers	**Number of Chairs**
1	**10**
2	**18**
3	**24**
4	**28**
5	**30**
6	**28**
7	**25**

   a. **Calculate the marginal and average product of labor for this production function.**

   The average product of labor, AP_L, is equal to $\frac{q}{L}$. The marginal product of labor, MP_L, is equal to $\frac{\Delta q}{\Delta L}$, the change in output divided by the change in labor input. For this production process we have:

L	q	AP_L	MP_L
0	0	—	—
1	10	10	10
2	18	9	8
3	24	8	6
4	28	7	4
5	30	6	2
6	28	4.7	–2
7	25	3.6	–3

b. Does this production function exhibit diminishing returns to labor? Explain.

Yes, this production function exhibits diminishing returns to labor. The marginal product of labor, the extra output produced by each additional worker, diminishes as workers are added, and this starts to occur with the second unit of labor.

c. Explain intuitively what might cause the marginal product of labor to become negative.

Labor's negative marginal product for $L > 5$ may arise from congestion in the chair manufacturer's factory. Since more laborers are using the same fixed amount of capital, it is possible that they could get in each other's way, decreasing efficiency and the amount of output. Firms also have to control the quality of their output, and the high congestion of labor may produce products that are not of a high enough quality to be offered for sale, which can contribute to a negative marginal product.

3. Fill in the gaps in the table below.

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	—	—
1	225		
2			300
3		300	
4	1140		
5		225	
6			225

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	—	—
1	225	225	225
2	600	375	300
3	900	300	300
4	1140	240	285
5	1365	225	273
6	1350	–15	225

4. A political campaign manager must decide whether to emphasize television advertisements or letters to potential voters in a reelection campaign. Describe the production function for campaign votes. How might information about this function (such as the shape of the isoquants) help the campaign manager to plan strategy?

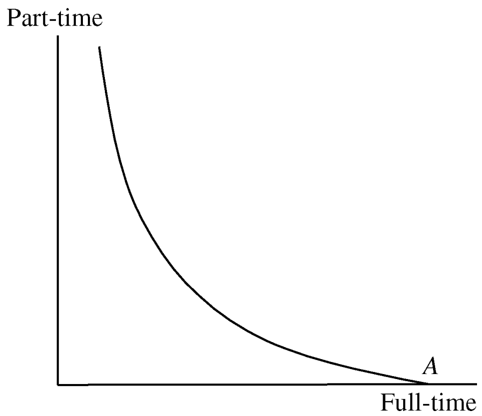
The output of concern to the campaign manager is the number of votes. The production function has two inputs, television advertising and letters. The use of these inputs requires knowledge of the substitution possibilities between them. If the inputs are perfect substitutes for example, the isoquants are straight lines, and the campaign manager should use only the less expensive input in this case.

If the inputs are not perfect substitutes, the isoquants will have a convex shape. The campaign manager should then spend the campaign's budget on the combination of the two inputs that maximize the number of votes.

5. **For each of the following examples, draw a representative isoquant. What can you say about the marginal rate of technical substitution in each case?**

   a. **A firm can hire only full-time employees to produce its output, or it can hire some combination of full-time and part-time employees. For each full-time worker let go, the firm must hire an increasing number of temporary employees to maintain the same level of output.**

   Place part-time workers on the vertical axis and full-time workers on the horizontal. The slope of the isoquant measures the number of part-time workers that can be exchanged for a full-time worker while still maintaining output. At the bottom end of the isoquant, at point *A*, the isoquant hits the full-time axis because it is possible to produce with full-time workers only and no part-timers. As we move up the isoquant and give up full-time workers, we must hire more and more part-time workers to replace each full-time worker. The slope increases (in absolute value) as we move up the isoquant. The isoquant is therefore convex and there is a diminishing marginal rate of technical substitution.



   b. **A firm finds that it can always trade two units of labor for one unit of capital and still keep output constant.**

   The marginal rate of technical substitution measures the number of units of capital that can be exchanged for a unit of labor while still maintaining output. If the firm can always trade two units of labor for one unit of capital then the *MRTS* of labor for capital is constant and equal to 1/2, and the isoquant is linear.

   c. **A firm requires exactly two full-time workers to operate each piece of machinery in the factory.**

   This firm operates under a fixed proportions technology, and the isoquants are L-shaped. The firm cannot substitute any labor for capital and still maintain output because it must maintain a fixed 2:1 ratio of labor to capital. The *MRTS* is infinite (or undefined) along the vertical part of the isoquant and zero on the horizontal part.

6. A firm has a production process in which the inputs to production are perfectly substitutable in the long run. Can you tell whether the marginal rate of technical substitution is high or low, or is further information necessary? Discuss.

Further information is necessary. The marginal rate of technical substitution, *MRTS*, is the absolute value of the slope of an isoquant. If the inputs are perfect substitutes, the isoquants will be linear. To calculate the slope of the isoquant, and hence the *MRTS*, we need to know the rate at which one input may be substituted for the other. In this case, we do not know whether the *MRTS* is high or low. All we know is that it is a constant number. We need to know the marginal product of each input to determine the *MRTS*.

7. The marginal product of labor in the production of computer chips is 50 chips per hour. The marginal rate of technical substitution of hours of labor for hours of machine capital is 1/4. What is the marginal product of capital?

The marginal rate of technical substitution is defined at the ratio of the two marginal products. Here, we are given the marginal product of labor and the marginal rate of technical substitution. To determine the marginal product of capital, substitute the given values for the marginal product of labor and the marginal rate of technical substitution in the following formula:

$$\frac{MP_L}{MP_K} = MRTS, \text{ or } \frac{50}{MP_K} = \frac{1}{4},$$

and therefore, $MP_K = 200$ computer chips per hour.

8. Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased and the other factor held constant?

a. $q = 3L + 2K$

This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 10. If L is 4 and K is 4 then q is 20. When the inputs are doubled, output will double. Each marginal product is constant for this production function. When L increases by 1, q will increase by 3. When K increases by 1, q will increase by 2.

b. $q = (2L + 2K)^{\frac{1}{2}}$

This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 2.8. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output increases by less than double. The marginal product of each input is decreasing. This can be determined using calculus by differentiating the production function with respect to one input while holding the other input constant. For example, the marginal product of labor is

$$\frac{\partial q}{\partial L} = \frac{2}{2(2L + 2K)^{\frac{1}{2}}}.$$

Since L is in the denominator, as L gets bigger, the marginal product gets smaller. If you do not know calculus, you can choose several values for L (holding K fixed at some level), find the corresponding q values and see how the marginal product changes. For example, if $L = 4$ and $K = 4$ then $q = 4$. If $L = 5$ and $K = 4$ then $q = 4.24$. If $L = 6$ and $K = 4$ then $q = 4.47$. Marginal product of labor falls from 0.24 to 0.23. Thus, MP_L decreases as L increases, holding K constant at 4 units.

c. $q = 3LK^2$

This function exhibits increasing returns to scale. For example, if L is 2 and K is 2, then q is 24. If L is 4 and K is 4 then q is 192. When the inputs are doubled, output more than doubles. Notice also that if we increase each input by the same factor λ then we get the following:

$$q' = 3(\lambda L)(\lambda K)^2 = \lambda^3 3LK^2 = \lambda^3 q.$$

Since λ is raised to a power greater than 1, we have increasing returns to scale.

The marginal product of labor is constant and the marginal product of capital is increasing. For any given value of K, when L is increased by 1 unit, q will go up by $3K^2$ units, which is a constant number. Using calculus, the marginal product of capital is $MP_K = 6LK$. As K increases, MP_K increases. If you do not know calculus, you can fix the value of L, choose a starting value for K, and find q. Now increase K by 1 unit and find the new q. Do this a few more times and you can calculate marginal product. This was done in part b above, and in part d below.

d. $q = L^{\frac{1}{2}} K^{\frac{1}{2}}$

This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 2. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output will exactly double. Notice also that if we increase each input by the same factor, λ, then we get the following:

$$q' = (\lambda L)^{\frac{1}{2}} (\lambda K)^{\frac{1}{2}} = \lambda L^{\frac{1}{2}} K^{\frac{1}{2}} = \lambda q.$$

Since λ is raised to the power 1, there are constant returns to scale.

The marginal product of labor is decreasing and the marginal product of capital is decreasing. Using calculus, the marginal product of capital is

$$MP_K = \frac{L^{\frac{1}{2}}}{2K^{\frac{1}{2}}}.$$

For any given value of L, as K increases, MP_K will decrease. If you do not know calculus then you can fix the value of L, choose a starting value for K, and find q. Let $L = 4$ for example. If K is 4 then q is 4, if K is 5 then q is 4.47, and if K is 6 then q is 4.90. The marginal product of the 5th unit of K is $4.47 - 4 = 0.47$, and the marginal product of the 6th unit of K is $4.90 - 4.47 = 0.43$. Hence we have diminishing marginal product of capital. You can do the same thing for labor.

e. $q = 4L^{\frac{1}{2}} + 4K$

This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 13.66. If L is 4 and K is 4 then q is 24. When the inputs are doubled, output increases by less than double.

The marginal product of labor is decreasing and the marginal product of capital is constant. For any given value of L, when K is increased by 1 unit, q goes up by 4 units, which is a constant number. To see that the marginal product of labor is decreasing, fix $K = 1$ and choose values for L. If $L = 1$ then $q = 8$, if $L = 2$ then $q = 9.66$, and if $L = 3$ then $q = 10.93$. The marginal product of the second unit of labor is $9.66 - 8 = 1.66$, and the marginal product of the third unit of labor is $10.93 - 9.66 = 1.27$. Marginal product of labor is diminishing.

9. The production function for the personal computers of DISK, Inc., is given by $q = 10K^{0.5}L^{0.5}$, where q is the number of computers produced per day, K is hours of machine time, and L is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function $q = 10K^{0.6}L^{0.4}$.

a. If both companies use the same amounts of capital and labor, which will generate more output?

Let q_1 be the output of DISK, Inc., q_2, be the output of FLOPPY, Inc., and X be the same equal amounts of capital and labor for the two firms. Then according to their production functions,

$$q_1 = 10X^{0.5}X^{0.5} = 10X^{(0.5+0.5)} = 10X$$

and

$$q_2 = 10X^{0.6}X^{0.4} = 10X^{(0.6+0.4)} = 10X.$$

Because $q_1 = q_2$, both firms generate the same output with the same inputs. Note that if the two firms both used the same amount of capital and the same amount of labor, but the amount of capital was not equal to the amount of labor, then the two firms would not produce the same levels of output. In fact, if $K > L$ then $q_2 > q_1$, and if $L > K$ then $q_1 > q_2$.

b. Assume that capital is limited to 9 machine hours, but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.

With capital limited to 9 machine hours, the production functions become $q_1 = 30L^{0.5}$ and $q_2 = 37.37L^{0.4}$. To determine the production function with the highest marginal productivity of labor, consider the following table:

L	q Firm 1	MP_L Firm 1	q Firm 2	MP_L Firm 2
0	0.0	—	0.00	—
1	30.00	30.00	37.37	37.37
2	42.43	12.43	49.31	11.94
3	51.96	9.53	57.99	8.68
4	60.00	8.04	65.06	7.07

For each unit of labor above 1, the marginal productivity of labor is greater for the first firm, DISK, Inc.

10. In Example 6.4, wheat is produced according to the production function $q = 100(K^{0.8}L^{0.2})$.

a. Beginning with a capital input of 4 and a labor input of 49, show that the marginal product of labor and the marginal product of capital are both decreasing.

For fixed labor and variable capital:

$K = 4 \Rightarrow q = (100)(4^{0.8})(49^{0.2}) = 660.22$

$K = 5 \Rightarrow q = (100)(5^{0.8})(49^{0.2}) = 789.25 \Rightarrow MP_K = 129.03$

$K = 6 \Rightarrow q = (100)(6^{0.8})(49^{0.2}) = 913.19 \Rightarrow MP_K = 123.94$

$K = 7 \Rightarrow q = (100)(7^{0.8})(49^{0.2}) = 1033.04 \Rightarrow MP_K = 119.85.$

For fixed capital and variable labor:

$L = 49 \Rightarrow q = (100)(4^{0.8})(49^{0.2}) = 660.22$

$L = 50 \Rightarrow q = (100)(4^{0.8})(50^{0.2}) = 662.89 \Rightarrow MP_L = 2.67$

$L = 51 \Rightarrow q = (100)(4^{0.8})(51^{0.2}) = 665.52 \Rightarrow MP_L = 2.63$

$L = 52 \Rightarrow q = (100)(4^{0.8})(52^{0.2}) = 668.11 \Rightarrow MP_L = 2.59.$

The marginal products of both capital and labor decrease as the variable input increases.

b. Does this production function exhibit increasing, decreasing, or constant returns to scale?

Constant (increasing, decreasing) returns to scale implies that proportionate increases in inputs lead to the same (more than, less than) proportionate increases in output. If we were to increase labor and capital by the same proportionate amount (λ) in this production function, output would change by the same proportionate amount:

$$q' = 100(\lambda K)^{0.8}(\lambda L)^{0.2}, \text{ or}$$
$$q' = 100K^{0.8}L^{0.2}\lambda^{(0.8+0.2)} = q\lambda$$

Therefore this production function exhibits constant returns to scale. You can also determine this if you plug in values for K and L and compute q, and then double the K and L values to see what happens to q. For example, let $K = 4$ and $L = 10$. Then $q = 480.45$. Now double both inputs to $K = 8$ and $L = 20$. The new value for q is 960.90, which is exactly twice as much output. Thus there are constant returns to scale.

11. Suppose life expectancy in years (L) is a function of two inputs, health expenditures (H) and nutrition expenditures (N) in hundreds of dollars per year. The production function is $L = cH^{0.8}N^{0.2}$.

a. Beginning with a health input of \$400 per year ($H = 4$) and a nutrition input of \$4900 per year ($N = 49$), show that the marginal product of health expenditures and the marginal product of nutrition expenditures are both decreasing.

When $H = 4$ and $N = 49$, $L = c(4^{0.8})(49^{0.2}) = 6.602c$. Holding N constant at 49, when $H = 5$, $L = 7.893c$, and when $H = 6$, $L = 9.132c$. The marginal product of H drops from 1.291c (7.893c − 6.602c) to 1.239c (9.132c − 7.893c). Therefore the marginal product of health expenditures is decreasing.

Now hold H constant at 4 and increase N to 50. $L = c(4^{0.8})(50^{0.2}) = 6.629c$. Increasing N to 51, $L = 6.655c$. The marginal product of N drops from 0.027c (6.629c − 6.602c) to 0.026c (6.655c − 6.629c), so the marginal product of nutrition expenditures is decreasing.

b. Does this production function exhibit increasing, decreasing, or constant returns to scale?

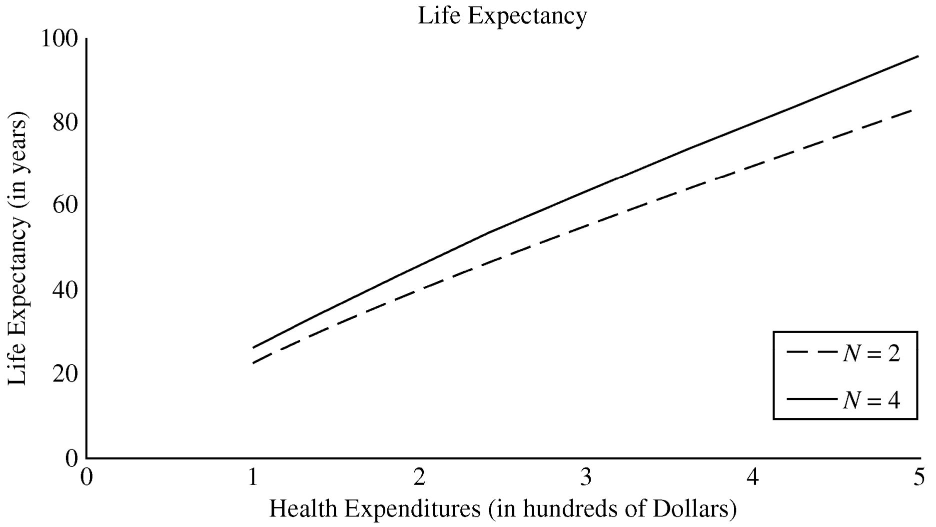
There are constant returns to scale. If both inputs are doubled, the new output is $L' = c(2H)^{0.8}(2N)^{0.2} = cH^{0.8}N^{0.2}(2^{0.8})(2^{0.2}) = 2cH^{0.8}N^{0.2} = 2L$. So when both inputs are doubled, life expectancy also doubles. Hence there are constant returns to scale.

c. Suppose that in a country suffering from famine, N is fixed at 2 and that $c = 20$. Plot the production function for life expectancy as a function of health expenditures, with L on the vertical axis and H on the horizontal axis.

The production function becomes $L = 20(H^{0.8})(2^{0.2}) = 22.974H^{0.8}$. To plot this, find the life expectancies for various levels of H, plot those points and draw a smooth curve through them. Here are some points: $H = 1$ and $L = 22.97$; $H = 2$ and $L = 40.00$; $H = 3$ and $L = 55.38$; $H = 4$ and $L = 69.64$; $H = 5$ and $L = 83.26$. This production function is plotted below as a dashed line.

d. Now suppose another nation provides food aid to the country suffering from famine so that N increases to 4. Plot the new production function.

The production function becomes $L = 20(H^{0.8})(4^{0.2}) = 26.390H^{0.8}$. Points to plot are $H = 1$ and $L = 26.39$; $H = 2$ and $L = 45.95$; $H = 3$ and $L = 63.55$; $H = 4$ and $L = 80.00$; $H = 5$ and $L = 95.63$. This production function is plotted below as a solid line.



e. Now suppose that $N = 4$ and $H = 2$. You run a charity that can provide either food aid or health aid to this country. Which would provide a greater benefit: increasing H by 1 or N by 1?

If $N = 4$ and $H = 2$, life expectancy is 45.95 years as calculated in part d. If N remains at 4 and H increases by 1 (so $H = 3$), life expectancy increases to 63.55 years as shown in part d. On the other hand, if H remains at 2 and N increases by 1 (so $N = 5$), life expectancy rises to $20(2^{0.8})(5^{0.2}) = 48.05$. It is clearly much more beneficial to increase H by 1, because life expectancy increases to 63.55 years. An increase in N by 1 raises life expectancy to only 48.05 years.

Chapter 7
The Cost of Production

■ Teaching Notes

This chapter is packed with new terms and concepts, and it will take some time to go through carefully. You might remind students that you are still building the underpinnings of supply and, for that purpose, it is critical to understand how firms make production decisions. Some key topics in the chapter are accounting versus economic costs; total, average, and marginal costs in the short and long run; and choosing the least-cost combination of inputs (graphically in the chapter, and mathematically in the appendix).

Other topics include economies of scope, the learning curve, and estimation of cost functions. You may omit some of these additional topics without disrupting the flow of the book if you want to save time for other issues later in the course.

To get started, it is important to distinguish between accounting and economic costs so that students will understand that zero (economic) profit is a reasonable long-run equilibrium in perfect competition (Chapter 8). Opportunity cost is crucial for understanding this distinction. Examples given in the chapter include the opportunity cost of using a building that a business owns, the cost of a business owner's time, and the opportunity cost of utilizing capital. The opportunity cost of capital, i.e., the rental rate on capital, may well be the same whether the firm owns or rents the capital, and is a source of confusion. It is important, therefore, to distinguish between the purchase price of capital equipment (or its depreciation as determined by accounting rules) and the opportunity cost of using the equipment. Also remind students that the rental rate on capital is the cost for the *flow of capital services* provided by the capital, not the total cost to purchase the capital. Give lots of examples. It is also useful to note that most costs are pretty straightforward explicit costs and are recognized as costs by both accountants and economists.

Sunk costs can also cause problems for students, and many confuse them with fixed costs. The difference is that fixed costs do not vary with output in the short run, but can be reduced or eliminated in the long run. Sunk costs have already been incurred (or committed to) and cannot be recovered or reduced. I like to give examples where people or firms incorrectly take sunk costs into account when making decisions. For instance, investors who will not sell stock that has declined in value until they at least "break even," and companies abandoning projects on which much has already been invested because the **total** amount spent will never be covered by future revenues. Another intriguing example is when a person attends a concert, even though the weather is terrible, because he paid for the ticket, but would not have attended if the ticket had been free. This relates to some of the behavioral economics material in Section 5.6 of Chapter 5.

Following the discussion of opportunity cost, the chapter looks at short-run costs. While the definitions of total, variable, fixed, average, and marginal costs and their graphical relationships can seem tedious and/or uninteresting to the student (and some instructors), they are important for understanding the derivation of the firm's supply curve in Chapter 8. Doing algebraic or numerical examples in table form is helpful for most students. Explain that each firm has a unique set of cost curves based on its own particular production function and the prices it has to pay for inputs. Discuss the importance of diminishing returns in explaining the shapes of the short-run cost curves. Point out that average total cost tends to be U-shaped in the short run, that marginal cost intersects average cost and average variable cost at their respective minimum points, and that the minimum of *AVC* occurs to the left of the minimum of *ATC* as in Figure 7.1. Draw these curves carefully and encourage your students to do the same.

Section 7.3 on long-run costs goes through the firm's cost-minimization problem. I like to point out that this isn't just a long-run problem. Even in the short run, firms have many variable inputs and must choose the least-cost combination of them. So the isocost/isoquant diagram is equally valid in the short run. You should rely heavily on the utility maximization material when covering cost minimization. The major difference is that the budget line (i.e., the isocost) is the objective function rather than the constraint and the indifference curve (i.e., the isoquant) is now the constraint and, of course, we are minimizing rather than maximizing.

I like to express the tangency condition for cost minimization in the form given in expression (7.4); that is, $MP_L/w = MP_K/r$. This has a nice intuitive interpretation and is something firms might actually be able to use to ascertain if they are using the least-cost combination of inputs. For example, suppose a roofing firm is using 3 workers and 2 nailing guns and can roof 100 square feet in an hour. If they had one more worker they could roof 120 square feet, or if they had one more nail gun they could roof 110 square feet. Workers are paid \$12 per hour and nail guns cost \$3 per hour. From this you can estimate the marginal products and determine that the firm is not at the cost-minimizing point. It should use relatively more nail guns.

After covering the cost-minimizing material, you can point out that cost functions are derived by solving the cost-minimization problem repeatedly for different output amounts. For each solution, calculate total cost as $C = wL + rK$, and plot this against output to get the total cost function. You can do this graphically using the firm's expansion path as in Figure 7.6. I like to illustrate this with an expansion path that is not linear, so I get a nonlinear cost function. You can then talk about economies and diseconomies of scale and how that affects the shape of the cost function. It can also help to distinguish economies of scale from returns to scale—a subtle distinction that often eludes students.

The relationship between short-run and long-run costs is also a difficulty for some students. Part of the problem is that students do not really understand what lies behind long-run costs. You need to emphasize that it takes time to move from one point to another along the long-run average cost curve. Each point represents the lowest average cost *after* all possible adjustments are made. Many students also have trouble with the fact that most points on the *LAC* curve do not correspond to minimum points on the corresponding *SAC* curves. For example, if the firm's chosen output is less than the output where *LAC* is minimized, the firm uses a plant larger than the one whose *SAC* is minimized at the chosen output. It purposely underutilizes a bigger plant because the bigger plant is more efficient (i.e., has a lower average cost) at the chosen output level than the smaller plant whose *SAC* is minimized at that output.

■ Questions for Review

1. **A firm pays its accountant an annual retainer of \$10,000. Is this an economic cost?**

   This is an explicit cost of purchasing the services of the accountant, and it is both an economic and an accounting cost. When the firm pays an annual retainer of \$10,000, there is a monetary transaction. The accountant trades his or her time in return for money. An annual retainer is an explicit cost and therefore an economic cost.

2. **The owner of a small retail store does her own accounting work. How would you measure the opportunity cost of her work?**

   The economic, or opportunity, cost of doing accounting work is measured by computing the monetary amount that the owner's time would be worth in its *next best use*. For example, if she could do accounting work for some other company instead of her own, her opportunity cost is the amount she could have earned in that alternative employment. Or if she is a great stand-up comic, her opportunity cost is what she could have earned in that occupation instead of doing her own accounting work.

3. **Please explain whether the following statements are true or false.**

   a. **If the owner of a business pays himself no salary, then the accounting cost is zero, but the economic cost is positive.**

   True. Since there is no monetary transaction, there is no accounting, or explicit, cost. However, since the owner of the business could be employed elsewhere, there is an economic cost. The economic cost is positive, reflecting the opportunity cost of the owner's time. The economic cost is the value of the owner's time in his next best alternative, or the amount that the owner would earn if he took the next best job.

   b. **A firm that has positive accounting profit does not necessarily have positive economic profit.**

   True. Accounting profit considers only the explicit, monetary costs. Since there may be some opportunity costs that were not fully realized as explicit monetary costs, it is possible that when the opportunity costs are added in, economic profit will become negative. This indicates that the firm's resources are not being put to their best use.

   c. **If a firm hires a currently unemployed worker, the opportunity cost of utilizing the worker's services is zero.**

   False. From the firm's point of view, the wage paid to the worker is an explicit cost whether she was previously unemployed or not. The firm's opportunity cost is equal to the wage, because if it did not hire this worker, it would have had to hire someone else at the same wage. The opportunity cost from the worker's point of view is the value of her time, which is unlikely to be zero. By taking this job, she cannot work at another job or take care of a child or elderly person at home. If her best alternative is working at another job, she gives up the wage she would have earned. If her best alternative is unpaid, such as taking care of a loved one, she will now have to pay someone else to do that job, and the amount she has to pay is her opportunity cost.

4. **Suppose that labor is the only variable input to the production process. If the marginal cost of production is diminishing as more units of output are produced, what can you say about the marginal product of labor?**

   The marginal product of labor must be increasing. The marginal cost of production measures the extra cost of producing one more unit of output. If this cost is diminishing, then it must be taking fewer units of labor to produce the extra unit of output. If fewer units of labor are required to produce a unit of output, then the marginal product (extra output produced by an extra unit of labor) must be increasing. Note also, that $MC = w/MP_L$, so that if MC is diminishing then MP_L must be increasing for any given w.

5. **Suppose a chair manufacturer finds that the marginal rate of technical substitution of capital for labor in her production process is substantially greater than the ratio of the rental rate on machinery to the wage rate for assembly-line labor. How should she alter her use of capital and labor to minimize the cost of production?**

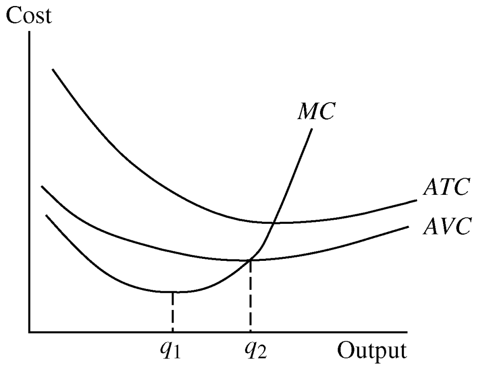
   The question states that the *MRTS* of capital for labor is greater than r/w. Note that this is different from the *MRTS* of labor for capital, which is what is used in Chapters 6 and 7. The *MRTS* of labor for capital equals MP_K/MP_L. So it follows that $MP_K/MP_L > r/w$ or, written another way, $MP_K/r > MP_L/w$. These two ratios should be equal to minimize cost. Since the manufacturer gets more marginal output per dollar from capital than from labor, she should use more capital and less labor to minimize the cost of production.

6. Why are isocost lines straight lines?

The isocost line represents all possible combinations of two inputs that may be purchased for a given total cost. The slope of the isocost line is the negative of the ratio of the input prices. If the input prices are fixed, their ratio is constant and the isocost line is therefore straight. Only if the ratio of the input prices changes as the quantities of the inputs change is the isocost line not straight.

7. Assume that the marginal cost of production is increasing. Can you determine whether the average variable cost is increasing or decreasing? Explain.

No. When marginal cost is increasing, average variable cost can be either increasing or decreasing as shown in the diagram below. Marginal cost begins increasing at output level q_1, but *AVC* is decreasing. This happens because *MC* is below *AVC* and is therefore pulling *AVC* down. *AVC* is decreasing for all output levels between q_1 and q_2. At q_2, *MC* cuts through the minimum point of *AVC*, and *AVC* begins to rise because *MC* is above it. Thus for output levels greater than q_2, *AVC* is increasing.



8. Assume that the marginal cost of production is greater than the average variable cost. Can you determine whether the average variable cost is increasing or decreasing? Explain.

Yes, the average variable cost is increasing. If marginal cost is above average variable cost, each additional unit costs more to produce than the average of the previous units, so the average variable cost is pulled upward. This is shown in the diagram above for output levels greater than q_2.

9. If the firm's average cost curves are U-shaped, why does its average variable cost curve achieve its minimum at a lower level of output than the average total cost curve?

Average total cost is equal to average fixed cost plus average variable cost: $ATC = AVC + AFC$. When graphed, the difference between the U-shaped average total cost and U-shaped average variable cost curves is the average fixed cost, and *AFC* is downward sloping at all output levels. When *AVC* is falling, *ATC* will also fall because both *AVC* and *AFC* are declining as output increases. When *AVC* reaches its minimum (the bottom of its U), *ATC* will continue to fall because *AFC* is falling. Even as *AVC* gradually begins to rise, *ATC* will still fall because of *AFC*'s decline. Eventually, however, as *AVC* rises more rapidly, the increases in *AVC* will outstrip the declines in *AFC*, and *ATC* will reach its minimum and then begin to rise.

10. If a firm enjoys economies of scale up to a certain output level, and cost then increases proportionately with output, what can you say about the shape of the long-run average cost curve?

When the firm experiences economies of scale, its long-run average cost curve is downward sloping. When costs increase proportionately with output, the firm's long-run average cost curve is horizontal. So this firm's long-run average cost curve has a rounded L-shape; first it falls and then it becomes horizontal as output increases.

11. How does a change in the price of one input change the firm's long-run expansion path?

The expansion path describes the cost-minimizing combination of inputs that the firm chooses for every output level. This combination depends on the ratio of input prices, so if the price of one input changes, the price ratio also changes. For example, if the price of an input increases, the intercept of the isocost line on that input's axis moves closer to the origin, and the slope of the isocost line (the price ratio) changes. As the price ratio changes, the firm substitutes away from the now more expensive input toward the cheaper input. Thus the expansion path bends toward the axis of the now cheaper input.

12. Distinguish between economies of scale and economies of scope. Why can one be present without the other?

Economies of scale refer to the production of *one* good and occur when total cost increases by a smaller proportion than output. Economies of scope refer to the production of *two or more goods* and occur when joint production is less costly than the sum of the costs of producing each good separately. There is no direct relationship between economies of scale and economies of scope, so production can exhibit one without the other. For example, there are economies of scale producing computers and economies of scale producing carpeting, but if one company produced both, there would likely be no synergies associated with joint production and hence no economies of scope.

13. Is the firm's expansion path always a straight line?

No. If the firm always uses capital and labor in the same proportion, the long run expansion path is a straight line. But if the optimal capital-labor ratio changes as output is increased, the expansion path is not a straight line.

14. What is the difference between economies of scale and returns to scale?

Economies of scale depend on the relationship between cost and output—i.e., how does cost change when output is doubled? Returns to scale depend on what happens to output when all inputs are doubled. The difference is that economies of scale reflect input proportions that change optimally as output is increased, while returns to scale are based on fixed input proportions (such as two units of labor for every unit of capital) as output increases.

■ Exercises

1. Joe quits his computer programming job, where he was earning a salary of $50,000 per year, to start his own computer software business in a building that he owns and was previously renting out for $24,000 per year. In his first year of business he has the following expenses: salary paid to himself, $40,000; rent, $0; other expenses, $25,000. Find the accounting cost and the economic cost associated with Joe's computer software business.

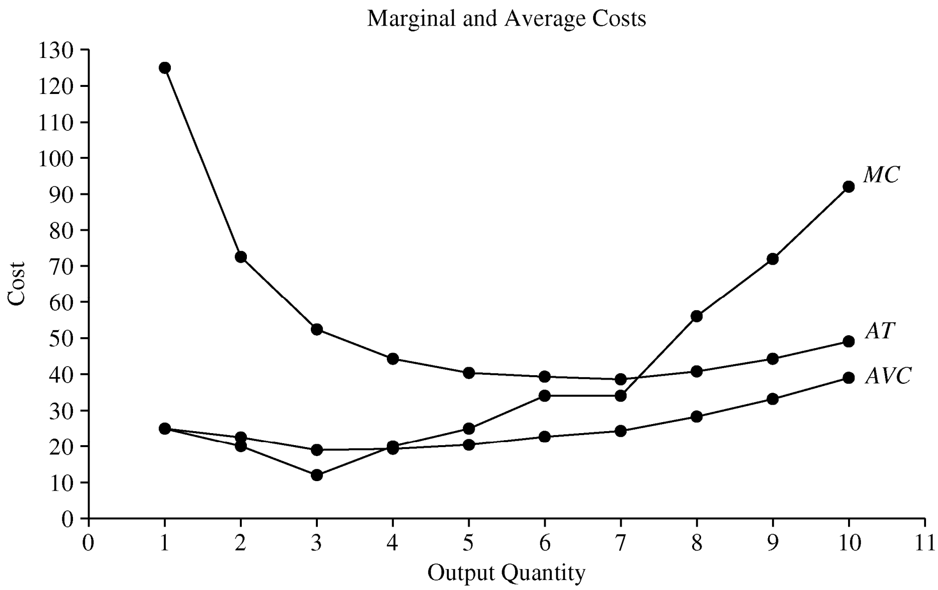
The accounting cost includes only the explicit expenses, which are Joe's salary and his other expenses: $40,000 + 25,000 = $65,000. Economic cost includes these explicit expenses plus opportunity costs. Therefore, economic cost includes the $24,000 Joe gave up by not renting the building and an extra $10,000 because he paid himself a salary $10,000 below market ($50,000 – 40,000). Economic cost is then $40,000 + 25,000 + 24,000 + 10,000 = $99,000.

2. a. Fill in the blanks in the table on page 271 of the textbook.

Units of Output	Fixed Cost	Variable Cost	Total Cost	Marginal Cost	Average Fixed Cost	Average Variable Cost	Average Total Cost
0	100	0	100	—	—	—	—
1	100	25	125	25	100	25	125
2	100	45	145	20	50	22.50	72.50
3	100	57	157	12	33.33	19.00	52.33
4	100	77	177	20	25.00	19.25	44.25
5	100	102	202	25	20.00	20.40	40.40
6	100	136	236	34	16.67	22.67	39.33
7	100	170	270	34	14.29	24.29	38.57
8	100	226	326	56	12.50	28.25	40.75
9	100	298	398	72	11.11	33.11	44.22
10	100	390	490	92	10.00	39.00	49.00

b. Draw a graph that shows marginal cost, average variable cost, and average total cost, with cost on the vertical axis and quantity on the horizontal axis.

Average total cost is U-shaped and reaches a minimum at an output of about 7. Average variable cost is also U-shaped and reaches a minimum at an output between 3 and 4. Notice that average variable cost is always below average total cost. The difference between the two costs is the average fixed cost. Marginal cost is first diminishing, up to a quantity of 3, and then increases as q increases above 3. Marginal cost should intersect average variable cost and average total cost at their respective minimum points, though this is not accurately reflected in the table or the graph. If specific functions had been given in the problem instead of just a series of numbers, then it would be possible to find the exact point of intersection between marginal and average total cost and marginal and average variable cost. The curves are likely to intersect at a quantity that is not a whole number, and hence are not listed in the table or represented exactly in the cost diagram.



3. A firm has a fixed production cost of $5000 and a constant marginal cost of production of $500 per unit produced.

a. What is the firm's total cost function? Average cost?

The variable cost of producing an additional unit, marginal cost, is constant at $500, so $VC = 500q$, and $AVC = \frac{VC}{q} = \frac{500q}{q} = 500$. Fixed cost is $5000 and therefore average fixed cost is $AFC = \frac{5000}{q}$. The total cost function is fixed cost plus variable cost or $TC = 5000 + 500q$.

Average total cost is the sum of average variable cost and average fixed cost: $ATC = 500 + \frac{5000}{q}$.

b. If the firm wanted to minimize the average total cost, would it choose to be very large or very small? Explain.

The firm would choose to be very large because average total cost decreases as q is increased. As q becomes extremely large, *ATC* will equal approximately 500 because the average fixed cost becomes close to zero.

4. Suppose a firm must pay an annual tax, which is a fixed sum, independent of whether it produces any output.

a. How does this tax affect the firm's fixed, marginal, and average costs?

This tax is a fixed cost because it does not vary with the quantity of output produced. If T is the amount of the tax and F is the firm's original fixed cost, the new total fixed cost increases to $TFC = T + F$. The tax does not affect marginal or variable cost because it does not vary with output. The tax increases both average fixed cost and average total cost by T/q.

b. Now suppose the firm is charged a tax that is proportional to the number of items it produces. Again, how does this tax affect the firm's fixed, marginal, and average costs?

Let t equal the per unit tax. When a tax is imposed on each unit produced, total variable cost increases by tq and fixed cost does not change. Average variable cost increases by t, and because fixed costs are constant, average total cost also increases by t. Further, because total cost increases by t for each additional unit produced, marginal cost increases by t.

5. A recent issue of *Business Week* reported the following:

During the recent auto sales slump, GM, Ford, and Chrysler decided it was cheaper to sell cars to rental companies at a loss than to lay off workers. That's because closing and reopening plants is expensive, partly because the auto makers' current union contracts obligate them to pay many workers even if they're not working.

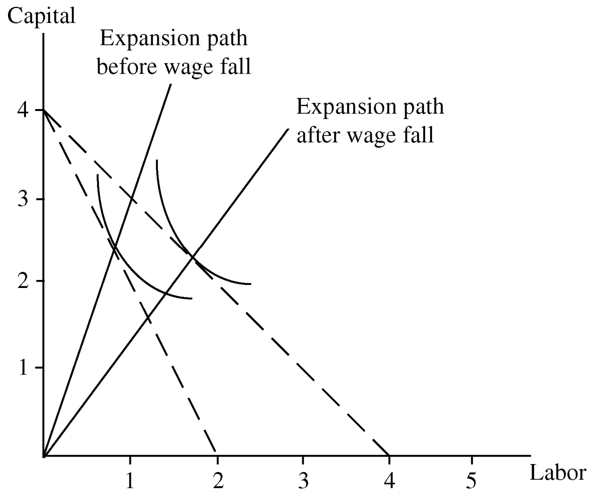
When the article discusses selling cars "at a loss," is it referring to accounting profit or economic profit? How will the two differ in this case? Explain briefly.

When the article refers to the car companies selling at a loss, it is referring to accounting profit. The article is stating that the price obtained for the sale of the cars to the rental companies was less than their accounting cost. Economic profit would be measured by the difference between the price and the opportunity cost of producing the cars. One major difference between accounting and economic cost in this case is the cost of labor. If the car companies must pay many workers even if they are not working, the wages paid to these workers are sunk. If the automakers have no alternative use for these workers (like doing repairs on the factory or preparing the companies' tax returns), the opportunity cost of using them to produce the rental cars is zero. Since the wages would be included in accounting costs, the accounting costs would be higher than the economic costs and would make the accounting profit lower than the economic profit.

6. **Suppose the economy takes a downturn, and that labor costs fall by 50% and are expected to stay at that level for a long time. Show graphically how this change in the relative price of labor and capital affects the firm's expansion path.**

The figure below shows a family of isoquants and two isocost curves. Units of capital are on the vertical axis and units of labor are on the horizontal axis. (***Note*:** The figure assumes that the production function underlying the isoquants implies linear expansion paths. However, the results do not depend on this assumption.)

If the price of labor decreases 50% while the price of capital remains constant, the isocost lines pivot outward. Because the expansion path is the set of points where the *MRTS* is equal to the ratio of prices, as the isocost lines become flatter, the expansion path becomes flatter and moves toward the labor axis. As a result the firm uses more labor relative to capital because labor has become less expensive.



7. **The cost of flying a passenger plane from point *A* to point *B* is $50,000. The airline flies this route four times per day at 7 AM, 10 AM, 1 PM, and 4 PM. The first and last flights are filled to capacity with 240 people. The second and third flights are only half full. Find the average cost per passenger for each flight. Suppose the airline hires you as a marketing consultant and wants to know which type of customer it should try to attract—the off-peak customer (the middle two flights) or the rush-hour customer (the first and last flights). What advice would you offer?**

The average cost per passenger is $50,000/240 = $208.33 for the full flights and $50,000/120 = $416.67 for the half full flights. The airline should focus on attracting more off-peak customers because there is excess capacity on the middle two flights. The marginal cost of taking another passenger on those two flights is almost zero, so the company will increase its profit if it can sell additional tickets for those flights, even if the ticket prices are less than average cost. The peak flights are already full, so attracting more customers at those times will not result in additional ticket sales.

8. You manage a plant that mass-produces engines by teams of workers using assembly machines. The technology is summarized by the production function $q = 5\ KL$, where q is the number of engines per week, K is the number of assembly machines, and L is the number of labor teams. Each assembly machine rents for r = \$10,000 per week, and each team costs w = \$5000 per week. Engine costs are given by the cost of labor teams and machines, plus \$2000 per engine for raw materials. Your plant has a fixed installation of 5 assembly machines as part of its design.

a. What is the cost function for your plant—namely, how much would it cost to produce q engines? What are average and marginal costs for producing q engines? How do average costs vary with output?

The short-run production function is $q = 5(5)L = 25L$, because K is fixed at 5. Thus, for any level of output q, the number of labor teams hired will be $L = \frac{q}{25}$. The total cost function is thus given by the sum of the costs of capital, labor, and raw materials:

$$TC(q) = rK + wL + 2000q = (10{,}000)(5) + (5{,}000)\left(\frac{q}{25}\right) + 2000q$$

$$TC(q) = 50{,}000 + 2200q.$$

The average cost function is then given by:

$$AC(q) = \frac{TC(q)}{q} = \frac{50{,}000 + 2200q}{q}.$$

and the marginal cost function is given by:

$$MC(q) = \frac{dTC}{dq} = 2200.$$

Marginal costs are constant at \$2200 per engine and average costs will decrease as quantity increases because the average fixed cost of capital decreases.

b. How many teams are required to produce 250 engines? What is the average cost per engine?

To produce $q = 250$ engines we need $L = \frac{q}{25}$, so $L = 10$ labor teams. Average costs are \$2400 as shown below:

$$AC(q = 250) = \frac{50{,}000 + 2200(250)}{250} = 2400.$$

c. You are asked to make recommendations for the design of a new production facility. What capital/labor (K/L) ratio should the new plant accommodate if it wants to minimize the total cost of producing at any level of output q?

We no longer assume that K is fixed at 5. We need to find the combination of K and L that minimizes cost at any level of output q. The cost-minimization rule is given by

$$\frac{MP_K}{r} = \frac{MP_L}{w}.$$

To find the marginal product of capital, observe that increasing K by 1 unit increases q by $5L$, so $MP_K = 5L$. Similarly, observe that increasing L by 1 unit increases q by $5K$, so $MP_L = 5K$. Mathematically,

$$MP_K = \frac{\partial q}{\partial K} = 5L \text{ and } MP_L = \frac{\partial q}{\partial L} = 5K.$$

Using these formulas in the cost-minimization rule, we obtain:

$$\frac{5L}{r} = \frac{5K}{w} \Rightarrow \frac{K}{L} = \frac{w}{r} = \frac{5000}{10{,}000} = \frac{1}{2}.$$

The new plant should accommodate a capital to labor ratio of 1 to 2, and this is the same regardless of the number of units produced.

9. The short-run cost function of a company is given by the equation $TC = 200 + 55q$, where TC is the total cost and q is the total quantity of output, both measured in thousands.

a. What is the company's fixed cost?

When $q = 0$, $TC = 200$, so fixed cost is equal to 200 (or \$200,000).

b. If the company produced 100,000 units of goods, what would be its average variable cost?

With 100,000 units, $q = 100$. Variable cost is $55q = (55)(100) = 5500$ (or \$5,500,000). Average variable cost is $\frac{VC}{q} = \frac{5500}{100} = 55$ (or \$55,000).

c. What would be its marginal cost of production?

With constant average variable cost, marginal cost is equal to average variable cost, which is 55 (or \$55,000).

d. What would be its average fixed cost?

At $q = 100$, average fixed cost is $\frac{FC}{q} = \frac{200}{100} = 2$ (or \$2000).

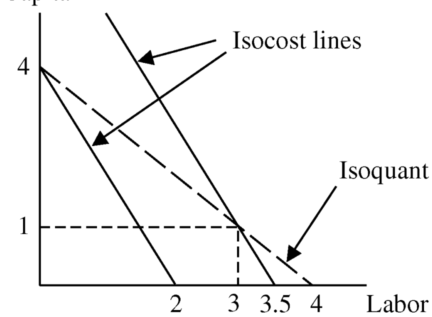
e. Suppose the company borrows money and expands its factory. Its fixed cost rises by \$50,000, but its variable cost falls to \$45,000 per 1000 units. The cost of interest (i) also enters into the equation. Each 1-point increase in the interest rate raises costs by \$3000. Write the new cost equation.

Fixed cost changes from 200 to 250, measured in thousands. Variable cost decreases from 55 to 45, also measured in thousands. Fixed cost also includes interest charges of $3i$. The cost equation is $TC = 250 + 45q + 3i$.

10. A chair manufacturer hires its assembly-line labor for \$30 an hour and calculates that the rental cost of its machinery is \$15 per hour. Suppose that a chair can be produced using 4 hours of labor or machinery in any combination. If the firm is currently using 3 hours of labor for each hour of machine time, is it minimizing its costs of production? If so, why? If not, how can it improve the situation? Graphically illustrate the isoquant and the two isocost lines for the current combination of labor and capital and for the optimal combination of labor and capital.

If the firm can produce one chair with either four hours of labor or four hours of machinery (i.e., capital), or any combination, then the isoquant is a straight line with a slope of −1 and intercepts at $K = 4$ and $L = 4$, as depicted by the dashed line.

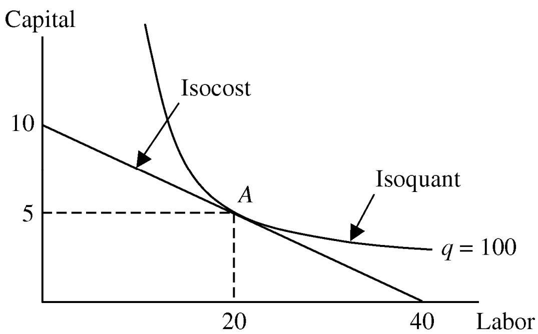
The isocost lines, $TC = 30L + 15K$, have slopes of $-30/15 = -2$ when plotted with capital on the vertical axis and intercepts at $K = TC/15$ and $L = TC/30$. The cost minimizing point is the corner solution where $L = 0$ and $K = 4$, so the firm is not currently minimizing its costs. At the optimal point, total cost is \$60. Two isocost lines are illustrated on the graph. The first one is further from the origin and represents the current higher cost (\$105) of using 3 labor and 1 capital. The firm will find it optimal to move to the second isocost line which is closer to the origin, and which represents a lower cost (\$60). In general, the firm wants to be on the lowest isocost line possible, which is the lowest isocost line that still intersects or touches the given isoquant.



11. Suppose that a firm's production function is $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. The cost of a unit of labor is \$20 and the cost of a unit of capital is \$80.

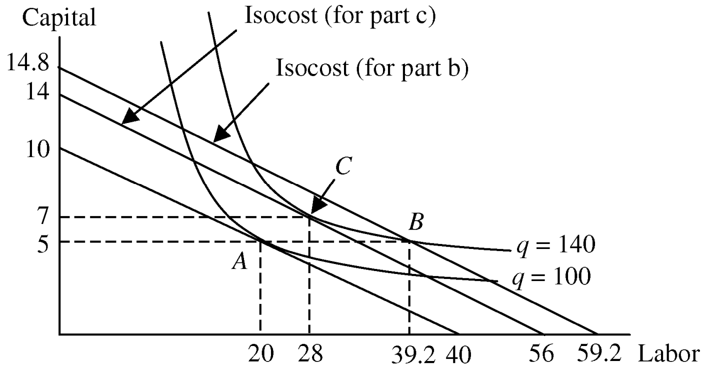
a. The firm is currently producing 100 units of output and has determined that the cost-minimizing quantities of labor and capital are 20 and 5, respectively. Graphically illustrate this using isoquants and isocost lines.

To graph the isoquant, set $q = 100$ in the production function and solve it for K. Solving for K: $K^{1/2} = \dfrac{q}{10L^{1/2}}$ Substitute 100 for q and square both sides. The isoquant is $K = 100/L$. Choose various combinations of L and K and plot them. The isoquant is convex. The optimal quantities of labor and capital are given by the point where the isocost line is tangent to the isoquant. The isocost line has a slope of $-1/4$, given labor is on the horizontal axis. The total cost is $TC = (\$20)(20) + (\$80)(5) = \$800$, so the isocost line has the equation $20L + 80K = 800$, or $K = 10 - 0.25L$, with intercepts $K = 10$ and $L = 40$. The optimal point is labeled A on the graph.



b. The firm now wants to increase output to 140 units. If capital is fixed in the short run, how much labor will the firm require? Illustrate this graphically and find the firm's new total cost.

The new level of labor is 39.2. To find this, use the production function $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$ and substitute 140 for output and 5 for capital; then solve for *L*. The new cost is TC = (\$20)(39.2) + (\$80)(5) = \$1184. The new isoquant for an output of 140 is above and to the right of the original isoquant. Since capital is fixed in the short run, the firm will move out horizontally to the new isoquant and new level of labor. This is point *B* on the graph below. This is not the long-run cost-minimizing point, but it is the best the firm can do in the short run with *K* fixed at 5. You can tell that this is not the long-run optimum because the isocost is not tangent to the isoquant at point *B*. Also there are points on the new (q = 140) isoquant that are below the new isocost (for part b) line. These points all involve hiring more capital and less labor.



c. Graphically identify the cost-minimizing level of capital and labor in the long run if the firm wants to produce 140 units.

This is point *C* on the graph above. When the firm is at point *B* it is not minimizing cost. The firm will find it optimal to hire more capital and less labor and move to the new lower isocost (for part c) line that is tangent to the q = 140 isoquant. Note that all three isocost lines are parallel and have the same slope.

d. If the marginal rate of technical substitution is $\frac{K}{L}$, find the optimal level of capital and labor required to produce the 140 units of output.

Set the marginal rate of technical substitution equal to the ratio of the input costs so that $\frac{K}{L} = \frac{20}{80} \Rightarrow K = \frac{L}{4}$. Now substitute this into the production function for *K*, set q equal to 140, and solve for *L*: $140 = 10L^{\frac{1}{2}}\left(\frac{L}{4}\right)^{\frac{1}{2}} \Rightarrow L = 28,\ K = 7.$ This is point *C* on the graph. The new cost is TC = (\$20)(28) + (\$80)(7) = \$1120, which is less than in the short run (part b), because the firm can adjust all its inputs in the long run.

12. A computer company's cost function, which relates its average cost of production *AC* to its cumulative output in thousands of computers *Q* and its plant size in terms of thousands of computers produced per year *q* (within the production range of 10,000 to 50,000 computers), is given by

$$\boldsymbol{AC = 10 - 0.1Q + 0.3q.}$$

a. Is there a learning curve effect?

The learning curve describes the relationship between the cumulative output and the inputs required to produce a unit of output. Average cost measures the input requirements per unit of output. Learning curve effects exist if average cost falls with increases in cumulative output. Here, average cost decreases by \$0.10 each time cumulative output Q increases by 1000. Therefore, there are learning curve effects.

b. Are there economies or diseconomies of scale?

There are diseconomies of scale. Holding cumulative output, Q, constant, there are diseconomies of scale if the average cost increases as annual output q increases. In this example, average cost increases by \$0.30 for each additional one thousand computers produced, so there are diseconomies of scale.

c. During its existence, the firm has produced a total of 40,000 computers and is producing 10,000 computers this year. Next year it plans to increase production to 12,000 computers. Will its average cost of production increase or decrease? Explain.

First, calculate average cost this year:

$$AC_1 = 10 - 0.1Q + 0.3q = 10 - (0.1)(40) + (0.3)(10) = 9.00.$$

Second, calculate the average cost next year:

$$AC_2 = 10 - (0.1)(50) + (0.3)(12) = 8.60.$$

(***Note*****:** Cumulative output has increased from 40,000 to 50,000, hence $Q = 50$ next year.) The average cost will decrease because of the learning effect, and despite the diseconomies of scale involved when annual output increases from 10 to 12 thousand computers.

13. Suppose the long-run total cost function for an industry is given by the cubic equation $TC = a + bq + cq^2 + dq^3$. Show (using calculus) that this total cost function is consistent with a U-shaped average cost curve for at least some values of *a*, *b*, *c*, and *d*.

To show that the cubic cost equation implies a U-shaped average cost curve, we use algebra, calculus, and economic reasoning to place sign restrictions on the parameters of the equation. These techniques are illustrated by the example below.

First, if output is equal to zero, then $TC = a$, where a represents fixed costs. In the short run, fixed costs are positive, $a > 0$, but in the long run, where all inputs are variable $a = 0$. Therefore, we restrict a to be zero.

Next, we know that average cost must be positive. Dividing TC by q, with $a = 0$:

$$AC = b + cq + dq^2.$$

This equation is simply a quadratic function. When graphed, it has two basic shapes: a U shape and a hill (upside down U) shape. We want the U, i.e., a curve with a minimum (minimum average cost), rather than a hill with a maximum.

At the minimum, the slope should be zero, thus the first derivative of the average cost curve with respect to q must be equal to zero. For a U-shaped AC curve, the second derivative of the average cost curve must be positive.

The first derivative is $c + 2dq$; the second derivative is $2d$. If the second derivative is to be positive, then $d > 0$. If the first derivative is to equal zero, then solving for c as a function of q and d yields: $c = -2dq$. Since d is positive, and the minimum AC must be at some point where q is positive, then c must be negative: $c < 0$.

To restrict b, we know that at its minimum, average cost must be positive. The minimum occurs when $c + 2dq = 0$. Solve for q as a function of c and d: $q = -c/2d > 0$. Next, substituting this value for q into the expression for average cost, and simplifying the equation:

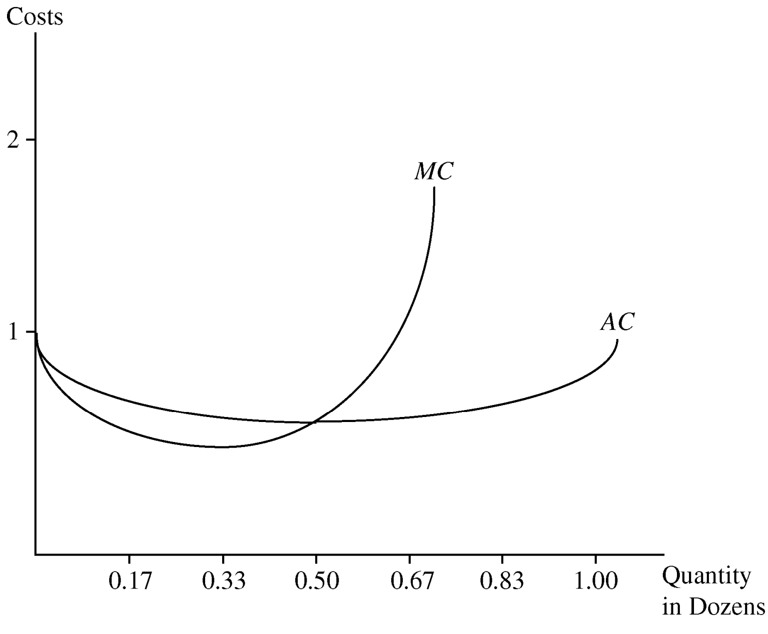
$$AC = b + cq + dq^2 = b + c\left(\frac{-c}{2d}\right) + d\left(\frac{-c}{2d}\right)^2, \text{ or}$$

$$AC = b - \frac{c^2}{2d} + \frac{c^2}{4d} = b - \frac{2c^2}{4d} + \frac{c^2}{4d} = b - \frac{c^2}{4d} > 0.$$

This implies $b > \frac{c^2}{4d}$. Because $c^2 > 0$ and $d > 0$, b must be positive.

In summary, for U-shaped long-run average cost curves, a must be zero, b and d must be positive, c must be negative, and $4db > c^2$. However, these conditions do not ensure that marginal cost is positive. To insure that marginal cost has a U shape and that its minimum is positive, use the same procedure, i.e., solve for q at minimum marginal cost: $q = -c/3d$. Then substitute into the expression for marginal cost: $b + 2cq + 3dq^2$. From this we find that c^2 must be less than $3bd$. Notice that parameter values that satisfy this condition also satisfy $4db > c^2$, but not the reverse, so $c^2 < 3bd$ is the more stringent requirement.

For example, let $a = 0$, $b = 1$, $c = -1$, $d = 1$. These values satisfy all the restrictions derived above. Total cost is $q - q^2 + q^3$, average cost is $1 - q + q^2$, and marginal cost is $1 - 2q + 3q^2$. Minimum average cost is where $q = 1/2$ and minimum marginal cost is where $q = 1/3$ (think of q as dozens of units, so no fractional units are produced). See the figure below.



14. A computer company produces hardware and software using the same plant and labor. The total cost of producing computer processing units H and software programs S is given by

$$\boldsymbol{TC = aH + bS - cHS}$$

where a, b, and c are positive. Is this total cost function consistent with the presence of economies or diseconomies of scale? With economies or diseconomies of scope?

If each product were produced by itself there would be neither economies nor diseconomies of scale. To see this, define the total cost of producing H alone (TC_H) to be the total cost when $S = 0$. Thus $TC_H = aH$. Similarly, $TC_S = bS$. In both cases, doubling the number of units produced doubles the total cost, so there are no economies or diseconomies of scale.

Economies of scope exist if $S_C > 0$, where, from equation (7.7) in the text:

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)}.$$

In our case, $C(q_1)$ is TC_H, $C(q_2)$ is TC_S, and $C(q_1, q_2)$ is TC. Therefore,

$$SC = \frac{aH + bS - (aH + bS - cHS)}{aH + bS - cHS} = \frac{cHS}{aH + bS - cHS}.$$

Because cHS (the numerator) and TC (the denominator) are both positive, it follows that $SC > 0$, and hence there are economies of scope.

Chapter 8
Profit Maximization and Competitive Supply

■ Teaching Notes

This chapter begins by explaining what economists mean by a competitive market and why it makes sense to assume that firms try to maximize profit. The chapter then covers the choice of optimal output in the short run, thereby revealing the underlying structure of short-run supply curves, the choice of output in the long run, and long-run competitive equilibrium. Along the way, the concepts of producer surplus and economic rent are introduced, and the chapter ends with development of the long-run industry supply curve.

Section 8.1 identifies the three basic assumptions of perfect competition and Section 8.2 discusses the assumption of profit maximization as the goal of the firm. Students often find some of these assumptions to be unrealistic and/or very restrictive, so it is important to acknowledge that the world is more complex, but remind them that our goal is prediction. Although the world does not work quite as simply as the model of perfect competition suggests, the model predicts quite well even when some of the assumptions hold loosely at best. To put perfect competition in perspective, it can be helpful to give a brief overview of monopoly, oligopoly, and monopolistic competition before or after presenting the assumptions of perfect competition. Restrict this discussion to identifying the number of firms in the industry, product differentiation, and barriers to entry and exit. This has the added benefit of stimulating student interest in future topics.

Section 8.3 derives the general result that any firm wanting to maximize profit should produce where marginal revenue is equal to marginal cost. It also identifies perfect competition as a special case where marginal revenue equals the market price because firms are price takers. You should clear up any confusion about the difference between market demand and the horizontal demand curve facing an individual firm. Students find the idea of a horizontal demand curve strange, so spend a few minutes explaining why it is reasonable under the conditions of perfect competition. If your students have had calculus, you can derive the marginal revenue equals marginal cost rule by differentiating the profit function with respect to q. If your students have not had calculus, then it may be helpful to work with tables containing cost and revenue information so they can see that profit is maximized where marginal revenue equals marginal cost. In either case, explain intuitively why profit cannot be at its maximum for a perfectly competitive firm if price is either greater or less than marginal cost. Figure 8.3 is useful for this purpose. Emphasize that the perfectly competitive firm chooses quantity and not price when maximizing profit.

Section 8.4 further explores the firm's decision to produce where price is equal to marginal cost. The two points where $P = MC$ in Figure 8.3 may require some additional explanation. If your students know calculus you can invoke second-order conditions. If not, explain intuitively that q_0 is the profit-*minimizing* output because the firm incurs losses on each unit it sells (since $P < MC$) up to that point. Students sometimes have difficulty determining the firm's profit in diagrams like those in Figures 8.3 and 8.4. The text explains that the distance between P and ATC is the average profit per unit, which is then multiplied by q to determine profit. I like to write this out algebraically as $(P - ATC)q = P \times q - ATC \times q = R - (TC/q) \times q = R - TC = \pi$. Finally, some students have difficulty with the shut-down rule. For some reason, they confuse the role of variable and fixed costs, so you may want to repeat this a couple of times, and a numerical example definitely helps. Although it may seem obvious, it is worth mentioning that maximizing profit is the same as minimizing loss.

Section 8.5 shows that the firm's short-run supply curve is the portion of its marginal cost curve that lies above its average variable cost curve. Carefully go over a diagram like the one in Figure 8.6 in considerable detail to make the case. This is important. Example 8.4 is interesting because it gives a "realistic" example of a firm's short-run marginal cost (and hence supply) curve where production is lumpy, leading to curves with steps in them.

In Section 8.6, you might mention that the short-run market supply is found by adding the firms' supply curves *horizontally*. You should give a numerical example, and you might consider using two supply curves such as $q_1 = -4 + P$ and $q_2 = -3 + 0.5P$ if you want to show the mechanics in more detail. Neither firm supplies the market when price is below 4; only firm 1 produces output when price is between 4 and 6; and both firms contribute to market supply at prices above 6. The short-run market supply has a kink at $P = 6$ ($q = 2$), and for prices above 6, the market supply is found by adding $Q = q_1 + q_2 = -7 + 1.5P$. Producer surplus is introduced briefly in this section. More will be made of it in Chapter 9, but it is useful to distinguish producer surplus from profit at this juncture.

Section 8.7 covers long-run profit maximization and the long-run competitive equilibrium. This is a good time to reiterate the definition of economic profit and to explain that zero economic profit is a lot different from zero accounting profit. Firms earning zero economic profit are doing as well as they could in their *best* alternative, so they are doing reasonably well and have no incentive to leave the industry. The most difficult part of this section is the material on cost differences among firms (starting on page 304), economic rent and producer surplus in the long run. Students find the ideas subtle and hard to grasp, so expect to slow down and give a number of examples to help clarify the concepts.

The final section of the chapter develops the industry long-run supply curve and distinguishes constant-cost, increasing-cost and decreasing-cost industries. Be sure to emphasize that constant-cost, increasing-cost and decreasing-cost have nothing to do with constant, increasing and decreasing returns to scale or with economies of scale. The names are so similar that this is a common mistake. The other problem you may encounter is the belief that the long-run supply curve is derived from the long-run marginal cost curve, similar to the short run case. While that may seem plausible to some, be sure to explain why it is not true.

■ Review Questions

1. **Why would a firm that incurs losses choose to produce rather than shut down?**

   Losses occur when revenues do not cover total costs. If revenues are greater than variable costs, but not total costs, the firm is better off producing in the short run rather than shutting down, even though it is incurring a loss. The reason is that the firm will be stuck will all its fixed cost and have no revenue if it shuts down, so its loss will equal its fixed cost. If it continues to produce, however, and revenue is greater than variable costs, the firm can pay for some of its fixed cost, so its loss is less than it would be if it shut down. In the long run, all costs are variable, and thus all costs must be covered if the firm is to remain in business.

2. **Explain why the industry supply curve is not the long-run industry marginal cost curve.**

   In the short run, a change in the market price induces the profit-maximizing firm to change its optimal level of output. This optimal output occurs where price is equal to marginal cost, as long as marginal cost exceeds average variable cost. Therefore, the short-run supply curve of the firm is its marginal cost curve, above average variable cost. (When the price falls below average variable cost, the firm will shut down.)

   In the long run, a change in the market price induces entry into or exit from the industry and may induce existing firms to change their optimal outputs as well. As a result, the prices firms pay for inputs can change, and these will cause the firms' marginal costs to shift up or down. Therefore, long-run supply is not the sum of the existing firms' long-run marginal cost curves. The long-run supply curve depends on the number of firms in the market and on how their costs change due to any changes in input costs.

As a simple example, consider a constant-cost industry where each firm has a U-shaped *LAC* curve. Here the input prices do not change, only the number of firms changes when industry price changes. Each firm has an increasing *LMC*, but the industry long-run supply is horizontal because any change in industry output comes about by firms entering or leaving the industry, not by existing firms moving up or down their *LMC* curves.

3. **In long-run equilibrium, all firms in the industry earn zero economic profit. Why is this true?**

   The theory of perfect competition explicitly assumes that there are no entry or exit barriers to new participants in an industry. With free entry, positive economic profits induce new entrants. As these firms enter, the supply curve shifts to the right, causing a fall in the equilibrium price of the product. Entry will stop, and equilibrium will be achieved, when economic profits have fallen to zero.

   Some firms may earn greater *accounting* profits than others because, for example, they own a superior source of an important input, but their *economic* profits will be the same. To be more concrete, suppose one firm can mine a critical input for $2 per pound while all other firms in the industry have to pay $3 per pound. The one firm will have an accounting cost advantage and will report higher accounting profits than other firms in the industry. But there is an opportunity cost associated with the company's input use, because other firms would be willing to pay up to $3 per pound to buy the input from the firm with the superior mine. Therefore, the company should include a $1 per pound opportunity cost for using its own input rather than selling it to other firms. Then, that firm's economic costs and economic profit will be the same as all the other firms in the industry. So all firms will earn zero economic profit in the long run.

4. **What is the difference between economic profit and producer surplus?**

   Economic profit is the difference between total revenue and total cost. Producer surplus is the difference between total revenue and total variable cost. So fixed cost is subtracted to find profit but not producer surplus, and thus profit equals producer surplus minus fixed cost (or producer surplus equals profit plus fixed cost).

5. **Why do firms enter an industry when they know that in the long run economic profit will be zero?**

   Firms enter an industry when they expect to earn economic profit, even if the profit will be short-lived. These short-run economic profits are enough to encourage entry because there is no cost to entering the industry, and some economic profit is better than none. Zero economic profit in the long run implies *normal* returns to the factors of production, including the labor and capital of the owner of the firm. So even when economic profit falls to zero, the firm will be doing as well as it could in any other industry, and then the owner will be indifferent to staying in the industry or exiting.

6. **At the beginning of the twentieth century, there were many small American automobile manufacturers. At the end of the century, there were only three large ones. Suppose that this situation is not the result of lax federal enforcement of antimonopoly laws. How do you explain the decrease in the number of manufacturers? (*Hint*: What is the inherent cost structure of the automobile industry?)**

   Automobile plants are highly capital-intensive, and consequently there are substantial economies of scale in production. So, over time, the automobile companies that produced larger quantities of cars were able to produce at lower average cost. They then sold their cars for less and eventually drove smaller (higher cost) companies out of business, or bought them to become even larger and more efficient. At very large levels of production, the economies of scale diminish, and diseconomies of scale may even occur. This would explain why more than one manufacturer remains.

7. **Because industry *X* is characterized by perfect competition, every firm in the industry is earning zero economic profit. If the product price falls, no firms can survive. Do you agree or disagree? Discuss.**

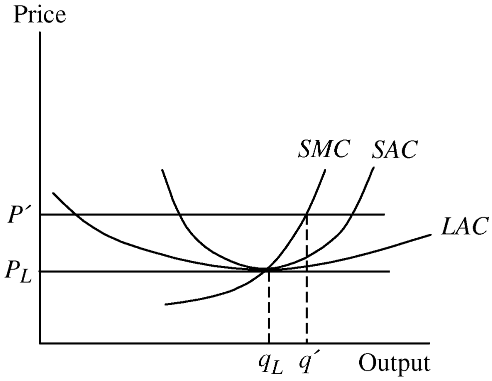
   Disagree. If the market price falls, all firms will suffer economic losses. They will cut production in the short run but continue in business as long as price is above average variable cost. In the long run, however, if price stays below average total cost, some firms will exit the industry. As firms leave industry *X*, the market supply decreases (i.e., shifts to the left). This causes the market price to increase. Eventually enough firms exit so that price increases to the point where profits return to zero for those firms still in the industry, and those firms will continue to survive and produce product *X*.

8. **An increase in the demand for movies also increases the salaries of actors and actresses. Is the long-run supply curve for films likely to be horizontal or upward sloping? Explain.**

   The long-run supply curve depends on the cost structure of the industry. Assuming there is a relatively fixed supply of actors and actresses, as more films are produced, higher salaries must be offered. Therefore the industry experiences increasing costs. In an increasing-cost industry, the long-run supply curve is upward sloping. Thus the supply curve for films would be upward sloping.

9. **True or false: A firm should always produce at an output at which long-run average cost is minimized. Explain.**

   False. In the long run, under perfect competition, firms will produce where long-run average cost is minimized. In the short run, however, it may be optimal to produce at a different level. For example, if price is above the long-run equilibrium price, the firm will maximize short-run profit by producing a greater amount of output than the level at which *LAC* is minimized as illustrated in the diagram. P_L is the long-run equilibrium price, and q_L is the output level that minimizes *LAC*. If price increases to P' in the short run, the firm maximizes profit by producing q', which is greater than q_L, because that is the output level at which *SMC* (short-run marginal cost) equals price.



10. **Can there be constant returns to scale in an industry with an upward-sloping supply curve? Explain.**

    Yes. Constant returns to scale means that proportional increases in all inputs yield the same proportional increase in output. However, when all firms increase their input use, the prices of some inputs may rise, because their supply curves are upward sloping. For example, production that uses rare or depleting inputs will see higher input costs as production increases. Doubling inputs will still yield double output, but because of rising input prices, production costs will more than double. In this case the industry is an increasing-cost industry, and it will have an upward-sloping long-run supply curve. Therefore, an industry can have both constant returns to scale and an upward-sloping industry supply curve.

11. What assumptions are necessary for a market to be perfectly competitive? In light of what you have learned in this chapter, why is each of these assumptions important?

The three primary assumptions of perfect competition are (1) all firms in the industry are price takers, (2) all firms produce identical products, and (3) there is free entry and exit of firms to and from the market. The first two assumptions are important because they imply that no firm has any market power and that each faces a horizontal demand curve. As a result, firms produce where price equals marginal cost, which defines their supply curves. With free entry and exit, positive (negative) economic profits encourage firms to enter (exit) the industry. Entry and exit affect industry supply and price. In the long run, entry or exit continues until price equals long-run average cost and firms earn zero economic profit.

12. Suppose a competitive industry faces an increase in demand (i.e., the demand curve shifts upward). What are the steps by which a competitive market insures increased output? Will your answer change if the government imposes a price ceiling?

If demand increases, price and profits increase in the short run. The price increase causes existing firms to increase output, and the positive profits induce new firms to enter the industry in the long run, shifting the supply curve to the right. This results in a new equilibrium with a higher quantity and a price (less than the short-run price) that earns all firms zero economic profit. With an effective price ceiling, price will not increase when demand increases, and firms will therefore not increase output. Also, without an increase in economic profit, no new firms enter, and there is no shift in the supply curve. So the result is very different with a price ceiling. Output does not increase as a result of the increase in demand. Instead there is a shortage of the product.

13. The government passes a law that allows a substantial subsidy for every acre of land used to grow tobacco. How does this program affect the long-run supply curve for tobacco?

A subsidy on land used to grow tobacco decreases every farmer’s average cost of producing tobacco and will lead existing tobacco growers to increase output. In addition, tobacco farmers will make positive economic profits that will encourage other firms to enter tobacco production. The result is that both the short-run and long-run supply curves for the industry will shift down and to the right.

14. A certain brand of vacuum cleaners can be purchased from several local stores as well as from several catalogues or websites.

a. If all sellers charge the same price for the vacuum cleaner, will they all earn zero economic profit in the long run?

Yes, all earn zero economic profit in the long run. If economic profit were greater than zero for, say, online sellers, then firms would enter the online industry and eventually drive economic profit for online sellers to zero. If economic profit were negative for catalogue sellers, some catalogue firms would exit the industry until economic profit returned to zero. So all must earn zero economic profit in the long run. Anything else will generate entry or exit until economic profit returns to zero.

b. If all sellers charge the same price and one local seller owns the building in which he does business, paying no rent, is this seller earning a positive economic profit?

No this seller would still earn zero economic profit. If he pays no rent then the accounting cost of using the building is zero, but there is still an opportunity cost, which represents the value of the building in its best alternative use.

c. Does the seller who pays no rent have an incentive to lower the price that he charges for the vacuum cleaner?

No he has no incentive to charge a lower price because he can sell as many units as he wants at the current market price. Lowering his price will only reduce his economic profit. Since all firms sell the identical good, they will all charge the same price for that good.

■ Exercises

1. The data in the table below give information about the price (in dollars) for which a firm can sell a unit of output and the total cost of production.

a. Fill in the blanks in the table.

b. Show what happens to the firm's output choice and profit if the price of the product falls from \$60 to \$50.

q	P	R $P = 60$	C	π $P = 60$	MC $P = 60$	MR $P = 60$	R $P = 50$	MR $P = 50$	π $P = 50$
0	60		100						
1	60		150						
2	60		178						
3	60		198						
4	60		212						
5	60		230						
6	60		250						
7	60		272						
8	60		310						
9	60		355						
10	60		410						
11	60		475						

The table below shows the firm's revenue and cost for the two prices.

q	P	R $P = 60$	C	π $P = 60$	MC $P = 60$	MR $P = 60$	R $P = 50$	MR $P = 50$	π $P = 50$
0	60	0	100	–100	—	—	0	—	–100
1	60	60	150	–90	50	60	50	50	–100
2	60	120	178	–58	28	60	100	50	–78
3	60	180	198	–18	20	60	150	50	–48
4	60	240	212	28	14	60	200	50	–12
5	60	300	230	70	18	60	250	50	20
6	60	360	250	110	20	60	300	50	50
7	60	420	272	148	22	60	350	50	78
8	60	480	310	170	38	60	400	50	90
9	60	540	355	185	45	60	450	50	95
10	60	600	410	190	55	60	500	50	90
11	60	660	475	185	65	60	550	50	75

At a price of \$60, the firm should produce ten units of output to maximize profit, which is \$190 when $q = 10$. This is also the point closest to where price equals marginal cost without having marginal cost exceed price. At a price of \$50, the firm should produce nine units to maximize profit, which will be \$95. Thus when price falls from \$60 to \$50, the firm's output drops from 10 to 9 units and profit falls from \$190 to \$95.

2. **Using the data in the table, show what happens to the firm's output choice and profit if the fixed cost of production increases from \$100 to \$150 and then to \$200. Assume that the price of the output remains at \$60 per unit. What general conclusion can you reach about the effects of fixed costs on the firm's output choice?**

   The table below shows the firm's revenue and cost information for fixed cost (F) of 100, 150, and 200.

   In all three cases, with fixed cost equal to 100, then 150, and then 200, the firm will produce 10 units of output because this is the point closest to where price equals marginal cost without having marginal cost exceed price. Fixed costs do not influence the optimal quantity, because they do not influence marginal cost. Higher fixed costs result in lower profits, but the highest profit always occurs at the same level of output, which is 10 units in this example.

q	*P*	*R*	*C* F = 100	*π* F = 100	*MC*	*C* F = 150	*π* F = 150	*C* F = 200	*π* F = 200
0	60	0	100	–100	—	150	–150	200	–200
1	60	60	150	–90	50	200	–140	250	–190
2	60	120	178	–58	28	228	–108	278	–158
3	60	180	198	–18	20	248	–68	298	–118
4	60	240	212	28	14	262	–22	312	–72
5	60	300	230	70	18	280	20	330	–30
6	60	360	250	110	20	300	60	350	10
7	60	420	272	148	22	322	98	372	48
8	60	480	310	170	38	360	120	410	70
9	60	540	355	185	45	405	135	455	85
10	60	600	410	190	55	460	140	510	90
11	60	660	475	185	65	525	135	575	85

3. Use the same information as in Exercise 1.

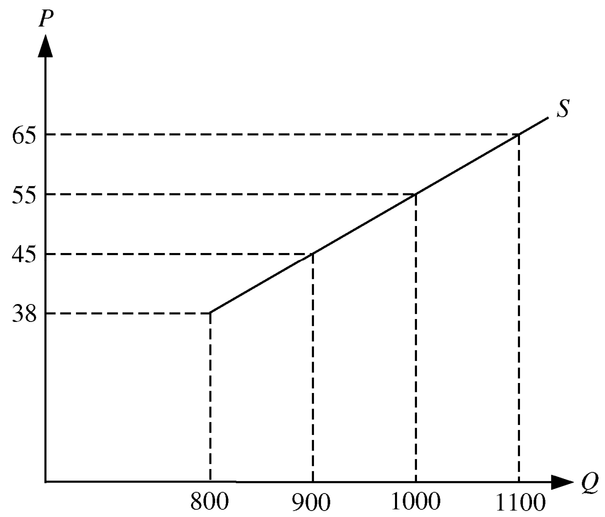
a. Derive the firm's short-run supply curve. (*Hint*: You may want to plot the appropriate cost curves.)

The firm's short-run supply curve is its marginal cost curve above average variable cost. The table below lists marginal cost, total cost, variable cost, fixed cost, and average variable cost. The firm will produce 8 or more units depending on the market price and will not produce in the 0 – 7 units of output range because in this range *MC* is less than *AVC*. When *MC* is below *AVC*, the firm minimizes losses by shutting down and producing nothing in the short run. The points on the firm's supply curve are therefore (38,8), (45,9), (55,10) and (65,11), where the first number inside the parentheses is price and the second is output *q*.

q	*C*	*MC*	*TVC*	*TFC*	*AVC*
0	100	—	0	100	—
1	150	50	50	100	50.0
2	178	28	78	100	39.0
3	198	20	98	100	32.7
4	212	14	112	100	28.0
5	230	18	130	100	26.0
6	250	20	150	100	25.0
7	272	22	172	100	24.6
8	310	38	210	100	26.3
9	355	45	255	100	28.3
10	410	55	310	100	31.0
11	475	65	375	100	34.1

b. If 100 identical firms are in the market, what is the industry supply curve?

For 100 firms with identical cost structures, the market supply curve is the horizontal summation of each firm's output at each price. From the table above we know that when $P = 38$, each firm will produce 8 units, because $MC = 38$ at an output of 8 units. Therefore, when $P = 38$, $Q = 800$ units would be supplied by all the firms in the industry. The other points we know are: $P = 45$ and $Q = 900$, $P = 55$ and $Q = 1000$, and $P = 65$ and $Q = 1100$. The industry supply curve is shown in the diagram below.



4. Suppose you are the manager of a watchmaking firm operating in a competitive market. Your cost of production is given by $C = 200 + 2q^2$, where q is the level of output and C is total cost. (The marginal cost of production is $4q$; the fixed cost is \$200.)

a. If the price of watches is \$100, how many watches should you produce to maximize profit?

Profits are maximized where price equals marginal cost. Therefore,

$$100 = 4q, \text{ or } q = 25.$$

b. What will the profit level be?

Profit is equal to total revenue minus total cost: $\pi = Pq - (200 + 2q^2)$. Thus,

$$\pi = (100)(25) - (200 + 2(25)^2) = \$1050.$$

c. At what minimum price will the firm produce a positive output?

The firm will produce in the short run if its revenues are greater than its total variable costs. The firm's short-run supply curve is its MC curve above minimum AVC. Here, $AVC = \frac{VC}{q} = \frac{2q^2}{q} = 2q$.

Also, $MC = 4q$. So, MC is greater than AVC for any quantity greater than 0. This means that the firm produces in the short run as long as price is positive.

5. Suppose that a competitive firm's marginal cost of producing output q is given by $MC(q) = 3 + 2q$. Assume that the market price of the firm's product is $9.

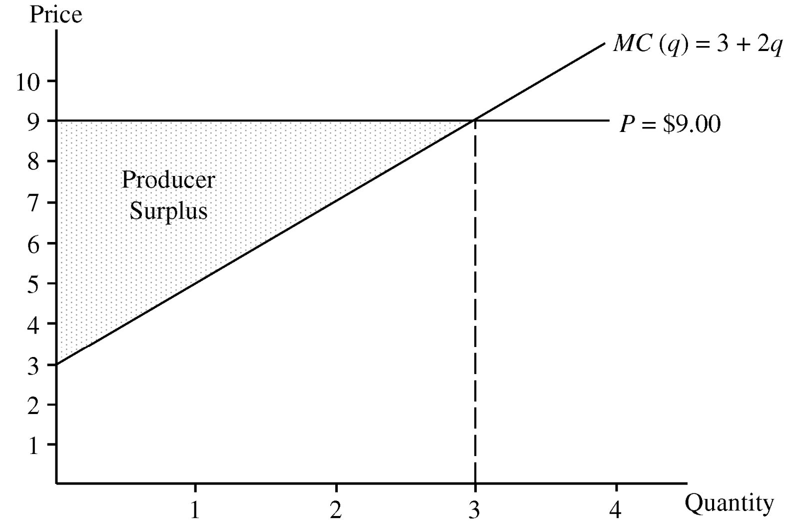
a. What level of output will the firm produce?

The firm should set the market price equal to marginal cost to maximize its profits:

$$9 = 3 + 2q, \text{ or } q = 3.$$

b. What is the firm's producer surplus?

Producer surplus is equal to the area below the market price, i.e., $9.00, and above the marginal cost curve, i.e., $3 + 2q$. Because MC is linear, producer surplus is a triangle with a base equal to 3 (since $q = 3$) and a height of $6 ($9 - 3 = 6$). The area of a triangle is $(1/2) \times (\text{base}) \times (\text{height})$. Therefore, producer surplus is $(0.5)(3)(6) = \$9$.



c. Suppose that the average variable cost of the firm is given by $AVC(q) = 3 + q$. Suppose that the firm's fixed costs are known to be $3. Will the firm be earning a positive, negative, or zero profit in the short run?

Profit is equal to total revenue minus total cost. Total cost is equal to total variable cost plus fixed cost. Total variable cost is equal to $AVC(q) \times q$. At $q = 3$, $AVC(q) = 3 + 3 = 6$, and therefore

$$TVC = (6)(3) = \$18.$$

Fixed cost is equal to $3. Therefore, total cost equals TVC plus TFC, or

$$C = \$18 + 3 = \$21.$$

Total revenue is price times quantity:

$$R = (\$9)(3) = \$27.$$

Profit is total revenue minus total cost:

$$\pi = \$27 - 21 = \$6.$$

Therefore, the firm is earning positive economic profits. More easily, you might recall that profit equals producer surplus minus fixed cost. Since we found that producer surplus was \$9 in part b, profit equals 9 – 3 or \$6.

6. A firm produces a product in a competitive industry and has a total cost function $C = 50 + 4q + 2q^2$ and a marginal cost function $MC = 4 + 4q$. At the given market price of \$20, the firm is producing 5 units of output. Is the firm maximizing its profit? What quantity of output should the firm produce in the long run?

If the firm is maximizing profit, then price will be equal to marginal cost. $P = MC$ results in $20 = 4 + 4q$, or $q = 4$. The firm is not maximizing profit; it is producing too much output. The current level of profit is

$$\pi = Pq - C = 20(5) - (50 + 4(5) + 2(5)^2) = -20,$$

and the profit maximizing level is

$$\pi = 20(4) - (50 + 4(4) + 2(4)^2) = -18.$$

Given no change in the price of the product or the cost structure of the firm, the firm should produce $q = 0$ units of output in the long run since economic profit is negative at the quantity where price equals marginal cost. The firm should exit the industry.

7. Suppose the same firm's cost function is $C(q) = 4q^2 + 16$.

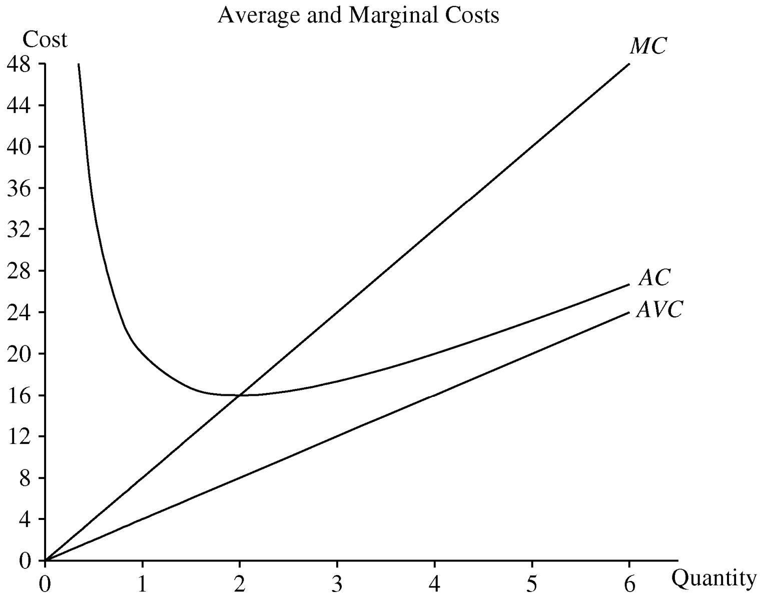
a. Find variable cost, fixed cost, average cost, average variable cost, and average fixed cost. (*Hint*: Marginal cost is given by $MC = 8q$.)

Variable cost is that part of total cost that depends on q (so $VC = 4q^2$) and fixed cost is that part of total cost that does not depend on q (so $FC = 16$).

$$VC = 4q^2$$
$$FC = 16$$
$$AC = \frac{C(q)}{q} = 4q + \frac{16}{q}$$
$$AVC = \frac{VC}{q} = 4q$$
$$AFC = \frac{FC}{q} = \frac{16}{q}$$

b. Show the average cost, marginal cost, and average variable cost curves on a graph.

Average cost is U-shaped. Average cost is relatively large at first because the firm is not able to spread the fixed cost over very many units of output. As output increases, average fixed cost falls quickly, leading to a rapid decline in average cost. Average cost will increase at some point because the average fixed cost will become very small and average variable cost is increasing as q increases. MC and AVC are linear in this example, and both pass through the origin. Average variable cost is everywhere below average cost. Marginal cost is everywhere above average variable cost. If the average is rising, then the marginal must be above the average. Marginal cost intersects average cost at its minimum point, which occurs at a quantity of 2 where *MC* and *AC* both equal $16.



c. Find the output that minimizes average cost.

Minimum average cost occurs at the quantity where *MC* is equal to *AC*:

$$AC = 4q + \frac{16}{q} = 8q = MC$$

$$\frac{16}{q} = 4q$$

$$16 = 4q^2$$

$$4 = q^2$$

$$2 = q.$$

d. At what range of prices will the firm produce a positive output?

The firm will supply positive levels of output in the short run as long as $P = MC > AVC$, or as long as the firm is covering its variable costs of production. In this case, marginal cost is above average variable cost at all output levels, so the firm will supply positive output at any positive price.

e. At what range of prices will the firm earn a negative profit?

The firm will earn negative profit when $P = MC < AC$, or at any price below minimum average cost. In part c we found that the minimum average cost occurs where $q = 2$. Plug $q = 2$ into the average cost function to find $AC = 16$. The firm will therefore earn negative profit if price is below 16.

f. At what range of prices will the firm earn a positive profit?

In part e we found that the firm would earn negative profit at any price below 16. The firm therefore earns positive profit as long as price is above 16.

8. A competitive firm has the following short-run cost function:

$$C(q) = q^3 - 8q^2 + 30q + 5.$$

a. Find MC, AC, and AVC and sketch them on a graph.

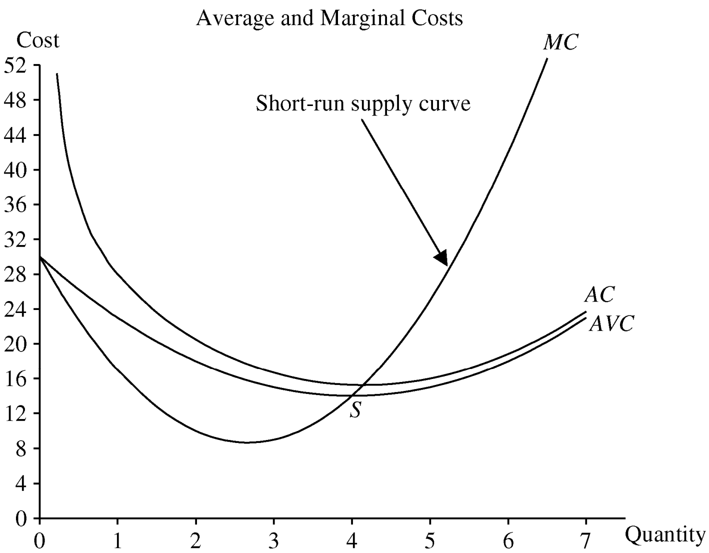
The functions can be calculated as follows:

$$MC = \frac{dC}{dq} = 3q^2 - 16q + 30$$

$$AC = \frac{C}{q} = q^2 - 8q + 30 + \frac{5}{q}$$

$$AVC = \frac{VC}{q} = q^2 - 8q + 30.$$

Graphically, all three cost functions are U-shaped in that cost initially declines as q increases, and then increases as q increases. Average variable cost is below average cost everywhere. Marginal cost is initially below AVC and then increases to intersect AVC at its minimum point, S. MC is also initially below AC and then intersects AC at its minimum point.



b. At what range of prices will the firm supply zero output?

The firm will find it profitable to produce in the short run as long as price is greater than or equal to average variable cost. If price is less than average variable cost then the firm will be better off shutting down in the short run, as it will only lose its fixed cost and not fixed plus some variable cost. Here we need to find the minimum average variable cost, which can be done in two different ways. You can either set marginal cost equal to average variable cost, or you can differentiate average variable cost with respect to q and set this equal to zero. In both cases, you can solve for q and then plug into AVC to find the minimum AVC. Here we will set AVC equal to MC:

$$AVC = q^2 - 8q + 30 = 3q^2 - 16q + 30 = MC$$
$$2q^2 = 8q$$
$$q = 4$$
$$AVC(q = 4) = 4^2 - 8 * 4 + 30 = 14.$$

This is point S on the graph. Hence the firm supplies zero output if $P <$ \$14.

c. Identify the firm's supply curve on your graph.

The firm's supply curve is the MC curve above the point where $MC = AVC$. The firm will produce at the point where price equals MC as long as MC is greater than or equal to AVC. This point is labeled S on the graph, so the short-run supply curve is the portion of MC that lies above point S.

d. At what price would the firm supply exactly 6 units of output?

The firm maximizes profit by choosing the level of output such that $P = MC$. To find the price where the firm would supply 6 units of output, set q equal to 6 and solve for MC:

$$P = MC = 3q^2 - 16q + 30 = 3(6^2) - 16(6) + 30 = 42.$$

9. a. Suppose that a firm's production function is $q = 9x^{\frac{1}{2}}$ in the short run, where there are fixed costs of \$1000, and x is the variable input whose cost is \$4000 per unit. What is the total cost of producing a level of output q? In other words, identify the total cost function $C(q)$.

Since the variable input costs \$4000 per unit, total variable cost is 4000 times the number of units used, or $4000x$. Therefore, the total cost of the inputs used is $C(x)$ = variable cost + fixed cost = $4000x + 1000$. Now rewrite the production function to express x in terms of q: $x = \dfrac{q^2}{81}$. We can then substitute this into the above cost function to find $C(q)$:

$$C(q) = \frac{4000q^2}{81} + 1000,$$

$$\text{or } C(q) = 49.3827q^2 + 1000.$$

b. Write down the equation for the supply curve.

The firm supplies output where $P = MC$ as long as $MC > AVC$. In this example, $MC = 98.7654q$ is always greater than $AVC = 49.3827q$, so the entire marginal cost curve is the supply curve. Therefore $P = 98.7654q$, or $q = .010125P$, is the firm's short-run supply curve.

c. If price is \$1000, how many units will the firm produce? What is the level of profit? Illustrate your answer on a cost curve graph.

Use the supply curve from part b: $q = 0.010125(1000) = 10.125$.

Profit $\pi = R - C = 1000(10.125) - [(4000(10.125)^2/81) + 1000] = \$4062.50$. Graphically, the firm produces where the price line hits the MC curve. Since profit is positive, this occurs at a quantity where price is greater than average cost. To find profit on the graph, take the difference between the revenue rectangle (price times quantity) and the cost rectangle (average cost times quantity). The area of the resulting rectangle is the firm's profit.

10. Suppose you are given the following information about a particular industry:

$Q^D = 6500 - 100P$	Market demand
$Q^S = 1200P$	Market supply
$C(q) = 722 + \dfrac{q^2}{200}$	Firm total cost function
$MC(q) = \dfrac{2q}{200}$	Firm marginal cost function.

Assume that all firms are identical, and that the market is characterized by perfect competition.

a. Find the equilibrium price, the equilibrium quantity, the output supplied by the firm, and the profit of each firm.

Equilibrium price and quantity are found by setting market demand equal to market supply: $6500 - 100P = 1200P$. Solve to find $P = \$5$ and substitute into either equation to find $Q = 6000$. To find the output for the firm set price equal to marginal cost: $5 = \frac{2q}{200}$ so $q = 500$. Profit is total revenue minus total cost or $\pi = Pq - \left(722 + \frac{q^2}{200}\right) = 5(500) - \left(722 + \frac{500^2}{200}\right) = \$528$. Notice that since the total output in the market is 6000, and each firm's output is 500, there must be 6000/500 = 12 firms in the industry.

b. Would you expect to see entry into or exit from the industry in the long run? Explain. What effect will entry or exit have on market equilibrium?

We would expect entry because firms in the industry are making positive economic profits. As new firms enter, market supply will increase (i.e., the market supply curve will shift down and to the right), which will cause the market equilibrium price to fall, all else the same. This, in turn, will reduce each firm's optimal output and profit. When profit falls to zero, no further entry will occur.

c. What is the lowest price at which each firm would sell its output in the long run? Is profit positive, negative, or zero at this price? Explain.

In the long run profit falls to zero, which means price falls to the minimum value of AC. To find the minimum average cost, set marginal cost equal to average cost and solve for q:

$$\frac{2q}{200} = \frac{722}{q} + \frac{q}{200}$$

$$\frac{q}{200} = \frac{722}{q}$$

$$q^2 = 722(200)$$

$$q = 380$$

$$AC(q = 380) = 3.8.$$

Therefore, the firm will not sell for any price less than \$3.80 in the long run. The long-run equilibrium price is therefore \$3.80, and at a price of \$3.80, each firm sells 380 units and earns an economic profit of zero because $P = AC$.

d. What is the lowest price at which each firm would sell its output in the short run? Is profit positive, negative, or zero at this price? Explain.

The firm will sell its output at any price above zero in the short run, because marginal cost is above average variable cost ($MC = q/100 > AVC = q/200$) for all positive prices. Profit is negative if price is just above zero.

11. Suppose that a competitive firm has a total cost function $C(q) = 450 + 15q + 2q^2$ and a marginal cost function $MC(q) = 15 + 4q$. If the market price is $P = \$115$ per unit, find the level of output produced by the firm. Find the level of profit and the level of producer surplus.

The firm should produce where price is equal to marginal cost so that $115 = 15 + 4q$, and therefore $q = 25$. Profit is $\pi = R - C = 115(25) - [450 + 15(25) + 2(25)^2] = \$800$.

Producer surplus is profit plus fixed cost, so $PS = 800 + 450 = \$1250$. Producer surplus can also be found graphically by calculating the area below price and above the marginal cost (supply) curve: $PS = (1/2)(25)(115 - 15) = \$1250$.

12. A number of stores offer film developing as a service to their customers. Suppose that each store offering this service has a cost function $C(q) = 50 + 0.5q + 0.08q^2$ and a marginal cost $MC = 0.5 + 0.16q$.

a. If the going rate for developing a roll of film is \$8.50, is the industry in long-run equilibrium? If not, find the price associated with long-run equilibrium.

Each firm's profit-maximizing quantity is where price equals marginal cost: $8.50 = 0.5 + 0.16q$. Thus $q = 50$. Profit is then $8.50(50) - [50 + 0.5(50) + 0.08(50)^2] = \$150$. The industry is not in long-run equilibrium because profit is greater than zero. In long-run equilibrium, firms produce where price is equal to minimum average cost and there is no incentive for entry or exit. To find the minimum average cost point, set marginal cost equal to average cost and solve for q:

$$MC = 0.5 + 0.16q = \frac{50}{q} + 0.5 + 0.08q = AC$$
$$0.08q^2 = 50$$
$$q = 25.$$

To find the long-run equilibrium price in the market, substitute $q = 25$ into either marginal cost or average cost to get $P = \$4.50$.

b. Suppose now that a new technology is developed which will reduce the cost of film developing by 25%. Assuming that the industry is in long-run equilibrium, how much would any one store be willing to pay to purchase this new technology?

The new total cost function and marginal cost function can be found by multiplying the old functions by 0.75 (or 75%). The new functions are:

$$C_{new}(q) = 0.75(50 + 0.5q + 0.08q^2) = 37.5 + 0.375q + 0.06q^2$$
$$MC_{new}(q) = 0.375 + 0.12q.$$

The firm will set marginal cost equal to price, which is \$4.50 in the long-run equilibrium. Solve for q to find that the firm will develop approximately 34 rolls of film (rounding down). If $q = 34$ then profit is \$33.39. This is the most the firm would be willing to pay per year for the new technology. It will pay this amount only if no other firms can adopt the new technology, because if all firms adopt the new technology, the long-run equilibrium price will fall to the minimum average cost of the new technology and profits will be driven to zero.

13. Consider a city that has a number of hot dog stands operating throughout the downtown area. Suppose that each vendor has a marginal cost of \$1.50 per hot dog sold and no fixed cost. Suppose the maximum number of hot dogs that any one vendor can sell is 100 per day.

a. If the price of a hot dog is \$2, how may hot dogs does each vendor want to sell?

Since marginal cost is equal to \$1.50 and the price is \$2, each hot dog vendor will want to sell as many hot dogs as possible, which is 100 per day.

b. If the industry is perfectly competitive, will the price remain at \$2 for a hot dog? If not, what will the price be?

Each hot dog vendor is making a profit of \$0.50 per hot dog at the current \$2 price: a total profit of \$50. Therefore, the price will not remain at \$2, because these positive economic profits will encourage new vendors to enter the market. As new firms start selling hot dogs, market supply will increase and price will drop until economic profits are driven to zero. That will happen when price falls to \$1.50, where price equals average cost. (Note that $AC = MC =$ \$1.50 for firms in this industry because fixed cost is zero and MC is constant at \$1.50).

c. If each vendor sells exactly 100 hot dogs a day and the demand for hot dogs from vendors in the city is $Q = 4400 - 1200P$, how many vendors are there?

At the current price of \$2, the total number of hot dogs demanded is $Q = 4400 - 1200(2) = 2000$, so there are 2000/100 = 20 vendors. In the long run, price will fall to \$1.50, and the number of hot dogs demanded will increase to $Q = 2600$. If each vendor sells 100 hot dogs, there will be 26 vendors in the long run.

d. Suppose the city decides to regulate hot dog vendors by issuing permits. If the city issues only 20 permits and if each vendor continues to sell 100 hot dogs a day, what price will a hot dog sell for?

If there are 20 vendors selling 100 hot dogs each, then the total number sold is 2000. If $Q = 2000$ then $P =$ \$2, from the demand curve.

e. Suppose the city decides to sell the permits. What is the highest price a vendor would pay for a permit?

At the price of \$2, each vendor is making a profit of \$50 per day as noted in part b. This is the most a vendor would pay per day for a permit.

14. A sales tax of \$1 per unit of output is placed on a particular firm whose product sells for \$5 in a competitive industry with many firms.

a. How will this tax affect the cost curves for the firm?

With a tax of \$1 per unit, all the firm's cost curves (except those based solely on fixed costs) shift up. Total cost becomes $TC + tq$, or $TC + q$ since the tax rate $t = 1$. Average variable cost becomes $AVC + 1$, Average cost is now $AC + 1$, and marginal cost becomes $MC + 1$. Average fixed cost does not change.

b. What will happen to the firm's price, output, and profit?

Because the firm is a price-taker in a competitive market, the imposition of the tax on only one firm does not change the market price. Since the firm's short-run supply curve is its marginal cost curve (above average variable cost), and the marginal cost curve has shifted up (and to the left), the firm supplies less to the market at every price. Profits are lower at every quantity.

c. Will there be entry or exit in the industry?

If the tax is placed on a single firm, that firm will go out of business in the long run because price will be below the firm's minimum average cost. One new firm will enter the market to take its place, assuming the new firm is not taxed like the original one.

15. A sales tax of 10% is placed on half the firms (the polluters) in a competitive industry. The revenue is paid to the remaining firms (the nonpolluters) as a 10% subsidy on the value of output sold.

a. Assuming that all firms have identical constant long-run average costs before the sales tax-subsidy policy, what do you expect to happen (in both the short run and the long run) to the price of the product, the output of firms, and industry output? (*Hint*: How does price relate to industry input?)

In long-run equilibrium, the price of the product equals each firm's minimum average cost. So, if a long-run competitive equilibrium existed before the sales tax-subsidy policy, price would have been equal to marginal cost and long-run minimum average cost. After the sales tax-subsidy policy is implemented, the net price received by polluters (after deducting the sales tax) falls below their minimum average cost. They will reduce output to the point where price minus the tax equals short-run marginal cost. The non-polluters will increase output because the net price they receive increases by 10%, and they will produce where net price equals short-run marginal cost. Total industry output may increase, decrease or stay the same in the short run. A likely scenario would be for the reduction in output from the polluters to be roughly equal to the increase in production from the non-polluters, keeping industry output and price about the same as they were before the sales tax-subsidy policy was implemented.

In the long run, all the polluters will exit the industry because their net price would be below the minimum average cost. Non-polluters, on the other hand, receive a price above minimum average cost because of the subsidy. They earn economic profits that will encourage the entry of other non-polluters. If this is a constant-cost industry, minimum average cost will remain unchanged as entry occurs. As more and more non-polluters enter the market, they will push the market price below the original equilibrium price because of the 10% subsidy. In fact, price should fall by about 10%. Total industry output will increase, although each individual firm will produce the same amount of output it did before the tax-subsidy policy.

b. Can such a policy *always* be achieved with a balanced budget in which tax revenues are equal to subsidy payments? Why or why not? Explain.

No. As the polluters exit and non-polluters enter the industry, revenues from the sales tax imposed on polluters decrease and subsidies paid to the non-polluters increase. This imbalance occurs when the first polluter leaves the industry and becomes larger and larger as more polluters exit and non-polluters enter the industry. If the taxes and subsidies are readjusted with every exiting and entering firm, then tax revenues from polluting firms will shrink and the non-polluters will get smaller and smaller subsidies. Taken to the extreme, both the sales tax and the subsidy will disappear.

Chapter 9
The Analysis of Competitive Markets

■ Teaching Notes

Now that students understand supply and demand more deeply, Chapter 9 shows how supply-demand analysis can be applied to a wide variety of economic issues and policies. In a sense, Chapter 9 picks up where Chapter 2 left off, but in more detail. The chapter begins with a review of consumer and producer surplus in Section 9.1. If you have postponed these topics, you should carefully explain the definition of each.

It is crucial for students to understand that the surplus measures are in dollars, which makes it possible to compare them and add or subtract them. I like to impress on students that it is remarkable that we are able to measure a consumer's gain or loss of utility in dollars, because that is exactly what consumer surplus does. Some students are more accepting of using consumer surplus as a measure of consumer welfare once they understand it is related to utility.

Another point you might want to stress is that the change in consumer surplus found from a market demand curve implicitly adds all the consumers' surpluses together, and this may mask how various subgroups do. If policy makers care more about certain groups, such as poor consumers, than others, an aggregate measure of consumer welfare may not tell the entire welfare story.

Some students may still be a bit perplexed about producer surplus, too. It is worth reiterating that producer surplus is revenue minus total variable cost, so it is different from profit in the short run. However, we are usually interested in the change in producer surplus. Since fixed costs do not change in the short run, the change in producer surplus is the change in profit. Profit is a more concrete concept than the nebulous producer surplus, so recognizing this connection can aid student understanding and acceptance of the change in producer surplus as a measure of welfare change. As with consumer surplus, when the change in producer surplus is calculated from a market supply curve, the effects on different subgroups (large and small farmers, for example) are obscured.

Section 9.2 describes the basic concept of efficiency in competitive markets but also discusses situations of market failure in which the competitive outcome is not efficient. Market failure is studied in much greater detail in the last two chapters of the book, but it is useful to introduce the idea here, especially if you do not plan to cover it later. Students are very attuned to issues such as pollution, and they will find this topic interesting and important. Also, if you do not bring this up, some perceptive students probably will, so you might as well beat them to it. If you want to spark a lively discussion, you can ask students about the market for human kidneys covered in Example 9.2.

Sections 9.3 to 9.6 present examples of government policies that cause the market equilibrium to differ from the competitive, efficient equilibrium. You can pick and choose among the topics covered—minimum prices in Section 9.3, price supports and production quotas in 9.4, import quotas and tariffs in 9.5, and taxes and subsidies in 9.6—depending on time constraints and personal preference. The presentation in each of these sections follows the same format: there is a general discussion of why market intervention leads to deadweight loss followed by the presentation of an important policy example.

The method used to determine the incidence of a tax or subsidy in Section 9.6 is a little different from many other textbooks. The typical method for, say, a specific tax levied on producers, is to add the tax to the industry supply curve so that the industry supply shifts upward by the amount of the tax. In this book, however, the effect is found by finding the point where the vertical difference between the supply and demand curves equals the tax. No curves are shifted. You might want to do it the other way also, because the book's method is a bit more abstract. Once students catch on, however, it is actually an easy method to use, and it has the added advantage that there are no new supply or demand curves to contend with when it comes time to calculate the changes in consumer and producer surplus.

Each of the last four sections is covered in one review question and applied in at least one exercise at the end of the chapter. Exercise 1 focuses on minimum wages; Exercises 4 and 5 reinforce discussion of price supports and production quotas; tariffs and quotas can be found in Exercises 3, 6, 7, 8, 11, and 12. Taxes and subsidies are discussed in Exercises 2, 9 and 14; Exercise 10 considers natural gas price controls (based on Example 9.1, which is a continuation of Example 2.10). Exercise 4 may be compared to Example 9.4 and discussed as an extension of Example 2.5.

■ Review Questions

1. **What is meant by *deadweight loss*? Why does a price ceiling usually result in a deadweight loss?**

   Deadweight loss refers to the benefits lost by consumers and/or producers when markets do not operate efficiently. The term deadweight denotes that these are benefits unavailable to any party. A price ceiling set below the equilibrium price in a perfectly competitive market will result in a deadweight loss because it reduces the quantity supplied by producers. Both producers and consumers lose surplus because less of the good is produced and consumed. The reduced (ceiling) price benefits consumers but hurts producers, so there is a transfer from one group to the other. The real culprit, then, and the primary source of the deadweight loss, is the reduction in the amount of the good in the market.

2. **Suppose the supply curve for a good is completely inelastic. If the government imposed a price ceiling below the market-clearing level, would a deadweight loss result? Explain.**

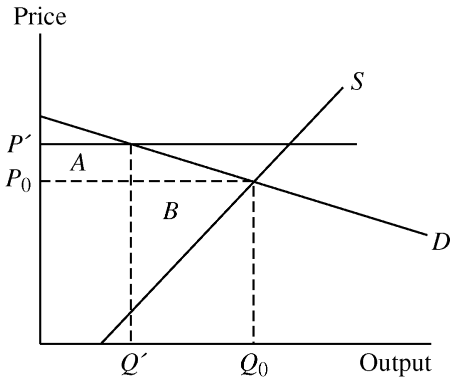
   When the supply curve is completely inelastic, it is vertical. In this case there is no deadweight loss because there is no reduction in the amount of the good produced. The imposition of the price ceiling transfers all lost producer surplus to consumers. Consumer surplus increases by the difference between the market-clearing price and the price ceiling times the market-clearing quantity. Consumers capture all decreases in total revenue, and no deadweight loss occurs.

3. **How can a price ceiling make consumers better off? Under what conditions might it make them worse off?**

   If the supply curve is highly inelastic a price ceiling will usually increase consumer surplus because the quantity available will not decline much, but consumers get to purchase the product at a reduced price. If the demand curve is inelastic, on the other hand, price controls may result in a net loss of consumer surplus because consumers who value the good highly are unable to purchase as much as they would like. (See Figure 9.3 on page 321 in the text.) The loss of consumer surplus is greater than the transfer of producer surplus to consumers. So consumers are made better off when demand is relatively elastic and supply is relatively inelastic, and they are made worse off when the opposite is true.

4. **Suppose the government regulates the price of a good to be no lower than some minimum level. Can such a minimum price make producers as a whole worse off? Explain.**

   With a minimum price set above the market-clearing price, some consumer surplus is transferred to producers because of the higher price, but some producer surplus is lost because consumers purchase less. If demand is highly elastic, the reduction in purchases can offset the higher price producers receive, making producers worse off. In the diagram below, the market-clearing price and quantity are P_0 and Q_0. The minimum price is set at P', and at this price consumers demand Q'. Assuming that suppliers produce Q' (and not the larger quantity indicated by the supply curve), producer surplus increases by area *A* due to the higher price, but decreases by the much larger area *B* because the quantity demanded drops sharply. The result is a reduction in producer surplus. Note that if suppliers produce more than Q', the loss in producer surplus is even greater because they will have unsold units.



5. How are production limits used in practice to raise the prices of the following goods or services: (a) taxi rides, (b) drinks in a restaurant or bar, (c) wheat or corn?

Municipal authorities usually regulate the number of taxis through the issuance of licenses or medallions. When the number of taxis is less than it would be without regulation, those taxis in the market may charge a higher-than-competitive price.

State authorities usually regulate the number of liquor licenses. By requiring that any bar or restaurant that serves alcohol have a liquor license and then limiting the number of licenses available, the state limits entry by new bars and restaurants. This limitation allows those establishments that have a license to charge a higher-than-competitive price for alcoholic beverages.

Federal authorities usually regulate the number of acres of wheat or corn in production by creating acreage limitation programs that give farmers financial incentives to leave some of their acreage idle. This reduces supply, driving up the price of wheat or corn.

6. Suppose the government wants to increase farmers' incomes. Why do price supports or acreage-limitation programs cost society more than simply giving farmers money?

Price supports and acreage limitations cost society more than the dollar cost of these programs because the higher price that results in either case will reduce quantity demanded and hence consumer surplus, leading to a deadweight loss because farmers are not able to capture the lost surplus. Giving farmers money does not result in any deadweight loss but is merely a redistribution of surplus from one group to the other.

7. Suppose the government wants to limit imports of a certain good. Is it preferable to use an import quota or a tariff? Why?

Changes in domestic consumer and producer surpluses are the same under import quotas and tariffs. There will be a loss in (domestic) total surplus in either case. However, with a tariff, the government can collect revenue equal to the tariff times the quantity of imports, and these revenues can be redistributed in the domestic economy to offset some of the domestic deadweight loss. Thus there is less of a loss to the domestic society as a whole with a tariff. With an import quota, foreign producers can capture the difference between the domestic and world price times the quantity of imports. Therefore, with an import quota, there is a loss to the domestic society as a whole. If the national government is trying to minimize domestic welfare loss, it should use a tariff.

8. The burden of a tax is shared by producers and consumers. Under what conditions will consumers pay most of the tax? Under what conditions will producers pay most of it? What determines the share of a subsidy that benefits consumers?

The burden of a tax and the benefits of a subsidy depend on the elasticities of demand and supply. If the absolute value of the ratio of the elasticity of demand to the elasticity of supply is small, the burden of the tax falls mainly on consumers. If the ratio is large, the burden of the tax falls mainly on producers. Similarly, the benefit of a subsidy accrues mostly to consumers (producers) if the ratio of the elasticity of demand to the elasticity of supply is small (large) in absolute value.

9. Why does a tax create a deadweight loss? What determines the size of this loss?

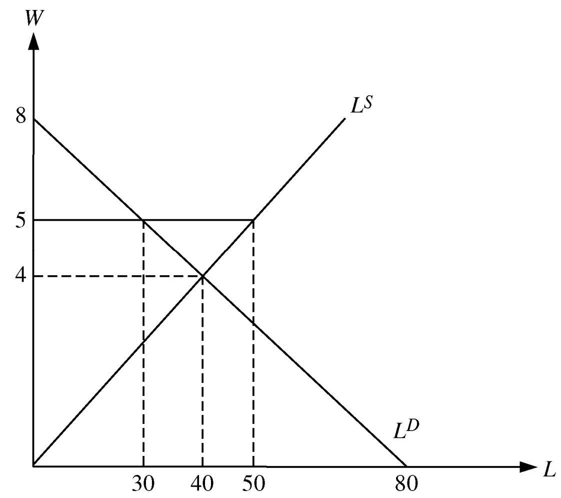
A tax creates deadweight loss by artificially increasing price above the free market level, thus reducing the equilibrium quantity. This reduction in quantity reduces consumer as well as producer surplus. The size of the deadweight loss depends on the elasticities of supply and demand and on the size of the tax. The more elastic supply and demand are, the larger will be the deadweight loss. Also, the larger the tax, the greater the deadweight loss.

■ Exercises

1. From time to time, Congress has raised the minimum wage. Some people suggested that a government subsidy could help employers finance the higher wage. This exercise examines the economics of a minimum wage and wage subsidies. Suppose the supply of low-skilled labor is given by $L^S = 10w$, where L^S is the quantity of low-skilled labor (in millions of persons employed each year), and w is the wage rate (in dollars per hour). The demand for labor is given by $L^D = 80 - 10w$.

a. What will be the free-market wage rate and employment level? Suppose the government sets a minimum wage of $5 per hour. How many people would then be employed?

In a free-market equilibrium, $L^S = L^D$. Solving yields $w = \$4$ and $L^S = L^D = 40$. If the minimum wage is \$5, then $L^S = 50$ and $L^D = 30$. The number of people employed will be given by the labor demand, so employers will hire only 30 million workers.



b. Suppose that instead of a minimum wage, the government pays a subsidy of $1 per hour for each employee. What will the total level of employment be now? What will the equilibrium wage rate be?

Let w_s denote the wage received by the sellers (i.e., the employees), and w_b the wage paid by the buyers (the firms). The new equilibrium occurs where the vertical difference between the supply and demand curves is $1 (the amount of the subsidy). This point can be found where

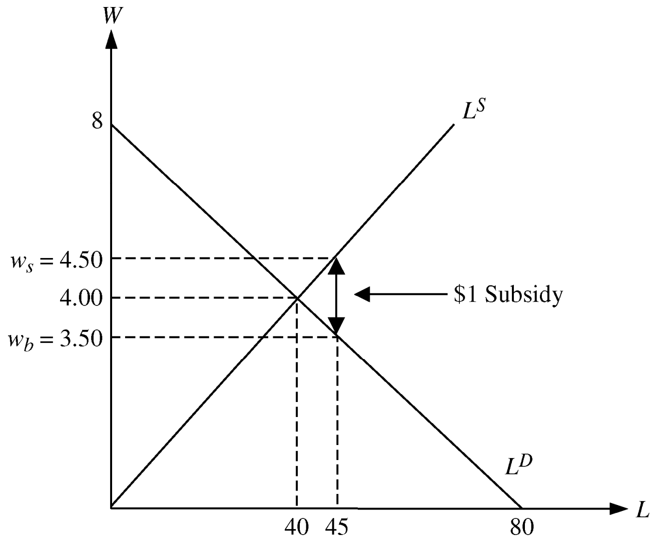
$$L^D(w_b) = L^S(w_s), \text{ and}$$

$$w_s - w_b = 1.$$

Write the second equation as $w_b = w_s - 1$. This reflects the fact that firms pay $1 less than the wage received by workers because of the subsidy. Substitute for w_b in the demand equation: $L^D(w_b) = 80 - 10(w_s - 1)$, so

$$L^D(w_b) = 90 - 10w_s.$$

Note that this is equivalent to an upward shift in demand by the amount of the $1 subsidy. Now set the new demand equal to supply: $90 - 10w_s = 10w_s$. Therefore, $w_s = \$4.50$, and $L^D = 90 - 10(4.50) = 45$. Employment increases to 45 (compared to 30 with the minimum wage), but wage drops to $4.50 (compared to $5.00 with the minimum wage). The net wage the firm pays falls to $3.50 due to the subsidy.



2. Suppose the market for widgets can be described by the following equations:

***Demand*: $P = 10 - Q$**   ***Supply*: $P = Q - 4$**

where P is the price in dollars per unit and Q is the quantity in thousands of units. Then:

a. What is the equilibrium price and quantity?

Equate supply and demand and solve for Q: $10 - Q = Q - 4$. Therefore $Q = 7$ thousand widgets.

Substitute Q into either the demand or the supply equation to obtain P.

$$P = 10 - 7 = \$3.00,$$

or

$$P = 7 - 4 = \$3.00.$$

b. Suppose the government imposes a tax of $1 per unit to reduce widget consumption and raise government revenues. What will the new equilibrium quantity be? What price will the buyer pay? What amount per unit will the seller receive?

With the imposition of a $1.00 tax per unit, the price buyers pay is $1 more than the price suppliers receive. Also, at the new equilibrium, the quantity bought must equal the quantity supplied. We can write these two conditions as

$$P_b - P_s = 1$$

$$Q_b = Q_s.$$

Let Q with no subscript stand for the common value of Q_b and Q_s. Then substitute the demand and supply equations for the two values of P:

$$(10 - Q) - (Q - 4) = 1$$

Therefore, $Q = 6.5$ thousand widgets. Plug this value into the demand equation, which is the equation for P_b, to find $P_b = 10 - 6.5 = \$3.50$. Also substitute $Q = 6.5$ into the supply equation to get $P_s = 6.5 - 4 = \$2.50$.

The tax raises the price in the market from $3.00 (as found in part a) to $3.50. Sellers, however, receive only $2.50 after the tax is imposed. Therefore the tax is shared equally between buyers and sellers, each paying $0.50.

c. Suppose the government has a change of heart about the importance of widgets to the happiness of the American public. The tax is removed and a subsidy of $1 per unit granted to widget producers. What will the equilibrium quantity be? What price will the buyer pay? What amount per unit (including the subsidy) will the seller receive? What will be the total cost to the government?

Now the two conditions that must be satisfied are

$$P_s - P_b = 1$$

$$Q_b = Q_s.$$

As in part b, let Q stand for the common value of quantity. Substitute the supply and demand curves into the first condition, which yields

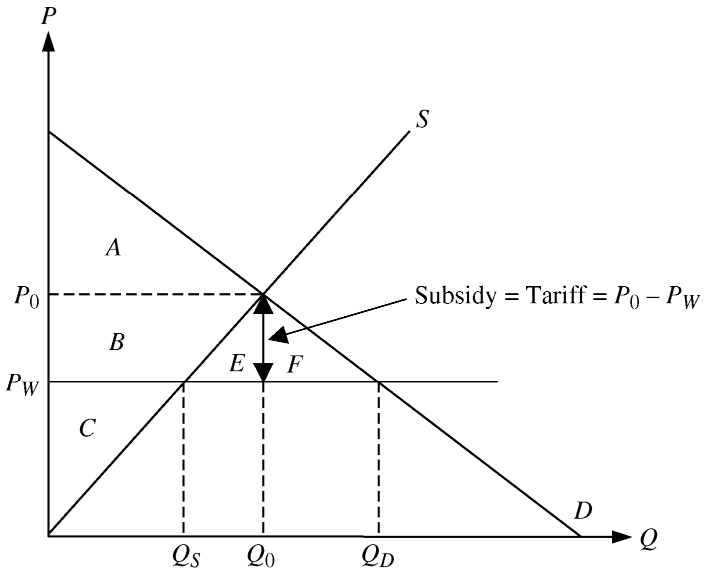
$$(Q - 4) - (10 - Q) = 1.$$

Therefore, $Q = 7.5$ thousand widgets. Using this quantity in the supply and demand equations, suppliers will receive $P_s = 7.5 - 4 = \$3.50$, and buyers will pay $P_b = 10 - 7.5 = \$2.50$. The total cost to the government is the subsidy per unit multiplied by the number of units. Thus the cost is ($1)(7.5) = $7.5 thousand, or $7500.

3. Japanese rice producers have extremely high production costs, due in part to the high opportunity cost of land and to their inability to take advantage of economies of large-scale production. Analyze two policies intended to maintain Japanese rice production: (1) a per-pound subsidy to farmers for each pound of rice produced, or (2) a per-pound tariff on imported rice. Illustrate with supply-and-demand diagrams the equilibrium price and quantity, domestic rice production, government revenue or deficit, and deadweight loss from each policy. Which policy is the Japanese government likely to prefer? Which policy are Japanese farmers likely to prefer?

We have to make some assumptions to answer this question. If you make different assumptions, you may get different answers. Assume that initially the Japanese rice market is open, meaning that foreign producers and domestic (Japanese) producers both sell rice to Japanese consumers. The world price of rice is P_W. This price is below P_0, which is the equilibrium price that would occur in the Japanese market if no imports were allowed. In the diagram below, S is the domestic supply, D is the domestic demand, and Q_0 is the equilibrium quantity that would prevail if no imports were allowed. The horizontal line at P_w is the world supply of rice, which is assumed to be perfectly elastic. Initially Japanese consumers purchase Q_D rice at the world price. Japanese farmers supply Q_S at that price, and $Q_D - Q_S$ is imported from foreign producers.

Now suppose the Japanese government pays a subsidy to Japanese farmers equal to the difference between P_0 and P_W. Then Japanese farmers would sell rice on the open market for P_w plus receive the subsidy of $P_0 - P_W$. Adding these together, the total amount Japanese farmers would receive is P_0 per pound of rice. At this price they would supply Q_0 pounds of rice. Consumers would still pay P_w and buy Q_D. Foreign suppliers would import $Q_D - Q_0$ pounds of rice. This policy would cost the government $(P_0-P_w)Q_0$, which is the subsidy per pound times the number of pounds supplied by Japanese farmers. It is represented on the diagram as areas $B + E$. Producer surplus increases from area C to $C + B$, so $\Delta PS = B$. Consumer surplus is not affected and remains as area $A + B + E + F$. Deadweight loss is area E, which is the cost of the subsidy minus the gain in producer surplus.



Instead, suppose the government imposes a tariff rather than paying a subsidy. Let the tariff be the same size as the subsidy, P_0-P_W. Now foreign firms importing rice into Japan will have to sell at the world price plus the tariff: $P_w + (P_0 - P_w) = P_0$. But at this price, Japanese farmers will supply Q_0, which is exactly the amount Japanese consumers wish to purchase. Therefore there will be no imports, and the government will not collect any revenue from the tariff. The increase in producer surplus equals area B, as it is in the case of the subsidy. Consumer surplus is area A, which is less than it is under the subsidy because consumers pay more (P_0) and consume less (Q_0). Consumer surplus decreases by $B + E + F$. Deadweight loss is $E + F$: the difference between the decrease in consumer surplus and the increase in producer surplus.

Under the assumptions made here, it seems likely that producers would not have a strong preference for either the subsidy or the tariff, because the increase in producer surplus is the same under both policies. The government might prefer the tariff because it does not require any government expenditure. On the other hand, the tariff causes a decrease in consumer surplus, and government officials who are elected by consumers might want to avoid that. Note that if the subsidy and tariff amounts were smaller than assumed above, some tariffs would be collected, but we would still get the same basic results.

4. In 1983, the Reagan Administration introduced a new agricultural program called the Payment-in-Kind Program. To see how the program worked, let's consider the wheat market.

a. Suppose the demand function is $Q^D = 28 - 2P$ and the supply function is $Q^S = 4 + 4P$, where P is the price of wheat in dollars per bushel, and Q is the quantity in billions of bushels. Find the free-market equilibrium price and quantity.

Equating demand and supply, $Q^D = Q^S$,

$$28 - 2P = 4 + 4P, \text{ or } P = \$4.00 \text{ per bushel.}$$

To determine the equilibrium quantity, substitute $P = 4$ into either the supply equation or the demand equation:

$$Q^S = 4 + 4(4) = 20 \text{ billion bushels}$$

and

$$Q^D = 28 - 2(4) = 20 \text{ billion bushels.}$$

b. Now suppose the government wants to lower the supply of wheat by 25% from the free-market equilibrium by paying farmers to withdraw land from production. However, the payment is made in wheat rather than in dollars—hence the name of the program. The wheat comes from vast government reserves accumulated from previous price support programs. The amount of wheat paid is equal to the amount that could have been harvested on the land withdrawn from production. Farmers are free to sell this wheat on the market. How much is now produced by farmers? How much is indirectly supplied to the market by the government? What is the new market price? How much do farmers gain? Do consumers gain or lose?

Because the free-market supply by farmers is 20 billion bushels, the 25% reduction required by the new Payment-In-Kind (PIK) Program means that the farmers now produce 15 billion bushels. To encourage farmers to withdraw their land from cultivation, the government must give them 5 billion bushels of wheat, which they sell on the market, so 5 billion bushels are indirectly supplied by the government.

Because the total quantity supplied to the market is still 20 billion bushels, the market price does not change; it remains at $4 per bushel. Farmers gain because they incur no costs for the 5 billion bushels received from the government. We can calculate these cost savings by taking the area under the supply curve between 15 and 20 billion bushels. These are the variable costs of producing the last 5 billion bushels that are no longer grown under the PIK Program. To find this area, first determine the prices when $Q = 15$ and when $Q = 20$. These values are $P = \$2.75$ and $P = \$4.00$. The total cost of producing the last 5 billion bushels is therefore the area of a trapezoid with a base of $20 - 15 = 5$ billion and an average height of $(2.75 + 4.00)/2 = 3.375$. The area is $5(3.375) =$ $16.875 billion, which is the amount farmers gain under the program.

The PIK program does not affect consumers in the wheat market because they purchase the same amount at the same price as they did in the free-market case.

c. Had the government not given the wheat back to the farmers, it would have stored or destroyed it. Do taxpayers gain from the program? What potential problems does the program create?

Taxpayers gain because the government does not incur costs to store or destroy the wheat. Although everyone seems to gain from the PIK program, it can only last while there are government wheat reserves. The program assumes that land removed from production may be restored to production when stockpiles of wheat are exhausted. If this cannot be done, consumers may eventually pay more for wheat-based products. Another potential problem is verifying that the land taken out of production is in fact capable of producing the amount of wheat paid to farmers under the PIK program. Farmers may try to game the system by removing less productive land.

5. About 100 million pounds of jelly beans are consumed in the United States each year, and the price has been about 50 cents per pound. However, jelly bean producers feel that their incomes are too low and have convinced the government that price supports are in order. The government will therefore buy up as many jelly beans as necessary to keep the price at $1 per pound. However, government economists are worried about the impact of this program because they have no estimates of the elasticities of jelly bean demand or supply.

a. Could this program cost the government *more* than $50 million per year? Under what conditions? Could it cost *less* than $50 million per year? Under what conditions? Illustrate with a diagram.

If the quantities demanded and supplied are very responsive to price changes, then a government program that doubles the price of jelly beans could easily cost more than $50 million. In this case, the change in price will cause a large change in quantity supplied, and a large change in quantity demanded. In Figure 9.5.a.i, the cost of the program is ($1)($Q_S - Q_D$). If $Q_S - Q_D$ is larger than 50 million, then the government will pay more than $50 million. If instead supply and demand are relatively inelastic, then the increase in price would result in small changes in quantity supplied and quantity demanded, and ($Q_S - Q_D$) would be less than $50 million as illustrated in Figure 9.5.a.ii.

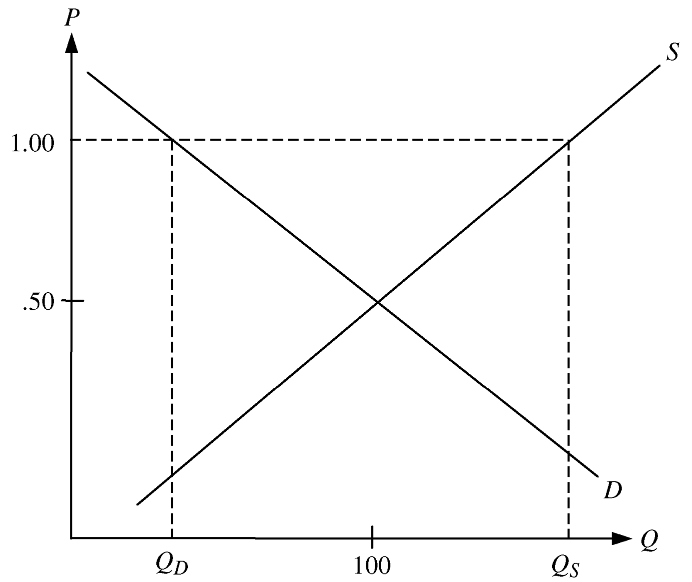


Figure 9.5.a.i

We can determine the combinations of supply and demand elasticities that yield either result. The elasticity of supply is $E_S = (\%\Delta Q_S)/(\%\Delta P)$, so the percentage change in quantity supplied is $\%\Delta Q_S = E_S(\%\Delta P)$. Since the price increase is 100% (from $0.50 to $1.00), $\%\Delta Q_S = 100E_S$. Likewise, the percentage change in quantity demanded is $\%\Delta Q_D = 100E_D$. The gap between Q_D and Q_S in percentage terms is $\%\Delta Q_S - \%\Delta Q_D = 100E_S - 100E_D = 100(E_S - E_D)$. If this gap is exactly 50% of the current 100 million pounds of jelly beans, the gap will be 50 million pounds, and the cost of the price support program will be exactly $50 million. So the program will cost $50 million if $100(E_S - E_D) = 50$, or

$(E_S - E_D) = 0.5$. If the difference between the elasticities is greater than one half, the program will cost more than \$50 million, and if the difference is less than one half, the program will cost less than \$50 million. So the supply and demand can each be fairly inelastic (for example, 0.3 and −0.4) and still trigger a cost greater than \$50 million.

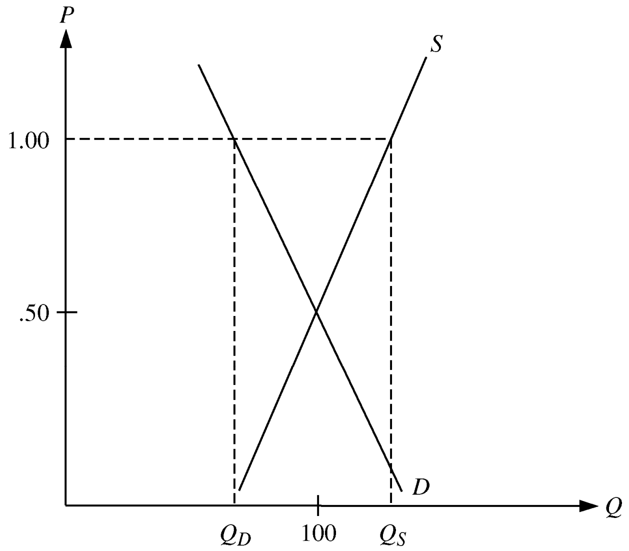


Figure 9.5.a.ii

b. Could this program cost consumers (in terms of lost consumer surplus) *more* than \$50 million per year? Under what conditions? Could it cost consumers *less* than \$50 million per year? Under what conditions? Again, use a diagram to illustrate.

When the demand curve is perfectly inelastic, the loss in consumer surplus is \$50 million, equal to (\$0.50)(100 million pounds). This represents the highest possible loss in consumer surplus, so the loss cannot be more than \$50 million per year. If the demand curve has any elasticity at all, the loss in consumer surplus will be less than \$50 million. In Figure 9.5.b, the loss in consumer surplus is area *A* plus area *B* if the demand curve is the completely inelastic *D* and only area *A* if the demand curve is *D*′.

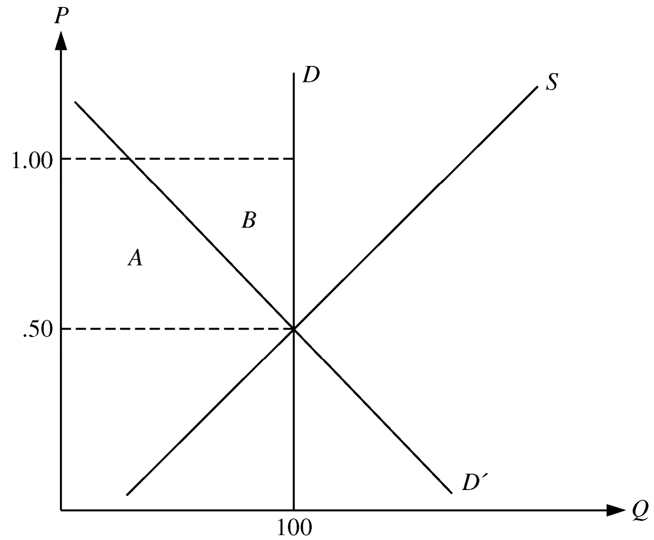


Figure 9.5.b

6. In Exercise 4 in Chapter 2 (page 62), we examined a vegetable fiber traded in a competitive world market and imported into the United States at a world price of \$9 per pound. U.S. domestic supply and demand for various price levels are shown in the following table.

Price	U.S. Supply (million pounds)	U.S. Demand (million pounds)
3	2	34
6	4	28
9	6	22
12	8	16
15	10	10
18	12	4

Answer the following questions about the U.S. market:

a. Confirm that the demand curve is given by $Q_D = 40 - 2P$, and that the supply curve is given by $Q_S = \frac{2}{3}P$.

To find the equation for demand, we need to find a linear function $Q_D = a + bP$ so that the line it represents passes through two of the points in the table such as (15, 10) and (12, 16). First, the slope, b, is equal to the "rise" divided by the "run,"

$$\frac{\Delta Q}{\Delta P} = \frac{10-16}{15-12} = -2 = b.$$

Second, substitute for b and one point, e.g., (15, 10), into the linear function to solve for the constant, a:

$$10 = a - 2(15), \text{ or } a = 40.$$

Therefore, $Q_D = 40 - 2P$.

Similarly, solve for the supply equation $Q_S = c + dP$ passing through two points such as (6, 4) and (3, 2). The slope, d, is

$$\frac{\Delta Q}{\Delta P} = \frac{4-2}{6-3} = \frac{2}{3}.$$

Solving for c:

$$4 = c + \left(\frac{2}{3}\right)(6), \text{ or } c = 0.$$

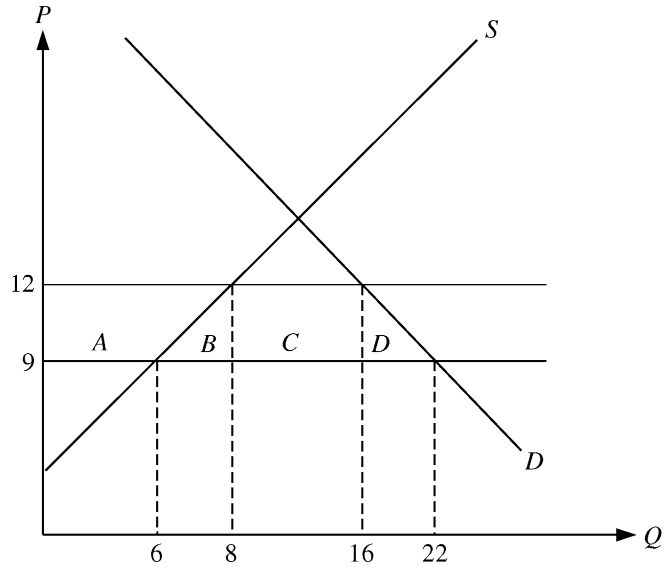
Therefore, $Q_S = \left(\frac{2}{3}\right)P$.

b. Confirm that if there were no restrictions on trade, the United States would import 16 million pounds.

If there were no trade restrictions, the world price of \$9.00 would prevail in the United States. From the table, we see that at \$9.00 domestic supply would be 6 million pounds. Similarly, domestic demand would be 22 million pounds. Imports provide the difference between domestic demand and domestic supply, so imports would be 22 – 6 = 16 million pounds.

c. If the United States imposes a tariff of $3 per pound, what will be the U.S. price and level of imports? How much revenue will the government earn from the tariff? How large is the deadweight loss?

With a $3.00 tariff, the U.S. price will be $12 (the world price plus the tariff). At this price, demand is 16 million pounds and U.S. supply is 8 million pounds, so imports are 8 million pounds (16 – 8). The government will collect $3(8) = $24 million, which is area *C* in the diagram below. To find deadweight loss, we must determine the changes in consumer and producer surpluses. Consumers lose area *A* + *B* + *C* + *D* because they pay the higher price of $12 and purchase fewer pounds of the fiber. U.S. producers gain area *A* because of the higher price and the greater quantity they sell. So the deadweight loss is the loss in consumer surplus minus the gain in producer surplus and the tariff revenue. Therefore, DWL = *B* + *D* = 0.5(12 – 9)(8 – 6) + 0.5(12 – 9)(22 – 16) = $12 million.



d. If the United States has no tariff but imposes an import quota of 8 million pounds, what will be the U.S. domestic price? What is the cost of this quota for U.S. consumers of the fiber? What is the gain for U.S. producers?

With an import quota of 8 million pounds, the domestic price will be $12. At $12, the difference between domestic demand and domestic supply is 8 million pounds, i.e., 16 million pounds minus 8 million pounds. Note you can also find the equilibrium price by setting demand equal to supply plus the quota so that

$$40 - 2P = \frac{2}{3}P + 8.$$

The cost of the quota to consumers is equal to area *A* + *B* + *C* + *D* in the figure above, which is the reduction in consumer surplus. This equals

$$(12 - 9)(16) + (0.5)(12 - 9)(22 - 16) = \$57 \text{ million.}$$

The gain to domestic producers (increase in producer surplus) is equal to area *A*, which is

$$(12 - 9)(6) + (0.5)(8 - 6)(12 - 9) = \$21 \text{ million.}$$

7. The United States currently imports all of its coffee. The annual demand for coffee by U.S. consumers is given by the demand curve $Q = 250 - 10P$, where Q is quantity (in millions of pounds) and P is the market price per pound of coffee. World producers can harvest and ship coffee to U.S. distributors at a constant marginal (= average) cost of \$8 per pound. U.S. distributors can in turn distribute coffee for a constant \$2 per pound. The U.S. coffee market is competitive. Congress is considering a tariff on coffee imports of \$2 per pound.

a. If there is no tariff, how much do consumers pay for a pound of coffee? What is the quantity demanded?

If there is no tariff then consumers will pay \$10 per pound of coffee, which is found by adding the \$8 that it costs to import the coffee plus the \$2 that it costs to distribute the coffee in the United States. In a competitive market, price is equal to marginal cost. At a price of \$10, the quantity demanded is 150 million pounds.

b. If the tariff is imposed, how much will consumers pay for a pound of coffee? What is the quantity demanded?

Now add \$2 per pound tariff to marginal cost, so price will be \$12 per pound, and quantity demanded is $Q = 250 - 10(12) = 130$ million pounds.

c. Calculate the lost consumer surplus.

Lost consumer surplus is $(12 - 10)(130) + 0.5(12 - 10)(150 - 130) = \$280$ million.

d. Calculate the tax revenue collected by the government.

The tax revenue is equal to the tariff of \$2 per pound times the 130 million pounds imported. Tax revenue is therefore \$260 million.

e. Does the tariff result in a net gain or a net loss to society as a whole?

There is a net loss to society because the gain (\$260 million) is less than the loss (\$280 million).

8. A particular metal is traded in a highly competitive world market at a world price of \$9 per ounce. Unlimited quantities are available for import into the United States at this price. The supply of this metal from domestic U.S. mines and mills can be represented by the equation $Q^S = 2/3P$, where Q^S is U.S. output in million ounces and P is the domestic price. The demand for the metal in the United States is $Q^D = 40 - 2P$, where Q^D is the domestic demand in million ounces.

In recent years the U.S. industry has been protected by a tariff of \$9 per ounce. Under pressure from other foreign governments, the United States plans to reduce this tariff to zero. Threatened by this change, the U.S. industry is seeking a voluntary restraint agreement that would limit imports into the United States to 8 million ounces per year.

a. Under the \$9 tariff, what was the U.S. domestic price of the metal?

With a \$9 tariff, the price of the imported metal in the U.S. market would be \$18; the \$9 tariff plus the world price of \$9. The \$18 price, however, is above the domestic equilibrium price. To determine the domestic equilibrium price, equate domestic supply and domestic demand:

$$\frac{2}{3}P = 40 - 2P, \text{ or } P = \$15.$$

Because the domestic price of \$15 is less than the world price plus the tariff, \$18, there will be no imports. The equilibrium quantity is found by substituting the price of \$15 into either the demand or supply equation. Using demand,

$$Q^D = 40 - (2)(15) = 10 \text{ million ounces.}$$

b. If the United States eliminates the tariff and the voluntary restraint agreement is approved, what will be the U.S. domestic price of the metal?

With the voluntary restraint agreement, the difference between domestic supply and domestic demand would be limited to 8 million ounces, i.e., $Q^D - Q^S = 8$. To determine the domestic price of the metal, set $Q^D - Q^S = 8$ and solve for P:

$$(40 - 2P) - \frac{2}{3}P = 8, \text{ or } P = \$12.$$

At a U.S. domestic price of \$12, $Q^D = 16$ and $Q^S = 8$; the difference of 8 million ounces will be supplied by imports.

9. Among the tax proposals regularly considered by Congress is an additional tax on distilled liquors. The tax would not apply to beer. The price elasticity of supply of liquor is 4.0, and the price elasticity of demand is –0.2. The cross-elasticity of demand for beer with respect to the price of liquor is 0.1.

a. If the new tax is imposed, who will bear the greater burden—liquor suppliers or liquor consumers? Why?

The fraction of the tax borne by consumers is given in Section 9.6 as $\frac{E_S}{E_S - E_D}$, where E_S is the own-price elasticity of supply and E_D is the own-price elasticity of demand. Substituting for E_S and E_D, the pass-through fraction is

$$\frac{4}{4 - (-0.2)} = \frac{4}{4.2} \approx 0.95.$$

Therefore, just over 95% of the tax is passed through to consumers because supply is highly elastic while demand is very inelastic. So liquor consumers will bear almost all the burden of the tax.

b. Assuming that beer supply is infinitely elastic, how will the new tax affect the beer market?

With an increase in the price of liquor (from the large pass-through of the liquor tax), some consumers will substitute away from liquor to beer because the cross-elasticity is positive. This will shift the demand curve for beer outward. With an infinitely elastic supply for beer (a horizontal supply curve), the equilibrium price of beer will not change, and the quantity of beer consumed will increase.

10. In Example 9.1 (page 322), we calculated the gains and losses from price controls on natural gas and found that there was a deadweight loss of \$5.68 billion. This calculation was based on a price of oil of \$50 per barrel.

a. If the price of oil were \$60 per barrel, what would be the free-market price of gas? How large a deadweight loss would result if the maximum allowable price of natural gas were \$3.00 per thousand cubic feet?

From Example 9.1, we know that the supply and demand curves for natural gas can be approximated as follows:

$$Q_S = 15.90 + 0.72P_G + 0.05P_O$$

and

$$Q_D = 0.02 - 1.8P_G + 0.69P_O,$$

where P_G is the price of natural gas in dollars per thousand cubic feet ($/mcf) and P_O is the price of oil in dollars per barrel ($/b).

With the price of oil at $60 per barrel, these curves become,

$$Q_S = 18.90 + 0.72P_G$$

and

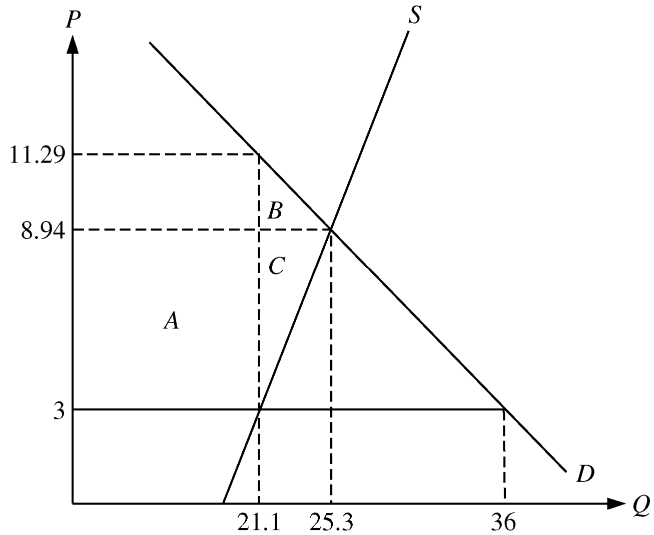
$$Q_D = 41.42 - 1.8P_G.$$

Setting quantity demanded equal to quantity supplied, find the free-market equilibrium price:

$$18.90 + 0.72P_G = 41.42 - 1.8P_G, \text{ or } P_G = \$8.94.$$

At this price, the equilibrium quantity is 25.3 trillion cubic feet (Tcf).

If a price ceiling of $3 is imposed, producers would supply only 18.9 + 0.72(3) = 21.1 Tcf, although consumers would demand 41.42 – 1.8(3) = 36.0 Tcf. See the diagram below. Area *A* is transferred from producers to consumers. The deadweight loss is *B* + *C*. To find area *B* we must first determine the price on the demand curve when quantity equals 21.1. From the demand equation, 21.1 = 41.42 − 1.8P_G. Therefore P_G = $11.29. Area *B* equals (0.5)(25.3 − 21.1)(11.29 − 8.94) = $4.9 billion, and area *C* is (0.5)(25.3 – 21.1)(8.94 – 3) = $12.5 billion. The deadweight loss is 4.9 + 12.5 = $17.4 billion.



b. What price of oil would yield a free-market price of natural gas of $3?

Set the original supply and demand equal to each other, and solve for P_O.

$$15.90 + 0.72P_G + 0.05P_O = 0.02 - 1.8P_G + 0.69P_O$$

$$0.64P_O = 15.88 + 2.52P_G$$

Substitute \$3 for the price of natural gas. Then

$$0.64P_O = 15.88 + 2.52(3), \text{ or } P_O = \$36.63.$$

11. Example 9.6 (page 342) describes the effects of the sugar quota. In 2011, imports were limited to 6.9 billion pounds, which pushed the domestic price to 36 cents per pound. Suppose imports were expanded to 10 billion pounds.

a. What would be the new U.S. domestic price?

Example 9.6 gives equations for the total market demand for sugar in the U.S. and the supply of U.S. producers:

$$Q_D = 29.73 - 0.19P$$

$$Q_S = -7.95 + 0.66P.$$

The difference between the domestic quantities demanded and supplied, $Q_D - Q_S$, is the amount of imported sugar that is restricted by the quota. If the quota is increased to 10 billion pounds, then $Q_D - Q_S = 10$ and we can solve for P:

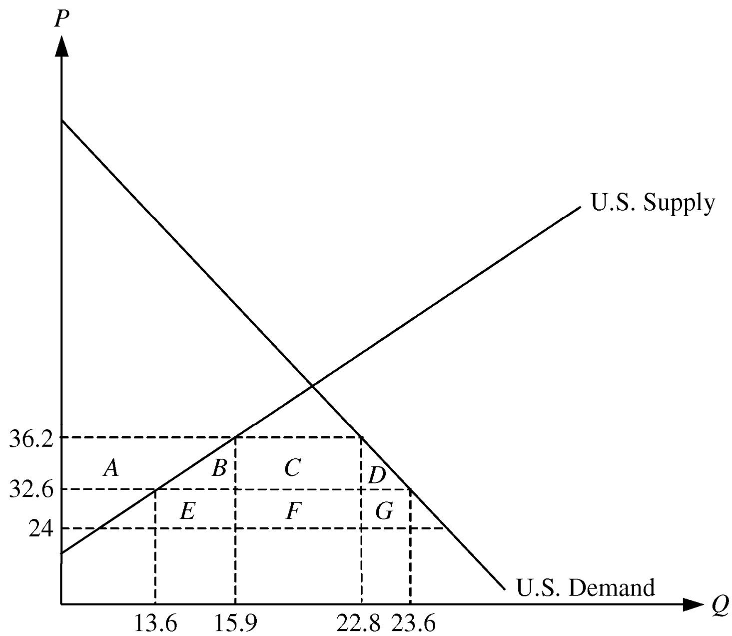
$$(29.73 - 0.19P) - (-7.95 + 0.66P) = 10$$

$$37.68 - 0.85P = 10$$

$$P = 32.6 \text{ cents per pound.}$$

At a price of 32.6 cents per pound, $Q_S = -7.95 + 0.66(32.6) = 13.6$ billion pounds, and $Q_D = Q_S + 10 = 13.6 + 10 = 23.6$ billion pounds.

b. How much would consumers gain and domestic producers lose?



The gain in consumer surplus is $A + B + C + D$. The loss to domestic producers is area A. The areas in billions of cents (i.e., tens of millions of dollars) are:

$A = (13.6)(36.2 - 32.6) + (0.5)(15.9 - 13.6)(36.2 - 32.6) = 53.10$

$B = (0.5)(15.9 - 13.6)(36.2 - 32.6) = 4.14$

$C = (22.8 - 15.9)(36.2 - 32.6) = 24.84$

$D = (0.5)(23.6 - 22.8)(36.2 - 32.6) = 1.44$

Thus, consumer surplus increases by 83.52, or $835.2 million, while domestic producer surplus decreases by 53.1, or $531 million.

c. What would be the effect on deadweight loss and foreign producers?

Domestic deadweight loss decreases by the difference between the increase in consumer surplus and the decrease in producer surplus, which is $835.2 – 531.0 = $304.2 million.

When the quota was 6.9 billion pounds, the profit earned by foreign producers was the difference between the domestic price and the world price (36.2 – 24) times the 6.9 billion units sold, for a total of 84.18, or $841.8 million. When the quota is increased to 10 billion pounds, domestic price falls to 32.6 cents per pound, and profit earned by foreigners is (32.6 – 24)(10) = 86, or $860 million. Profit earned by foreigners therefore increases by $18.2 million. On the diagram above, this is area $(E + F + G) - (C + F) = E + G - C$. The deadweight loss of the quota, including foreign producer surplus, decreases by area $B + D + E + G$. Area $E = 19.78$ and $G = 6.88$, so the decrease in deadweight loss = 4.14 + 1.44 + 19.78 + 6.88 = 32.24, or $322.4 million.

12. The domestic supply and demand curves for hula beans are as follows:

***Supply*: $P = 50 + Q$** ***Demand*: $P = 200 - 2Q$**

where P is the price in cents per pound and Q is the quantity in millions of pounds. The U.S. is a small producer in the world hula bean market, where the current price (which will not be affected by anything we do) is 60 cents per pound. Congress is considering a tariff of 40 cents per pound. Find the domestic price of hula beans that will result if the tariff is imposed. Also compute the dollar gain or loss to domestic consumers, domestic producers, and government revenue from the tariff.

To analyze the influence of a tariff on the domestic hula bean market, start by solving for domestic equilibrium price and quantity. First, equate supply and demand to determine equilibrium quantity without the tariff:

$$50 + Q = 200 - 2Q, \text{ or } Q_{EQ} = 50.$$

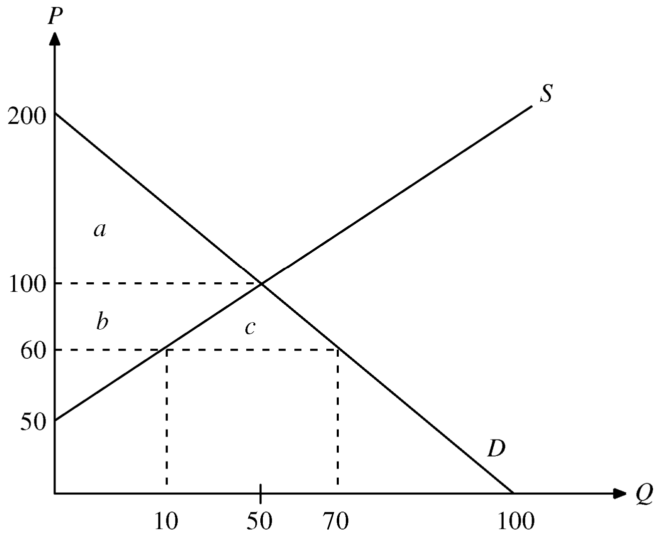
Thus the equilibrium quantity is 50 million pounds. Substituting Q_{EQ} of 50 into either the supply or demand equation to determine price, we find:

$$P_S = 50 + 50 = 100 \text{ and } P_D = 200 - (2)(50) = 100.$$

The equilibrium price P is $1 (100 cents). However, the world market price is 60 cents. At this price, the domestic quantity supplied is $60 = 50 + Q_S$, or $Q_S = 10$, and similarly, domestic demand at the world price is $60 = 200 - 2Q_D$, or $Q_D = 70$. Imports are equal to the difference between domestic demand and supply, or 60 million pounds. If Congress imposes a tariff of 40 cents, the effective price of imports increases to $1. At $1, domestic producers satisfy domestic demand and imports fall to zero.

As shown in the figure below, consumer surplus before the imposition of the tariff is equal to area $a + b + c$, or (0.5)(70)(200 − 60) = 4900 million cents, or \$49 million. After the tariff, the price rises to \$1.00 and consumer surplus falls to area a, or (0.5)(50)(200 − 100) = \$25 million, a loss of \$24 million. Producer surplus increases by area b, or (10)(100 − 60) + (0.5)(50 − 10)(100 − 60) = \$12 million.

Finally, because domestic production is equal to domestic demand at \$1, no hula beans are imported and the government receives no revenue. The difference between the loss of consumer surplus and the increase in producer surplus is deadweight loss, which in this case is equal to \$24 − 12 = \$12 million (area c).



13. Currently, the social security payroll tax in the United States is evenly divided between employers and employees. Employers must pay the government a tax of 6.2% of the wages they pay, and employees must pay 6.2% of the wages they receive. Suppose the tax were changed so that employers paid the full 12.4% and employees paid nothing. Would employees then be better off?

If the labor market is competitive (i.e., both employers and employees take the wage as given), then shifting all the tax onto employers will have no effect on the amount of labor employed or on employees' after tax wages. We know this because the incidence of a tax is the same regardless of who officially pays it. As long as the total tax doesn't change, the same amount of labor will be employed, and the wages paid by employers and received by employees (after tax) will not change. Hence, employees would be no better or worse off if employers paid the full amount of the social security tax.

14. You know that if a tax is imposed on a particular product, the burden of the tax is shared by producers and consumers. You also know that the demand for automobiles is characterized by a stock adjustment process. Suppose a special 20% sales tax is suddenly imposed on automobiles. Will the share of the tax paid by consumers rise, fall, or stay the same over time? Explain briefly. Repeat for a 50-cents-per-gallon gasoline tax.

For products with demand characterized by a stock adjustment process, short-run demand is more elastic than long-run demand because consumers can delay their purchases of these goods in the short run. For example, when price rises, consumers may continue using the older version of the product that they currently own. However, in the long run, a new product will be purchased as the old one wears out. Thus the long-run demand curve is more inelastic than the short-run one.

Consider the effect of imposing a 20% sales tax on automobiles in the short and long run. The portion of the tax that will be borne by consumers is given by the pass-through fraction, $E_S/(E_S - E_D)$. Assuming that the elasticity of supply, E_S, is the same in the short and long run, as demand becomes less elastic in the long run, the elasticity of demand, E_D, will become smaller in absolute value. Therefore the pass-through fraction will increase, and the share of the automobile tax paid by consumers will rise over time.

Unlike the automobile market, the gasoline demand curve is not characterized by a stock adjustment effect. Long-run demand will be more elastic than short-run demand, because in the long run consumers can make adjustments such as buying more fuel-efficient cars and taking public transportation that will reduce their use of gasoline. As the demand becomes more elastic in the long run, the pass-through fraction will fall, and therefore the share of the gas tax paid by consumers will fall over time.

15. In 2011, Americans smoked 16 billion packs of cigarettes. They paid an average retail price of \$5.00 per pack.

a. Given that the elasticity of supply is 0.5 and the elasticity of demand is –0.4, derive linear demand and supply curves for cigarettes.

Let the demand curve be of the general linear form $Q = a - bP$ and the supply curve be $Q = c + dP$, where a, b, c, and d are positive constants that we have to find from the information given above. To begin, recall the formula for the price elasticity of demand

$$E_P^D = \frac{P}{Q}\frac{\Delta Q}{\Delta P}.$$

We know the values of the elasticity, P, and Q, which means we can solve for the slope, which is $-b$ in the above formula for the demand curve.

$$-0.4 = \left(\frac{5.00}{16}\right)(-b)$$

$$b = 0.4\left(\frac{16}{5.00}\right) = 1.28.$$

To find the constant a, substitute for Q, P, and b in the demand curve formula: $16 = a - 1.28(5.00)$. Solving yields $a = 22.4$. The equation for demand is therefore $Q = 22.4 - 1.28P$. To find the supply curve, recall the formula for the elasticity of supply and follow the same method as above:

$$E_P^S = \frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

$$0.5 = \left(\frac{5.00}{16}\right)(d)$$

$$d = 0.5\left(\frac{16}{5.00}\right) = 1.6$$

To find the constant c, substitute for Q, P, and d in the supply formula, which yields $16 = c + 1.6(5.00)$. Therefore $c = 8$, and the equation for the supply curve is $Q = 8 + 1.6P$.

b. Cigarettes are subject to a federal tax, which was about \$1.00 per pack in 2011. What does this tax do to the market-clearing price and quantity?

The tax drives a wedge between supply and demand. At the new equilibrium, the price buyers pay, P_b, will be \$1.00 higher than the price sellers receive, P_s. Also, the quantity buyers demand at P_b must equal the quantity supplied at price P_s. These two conditions are:

$$P_b - P_s = 1.00 \quad \text{and} \quad 22.4 - 1.28P_b = 8 + 1.6\,P_s.$$

Solving these simultaneously, P_s = \$4.56 and P_b = \$5.56. The new quantity will be $Q = 22.4 - 1.28(5.56) = 15.3$ billion packs. So the price consumers pay will increase from \$5.00 to \$5.56 (a 56-cent increase) and consumption will fall from 16 to 15.3 billion packs per year (a drop of 700 million packs per year).

c. How much of the federal tax will consumers pay? What part will producers pay?

Consumers pay \$5.56 – 5.00 = \$0.56 and producers pay the remaining \$5.00 – 4.56 = \$0.44 per pack. We could also find these amounts using the pass-through formula. The fraction of the tax paid by consumers is $E_S/(E_S - E_D) = 0.5/[0.5 - (-0.4)] = 0.5/0.9 = 0.56$. Therefore, consumers will pay 56% of the \$1.00 tax, which is 56 cents, and suppliers will pay the remaining 44 cents.

PART THREE

Market Structure and Competitve Strategy

Chapter 10
Market Power: Monopoly and Monopsony

■ Teaching Notes

This chapter covers both monopoly and monopsony in order to highlight the similarity between the two types of market power. You might remind students that this is the other end of the market structure spectrum from perfect competition. Students seem to have an easier time accepting the assumptions underlying monopoly and monopsony, but you might want to remind them again that economic models are simplified versions of the real world, and there are very few companies that would qualify as pure monopolies or monopsonies.

The chapter begins with a discussion of monopoly power and the monopoly model in Sections 10.1–10.4. Section 10.5 introduces monopsony and offers a comparison of monopoly and monopsony. Section 10.6 discusses sources of monopsony power and the social costs of monopsony power, while Section 10.7 concludes with a discussion of antitrust law. If you are pressed for time you might choose to cover the first four sections on monopoly and skip the remainder of the chapter. Section 10.7 can be covered even if you choose to skip Sections 10.5 and 10.6. The last part of 10.1 on the multiplant firm can also be skipped if necessary, but it is interesting in its own right and particularly useful for deriving the monopoly marginal cost curve when comparing monopoly to perfect competition in Section 10.4.

Although Chapter 8 presented the general rule for profit maximization, you should review marginal revenue and the $MR = MC$ rule. Even though monopolists are price setters, we model them as choosing quantity, and this sometimes confuses students, so you may want to clear this up early in the chapter. The relationship between marginal revenue and price elasticity of demand is quite useful, and I like to derive the relationship $MR = P(1 + 1/E_d)$ following the steps on page 363. This can then be used to draw a demand and MR curve as in Figure 10.1 and to show how MR is related to price elasticity along a linear demand curve. Point out that marginal revenue is positive (because it equals a positive MC) at the profit-maximizing quantity, and therefore demand at that quantity must be elastic. This is true whether demand is linear or not.

Equation 10.1 on page 363 (and its close cousin, equation 10.2 on page 364) is also useful and leads directly to the Lerner Index in Section 10.2. This provides fruitful ground for discussion of a monopolist's market power. For example, if E_d is large (e.g., because of close substitutes), then (1) the demand curve is relatively flat, (2) the marginal revenue curve is relatively flat (although steeper than the demand curve), and (3) the monopolist has little power to raise price above marginal cost. To reinforce these points, introduce a non-linear demand curve by, for example, showing the location of the marginal revenue curve for a demand curve with an elasticity of –2. Once this concept has been clearly presented, the discussion of the effect of an excise tax on a monopolist with non-linear demand (Figure 10.5) will not seem out of place.

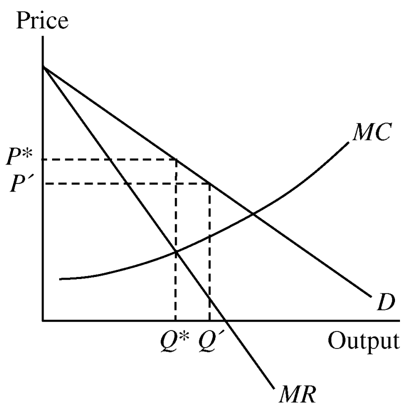
The social costs of market power are a good topic for class discussion. When discussing the deadweight loss from monopoly in Figure 10.10 be sure to stress that the analysis assumes there are no economies of scale for the monopolist. You might ask students whether it would be possible for the monopoly price to be less than the competitive price and, if so, to draw a diagram where this occurs. The material on price regulation in Section 10.4 (see Figure 10.11) is a bit complicated, but the results may surprise students, because a price ceiling can lead a monopolist to reduce price *and* increase output, which reduces deadweight loss. Since students have just seen how price ceilings cause deadweight loss in Chapter 9, you can use this result to show how economic theory helps us understand why things may work one way in some situations and another way in other situations. Also, Exercises 9, 13, and 15 involve "kinked" marginal revenue curves, so be sure to go over Figure 10.11 if you plan to assign those problems.

Students find the monopsony material confusing because everything seems backwards or upside down. The firm is not selling a product or service; it is buying. Try to draw parallels between monopoly and monopsony wherever possible, but expect some difficulties. One interesting application is the effect of a minimum wage law when the employer is a monopsonist. Rather than reducing employment and causing unemployment as in a competitive labor market, a minimum wage can raise wages *and* employment if the labor market is characterized by monopsony. Of course, not many companies have monopsony power in the labor markets where they hire, especially with today's highly mobile workers.

■ Review Questions

1. **A monopolist is producing at a point at which marginal cost exceeds marginal revenue. How should it adjust its output to increase profit?**

   When marginal cost is greater than marginal revenue, the cost of producing the last unit is greater than the additional revenue from the sale of the last unit, so the firm loses money on that unit. The firm would increase profit by not producing as many units. It should reduce production, thereby decreasing marginal cost and increasing marginal revenue, until marginal cost is equal to marginal revenue. The diagram shows this situation. The firm is producing an output like Q', where $MC > MR$. The firm should decrease output until it reaches the profit-maximizing output Q^*.



2. **We write the percentage markup of prices over marginal cost as $(P - MC)/P$. For a profit-maximizing monopolist, how does this markup depend on the elasticity of demand? Why can this markup be viewed as a measure of monopoly power?**

   Equation 10.1 on page 363 shows that the markup percentage is equal to the negative inverse of the price elasticity of demand.

$$\frac{P - MC}{P} = -\frac{1}{E_d}.$$

   Therefore, as demand becomes more elastic (E_d becomes more negative), the markup percentage becomes smaller. For example, if E_d changes from –2 to –5, the markup decreases from 0.5 to 0.2. This tells us that the firm has less power to mark up its price above marginal cost when it faces a more elastic demand. Therefore, the markup percentage can be viewed as a measure of monopoly power.

3. Why is there no market supply curve under conditions of monopoly?

The monopolist's output decision depends not only on marginal cost, but also on the demand curve. Shifts in demand do not trace out a series of prices and quantities that we can identify as the supply curve for the firm. Instead, shifts in demand lead to changes in price, output, or both. Thus there is no one-to-one correspondence between the price and the seller's quantity; therefore, a monopolized market lacks a supply curve.

4. Why might a firm have monopoly power even if it is not the only producer in the market?

A firm can have some monopoly power if its product is differentiated from other firms' products, and if some consumers prefer its product to other firms' products. A firm might also be located more conveniently for some consumers. These differences allow the firm to charge a price above its marginal cost and different from its rivals.

5. What are some of the different types of barriers to entry that give rise to monopoly power? Give an example of each.

There are several types of barriers to entry, including exclusive rights (e.g., patents, copyrights, and licenses), control of an essential resource, and economies of scale. Exclusive rights are legally granted property rights to produce or distribute a good or service. Complete control over an essential raw material such as bauxite to produce aluminum or oil to produce gasoline prevents other firms from producing the same product. Large economies of scale lead to "natural monopolies" because the largest producer can charge a lower price, driving competition from the market. For example, in the production of aluminum, there is evidence to suggest that there are scale economies in the conversion of bauxite to alumina. (See *U.S. v. Aluminum Company of America*, 148 F.2d 416 [1945], discussed in Exercise 10, below.)

6. What factors determine the amount of monopoly power an individual firm is likely to have? Explain each one briefly.

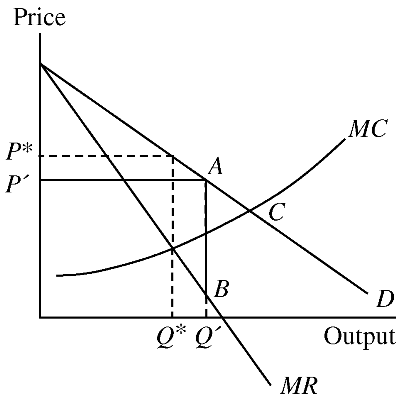
Three factors determine the firm's elasticity of demand and hence its market power: (1) the elasticity of market demand, (2) the number of firms in the market, and (3) the interaction among firms in the market. The elasticity of market demand depends on the uniqueness of the product, i.e., how easy it is for consumers to substitute for the product. As the number of firms in the market increases, the demand elasticity facing each firm increases because customers have more choices. The number of firms in the market is determined by how easy it is to enter the industry (the height of barriers to entry). Finally, the ability to raise price above marginal cost depends on how other firms react to the firm's price changes. If other firms match price changes, customers have little incentive to switch to another supplier, and this increases market power.

7. Why is there a social cost to monopoly power? If the gains to producers from monopoly power could be redistributed to consumers, would the social cost of monopoly power be eliminated? Explain briefly.

When the firm exploits its monopoly power by charging a price above marginal cost, consumers buy less at the higher price, and consumer surplus decreases. Some of the lost consumer surplus is not captured by the seller, however, because the quantity produced and consumed decreases at the higher price, and this is a deadweight loss to society. Therefore, if the gains to producers were redistributed to consumers, society would still suffer the deadweight loss.

8. Why will a monopolist's output increase if the government forces it to lower its price? If the government wants to set a price ceiling that maximizes the monopolist's output, what price should it set?

By restricting price to be below the monopolist's profit-maximizing price, the government can change the shape of the firm's marginal revenue curve. When a price ceiling is imposed, *MR* is equal to the price ceiling for all quantities less than or equal to the quantity demanded at the price ceiling. For example, in the diagram the price ceiling is set at P', which is below the profit-maximizing price P^*. The *MR* curve becomes the line $P'A$ and then jumps down to *B* and follows the original *MR* curve beyond that point. The optimal output for the monopolist is then Q', which is greater than the profit-maximizing output.



If the government wants to maximize output, it should set a price ceiling at the point where the demand curve and the marginal cost curve intersect, point *C* in the diagram. Then, when the firm produces where $MR = MC$, it will be producing the output level at which $P = MC$, where *P* is the price ceiling. In this way, the government can induce the monopolist to produce the competitive level of output. If the price ceiling is set below this point, the monopolist will decrease output below the competitive level.

9. How should a monopsonist decide how much of a product to buy? Will it buy more or less than a competitive buyer? Explain briefly.

The marginal expenditure is the change in the total expenditure as the purchased quantity changes. For a firm competing with many firms for inputs, the marginal expenditure is equal to the average expenditure (price). For a monopsonist, the marginal expenditure curve lies above the average expenditure curve because the decision to buy an extra unit raises the price that must be paid for all units, including the last unit. All firms should buy inputs so that the marginal value of the last unit is equal to the marginal expenditure on that unit. This is true for both the competitive buyer and the monopsonist. However, because the monopsonist's marginal expenditure curve lies above the average expenditure curve and because the marginal value curve is downward sloping, the monopsonist buys less than a firm would buy in a competitive market.

10. What is meant by the term "monopsony power"? Why might a firm have monopsony power even if it is not the only buyer in the market?

Monopsony power refers to a buyer's ability to affect the price of a good and to purchase the good for a lower price than in a competitive market. Any buyer facing an upward-sloping supply curve has some monopsony power. In a competitive market, the seller faces a perfectly elastic market demand curve and the buyer faces a perfectly elastic market supply curve. Thus, any characteristic of the market (e.g., a small number of buyers or buyers who engage in collusive behavior) that leads to a less-than-perfectly-elastic supply curve gives the buyer some monopsony power, even if it is not the only buyer in the market.

11. What are some sources of monopsony power? What determines the amount of monopsony power an individual firm is likely to have?

The individual firm’s monopsony power depends on the characteristics of the “buying-side” of the market. There are three characteristics that enhance monopsony power: (1) the elasticity of market supply, (2) the number of buyers, and (3) how the buyers interact. First, if market supply is very inelastic, then the buyer will enjoy more monopsony power. When supply is very elastic, marginal expenditure and average expenditure do not differ by much, so price will be closer to the competitive price. Second, the fewer the number of buyers, the greater the monopsony power. Third, if buyers are able to collude and/or they do not compete very aggressively with each other, then each will enjoy more monopsony power.

12. Why is there a social cost to monopsony power? If the gains to buyers from monopsony power could be redistributed to sellers, would the social cost of monopsony power be eliminated? Explain briefly.

With monopsony power, the price is lower and the quantity is less than under competitive buying conditions. Because of the lower price and reduced sales, sellers lose revenue. Only part of this lost revenue is transferred to the buyer as consumer surplus, and the net loss in total surplus is deadweight loss. Even if the consumer surplus could be redistributed to sellers, the deadweight loss persists. This inefficiency will remain because quantity is reduced below the level where price is equal to marginal cost.

13. How do the antitrust laws limit market power in the United States? Give examples of major provisions of these laws.

Antitrust laws limit market power by proscribing a firm’s behavior in attempting to maximize profit. Section 1 of the Sherman Act prohibits *every* restraint of trade, including any attempt to fix prices by buyers or sellers. Section 2 of the Sherman Act prohibits behavior that leads to monopolization. The Clayton Act, with the Robinson-Patman Act, prohibits price discrimination and exclusive dealing (sellers prohibiting buyers from buying goods from other sellers). The Clayton Act also limits mergers when they could substantially lessen competition. The Federal Trade Commission Act makes it illegal to use unfair or deceptive practices.

14. Explain briefly how the U.S. antitrust laws are actually enforced.

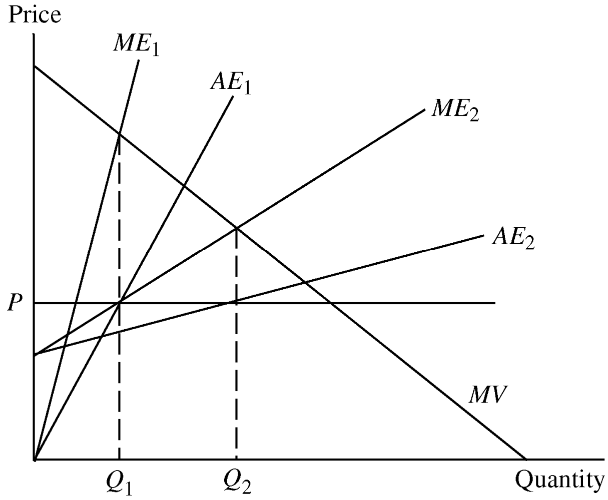
Antitrust laws are enforced in three ways: (1) through the Antitrust Division of the Justice Department whenever firms violate federal statutes, (2) through the Federal Trade Commission whenever firms violate the Federal Trade Commission Act, and (3) through civil suits. The Justice Department can seek to impose fines or jail terms on managers or owners, or it can seek to reorganize a firm, as it did in its case against AT&T. The FTC can seek a voluntary understanding to comply with the law or a formal Commission order. Individuals or companies can sue in federal court for awards equal to three times the damage arising from anticompetitive behavior.

■ Exercises

1. **Will an increase in the demand for a monopolist's product always result in a higher price? Explain. Will an increase in the supply facing a monopsonist buyer always result in a lower price? Explain.**

   As illustrated in Figure 10.4b in the textbook, an increase in demand for a monopolist's product need not *always* result in a higher price. Under the conditions portrayed in Figure 10.4b, the monopolist supplies different quantities at the same price.

   Similarly, an increase in supply facing a monopsonist need not *always* result in a higher price. Suppose the average expenditure curve shifts from AE_1 to AE_2, as illustrated in the figure. With the shift in the average expenditure curve, the marginal expenditure curve shifts from ME_1 to ME_2. The ME_1 curve intersects the marginal value curve (demand curve) at Q_1, resulting in a price of P. When the AE curve shifts, the ME_2 curve intersects the marginal value curve at Q_2 resulting in the same price at P.



2. **Caterpillar Tractor, one of the largest producers of farm machinery in the world, has hired you to advise it on pricing policy. One of the things the company would like to know is how much a 5% increase in price is likely to reduce sales. What would you need to know to help the company with this problem? Explain why these facts are important.**

   As a large producer of farm equipment, Caterpillar Tractor has some market power and should consider the entire demand curve when choosing prices for its products. As their advisor, you should focus on the determination of the elasticity of demand for the company's tractors. There are at least four important factors to be considered. First, how similar are the products offered by Caterpillar's competitors? If they are close substitutes, a small increase in price could induce customers to switch to the competition. Second, how will Caterpillar's competitors respond to a price increase? If the other firms are likely to match Caterpillar's increase, Caterpillar's sales will not fall nearly as much as they would were the other firms not to match the price increase. Third, what is the age of the existing stock of tractors? With an older population of tractors, farmers will want to replace their aging stock, and their demands will be less elastic. In this case, a 5% price increase induces a smaller drop in sales than would occur with a younger stock of tractors that are not in need of replacement. Finally, because farm tractors are a capital input in agricultural production, what is the expected profitability of the agricultural sector? If farm incomes are expected to fall, an increase in tractor prices would cause a greater decline in sales than would occur if farm incomes were high.

3. A monopolist firm faces a demand with constant elasticity of −2.0. It has a constant marginal cost of $20 per unit and sets a price to maximize profit. If marginal cost should increase by 25%, would the price charged also rise by 25%?

The monopolist's pricing rule is: $\frac{P-MC}{P} = -\frac{1}{E_d}$, or alternatively, $P = \frac{MC}{\left(1+\left(\frac{1}{E_d}\right)\right)}$. Therefore, price should be set so that $P = \frac{MC}{\left(1+\frac{1}{-2}\right)} = 2MC$. With $MC = 20$, the optimal price is $P = 2(20) = \$40$.

If MC increases by 25% to $25, the new optimal price is $P = 2(25) = \$50$, a 25% increase. So if marginal cost increases by 25%, the price also increases by 25%.

4. A firm faces the following average revenue (demand) curve:

$$\boldsymbol{P = 120 - 0.02Q}$$

where Q is weekly production and P is price, measured in cents per unit. The firm's cost function is given by $C = 60Q + 25{,}000$. Assume that the firm maximizes profits.

a. What is the level of production, price, and total profit per week?

The profit-maximizing output is found by setting marginal revenue equal to marginal cost. Given a linear demand curve in inverse form, $P = 120 - 0.02Q$, we know that the marginal revenue curve has the same intercept and twice the slope of the demand curve. Thus, the marginal revenue curve for the firm is $MR = 120 - 0.04Q$. Marginal cost is the slope of the total cost curve. The slope of $TC = 60Q + 25{,}000$ is 60, so MC is constant and equal to 60. Setting $MR = MC$ to determine the profit-maximizing quantity:

$$120 - 0.04Q = 60, \text{ or}$$

$$Q = 1500.$$

Substituting the profit-maximizing quantity into the inverse demand function to determine the price:

$$P = 120 - (0.02)(1500) = 90 \text{ cents.}$$

Profit equals total revenue minus total cost:

$$\pi = (90)(1500) - (25{,}000 + (60)(1500)), \text{ so}$$

$$\pi = 20{,}000 \text{ cents per week, or \$200 per week.}$$

b. If the government decides to levy a tax of 14 cents per unit on this product, what will be the new level of production, price, and profit?

Suppose initially that consumers must pay the tax to the government. Since the total price (including the tax) that consumers would be willing to pay remains unchanged, we know that the demand function is

$$P^* + t = 120 - 0.02Q, \text{ or}$$

$$P^* = 120 - 0.02Q - t,$$

where P^* is the price received by the suppliers and t is the tax per unit. Because the tax increases the price consumers pay for each unit, total revenue for the monopolist decreases by tQ. You can see this most easily by expressing $R = P^*Q$, which means tQ is subtracted from revenue. Marginal revenue, the revenue on each additional unit, decreases by t:

$$MR = 120 - 0.04Q - t$$

where $t = 14$ cents. To determine the profit-maximizing level of output with the tax, equate marginal revenue with marginal cost:

$$120 - 0.04Q - 14 = 60, \text{ or}$$

$$Q = 1150 \text{ units.}$$

Substituting Q into the demand function to determine price:

$$P^* = 120 - (0.02)(1150) - 14 = 83 \text{ cents.}$$

Profit is total revenue minus total cost:

$$= (83)(1150) - [(60)(1150) + 25{,}000] = 1450 \text{ cents, or}$$

$$\$14.50 \text{ per week.}$$

***Note*:** The price facing the consumer after the imposition of the tax is 83 + 14 = 97 cents. Compared to the 90-cent price before the tax is imposed, consumers and the monopolist each pay 7 cents of the tax.

If the monopolist had to pay the tax instead of the consumer, we would arrive at the same result. The monopolist's cost function would then be

$$TC = 60Q + 25{,}000 + tQ = (60 + t)Q + 25{,}000.$$

The slope of the cost function is $(60 + t)$, so $MC = 60 + t$. We set this MC equal to the marginal revenue function from part a:

$$120 - 0.04Q = 60 + 14, \text{ or}$$

$$Q = 1150.$$

Thus, it does not matter who sends the tax payment to the government. The burden of the tax is shared by consumers and the monopolist in exactly the same way.

5. The following table shows the demand curve facing a monopolist who produces at a constant marginal cost of $10:

Price	**Quantity**
18	**0**
16	**4**
14	**8**
12	**12**
10	**16**
8	**20**
6	**24**
4	**28**
2	**32**
0	**36**

a. Calculate the firm's marginal revenue curve.

To find the marginal revenue curve, we first derive the inverse demand curve. The intercept of the inverse demand curve on the price axis is 18. The slope of the inverse demand curve is the change in price divided by the change in quantity. For example, a decrease in price from 18 to 16 yields an increase in quantity from 0 to 4. Therefore, the slope of the inverse demand is $\frac{\Delta P}{\Delta Q} = \frac{-2}{4} = -0.5$, and the demand curve is therefore

$$P = 18 - 0.5Q.$$

The marginal revenue curve corresponding to a linear demand curve is a line with the same intercept as the inverse demand curve and a slope that is twice as steep. Therefore, the marginal revenue curve is

$$MR = 18 - Q.$$

b. What are the firm's profit-maximizing output and price? What is its profit?

The monopolist's profit-maximizing output occurs where marginal revenue equals marginal cost. Marginal cost is a constant \$10. Setting MR equal to MC to determine the profit-maximizing quantity:

$$18 - Q = 10, \text{ or } Q = 8.$$

To find the profit-maximizing price, substitute this quantity into the demand equation:

$$P = 18 - (0.5)(8) = \$14.$$

Total revenue is price times quantity:

$$TR = (14)(8) = \$112.$$

The profit of the firm is total revenue minus total cost, and total cost is equal to average cost times the level of output produced. Since marginal cost is constant, average variable cost is equal to marginal cost. Ignoring any fixed costs, total cost is $10Q$ or 80, and profit is

$$\pi = 112 - 80 = \$32.$$

c. What would the equilibrium price and quantity be in a competitive industry?

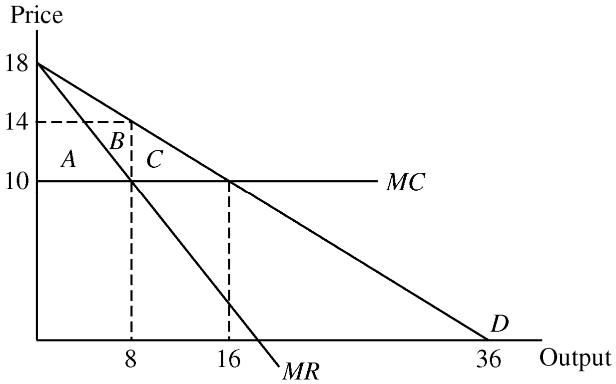
For a competitive industry, price would equal marginal cost at equilibrium. Setting the expression for price equal to a marginal cost of 10:

$$18 - 0.5Q = 10, \text{ so that } Q = 16 \text{ and } P = \$10.$$

Note the increase in the equilibrium quantity and decrease in price compared to the monopoly solution.

d. What would the social gain be if this monopolist were forced to produce and price at the competitive equilibrium? Who would gain and lose as a result?

The social gain arises from the elimination of deadweight loss. When price drops from \$14 to \$10, consumer surplus increases by area $A + B + C = 8(14 - 10) + (0.5)(16 - 8)(14 - 10) = \$48$. Producer surplus decreases by area $A + B = 8(14 - 10) = \$32$. So consumers gain \$48 while producers lose \$32. Deadweight loss decreases by the difference, \$48 − 32 = \$16. Thus the social gain if the monopolist were forced to produce and price at the competitive level is \$16.



6. Suppose that an industry is characterized as follows:

$C = 100 + 2q^2$	**each firm's total cost function**
$MC = 4q$	**firm's marginal cost function**
$P = 90 - 2Q$	**industry demand curve**
$MR = 90 - 4Q$	**industry marginal revenue curve**

a. If there is *only one firm* in the industry, find the monopoly price, quantity, and level of profit.

If there is only one firm in the industry, then the firm will act like a monopolist and produce at the point where marginal revenue is equal to marginal cost:

$$90 - 4Q = 4Q$$

$$Q = 11.25.$$

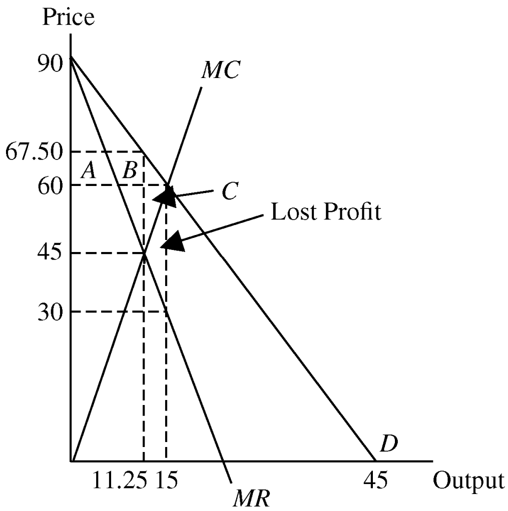
For a quantity of 11.25, the firm will charge a price $P = 90 - 2(11.25) = \$67.50$. $= PQ - C =$ \$67.50(11.25) − [100 + 2(11.25)^2] = \$406.25.

b. Find the price, quantity, and level of profit if the industry is competitive.

If the industry is competitive, price will equal marginal cost. Therefore $90 - 2Q = 4Q$, or $Q = 15$. At a quantity of 15, price is equal to $P = 90 - 2(15) = \$60$. The industry's profit is = \$60(15) − [100 + 2(15)^2] = \$350.

c. Graphically illustrate the demand curve, marginal revenue curve, marginal cost curve, and average cost curve. Identify the difference between the profit level of the monopoly and the profit level of the competitive industry in two different ways. Verify that the two are numerically equivalent.

The graph below illustrates the demand curve, marginal revenue curve, and marginal cost curve. The average cost curve is not shown because it makes the diagram too cluttered. *AC* reaches its minimum value of \$28.28 and intersects the marginal cost curve at a quantity of 7.07. The profit that is lost by having the firm produce at the competitive solution as compared to the monopoly solution is the difference of the two profit levels as calculated in parts a and b: \$406.25 − 350 = \$56.25. On the graph below, this difference is represented by the lost profit area, which is the triangle below the marginal cost curve and above the marginal revenue curve, between the quantities of 11.25 and 15. This is lost profit because for each of these 3.75 units, extra revenue earned was less than extra cost incurred. This area is (0.5)(3.75)(60–30) = \$56.25. Another way to find this difference is to use the fact that the change in producer surplus equals the change in profit. Going from the monopoly price to the competitive price, producer surplus is reduced by areas *A* + *B* and increased by area *C*. *A* + *B* is a rectangle with area (11.25)(67.50 − 60) = \$84.375. Area *C* equals (0.5)(3.75)(60 − 45) = \$28.125. The difference is \$84.375 − 28.125 = \$56.25. A final method of graphically illustrating the difference in the two profit levels is to draw in the average cost curve and identify the two profit rectangles, one for the monopoly output and the other for the competitive output. The area of each profit rectangle is the difference between price and average cost multiplied by quantity, (*P* − *AC*)*Q*. The difference between the areas of the two profit rectangles is \$56.25.



7. **Suppose a profit-maximizing monopolist is producing 800 units of output and is charging a price of \$40 per unit.**

   a. **If the elasticity of demand for the product is −2, find the marginal cost of the last unit produced.**

   The monopolist's pricing rule as a function of the elasticity of demand is:

$$\frac{(P - MC)}{P} = -\frac{1}{E_d}$$

   or alternatively,

$$P\left(1 + \frac{1}{E_d}\right) = MC$$

   Substitute −2 for the elasticity and 40 for price, and then solve for MC = \$20.

   b. **What is the firm's percentage markup of price over marginal cost?**

   (P − MC)/P = (40 − 20)/40 = 0.5, so the markup is 50% of the price.

c. Suppose that the average cost of the last unit produced is \$15 and the firm's fixed cost is \$2000. Find the firm's profit.

Total revenue is price times quantity, or \$40(800) = \$32,000. Total cost is equal to average cost times quantity, or \$15(800) = \$12,000. Profit is therefore \$32,000 − 12,000 = \$20,000. Fixed cost is already included in average cost, so we do not use the \$2000 fixed cost figure separately.

8. A firm has two factories for which costs are given by:

$$\text{Factory \#1: } C_1(Q_1) = 10Q_1^2$$
$$\text{Factory \#2: } C_2(Q_2) = 20Q_2^2$$

The firm faces the following demand curve:

$$\boldsymbol{P = 700 - 5Q}$$

where Q is total output – i.e., $Q = Q_1 + Q_2$.

a. On a diagram, draw the marginal cost curves for the two factories, the average and marginal revenue curves, and the total marginal cost curve (i.e., the marginal cost of producing $Q = Q_1 + Q_2$). Indicate the profit-maximizing output for each factory, total output, and price.

The average revenue curve is the demand curve,

$$P = 700 - 5Q.$$

For a linear demand curve, the marginal revenue curve has the same intercept as the demand curve and a slope that is twice as steep:

$$MR = 700 - 10Q.$$

Next, determine the marginal cost of producing Q. To find the marginal cost of production in Factory 1, take the derivative of the cost function with respect to Q_1:

$$MC_1 = \frac{dC_1(Q_1)}{dQ_1} = 20Q_1.$$

Similarly, the marginal cost in Factory 2 is

$$MC_2 = \frac{dC_2(Q_2)}{dQ_2} = 40Q_2.$$

We know that total output should be divided between the two factories so that the marginal cost is the same in each factory. Let MC_T be this common marginal cost value. Then, rearranging the marginal cost equations in inverse form and horizontally summing them, we obtain total marginal cost, MC_T:

$$Q = Q_1 + Q_2 = \frac{MC_1}{20} + \frac{MC_2}{40} = \frac{3MC_T}{40}, \text{ or}$$
$$MC_T = \frac{40Q}{3}.$$

Profit maximization occurs where $MC_T = MR$. See the figure below for the profit-maximizing output for each factory, total output Q_T, and price P_M.

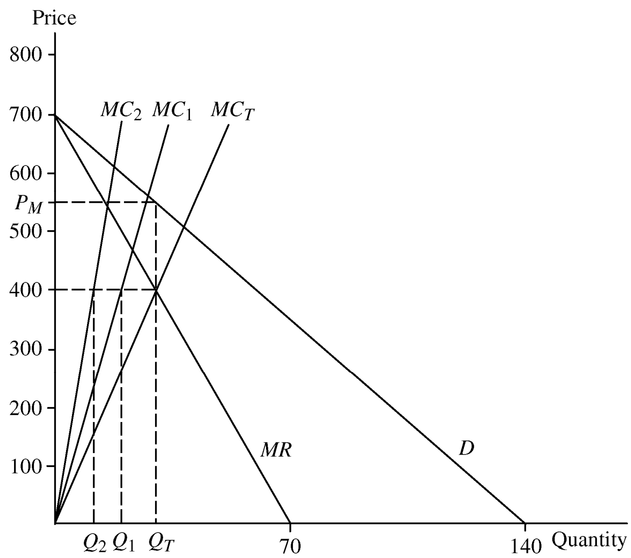


Figure 10.8.a

b. Calculate the values of Q_1, Q_2, Q, and P that maximize profit.

To calculate the total output Q that maximizes profit, set $MC_T = MR$:

$$\frac{40Q}{3} = 700 - 10Q, \text{ or } Q = 30.$$

When $Q = 30$, marginal revenue is $MR = 700 - (10)(30) = 400$. At the profit-maximizing point, $MR = MC_1 = MC_2$. Therefore,

$$MC_1 = 400 = 20Q_1, \text{ or } Q_1 = 20 \text{ and}$$

$$MC_2 = 400 = 40Q_2, \text{ or } Q_2 = 10.$$

To find the monopoly price, P, substitute for Q in the demand equation:

$$P = 700 - 5(30), \text{ or}$$

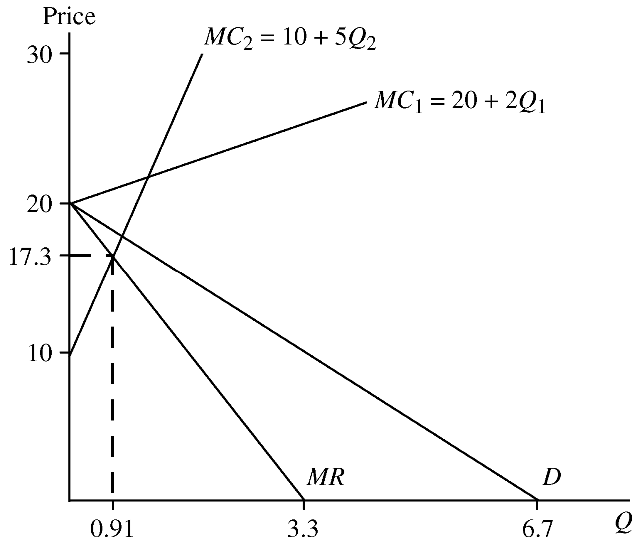
$$P_M = \$550.$$

c. Suppose that labor costs increase in Factory 1 but not in Factory 2. How should the firm adjust (i.e., raise, lower, or leave unchanged) the following: Output in Factory 1? Output in Factory 2? Total output? Price?

An increase in labor costs will lead to a horizontal shift to the left in MC_1, causing MC_T to shift to the left as well (since it is the horizontal sum of MC_1 and MC_2). The new MC_T curve will intersect the MR curve at a lower total quantity and higher marginal revenue. You can see this in Figure 10.8.a above. At a higher level of marginal revenue, Q_2 is greater than at the original level of MR. Since Q_T falls and Q_2 rises, Q_1 must fall. Since Q_T falls, price must rise.

9. A drug company has a monopoly on a new patented medicine. The product can be made in either of two plants. The costs of production for the two plants are $MC_1 = 20 + 2Q_1$ and $MC_2 = 10 + 5Q_2$. The firm's estimate of demand for the product is $P = 20 - 3(Q_1 + Q_2)$. How much should the firm plan to produce in each plant? At what price should it plan to sell the product?

First, notice that *only* MC_2 is relevant because the marginal cost curve of the first plant lies above the demand curve.



This means that the demand curve becomes $P = 20 - 3Q_2$. With an inverse linear demand curve, we know that the marginal revenue curve has the same vertical intercept but twice the slope, or $MR = 20 - 6Q_2$. To determine the profit-maximizing level of output, equate MR and MC_2:

$$20 - 6Q_2 = 10 + 5Q_2, \text{ or } Q_2 = 0.91.$$

Also, $Q_1 = 0$, and therefore total output is $Q = 0.91$. Price is determined by substituting the profit-maximizing quantity into the demand equation:

$$P = 20 - 3(0.91) = \$17.27.$$

10. One of the more important antitrust cases of the 20th century involved the Aluminum Company of America (Alcoa) in 1945. At that time, Alcoa controlled about 90% of primary aluminum production in the United States, and the company had been accused of monopolizing the aluminum market. In its defense, Alcoa argued that although it indeed controlled a large fraction of the primary market, secondary aluminum (i.e., aluminum produced from the recycling of scrap) accounted for roughly 30% of the total supply of aluminum and that many competitive firms were engaged in recycling. Therefore, Alcoa argued, it did not have much monopoly power.

a. Provide a clear argument *in favor* of Alcoa's position.

Although Alcoa controlled about 90% of primary aluminum production, secondary production by recyclers accounted for 30% of the total aluminum supply. Therefore, Alcoa actually controlled about 63% (90% of the 70% that did not come from recyclers) of the aluminum supply. Alcoa's ability to raise prices was constrained by the recyclers because with a higher price, a much larger proportion of aluminum supply could come from these secondary sources, as there was a large stock of potential scrap supply in the economy. Therefore, the price elasticity of demand for Alcoa's primary aluminum was much higher (in absolute value) than might be expected, given Alcoa's dominant position in primary aluminum production. In addition, other metals such as copper and steel are feasible substitutes for aluminum in some applications. Again, the demand elasticity Alcoa faced might be higher than would otherwise be expected.

b. Provide a clear argument *against* Alcoa's position.

While Alcoa could not raise its price by very much at any one time, the stock of potential aluminum supply is limited. Therefore, by keeping a stable high price, Alcoa could reap monopoly profits. Also, since Alcoa had originally produced most of the metal reappearing as recycled scrap, it would have considered the effect of scrap reclamation on future prices. Therefore, it exerted effective monopolistic control over the secondary metal supply.

c. The 1945 decision by Judge Learned Hand has been called "one of the most celebrated judicial opinions of our time." Do you know what Judge Hand's ruling was?

Judge Hand ruled against Alcoa but did not order it to divest itself of any of its United States production facilities. The two remedies imposed by the court were (1) that Alcoa was barred from bidding for two primary aluminum plants constructed by the government during World War II (they were sold to Reynolds and Kaiser), and (2) that it divest itself of its Canadian subsidiary, which became Alcan.

11. A monopolist faces the demand curve $P = 11 - Q$, where P is measured in dollars per unit and Q in thousands of units. The monopolist has a constant average cost of \$6 per unit.

a. Draw the average and marginal revenue curves and the average and marginal cost curves. What are the monopolist's profit-maximizing price and quantity? What is the resulting profit? Calculate the firm's degree of monopoly power using the Lerner index.

Because demand (average revenue) is $P = 11 - Q$, the marginal revenue function is $MR = 11 - 2Q$. Also, because average cost is constant, marginal cost is constant and equal to average cost, so $MC = 6$.

To find the profit-maximizing level of output, set marginal revenue equal to marginal cost:

$$11 - 2Q = 6, \text{ or } Q = 2.5.$$

That is, the profit-maximizing quantity equals 2500 units. Substitute the profit-maximizing quantity into the demand equation to determine the price:

$$P = 11 - 2.5 = \$8.50.$$

Profits are equal to total revenue minus total cost,

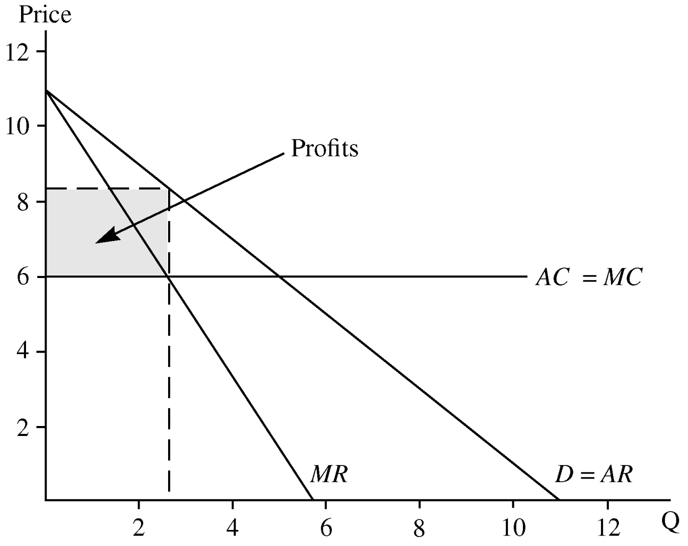
$$\pi = TR - TC = PQ - (AC)(Q), \text{ or}$$

$$\pi = (8.50)(2.5) - (6)(2.5) = 6.25, \text{ or } \$6250.$$

The diagram below shows the demand, MR, AC, and MC curves along with the optimal price and quantity and the firm's profits.

The degree of monopoly power according to the Lerner Index is:

$$\frac{P - MC}{P} = \frac{8.5 - 6}{8.5} = 0.294.$$



b. A government regulatory agency sets a price ceiling of $7 per unit. What quantity will be produced, and what will the firm's profit be? What happens to the degree of monopoly power?

To determine the effect of the price ceiling on the quantity produced, substitute the ceiling price into the demand equation.

$$7 = 11 - Q, \text{ or } Q = 4.$$

Therefore, the firm will choose to produce 4000 units rather than the 2500 units without the price ceiling. Also, the monopolist will choose to sell its product at the $7 price ceiling because $7 is the highest price that it can charge, and this price is still greater than the constant marginal cost of $6, resulting in positive monopoly profit.

Profits are equal to total revenue minus total cost:

$$\pi = 7(4000) - 6(4000) = \$4000.$$

The degree of monopoly power falls to

$$\frac{P - MC}{P} = \frac{7 - 6}{7} = 0.143.$$

c. What price ceiling yields the largest level of output? What is that level of output? What is the firm's degree of monopoly power at this price?

If the regulatory authority sets a price below $6, the monopolist would prefer to go out of business because it cannot cover its average variable costs. At any price above $6, the monopolist would produce less than the 5000 units that would be produced in a competitive industry. Therefore, the regulatory agency should set a price ceiling of $6, thus making the monopolist face a horizontal effective demand curve up to $Q = 5$ (i.e., 5000 units). To ensure a positive output (so that the monopolist is not indifferent between producing 5000 units and shutting down), the price ceiling should be set at $6 + δ, where δ is small.

Thus, 5000 is the maximum output that the regulatory agency can extract from the monopolist by using a price ceiling. The degree of monopoly power is

$$\frac{P-MC}{P}=\frac{6+\delta-6}{6}=\frac{\delta}{6}\to 0 \text{ as } \delta \to 0.$$

12. Michelle's Monopoly Mutant Turtles (*MMMT*) has the exclusive right to sell Mutant Turtle t-shirts in the United States. The demand for these t-shirts is $Q = 10{,}000/P^2$. The firm's short-run cost is $SRTC = 2000 + 5Q$, and its long-run cost is $LRTC = 6Q$.

a. What price should *MMMT* charge to maximize profit in the short run? What quantity does it sell, and how much profit does it make? Would it be better off shutting down in the short run?

MMMT should offer enough t-shirts so that $MR = MC$. In the short run, marginal cost is the change in *SRTC* as the result of the production of another t-shirt, i.e., $SRMC = 5$, the slope of the *SRTC* curve. Demand is:

$$Q=\frac{10{,}000}{P^2},$$

or, in inverse form,

$$P = 100Q^{-1/2}.$$

Total revenue is $TR = PQ = 100Q^{1/2}$. Taking the derivative of *TR* with respect to Q, $MR = 50Q^{-1/2}$. Equating *MR* and *MC* to determine the profit-maximizing quantity:

$$5 = 50Q^{-1/2}, \text{ or } Q = 100.$$

Substituting $Q = 100$ into the demand function to determine price:

$$P = (100)(100^{-1/2}) = \$10.$$

The profit at this price and quantity is equal to total revenue minus total cost:

$$\pi = 10(100) - [2000 + 5(100)] = -\$1500.$$

Although profit is negative, price is above the average variable cost of 5, and therefore the firm should not shut down in the short run. Since most of the firm's costs are fixed, the firm loses \$2000 if nothing is produced. If the profit-maximizing (i.e., loss-minimizing) quantity is produced, the firm loses only \$1500.

b. What price should *MMMT* charge in the long run? What quantity does it sell and how much profit does it make? Would it be better off shutting down in the long run?

In the long run, marginal cost is equal to the slope of the *LRTC* curve, which is 6.

Equating marginal revenue and long run marginal cost to determine the profit-maximizing quantity:

$$50Q^{-1/2} = 6, \text{ or } Q = 69.444$$

Substituting $Q = 69.444$ into the demand equation to determine price:

$$P = (100)(69.444)^{-1/2} = (100)(1/8.333) = 12$$

Total revenue is $TR = 12(69.444) = \$833.33$ and total cost is $LRTC = 6(69.444) = \$416.67$. Profit is therefore $\$833.33 - 416.67 = \$416.66$. The firm should remain in business in the long run.

c. Can we expect *MMMT* to have lower marginal cost in the short run than in the long run? Explain why.

In the long run, *MMMT* can change all its inputs when it changes output level. Therefore, *LRMC* includes the costs of all inputs that are fixed in the short run but variable in the long run. These costs do not appear in *SRMC*. As a result we can expect *SRMC* to be lower than *LRMC* in many cases.

13. You produce widgets for sale in a perfectly competitive market at a market price of \$10 per widget. Your widgets are manufactured in two plants, one in Massachusetts and the other in Connecticut. Because of labor problems in Connecticut, you are forced to raise wages there so that marginal costs in that plant increase. In response to this, should you shift production and produce more in your Massachusetts plant?

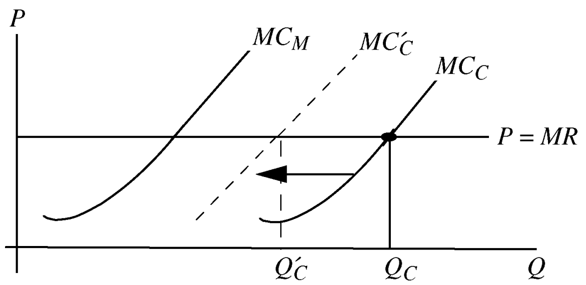
No, production should not shift to the Massachusetts plant, although production in the Connecticut plant should be reduced. To maximize profits, a multiplant firm will schedule production so that the following two conditions are met:

- Marginal costs of production at each plant are equal.
- Marginal revenue of the last unit sold is equal to the marginal cost at each plant.

These two rules can be summarized as $MR = MC_1 = MC_2$, where the subscripts indicate plants. The firm in this example has two plants and sells in a perfectly competitive market. In a perfectly competitive market $P = MR$. Therefore, production among the plants should be allocated such that:

$$P = MC_c(Q_c) = MC_m(Q_m),$$

where the subscripts denote plant locations (*c* for Connecticut, etc.). The marginal costs of production have increased in Connecticut but have not changed in Massachusetts. *MC* shifts up and to the left in Connecticut, so production in the Connecticut plant should drop from Q_c to Q_c' in the diagram below. Since costs have not changed in Massachusetts, the level of Q_m that sets $MC_m(Q_m) = P$, should not change.



14. The employment of teaching assistants (TAs) by major universities can be characterized as a monopsony. Suppose the demand for TAs is $W = 30{,}000 - 125n$, where W is the wage (as an annual salary), and n is the number of TAs hired. The supply of TAs is given by $W = 1000 + 75n$.

a. If the university takes advantage of its monopsonist position, how many TAs will it hire? What wage will it pay?

The supply curve is equivalent to the average expenditure curve. With a supply curve of $W = 1000 + 75n$, the total expenditure is $Wn = 1000n + 75n^2$. Taking the derivative of the total expenditure function with respect to the number of TAs, the marginal expenditure curve is $ME = 1000 + 150n$. As a monopsonist, the university would equate marginal value (demand) with marginal expenditure to determine the number of TAs to hire:

$$30{,}000 - 125n = 1000 + 150n, \text{ or } n = 105.5.$$

Substituting $n = 105.5$ into the supply curve to determine the wage:

$$1000 + 75(105.5) = \$8{,}912.50 \text{ annually.}$$

b. If, instead, the university faced an infinite supply of TAs at the annual wage level of $10,000, how many TAs would it hire?

With an infinite number of TAs at $10,000, the supply curve is horizontal at $10,000. Total expenditure is $10{,}000(n)$, and marginal expenditure is 10,000. Equating marginal value and marginal expenditure:

$$30{,}000 - 125n = 10{,}000, \text{ or } n = 160.$$

15. Dayna's Doorstops, Inc. (*DD*) is a monopolist in the doorstop industry. Its cost is $C = 100 - 5Q + Q^2$, and demand is $P = 55 - 2Q$.

a. What price should *DD* set to maximize profit? What output does the firm produce? How much profit and consumer surplus does *DD* generate?

To maximize profit, *DD* should equate marginal revenue and marginal cost. Given a demand of $P = 55 - 2Q$, we know that total revenue, PQ, is $55Q - 2Q^2$. Marginal revenue is found by taking the first derivative of total revenue with respect to Q or:

$$MR = \frac{dTR}{dQ} = 55 - 4Q.$$

Similarly, marginal cost is determined by taking the first derivative of the total cost function with respect to Q or:

$$MC = \frac{dTC}{dQ} = 2Q - 5.$$

Equating MC and MR to determine the profit-maximizing quantity,

$$55 - 4Q = 2Q - 5, \text{ or } Q = 10.$$

Substitute $Q = 10$ into the demand equation to find the profit-maximizing price:

$$P = 55 - 2(10) = \$35.$$

Profits are equal to total revenue minus total cost:

$$\pi = (35)(10) - [100 - 5(10) + 10^2] = \$200.$$

Consumer surplus is equal to one-half times the profit-maximizing quantity, 10, times the difference between the demand intercept (55) and the monopoly price (35):

$$CS = (0.5)(10)(55 - 35) = \$100.$$

b. What would output be if *DD* acted like a perfect competitor and set $MC = P$? What profit and consumer surplus would then be generated?

In competition, profits are maximized at the point where price equals marginal cost. So set price (as given by the demand curve) equal to MC:

$$55 - 2Q = 2Q - 5, \text{ or}$$
$$Q = 15.$$

Substituting $Q = 15$ into the demand equation to determine the price:

$$P = 55 - 2(15) = \$25.$$

Profits are total revenue minus total cost or:

$$\pi = (25)(15) - [100 - 5(15) + 15^2] = \$125.$$

Consumer surplus is

$$CS = (0.5)(15)(55 - 25) = \$225.$$

So consumer surplus increases by \$125 and producer surplus decreases by \$75.

c. What is the deadweight loss from monopoly power in part a?

The deadweight loss is equal to the area below the demand curve, above the marginal cost curve, and between the quantities of 10 and 15, or numerically

$$DWL = (0.5)(35 - 15)(15 - 10) = \$50.$$

d. Suppose the government, concerned about the high price of doorstops, sets a maximum price at \$27. How does this affect price, quantity, consumer surplus, and *DD*'s profit? What is the resulting deadweight loss?

With the price ceiling, the maximum price that *DD* may charge is \$27.00. Note that when a ceiling price is set below the monopoly price the ceiling price is the firm's marginal revenue for each unit sold up to the quantity demanded at the ceiling price.

Substitute the ceiling price of \$27.00 into the demand equation to determine the effect on the equilibrium quantity sold:

$$27 = 55 - 2Q, \text{ or } Q = 14.$$

Compared to part a, price drops from \$35 to \$27 and output increases from 10 to 14.

Consumer surplus is

$$CS = (0.5)(14)(55 - 27) = \$196.$$

Profits are

$$\pi = (27)(14) - [100 - 5(14) + 14^2] = \$152.$$

So *CS* increases from \$100 to \$196 and profit falls from \$200 to \$152.

The deadweight loss is $DWL = (0.5)(15 - 14)(27 - 23) = \$2$

e. Now suppose the government sets the maximum price at \$23. How does this decision affect price, quantity, consumer surplus, *DD*'s profit, and deadweight loss?

With a ceiling price set below the competitive price, *DD*'s output will be less than the competitive output of 15. Equate marginal revenue (the ceiling price) and marginal cost to determine the profit-maximizing level of output:

$$23 = 2Q - 5, \text{ or } Q = 14.$$

With the government-imposed maximum price of \$23, profits are

$$\pi = (23)(14) - [100 - 5(14) + 14^2] = \$96.$$

Consumer surplus is realized on 14 doorsteps. Therefore, it is equal to the consumer surplus in part d, which was \$196, plus an additional area due to the fact that price is now \$23 instead of \$27. The additional amount is $(27 - 23)(14) = \$56$. Therefore, consumer surplus is $\$196 + 56 = \$252$.

Compared to part d, price is \$4 less and quantity is the same. Consumer surplus increases by \$56 and *DD*'s profit decreases by \$56, which is also the drop in producer surplus. Since the increase in consumer surplus equals the drop in producer surplus, deadweight loss is the same as before at \$2.00.

f. Finally, consider a maximum price of \$12. What will this do to quantity, consumer surplus, profit, and deadweight loss?

With a maximum price of only \$12, output decreases considerably:

$$12 = 2Q - 5, \text{ or } Q = 8.5.$$

Profits are

$$\pi = (12)(8.5) - [100 - 5(8.5) + 8.5^2] = -\$27.75.$$

Even though the firm is making losses, it will continue to produce in the short run because revenue (\$102) is greater than total variable cost (\$29.75).

Consumer surplus is realized on only 8.5 units. Note that the consumer buying the last unit would have been willing to pay a price of \$38 ($38 = 55 - 2(8.5)$). Therefore,

$$CS = (0.5)(8.5)(55 - 38) + (8.5)(38 - 12) = \$293.25.$$

$$DWL = (0.5)(15 - 8.5)(38 - 12) = \$84.50.$$

The result of this low price is that output falls, consumer surplus increases, profit drops, and deadweight loss increases. In the long run, the firm will shut down, and then output, consumer surplus, and profit will all drop to zero.

16. There are 10 households in Lake Wobegon, Minnesota, each with a demand for electricity of $Q = 50 - P$. Lake Wobegon Electric's (*LWE*) cost of producing electricity is $TC = 500 + Q$.

a. If the regulators of *LWE* want to make sure that there is no deadweight loss in this market, what price will they force *LWE* to charge? What will output be in that case? Calculate consumer surplus and *LWE*'s profit with that price.

The first step in solving the regulator's problem is to determine the market demand for electricity in Lake Wobegon. The quantity demanded in the market is the sum of the quantity demanded by each individual at any given price. Graphically, we horizontally sum each household's demand for electricity to arrive at market demand, and mathematically

$$Q_M = \sum_{i=1}^{10} Q_i = 10(50 - P) = 500 - 10P \Rightarrow P = 50 - 0.1Q.$$

To avoid deadweight loss, the regulators will set price equal to marginal cost. Given $TC = 500 + Q$, $MC = 1$ (the slope of the total cost curve). Setting price equal to marginal cost, and solving for quantity:

$$50 - 0.1Q = 1, \text{ or}$$

$$Q = 490.$$

Profits are equal to total revenue minus total costs:

$$\pi = (1)(490) - (500 + 490) = -\$500 \text{ (a loss).}$$

Total consumer surplus is:

$$CS = (0.5)(490)(50 - 1) = \$12{,}005, \text{ or } \$1200.50 \text{ per household.}$$

b. If regulators want to ensure that *LWE* doesn't lose money, what is the lowest price they can impose? Calculate output, consumer surplus, and profit. Is there any deadweight loss?

To guarantee that *LWE* does not lose money, regulators will allow *LWE* to charge the average cost of production, where

$$AC = \frac{TC}{Q} = \frac{500}{Q} + 1.$$

To determine the equilibrium price and quantity under average cost pricing, set price equal to average cost:

$$50 - 0.1Q = \frac{500}{Q} + 1.$$

Solving for Q yields the following quadratic equation:

$$0.1Q^2 - 49Q + 500 = 0.$$

Note**:** if $aQ^2 + bQ + c = 0$, then

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Using this quadratic formula:

$$Q = \frac{49 \pm \sqrt{49^2 - (4)(0.1)(500)}}{(2)(0.1)}$$

there are two solutions: 10.4 and 479.6. Note that at a quantity of 10.4, marginal revenue is greater than marginal cost, and the firm will gain by producing more output. Also, note that the larger quantity results in a lower price and hence a larger consumer surplus. Therefore, $Q = 479.6$ and $P = \$2.04$. At this quantity and price, profit is zero (given some slight rounding error). Consumer surplus is

$$CS = (0.5)(479.6)(50 - 2.04) = \$11{,}500.81, \text{ or } \$1150.08 \text{ per household.}$$

Deadweight loss is

$$DWL = (0.5)(490 - 479.6)(2.04 - 1) = \$5.41.$$

c. Kristina knows that deadweight loss is something that this small town can do without. She suggests that each household be required to pay a fixed amount just to receive any electricity at all, and then a per-unit charge for electricity. Then *LWE* can break even while charging the price calculated in part a. What fixed amount would each household have to pay for Kristina's plan to work? Why can you be sure that no household will choose instead to refuse the payment and go without electricity?

Fixed costs are \$500. If each household pays \$50, the fixed costs are covered and the utility can charge marginal cost for electricity. Because consumer surplus per household under marginal cost pricing is \$1200.50, each would be willing to pay the \$50.

17. A certain town in the Midwest obtains all of its electricity from one company, Northstar Electric. Although the company is a monopoly, it is owned by the citizens of the town, all of whom split the profits equally at the end of each year. The *CEO* of the company claims that because all of the profits will be given back to the citizens, it makes economic sense to charge a monopoly price for electricity. True or false? Explain.

The *CEO*'s claim is false. If the company charges the monopoly price it will produce a smaller quantity than the competitive equilibrium. Therefore, even though all of the monopoly profits are given back to the citizens, there is still a deadweight loss associated with the fact that too little electricity is produced and consumed.

18. A monopolist faces the following demand curve: $Q = 144/P^2$, where Q is the quantity demanded and P is price. Its *average variable* cost is $AVC = Q^{1/2}$, and its *fixed cost* is 5.

a. What are its profit-maximizing price and quantity? What is the resulting profit?

To maximize profit the monopolist should set marginal revenue equal to marginal cost. To find marginal revenue, first rewrite the demand function as a function of Q so that you can then express total revenue as a function of Q and calculate marginal revenue:

$$Q = \frac{144}{P^2} \Rightarrow P^2 = \frac{144}{Q} \Rightarrow P = \sqrt{\frac{144}{Q}} = \frac{12}{\sqrt{Q}} \Rightarrow P = 12Q^{-0.5}$$

$$R = PQ = \frac{12}{\sqrt{Q}}Q = 12\sqrt{Q} = 12Q^{0.5}$$

$$MR = \frac{dR}{dQ} = 0.5(12Q^{-0.5}) = 6Q^{-0.5} = \frac{6}{\sqrt{Q}}.$$

To find marginal cost, first find total cost, which is equal to fixed cost plus variable cost. Fixed cost is 5, and variable cost is equal to average variable cost times Q. Therefore, total cost and marginal cost are:

$$TC = 5 + (Q^{\frac{1}{2}})Q = 5 + Q^{\frac{3}{2}}$$

$$MC = \frac{dTC}{dQ} = \frac{3}{2}Q^{\frac{1}{2}} = \frac{3\sqrt{Q}}{2}.$$

To find the profit-maximizing level of output, set $MR = MC$:

$$\frac{6}{\sqrt{Q}} = \frac{3\sqrt{Q}}{2} \Rightarrow Q = 4.$$

Now find price and profit:

$$P = \frac{12}{\sqrt{Q}} = \frac{12}{\sqrt{4}} = \$6$$

$$\pi = PQ - TC = (6)(4) - (5 + 4^{\frac{3}{2}}) = \$11.$$

b. Suppose the government regulates the price to be no greater than $4 per unit. How much will the monopolist produce? What will its profit be?

The price ceiling truncates the demand curve that the monopolist faces at $P = \$4$, so $Q = \frac{144}{4^2} = 9.$ Therefore, if the monopolist produces 9 units or less, the price must be $4. Because of the regulation, the demand curve now has two parts:

$$P = \begin{cases} \$4, \text{ if } Q \le 9 \\ 12Q^{-1/2}, \text{ if } Q > 9. \end{cases}$$

Thus, total revenue and marginal revenue also have two parts:

$$TR = \begin{cases} 4Q, \text{ if } Q \le 9 \\ 12\text{Q}^{1/2}, \text{ if } Q > 9 \end{cases}, \text{ and}$$

$$MR = \begin{cases} \$4, \text{ if } Q \le 9 \\ 6Q^{-1/2}, \text{ if } Q > 9. \end{cases}$$

To find the profit-maximizing level of output, set marginal revenue equal to marginal cost, so that for $P = 4$,

$$4 = \frac{3}{2}\sqrt{Q}, \text{ or } \sqrt{Q} = \frac{8}{3}, \text{ or } Q = 7.11.$$

If the monopolist produces an integer number of units, the profit-maximizing production level is 7 units, price is $4, revenue is $28, total cost is $23.52, and profit is $4.48. There is a shortage of two units, since the quantity demanded at the price of $4 is 9 units.

c. Suppose the government wants to set a ceiling price that induces the monopolist to produce the largest possible output. What price will accomplish this goal?

To maximize output, the regulated price should be set so that demand equals marginal cost, which occurs where

$$\frac{12}{\sqrt{Q}} = \frac{3\sqrt{Q}}{2} \Rightarrow Q = 8 \text{ and } P = \$4.24.$$

The regulated price becomes the monopolist's marginal revenue up to a quantity of 8. So *MR* is a horizontal line with an intercept equal to the regulated price of $4.24. To maximize profit, the firm produces where marginal cost is equal to marginal revenue, which results in a quantity of 8 units.

The monopolist's profit is

$$\pi = TR - TC = (4.24)(8) - (5 + 8^{3/2}) = 33.92 - 27.63 = \$6.29.$$

Chapter 11
Pricing With Market Power

■ Teaching Notes

Chapter 11 begins with a discussion of the basic objective of every pricing strategy employed by a firm with market power, which is to capture as much consumer surplus as possible and convert it into additional profit for the firm. The remainder of the chapter explores different methods of capturing this surplus. Section 11.2 discusses first-, second-, and third-degree price discrimination, Section 11.3 covers intertemporal price discrimination and peak-load pricing, Section 11.4 discusses two-part tariffs, Section 11.5 explores bundling and tying, and Section 11.6 considers advertising. If you are pressed for time, you can pick and choose between Sections 11.3 to 11.6. The chapter contains a wide array of examples of how price discrimination is applied in different types of markets, not only in the formal examples but also in the body of the text. Although the graphs can seem very complicated to students, the challenge of figuring out how to price discriminate in a specific case can be quite stimulating and can promote many interesting class discussions. The Appendix to the chapter covers transfer pricing, which is particularly relevant in a business-oriented course. Should you choose to include the Appendix, make sure students have an intuitive feel for the model before presenting the algebra or geometry.

When introducing this chapter, highlight the requirements for profitable price discrimination: (1) supply-side market power, (2) the ability to separate customers, and (3) differing demand elasticities for different classes of customers. The material on first-degree price discrimination begins with the concept of a reservation price. The text uses reservation prices throughout the chapter, so be sure students understand this concept. The calculation of variable profit as the yellow area in Figure 11.2 may be confusing to students. You can point out that it is the same as the more familiar area between price P^* and MC because the area of the yellow triangle in the upper left between prices P_{max} and P^* is the same as the area of the lavender triangle between P^* and the intersection of MC and MR. You might want to remind students that variable profit is the same thing as producer surplus. Be sure to show that with first-degree price discrimination the monopolist captures deadweight loss and all consumer surplus, so the end result is like perfect competition except that producers get all the surplus. Also, stress that with perfect price discrimination the marginal revenue curve coincides with the demand curve.

You might want to follow first-degree price discrimination with a discussion of third-degree, rather than second-degree, price discrimination. Some instructors find that third-degree price discrimination flows more naturally from the discussion of imperfect price discrimination starting on page 402. When you do cover second-degree price discrimination, you might note that many utilities currently charge *higher* prices for larger blocks to encourage conservation. (Use your own electricity bill as an example if applicable.) The geometry of third-degree price discrimination in Figure 11.5 is difficult for most students; therefore, they need a careful explanation of the intuition behind the model. Slowly introduce the algebra so that students can see that the profit-maximizing quantities in each market are those where marginal revenue equals marginal cost. You might consider dividing Figure 11.5 into three graphs. The first shows demand and MR in market 1, the second shows demand and MR in market 2, and the last contains the total MR and MC curves. Find the intersection of MR_T and MC in the third diagram and then trace this back to the other two diagrams to determine the quantity and price in each market. Section 11.2 concludes with Examples 11.1 and 11.2. Because of the prevalence of coupons, rebates, and airline travel, all students will be able to relate to these examples.

When presenting intertemporal price discrimination and peak-load pricing, begin by comparing the similarities with third-degree price discrimination. Discuss the difference between these two forms of exploiting monopoly power and third-degree price discrimination. Here, marginal revenue and cost are equal within customer class but need not be equal across classes.

Students easily grasp the case of a two-part tariff with a single customer. Fewer will understand the case with two customers. Fewer still will understand the case of many different customers, so you need to be especially careful when discussing this case. Example 11.4 shows how cellular phone companies combine third-degree price discrimination with a two-part tariff.

When discussing bundling, point out that in Figure 11.12 prices are on both axes. To introduce mixed bundling, consider starting with Example 11.5 and a menu from a local restaurant. Make sure students understand when bundling is profitable (when demands are negatively correlated) and that mixed bundling can be more profitable than either selling separately or pure bundling (when demands are only somewhat negatively correlated and/or marginal production costs are significant). To distinguish tying from bundling, point out that with tying the first product is typically useless without the second product.

Section 11.6 on advertising is best suited for business-oriented courses. If you cover this section, you might start by noting that firms often prefer non-price competition because it is easy for a rival firm to match another firm's price, but not so simple to match its advertising. This is especially true because advertising takes time to prepare and involves creativity, so even if another firm tries to compete on the basis of advertising it may have a difficult time countering the unique appeal of a well-conceived advertising campaign. The rule of thumb given in Equation 11.4 is also known as the Dorfman-Steiner condition. The original article by Robert Dorfman and Peter O. Steiner also develops a similar condition for choosing optimal product quality.[1]

■ Review Questions

1. **Suppose a firm can practice perfect first-degree price discrimination. What is the lowest price it will charge, and what will its total output be?**

   When a firm practices perfect first-degree price discrimination, each unit is sold at the reservation price of each consumer (assuming each consumer purchases one unit). Because each unit is sold at the consumer's reservation price, marginal revenue is simply the price at which each unit is sold, and thus the demand curve is the firm's marginal revenue curve. The profit-maximizing output is therefore where $MR = MC$, which is the point where the marginal cost curve intersects the demand curve. Thus the price of the last unit sold equals the marginal cost of producing that unit, and the firm produces the perfectly competitive level of output.

2. **How does a car salesperson practice price discrimination? How does the ability to discriminate correctly affect his or her earnings?**

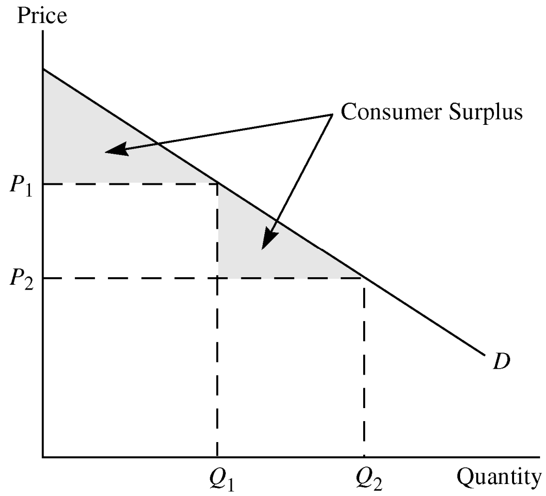
   By sizing up the customer, the salesperson determines the customer's reservation price. Through a process of bargaining, a sales price is determined. If the salesperson has misjudged the reservation price of the customer, either the sale is lost because the customer's reservation price is lower than the price offered by the salesperson or profit is lost because the customer's reservation price is higher than the salesperson's offer. Thus, the salesperson's commission is positively correlated to his or her ability to determine the reservation price of each customer.

[1] Robert Dorfman and Peter O. Steiner, "Optimal Advertising and Optimal Quality," *American Economic Review*, December 1954, **44**:5, 826–836.

3. Electric utilities often practice second-degree price discrimination. Why might this improve consumer welfare?

Consumer surplus may be higher under block pricing than under monopoly pricing because more output is produced. For example, assume there are two prices, P_1 and P_2, with $P_1 > P_2$ as shown in the diagram below. Customers with reservation prices above P_1 pay P_1, capturing surplus equal to the area bounded by the demand curve and P_1. For simplicity, suppose P_1 is the monopoly price; then the area between demand and P_1 is also the consumer surplus under monopoly.

Under block pricing, however, customers with reservation prices between P_1 and P_2 capture additional surplus equal to the area bounded by the demand curve, the difference between P_1 and P_2, and the difference between Q_1 and Q_2. Hence block pricing under these assumptions improves consumer welfare.



4. Give some examples of third-degree price discrimination. Can third-degree price discrimination be effective if the different groups of consumers have different levels of demand but the same price elasticities?

To engage in third-degree price discrimination, the producer must separate customers into distinct market segments and prevent reselling of the product from customers in one market to customers in another market (arbitrage). While examples in this chapter stress the techniques for separating customers, there are also techniques for preventing resale. For example, airlines restrict the use of their tickets by printing the name of the passenger on the ticket. Other examples include dividing markets by age and gender, e.g., charging different prices for movie tickets to different age groups. If customers in the separate markets have the same price elasticities, then from Equation 11.2 we know that the prices are the same in all markets. While the producer can effectively separate the markets, there is no profit incentive to do so.

5. Show why optimal third-degree price discrimination requires that marginal revenue for each group of consumers equals marginal cost. Use this condition to explain how a firm should change its prices and total output if the demand curve for one group of consumers shifts outward, causing marginal revenue for that group to increase.

We know that firms maximize profits by choosing output so marginal revenue is equal to marginal cost. If *MR* for one market is greater than *MC*, then the firm should increase sales in that market, thus lowering price and possibly raising *MC*. Similarly, if *MR* for one market is less than *MC*, the firm should decrease sales by raising the price in that market. By equating *MR* and *MC* in each market, marginal revenue is equal in all markets.

Determining how prices and outputs should change when demand in one market increases is actually quite complicated and depends on the shapes of the demand and marginal cost curves. If all demand curves are linear and marginal cost is upward sloping, here's what happens when demand increases in market 1.

Since $MR_1 = MC$ before the demand shift, MR_1 will be greater than MC after the shift. To bring MR_1 and MC back to equality, the firm should increase both price and sales in market 1. It can raise price and still sell more because demand has increased.

In addition, the firm must increase the *MR*s in its other markets so that they equal the new larger value of MR_1. This is done by decreasing output and raising prices in the other markets. The firm increases total output and shifts sales to the market experiencing increased demand and away from other markets.

6. **When pricing automobiles, American car companies typically charge a much higher percentage markup over cost for "luxury option" items (such as leather trim, etc.) than for the car itself or for more "basic" options such as power steering and automatic transmission. Explain why.**

This can be explained partially as an instance of third-degree price discrimination. Consider leather seats, for example. Some people would like leather seats but are not willing to pay a lot for them, so their demand is highly elastic. Others have a strong preference for leather, and their demand is not very elastic. Rather than selling leather seats to both groups at different prices (as in the pure case of third-degree price discrimination), the car companies sell leather seats at a high markup over cost and cloth seats at a low markup over cost. Consumers with a low elasticity and strong preference for leather seats buy the leather seats while those with a high elasticity for leather seats buy the cloth seats instead. This is like the case of supermarkets selling brand name items for higher prices and markups than similar store brand items. Thus the pricing of automobile options can be explained if the "luxury" options are purchased by consumers with low elasticities of demand relative to consumers of more "basic" options.

7. **How is peak-load pricing a form of price discrimination? Can it make consumers better off? Give an example.**

Price discrimination involves separating customers into distinct markets. There are several ways of segmenting markets: by customer characteristics, by geography, and by time. In peak-load pricing, sellers charge different prices to customers at different times, setting higher prices when demand is high. This is a form of price discrimination because consumers with highly elastic demands wait to purchase the product at lower prices during off-peak times while consumers with less elastic demands pay the higher prices during peak times. Peak-load pricing can increase total consumer surplus because consumers with highly elastic demands consume more of the product at lower prices during off-peak times than they would have if the company had charged one price at all times. An example is telephone pricing. Most phone companies charge lower (or zero) prices for long distance calls in the evening and weekends than during normal business hours. Callers with more elastic demands wait until the evenings and weekends to make their calls while businesses, whose demands are less elastic, pay the higher daytime prices. When these pricing plans were first introduced, the number of long distance calls made by households increased dramatically, and those consumers were clearly made better off.

8. **How can a firm determine an optimal two-part tariff if it has two customers with different demand curves? (Assume that it knows the demand curves.)**

If all customers had the same demand curve, the firm would set a price equal to marginal cost and a fee equal to consumer surplus. When consumers have different demand curves, and therefore different levels of consumer surplus, the firm should set price above marginal cost and charge a fee equal to the consumer surplus of the consumer with the smaller demand. One way to do this is to choose a price P that is just above MC and then calculate the fee T that can be charged so that both consumers buy the product.

Then calculate the profit that will be earned with this combination of P and T. Now try a slightly higher price and go through the process of finding the new T and profit. Keep doing this as long as profit is increasing. When profit hits its peak you have found the optimal price and fee.

9. **Why is the pricing of a Gillette safety razor a form of two-part tariff? Must Gillette be a monopoly producer of its blades as well as its razors? Suppose you were advising Gillette on how to determine the two parts of the tariff. What procedure would you suggest?**

   By selling the razor and the blades separately, the pricing of a Gillette safety razor can be thought of as a two-part tariff, where the entry fee is the price of the razor and the usage fee is the price of the blades. In the simplest case where all consumers have identical demand curves, Gillette should set the blade price equal to marginal cost, and the razor price equal to total consumer surplus for each consumer. Since blade price equals marginal cost it does not matter if Gillette has a monopoly in the production of blades. The determination of the two parts of the tariff is more complicated the greater the variety of consumers with different demands, and there is no simple formula to calculate the optimal two-part tariff. The key point to consider is that as the price of the razor becomes smaller, more consumers will buy the razor, but the profit per razor will fall. However, more razor owners mean more blade sales and greater profit from blade sales because the price for blades is above marginal cost. Arriving at the optimal two-part tariff might involve experimentation with different razor and blade prices. You might want to advise Gillette to try different price combinations in different geographic regions of the country to see which combination results in the largest profit.

10. **In the town of Woodland, California, there are many dentists but only one eye doctor. Are senior citizens more likely to be offered discount prices for dental exams or for eye exams? Why?**

    The dental market is competitive, whereas the eye doctor is a local monopolist. Only firms with market power can practice price discrimination, which means senior citizens are more likely to be offered discount prices from the eye doctor. Dentists are already charging a price close to marginal cost, so they are not able to offer senior discounts.

11. **Why did MGM bundle *Gone with the Wind* and *Getting Gertie's Garter*? What characteristic of demands is needed for bundling to increase profits?**

    MGM bundled *Gone with the Wind* and *Getting Gertie's Garter* to maximize revenues and profits. Because MGM could not price discriminate by charging a different price to each customer according to the customer's price elasticity, it chose to bundle the two films and charge theaters for showing both films. Demands must be negatively correlated for bundling to increase profits.

12. **How does mixed bundling differ from pure bundling? Under what conditions is mixed bundling preferable to pure bundling? Why do many restaurants practice mixed bundling (by offering a complete dinner as well as an à la carte menu) instead of pure bundling?**

    Pure bundling involves selling products only as a package. Mixed bundling is selling the products both together and separately. Mixed bundling may yield higher profits when demands for the individual products do not have a strong negative correlation, marginal costs are high, or both. Restaurants can maximize profits by offering both à la carte menus and full dinners. By charging higher prices for individual items, restaurants capture consumer surplus from diners who value some dishes much more highly than others, while charging less for a bundled complete dinner allows them to capture consumer surplus from diners who attach moderate values to all dishes.

13. How does tying differ from bundling? Why might a firm want to practice tying?

Tying involves the sale of two or more goods or services that must be used as complements. Bundling can involve complements or substitutes. Tying allows the firm to monitor customer demand and more effectively determine profit-maximizing prices for the tied products. For example, a microcomputer firm might sell its computer, the tying product, with minimum memory and a unique architecture, then sell extra memory, the tied product, above marginal cost.

14. Why is it incorrect to advertise up to the point that the last dollar of advertising expenditures generates another dollar of sales? What is the correct rule for the marginal advertising dollar?

If the firm increases advertising expenditures to the point that the last dollar of advertising generates another dollar of sales, it will not be maximizing profits, because the firm is ignoring additional production costs. The correct rule is to advertise so that the additional revenue generated by an additional dollar of advertising equals the additional dollar spent on advertising plus the marginal production cost of the increased quantity sold.

15. How can a firm check that its advertising-to-sales ratio is not too high or too low? What information does it need?

The firm can check whether its advertising-to-sales ratio is profit maximizing by comparing it with the negative of the ratio of the advertising elasticity of demand to the price elasticity of demand. The two ratios are equal when the firm is using the profit-maximizing price and advertising levels. The firm must know both the advertising elasticity of demand and the price elasticity of demand to do this.

■ Exercises

1. Price discrimination requires the ability to sort customers and the ability to prevent arbitrage. Explain how the following can function as price discrimination schemes and discuss both sorting and arbitrage:

a. Requiring airline travelers to spend at least one Saturday night away from home to qualify for a low fare.

The requirement of staying over Saturday night separates business travelers, who prefer to return home for the weekend, from tourists, who usually travel on the weekend. Arbitrage is not possible when the ticket specifies the name of the traveler.

b. Insisting on delivering cement to buyers and basing prices on buyers' locations.

By basing prices on the buyer's location, customers are sorted by geography. Prices may then include transportation charges, which the customer pays for whether delivery is received at the buyer's location or at the cement plant. Since cement is heavy and bulky, transportation charges may be large. Note that this pricing strategy sometimes leads to what is called "basing-point" pricing, where all cement producers use the same base point and calculate transportation charges from that base point. Every seller then quotes individual customers the same price. This pricing system is often viewed as a method to facilitate collusion among sellers. For example, in *FTC v. Cement Institute*, 333 U.S. 683 [1948], the Court found that sealed bids by eleven companies for a 6000-barrel government order in 1936 all quoted $3.286854 per barrel.

c. Selling food processors along with coupons that can be sent to the manufacturer for a $10 rebate.

Rebate coupons for food processors separate consumers into two groups: (1) customers who are less price sensitive (those who have a lower elasticity of demand) and do not fill out the forms necessary to request the rebate; and (2) customers who are more price sensitive (those who have a higher demand elasticity) and do the paperwork to request the rebate. The latter group could buy the food processors, send in the rebate coupons, and resell the processors at a price just below the retail price without the rebate. To prevent this type of arbitrage, sellers could limit the number of rebates per household.

d. Offering temporary price cuts on bathroom tissue.

A temporary price cut on bathroom tissue is a form of intertemporal price discrimination. During the price cut, price-sensitive consumers buy greater quantities of tissue than they would otherwise and store it for later use. Non-price-sensitive consumers buy the same amount of tissue that they would buy without the price cut. Arbitrage is possible, but the profits on reselling bathroom tissue probably are so small that they do not compensate for the cost of storage, transportation, and resale.

e. Charging high-income patients more than low-income patients for plastic surgery.

The plastic surgeon might not be able to separate high-income patients from low-income patients, but he or she can guess. One strategy is to quote a high price initially, observe the patient's reaction, and then negotiate the final price. Many medical insurance policies do not cover elective plastic surgery. Since plastic surgery cannot be transferred from low-income patients to high-income patients, arbitrage does not present a problem.

2. If the demand for drive-in movies is more elastic for couples than for single individuals, it will be optimal for theaters to charge one admission fee for the driver of the car and an extra fee for passengers. True or false? Explain.

True. This is a two-part tariff problem where the entry fee is a charge for the car plus driver and the usage fee is a charge for each additional passenger other than the driver. Assume that the marginal cost of showing the movie is zero, i.e., all costs are fixed and do not vary with the number of cars. The theater should set its entry fee to capture the consumer surplus of the driver, a single viewer, and should charge a positive price for each passenger.

3. In Example 11.1 (page 408), we saw how producers of processed foods and related consumer goods use coupons as a means of price discrimination. Although coupons are widely used in the United States, that is not the case in other countries. In Germany, coupons are illegal.

a. Does prohibiting the use of coupons in Germany make German *consumers* better off or worse off?

In general, we cannot tell whether consumers will be better off or worse off. Total consumer surplus can increase or decrease with price discrimination, depending on the number of prices charged and the distribution of consumer demand. Here is an example where coupons increase consumer surplus. Suppose a company sells boxes of cereal for $4, and 1,000,000 boxes are sold per week before issuing coupons. Then it offers a coupon good for $1 off the price of a box of cereal. As a result, 1,500,000 boxes are sold per week and 750,000 coupons are redeemed. Half a million new buyers buy the product for a net price of $3 per box, and 250,000 consumers who used to pay $4 redeem coupons and save $1 per box. Both these groups gain consumer surplus while the 750,000 who continue paying $4 per box do not gain or lose. In a case like this, German consumers would be worse off if coupons were prohibited.

Things get messy if the producer raises the price of its product when it offers the coupons. For example, if the company raised its price to \$4.50 per box, some of the original buyers might no longer purchase the cereal because the cost of redeeming the coupon is too high for them, and the higher price for the cereal leads them to a competitor's product. Others continue to purchase the cereal at the higher price. Both of these groups lose consumer surplus. However, some who were buying at \$4 redeem the coupon and pay a net price of \$3.50, and others who did not buy originally now buy the product at the net price of \$3.50. Both of these groups gain consumer surplus. So consumers as a whole may or may not be better off with the coupons. In this case we cannot say for sure whether German consumers would be better or worse off with a ban on coupons.

b. Does prohibiting the use of coupons make German *producers* better off or worse off?

Prohibiting the use of coupons will make German producers worse off, or at least not better off. Producers use coupons only if it increases profits, so prohibiting coupons hurts those producers who would have found their use profitable and has no effect on producers who would not have used them anyway.

4. Suppose that BMW can produce any quantity of cars at a constant marginal cost equal to \$20,000 and a fixed cost of \$10 billion. You are asked to advise the CEO as to what prices and quantities BMW should set for sales in Europe and in the United States. The demand for BMWs in each market is given by:

$$\boldsymbol{Q_E = 4{,}000{,}000 - 100P_E \text{ and } Q_U = 1{,}000{,}000 - 20P_U}$$

where the subscript *E* denotes Europe and the subscript *U* denotes the United States. Assume that BMW can restrict U.S. sales to authorized BMW dealers only.

a. What quantity of BMWs should the firm sell in each market, and what should the price be in each market? What should the total profit be?

BMW should choose the levels of Q_E and Q_U so that $MR_E = MR_U = MC$.

To find the marginal revenue expressions, solve for the inverse demand functions:

$$P_E = 40{,}000 - 0.01Q_E \text{ and } P_U = 50{,}000 - 0.05Q_U.$$

Since demand is linear in both cases, the marginal revenue function for each market has the same intercept as the inverse demand curve and twice the slope:

$$MR_E = 40{,}000 - 0.02Q_E \text{ and } MR_U = 50{,}000 - 0.1Q_U.$$

Marginal cost is constant and equal to \$20,000. Setting each marginal revenue equal to 20,000 and solving for quantity yields:

$$40{,}000 - 0.02Q_E = 20{,}000, \text{ or } Q_E = 1{,}000{,}000 \text{ cars in Europe, and}$$
$$50{,}000 - 0.1Q_U = 20{,}000, \text{ or } Q_U = 300{,}000 \text{ cars in the United States}$$

Substituting Q_E and Q_U into their respective inverse demand equations, we may determine the price of cars in each market:

$$P_E = 40{,}000 - 0.01(1{,}000{,}000) = \$30{,}000 \text{ in Europe, and}$$
$$P_U = 50{,}000 - 0.05(300{,}000) = \$35{,}000 \text{ in the United States}$$

Profit is therefore:

$$\begin{aligned}\pi &= TR - TC = (30{,}000)(1{,}000{,}000) + (35{,}000)(300{,}000) \\ &\quad - [10{,}000{,}000{,}000 + 20{,}000(1{,}300{,}000)] \\ \pi &= \$4.5 \text{ billion.}\end{aligned}$$

b. If BMW were forced to charge the same price in each market, what would be the quantity sold in each market, the equilibrium price, and the company's profit?

If BMW must charge the same price in both markets, they must find total demand, $Q = Q_E + Q_U$, where each price is replaced by the common price P:

$$Q = 5{,}000{,}000 - 120P, \text{ or in inverse form, } P = \frac{5{,}000{,}000}{120} - \frac{Q}{120}.$$

Marginal revenue has the same intercept as the inverse demand curve and twice the slope:

$$MR = \frac{5{,}000{,}000}{120} - \frac{Q}{60}.$$

To find the profit-maximizing quantity, set marginal revenue equal to marginal cost:

$$\frac{5{,}000{,}000}{120} - \frac{Q}{60} = 20{,}000, \text{ or } Q^* = 1{,}300{,}000 \text{ cars.}$$

Substituting Q^* into the inverse demand equation to determine price:

$$P = \frac{5{,}000{,}000}{120} - \left(\frac{1{,}300{,}000}{120}\right) = \$30{,}833.33.$$

Substitute into the demand equations for the European and American markets to find the quantity sold in each market:

$Q_E = 4{,}000{,}000 - (100)(30{,}833.3)$, or $Q_E = 916{,}667$ cars in Europe, and

$Q_U = 1{,}000{,}000 - (20)(30{,}833.3)$, or $Q_U = 383{,}333$ cars in the United States

Profit is $\pi = \$30{,}833.33(1{,}300{,}000) - [10{,}000{,}000{,}000 + 20{,}000(1{,}300{,}000)]$, or

$\pi = \$4.083$ billion.

U.S. consumers would gain and European consumers would lose if BMW were forced to sell at the same price in both markets, because Americans would pay $4166.67 less and Europeans would pay $833.33 more for each BMW. Also, BMW's profits would drop by more than $400 million.

5. A monopolist is deciding how to allocate output between two geographically separated markets (East Coast and Midwest). Demand and marginal revenue for the two markets are:

$$P_1 = 15 - Q_1 \qquad MR_1 = 15 - 2Q_1$$

$$P_2 = 25 - 2Q_2 \qquad MR_2 = 25 - 4Q_2$$

The monopolist's total cost is $C = 5 + 3(Q_1 + Q_2)$. What are price, output, profits, marginal revenues, and deadweight loss (i) if the monopolist can price discriminate? (ii) if the law prohibits charging different prices in the two regions?

(i) Choose quantity in each market such that marginal revenue is equal to marginal cost. The marginal cost is equal to 3 (the slope of the total cost curve). The profit-maximizing quantities in the two markets are:

$15 - 2Q_1 = 3$, or $Q_1 = 6$ on the East Coast, and

$25 - 4Q_2 = 3$, or $Q_2 = 5.5$ in the Midwest.

Substituting into the respective demand equations, prices for the two markets are:

$P_1 = 15 - 6 = \$9$, and $P_2 = 25 - 2(5.5) = \$14$.

Noting that the total quantity produced is 11.5, profit is

$$\pi = 9(6) + 14(5.5) - [5 + 3(11.5)] = \$91.50.$$

When MC is constant and demand is linear, the monopoly deadweight loss is

$$DWL = (0.5)(Q_C - Q_M)(P_M - P_C),$$

where the subscripts C and M stand for the competitive and monopoly levels, respectively. Here, $P_C = MC = 3$ and Q_C in each market is the amount that is demanded when $P = \$3$. The deadweight losses in the two markets are

$$DWL_1 = (0.5)(12 - 6)(9 - 3) = \$18, \text{ and}$$
$$DWL_2 = (0.5)(11 - 5.5)(14 - 3) = \$30.25.$$

Therefore, the total deadweight loss is \$48.25.

(ii) Without price discrimination the monopolist must charge a single price for the entire market. To maximize profit, find the quantity such that marginal revenue is equal to marginal cost. Adding demand equations, we find that the total demand curve has a kink at $Q = 5$:

$$P = \begin{cases} 25 - 2Q, & \text{if } Q \le 5 \\ 18.33 - 0.67Q, & \text{if } Q > 5. \end{cases}$$

This implies marginal revenue equations of

$$MR = \begin{cases} 25 - 4Q, & \text{if } Q \le 5 \\ 18.33 - 1.33Q, & \text{if } Q > 5. \end{cases}$$

With marginal cost equal to 3, $MR = 18.33 - 1.33Q$ is relevant here because the marginal revenue curve "kinks" when $P = \$15$. To determine the profit-maximizing quantity, equate marginal revenue and marginal cost:

$$18.33 - 1.33Q = 3, \text{ or } Q = 11.5.$$

Substituting the profit-maximizing quantity into the demand equation to determine price:

$$P = 18.33 - (0.67)(11.5) = \$10.67.$$

With this price, $Q_1 = 4.33$ and $Q_2 = 7.17$. (Note that at these quantities $MR_1 = 6.34$ and $MR_2 = -3.68$). Profit is

$$\pi = 10.67(11.5) - [5 + 3(11.5)] = \$83.21.$$

Deadweight loss in the first market is

$$DWL_1 = (0.5)(12 - 4.33)(10.67 - 3) = \$29.41.$$

Deadweight loss in the second market is

$$DWL_2 = (0.5)(11 - 7.17)(10.67 - 3) = \$14.69.$$

Total deadweight loss is \$44.10. Without price discrimination, profit is lower, deadweight loss is lower, and total output is unchanged. The big winners are consumers in market 2 who now pay \$10.67 instead of \$14. DWL in market 2 drops from \$30.25 to \$14.69. Consumers in market 1 and the monopolist are worse off when price discrimination is not allowed.

6. Elizabeth Airlines (EA) flies only one route: Chicago-Honolulu. The demand for each flight is $Q = 500 - P$. EA's cost of running each flight is \$30,000 plus \$100 per passenger.

a. What is the profit-maximizing price that EA will charge? How many people will be on each flight? What is EA's profit for each flight?

First, find the demand curve in inverse form: $P = 500 - Q$.

Marginal revenue for a linear demand curve has twice the slope, so $MR = 500 - 2Q$.

Setting marginal revenue equal to marginal cost (where $MC = \$100$) yields

$$500 - 2Q = 100, \text{ or } Q = 200 \text{ people per flight.}$$

Substitute $Q = 200$ into the demand equation to find the profit-maximizing price:

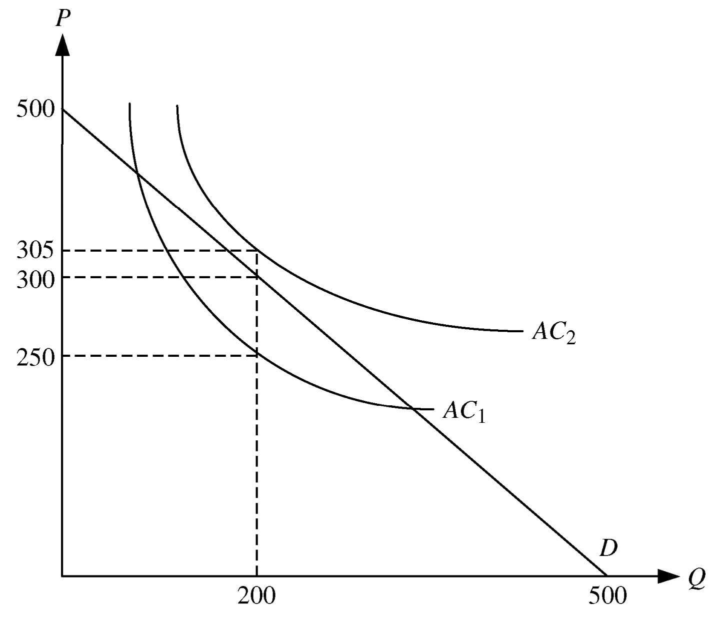
$$P = 500 - 200, \text{ or } P = \$300 \text{ per ticket.}$$

Profit equals total revenue minus total costs:

$$\pi = (300)(200) - [30{,}000 + (100)(200)] = \$10{,}000 \text{ per flight.}$$

b. EA learns that the fixed costs per flight are in fact \$41,000 instead of \$30,000. Will the airline stay in business for long? Illustrate your answer using a graph of the demand curve that EA faces, EA's average cost curve when fixed costs are \$30,000, and EA's average cost curve when fixed costs are \$41,000.

An increase in fixed costs will not change the profit-maximizing price and quantity. If the fixed cost per flight is \$41,000, EA will lose \$1000 on each flight. However, EA will not shut down immediately because doing so would leave it with a loss of \$41,000 (the fixed costs). If conditions do not improve, EA should shut down as soon as it can shed its fixed costs by selling off its planes and other fixed assets. AC_1 in the diagram below is EA's average cost curve when fixed costs are \$30,000, and AC_2 is when fixed costs are \$41,000.



c. Wait! EA finds out that two different types of people fly to Honolulu. Type *A* consists of business people with a demand of $Q_A = 260 - 0.4P$. Type *B* consists of students whose total demand is $Q_B = 240 - 0.6P$. Because the students are easy to spot, EA decides to charge them different prices. Graph each of these demand curves and their horizontal sum. What price does EA charge the students? What price does it charge other customers? How many of each type are on each flight?

Writing the demand curves in inverse form for the two markets:

$$P_A = 650 - 2.5Q_A \text{ and}$$
$$P_B = 400 - 1.667Q_B.$$

Marginal revenue curves have twice the slope of linear demand curves, so we have:

$$MR_A = 650 - 5Q_A, \text{ and}$$
$$MR_B = 400 - 3.33Q_B.$$

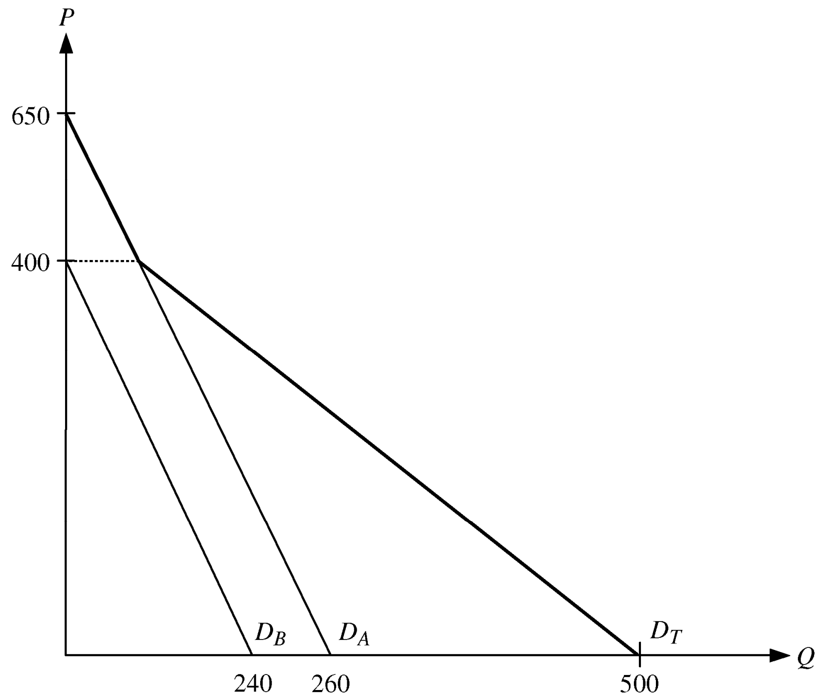
To determine the profit-maximizing quantities, set marginal revenue equal to marginal cost in each market:

$$650 - 5Q_A = 100, \text{ or } Q_A = 110, \text{ and}$$
$$400 - 3.33Q_B = 100, \text{ or } Q_B = 90.$$

Substitute the profit-maximizing quantities into the respective demand curves:

$$P_A = 650 - 2.5(110) = \$375, \text{ and}$$
$$P_B = 400 - 1.667(90) = \$250.$$

When EA is able to distinguish the two groups, the airline finds it profit-maximizing to charge a higher price to the Type *A* travelers, i.e., those who have a less elastic demand at any price.



d. What would EA's profit be for each flight? Would the airline stay in business? Calculate the consumer surplus of each consumer group. What is the total consumer surplus?

With price discrimination, profit per flight is positive, so EA will stay in business:

$$\pi = 250(90) + 375(110) - [41{,}000 + 100(90 + 110)] = \$2750.$$

Consumer surplus for Type A and Type B travelers are

$$CS_A = (0.5)(110)(650 - 375) = \$15{,}125, \text{ and}$$
$$CS_B = (0.5)(90)(400 - 250) = \$6750.$$

Total consumer surplus is therefore \$21,875.

e. Before EA started price discriminating, how much consumer surplus was the Type *A* demand getting from air travel to Honolulu? Type *B*? Why did total consumer surplus decline with price discrimination, even though total quantity sold remained unchanged?

When price was \$300, Type A travelers demanded 140 seats, and consumer surplus was

$$(0.5)(140)(650 - 300) = \$24{,}500.$$

Type B travelers demanded 60 seats at $P = \$300$; their consumer surplus was

$$(0.5)(60)(400 - 300) = \$3000.$$

Consumer surplus was therefore \$27,500, which is greater than the consumer surplus of \$21,875 with price discrimination. Although the total quantity is unchanged by price discrimination, price discrimination has allowed EA to extract consumer surplus from business passengers (Type B) who value travel most and have less elastic demand than students.

7. Many retail video stores offer two alternative plans for renting films:

- ***A two-part tariff*: Pay an annual membership fee (e.g., \$40) and then pay a small fee for the daily rental of each film (e.g., \$2 per film per day).**
- ***A straight rental fee*: Pay no membership fee, but pay a higher daily rental fee (e.g., \$4 per film per day).**

What is the logic behind the two-part tariff in this case? Why offer the customer a choice of two plans rather than simply a two-part tariff?

By employing this strategy, the firm allows consumers to sort themselves into two groups, or markets (assuming that subscribers do not rent to nonsubscribers): high-volume consumers who rent many movies per year (here, more than 20) and low-volume consumers who rent only a few movies per year (less than 20). If only a two-part tariff is offered, the firm has the problem of determining the profit-maximizing entry and rental fees with many different consumers. A high entry fee with a low rental fee discourages low-volume consumers from subscribing. A low entry fee with a high rental fee encourages low-volume consumer membership, but discourages high-volume customers from renting. Instead of forcing customers to pay both an entry and rental fee, the firm effectively charges two different prices to two types of customers.

8. Sal's satellite company broadcasts TV to subscribers in Los Angeles and New York. The demand functions for each of these two groups are

$$Q_{NY} = 60 - 0.25P_{NY} \qquad\qquad Q_{LA} = 100 - 0.50P_{LA}$$

where Q is in thousands of subscriptions per year and P is the subscription price per year. The cost of providing Q units of service is given by

$$\mathbf{C = 1000 + 40Q}$$

where $Q = Q_{NY} + Q_{LA}$.

a. What are the profit-maximizing prices and quantities for the New York and Los Angeles markets?

Sal should pick quantities in each market so that the marginal revenues are equal to one another and equal to marginal cost. To determine marginal revenues in each market, first solve for price as a function of quantity:

$$P_{NY} = 240 - 4Q_{NY}, \text{ and}$$

$$P_{LA} = 200 - 2Q_{LA}.$$

Since marginal revenue curves have twice the slope of their demand curves, the marginal revenue curves for the respective markets are:

$$MR_{NY} = 240 - 8Q_{NY}, \text{ and}$$

$$MR_{LA} = 200 - 4Q_{LA}.$$

Set each marginal revenue equal to marginal cost, which is $40, and determine the profit-maximizing quantity in each submarket:

$$40 = 240 - 8Q_{NY}, \text{ or } Q_{NY} = 25 \text{ thousand, and}$$

$$40 = 200 - 4Q_{LA}, \text{ or } Q_{LA} = 40 \text{ thousand.}$$

Determine the price in each submarket by substituting the profit-maximizing quantity into the respective demand equation:

$$P_{NY} = 240 - 4(25) = \$140, \text{ and}$$

$$P_{LA} = 200 - 2(40) = \$120.$$

b. As a consequence of a new satellite that the Pentagon recently deployed, people in Los Angeles receive Sal's New York broadcasts, and people in New York receive Sal's Los Angeles broadcasts. As a result, anyone in New York or Los Angeles can receive Sal's broadcasts by subscribing in either city. Thus Sal can charge only a single price. What price should he charge, and what quantities will he sell in New York and Los Angeles?

Sal's combined demand function is the horizontal summation of the *LA* and *NY* demand functions. Above a price of $200 (the vertical intercept of the *LA* demand function), the total demand is just the New York demand function, whereas below a price of $200, we add the two demands:

$$Q_T = 60 - 0.25P + 100 - 0.50P, \text{ or } Q_T = 160 - 0.75P.$$

Solving for price gives the inverse demand function:

$$P = 213.33 - 1.333Q,$$

and therefore, $MR = 213.33 - 2.667Q$.

Setting marginal revenue equal to marginal cost:

$$213.33 - 2.667Q = 40, \text{ or } Q = 65 \text{ thousand.}$$

Substitute $Q = 65$ into the inverse demand equation to determine price:

$$P = 213.33 - 1.333(65), \text{ or } P = \$126.67.$$

Although a price of \$126.67 is charged in both markets, different quantities are purchased in each market.

$$Q_{NY} = 60 - 0.25(126.67) = 28.3 \text{ thousand and}$$
$$Q_{LA} = 100 - 0.50(126.67) = 36.7 \text{ thousand.}$$

Together, 65 thousand subscriptions are purchased at a price of \$126.67 each.

c. In which of the above situations, a or b, is Sal better off? In terms of consumer surplus, which situation do people in New York prefer and which do people in Los Angeles prefer? Why?

Sal is better off in the situation with the highest profit, which occurs in part a with price discrimination. Under price discrimination, profit is equal to:

$$\pi = P_{NY}Q_{NY} + P_{LA}Q_{LA} - [1000 + 40(Q_{NY} + Q_{LA})], \text{ or}$$
$$\pi = \$140(25) + \$120(40) - [1000 + 40(25 + 40)] = \$4700 \text{ thousand.}$$

Under the market conditions in b, profit is:

$$\pi = PQ_T - [1000 + 40Q_T], \text{ or}$$
$$\pi = \$126.67(65) - [1000 + 40(65)] = \$4633.33 \text{ thousand.}$$

Therefore, Sal is better off when the two markets are separated.

Under the market conditions in a, the consumer surpluses in the two cities are:

$$CS_{NY} = (0.5)(25)(240 - 140) = \$1250 \text{ thousand, and}$$
$$CS_{LA} = (0.5)(40)(200 - 120) = \$1600 \text{ thousand.}$$

Under the market conditions in b, the respective consumer surpluses are:

$$CS_{NY} = (0.5)(28.3)(240 - 126.67) = \$1603.67 \text{ thousand, and}$$
$$CS_{LA} = (0.5)(36.7)(200 - 126.67) = \$1345.67 \text{ thousand.}$$

New Yorkers prefer b because their price is \$126.67 instead of \$140, giving them a higher consumer surplus. Customers in Los Angeles prefer a because their price is \$120 instead of \$126.67, and their consumer surplus is greater in a.

9. You are an executive for Super Computer, Inc. (SC), which rents out super computers. SC receives a fixed rental payment per time period in exchange for the right to unlimited computing at a rate of P cents per second. SC has two types of potential customers of equal number—10 businesses and 10 academic institutions. Each business customer has the demand function $Q = 10 - P$, where Q is in millions of seconds per month; each academic institution has the demand $Q = 8 - P$. The marginal cost to SC of additional computing is 2 cents per second, regardless of volume.

a. Suppose that you could separate business and academic customers. What rental fee and usage fee would you charge each group? What would be your profits?

For academic customers, consumer surplus at a price equal to marginal cost is

$$(0.5)(6)(8 - 2) = 18 \text{ million cents per month or } \$180{,}000 \text{ per month.}$$

Therefore, charge each academic customer \$180,000 per month as the rental fee and two cents per second in usage fees, i.e., the marginal cost. Each academic customer will yield a profit of \$180,000 for total profits of \$1,800,000 per month.

For business customers, consumer surplus is

$$(0.5)(8)(10-2) = 32 \text{ million cents or } \$320{,}000 \text{ per month.}$$

Therefore, charge \$320,000 per month as a rental fee and 2 cents per second in usage fees. Each business customer will yield a profit of \$320,000 per month for total profits of \$3,200,000 per month.

Total profits will be \$5 million per month minus any fixed costs.

b. Suppose you were unable to keep the two types of customers separate and charged a zero rental fee. What usage fee would maximize your profits? What would be your profits?

Total demand for the two types of customers with ten customers per type is

$$Q = (10)(10-P) + (10)(8-P) = 180 - 20P.$$

Solving for price as a function of quantity:

$$P = 9 - \frac{Q}{20}, \text{ which implies } MR = 9 - \frac{Q}{10}.$$

To maximize profits, set marginal revenue equal to marginal cost,

$$9 - \frac{Q}{10} = 2, \text{ or } Q = 70 \text{ million seconds.}$$

At this quantity, the profit-maximizing price, or usage fee, is 5.5 cents per second.

$$\pi = (5.5-2)(70) = 245 \text{ million cents per month, or } \$2.45 \text{ million per month.}$$

c. Suppose you set up one two-part tariff—that is, you set one rental and one usage fee that both business and academic customers pay. What usage and rental fees would you set? What would be your profits? Explain why price would not be equal to marginal cost.

With a two-part tariff and no price discrimination, set the rental fee (RENT) to be equal to the consumer surplus of the academic institution (if the rental fee were set equal to that of business, academic institutions would not purchase any computer time):

$$\text{RENT} = CS_A = (0.5)(8-P^*)(8-P^*) = (0.5)(8-P^*)^2,$$

where P^* is the optimal usage fee. Let Q_A and Q_B be the total amount of computer time used by the 10 academic and the 10 business customers, respectively. Then total revenue and total costs are:

$$TR = (20)(\text{RENT}) + (Q_A + Q_B)(P^*)$$
$$TC = 2(Q_A + Q_B).$$

Substituting for quantities in the profit equation with total quantity in the demand equation:

$$\pi = (20)(\text{RENT}) + (Q_A + Q_B)(P^*) - (2)(Q_A + Q_B), \text{ or}$$
$$\pi = (10)(8-P^*)^2 + (P^* - 2)(180 - 20P^*).$$

Differentiating with respect to price and setting it equal to zero:

$$\frac{d\pi}{dP^*} = -20P^* + 60 = 0.$$

Solving for price, P^* = 3 cents per second. At this price, the rental fee is

$$(0.5)(8 - 3)^2 = 12.5 \text{ million cents or \$125,000 per month.}$$

At this price

$$Q_A = (10)(8 - 3) = 50 \text{ million seconds, and}$$
$$Q_B = (10)(10 - 3) = 70 \text{ million seconds.}$$

The total quantity is 120 million seconds. Profits are rental fees plus usage fees minus total cost: $\pi = (20)(12.5) + (3)(120) - (2)(120) = 370$ million cents, or \$3.7 million per month, which is greater than the profit in part b where a rental fee of zero is charged. Price does not equal marginal cost, because SC can make greater profits by charging a rental fee and a higher-than-marginal-cost usage fee.

10. As the owner of the only tennis club in an isolated wealthy community, you must decide on membership dues and fees for court time. There are two types of tennis players. "Serious" players have demand

$$Q_1 = 10 - P$$

where Q_1 is court hours per week and P is the fee per hour for each individual player. There are also "occasional" players with demand

$$Q_2 = 4 - 0.25P.$$

Assume that there are 1000 players of each type. Because you have plenty of courts, the marginal cost of court time is zero. You have fixed costs of \$10,000 per week. Serious and occasional players look alike, so you must charge them the same prices.

a. Suppose that to maintain a "professional" atmosphere, you want to limit membership to serious players. How should you set the *annual* membership dues and court fees (assume 52 weeks per year) to maximize profits, keeping in mind the constraint that only serious players choose to join? What would profits be (per week)?

In order to limit membership to serious players, the club owner should charge an entry fee, T, equal to the total consumer surplus of serious players and a usage fee P equal to marginal cost of zero. With individual demands of $Q_1 = 10 - P$, individual consumer surplus is equal to:

$$(0.5)(10 - 0)(10 - 0) = \$50, \text{ or}$$
$$(50)(52) = \$2600 \text{ per year.}$$

An entry fee of \$2600 maximizes profits by capturing all consumer surplus. The profit-maximizing court fee is set to zero, because marginal cost is equal to zero. The entry fee of \$2600 is higher than the occasional players are willing to pay (higher than their consumer surplus at a court fee of zero); therefore, this strategy will limit membership to the serious players. Weekly profits would be

$$\pi = (50)(1000) - 10{,}000 = \$40{,}000.$$

b. A friend tells you that you could make greater profits by encouraging both types of players to join. Is your friend right? What annual dues and court fees would maximize weekly profits? What would these profits be?

When there are two classes of customers, serious and occasional players, the club owner maximizes profits by charging court fees above marginal cost and by setting the entry fee (annual dues) equal to the remaining consumer surplus of the consumer with the lesser demand, in this case, the occasional player. The entry fee, T, equals the consumer surplus remaining after the court fee P is assessed:

$$T = 0.5Q_2(16 - P), \text{ where}$$
$$Q_2 = 4 - 0.25P.$$

Therefore,

$$T = 0.5(4 - 0.25P)(16 - P) = 32 - 4P + 0.125P^2.$$

Total entry fees paid by all players would be

$$2000T = 2000(32 - 4P + 0.125P^2) = 64,000 - 8000P + 250P^2.$$

Revenues from court fees equal

$$P(1000Q_1 + 1000Q_2) = P[1000(10 - P) + 1000(4 - 0.25P)] = 14,000P - 1250P^2.$$

Therefore, total revenue from entry fees and court fees is

$$TR = 64,000 + 6000P - 1000P^2.$$

Marginal cost is zero, so we want to maximize total revenue. To do this, differentiate total revenue with respect to price and set the derivative to zero:

$$\frac{dTR}{dP} = 6000 - 2000P = 0.$$

Solving for the optimal court fee, P = \$3.00 per hour. Serious players will play 10 – 3 = 7 hours per week, and occasional players will demand 4 – 0.25(3) = 3.25 hours of court time per week. Total revenue is then 64,000 + 6000(3) – 1000(3)^2 = \$73,000 per week. So profit is \$73,000 – 10,000 = \$63,000 per week, which is greater than the \$40,000 profit when only serious players become members. Therefore, your friend is right; it is more profitable to encourage both types of players to join.

c. Suppose that over the years young, upwardly mobile professionals move to your community, all of whom are serious players. You believe there are now 3000 serious players and 1000 occasional players. Would it still be profitable to cater to the occasional player? What would be the profit-maximizing annual dues and court fees? What would profits be per week?

An entry fee of \$50 per week would attract only serious players. With 3000 serious players, total revenues would be \$150,000 and profits would be \$140,000 per week. With both serious and occasional players, we may follow the same procedure as in part b. Entry fees would be equal to 4000 times the consumer surplus of the occasional player:

$$T = 4000(32 - 4P + 0.125P^2) = 128,000 - 16,000P + 500P^2$$

Court fees are

$$P(3000Q_1 + 1000Q_2) = P[3000(10 - P) + 1000(4 - 0.25P)] = 34{,}000P - 3250P^2, \text{ and}$$

$$TR = 128{,}000 + 18{,}000P - 2750P^2.$$

$$\frac{dTR}{dP} = 18{,}000 - 5500P = 0, \text{ so } P = \$3.27 \text{ per hour.}$$

With a court fee of \$3.27 per hour, total revenue is 128,000 + 18,000(3.27) − 2750(3.27)^2 = \$157,455 per week. Profit is \$157,455 − 10,000 = \$147,455 per week, which is more than the \$140,000 with serious players only. So you should set the entry fee and court fee to attract both types of players. The annual dues (i.e., the entry fee) should equal 52 times the weekly consumer surplus of the occasional player, which is 52[32 – 4(3.27) + 0.125(3.27)^2] = \$1053. The club's annual profit will be 52(147,455) = \$7.67 million per year.

11. Look again at Figure 11.12 (p. 420), which shows the reservation prices of three consumers for two goods. Assuming that marginal production cost is zero for both goods, can the producer make the most money by selling the goods separately, by using pure bundling, or by using mixed bundling? What prices should be charged?

The following tables summarize the reservation prices of the three consumers as shown in Figure 11.12 in the text and the profits from the three pricing strategies:

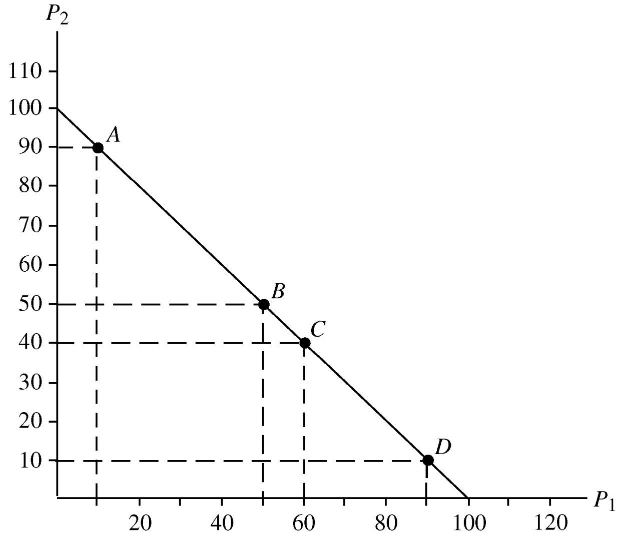
	Reservation Price		
	For 1	For 2	Total
Consumer *A*	\$ 3.25	\$ 6.00	\$ 9.25
Consumer *B*	\$ 8.25	\$ 3.25	\$11.50
Consumer *C*	\$10.00	\$10.00	\$20.00

	Price 1	Price 2	Bundled	Profit
Sell Separately	\$ 8.25	\$6.00	—	\$28.50
Pure Bundling	—	—	\$ 9.25	\$27.75
Mixed Bundling	\$10.00	\$6.00	\$11.50	\$29.00

The profit-maximizing strategy is to use mixed bundling. When each item is sold separately, two of Product 1 are sold (to Consumers *B* and *C*) at \$8.25, and two of Product 2 are sold (to Consumers *A* and *C*) at \$6.00. In the pure bundling case, three bundles are purchased at a price of \$9.25. This is more profitable than selling two bundles (to Consumers *B* and *C*) at \$11.50. With mixed bundling, one Product 2 is sold to *A* at \$6.00 and two bundles are sold (to *B* and *C*) at \$11.50. Other possible mixed bundling prices yield lower profits. Mixed bundling is often the ideal strategy when demands are only somewhat negatively correlated and/or when marginal production costs are significant.

12. Look again at Figure 11.17 (p. 424). Suppose that the marginal costs c_1 and c_2 were zero. Show that in this case, pure bundling, not mixed bundling, is the most profitable pricing strategy. What price should be charged for the bundle? What will the firm's profit be?

Figure 11.17 in the text is reproduced below. With marginal costs both equal to zero, the firm wants to maximize revenue. The firm should set the bundle price at \$100, since this is the sum of the reservation prices for all consumers. At this price all customers purchase the bundle, and the firm's revenues are \$400. This revenue is greater than setting $P_1 = P_2 = \$89.95$ and setting $P_B = \$100$ with the mixed bundling strategy. With mixed bundling, the firm sells one unit of Product 1, one unit of Product 2, and two bundles. Total revenue is \$379.90, which is less than \$400. Since marginal cost is zero and demands are negatively correlated, pure bundling is the best strategy.



13. Some years ago, an article appeared in the *New York Times* about IBM's pricing policy. The previous day, IBM had announced major price cuts on most of its small and medium-sized computers. The article said:

> **IBM probably has no choice but to cut prices periodically to get its customers to purchase more and lease less. If they succeed, this could make life more difficult for IBM's major competitors. Outright purchases of computers are needed for ever larger IBM revenues and profits, says Morgan Stanley's Ulric Weil in his new book, *Information Systems in the 80's*. Mr. Weil declares that IBM cannot revert to an emphasis on leasing.**

a. Provide a brief but clear argument in *support* of the claim that IBM should try to "get its customers to purchase more and lease less."

If we assume there is no resale market, there are at least three arguments that could be made in support of the claim that IBM should try to "get its customers to purchase more and lease less." First, when customers purchase computers, they are "locked into" the product. They do not have the option of not renewing the lease when it expires. Second, by getting customers to purchase a computer instead of leasing it, IBM leads customers to make a stronger economic decision for IBM and against its competitors. Thus it would be easier for IBM to eliminate its competitors if all its customers purchased, rather than leased, computers. Third, computers have a high obsolescence rate. If IBM believes that this rate is higher than what their customers perceive it is, the lease charges would be higher than what the customers would be willing to pay, and it would be more profitable to sell the computers rather than lease them.

b. Provide a brief but clear argument *against* this claim.

The primary argument for leasing computers instead of selling them is due to IBM's monopoly power, which would enable IBM to charge a two-part tariff that would extract some consumer surplus and increase its profits. For example, IBM could charge a fixed leasing fee plus a charge per unit of computing time used. Such a scheme would not be possible if the computers were sold outright.

c. What factors determine whether leasing or selling is preferable for a company like IBM? Explain briefly.

There are at least three factors that determine whether leasing or selling is preferable for IBM. The first is the amount of consumer surplus that IBM can extract if the computers are leased and a two-part tariff scheme is applied. The second factor is the relative discount rate on cash flows: if IBM has a higher discount rate than its customers, it might prefer to sell; if IBM has a lower discount rate, it might prefer to lease. A third factor is the vulnerability of IBM's competitors. Selling computers forces customers to make more of a financial commitment to one company over the rest, while with a leasing arrangement customers have more flexibility. Thus, if IBM feels it has the requisite market power, it might prefer to sell computers instead of lease them.

14. You are selling two goods, 1 and 2, to a market consisting of three consumers with reservation prices as follows:

	Reservation Price ($)	
Consumer	**For 1**	**For 2**
A	**20**	**100**
B	**60**	**60**
C	**100**	**20**

The unit cost of each product is $30.

a. Compute the optimal prices and profits for (i) selling the goods separately, (ii) pure bundling, and (iii) mixed bundling.

The optimal prices and resulting profits for each strategy are:

	Price 1	Price 2	Bundled Price	Profit
Sell Separately	$100.00	$100.00	—	$140.00
Pure Bundling	—	—	$120.00	$180.00
Mixed Bundling	$ 99.95	$ 99.95	$120.00	$199.90

You can try other prices to confirm that these are the best. For example, if you sell separately and charge $60 for good 1 and $60 for good 2, then *B* and *C* will buy good 1, and *A* and *B* will buy good 2. Since marginal cost for each unit is $30, profit for each unit is $60 – 30 = $30 for a total profit of $120.

b. Which strategy would be most profitable? Why?

Mixed bundling is best because, for each good, marginal production cost ($30) exceeds the reservation price for one consumer. For example, Consumer *A* has a reservation price of $100 for good 2 and only $20 for good 1. The firm responds by offering good 2 at a price just below Consumer *A*'s reservation price, so *A* would earn a small positive surplus by purchasing good 2 alone, and by charging a price for the bundle so that Consumer *A* would earn zero surplus by choosing the bundle. The result is that Consumer *A* chooses to purchase good 2 and not the bundle. Consumer *C*'s choice is symmetric to Consumer *A*'s choice. Consumer *B* chooses the bundle because the bundle's price is equal to the reservation price and the separate prices for the goods are both above the reservation price for either.

15. Your firm produces two products, the demands for which are independent. Both products are produced at zero marginal cost. You face four consumers (or groups of consumers) with the following reservation prices:

Consumer	Good 1 ($)	Good 2 ($)
A	**25**	**100**
B	**40**	**80**
C	**80**	**40**
D	**100**	**25**

a. Consider three alternative pricing strategies: (i) selling the goods separately; (ii) pure bundling; (iii) mixed bundling. For *each strategy*, determine the optimal prices to be charged and the resulting profits. Which strategy would be best?

For each strategy, the optimal prices and profits are

	Price 1	Price 2	Bundled Price	Profit
Sell Separately	$80.00	$80.00	—	$320.00
Pure Bundling	—	—	$120.00	$480.00
Mixed Bundling	$94.95	$94.95	$120.00	$429.90

You can try other prices to verify that $80 for each good is optimal if the goods are sold separately. For example if P_1 = $100 and P_2 = $80, then one unit of good 1 is sold for $100 and two units of 2 for $80, for a profit of $260. Note that in the case of mixed bundling, the price of each good must be set at $94.95 and not $99.95 since the bundle is $5 cheaper than the sum of the reservation prices for Consumers *A* and *D*. If the price of each good is set at $99.95 then neither Consumer *A* nor *D* will buy the individual good because they only save 5 cents off of their reservation price, as opposed to $5 for the bundle. Pure bundling dominates mixed bundling, because with zero marginal costs, there is no reason to exclude purchases of both goods by all consumers.

b. Now suppose that the production of each good entails a marginal cost of $30. How does this information change your answers to a? Why is the optimal strategy now different?

With marginal cost of $30, the optimal prices and profits are:

	Price 1	Price 2	Bundled Price	Profit
Sell Separately	$80.00	$80.00	—	$200.00
Pure Bundling	—	—	$120.00	$240.00
Mixed Bundling	$94.95	$94.95	$120.00	$249.90

Mixed bundling is the best strategy. Since the marginal cost is above the reservation price of Consumers *A* and *D*, the firm can benefit by using mixed bundling to encourage them to buy only one good.

16. A cable TV company offers, in addition to its basic service, two products: a sports channel (product 1) and a movie channel (product 2). Subscribers to the basic service can subscribe to these additional services individually at the monthly prices P_1 and P_2, respectively, or they can buy the two as a bundle for the price P_B, where $P_B < P_1 + P_2$. They can also forego the additional services and simply buy the basic service. The company's marginal cost for these additional services is *zero*. Through market research, the cable company has estimated the reservation prices for these two services for a representative group of consumers in the company's service area. These reservation prices are plotted (as *x*'s) in Figure 11.21, as are the prices P_1, P_2, and P_B that the cable company is currently charging. The graph is divided into regions, I, II, III, and IV.

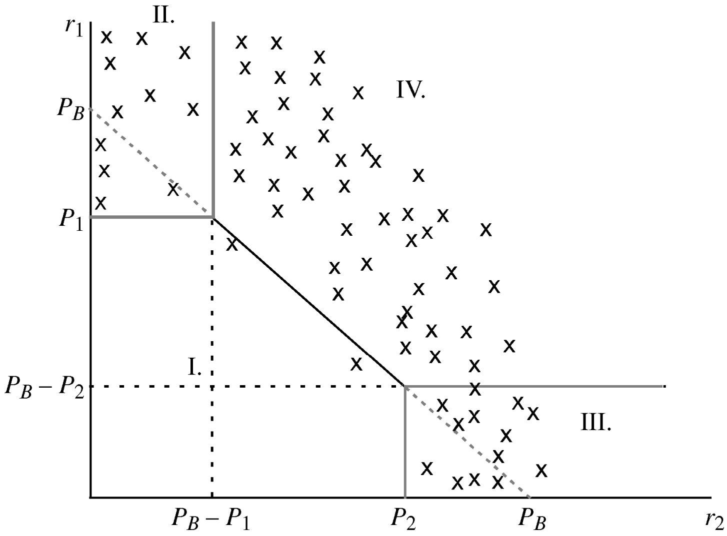


Figure 11.21

a. Which products, if any, will be purchased by the consumers in region I? In region II? In region III? In region IV? Explain briefly.

Product 1 = sports channel. Product 2 = movie channel.

Region	Purchase	Reservation Prices
I	nothing	$r_1 < P_1$, $r_2 < P_2$, $r_1 + r_2 < P_B$
II	sports channel	$r_1 > P_1$, $r_2 < P_B - P_1$
III	movie channel	$r_2 > P_2$, $r_1 < P_B - P_2$
IV	both channels	$r_1 > P_B - P_2$, $r_2 > P_B - P_1$, $r_1 + r_2 > P_B$

To see why consumers in regions II and III do not buy the bundle, reason as follows: for region II, $r_1 > P_1$, so the consumer will buy product 1. If she bought the bundle, she would pay an additional $P_B - P_1$. Since her reservation price for product 2 is less than $P_B - P_1$, she will choose to buy only product 1. Similar reasoning applies to region III.

Consumers in region I purchase nothing because the sum of their reservation values is less than the bundled price and each reservation value is lower than the respective price.

In region IV the sum of the reservation values for the consumers are higher than the bundle price, so these consumers would rather purchase the bundle than nothing. To see why the consumers in this region cannot do better than purchase either of the products separately, reason as follows: since $r_1 > P_B - P_2$ the consumer is better off purchasing both products than just product 2, likewise since $r_2 > P_B - P_1$, the consumer is better off purchasing both products rather than just product 1.

b. Note that as drawn in the figure, the reservation prices for the sports channel and the movie channel are negatively correlated. Why would you, or why would you not, expect consumers' reservation prices for cable TV channels to be negatively correlated?

Reservation prices may be negatively correlated if people's tastes differ in the following way: the more avidly a person likes sports, the less he or she will care for movies, and vice versa. Reservation prices would not be negatively correlated if people who were willing to pay a lot of money to watch sports were also willing to pay a lot of money to watch movies.

c. The company's vice president has said: "Because the marginal cost of providing an additional channel is zero, mixed bundling offers no advantage over pure bundling. Our profits would be just as high if we offered the sports channel and the movie channel together as a bundle, and only as a bundle." Do you agree or disagree? Explain why.

It depends. By offering only the bundled product, the company would lose customers below the bundled price line in regions II and III. At the same time, those consumers above the bundled price line in these regions would buy both channels rather than only one because the sum of their reservation prices exceeds the bundle price, and the channels are not offered separately. The net effect on revenues is therefore indeterminate. The exact solution depends on the distribution of consumers in those regions.

d. Suppose the cable company continues to use mixed bundling to sell these two services. Based on the distribution of reservation prices shown in Figure 11.21, do you think the cable company should alter any of the prices it is now charging? If so, how?

The cable company could raise P_B, P_1, and P_2 slightly without losing any customers. Alternatively, it could raise prices even past the point of losing customers as long as the additional revenue from the remaining customers made up for the revenue loss from the lost customers.

17. Consider a firm with monopoly power that faces the demand curve

$$\mathbf{P = 100 - 3Q + 4A^{1/2}}$$

and has the total cost function

$$\mathbf{C = 4Q^2 + 10Q + A}$$

where *A* is the level of advertising expenditures, and *P* and *Q* are price and output.

a. Find the values of *A*, *Q*, and *P* that maximize the firm's profit.

Profit (π) is equal to total revenue, *TR*, minus total cost, *TC*. Here,

$$TR = PQ = (100 - 3Q + 4A^{1/2})Q = 100Q - 3Q^2 + 4QA^{1/2} \text{ and}$$
$$TC = 4Q^2 + 10Q + A.$$

Therefore,

$$\pi = 100Q - 3Q^2 + 4QA^{1/2} - 4Q^2 - 10Q - A, \text{ or}$$
$$\pi = 90Q - 7Q^2 + 4QA^{1/2} - A.$$

The firm wants to choose its level of output and advertising expenditures to maximize its profits:

$$Max\ \pi = 90Q - 7Q^2 + 4A^{1/2} - A.$$

The necessary conditions for an optimum are:

(1) $\dfrac{\partial \pi}{\partial Q} = 90 - 14Q + 4A^{1/2} = 0,$ and

(2) $\dfrac{\partial \pi}{\partial A} = 2QA^{-1/2} - 1 = 0.$

From equation (2), we obtain

$$A^{1/2} = 2Q.$$

Substituting this into equation (1), gives

$$90 - 14Q + 4(2Q) = 0, \text{ or } Q^* = 15.$$

Then,

$$A^* = (4)(15^2) = \$900,$$

which implies

$$P^* = 100 - (3)(15) + (4)(900^{1/2}) = \$175.$$

b. Calculate the Lerner index, $L = (P - MC)/P$, for this firm at its profit-maximizing levels of A, Q, and P.

The degree of monopoly power is given by the formula $\dfrac{P - MC}{P}$. Marginal cost is $8Q + 10$ (the derivative of total cost with respect to quantity). At the optimum, $Q = 15$, and thus $MC = (8)(15) + 10 = 130$. Therefore, the Lerner index is

$$L = \frac{175 - 130}{175} = 0.257.$$

Chapter 12
Monopolistic Competition and Oligopoly

■ Teaching Notes

Students viewing this material for the first time can be overwhelmed because of the number of models presented. Chapter 12 discusses ten models (with some overlap): monopolistic competition, Cournot-Nash, Stackelberg, Bertrand, price competition with differentiated products, prisoners' dilemma, kinked demand, price leadership, dominant firm, and cartels. The reason for all the models, of course, is that there is no single oligopoly model. I try to convince students that since oligopoly theory is still evolving, it is an exciting area to study. Nonetheless, there is a great deal of material here, and you may want to concentrate on the more basic models, e.g., monopolistic competition, Cournot-Nash, prisoners' dilemma, and cartels. You can otherwise pick and choose among the models as time and interest dictates.

When introducing the material in this chapter, start by reminding students that these market structures lie between perfect competition and monopoly. When presenting monopolistic competition, focus on why positive profits encourage entry and on the similarities and differences of this model with competition and monopoly. The example of brand competition in cola and coffee markets presented at the end of Section 12.1 facilitates a class discussion of the costs and benefits of freedom of choice among a vast array of brand names and trademarks.

The key to the Cournot-Nash duopoly model in Section 12.2 is an understanding of the way firms react to each other and the resulting reaction functions. Stress that reaction functions are graphed on axes that both represent quantities (see Figure 12.4). Once they understand reaction functions, students will be better able to follow the assumptions, reasoning, and results of the Cournot-Nash, Stackelberg, and Bertrand models. Be sure to compare the competitive, Cournot-Nash, and collusive (monopoly) equilibria as shown in Figure 12.5. Figure 12.5 gives the impression that the duopolists always have symmetric reaction curves. Exercise 2 shows that if the cost structures are not identical, the reaction curves are asymmetric.

An alternative to jumping right into the Cournot-Nash material is to start with the kinked demand model in Section 12.5. You can use this to discuss the importance of other firms' reactions by drawing two complete demand curves through the current price-quantity point. In fact, there are many possible demand curves depending on how other firms react to a price change. For example, other firms may change price by half the amount or double the amount of the original firm's change. There may be uncertainty about how other firms will react, which you could relate back to the material in Chapter 5 if you covered that chapter. Deriving the kinked demand and associated marginal revenue curve will be easier if you went through the effect of a price ceiling on a monopolist in Chapter 10, where there is also a kinked demand.

Section 12.4 on the prisoners' dilemma is a student favorite. You might add some other examples such as advertising decisions where the primary effect of advertising is to take sales away from the other firm. Both firms have "high advertising" as a dominant strategy even though both would be more profitable if they each chose "low advertising" instead. Another good example is arms races between nations. Neither wants to be the one with a low number of weapons, so each stockpiles large amounts of weapons. Examples 12.2 and 12.3 deal with a pricing decision facing Procter & Gamble and together are a nice example of a prisoners' dilemma as well as an illustration of a pricing problem in foreign markets.

Section 12.5 looks at price signaling and price leadership. Another type of price signaling you could mention in class occurs when a firm announces that it will match any competitor's price. This is quite common in retail chains selling electronic equipment, office supplies, food, etc. While such promises sound good to consumers, they may also be a way to signal willingness to set prices at or near collusive levels. If firm *A* makes such a pledge, firm *B* knows that cutting prices will not be profitable, and it thus has an incentive to keep prices high.

Section 12.6 discusses cartels—an issue that most students find interesting. You can refer back to the prisoners' dilemma model to explain why cheating in cartels is often a problem. This section ends with two topics that invoke opinions from almost every student: a discussion of OPEC and Example 12.6, "The Cartelizaton of Intercollegiate Athletics."

■ Review Questions

1. **What are the characteristics of a monopolistically competitive market? What happens to the equilibrium price and quantity in such a market if one firm introduces a new, improved product?**

   The two primary characteristics of a monopolistically competitive market are that (1) firms compete by selling differentiated products that are highly, but not perfectly, substitutable and (2) there is free entry and exit from the market. When a new firm enters a monopolistically competitive market or one firm introduces an improved product, the demand curve for each of the other firms shifts inward, reducing the price and quantity received by those incumbents. Thus, the introduction of a new product by a firm will reduce the price received and quantity sold of existing products.

2. **Why is the firm's demand curve flatter than the total market demand curve in monopolistic competition? Suppose a monopolistically competitive firm is making a profit in the short run. What will happen to its demand curve in the long run?**

   The flatness or steepness of the firm's demand curve is a function of the elasticity of demand for the firm's product. The elasticity of the firm's demand curve is greater than the elasticity of market demand because it is easier for consumers to switch to another firm's highly substitutable product than to switch consumption to an entirely different product. Profit in the short run induces other firms to enter. As new firms enter, the incumbent firm's demand and marginal revenue curves shift to the left, reducing the profit-maximizing quantity. In the long run profits fall to zero, leaving no incentive for more firms to enter.

3. **Some experts have argued that too many brands of breakfast cereal are on the market. Give an argument to support this view. Give an argument against it.**

   Pro: Too many brands of any single product signals excess capacity, implying that each firm is producing an output level smaller than the level that would minimize average cost. Limiting the number of brands would therefore enhance overall economic efficiency.

   Con: Consumers value the freedom to choose among a wide variety of competing products. Even if costs are slightly higher as a result of the large number of brands available, the benefits to consumers outweigh the extra costs.

   (***Note*:** In 1972 the Federal Trade Commission filed suit against Kellogg, General Mills, and General Foods. It charged that these firms attempted to suppress entry into the cereal market by introducing 150 heavily advertised brands between 1950 and 1970, crowding competitors off grocers' shelves. This case was eventually dismissed in 1982.)

4. Why is the Cournot equilibrium stable? (i.e., Why don't firms have any incentive to change their output levels once in equilibrium?) Even if they can't collude, why don't firms set their outputs at the joint profit-maximizing levels (i.e., the levels they would have chosen had they colluded)?

A Cournot equilibrium is stable because each firm is producing the amount that maximizes its profit, *given what its competitors are producing*. If all firms behave this way, no firm has an incentive to change its output. Without collusion, firms find it difficult to agree tacitly to reduce output.

If all firms were producing at the joint profit-maximizing level, each would have an incentive to increase output, because that would increase each firm's profit at the expense of the firms that were limiting sales. But when each firm increases output, they all end up back at the Cournot equilibrium. Thus it is very difficult to reach the joint profit-maximizing level without overt collusion, and even then it may be difficult to prevent cheating among the cartel members.

5. In the Stackelberg model, the firm that sets output first has an advantage. Explain why.

The Stackelberg leader gains an advantage because the second firm must accept the leader's large output as given and produce a smaller output for itself. If the second firm decided to produce a larger quantity, this would reduce price and profit for both firms. The first firm knows that the second firm will have no choice but to produce a smaller output in order to maximize profit, and thus the first firm is able to capture a larger share of industry profits.

6. What do the Cournot and Bertrand models have in common? What is different about the two models?

Both are oligopoly models in which firms produce a homogeneous good. In the Cournot model, each firm assumes its rivals will not change the quantity they produce. In the Bertrand model, each firm assumes its rivals will not change the price they charge. In both models, each firm takes some aspect of its rivals' behavior (either quantity or price) as fixed when making its own decision. The difference between the two is that in the Bertrand model firms end up producing where price equals marginal cost, whereas in the Cournot model the firms will produce more than the monopoly output but less than the competitive output.

7. Explain the meaning of a Nash equilibrium when firms are competing with respect to price. Why is the equilibrium stable? Why don't the firms raise prices to the level that maximizes joint profits?

A Nash equilibrium in price competition occurs when each firm chooses its price, assuming *its competitors' prices will not change*. In equilibrium, each firm is doing the best it can, conditional on its competitors' prices. The equilibrium is stable because each firm is maximizing its profit, and therefore no firm has an incentive to raise or lower its price.

No individual firm would raise its price to the level that maximizes joint profit if the other firms do not do the same, because it would lose sales to the firms with lower prices. It is also difficult to collude. A cartel agreement is difficult to enforce because each firm has an incentive to cheat. By lowering price, the cheating firm can increase its market share and profits. A second reason that firms do not collude is that such behavior violates antitrust laws. In particular, price fixing violates Section 1 of the Sherman Act. Of course, there are attempts to circumvent antitrust laws through tacit collusion.

8. **The kinked demand curve describes price rigidity. Explain how the model works. What are its limitations? Why does price rigidity occur in oligopolistic markets?**

According to the kinked demand curve model, each firm faces a demand curve that is kinked at the currently prevailing price. Each firm believes that if it raises its price, the other firms will not raise their prices, and thus many of the firm's customers will shift their purchases to competitors. This reasoning implies a highly elastic demand for price increases. On the other hand, each firm believes that if it lowers its price, its competitors will also lower their prices, and the firm will not increase sales by much. This implies a demand curve that is less elastic for price decreases than for price increases. This kink in the demand curve leads to a discontinuity in the marginal revenue curve, so only large changes in marginal cost lead to changes in price.

A major limitation is that the kinked-demand model does not explain *how* the starting price is determined. Price rigidity may occur in oligopolistic markets because firms want to avoid destructive price wars. Managers learn from experience that cutting prices does not lead to lasting increases in profits. As a result, firms are reluctant to "rock the boat" by changing prices even when costs change.

9. **Why does price leadership sometimes evolve in oligopolistic markets? Explain how the price leader determines a profit-maximizing price.**

Since firms cannot explicitly coordinate on setting price, they use implicit means. One form of implicit collusion is to follow a price leader. The price leader, often the largest or dominant firm in the industry, determines its profit-maximizing quantity by calculating the demand curve it faces as follows: at each price, it subtracts the quantity supplied by all other firms from the market demand, and the residual is its demand curve. The leader chooses the quantity that equates its marginal revenue with its marginal cost and sets price to sell this quantity. The other firms (the followers) match the leader's price and supply the remainder of the market.

10. **Why has the OPEC oil cartel succeeded in raising prices substantially while the CIPEC copper cartel has not? What conditions are necessary for successful cartelization? What organizational problems must a cartel overcome?**

Successful cartelization requires two characteristics: demand should be relatively inelastic, and the cartel must be able to control most of the supply. OPEC succeeded in the short run because the short-run demand and supply of oil were both inelastic. CIPEC has not been successful because both demand and non-CIPEC supply were highly responsive to price. A cartel faces two organizational problems: agreement on a price and a division of the market among cartel members, and it must monitor and enforce the agreement.

■ Exercises

1. **Suppose all firms in a monopolistically competitive industry were merged into one large firm. Would that new firm produce as many different brands? Would it produce only a single brand? Explain.**

Monopolistic competition is defined by product differentiation. Each firm earns economic profit by distinguishing its brand from all other brands. This distinction can arise from underlying differences in the product or from differences in advertising. If these competitors merge into a single firm, the resulting monopolist would not produce as many brands, since too much brand competition is internecine (mutually destructive). However, it is unlikely that only one brand would be produced after the merger. Producing several brands with different prices and characteristics is one method of splitting the market into sets of customers with different tastes and price elasticities. The monopolist can sell to more consumers and maximize overall profit by producing multiple brands and practicing a form of price discrimination.

2. Consider two firms facing the demand curve $P = 50 - 5Q$, where $Q = Q_1 + Q_2$. The firms' cost functions are $C_1(Q_1) = 20 + 10Q_1$ and $C_2(Q_2) = 10 + 12Q_2$.

a. Suppose both firms have entered the industry. What is the joint profit-maximizing level of output? How much will each firm produce? How would your answer change if the firms have not yet entered the industry?

If the firms collude, they face the market demand curve, so their marginal revenue curve is:

$$MR = 50 - 10Q.$$

Set marginal revenue equal to marginal cost (the marginal cost of Firm 1, since it is lower than that of Firm 2) to determine the profit-maximizing quantity, Q:

$$50 - 10Q = 10, \text{ or } Q = 4.$$

Substituting $Q = 4$ into the demand function to determine price:

$$P = 50 - 5(4) = \$30.$$

The question now is how the firms will divide the total output of 4 among themselves. The joint profit-maximizing solution is for Firm 1 to produce all of the output because its marginal cost is less than Firm 2's marginal cost. We can ignore fixed costs because both firms are already in the market and will be saddled with their fixed costs no matter how many units each produces. If Firm 1 produces all 4 units, its profit will be

$$\pi_1 = (30)(4) - (20 + (10)(4)) = \$60.$$

The profit for Firm 2 will be:

$$\pi_2 = (30)(0) - (10 + (12)(0)) = -\$10.$$

Total industry profit will be:

$$\pi_T = \pi_1 + \pi_2 = 60 - 10 = \$50.$$

Firm 2, of course, will not like this. One solution is for Firm 1 to pay Firm 2 \$35 so that both earn a profit of \$25, although they may well disagree about the amount to be paid. If they split the output evenly between them, so that each firm produces 2 units, then total profit would be \$46 (\$20 for Firm 1 and \$26 for Firm 2). This does not maximize total profit, but Firm 2 would prefer it to the \$25 it gets from an even split of the maximum \$50 profit. So there is no clear-cut answer to this question.

If Firm 1 were the only entrant, its profits would be \$60 and Firm 2's would be 0.

If Firm 2 were the only entrant, then it would equate marginal revenue with its marginal cost to determine its profit-maximizing quantity:

$$50 - 10Q_2 = 12, \text{ or } Q_2 = 3.8.$$

Substituting Q_2 into the demand equation to determine price:

$$P = 50 - 5(3.8) = \$31.$$

The profits for Firm 2 would be:

$$\pi_2 = (31)(3.8) - (10 + (12)(3.8)) = \$62.20,$$

and Firm 1 would earn 0. Thus, Firm 2 would make a larger profit than Firm 1 if it were the only firm in the market, because Firm 2 has lower fixed costs.

b. What is each firm's equilibrium output and profit if they behave noncooperatively? Use the Cournot model. Draw the firms' reaction curves and show the equilibrium.

In the Cournot model, Firm 1 takes Firm 2's output as given and maximizes profits. Firm 1's profit function is

$$\pi_1 = (50 - 5Q_1 - 5Q_2)Q_1 - (20 + 10Q_1), \text{ or}$$
$$\pi_1 = 40Q_1 - 5Q_1^2 - 5Q_1Q_2 - 20.$$

Setting the derivative of the profit function with respect to Q_1 to zero, we find Firm 1's reaction function:

$$\frac{\partial \pi_1}{\partial Q_1} = 40 - 10Q_1 - 5Q_2 = 0, \text{ or } Q_1 = 4 - \left(\frac{Q_2}{2}\right).$$

Similarly, Firm 2's reaction function is $Q_2 = 3.8 - \left(\frac{Q_1}{2}\right)$.

To find the Cournot equilibrium, substitute Firm 2's reaction function into Firm 1's reaction function:

$$Q_1 = 4 - \left(\frac{1}{2}\right)\left(3.8 - \frac{Q_1}{2}\right), \text{ or } Q_1 = 2.8.$$

Substituting this value for Q_1 into the reaction function for Firm 2, we find

$$Q_2 = 2.4.$$

Substituting the values for Q_1 and Q_2 into the demand function to determine the equilibrium price:

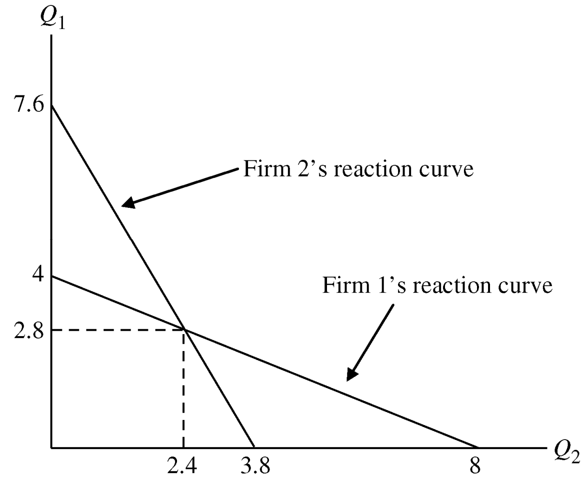
$$P = 50 - 5(2.8 + 2.4) = \$24.$$

The profits for Firms 1 and 2 are equal to

$$\pi_1 = (24)(2.8) - (20 + (10)(2.8)) = \$19.20, \text{ and}$$

$$\pi_2 = (24)(2.4) - (10 + (12)(2.4)) = \$18.80.$$

The firms' reaction curves and the Cournot equilibrium are shown below.



c. How much should Firm 1 be willing to pay to purchase Firm 2 if collusion is illegal but a takeover is not?

To determine how much Firm 1 will be willing to pay to purchase Firm 2, we must compare Firm 1's profits in the monopoly situation versus it profits in an oligopoly. The difference between the two will be what Firm 1 is willing to pay for Firm 2.

From part a, Firm 1's profit when it sets marginal revenue equal to its marginal cost is \$60. This is what the firm would earn if it was a monopolist. From part b, profit is \$19.20 for Firm 1 when the firms compete against each other in a Cournot-type market. Firm 1 should therefore be willing to pay up to \$60 − 19.20 = \$40.80 for Firm 2.

3. A monopolist can produce at a constant average (and marginal) cost of $AC = MC =$ \$5. It faces a market demand curve given by $Q = 53 - P$.

a. Calculate the profit-maximizing price and quantity for this monopolist. Also calculate its profits.

First solve for the inverse demand curve, $P = 53 - Q$. Then the marginal revenue curve has the same intercept and twice the slope:

$$MR = 53 - 2Q.$$

Marginal cost is a constant \$5. Setting $MR = MC$, find the optimal quantity:

$$53 - 2Q = 5, \text{ or } Q = 24.$$

Substitute $Q = 24$ into the demand function to find price:

$$P = 53 - 24 = \$29.$$

Assuming fixed costs are zero, profits are equal to

$$\pi = TR - TC = (29)(24) - (5)(24) = \$576.$$

b. Suppose a second firm enters the market. Let Q_1 be the output of the first firm and Q_2 be the output of the second. Market demand is now given by

$$Q_1 + Q_2 = 53 - P.$$

Assuming that this second firm has the same costs as the first, write the profits of each firm as functions of Q_1 and Q_2.

When the second firm enters, price can be written as a function of the output of both firms: $P = 53 - Q_1 - Q_2$. We may write the profit functions for the two firms:

$$\pi_1 = PQ_1 - C(Q_1) = (53 - Q_1 - Q_2)Q_1 - 5Q_1, \text{ or } \pi_1 = 48Q_1 - Q_1^2 - Q_1Q_2$$

and

$$\pi_2 = PQ_2 - C(Q_2) = (53 - Q_1 - Q_2)Q_2 - 5Q_2, \text{ or } \pi_2 = 48Q_2 - Q_2^2 - Q_1Q_2.$$

c. Suppose (as in the Cournot model) that each firm chooses its profit-maximizing level of output on the assumption that its competitor's output is fixed. Find each firm's "reaction curve" (i.e., the rule that gives its desired output in terms of its competitor's output).

Under the Cournot assumption, each firm treats the output of the other firm as a constant in its maximization calculations. Therefore, Firm 1 chooses Q_1 to maximize π_1 in part b with Q_2 being treated as a constant. The change in π_1 with respect to a change in Q_1 is

$$\frac{\partial \pi_1}{\partial Q_1} = 48 - 2Q_1 - Q_2 = 0, \text{ or } Q_1 = 24 - \frac{Q_2}{2}.$$

This equation is the reaction function for Firm 1, which generates the profit- maximizing level of output, given the output of Firm 2. Because the problem is symmetric, the reaction function for Firm 2 is

$$Q_2 = 24 - \frac{Q_1}{2}.$$

d. Calculate the Cournot equilibrium (i.e., the values of Q_1 and Q_2 for which each firm is doing as well as it can given its competitor's output). What are the resulting market price and profits of each firm?

Solve for the values of Q_1 and Q_2 that satisfy both reaction functions by substituting Firm 2's reaction function into the function for Firm 1:

$$Q_1 = 24 - \left(\frac{1}{2}\right)\left(24 - \frac{Q_1}{2}\right), \text{ or } Q_1 = 16.$$

By symmetry, $Q_2 = 16$.

To determine the price, substitute Q_1 and Q_2 into the demand equation:

$$P = 53 - 16 - 16 = \$21.$$

Profit for Firm 1 is therefore

$$\pi_i = PQ_i - C(Q_i) = \pi_i = (21)(16) - (5)(16) = \$256.$$

Firm 2's profit is the same, so total industry profit is $\pi_1 + \pi_2 = \$256 + \$256 = \$512$.

e. Suppose there are N firms in the industry, all with the same constant marginal cost, $MC = \$5$. Find the Cournot equilibrium. How much will each firm produce, what will be the market price, and how much profit will each firm earn? Also, show that as N becomes large, the market price approaches the price that would prevail under perfect competition.

If there are N identical firms, then the price in the market will be

$$P = 53 - (Q_1 + Q_2 + \cdots + Q_N).$$

Profits for the ith firm are given by

$$\pi_i = PQ_i - C(Q_i),$$
$$\pi_i = 53Q_i - Q_1Q_i - Q_2Q_i - \cdots - Q_i^2 - \cdots - Q_NQ_i - 5Q_i.$$

Differentiating to obtain the necessary first-order condition for profit maximization,

$$\frac{\partial \pi_i}{\partial Q_i} = 53 - Q_1 - Q_2 - \cdots - 2Q_i - \cdots - Q_N - 5 = 0.$$

Solving for Q_i,

$$Q_i = 24 - \frac{1}{2}(Q_1 + \cdots + Q_{i-1} + Q_{i+1} + \cdots + Q_N).$$

If all firms face the same costs, they will all produce the same level of output, i.e., $Q_i = Q^*$. Therefore,

$$Q^* = 24 - \frac{1}{2}(N-1)Q^*, \text{ or } 2Q^* = 48 - (N-1)Q^*, \text{ or}$$
$$(N+1)Q^* = 48, \text{ or } Q^* = \frac{48}{(N+1)}.$$

Now substitute $Q = NQ^*$ for total output in the demand function:

$$P = 53 - N\left(\frac{48}{N+1}\right).$$

Total profits are

$$\pi_T = PQ - C(Q) = P(NQ^*) - 5(NQ^*)$$

or

$$\pi_T = \left[53 - N\left(\frac{48}{N+1}\right)\right](N)\left(\frac{48}{N+1}\right) - 5N\left(\frac{48}{N+1}\right) \text{ or}$$
$$\pi_T = \left[48 - (N)\left(\frac{48}{N+1}\right)\right](N)\left(\frac{48}{N+1}\right)$$

or

$$\pi_T = (48)\left(\frac{N+1-N}{N+1}\right)(48)\left(\frac{N}{N+1}\right) = (2,304)\left(\frac{N}{(N+1)}\right)$$

Notice that with N firms

$$Q = 48\left(\frac{N}{N+1}\right)$$

and that, as N increases ($N \to \infty$)

$$Q = 48.$$

Similarly, with

$$P = 53 - 48\left(\frac{N}{N+1}\right),$$

as $N \to \infty$,

$$P = 53 - 48 = 5.$$

Finally,

$$\pi_T = 2,304\left(\frac{N}{(N+1)^2}\right),$$

so as $N \to \infty$,

$$\pi_T = \$0.$$

In perfect competition, we know that profits are zero and price equals marginal cost. Here, $\pi_T = \$0$ and $P = MC = 5$. Thus, when N approaches infinity, this market approaches a perfectly competitive one.

4. This exercise is a continuation of Exercise 3. We return to two firms with the same constant average and marginal cost, $AC = MC = 5$, facing the market demand curve $Q_1 + Q_2 = 53 - P$. Now we will use the Stackelberg model to analyze what will happen if one of the firms makes its output decision before the other.

a. Suppose Firm 1 is the Stackelberg leader (i.e., makes its output decisions before Firm 2). Find the reaction curves that tell each firm how much to produce in terms of the output of its competitor.

Firm 2's reaction curve is the same as determined in part c of Exercise 3:

$Q_2 = 24 - \left(\frac{Q_1}{2}\right)$. Firm 1 does not have a reaction function because it makes its output decision before Firm 2, so there is nothing to react to. Instead, Firm 1 uses its knowledge of Firm 2's reaction function when determining its optimal output as shown in part b below.

b. How much will each firm produce, and what will its profit be?

Firm 1, the Stackelberg leader, will choose its output, Q_1, to maximize its profits, subject to the reaction function of Firm 2:

$$\max \pi_1 = PQ_1 - C(Q_1),$$

subject to

$$Q_2 = 24 - \left(\frac{Q_1}{2}\right).$$

Substitute for Q_2 in the demand function and, after solving for P, substitute for P in the profit function:

$$\max \pi_1 = \left(53 - Q_1 - \left(24 - \frac{Q_1}{2}\right)\right)(Q_1) - 5Q_1.$$

To determine the profit-maximizing quantity, we find the change in the profit function with respect to a change in Q_1:

$$\frac{d\pi_1}{dQ_1} = 53 - 2Q_1 - 24 + Q_1 - 5.$$

Set this expression equal to 0 to determine the profit-maximizing quantity:

$$53 - 2Q_1 - 24 + Q_1 - 5 = 0, \text{ or } Q_1 = 24.$$

Substituting $Q_1 = 24$ into Firm 2's reaction function gives Q_2:

$$Q_2 = 24 - \frac{24}{2} = 12.$$

Substitute Q_1 and Q_2 into the demand equation to find the price:

$$P = 53 - 24 - 12 = \$17.$$

Profits for each firm are equal to total revenue minus total costs, or

$$\pi_1 = (17)(24) - (5)(24) = \$288, \text{ and}$$

$$\pi_2 = (17)(12) - (5)(12) = \$144.$$

Total industry profit, $\pi_T = \pi_1 + \pi_2 = \$288 + \$144 = \$432$.

Compared to the Cournot equilibrium, total output has increased from 32 to 36, price has fallen from \$21 to \$17, and total profits have fallen from \$512 to \$432. Profits for Firm 1 have risen from \$256 to \$288, while the profits of Firm 2 have declined sharply from \$256 to \$144.

5. Two firms compete in selling identical widgets. They choose their output levels Q_1 and Q_2 simultaneously and face the demand curve

$$P = 30 - Q$$

where $Q = Q_1 + Q_2$. Until recently, both firms had *zero marginal costs*. Recent environmental regulations have increased Firm 2's marginal cost to \$15. Firm 1's marginal cost remains constant at zero. True or false: As a result, the market price will rise to the *monopoly* level.

Surprisingly, this is true. However, it occurs only because the marginal cost for Firm 2 is \$15 or more. If the market were monopolized before the environmental regulations, the marginal revenue for the monopolist would be

$$MR = 30 - 2Q.$$

Profit maximization implies $MR = MC$, or $30 - 2Q = 0$. Therefore, $Q = 15$, and (using the demand curve) $P = \$15$.

The situation after the environmental regulations is a Cournot game where Firm 1's marginal costs are zero and Firm 2's marginal costs are \$15. We need to find the best response functions:

Firm 1's revenue is

$$PQ_1 = (30 - Q_1 - Q_2)Q_1 = 30Q_1 - Q_1^2 - Q_1Q_2,$$

and its marginal revenue is given by:

$$MR_1 = 30 - 2Q_1 - Q_2.$$

Profit maximization implies $MR_1 = MC_1$ or

$$30 - 2Q_1 - Q_2 = 0 \Rightarrow Q_1 = 15 - \frac{Q_2}{2},$$

which is Firm 1's best response function.

Firm 2's revenue function is symmetric to that of Firm 1 and hence

$$MR_2 = 30 - Q_1 - 2Q_2.$$

Profit maximization implies $MR_2 = MC_2$, or

$$30 - 2Q_2 - Q_1 = 15 \Rightarrow Q_2 = 7.5 - \frac{Q_1}{2},$$

which is Firm 2's best response function.

Cournot equilibrium occurs at the intersection of the best response functions. Substituting for Q_1 in the response function for Firm 2 yields:

$$Q_2 = 7.5 - 0.5\left(15 - \frac{Q_2}{2}\right).$$

Thus $Q_2 = 0$ and $Q_1 = 15$. $P = 30 - Q_1 - Q_2 = \$15$, which is the monopoly price.

6. Suppose that two identical firms produce widgets and that they are the only firms in the market. Their costs are given by $C_1 = 60Q_1$ and $C_2 = 60Q_2$, where Q_1 is the output of Firm 1 and Q_2 the output of Firm 2. Price is determined by the following demand curve:

$$P = 300 - Q$$

where $Q = Q_1 + Q_2$.

a. Find the Cournot-Nash equilibrium. Calculate the profit of each firm at this equilibrium.

Profit for Firm 1, $TR_1 - TC_1$, is equal to

$$\pi_1 = 300Q_1 - Q_1^2 - Q_1Q_2 - 60Q_1 = 240Q_1 - Q_1^2 - Q_1Q_2.$$

Therefore,

$$\frac{\partial \pi_1}{\partial Q_1} = 240 - 2Q_1 - Q_2.$$

Setting this equal to zero and solving for Q_1 in terms of Q_2:

$$Q_1 = 120 - 0.5Q_2.$$

This is Firm 1's reaction function. Because Firm 2 has the same cost structure, Firm 2's reaction function is

$$Q_2 = 120 - 0.5Q_1.$$

Substituting for Q_2 in the reaction function for Firm 1, and solving for Q_1, we find

$$Q_1 = 120 - (0.5)(120 - 0.5Q_1), \text{ or } Q_1 = 80.$$

By symmetry, $Q_2 = 80$. Substituting Q_1 and Q_2 into the demand equation to determine the equilibrium price:

$$P = 300 - 80 - 80 = \$140.$$

Substituting the values for price and quantity into the profit functions,

$$\pi_1 = (140)(80) - (60)(80) = \$6400, \text{ and}$$

$$\pi_2 = (140)(80) - (60)(80) = \$6400.$$

Therefore, profit is \$6400 for both firms in the Cournot-Nash equilibrium.

b. Suppose the two firms form a cartel to maximize joint profits. How many widgets will be produced? Calculate each firm's profit.

Given the demand curve is $P = 300 - Q$, the marginal revenue curve is $MR = 300 - 2Q$. Profit will be maximized by finding the level of output such that marginal revenue is equal to marginal cost:

$$300 - 2Q = 60, \text{ or } Q = 120.$$

When total output is 120, price will be \$180, based on the demand curve. Since both firms have the same marginal cost, they will split the total output equally, so they each produce 60 units.

Profit for each firm is:

$$\pi = 180(60) - 60(60) = \$7200.$$

c. Suppose Firm 1 were the only firm in the industry. How would market output and Firm 1's profit differ from that found in part b above?

If Firm 1 were the only firm, it would produce where marginal revenue is equal to marginal cost, as found in part b. In this case Firm 1 would produce the entire 120 units of output and earn a profit of $14,400.

d. Returning to the duopoly of part b, suppose Firm 1 abides by the agreement, but Firm 2 cheats by increasing production. How many widgets will Firm 2 produce? What will be each firm's profits?

Assuming their agreement is to split the market equally, Firm 1 produces 60 widgets. Firm 2 cheats by producing its profit-maximizing level, given $Q_1 = 60$. Substituting $Q_1 = 60$ into Firm 2's reaction function:

$$Q_2 = 120 - \frac{60}{2} = 90.$$

Total industry output, Q_T, is equal to Q_1 plus Q_2:

$$Q_T = 60 + 90 = 150.$$

Substituting Q_T into the demand equation to determine price:

$$P = 300 - 150 = \$150.$$

Substituting Q_1, Q_2, and P into the profit functions:

$$\pi_1 = (150)(60) - (60)(60) = \$5400, \text{ and}$$

$$\pi_2 = (150)(90) - (60)(90) = \$8100.$$

Firm 2 increases its profits at the expense of Firm 1 by cheating on the agreement.

7. Suppose that two competing firms, *A* and *B*, produce a homogeneous good. Both firms have a marginal cost of *MC* = $50. Describe what would happen to output and price in each of the following situations if the firms are at (i) Cournot equilibrium, (ii) collusive equilibrium, and (iii) Bertrand equilibrium.

a. Because Firm *A* must increase wages, its *MC* increases to $80.

(i) In a Cournot equilibrium you must think about the effect on the reaction functions, as illustrated in Figure 12.5 of the text. When Firm *A* experiences an increase in marginal cost, its reaction function will shift inward. The quantity produced by Firm *A* will decrease and the quantity produced by Firm *B* will increase. Total quantity produced will decrease and price will increase.

(ii) In a collusive equilibrium, the two firms will collectively act like a monopolist. When the marginal cost of Firm *A* increases, Firm *A* will reduce its production to zero, because Firm *B* can produce at a lower marginal cost. Because Firm *B* can produce the entire industry output at a marginal cost of $50, there will be no change in output or price. However, the firms will have to come to some agreement on how to share the profit earned by *B*.

(iii) Before the increase in Firm *A*'s costs, both firms would charge a price equal to marginal cost (*P* = $50) because the good is homogeneous. After Firm *A*'s marginal cost increases, Firm *B* will raise its price to $79.99 (or some price just below $80) and take all sales away from Firm *A*. Firm *A* would lose money on each unit sold at any price below its marginal cost of $80, so it will produce nothing.

b. The marginal cost of both firms increases.

(i) Again refer to Figure 12.5. The increase in the marginal cost of both firms will shift both reaction functions inward. Both firms will decrease quantity produced and price will increase.

(ii) When marginal cost increases, both firms will produce less and price will increase, as in the monopoly case.

(iii) Price will increase to the new level of marginal cost and quantity will decrease.

c. The demand curve shifts to the right.

(i) This is the opposite of the case in part b. In this situation, both reaction functions will shift outward and both will produce a higher quantity. Price will tend to increase.

(ii) Both firms will increase the quantity produced as demand and marginal revenue increase. Price will also tend to increase.

(iii) Both firms will supply more output. Given that marginal cost remains the same, the price will not change.

8. Suppose the airline industry consisted of only two firms: American and Texas Air Corp. Let the two firms have identical cost functions, *C*(*q*) = 40*q*. Assume the demand curve for the industry is given by *P* = 100 − *Q* and that each firm expects the other to behave as a Cournot competitor.

a. Calculate the Cournot-Nash equilibrium for each firm, assuming that each chooses the output level that maximizes its profits when taking its rival's output as given. What are the profits of each firm?

First, find the reaction function for each firm; then solve for price, quantity, and profit. Profit for Texas Air, π_1, is equal to total revenue minus total cost:

$$\pi_1 = (100 - Q_1 - Q_2)Q_1 - 40Q_1, \text{ or}$$
$$\pi_1 = 100Q_1 - Q_1^2 - Q_1Q_2 - 40Q_1, \text{ or } \pi_1 = 60Q_1 - Q_1^2 - Q_1Q_2.$$

The change in π_1 with respect to Q_1 is

$$\frac{\partial \pi_1}{\partial Q_1} = 60 - 2Q_1 - Q_2.$$

Setting the derivative to zero and solving for Q_1 gives Texas Air's reaction function:

$$Q_1 = 30 - 0.5Q_2.$$

Because American has the same cost structure, American's reaction function is

$$Q_2 = 30 - 0.5Q_1.$$

Substituting for Q_2 in the reaction function for Texas Air,

$$Q_1 = 30 - 0.5(30 - 0.5Q_1), \text{ or } Q_1 = 20.$$

By symmetry, $Q_2 = 20$. Industry output, Q_T, is Q_1 plus Q_2, or

$$Q_T = 20 + 20 = 40.$$

Substituting industry output into the demand equation, we find $P = \$60$. Substituting Q_1, Q_2, and P into the profit function, we find

$$\pi_1 = \pi_2 = 60(20) - 20^2 - (20)(20) = \$400.$$

b. What would be the equilibrium quantity if Texas Air had constant marginal and average costs of \$25 and American had constant marginal and average costs of \$40?

By solving for the reaction functions under this new cost structure, we find that profit for Texas Air is equal to

$$\pi_1 = 100Q_1 - Q_1^2 - Q_1Q_2 - 25Q_1 = 75Q_1 - Q_1^2 - Q_1Q_2.$$

The change in profit with respect to Q_1 is

$$\frac{\partial \pi_1}{\partial Q_1} = 75 - 2Q_1 - Q_2.$$

Set the derivative to zero, and solve for Q_1 in terms of Q_2,

$$Q_1 = 37.5 - 0.5Q_2.$$

This is Texas Air's reaction function. Since American has the same cost structure as in part a, American's reaction function is the same as before:

$$Q_2 = 30 - 0.5Q_1.$$

To determine Q_1, substitute for Q_2 in the reaction function for Texas Air and solve for Q_1:

$$Q_1 = 37.5 - (0.5)(30 - 0.5Q_1), \text{ so } Q_1 = 30.$$

Texas Air finds it profitable to increase output in response to a decline in its cost structure.

To determine Q_2, substitute for Q_1 in the reaction function for American:

$$Q_2 = 30 - (0.5)(30) = 15.$$

American has cut back slightly in its output in response to the increase in output by Texas Air.

Total quantity, Q_T, is $Q_1 + Q_2$, or

$$Q_T = 30 + 15 = 45.$$

Compared to part a, the equilibrium quantity has risen slightly.

c. Assuming that both firms have the original cost function, $C(q) = 40q$, how much should Texas Air be willing to invest to lower its marginal cost from 40 to 25, assuming that American will not follow suit? How much should American be willing to spend to reduce its marginal cost to 25, assuming that Texas Air will have marginal costs of 25 regardless of American's actions?

Recall that profits for both firms were \$400 under the original cost structure. With constant average and marginal costs of \$25, we determined in part b that Texas Air would produce 30 units and American 15. Industry price would then be $P = 100 - 30 - 15 = \$55$. Texas Air's profits would be

$$(55)(30) - (25)(30) = \$900.$$

The difference in profit is \$500. Therefore, Texas Air should be willing to invest up to \$500 to *lower costs* from 40 to 25 per unit (assuming American does not follow suit).

To determine how much American would be willing to spend to reduce its average costs, calculate the difference in American's profits, assuming Texas Air's average cost is \$25. First, without investment, American's profits would be:

$$(55)(15) - (40)(15) = \$225.$$

Second, with investment by both firms, the reaction functions would be:

$$Q_1 = 37.5 - 0.5Q_2 \text{ and}$$

$$Q_2 = 37.5 - 0.5Q_1.$$

To determine Q_1, substitute for Q_2 in the first reaction function and solve for Q_1:

$$Q_1 = 37.5 - (0.5)(37.5 - 0.5Q_1), \text{ which implies } Q_1 = 25.$$

Since the firms are symmetric, Q_2 is also 25.

Substituting industry output into the demand equation to determine price:

$$P = 100 - 50 = \$50.$$

Therefore, American's profits when both firms have $MC = AC = 25$ are

$$\pi_2 = (50)(25) - (25)(25) = \$625.$$

The difference in profit with and without the cost-saving investment for American is \$400. American would be willing to invest up to \$400 to reduce its marginal cost to 25 if Texas Air also has marginal costs of 25.

9. Demand for light bulbs can be characterized by $Q = 100 - P$, where Q is in millions of boxes of lights sold and P is the price per box. There are two producers of lights, Everglow and Dimlit. They have identical cost functions:

$$\boldsymbol{C_i = 10Q_i + \frac{1}{2}Q_i^2 \ (i = E, D) \qquad\qquad Q = Q_E + Q_D.}$$

a. Unable to recognize the potential for collusion, the two firms act as short-run perfect competitors. What are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

Given that the total cost function is $C_i = 10Q_i + 1/2Q_i^2$, the marginal cost curve for each firm is $MC_i = 10 + Q_i$. In the short run, perfectly competitive firms determine the optimal level of output by taking price as given and setting price equal to marginal cost. There are two ways to solve this problem. One way is to set price equal to marginal cost for each firm so that:

$$P = 100 - Q_1 - Q_2 = 10 + Q_1$$
$$P = 100 - Q_1 - Q_2 = 10 + Q_2.$$

Given we now have two equations and two unknowns, we can solve for Q_1 and Q_2 simultaneously. Solve the second equation for Q_2 to get

$$Q_2 = \frac{90 - Q_1}{2},$$

and substitute into the other equation to get

$$100 - Q_1 - \frac{90 - Q_1}{2} = 10 + Q_1.$$

This yields a solution where $Q_1 = 30$, $Q_2 = 30$, and $P = \$40$. You can verify that $P = MC$ for each firm. Profit is total revenue minus total cost or

$$\pi_i = 40(30) - [10(30) + 0.5(30)^2] = \$450 \text{ million.}$$

The other way to solve the problem is to find the market supply curve by summing the marginal cost curves, which yields $Q_M = 2P - 20$. Set supply equal to demand to find $P = \$40$ and $Q = 60$ in the market, or 30 per firm since they are identical.

b. Top management in both firms is replaced. Each new manager independently recognizes the oligopolistic nature of the light bulb industry and plays Cournot. What are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

To determine the Cournot-Nash equilibrium, we first calculate the reaction function for each firm, then solve for price, quantity, and profit. Profits for Everglow are equal to $TR_E - TC_E$, or

$$\pi_E = (100 - Q_E - Q_D)Q_E - (10Q_E + 0.5Q_E^2) = 90Q_E - 1.5Q_E^2 - Q_E Q_D.$$

The change in profit with respect to Q_E is

$$\frac{\partial \pi_E}{\partial Q_E} = 90 - 3Q_E - Q_D.$$

To determine Everglow's reaction function, set the change in profits with respect to Q_E equal to 0 and solve for Q_E:

$$90 - 3Q_E - Q_D = 0, \text{ or}$$

$$Q_E = \frac{90 - Q_D}{3}.$$

Because Dimlit has the same cost structure, Dimlit's reaction function is

$$Q_D = \frac{90 - Q_E}{3}.$$

Substituting for Q_D in the reaction function for Everglow, and solving for Q_E:

$$Q_E = \frac{90 - \frac{90 - Q_E}{3}}{3}$$

$$3Q_E = 90 - 30 + \frac{Q_E}{3}$$

$$Q_E = 22.5.$$

By symmetry, $Q_D = 22.5$, and total industry output is 45.

Substituting industry output into the demand equation gives P:

$$45 = 100 - P, \text{ or } P = \$55.$$

Each firm's profit equals total revenue minus total cost:

$$B_I = 55(22.5) - [10(22.5) + 0.5(22.5)^2] = \$759.4 \text{ million.}$$

c. Suppose the Everglow manager guesses correctly that Dimlit is playing Cournot, so Everglow plays Stackelberg. What are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

Recall Everglow's profit function:

$$\pi_E = (100 - Q_E - Q_D)Q_E - (10Q_E + 0.5Q_E^2).$$

If Everglow sets its quantity first, knowing Dimlit's reaction function $\left(\text{i.e., } Q_D = 30 - \dfrac{Q_E}{3}\right)$, we may determine Everglow's profit by substituting for Q_D in its profit function. We find

$$\pi_E = 60Q_E - \frac{7Q_E^2}{6}.$$

To determine the profit-maximizing quantity, differentiate profit with respect to Q_E, set the derivative to zero and solve for Q_E:

$$\frac{\partial \pi_E}{\partial Q_E} = 60 - \frac{7Q_E}{3} = 0, \text{ or } Q_E = 25.7.$$

Substituting this into Dimlit's reaction function, $Q_D = 30 - \dfrac{25.7}{3} = 21.4$. Total industry output is therefore 47.1 and P = \$52.90. Profit for Everglow is

$$\pi_E = (52.90)(25.7) - [10(25.7) + 0.5(25.7)^2] = \$772.3 \text{ million.}$$

Profit for Dimlit is

$$\pi_D = (52.90)(21.4) - [10(21.4) + 0.5(21.4)^2] = \$689.1 \text{ million.}$$

d. If the managers of the two companies collude, what are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

Because the firms are identical, they should split the market equally, so each produces $Q/2$ units, where Q is the total industry output. Each firm's total cost is therefore

$$C_i = 10\left(\frac{Q}{2}\right) + \frac{1}{2}\left(\frac{Q}{2}\right)^2,$$

and total industry cost is

$$TC = 2C_i = 10Q + \left(\frac{Q}{2}\right)^2.$$

Hence, industry marginal cost is

$$MC = 10 + 0.5Q.$$

With inverse industry demand given by $P = 100 - Q$, industry marginal revenue is

$$MR = 100 - 2Q.$$

Setting $MR = MC$, we have

$$100 - 2Q = 10 + 0.5Q, \text{ and so } Q = 36,$$

which means $Q_E = Q_D = Q/2 = 18$.

Substituting Q in the demand equation to determine price:

$$P = 100 - 36 = \$64.$$

The profit for each firm is equal to total revenue minus total cost:

$$\pi_i = 64(18) - [10(18) + 0.5(18)^2] = \$810 \text{ million.}$$

Note that you can also solve for the optimal quantities by treating the two firms as a monopolist with two plants. In that case, the optimal outputs satisfy the condition $MR = MC_E = MC_D$. Setting marginal revenue equal to each marginal cost function gives the following two equations:

$$MR = 100 - 2(Q_E + Q_D) = 10 + Q_E = MC_E$$

$$MR = 100 - 2(Q_E + Q_D) = 10 + Q_D = MC_D.$$

Solving simultaneously, we get the same solution as before; that is, $Q_E = Q_D = 18$.

10. Two firms produce luxury sheepskin auto seat covers, Western Where (WW) and B.B.B. Sheep (BBBS). Each firm has a cost function given by

$$C(q) = 30q + 1.5q^2$$

The market demand for these seat covers is represented by the inverse demand equation

$$P = 300 - 3Q$$

where $Q = q_1 + q_2$, total output.

a. If each firm acts to maximize its profits, taking its rival's output as given (i.e., the firms behave as Cournot oligopolists), what will be the equilibrium quantities selected by each firm? What is total output, and what is the market price? What are the profits for each firm?

Find the best response functions (the reaction curves) for both firms by setting marginal revenue equal to marginal cost (alternatively you can set up the profit function for each firm and differentiate with respect to the quantity produced for that firm):

$$R_1 = P\,q_1 = (300 - 3(q_1 + q_2))\,q_1 = 300q_1 - 3q_1^2 - 3q_1q_2.$$

$$MR_1 = 300 - 6q_1 - 3q_2$$

$$MC_1 = 30 + 3q_1$$

$$300 - 6q_1 - 3q_2 = 30 + 3q_1$$

$$q_1 = 30 - (1/3)q_2.$$

By symmetry, BBBS's best response function will be:

$$q_2 = 30 - (1/3)q_1.$$

Cournot equilibrium occurs at the intersection of these two best response functions, which is:

$$q_1 = q_2 = 22.5.$$

Thus,

$$Q = q_1 + q_2 = 45$$

$$P = 300 - 3(45) = \$165.$$

Profit for both firms will be equal and given by:

$$\pi = R - C = (165)(22.5) - [30(22.5) + 1.5(22.5^2)] = \$2278.13.$$

b. It occurs to the managers of WW and BBBS that they could do a lot better by colluding. If the two firms collude, what will be the profit-maximizing choice of output? The industry price? The output and the profit for each firm in this case?

In this case the firms should each produce half the quantity that maximizes total industry profits (i.e., half the monopoly output). Note that if the two firms had different cost functions, then it would not be optimal for them to split the monopoly output evenly.

Joint profits will be $(300 - 3Q)Q - 2[30(Q/2) + 1.5(Q/2)^2] = 270Q - 3.75Q^2$, which will be maximized at $Q = 36$. You can find this quantity by differentiating the profit function with respect to Q, setting the derivative equal to zero, and solving for Q: $d\pi/dQ = 270 - 7.5Q = 0$, so $Q = 36$.

The optimal output for each firm is $q_1 = q_2 = 36/2 = 18$, and the optimal price for the firms to charge is $P = 300 - 3(36) = \$192$.

Profit for each firm will be $\pi = (192)(18) - [30(18) + 1.5(18^2)] = \$2430$.

c. The managers of these firms realize that explicit agreements to collude are illegal. Each firm must decide on its own whether to produce the Cournot quantity or the cartel quantity. To aid in making the decision, the manager of WW constructs a payoff matrix like the one below. Fill in each box with the profit of WW and the profit of BBBS. Given this payoff matrix, what output strategy is each firm likely to pursue?

To fill in the payoff matrix, we have to calculate the profit each firm would make with each of the possible output level combinations. We already know the profits if both choose the Cournot output or both choose the cartel output. If WW produces the Cournot level of output (22.5) and BBBS produces the collusive level (18), then:

$$Q = q_1 + q_2 = 22.5 + 18 = 40.5$$

$$P = 300 - 3(40.5) = \$178.50.$$

$$\text{Profit for WW} = (178.5)(22.5) - [30(22.5) + 1.5(22.5^2)] = \$2581.88.$$

$$\text{Profit for BBBS} = (178.5)(18) - [30(18) + 1.5(18^2)] = \$2187.$$

If WW chooses the collusive output level and BBBS chooses the Cournot output, profits will be reversed. Rounding off profits to whole dollars, the payoff matrix is as follows.

Profit Payoff Matrix (WW profit, BBBS profit)		**BBBS** Produce Cournot q	**BBBS** Produce Cartel q
W	Produce Cournot q	2278, 2278	2582, 2187
W	Produce Cartel q	2187, 2582	2430, 2430

For each firm, the Cournot output dominates the cartel output, because each firm's profit is higher when it chooses the Cournot output, *regardless of the other firm's output*. For example, if WW chooses the Cournot output, BBBS earns \$2278 if it chooses the Cournot output but only \$2187 if it chooses the cartel output. On the other hand, if WW chooses the cartel output, BBBS earns \$2582 with the Cournot output, which is better than the \$2430 profit it would make with the cartel output. So no matter what WW chooses, BBBS is always better off choosing the Cournot output. Therefore, producing at the Cournot output levels will be the Nash Equilibrium in this industry.

This is a prisoners' dilemma game, because both firms would make greater profits if they both produced the cartel output. The cartel profit of \$2430 is greater than the Cournot profit of \$2278. The problem is that each firm has an incentive to cheat and produce the Cournot output instead of the cartel output. For example, if the firms are colluding and WW continues to produce the cartel output but BBBS increases output to the Cournot level, BBBS increases its profit from \$2430 to \$2582. When both firms do this, however, they wind up back at the Nash-Cournot equilibrium where each produces the Cournot output level and each makes a profit of only \$2278.

d. Suppose WW can set its output level *before* BBBS does. How much will WW choose to produce in this case? How much will BBBS produce? What is the market price, and what is the profit for each firm? Is WW better off by choosing its output first? Explain why or why not.

WW will use the Stackelberg strategy. WW knows that BBBS will choose a quantity q_2, which will be its best response to q_1 or:

$$q_2 = 30 - \frac{1}{3}q_1.$$

WW's profits will be:

$$\pi = Pq_1 - C_1 = (300 - 3q_1 - 3q_2)q_1 - (30q_1 + 1.5q_1^2)$$

$$\pi = Pq_1 - C_1 = (300 - 3q_1 - 3(30 - \frac{1}{3}q_1))q_1 - (30q_1 + 1.5q_1^2)$$

$$\pi = 180q_1 - 3.5q_1^2$$

Profit maximization implies:

$$\frac{d\pi}{dq_1} = 180 - 7q_1 = 0.$$

This results in $q_1 = 25.7$ and $q_2 = 21.4$. The equilibrium price and profits will be:

$$P = 300 - 3(q_1 + q_2) = 300 - 3(25.7 + 21.4) = \$158.70$$

$$\pi_1 = (158.70)(25.7) - [(30)(25.7) + 1.5(25.7)^2] = \$2316.86$$

$$\pi_2 = (158.70)(21.4) - [(30)(21.4) + 1.5(21.4)^2] = \$2067.24.$$

WW is able to benefit from its first-mover advantage by committing to a high level of output. Since BBBS moves after WW has selected its output, BBBS can only react to the output decision of WW. If WW produces its Cournot output as a leader, BBBS produces its Cournot output as a follower. Hence WW cannot do worse as a leader than it does in the Cournot game. When WW produces more, BBBS produces less, raising WW's profits.

11. Two firms compete by choosing price. Their demand functions are

$$\mathbf{Q_1 = 20 - P_1 + P_2 \quad \text{and} \quad Q_2 = 20 + P_1 - P_2}$$

where P_1 and P_2 are the prices charged by each firm, respectively, and Q_1 and Q_2 are the resulting demands. Note that the demand for each good depends only on the difference in prices; if the two firms colluded and set the same price, they could make that price as high as they wanted, and earn infinite profits. Marginal costs are zero.

a. Suppose the two firms set their prices at the *same time*. Find the resulting Nash equilibrium. What price will each firm charge, how much will it sell, and what will its profit be? (*Hint*: Maximize the profit of each firm with respect to its price.)

To determine the Nash equilibrium in prices, first calculate the reaction function for each firm, then solve for price. With zero marginal cost, profit for Firm 1 is:

$$\pi_1 = P_1Q_1 = P_1(20 - P_1 + P_2) = 20P_1 - P_1^2 + P_2P_1.$$

Marginal revenue is the slope of the total revenue function (here it is the derivative of the profit function with respect to P_1 because total cost is zero):

$$MR_1 = 20 - 2P_1 + P_2.$$

At the profit-maximizing price, $MR_1 = 0$. Therefore,

$$P_1 = \frac{20 + P_2}{2}.$$

This is Firm 1's reaction function. Because Firm 2 is symmetric to Firm 1, its reaction function is $P_2 = \frac{20 + P_1}{2}$. Substituting Firm 2's reaction function into that of Firm 1:

$$P_1 = \frac{20 + \frac{20 + P_1}{2}}{2} = 10 + 5 + \frac{P_1}{4}, \text{ so } P_1 = \$20$$

By symmetry, $P_2 = \$20$.

To determine the quantity produced by each firm, substitute P_1 and P_2 into the demand functions:

$$Q_1 = 20 - 20 + 20 = 20, \text{ and}$$

$$Q_2 = 20 + 20 - 20 = 20.$$

Profits for Firm 1 are $P_1Q_1 = \$400$, and, by symmetry, profits for Firm 2 are also \$400.

b. Suppose Firm 1 sets its price *first* and then Firm 2 sets its price. What price will each firm charge, how much will it sell, and what will its profit be?

If Firm 1 sets its price first, it takes Firm 2's reaction function into account. Firm 1's profit function is:

$$\pi_1 = P_1\left(20 - P_1 + \frac{20 + P_1}{2}\right) = 30P_1 - \frac{P_1^2}{2}.$$

To determine the profit-maximizing price, find the change in profit with respect to a change in price:

$$\frac{d\pi_1}{dP_1} = 30 - P_1.$$

Set this expression equal to zero to find the profit-maximizing price:

$$30 - P_1 = 0, \text{ or } P_1 = \$30.$$

Substitute P_1 in Firm 2's reaction function to find P_2:

$$P_2 = \frac{20 + 30}{2} = \$25.$$

At these prices,

$$Q_1 = 20 - 30 + 25 = 15, \text{ and}$$

$$Q_2 = 20 + 30 - 25 = 25.$$

Profits are

$$\pi_1 = (30)(15) = \$450 \text{ and}$$

$$\pi_2 = (25)(25) = \$625.$$

If Firm 1 must set its price first, Firm 2 is able to undercut Firm 1 and gain a larger market share. However, both firms make greater profits than they did in part a, where they chose prices simultaneously.

c. Suppose you are one of these firms and that there are three ways you could play the game: (i) Both firms set price at the same time; (ii) You set price first; or (iii) Your competitor sets price first. If you could choose among these options, which would you prefer? Explain why.

Compare the Nash profits in part a, \$400, with the profits in part b, \$450 for the firm that sets price first and \$625 for the follower. Clearly it is best to be the follower, so you should choose option (iii). From the reaction functions, we know that the price leader raises price and provokes a price increase by the follower. By being able to move second, however, the follower increases price by less than the leader, and hence undercuts the leader. Both firms enjoy increased profits, but the follower does better.

12. The dominant firm model can help us understand the behavior of some cartels. Let's apply this model to the OPEC oil cartel. We will use isoelastic curves to describe world demand *W* and noncartel (competitive) supply *S*. Reasonable numbers for the price elasticities of world demand and noncartel supply are −1/2 and 1/2, respectively. Then, expressing *W* and *S* in millions of barrels per day (*mb/d*), we could write

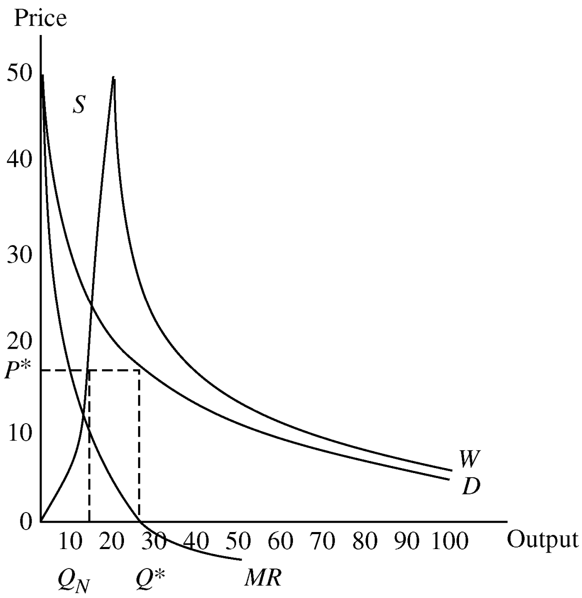
$$\boldsymbol{W = 160P^{-\frac{1}{2}}} \quad \textbf{and} \quad \boldsymbol{S = 3\frac{1}{3}P^{\frac{1}{2}}}.$$

Note that OPEC's net demand is *D* = *W* − *S*.

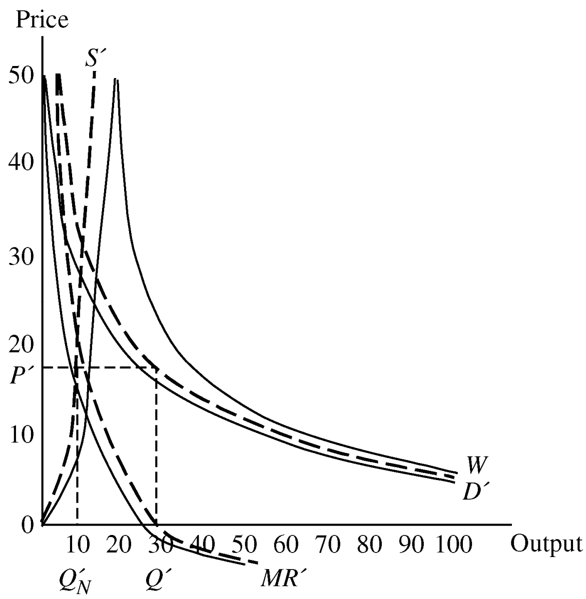
a. Draw the world demand curve *W*, the non-OPEC supply curve *S*, OPEC's net demand curve *D*, and OPEC's marginal revenue curve. For purposes of approximation, assume OPEC's production cost is zero. Indicate OPEC's optimal price, OPEC's optimal production, and non-OPEC production on the diagram. Now, show on the diagram how the various curves will shift and how OPEC's optimal price will change if non-OPEC supply becomes more expensive because reserves of oil start running out.

OPEC's initial net demand curve is $D = 160P^{-1/2} - 3\frac{1}{3}P^{1/2}$. Marginal revenue is quite difficult to find. If you were going to determine it analytically, you would have to solve OPEC's net demand curve for *P*. Then take that expression and multiply by *Q* (=*D*) to get total revenue as a function of output. Finally, you would take the derivative of revenue with respect to *Q*. The *MR* curve would look approximately like that shown in the figure below.

OPEC's optimal production, Q^*, occurs where $MR = 0$ (since production cost is assumed to be zero), and OPEC's optimal price, P^*, is found from the net demand curve at Q^*. Non-OPEC production, Q_N, can be read off the non-OPEC supply curve, S, at price P^*.



Now, if non-OPEC oil becomes more expensive, the supply curve S shifts to S'. This shifts OPEC's net demand curve outward from D to D', which in turn creates a new marginal revenue curve, MR', and a new optimal OPEC production level of Q', yielding a new higher price of P'. At this new price, non-OPEC production is Q_N'. The new S, D, and MR curves are dashed lines. Unfortunately, the diagram is difficult to sort out, but OPEC's new optimal output has increased to around 30, non-OPEC supply has dropped to about 10, and the optimal price has increased slightly.



b. Calculate OPEC's optimal (profit-maximizing) price. (*Hint*: Because OPEC's cost is zero, just write the expression for OPEC revenue and find the price that maximizes it.)

Since costs are zero, OPEC will choose a price that maximizes total revenue:

$$\text{Max } \pi = PQ = P(W - S)$$

$$\pi = P\left(160P^{-1/2} - 3\frac{1}{3}P^{1/2}\right) = 160P^{1/2} - 3\frac{1}{3}P^{3/2}.$$

To determine the profit-maximizing price, take the derivative of profit with respect to price and set it equal to zero:

$$\frac{\partial \pi}{\partial P} = 80P^{-1/2} - \left(3\frac{1}{3}\right)\left(\frac{3}{2}\right)P^{1/2} = 80P^{-1/2} - 5P^{1/2} = 0.$$

Solving for P,

$$5P^{1/2} = \frac{80}{P^{1/2}}, \text{ or } P = \$16.$$

At this price, $W = 40$, $S = 13.33$, and $D = 26.67$ as shown in the first diagram.

c. Suppose the oil-consuming countries were to unite and form a "buyers' cartel" to gain monopsony power. What can we say, and what can't we say, about the impact this action would have on price?

If the oil-consuming countries unite to form a buyers' cartel, then we have a monopoly (OPEC) facing a monopsony (the buyers' cartel). As a result, there are no well-defined demand or supply curves. We expect that the price will fall below the monopoly price when the buyers also collude, because monopsony power offsets some monopoly power. However, economic theory cannot determine the exact price that results from this bilateral monopoly because the price depends on the bargaining skills of the two parties, as well as on other factors such as the elasticities of supply and demand.

13. Suppose the market for tennis shoes has one dominant firm and five fringe firms. The market demand is $Q = 400 - 2P$. The dominant firm has a constant marginal cost of 20. The fringe firms each have a marginal cost of $MC = 20 + 5q$.

a. Verify that the total supply curve for the five fringe firms is $Q_f = P - 20$.

The total supply curve for the five firms is found by horizontally summing the five marginal cost curves, or in other words, adding up the quantity supplied by each firm for any given price. Rewrite each fringe firm's marginal cost curve as follows:

$$MC = 20 + 5q = P$$

$$5q = P - 20$$

$$q = \frac{P}{5} - 4$$

Since each firm is identical, the supply curve is five times the supply of one firm for any given price:

$$Q_f = 5\left(\frac{P}{5} - 4\right) = P - 20.$$

b. Find the dominant firm's demand curve.

The dominant firm's demand curve is given by the difference between the market demand and the fringe total supply curve:

$$Q_D = 400 - 2P - (P - 20) = 420 - 3P.$$

c. Find the profit-maximizing quantity produced and price charged by the dominant firm, and the quantity produced and price charged by each of the fringe firms.

The dominant firm will set marginal revenue equal to marginal cost. The marginal revenue curve has the same intercept and twice the slope of the linear inverse demand curve, which is shown below:

$$Q_D = 420 - 3P$$

$$P = 140 - \frac{1}{3}Q_D$$

$$MR = 140 - \frac{2}{3}Q_D.$$

Now set marginal revenue equal to marginal cost to find the profit-maximizing quantity for the dominant firm, and the price charged by the dominant firm:

$$MR = 140 - \frac{2}{3}Q_D = 20 = MC$$

$$Q_D = 180, \text{ and } P = \$80.$$

Each fringe firm will charge the same \$80 price as the dominant firm, and the total output produced by the five fringe firms will be $Q_f = P - 20 = 60.$ Each fringe firm will therefore produce 12 units.

d. Suppose there are ten fringe firms instead of five. How does this change your results?

We need to find the new fringe supply curve, dominant firm demand curve, and dominant firm marginal revenue curve as above. The new total fringe supply curve is $Q_f = 2P - 40.$ The new dominant firm demand curve is $Q_D = 440 - 4P.$ The new dominant firm marginal revenue curve is $MR = 110 - \frac{Q}{2}.$ The dominant firm will produce where marginal revenue is equal to marginal cost which occurs at 180 units. Substituting a quantity of 180 into the demand curve faced by the dominant firm results in a price of \$65. Substituting the price of \$65 into the total fringe supply curve results in a total fringe quantity supplied of 90, so that each fringe firm will produce 9 units. Increasing the number of fringe firms reduces market price from \$80 to \$65, increases total market output from 240 to 270 units, and reduces the market share of the dominant firm from 75% to 67% (although the dominant firm continues to sell 180 units).

e. Suppose there continue to be five fringe firms but that each manages to reduce its marginal cost to $MC = 20 + 2q$. How does this change your results?

Follow the same method as in earlier parts of this problem. Rewrite the fringe marginal cost curve as

$$q = \frac{P}{2} - 10.$$

The new total fringe supply curve is five times the individual fringe supply curve, which is also the fringe marginal cost curve:

$$Q_f = \frac{5}{2}P - 50.$$

The new dominant firm demand curve is found by subtracting the fringe supply curve from the market demand curve to get $Q_D = 450 - 4.5P$.

The new inverse demand curve for the dominant firm is therefore,

$$P = 100 - \frac{Q}{4.5}.$$

The dominant firm's new marginal revenue curve is

$$MR = 100 - \frac{2Q}{4.5}.$$

Set $MR = MC = 20$. The dominant firm will produce 180 units and will charge a price of

$$P = 100 - \frac{180}{4.5} = \$60.$$

Therefore, price drops from \$80 to \$60. The fringe firms will produce a total of $\frac{5}{2}(60) - 50 = 100$ units, so total industry output increases from 240 to 280. The market share of the dominant firm drops from 75% to 64%.

14. A lemon-growing cartel consists of four orchards. Their total cost functions are:

$$\boldsymbol{TC_1 = 20 + 5Q_1^2}$$
$$\boldsymbol{TC_2 = 25 + 3Q_2^2}$$
$$\boldsymbol{TC_3 = 15 + 4Q_3^2}$$
$$\boldsymbol{TC_4 = 20 + 6Q_4^2}$$

***TC* is in hundreds of dollars, and *Q* is in cartons per month picked and shipped.**

a. Tabulate total, average, and marginal costs for each firm for output levels between 1 and 5 cartons per month (i.e., for 1, 2, 3, 4, and 5 cartons).

The following tables give total, average, and marginal costs for each firm.

	Firm 1			Firm 2		
Units	***TC***	***AC***	***MC***	***TC***	***AC***	***MC***
0	20	—	—	25	—	—
1	25	25	5	28	28	3
2	40	20	15	37	18.5	9
3	65	21.67	25	52	17.33	15
4	100	25	35	73	18.25	21
5	145	29	45	100	20	27

	Firm 3			Firm 4		
Units	***TC***	***AC***	***MC***	***TC***	***AC***	***MC***
0	15	—	—	20	—	—
1	19	19	4	26	26	6
2	31	15.5	12	44	22	18
3	51	17	20	74	24.67	30
4	79	19.75	28	116	29	42
5	115	23	36	170	34	54

b. If the cartel decided to ship 10 cartons per month and set a price of $25 per carton, how should output be allocated among the firms?

The cartel should assign production such that the lowest marginal cost is achieved for each unit, i.e.,

Cartel Unit Assigned	Firm Assigned	*MC*
1	2	3
2	3	4
3	1	5
4	4	6
5	2	9
6	3	12
7	1	15
8	2	15
9	4	18
10	3	20

Therefore, Firms 1 and 4 produce two units each and Firms 2 and 3 produce three units each.

c. At this shipping level, which firm has the most incentive to cheat? Does any firm *not* have an incentive to cheat?

At this level of output, Firm 2 has the lowest marginal cost for producing one more unit beyond its allocation, i.e., $MC = 21$ for the fourth unit for Firm 2. In addition, $MC = 21$ is less than the price of $25. For all other firms, the next unit has a marginal cost equal to or greater than $25. Firm 2 has the most incentive to cheat, while Firms 3 and 4 have no incentive to cheat, and Firm 1 is indifferent.

Chapter 13
Game Theory and Competitive Strategy

■ Teaching Notes

Chapter 13 continues the discussion of strategic decisions begun in Chapter 12 using two-player games. Like Chapter 12, there are many different models presented, which students can find quite confusing. You will need to explain when each type of model is used and how they differ from each other. If you are pressed for time, you might want to cover only the first three or four sections. Section 13.2 reviews dominant strategies and 13.3 reviews and expands on the Nash equilibrium and related topics including multiple equilibria, the prisoners' dilemma, maximin strategies, and a brief discussion of mixed strategies. Sections 13.4 through 13.8 introduce advanced topics, such as repeated games, sequential games (including first-mover advantage, threats, commitments, credibility, bargaining, and entry deterrence), and auctions. The presentation throughout the chapter focuses on the intuition behind each model or strategy.

Two concepts pervade this chapter: rationality and equilibrium. Assuming the players are rational means that each player maximizes his or her own payoff whether it hurts or helps other players, and each player assumes the other players are also maximizing their payoffs. Rationality underlies many of the equilibria in the chapter and the Nash equilibrium is used extensively, so be sure students understand this equilibrium concept. If your students are having trouble finding the Nash equilibrium (or equilibria) in a game, try the following. Have them circle player 1's highest payoff for each of player 2's actions. This will identify player 1's best response for each of player 2's actions and will result in a circle around the highest payoff for player 1 in each column. Then use a box around player 2's highest payoff for each of player 1's actions. This will result in one box in each row signifying player 2's highest payoff in that row. Any cell in the payoff matrix that has both a circle and a box is then a Nash equilibrium in pure strategies.

When you cover the prisoners' dilemma model, be sure that students understand what we mean by a prisoners' dilemma. Some may think a prisoners' dilemma exists when a non-Nash pair of actions give a higher *total* payoff to the two players than the sum of the Nash payoffs. Be sure they understand that *each* player must receive a higher payoff with the non-Nash actions. If you want, you can use this opportunity to talk about the case where the sum of the non-Nash payoffs is greater than the sum of the Nash equilibrium payoffs and discuss side payments. If you are looking for further examples of prisoners' dilemma games, you could introduce the issue of common property resources such as fisheries. This topic will come up again in Chapter 18, but the coverage there does not make use of game theory.

The analysis in the last five sections of the chapter is more demanding, but the examples are more detailed. Section 13.4 examines repeated games, and it will be important to discuss the role of rationality in the achievement of an equilibrium in both finite and infinite-horizon games. Example 13.2 points out conditions that lead to stability in repeated games, while Example 13.3 presents an unstable case. Sections 13.5, 13.6, and 13.7 introduce strategy in the context of sequential games. To capture students' attention, discuss the phenomenal success of Wal-Mart in its attempt to preempt the entry of other discount stores in rural areas (see Example 13.4). Other strategies for deterring entry include the use of new capacity and R&D (see Examples 13.5 and 13.6).

First-mover advantage seems like a simple concept, but students often do not appreciate the subtleties of determining whether a first-mover advantage exists. For example in Table 13.10 on page 504, Firm 1 has a first-mover advantage, but some students will think that the firm should choose an output of 10 because it can earn a profit of 125 by doing so. They ignore the fact that Firm 2 also gets to make a decision and will not oblige Firm 1 by choosing to produce 7.5 units. Rather, Firm 2 will choose what is best for it, which is to produce 10 units, and Firm 1's profit will be 100, not 125. It is also useful to remind students that going first can be a disadvantage in many games, because it allows the other player to pick the action that is best against the first-mover's action. For example, in the children's game of rock, paper, scissors, if one player must go first he is sure to lose. Most sports games also do not favor first movers. Imagine in football announcing your play in advance to the defense.

The last section on auctions is one that usually holds a great deal of student interest. Most are not aware of the different types of auctions (especially Dutch and second-price auctions) or how prevalent auctions are. Students are familiar with eBay, however, so almost all have had some experience buying or selling in an auction. The winner's curse and collusion are favorite topics.

■ Review Questions

1. **What is the difference between a cooperative and a noncooperative game? Give an example of each.**

   In a noncooperative game the players do not formally communicate in an effort to coordinate their actions. They are aware of one another's existence, and typically know each other's payoffs, but they act independently. The primary difference between a cooperative and a noncooperative game is that binding contracts, i.e., agreements between the players to which both parties must adhere, is possible in the former, but not in the latter. An example of a cooperative game would be a formal cartel agreement, such as OPEC, or a joint venture. A noncooperative game example would be a research and development race to obtain a patent.

2. **What is a dominant strategy? Why is an equilibrium stable in dominant strategies?**

   A dominant strategy is one that is best no matter what action is taken by the other player in the game. When both players have dominant strategies, the outcome is stable because neither player has an incentive to change.

3. **Explain the meaning of a Nash equilibrium. How does it differ from an equilibrium in dominant strategies?**

   A Nash equilibrium is an outcome where both players correctly believe that they are doing the best they can, *given the action of the other player*. A game is in equilibrium if neither player has an incentive to change his or her choice, unless there is a change by the other player. The key feature that distinguishes a Nash equilibrium from an equilibrium in dominant strategies is the *dependence* on the opponent's behavior. An equilibrium in dominant strategies results if each player has a best choice, regardless of the other player's choice. Every dominant strategy equilibrium is a Nash equilibrium but the reverse does not hold.

4. **How does a Nash equilibrium differ from a game's maximin solution? When is a maximin solution a more likely outcome than a Nash equilibrium?**

   A maximin strategy is one in which a player determines the worst outcome that can occur for each of his or her possible actions. The player then chooses the action that maximizes the minimum gain that can be earned. If both players use maximin strategies, the result is a maximin solution to the game rather than a Nash equilibrium. Unlike the Nash equilibrium, the maximin solution does not require players to react to an opponent's choice. Using a maximin strategy is conservative and usually is not profit maximizing, but it can be a good choice if a player thinks his or her opponent may not behave rationally. The maximin solution is more likely than the Nash solution in cases where there is a higher probability of irrational (non-optimizing) behavior.

5. **What is a "tit-for-tat" strategy? Why is it a rational strategy for the infinitely repeated prisoners' dilemma?**

   A player following a tit-for-tat strategy will cooperate as long as his or her opponent is cooperating and will switch to a noncooperative action if the opponent stops cooperating. When the competitors assume that they will be repeating their interaction in *every* future period, the long-term gains from cooperating will outweigh any short-term gains from not cooperating. Because tit-for-tat encourages cooperation in infinitely repeated games, it is rational.

6. **Consider a game in which the prisoners' dilemma is repeated 10 times and both players are rational and fully informed. Is a tit-for-tat strategy optimal in this case? Under what conditions would such a strategy be optimal?**

   Since cooperation will unravel from the last period back to the first period, the tit-for-tat strategy is not optimal when there is a finite number of periods and both players anticipate the competitor's response in *every* period. Given that there is no response possible in the eleventh period for action in the tenth (and last) period, cooperation breaks down in the last period. Then, knowing that there is no cooperation in the last period, players should maximize their self-interest by not cooperating in the second-to-last period, and so on back to the first period. This unraveling occurs because both players assume that the other player has considered *all* consequences in *all* periods. However, if one player thinks the other may be playing tit-for-tat "blindly" (i.e., with limited rationality in the sense that he or she has not fully anticipated the consequences of the tit-for-tat strategy in the final period), then the tit-for-tat strategy can be optimal, and the rational player can reap higher payoffs during the first nine plays of the game and wait until the final period to earn the highest payoff by switching to the noncooperative action.

7. **Suppose you and your competitor are playing the pricing game shown in Table 13.8 (page 498). Both of you must announce your prices at the same time. Can you improve your outcome by promising your competitor that you will announce a high price?**

   If the game is to be played only a few times, there is little to gain. If you are Firm 1 and promise to announce a high price, Firm 2 will undercut you and you will end up with a payoff of −50. However, next period you will undercut too, and both firms will earn 10. If the game is played many times, there is a better chance that Firm 2 will realize that if it matches your high price, the long-term payoff of 50 each period is better than 100 in the first period and 10 in every period thereafter.

8. **What is meant by "first-mover advantage"? Give an example of a gaming situation with a first-mover advantage.**

   A first-mover advantage can occur in a game where the first player to act receives a higher payoff than he or she would have received with simultaneous moves by both players. The first-mover signals his or her choice to the opponent, and the opponent must choose a response, *given this signal*. The first-mover goes on the offensive and the second-mover responds defensively. In many recreational games, from chess to tic-tac-toe, the first-mover has an advantage. In many markets, the first firm to introduce a product can set the standard for competitors to follow. In some cases, the standard-setting power of the first mover becomes so pervasive in the market that the brand name of the product becomes synonymous with the product, e.g., "Kleenex," the name of Kleenex-brand facial tissue, is used by many consumers to refer to facial tissue of any brand.

9. What is a "strategic move"? How can the development of a certain kind of reputation be a strategic move?

A strategic move involves a commitment to reduce one's options. The strategic move might not seem rational outside the context of the game in which it is played, but it is rational given the anticipated response of the other player. Random responses to an opponent's action may not appear to be rational, but developing a reputation for being unpredictable could lead to higher payoffs in the long run. Another example would be making a promise to give a discount to all previous consumers if you give a discount to one. Such a move makes the firm vulnerable, but the goal of such a strategic move is to signal to rivals that you *won't* be discounting price and hope that your rivals follow suit.

10. Can the threat of a price war deter entry by potential competitors? What actions might a firm take to make this threat credible?

Both the incumbent and the potential entrant know that a price war will leave both worse off, so normally, such a threat is not credible. Thus, the incumbent must make his or her threat of a price war believable by signaling to the potential entrant that a price war *will* result if entry occurs. One strategic move is to increase capacity, signaling a lower future price. Even though this decreases current profit because of the additional fixed costs associated with the increased capacity, it can increase future profits by discouraging entry. Another possibility is to develop a reputation for starting price wars. Although the price wars will reduce profits, they may prevent future entry and hence increase future profits.

11. A strategic move limits one's flexibility and yet gives one an advantage. Why? How might a strategic move give one an advantage in bargaining?

A strategic move influences conditional behavior by the opponent. If the game is well understood, and the opponent's reaction can be predicted, a strategic move can give the player an advantage in bargaining. If a bargaining game is played only once (so no reputations are involved), one player might act strategically by committing to something unpleasant if he does not adhere to a bargaining position he has taken. For example, a potential car buyer might announce to the car dealer that he will pay no more than $20,000 for a particular car. To make this statement credible, the buyer might sign a contract promising to pay a friend $10,000 if he pays more than $20,000 for the car. If bargaining is repeated, players might act strategically to establish reputations for future negotiations.

12. Why is the winner's curse potentially a problem for a bidder in a common-value auction but not in a private-value auction?

The winner's curse occurs when the winner of a common-value auction pays more than the item is worth, because the winner was overly optimistic and, as a consequence, bid too high for the item. In a private-value auction, you know what the item is worth to you, i.e., you know your own reservation price, and will bid accordingly. Once the price exceeds your reservation price, you will no longer bid. If you win, it is because the winning bid was below your reservation price. In a common-value auction, however, you do not know the exact value of the good you are bidding on. Some bidders will overestimate and some will underestimate the value of the good, and the winner will tend to be the one who has most overestimated the good's value.

■ Exercises

1. **In many oligopolistic industries, the same firms compete over a long period of time, setting prices and observing each other's behavior repeatedly. Given the large number of repetitions, why don't collusive outcomes typically result?**

   First of all, collusion is illegal in most instances, so overt collusion is difficult and risky. However, if games are repeated indefinitely and all players know all payoffs, rational behavior can lead to apparently collusive outcomes, i.e., the same outcomes that would have resulted if the players had actively colluded. This may not happen in practice for a number of reasons. For one thing, all players might not know all payoffs. Sometimes the payoffs of other firms can only be known by engaging in extensive, possibly illegal, and costly information exchanges or by making moves and observing rivals' responses. Also, successful collusion (or a collusive-like outcome) encourages entry. Perhaps the greatest problem in maintaining a collusive outcome is that changes in market conditions change the optimal collusive price and quantity. Firms may not always agree on how the market has changed or what the best price and quantity are. This makes it difficult to coordinate decisions and increases the ability of one or more firms to cheat without being discovered.

2. **Many industries are often plagued by overcapacity: Firms simultaneously invest in capacity expansion, so that total capacity far exceeds demand. This happens not only in industries in which demand is highly volatile and unpredictable, but also in industries in which demand is fairly stable. What factors lead to overcapacity? Explain each briefly.**

   In Chapter 12, we found that excess capacity may arise in industries with easy entry and differentiated products. In the monopolistic competition model, downward-sloping demand curves for each firm lead to output with average cost above minimum average cost. The difference between the resulting output and the output at minimum long-run average cost is defined as excess capacity. In this chapter, we saw that overcapacity can be used to deter new entry; that is, investments in capacity expansion can convince potential competitors that entry would be unprofitable.

3. **Two computer firms, *A* and *B*, are planning to market network systems for office information management. Each firm can develop either a fast, high-quality system (High), or a slower, low-quality system (Low). Market research indicates that the resulting profits to each firm for the alternative strategies are given by the following payoff matrix:**

		Firm *B*	
		High	***Low***
Firm *A*	***High***	**50, 40**	**60, 45**
	Low	**55, 55**	**15, 20**

   a. **If both firms make their decisions at the same time and follow *maximin* (low-risk) strategies, what will the outcome be?**

      With a maximin strategy, a firm determines the worst outcome for each action, then chooses the action that maximizes the payoff among the worst outcomes. If Firm *A* chooses *High*, the worst payoff would occur if Firm *B* chooses *High*: *A*'s payoff would be 50. If Firm *A* chooses *Low*, the worst payoff would occur if Firm *B* chooses *Low*: *A*'s payoff would be 15. With a maximin strategy, *A* therefore chooses *High*. If Firm *B* chooses *Low*, the worst payoff would be 20, and if *B* chooses *High*, the worst payoff would be 40. With a maximin strategy, *B* therefore chooses *High*. So under maximin, both *A* and *B* produce a high-quality system.

b. Suppose that both firms try to maximize profits, but that Firm *A* has a head start in planning and can commit first. Now what will be the outcome? What will be the outcome if Firm *B* has the head start in planning and can commit first?

If Firm *A* can commit first, it will choose *High*, because it knows that Firm *B* will rationally choose *Low*, since *Low* gives a higher payoff to *B* (45 vs. 40). This gives Firm *A* a payoff of 60. If Firm *A* instead committed to *Low*, *B* would choose *High* (55 vs. 20), giving A 55 instead of 60. If Firm *B* can commit first, it will choose *High*, because it knows that Firm *A* will rationally choose *Low*, since *Low* gives a higher payoff to *A* (55 vs. 50). This gives Firm *B* a payoff of 55, which is the best it can do.

c. Getting a head start costs money. (You have to gear up a large engineering team.) Now consider the *two-stage* game in which, *first*, each firm decides how much money to spend to speed up its planning, and, *second*, it announces which product (*H* or *L*) it will produce. Which firm will spend more to speed up its planning? How much will it spend? Should the other firm spend *anything* to speed up its planning? Explain.

In this game, there is an advantage to being the first mover. If *A* moves first, its profit is 60. If it moves second, its profit is 55, a difference of 5. Thus, it would be willing to spend up to 5 for the option of announcing first. On the other hand, if *B* moves first, its profit is 55. If it moves second, its profit is 45, a difference of 10, and thus it would be willing to spend up to 10 for the option of announcing first.

If Firm *A* knows that Firm *B* is spending to speed up its planning, *A* should not spend anything to speed up its own planning. If Firm *A* also sped up its planning and both firms chose to produce the high-quality system, both would earn lower payoffs. Therefore, Firm *A* should not spend any money to speed up the introduction of its product. It should let *B* go first and earn 55 instead of 60.

4. Two firms are in the chocolate market. Each can choose to go for the high end of the market (high quality) or the low end (low quality). Resulting profits are given by the following payoff matrix:

		Firm 2: *Low*	Firm 2: *High*
Firm 1	*Low*	**–20, –30**	**900, 600**
	High	**100, 800**	**50, 50**

a. What outcomes, if any, are Nash equilibria?

A Nash equilibrium exists when neither party has an incentive to alter its strategy, taking the other's strategy as given. If Firm 2 chooses Low and Firm 1 chooses High, neither will have an incentive to change (100 > –20 for Firm 1 and 800 > 50 for Firm 2). Also, if Firm 2 chooses High and Firm 1 chooses Low, neither will have an incentive to change (900 > 50 for Firm 1 and 600 > –30 for Firm 2). Both outcomes are Nash equilibria. Both firms choosing Low, for example, is not a Nash equilibrium because if Firm 1 chooses Low then Firm 2 is better off by switching to High since 600 is greater than –30.

b. If the managers of both firms are conservative and each follows a maximin strategy, what will be the outcome?

If Firm 1 chooses Low, its worst payoff is –20, and if it chooses High, its worst payoff is 50. Therefore, with a conservative maximin strategy, Firm 1 chooses High. Similarly, if Firm 2 chooses Low, its worst payoff is −30, and if it chooses High, its worst payoff is 50. Therefore, Firm 2 chooses High. Thus both firms choose High, yielding a payoff of 50 for each.

c. What is the cooperative outcome?

The cooperative outcome would maximize *joint* payoffs. This would occur if Firm 1 goes for the low end of the market and Firm 2 goes for the high end of the market. The joint payoff is 1500 (Firm 1 gets 900 and Firm 2 gets 600).

d. Which firm benefits most from the cooperative outcome? How much would that firm need to offer the other to persuade it to collude?

Firm 1 benefits most from cooperation. The difference between its best payoff under cooperation and the next best payoff is 900 − 100 = 800. To persuade Firm 2 to choose Firm 1's best option, Firm 1 must offer at least the difference between Firm 2's payoff under cooperation, 600, and its best payoff, 800, i.e., 200. However, Firm 2 realizes that Firm 1 benefits much more from cooperation and will try to extract as much as it can from Firm 1 (up to 800).

5. Two major networks are competing for viewer ratings in the 8:00–9:00 PM and 9:00–10:00 PM slots on a given weeknight. Each has two shows to fill these time periods and is juggling its lineup. Each can choose to put its "bigger" show first or to place it second in the 9:00–10:00 PM slot. The combination of decisions leads to the following "ratings points" results:

		Network 2	
		First	**Second**
Network 1	**First**	**20, 30**	**18, 18**
	Second	**15, 15**	**30, 10**

a. Find the Nash equilibria for this game, assuming that both networks make their decisions at the same time.

A Nash equilibrium exists when neither party has an incentive to alter its strategy, taking the other's strategy as given. By inspecting each of the four combinations, we find that (First, First) is the only Nash equilibrium, yielding payoffs of (20, 30). There is no incentive for either network to change from this outcome. Suppose, instead, you thought (First, Second) was an equilibrium. Then Network 1 has an incentive to switch to Second (because 30 > 18), and Network 2 would want to switch to First (since 30 > 18), so (First, Second) cannot be an equilibrium.

b. If each network is risk averse and uses a maximin strategy, what will be the resulting equilibrium?

This conservative strategy of maximizing the minimum gain focuses on limiting the extent of the worst possible outcome. If Network 1 plays First, the worst payoff is 18. If Network 1 plays Second, the worst payoff is 15. Under maximin, Network 1 plays First. If Network 2 plays First, the worst payoff is 15. If Network 2 plays Second, the worst payoff is 10. So Network 2 plays First, which is a dominant strategy. The maximin equilibrium is (First, First) with a payoff of (20, 30): the same as the Nash equilibrium in this particular case.

c. What will be the equilibrium if Network 1 makes its selection first? If Network 2 goes first?

Network 2 will play First regardless of what Network 1 chooses and regardless of who goes first, because First is a dominant strategy for Network 2. Knowing this, Network 1 would play First if it could make its selection first, because 20 is greater than 15. If Network 2 goes first, it will play

First, its dominant strategy. So the outcome of the game is the same regardless of who goes first. The equilibrium is (First, First), which is the same as the Nash equilibrium, so there is no first-mover advantage in this game.

d. Suppose the network managers meet to coordinate schedules and Network 1 promises to schedule its big show first. Is this promise credible? What would be the likely outcome?

A move is credible if, once declared, *there is no incentive to change*. If Network 1 chooses First, then Network 2 will also choose First. This is the Nash equilibrium, so neither network would want to change its decision. Therefore, Network 1's promise is credible, although it is not very relevant since Network 2 will choose First no matter what Network 1 promises.

6. Two competing firms are each planning to introduce a new product. Each will decide whether to produce Product *A*, Product *B*, or Product *C*. They will make their choices at the same time. The resulting payoffs are shown below.

		Firm 2		
		A	***B***	***C***
	A	**–10, –10**	**0, 10**	**10, 20**
Firm 1	***B***	**10, 0**	**–20, –20**	**–5, 15**
	C	**20, 10**	**15, –5**	**–30, –30**

a. Are there any Nash equilibria in pure strategies? If so, what are they?

There are two Nash equilibria in pure strategies. Each one involves one firm introducing Product *A* and the other firm introducing Product *C*. We can write these two strategy pairs as (*A*, *C*) and (*C*, *A*), where the first strategy is for Firm 1. The payoffs for these two strategies are, respectively, (10, 20) and (20, 10).

b. If both firms use maximin strategies, what outcome will result?

Recall that maximin strategies maximize the minimum payoff for both players. If Firm 1 chooses *A*, the worst payoff is –10, with *B* the worst payoff is –20, and with *C* the worst is –30. So Firm 1 would choose *A* because –10 is better than the other two payoff amounts. The same reasoning applies for Firm 2. Thus (*A*, *A*) will result, and payoffs will be (–10, –10). Each player is much worse off than at either of the pure-strategy Nash equilibria.

c. If Firm 1 uses a maximin strategy and Firm 2 knows this, what will Firm 2 do?

If Firm 1 plays its maximin strategy of *A*, and Firm 2 knows this, then Firm 2 would get the highest payoff by playing *C*. Notice that when Firm 1 plays conservatively, the outcome that results gives Firm 2 the higher payoff of the two Nash equilibria.

7. We can think of U.S. and Japanese trade policies as a prisoners' dilemma. The two countries are considering policies to open or close their import markets. The payoff matrix is shown below.

		Japan	
		Open	**Close**
U.S.	**Open**	**10, 10**	**5, 5**
	Close	**–100, 5**	**1, 1**

a. Assume that each country knows the payoff matrix and believes that the other country will act in its own interest. Does either country have a dominant strategy? What will be the equilibrium policies if each country acts rationally to maximize its welfare?

Open is a dominant strategy for both countries. If Japan chooses Open, the U.S. does best by choosing Open. If Japan chooses Close, the U.S. does best by choosing Open. Therefore, the U.S. should choose Open, no matter what Japan does. If the U.S. chooses Open, Japan does best by choosing Open. If the U.S. chooses Close, Japan does best by choosing Open. Therefore, both countries will choose to have Open policies in equilibrium.

b. Now assume that Japan is not certain that the United States will behave rationally. In particular, Japan is concerned that U.S. politicians may want to penalize Japan even if that does not maximize U.S. welfare. How might this concern affect Japan's choice of strategy? How might this change the equilibrium?

The irrationality of U.S. politicians could change the equilibrium to (Close, Open). If the U.S. wants to penalize Japan they will choose Close, but Japan's strategy will not be affected since choosing Open is still Japan's dominant strategy.

8. You are a duopolist producer of a homogeneous good. Both you and your competitor have zero marginal costs. The market demand curve is

$$P = 30 - Q$$

where $Q = Q_1 + Q_2$. Q_1 is your output and Q_2 your competitor's output. Your competitor has also read this book.

a. Suppose you will play this game only once. If you and your competitor must announce your outputs at the same time, how much will you choose to produce? What do you expect your profit to be? Explain.

These are some of the cells in the payoff matrix, with profits rounded to dollars:

	Firm 2's Output						
Firm 1's Output	**0**	**2.5**	**5**	**7.5**	**10**	**12.5**	**15**
0	0,0	0,69	0,125	0,169	0,200	0,219	0,225
2.5	69,0	63,63	56,113	50,150	44,175	38,188	31,188
5	125,0	113,56	100,100	88,131	75,150	63,156	50,150
7.5	169,0	150,50	131,88	113,113	94,125	75,125	56,113
10	200,0	175,44	150,75	125,94	100,100	75,94	50,75
12.5	219,0	188,38	156,63	125,75	94,75	63,63	31,38
15	225,0	188,31	150,50	113,56	75,50	38,31	0,0

If both firms must announce output at the same time, both firms believe that the other firm is behaving rationally, and each firm treats the output of the other firm as a fixed number, a Cournot equilibrium will result.

For Firm 1, total revenue will be

$$TR_1 = (30 - (Q_1 + Q_2))Q_1, \text{ or } TR_1 = 30Q_1 - Q_1^2 - Q_1Q_2.$$

Marginal revenue for Firm 1 is the derivative of total revenue with respect to Q_1,

$$\frac{\partial TR}{\partial Q_1} = 30 - 2Q_1 - Q_2.$$

Because the firms are identical, marginal revenue for Firm 2 will be symmetric to that of Firm 1:

$$\frac{\partial TR}{\partial Q_2} = 30 - 2Q_2 - Q_1.$$

To find the profit-maximizing level of output for both firms, set marginal revenue equal to marginal cost, which is zero. The reaction functions are:

$$Q_1 = 15 - \frac{Q_2}{2} \text{ and}$$

$$Q_2 = 15 - \frac{Q_1}{2}.$$

With two equations and two unknowns, we may solve for Q_1 and Q_2:

$$Q_1 = 15 - (0.5)\left(15 - \frac{Q_1}{2}\right), \text{ or } Q_1 = 10, \text{ and by symmetry, } Q_2 = 10.$$

Substitute Q_1 and Q_2 into the demand equation to determine price:

$$P = 30 - (10 + 10), \text{ or } P = \$10.$$

Since no costs are given, profits for each firm will be equal to total revenue:

$$\pi_1 = TR_1 = (10)(10) = \$100 \text{ and}$$

$$\pi_2 = TR_2 = (10)(10) = \$100.$$

Thus, the equilibrium occurs when both firms produce 10 units of output and both firms earn \$100. Looking back at the payoff matrix, note that the outcome (100, 100) is indeed a Nash equilibrium: neither firm will have an incentive to deviate, given the other firm's choice.

b. Suppose you are told that you must announce your output *before* your competitor does. How much will you produce in this case, and how much do you think your competitor will produce? What do you expect your profit to be? Is announcing first an advantage or a disadvantage? Explain briefly. How much would you pay for the option of announcing either first or second?

If you must announce first, you would announce an output of 15, knowing that your competitor would announce an output of 7.5. (***Note*:** This is the Stackelberg equilibrium.)

$$TR_1 = (30 - (Q_1 + Q_2))Q_1 = 30Q_1 - Q_1^2 - Q_1\left(15 - \frac{Q_1}{2}\right) = 15Q_1 - \frac{Q_1^2}{2}.$$

Therefore, setting $MR = MC = 0$ implies:

$$15 - Q_1 = 0, \text{ or } Q_1 = 15, \text{ and}$$

$$Q_2 = 7.5.$$

You can check this solution using the payoff matrix above. If you announce a quantity of 15, the best Firm 2 can do is to produce 7.5 units. At that output, your competitor is maximizing profits, given that you are producing 15. At these outputs, price is equal to

$$P = 30 - 15 - 7.5 = \$7.50.$$

Your profit would be

$$(7.50)(15) = \$112.50.$$

Your competitor's profit would be

$$(7.50)(7.5) = \$56.25.$$

Announcing first is an advantage in this game. The difference in profits between announcing first and announcing second is \$56.25. You would be willing to pay up to this difference for the option of announcing first.

c. Suppose instead that you are to play the first round of a series of 10 rounds (with the same competitor). In each round, you and your competitor announce your outputs at the same time. You want to maximize the sum of your profits over the 10 rounds. How much will you produce in the first round? How much do you expect to produce in the tenth round? In the ninth round? Explain briefly.

Given that your competitor has also read this book, you can assume that he or she will be acting rationally. You should begin with the Cournot output (10 units) and continue with the Cournot output in each round, including the ninth and tenth rounds. Any deviation from this output will reduce the sum of your profits over the ten rounds.

d. Once again you will play a series of 10 rounds. This time, however, in each round your competitor will announce its output before you announce yours. How will your answers to c change in this case?

If your competitor always announces first, it might be more profitable to behave by reacting "irrationally" in a single period. For example, in the first round your competitor will announce an output of 15, as in b. Rationally, you would respond with an output of 7.5. If you behave this way in every round, your total profits for all ten rounds will be \$562.50. Your competitor's profits will be \$1125. However, if you respond with an output of 15 every time your competitor announces an output of 15, profits will be reduced to zero for both of you in that period. If your competitor fears, or learns, that you will respond in this way, he or she will be better off by choosing the Cournot output of 10, and your profits after that point will be \$75 per period (or \$100 if you also switch to the Cournot output). Whether this strategy is profitable depends on your opponent's expectations about your behavior, as well as how you value future profits relative to current profits.

(***Note*:** A problem could develop in the last period, however, because your competitor will know that you realize that there are no more long-term gains to be had from behaving strategically. Thus, your competitor will announce an output of 15, knowing that you will respond with an output of 7.5. Furthermore, knowing that you will not respond strategically in the last period, there are also no long-term gains to be made in the ninth period from behaving strategically. Therefore, in the ninth period, your competitor will announce an output of 15, and you should respond rationally with an output of 7.5, and so on.)

9. You play the following bargaining game. Player *A* moves first and makes Player *B* an offer for the division of \$100. (For example, Player *A* could suggest that she take \$60 and Player *B* take \$40.) Player *B* can accept or reject the offer. If he rejects it, the amount of money available drops to \$90, and he then makes an offer for the division of this amount. If Player *A* rejects this offer, the amount of money drops to \$80 and Player *A* makes an offer for its division. If Player *B* rejects this offer, the amount of money drops to 0. Both players are rational, fully informed, and want to maximize their payoffs. Which player will do best in this game?

Solve the game by starting at the end and working backwards. If *B* rejects *A*'s offer at the third round, *B* gets 0. When *A* makes an offer at the third round, *B* will accept even a minimal amount, such as \$1. So *A* should offer \$1 at this stage and take \$79 for herself. In the second stage, *B* knows that *A* will turn down any offer giving her less than \$79, so *B* must offer \$80 to A, leaving \$10 for *B*. At the first stage, *A* knows *B* will turn down any offer giving him less than \$10. So *A* can offer \$11 to *B* and keep \$89 for herself. *B* will take that offer, since *B* can never do any better by rejecting and waiting. The following table summarizes this. Player *A* does much better than *B*, because she goes first.

Round	Money Available	Offering Party	Amount to *A*	Amount to *B*
1	\$100	*A*	\$89	\$11
2	\$ 90	*B*	\$80	\$10
3	\$ 80	*A*	\$79	\$ 1
End	\$ 0		\$ 0	\$ 0

10. Defendo has decided to introduce a revolutionary video game. As the first firm in the market, it will have a monopoly position for at least some time. In deciding what type of manufacturing plant to build, it has the choice of two technologies. Technology *A* is publicly available and will result in annual costs of

$$\mathbf{C^A(q) = 10 + 8q}$$

Technology *B* is a proprietary technology developed in Defendo's research labs. It involves a higher fixed cost of production but lower marginal costs:

$$\mathbf{C^B(q) = 60 + 2q}$$

Defendo must decide which technology to adopt. Market demand for the new product is $P = 20 - Q$, where Q is total industry output.

a. Suppose Defendo were certain that it would maintain its monopoly position in the market for the entire product lifespan (about five years) without threat of entry. Which technology would you advise Defendo to adopt? What would be Defendo's profit given this choice?

Defendo has two choices: Technology *A* with a marginal cost of 8 and Technology *B* with a marginal cost of 2. Given the inverse demand curve is $P = 20 - Q$, total revenue, PQ, is equal to $20Q - Q^2$ for both technologies. Marginal revenue is $20 - 2Q$. To determine the profits for each technology, equate marginal revenue and marginal cost:

$$20 - 2Q_A = 8, \text{ or } Q_A = 6, \text{ and}$$

$$20 - 2Q_B = 2, \text{ or } Q_B = 9.$$

Substituting the profit-maximizing quantities into the demand equation to determine the profit-maximizing prices, we find:

$$P_A = 20 - 6 = \$14, \text{ and}$$

$$P_B = 20 - 9 = \$11.$$

To determine the profits for each technology, subtract total cost from total revenue:

$$\pi_A = (14)(6) - (10 + (8)(6)) = \$26, \text{ and}$$

$$\pi_B = (11)(9) - (60 + (2)(9)) = \$21.$$

To maximize profits, Defendo should choose Technology *A*, the publicly available option.

b. Suppose Defendo expects its archrival, Offendo, to consider entering the market shortly after Defendo introduces its new product. Offendo will have access only to Technology *A*. If Offendo does enter the market, the two firms will play a Cournot game (in quantities) and arrive at the Cournot-Nash equilibrium.

i. If Defendo adopts Technology *A* and Offendo enters the market, what will be the profit of each firm? Would Offendo choose to enter the market given these profits?

If both firms play Cournot, each will choose its best output, taking the other's strategy as given. Letting D = Defendo and O = Offendo, the demand function will be

$$P = 20 - Q_D - Q_O.$$

Profit for Defendo will be

$$\pi_D = (20 - Q_D - Q_O)Q_D - (10 + 8Q_D), \text{ or } \pi_D = 12Q_D - Q_D^2 - Q_DQ_O - 10$$

To determine the profit-maximizing quantity, set the first derivative of profits with respect to Q_D equal to zero and solve for Q_D:

$$\frac{\partial \pi_D}{\partial Q_D} = 12 - 2Q_D - Q_O = 0, \text{ or } Q_D = 6 - 0.5Q_O.$$

This is Defendo's reaction function. Because both firms use the same technology, Offendo's reaction function is analogous:

$$Q_O = 6 - 0.5Q_D.$$

Substituting Offendo's reaction function into Defendo's reaction function and solving for Q_D:

$$Q_D = 6 - (0.5)(6 - 0.5Q_D), \text{ or } Q_D = 4.$$

Substituting into Defendo's reaction function and solving for Q_O:

$$Q_O = 6 - (0.5)(4) = 4.$$

Total industry output is therefore 8 video games. To determine price, substitute Q_D and Q_O into the demand function:

$$P = 20 - 4 - 4 = \$12.$$

The profits for each firm are equal to total revenue minus total costs:

$$\pi_D = (12)(4) - (10 + (8)(4)) = \$6, \text{ and}$$

$$\pi_O = (12)(4) - (10 + (8)(4)) = \$6.$$

Therefore, Offendo would enter the market because it would make positive economic profits.

ii. If Defendo adopts Technology *B* and Offendo enters the market, what will be the profit of each firm? Would Offendo choose to enter the market given these profits?

Profit for Defendo will be

$$\pi_D = (20 - Q_D - Q_O)Q_D - (60 + 2Q_D), \text{ or } \pi_D = 18Q_D - Q_D^2 - Q_DQ_O - 60.$$

The change in profit with respect to Q_D is

$$\frac{\partial \pi_D}{\partial Q_D} = 18 - 2Q_D - Q_O$$

To determine the profit-maximizing quantity, set this derivative to zero and solve for Q_D:

$$18 - 2Q_D - Q_O = 0, \text{ or } Q_D = 9 - 0.5Q_O.$$

This is Defendo's reaction function. Substituting Offendo's reaction function (from part i above) into Defendo's reaction function and solving for Q_D:

$$Q_D = 9 - 0.5(6 - 0.5Q_D), \text{ or } Q_D = 8.$$

Substituting Q_D into Offendo's reaction function yields

$$Q_O = 6 - (0.5)(8), \text{ or } Q_O = 2.$$

To determine the industry price, substitute the profit-maximizing quantities for Defendo and Offendo into the demand function:

$$P = 20 - 8 - 2 = \$10.$$

The profit for each firm is equal to total revenue minus total cost, or:

$$\pi_D = (10)(8) - (60 + (2)(8)) = \$4, \text{ and}$$

$$\pi_O = (10)(2) - (10 + (8)(2)) = -\$6.$$

With negative profit, Offendo *would not* enter the industry.

iii. Which technology would you advise Defendo to adopt given the threat of possible entry? What will be Defendo's profit given this choice? What will be consumer surplus given this choice?

With Technology *A* and Offendo's entry, Defendo's profit would be \$6. With Technology *B* and no entry by Defendo, Defendo's profit would be \$4. I would advise Defendo to stick with Technology *A* and let Offendo enter the market. Under this advice, total output is 8 and price is \$12. Consumer surplus is

$$(0.5)(8)(20 - 12) = \$32.$$

c. What happens to social welfare (the sum of consumer surplus and producer profit) as a result of the threat of entry in this market? What happens to equilibrium price? What might this imply about the role of *potential* competition in limiting market power?

From part a we know that, under monopoly, $Q = 6$, $P = \$14$, and profit is \$26. Consumer surplus is

$$(0.5)(6)(20 - 14) = \$18.$$

Social welfare is defined here as the sum of consumer surplus plus profits, or

$$\$18 + 26 = \$44.$$

With entry, $Q = 8$, $P = \$12$, and profits sum to \$12. Consumer surplus is

$$(0.5)(8)(20 - 12) = \$32.$$

Social welfare is \$44 – equal to \$32 (consumer surplus) plus \$12 (industry profit). Social welfare does not change with entry, but entry shifts surplus from producers to consumers. The equilibrium price falls with entry, and therefore *potential* competition can *limit* market power.

Note that Defendo has one other option: to increase quantity from the monopoly level of 6 to discourage entry by Offendo. If Defendo increases output from 6 to 8 under Technology *A*, Offendo is unable to earn a positive profit. With an output of 8, Defendo's profit decreases from \$26 to

$$(8)(12) - (10 + (8)(8)) = \$22.$$

As before, with an output of 8, consumer surplus is \$32; social welfare is \$54. In this case, social welfare rises when output is increased to discourage entry.

11. Three contestants, *A*, *B*, and *C*, each has a balloon and a pistol. From fixed positions, they fire at each other's balloons. When a balloon is hit, its owner is out. When only one balloon remains, its owner gets a \$1000 prize. At the outset, the players decide by lot the order in which they will fire, and each player can choose any remaining balloon as his target. Everyone knows that *A* is the best shot and always hits the target, that *B* hits the target with probability 0.9, and that *C* hits the target with probability 0.8. Which contestant has the highest probability of winning the \$1000? Explain why.

Surprisingly, *C* has the highest probability of winning, even though both *A* and *B* are better shots. The reason is that each contestant wants to remove the shooter with the highest probability of success. By following this strategy, each improves his chance of winning the game. *A* targets *B* because by removing *B* from the game, *A*'s chance of winning becomes greater. *B* will target *A* because if *B* targets *C* instead and hits *C*, then *A* will hit *B*'s balloon for sure and win the game. *C* will follow a similar strategy, because if *C* targets *B* and hits *B*, then *A* will hit *C*'s balloon and win the game. Therefore, both *B* and *C* increase their chance of winning by eliminating *A* first. Similarly, *A* increases his chance of winning by eliminating *B* first. No one shoots at *C* first. A complete probability tree can be constructed to show that *A*'s chance of winning is 8%, *B*'s chance of winning is 32%, and *C*'s chance of winning is 60%.

But wait, *C* can actually do better than this by intentionally missing his shot if both *A* and *B* are still in the game. Suppose *C* goes first followed by *A* and *B*. If *C* hits *A*'s balloon, then *B* will go next and has a 90% chance of hitting *C*'s balloon, so *C*'s chance of winning is small. But if *C* misses *A*'s balloon, intentionally or otherwise, then *A* shoots next and will knock out *B* for sure. Then it will be *C*'s turn again, and he will have an 80% chance of hitting *A*'s balloon and winning the game. A complete probability tree using this strategy shows that *A*'s chance of winning increases slightly to 11%, *B*'s chance of winning drops dramatically to 8%, and *C*'s probability of winning jumps up to 81%.

12. An antique dealer regularly buys objects at hometown auctions whose bidders are limited to other dealers. Most of her successful bids turn out to be financially worthwhile because she is able to resell the antiques for a profit. On occasion, however, she travels to a nearby town to bid in an auction that is open to the public. She often finds that on the rare occasions in which she does bid successfully, she is disappointed—the antique cannot be sold at a profit. Can you explain the difference in her success between the two sets of circumstances?

When she bids at the hometown auction that is limited to other dealers, she is bidding against people who are all going to resell the antique if they win the bid. In this case, all the bidders are limiting their bids to prices that will tend to earn them a profit.

A rational dealer will not place a bid that is higher than the price she or he can expect to resell the antique for. Given that all dealers are rational, the winning bid will tend to be below the expected resale price.

When she bids in the auction that is open to the public, however, she is bidding against the people who are likely to come into her shop. You can assume that local antique lovers will frequent these auctions as well as the local antique shops. In the case where she wins the bid at one of these open auctions, the other participants have decided that the price is too high. In this case, they will not come into her shop and pay any higher price which would earn her a profit. She will only tend to profit in this case if she is able to resell to a customer from out of the area, or who was not at the auction, and who has a sufficiently high reservation price. In any event, the winning bid price will tend to be higher because she is bidding against customers rather than dealers, and when she wins, there is a good chance she has overestimated the value of the antique. This is an example of the winner's curse.

13. You are in the market for a new house and have decided to bid for a house at auction. You believe that the value of the house is between $125,000 and $150,000, but you are uncertain as to where in the range it might be. You do know, however, that the seller has reserved the right to withdraw the house from the market if the winning bid is not satisfactory.

a. Should you bid in this auction? Why or why not?

Yes you should bid if you are confident about your estimate of the value of the house and/or if you allow for the possibility of being wrong so as to avoid the winner's curse. To allow for the possibility of being wrong, you should reduce your high bid by an amount equal to the expected error of the winning bidder. If you have experience at auctions, you will have information on how likely you are to enter a wrong bid and can then adjust your high bid accordingly.

b. Suppose you are a building contractor. You plan to improve the house and then to resell it at a profit. How does this situation affect your answer to part a? Does it depend on the extent to which your skills are uniquely suitable to improving this particular house?

In addition to the winner's curse, there is another important issue you need to consider. In part a you were buying the house for your own use and were bidding against other similar buyers. In this case, you are buying the house to resell it, but you are bidding against others who want to buy the house to live in. These are the same people who will be the buyers to whom you will want to resell the house after you have made improvements to it. So if you win the bidding, you will have paid more for the house than these people were willing to pay. This is not an encouraging start to your venture. The only way you will make a profit, then, is if you can do the improvements more efficiently than the typical buyer can do them (either by paying to have the work done or by doing the work themselves). Thus, the more you are uniquely qualified for making the improvements, the more you can pay for the house and still make a profit.

Chapter 14
Markets for Factor Inputs

■ Teaching Notes

Chapters 14 and 15 examine the markets for labor and capital. Although the discussion in this chapter is general, most of the examples refer to labor as the only variable input to production, with the exception of Example 14.1, which discusses "The Demand for Jet Fuel" by airlines. Section 14.1 covers demand and supply in a competitive factor market, and Section 14.2 discusses the competitive factor market equilibrium and economic rent. Section 14.3 explores factor markets when the buyer has monopsony power, and Section 14.4 discusses the case of monopoly power on the part of the seller of the factor.

An understanding of this chapter relies on concepts from Chapters 4, 6 through 8, and 10. If you have just covered Chapters 11–13, you might begin by reviewing marginal product, marginal revenue, and cost minimization. You should then discuss marginal revenue product and the profit-maximizing condition, $MRP_L = w$. Explain why we are interested only in the portion of the *MP* curve below the average product curve (the downward-sloping portion). The derivation of the firm's demand curve for labor is straightforward when labor is the only factor but becomes more complicated when there are several variable inputs. In particular, you might explain why the MRP_L curve shifts as the firm substitutes one input for another in production in response to a price change by noting that the MRP_L curve is drawn for a fixed level of the other variable inputs.

When presenting the market labor demand curve, explain that since the input prices change as more inputs are demanded, the market demand curve is not simply the summation of individual demand curves. You can extend the presentation of price elasticity of input demand (see Example 14.1) by discussing the conditions leading to price sensitivity. Elasticity is greater (1) when the elasticity of demand for the product is higher, (2) when it is easy to substitute one input for another, and (3) when the elasticity of supply is higher for other inputs. Elasticity of supply, which was discussed in Chapter 2, is reintroduced in Example 14.2. You should also distinguish between short-run and long-run elasticity (see Figure 14.6).

If you have already covered substitution and income effects, your students will be ready for the derivation of the backward-bending supply curve for labor. Although Figure 14.9 is a straightforward application of these tools, students are often confused by the plotting of income against leisure. Point out that this is just another type of utility maximization problem where the two goods are leisure and income. Income can be thought of as the consumption of goods other than leisure, in that more income buys more goods. You can also implicitly assume that the price of other goods is $1 and the price of leisure is the wage. The supply of labor curve is derived by changing the wage and finding the new level of hours worked. An individual's supply curve of labor is backward bending only when the income effect dominates the substitution effect and leisure is a normal good. Show typical supply curves for each group in Table 14.2. For an experimental study of the labor-leisure trade-off see Battalio, Green, and Kagel, "Income-Leisure Tradeoff of Animal Workers," *American Economic Review* (September 1981).

Section 14.2 brings together labor demand and supply for both competitive and monopolistic product markets. Although economic rent was initially presented in Chapter 8, it is reintroduced with more detail here. In Section 14.3, carefully explain why the marginal expenditure curve is above the average expenditure curve for a monopsonist (see Figure 14.14). You can discuss how a monopsonist would price discriminate, e.g., pay a different wage rate to each employee. With perfect price discrimination, the marginal expenditure curve would coincide with the average expenditure curve. Although monopsony exists in some markets, the exercise of monopsony power is rare because of factor mobility. However, the employment of athletes by the owners of professional teams provides a good example (see Example 14.4, "Monopsony Power in the Market for Baseball Players"). On this same topic, see Sommers and Quinton, "Pay and Performance in Major League Baseball: The Case of the First Family of Free Agents," *Journal of Human Resources* (Summer 1982). Section 14.4 discusses labor unions to explore monopoly power on the part of the seller of the input.

■ Review Questions

1. **Why is a firm's demand for labor curve more inelastic when the firm has monopoly power in the output market than when the firm is producing competitively?**

   The firm's demand curve for labor is determined by the incremental revenue from hiring an additional unit of labor, known as the marginal revenue product of labor. $MRP_L = (MP_L)(MR)$ is the additional output ("product") that the last worker produced, times the additional revenue earned by selling that output. In a competitive industry, the marginal revenue curve is perfectly elastic and equal to price. For a monopolist, marginal revenue is downward sloping. As more labor is hired and more output is produced, the monopolist will charge a lower price and marginal revenue will diminish. All else the same, marginal revenue product will therefore fall more quickly for the monopolist. This implies that the marginal revenue product curve (the demand curve for labor) will be steeper for the monopolist and hence more inelastic than for the competitive firm.

2. **Why might a labor supply curve be backward bending?**

   A backward-bending supply curve for labor may occur when the income effect of an increase in the wage rate dominates the substitution effect. Individuals make labor supply decisions by choosing the most satisfying combination of income (with which to consume goods and services) and leisure. With a larger wage, the individual can afford to work fewer hours: the income effect. But as the wage rate increases, the value of leisure time (the opportunity cost of leisure) increases, thus inducing the individual to consume less leisure and work longer hours: the substitution effect. Because the two effects work in opposite directions, the labor supply curve is backward bending if the income effect triggered by a higher wage is greater than the substitution effect of the higher wage.

3. **How is a computer company's demand for computer programmers a derived demand?**

   A computer company's demand for inputs, including programmers, depends on how many computers it sells. The firm's demand for programming labor depends on (is derived from) the demand it faces in its market for computers. As demand for computers shifts, the demand for programmers shifts.

4. **Compare the hiring choices of a monopsonistic and a competitive employer of workers. Which will hire more workers, and which will pay the higher wage? Explain.**

   Since the decision to hire another worker means the monopsonist must pay a higher wage to *all* workers, and not just to the last worker hired, its marginal expenditure curve lies above the input supply curve (the average expenditure curve). The monopsonist's profit-maximizing input demand, where the marginal expenditure curve intersects the marginal revenue product curve, will be less than the competitor's profit-maximizing input choice, where the average expenditure curve intersects the demand curve, assuming that both firms have the same demand curve for labor. Therefore, the monopsonist hires less labor, and the wage paid will be less than in a competitive market.

5. **Rock musicians sometimes earn several million dollars per year. Can you explain such large incomes in terms of economic rent?**

   Economic rent is the difference between the actual payment to the factor of production and the minimum amount that the factor is willing to accept. In this case, you might assume that there are a limited number of top-quality rock musicians who will continue to play rock music almost no matter what they are paid. This results in a perfectly inelastic supply curve, or something close to it. Given the high demand for rock music, the wage will be very high and there will be a lot of economic rent. If there was a larger supply of top-quality rock musicians, or a more elastic supply, then the economic rent would be smaller.

6. **What happens to the demand for one input when the use of a complementary input increases?**

   If the demand for the complementary input increases, the demand for the given input will increase as well. When demand for the complementary input increases, there is an increase in the quantity hired and possibly the price paid. Both of these changes will increase the *MRP* of the given input, and hence will increase the quantity hired and possibly the price paid. Whether the prices of the inputs increase depends on the degree of monopsony power of the input buyers.

7. **For a monopsonist, what is the relationship between the supply of an input and the marginal expenditure on it?**

   The decision to increase employment means the monopsonist must pay *all* units the higher price, and not just the last unit hired. Therefore, its marginal expenditure curve lies above the input supply curve (the average expenditure curve). Hiring more labor will increase the marginal expenditure, which will increase the average expenditure. If the average expenditure is increasing, then the marginal expenditure must be greater than the average expenditure.

8. **Currently the National Football League has a system for drafting college players by which each player is picked by only one team. The player must sign with that team or not play in the league. What would happen to the wages of both newly drafted and more experienced football players if the draft system were repealed and all teams could compete for college players?**

   The National Football League draft and reserve clause creates a monopsonist cartel among the owners of NFL teams. If the draft system were repealed, competition among teams would increase wages of new football players to the point where the marginal revenue product of each player would be equal to the player's wage. It might not have much effect on experienced players because they already have the right to become free agents and sell their services to the team willing to pay the most.

9. **The government wants to encourage individuals on welfare to become employed. It is considering two possible incentive programs:**

   a. **Give firms $2 per hour for every individual on welfare who is hired.**

   b. **Give each firm that hires one or more welfare workers a payment of $1000 per year, irrespective of the number of hires.**

To what extent is each of these programs likely to be effective at increasing the employment opportunities for welfare workers?

Firms will hire additional labor as long as the extra benefit is greater than the extra cost of hiring an additional worker, or until $MRP_L = w$. Option a would be effective because if the firm receives \$2 per hour for every welfare worker hired, then the effective wage paid falls to $w - 2$ and the firm will find it optimal to hire more labor until the benefits (MRP_L) again equal the costs ($w - 2$) at the margin. Option b would also increase employment among welfare workers. However, it is not likely to be as effective as option a because the firm receives one lump sum payment regardless of the number of welfare workers hired. In this case the firm has an incentive to hire only one welfare worker.

10. **A small specialty cookie company whose only variable input is labor finds that the average worker can produce 50 cookies per day, the cost of the average worker is \$64 per day, and the price of a cookie is \$1. Is the company maximizing its profit? Explain.**

The marginal product of labor is 50 (cookies per day) and the cookie price is \$1 (per cookie) so the marginal revenue product is \$50/day. Since this is less than the wage of \$64 per day the cookie company is not maximizing profit. They are employing too much labor since the cost of labor (\$64) is greater than the benefit of labor (\$50) at the margin, and they are therefore also producing too many cookies.

11. **A firm uses both labor and machines in production. Explain why an increase in the average wage rate causes both a movement along the labor demand curve and a shift of the curve.**

An increase in the wage rate causes a movement up and to the left along the labor demand curve (the marginal revenue product curve), because the firm will want to hire fewer workers when the wage increases. However, when the wage increases, the marginal cost of producing the firm's product increases, and the firm will reduce output. When output falls, the firm will demand fewer machines, and the reduction in number of machines used will cause the marginal product of labor curve to shift to the left, assuming machines and labor are complementary. This reduces the demand for labor (shifts MRP_L to the left) and causes the firm to hire even less labor at the new wage rate.

■ Exercises

1. **Suppose that the wage rate is \$16 per hour and the price of the product is \$2. Values for output and labor are in units per hour.**

q	*L*
0	0
20	1
35	2
47	3
57	4
65	5
70	6

a. Find the profit-maximizing quantity of labor.

From the information given above, calculate the marginal product of labor (the extra output produced by hiring one more unit of labor) and then multiply by price to get the marginal revenue product of labor. To maximize profit, the firm should hire labor only as long as the marginal revenue product of labor is greater than or equal to the wage of $16. From the table below, the firm will hire 5 units of labor per hour.

q	L	MP_L	MRP_L
0	0	—	—
20	1	20	$40
35	2	15	$30
47	3	12	$24
57	4	10	$20
65	5	8	$16
70	6	5	$10

b. Suppose that the price of the product remains at $2 but that the wage rate increases to $21. Find the new profit-maximizing level of *L*.

The above table does not change for this part of the problem. However, the firm no longer wants to hire 5 units of labor because the benefit of the 5th unit ($16 per hour) is less than the cost of the 5th unit ($21 per hour). The firm would hire only 3 units of labor per hour since the benefit exceeds the cost at this level. The firm would not hire 4 units because the cost ($21) is greater than the benefit ($20). If the firm could hire fractional units of labor, some amount between 3 and 4 units per hour would be optimal.

c. Suppose that the price of the product increases to $3 and the wage remains at $16 per hour. Find the new profit-maximizing *L*.

A change in the price of the product will not change the marginal product of labor, but it will change the marginal revenue product of labor. The new marginal revenue product of labor is given in the table below. The firm will still want to hire 5 units of labor, as in part a above. It will not hire the 6th unit because the extra benefit ($15) is less than the extra cost ($16). Profit will be greater than in part a.

q	L	MP_L	MRP_L
0	0	—	—
20	1	20	60
35	2	15	45
47	3	12	36
57	4	10	30
65	5	8	24
70	6	5	15

d. Suppose that the price of the product remains at \$2 and the wage at \$16, but that there is a technological breakthrough that increases output by 25% for any given level of labor. Find the new profit-maximizing *L*.

The technological breakthrough changes the number of units of output produced by any given number of units of labor, and hence changes the marginal product and the marginal revenue product of labor. The new output values are found by multiplying the old values by 1.25. This new information is given in the table below. The firm will still choose to hire 5 units of labor because $MRP_L > w$ when $L = 5$ while $MRP_L < w$ when $L = 6$. Profit will be greater than in part a.

q	*L*	MP_L	MRP_L
0	0	—	—
25	1	25	50
43.75	2	18.75	37.5
58.75	3	15	30
71.25	4	12.5	25
81.25	5	10	20
87.5	6	6.25	12.5

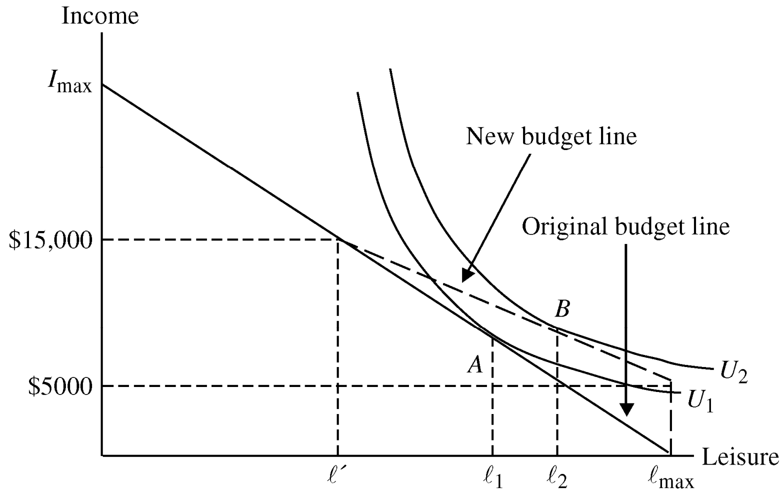
2. Assume that workers whose incomes are less than \$10,000 currently pay no federal income taxes. Suppose a new government program guarantees each worker \$5000, whether or not he or she earns any income. For all earned income up to \$10,000, the worker must pay a 50% tax. Draw the budget line facing the worker under this new program. How is the program likely to affect the labor supply curve of workers?

Initially the first \$10,000 of income was untaxed. We are not told what the tax rate was for incomes above \$10,000, so let's assume it was zero to make things simple.

With the new program, everyone gets \$5000 regardless of earned income, and this \$5000 is not taxed. However, the first \$10,000 of earned income is taxed at the 50% rate. Again, for simplicity, assume that earned income above \$10,000 is untaxed.

The worker's original income-leisure budget line before the new program is shown as the solid line in the diagram below. With the new program, the budget line (dashed line) shifts up by the \$5000 government grant when the worker does no work at all and takes the maximum amount of leisure, ℓ_{max}. As the number of hours worked increases (i.e., leisure decreases), the budget line has half the slope of the original line because earned income is taxed at 50%. When the worker earns \$10,000 (so that total income is \$15,000 including the \$5000 grant), the new budget line coincides with the original line. This occurs at leisure level ℓ' in the diagram below. For levels of leisure below that (i.e., for a greater number of hours worked) the two budget lines are identical. The result is that the new program will have no effect if the worker originally earned more than \$15,000 per year, but will probably reduce the amount of time worked (i.e., increase leisure) if the worker earned less than \$15,000.

To see the effect of the new program, draw indifference curves that originally lead the worker to choose more than ℓ' of leisure. Indifference curve U_1 in the diagram is tangent to the original budget line at point *A*, and the worker chooses ℓ_1 hours of leisure. After the new program goes into effect, the worker's utility increases to U_2, which is tangent to the new budget line at point *B* where the worker chooses ℓ_2 hours of leisure. The new program induces the worker to take more hours of leisure and hence to work fewer hours. This is what we would expect, because the $5000 grant triggers an income effect, which causes the worker to work fewer hours. In addition, by taxing earned income, the effective wage rate is cut in half. This reduces the cost of leisure, and leads the worker to substitute leisure for income, and thus work fewer hours. So both the income and substitution effects lead to less work.



3. Using your knowledge of marginal revenue product, explain the following:

a. A famous tennis star is paid $200,000 for appearing in a 30-second television commercial. The actor who plays his doubles partner is paid $500.

Marginal revenue product of labor, MRP_L, is equal to marginal revenue from an incremental unit of output multiplied by the marginal product from an incremental unit of labor, or in other words, the extra revenue generated by having the tennis star appear in the ad. The famous tennis star is able to help increase revenues far more than the actor, so he is paid much more than the actor. The wage of the actor is determined by the supply and demand of actors willing to play tennis with tennis stars.

b. The president of an ailing savings and loan is paid *not* to stay in his job for the last two years of his contract.

The marginal revenue product of the president of the ailing savings and loan is either low or negative, and therefore the savings and loan is better off by paying the president not to show up. They have calculated that they will lose less (or gain more) by paying the president to leave and hiring someone else whose *MRP* is much greater.

c. A jumbo jet carrying 400 passengers is priced higher than a 250-passenger model even though both aircraft cost the same to manufacture.

The marginal product of the jumbo jet is 400 while that of the smaller jet is only 250. This means the jumbo jet's *MRP* is greater, so it will generate more revenue per flight than the smaller jet. This makes the jumbo jet more valuable to the airline, and therefore the airline is willing to pay more for it.

4. **The demands for the factors of production listed below have increased. What can you conclude about changes in the demands for the related consumer goods? If demands for the consumer goods remain unchanged, what other explanation is there for an increase in derived demands for these items?**

   a. **Computer memory chips**

   In general, an increase in the demand for a good increases the demand for its factor inputs. The converse is not necessarily true; i.e., an increase in the demand for factor inputs does not necessarily mean there was an increase in the demand for the final product. The demand for an input may increase due to a change in the use of other inputs in the production process. As the price of another input increases, the quantity demanded of that input falls, and the demand for substitutable inputs rises.

   In this case, the increase in demand for computer memory chips was probably caused by an increase in the demand for computers, tablets, and other such devices, assuming that computer memory chips are used only in these products and that there are no substitutes for the memory chips. If the demand for the final products did not change, then perhaps the prices of other inputs fell, making it less costly to produce the products. This would cause the supply of the products to increase, which would result in a drop in their prices and an increase in the quantity sold. With an increase in sales, the demand for all inputs, including memory chips, would increase.

   b. **Jet fuel for passenger planes**

   There are no substitutes for jet fuel, so the increase in demand for jet fuel most likely was caused by an increase in the demand for jet travel. Another possibility would be if other input prices, such as wages of airline employees, decreased. This would reduce the cost of flying and lead airlines to supply more flights that would, in turn, increase the demand for jet fuel.

   c. **Paper used for newsprint**

   Given that the paper is being used to print newspapers, it seems likely that there was an increase in the demand for newspapers. However, if not, perhaps the cause was a drop in other input prices such as ink and wages. As in parts a and b, this would increase the supply of newspapers, and hence the demand for newsprint.

   d. **Aluminum used for beverage cans**

   With an increase in demand for cold drinks in the summer, the seasonal demand for aluminum increases, so this is one possible explanation. Alternatively, if glass or plastic containers have become more expensive, then the demand for aluminum would increase. Finally, changes in the market for recycled aluminum may affect the demand for new aluminum.

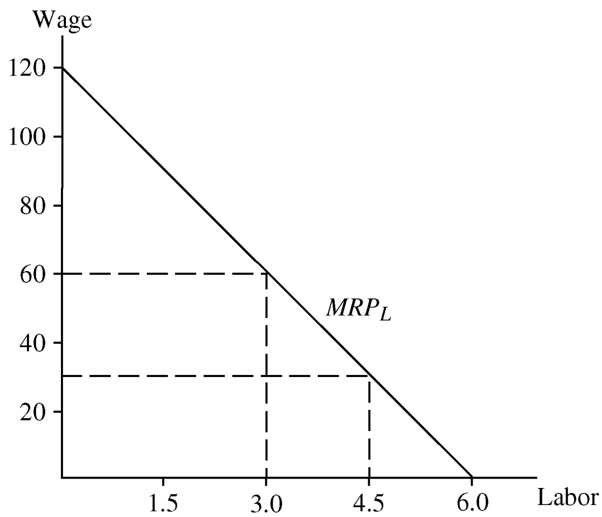
5. **Suppose there are two groups of workers, unionized and nonunionized. Congress passes a law that requires all workers to join the union. What do you expect to happen to the wage rates of formerly nonunionized workers? Of those workers who were originally unionized? What have you assumed about the union's behavior?**

   In general, we expect that nonunionized workers are earning lower wages than unionized workers. If all workers are forced to join the union, it would be reasonable to expect that the formerly nonunionized workers will now receive higher wages and the unionized workers will receive a wage that could go either way. There are a couple of issues to consider. First, the union now has more monopoly power because there are no nonunion workers to act as substitutes for union workers. This gives more power to the union, which means higher wages can be negotiated. However, the union now has more members to satisfy. If wages are kept at a high level, there will be fewer jobs, and hence some previously nonunionized workers may end up with no job. The union may wish to trade off some of the wage for a guarantee of more jobs. The average income of all workers will rise if labor demand is inelastic and will fall if labor demand is elastic.

6. Suppose a firm's production function is given by $Q = 12L - L^2$, for $L = 0$ to 6, where L is labor input per day and Q is output per day. Derive and draw the firm's demand for labor curve if the firm's output sells for \$10 in a competitive market. How many workers will the firm hire when the wage rate is \$30 per day? \$60 per day? (*Hint*: The marginal product of labor is $12 - 2L$.)

The marginal revenue product of labor is the demand for labor and is equal to marginal revenue multiplied by the marginal product of labor: $MRP_L = (MR)(MP_L)$. In a competitive market, price is equal to marginal revenue, so $MR = 10$. We are given $MP_L = 12 - 2L$ (the slope of the production function). Therefore, the $MRP_L = (10)(12 - 2L) = 120 - 20L$, as shown in the diagram.

The firm's profit-maximizing quantity of labor occurs where $MRP_L = w$. If $w = 30$, then $30 = 120 - 20L$ at the optimum. Solving for L yields 4.5 hours per day as shown in the diagram. Similarly, if $w = 60$, solving for L yields 3 hours per day.



7. The only legal employer of military soldiers in the United States is the federal government. If the government uses its knowledge of its monopsonistic position, what criteria will it employ when determining how many soldiers to recruit? What happens if a mandatory draft is implemented?

Acting as a monopsonist in hiring soldiers, the federal government would hire soldiers until the marginal value of the last soldier is equal to the marginal expenditure of hiring the last soldier. There are two implications of the government's monopsony power: fewer soldiers are hired, and they are paid less than their marginal value. If a mandatory draft is implemented, the government can "hire" (i.e., draft) as many soldiers as it wants at whatever wage it offers. We would expect the government to pay a lower wage to draftees than to volunteer soldiers and to hire up to the point where the marginal value of the last draftee is equal to the draftee wage. This would result in a larger number of soldiers than under the volunteer system.

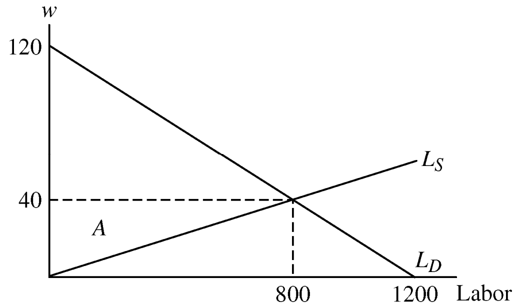
8. The demand for labor by an industry is given by the curve $L = 1200 - 10w$, where L is the labor demanded per day and w is the wage rate. The supply curve is given by $L = 20w$. What is the equilibrium wage rate and quantity of labor hired? What is the economic rent earned by workers?

Set the supply of labor equal to the demand for labor, and solve for the equilibrium wage rate:

$$20w = 1200 - 10w, \text{ or } w = \$40.$$

Substitute into either the labor supply or labor demand equation to find the equilibrium quantity of labor. For example, $L_D = 1200 - 10(40) = 800$.

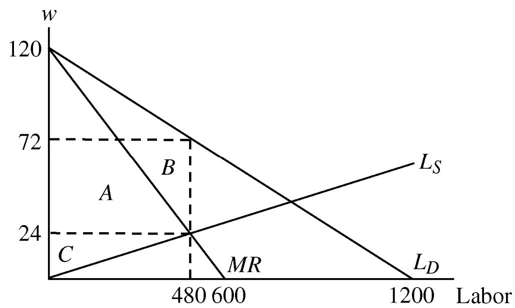
Economic rent is area *A* in the diagram. This triangle's area is (0.5)(800)($40) = $16,000.



9. Using the same information as in Exercise 8, suppose now that the only labor available is controlled by a monopolistic labor union that wishes to maximize the rent earned by union members. What will be the quantity of labor employed and the wage rate? How does your answer compare with your answer to Exercise 8? Discuss. (*Hint*: The union's marginal revenue curve is given by $MR = 120 - 0.2L$.)

To maximize rent, the union will choose the number of labor-days so that the marginal revenue of the last day of labor "sold" (the additional wages earned) is equal to the extra cost of inducing a worker to work the additional day. This involves choosing the quantity of labor at the point where the marginal revenue curve crosses the labor supply curve. To find marginal revenue, first find the inverse demand for labor:

$$w = 120 - 0.1L.$$



Marginal revenue has twice the slope of the labor demand curve, so $MR = 120 - 0.2L$ as given in the hint. Now solve the labor supply function for w: $w = 0.05L$.

Setting *MR* equal to labor supply yields the rent-maximizing quantity of labor:

$$120 - 0.2L = 0.05L, \text{ or } L = 480.$$

Therefore, to maximize the rent that its members earn, the union should limit employment to 480 members.

The wage that the union should charge for its members' labor is found from the labor demand curve:

$$w = 120 - 0.1(480) = \$72 \text{ per day.}$$

Marginal revenue ($24) is less than the wage, because when more workers are hired, all workers receive a lower wage.

The total rent that employed union members will receive is equal to area $A + B + C$:

$$\text{Rent} = (480)(72 - 24) + (0.5)(480)(24) = \$28{,}800.$$

So total rent and the wage are considerably higher, and the number of workers employed is much less than in the competitive case (Exercise 8).

10. A firm uses a single input, labor, to produce output q according to the production function $q = 8\sqrt{L}$. The commodity sells for $150 per unit and the wage rate is $75 per hour.

a. Find the profit-maximizing quantity of L.

There are two (equivalent) methods of solving this problem. Most generally, define the profit function, where revenues and costs are expressed in terms of the input, calculate the first order necessary condition (the first derivative of the profit function), and solve for the optimal quantity of the input. Alternatively, use the rule that the firm will hire labor up until the point where the marginal revenue product ($p \times MP_L$) equals the wage rate. Using the first method:

$$\pi = TR - TC = pq - wL$$
$$\pi = 150(8L^{0.5}) - 75L$$
$$\frac{d\pi}{dL} = 600L^{-0.5} - 75 = 0$$
$$L = \frac{600^2}{75^2} = 64$$

b. Find the profit-maximizing quantity of q.

From part a, the profit maximizing quantity of labor is 64. Substitute this quantity of labor into the production function to find $q = 8L^{0.5} = 8\sqrt{64} = 64$.

c. What is the maximum profit?

Profit is total revenue minus total cost or = (150)(64) – (75)(64) = $4800.

d. Suppose now that the firm is taxed $30 per unit of output and that the wage rate is subsidized at a rate of $15 per hour. Assume that the firm is a price taker, so the price of the product remains at $150. Find the new profit-maximizing levels of L, q, and profit.

After the $30 tax per unit of output is paid, the firm receives $150 – 30 = $120 per unit of output sold. This is the relevant price for the profit maximizing decision. The input cost is now $75 – 15 = $60 per unit of labor after the subsidy is received. The profit maximizing values can be found as in parts a–c above:

$$\pi = TR - TC = pq - wL$$
$$\pi = 120(8L^{0.5}) - 60L$$
$$\frac{d\pi}{dL} = 480L^{-0.5} - 60 = 0$$
$$L = \frac{480^2}{60^2} = 64$$

The optimal quantity of labor is still 64 hours, and thus the optimal output remains at $q = 64$ units. The company's profit is = (120)(64) – (60)(64) = \$3840. So the combination of tax and subsidy leads the firm to produce the same output as before, but the firm's profit is less.

e. Now suppose that the firm is required to pay a 20% tax on its profits. Find the new profit-maximizing levels of *L*, *q*, and profit.

Assuming that the tax and subsidy in part d no longer apply, the product's price is \$150 and the wage rate is \$75 as in parts a–c. The profit maximizing values can be found as above; only here after-tax profit is 80% of total revenue minus total cost.

$$\pi = 0.8(TR - TC) = 0.8(pq - wL)$$

$$\pi = 0.8(150(8L^{0.5}) - 75L) = 120(8L^{0.5}) - 60L$$

$$\frac{d\pi}{dL} = 480L^{-0.5} - 60 = 0$$

$$L = \frac{480^2}{60^2} = 64$$

The 20% tax on profit leads the firm to use the same amount of labor and produce the same level of output ($q = 64$) as before. The firm's after-tax profit is now = 0.8[(150)(64) – (75)(64)] = \$3840. So the firm earns the same profit after taxes as it does under the tax and subsidy scheme in part d.

Chapter 15
Investment, Time, and Capital Markets

■ Teaching Notes

The primary focus of this chapter is on how firms make capital investment decisions, though the chapter also includes some topical applications of the net present value criterion. You will notice that the chapter does not derive the rate of time preference; instead, it introduces students to financial decision-making. The key sections to cover are 15.1, 15.2, and 15.4, which cover stocks and flows, discounted present value, and the net present value criterion, respectively. Section 15.4 also discusses real and nominal discount rates. You can then pick and choose among the remaining sections depending on time and interest. Each of the special topics is briefly described below.

Section 15.3 explains how to find the value of a bond (including a perpetuity) using present discounted value calculations. Once students understand the effective yield on a bond, you could introduce the internal rate of return, *IRR*, and then discuss why the net present value, *NPV*, is superior to the *IRR* criterion for making capital investment decisions. For a comparison of *IRR* and *NPV* see, for example, Brealey and Myers, *Principles of Corporate Finance* (McGraw-Hill).

Section 15.5 discusses risk and the risk-free discount rate. You can motivate the discussion of risk by considering the probability of default by different classes of borrowers (this introduces the discussion of credit markets that is touched on in Section 17.1). This section introduces students to the Capital Asset Pricing Model. To understand the *CAPM*, it helps if students are familiar with Chapter 5, particularly Section 5.4, "The Demand for Risky Assets." The biggest stumbling block is the definition of β. If students have an intuitive feel for β, they may use Equation (15.7) to calculate the appropriate discount rate for a firm.

Section 15.6 applies the *NPV* criterion to investment decisions by consumers, leading to a wealth of applications. Example 15.5 presents Hausman's analysis of the decision to purchase an air conditioner, which comes to some interesting conclusions about the discount rates consumers use. You might use this as the basis for class discussion about whether the results of the study seem reasonable.

Section 15.7 considers investments in human capital. Human capital theory is a topic that bridges Chapters 14 and 15 since greater investment in human capital typically leads to higher wages for the worker. Students are usually quite interested in the calculation of the *NPV* of a college education and the return on an MBA degree.

Section 15.8 examines depletable resources and presents Hotelling's model of exhaustible resources. This example is a particularly good topic for class discussion given current oil prices. If you want to explore the resources area in further detail, you could also consider renewable resources such as timber. A straightforward treatment can be found in books on mathematical techniques in economics such as Chiang and Wainwright, *Fundamental Methods of Mathematical Economics* (McGraw-Hill, 2005). Note that these problems involve calculus but may be solved geometrically.

Section 15.9 discusses how interest rates are determined in the market for loanable funds and defines a variety of interest rates including the federal funds and commercial paper rates. If you have introduced students to the marginal rate of time preference, you can complete the analysis by introducing the simple two-period model of consumption and savings, since that model does not appear anywhere in the text. The axes are consumption (in dollars) today and at some future date such as next year. The budget line shows the rate at which consumption today may be transformed into future consumption at interest rate *R*. Indifference curves depend on the consumer's rate of time preference and determine the optimal consumption today and in the future. Covering this model would be especially useful if your students were concerned that savings did not appear in the consumer choice model back in Chapters 3 and 4.

■ Review Questions

1. **A firm uses cloth and labor to produce shirts in a factory that it bought for $10 million. Which of its factor inputs are measured as flows and which as stocks? How would your answer change if the firm had leased a factory instead of buying one? Is its output measured as a flow or a stock? What about profit?**

   Inputs that are purchased and used up during a particular time period are flows. Flow variables can be measured over time periods such as hours, days, weeks, months, or years. Inputs measured at a particular point in time are stocks. All stock variables have an associated flow variable. At any particular time, a firm will have a stock of buildings and machines that it owns, which are stock variables. During a given time period, as the firm uses its capital stock, that stock will depreciate, and this depreciation is a flow. In this question, cloth and labor are flows, while the factory is a stock. If the firm instead leases the building, the factory is still a stock variable that is owned in this case by someone else. The firm would pay rent during a particular time period, which would be a flow just like depreciation when the firm owns the capital stock. Output is always a flow variable that is measured over some time period. Since profit is the difference between revenues and costs over some given time period, it is also a flow.

2. **How do investors calculate the net present value of a bond? If the interest rate is 5%, what is the present value of a perpetuity that pays $1000 per year forever?**

   The present value of a bond is the sum of discounted values of each payment to the bond holder over the life of the bond. This includes the payment of interest in each period and the repayment of the principal at the end of the bond's life. A perpetuity involves the payment of interest in every future period forever, with no repayment of the principal. The present discounted value of a perpetuity is $PDV = \frac{A}{R}$, where *A* is the annual payment and *R* is the annual interest rate. If *A* = $1000 and *R* = 0.05, $PDV = \frac{\$1000}{0.05} = \$20{,}000.$

3. **What is the *effective yield* on a bond? How does one calculate it? Why do some corporate bonds have higher effective yields than others?**

   The effective yield is the interest rate that equates the present value of a bond's payment stream with the bond's current market price. The present discounted value of a payment made in the future is

   $$PDV = FV/(1 + R)^t,$$

   where *t* is the length of time before payment. The bond's selling price is its *PDV*. The payments it makes are the future values, *FV*, paid in time *t*. Thus the selling price *P* equals $P = \frac{A}{1+R} + \frac{A}{(1+R)^2} + \cdots + \frac{A}{(1+R)^N} + \frac{I}{(1+R)^N}$, where *A* is the annual interest payment, *I* is the principal repayment, and N is the number of years until maturity. We must solve for R, which is the bond's effective yield. The effective yield is determined by the interaction of buyers and sellers in the bond market. Some corporate bonds have higher effective yields because they are thought to be a more risky investment, and hence buyers must be rewarded with a higher rate of return so that they will be willing to hold the bonds. Higher rates of return imply a lower present discounted value. If bonds have the same payment stream, the bonds of riskier firms will sell for less than the bonds of less risky firms.

4. **What is the Net Present Value (*NPV*) criterion for investment decisions? How does one calculate the *NPV* of an investment project? If all cash flows for a project are certain, what discount rate should be used to calculate *NPV*?**

   The Net Present Value criterion for investment decisions says to invest if the present discounted value of the expected future cash flows from an investment is larger than the cost of the investment (Section 15.4). We calculate the *NPV* by (1) determining the present discounted value of all future cash flows and (2) subtracting the discounted value of all costs, present and future. To discount both income and cost, the firm should use a discount rate that reflects its opportunity cost of capital, the next highest return on an alternative investment of similar riskiness. Therefore, the risk-free interest rate should be used if the cash flows are certain.

5. **You are retiring from your job and are given two options. You can accept a lump sum payment from the company, or you can accept a smaller annual payment that will continue for as long as you live. How would you decide which option is best? What information do you need?**

   The best option is the one that has the highest present discounted value. The lump sum payment has a present discounted value equal to the amount of the lump sum payment. To calculate the present discounted value of the payment stream, you need to know approximately how many years you might live. If you made a guess of 25 years you could then discount each of the 25 future payments back to the current year and add them up to see how this sum compares to the lump sum payment. The discount factor would be the rate of return you expect to earn each year when you invest the lump sum payment. Finally, you must consider the time and risks involved in managing a lump sum on your own and decide if it is better or easier to just take the annual payments. The critical information you need to make this decision is how long you will live and the rate of return you will earn on your investment each year. Unfortunately, neither is knowable with certainty.

6. **You have noticed that bond prices have been rising over the past few months. All else equal, what does this suggest has been happening to interest rates? Explain.**

   This suggests that interest rates have been falling because bond prices and interest rates are inversely related. When the price of a bond (with a fixed stream of future payments) rises, then the effective yield on the bond falls. So if bond prices have been rising, the yields on bonds must be falling, and the only way people will be willing to hold bonds whose yields have fallen is if interest rates in general have also fallen.

7. **What is the difference between a real discount rate and a nominal discount rate? When should a real discount rate be used in an *NPV* calculation and when should a nominal rate be used?**

   The real discount rate is net of inflation, whereas the nominal discount rate includes inflationary expectations. The real discount rate is approximately equal to the nominal discount rate minus the expected rate of inflation. If cash flows are in real terms, the appropriate discount rate is the real rate, but if the cash flows are in nominal terms, a nominal discount rate should be used. For example, in applying the *NPV* criterion to a manufacturing decision, if future prices of inputs and outputs are projected using current dollars (and have not been increased to account for future inflation), a real discount rate should be used to determine whether the *NPV* is positive. On the other hand, if the future dollar amounts have been adjusted to account for inflation, then a nominal discount rate should be used. In sum, all numbers should either be expressed in real terms or nominal terms; not a mix of real and nominal.

8. How is risk premium used to account for risk in *NPV* calculations? What is the difference between diversifiable and nondiversifiable risk? Why should only nondiversifiable risk enter into the risk premium?

To determine the present discounted value of a cash flow, the discount rate should reflect the riskiness of the project generating the cash flow. The risk premium is the difference between a discount rate that reflects the riskiness of the cash flow and a discount rate for a risk-free flow, e.g., the discount rate associated with a short-term government bond. The higher the riskiness of a project, the higher the risk premium should be.

Diversifiable risk can be eliminated by investing in many projects. Hence, an efficient capital market will not compensate an investor for taking on risk that can be eliminated costlessly through diversification. Nondiversifiable risk is that part of a project's risk that cannot be eliminated by investing in a large number of other projects. It is that part of a project's risk which is correlated with the portfolio of all projects available in the market. Since investors can eliminate diversifiable risk, they cannot expect to earn a risk premium on diversifiable risk. Only nondiversifiable risk should enter into the risk premium.

9. What is meant by the "market return" in the Capital Asset Pricing Model (*CAPM*)? Why is the market return greater than the risk-free interest rate? What does an asset's "beta" measure in the *CAPM*? Why should high-beta assets have a higher expected return than low-beta assets?

In the Capital Asset Pricing Model (*CAPM*), the market return is the rate of return on the portfolio of all risky assets in the market. Thus the market return reflects only nondiversifiable risk.

Since the market portfolio has no diversifiable risk, the market return reflects the risk premium associated with holding one unit of nondiversifiable risk. The market rate of return is greater than the risk-free rate of return, because risk-averse investors must be compensated with higher average returns for holding a risky asset.

An asset's beta reflects the sensitivity (covariance) of the asset's return with the return on the market portfolio. An asset with a high beta will have a greater expected return than a low-beta asset, because the high-beta asset has greater nondiversifiable risk than the low-beta asset.

10. Suppose you are deciding whether to invest $100 million in a steel mill. You know the expected cash flows for the project, but they are risky—steel prices could rise or fall in the future. How would the *CAPM* help you select a discount rate for an *NPV* calculation?

To evaluate the net present value of a $100 million investment in a steel mill, you should use the stock market's current evaluation of firms that own steel mills as a guide to selecting the appropriate discount rate. For example, you would (1) identify nondiversified steel companies that are primarily involved in steel production, (2) determine the beta associated with stocks issued by those companies (this can be done statistically or by relying on a financial service that publishes stock betas, such as *Value Line*), and (3) take a weighted average of these betas, where the weights are equal to each firm's assets divided by the sum of all the nondiversified steel firms' assets. With an estimate of beta, plus estimates of the expected market and risk-free rates of return, you could infer the discount rate using Equation (15.7) in the text: Discount rate $= r_f + \beta(r_m - r_f)$.

11. How does a consumer trade off current and future costs when selecting an air conditioner or other major appliance? How could this selection be aided by an *NPV* calculation?

There are two major costs to be considered when purchasing a durable good: the initial purchase price and the cost of operating (and perhaps repairing) the appliance over its lifetime. Since these costs occur at different points in time, consumers should calculate the present discounted value of all the future costs and add that to the purchase price to determine the total cost of the appliance. This is what an *NPV* calculation does, so selecting the best appliance can be aided by doing an *NPV* calculation using the consumer's opportunity cost of money as the discount rate.

12. What is meant by the "user cost" of producing an exhaustible resource? Why does price minus extraction cost rise at the rate of interest in a competitive market for an exhaustible resource?

In addition to the marginal cost of extracting the resource and preparing it for sale, there is an additional opportunity cost arising from the depletion of the resource, because producing and selling a unit today makes that unit unavailable for production and sale in the future. User cost is the difference between price and the marginal cost of production. User cost rises over time because as reserves of the resource become depleted, the remaining reserves become more valuable.

Given constant demand over time, the price of the resource minus its marginal cost of extraction, $P - MC$, should rise over time at the rate of interest. If $P - MC$ rises faster than the rate of interest, no extraction should occur in the present period, because holding the resource for another year would earn a higher profit a year from now than selling the resource now and investing the proceeds for another year. If $P - MC$ rises more slowly than the rate of interest, current extraction should increase, thus increasing supply, lowering the equilibrium price, and decreasing the return on producing the resource. In equilibrium, the price of an exhaustible resource rises at the rate of interest.

13. What determines the supply of loanable funds? The demand for loanable funds? What might cause the supply or demand for loanable funds to shift? How would such a shift affect interest rates?

The supply of loanable funds is determined by the interest rate offered to savers. A higher interest rate induces households to consume less today (save) in favor of greater consumption in the future. The demand for loanable funds comes from consumers who wish to consume more today than tomorrow or from investors who wish to borrow money. Demand depends on the interest rate at which these two groups can borrow. Several factors can shift the demand and supply of loanable funds. For example, a recession decreases demand at all interest rates, shifting the demand curve inward and causing the equilibrium interest rate to fall. On the other hand, the supply of loanable funds will shift out if the Federal Reserve increases the money supply, again causing the interest rate to fall.

■ Exercises

1. Suppose the interest rate is 10%. If \$100 is invested at this rate today, how much will it be worth after one year? After two years? After five years? What is the value today of \$100 paid one year from now? Paid two years from now? Paid five years from now?

The future value, *FV*, of \$100 invested today at an interest rate of 10% is

$$FV = \$100 + (\$100)(0.10) = \$110.$$

Two years from now we will earn interest on the original \$100 (interest = \$10) *and* we will earn interest on the interest from the first year, i.e., (\$10)(0.10) = \$1. Thus our investment will be worth \$100 + \$10 (from the first year) + \$10 (from the second year) + \$1 (interest on the first year's interest) = \$121.

Algebraically, $FV = PDV(1 + R)^t$, where PDV is the present discounted value (or initial amount) of the investment, R is the interest rate, and t is the number of years. After two years,

$$FV = PDV(1 + R)^t = (\$100)(1.1)^2 = (\$100)(1.21) = \$121.00.$$

After five years

$$FV = PDV(1 + R)^t = (\$100)(1.1)^5 = (\$100)(1.61051) = \$161.05.$$

To find the present discounted value of \$100 paid one year from now, we must determine how much we would need to invest today at 10% to have \$100 one year from now. Using the formula for FV, solve for PDV as a function of FV:

$$PDV = (FV)(1 + R)^{-t}.$$

With $t = 1$, $R = 0.10$, and $FV = \$100$,

$$PDV = (100)(1.1)^{-1} = \$90.91.$$

With $t = 2$, $PDV = (100)(1.1)^{-2} = \$82.64$,

With $t = 5$, $PDV = (100)(1.1)^{-5} = \$62.09$.

2. **You are offered the choice of two payment streams: (a) \$150 paid one year from now and \$150 paid two years from now; (b) \$130 paid one year from now and \$160 paid two years from now. Which payment stream would you prefer if the interest rate is 5%? If it is 15%?**

To compare two income streams, calculate the present discounted value of each and choose the one with the higher value. Use the formula $PDV = FV(1 + R)^{-t}$ for each cash flow. First, use an interest rate of 5%. Stream (a) has two payments:

$$PDV_a = FV_1(1 + R)^{-1} + FV_2(1 + R)^{-2}$$

$$PDV_a = (\$150)(1.05)^{-1} + (\$150)(1.05)^{-2}, \text{ or}$$

$$PDV_a = \$142.86 + 136.05 = \$278.91.$$

Stream (b) has two payments:

$$PDV_b = (\$130)(1.05)^{-1} + (\$160)(1.05)^{-2}, \text{ or}$$

$$PDV_b = \$123.81 + \$145.12 = \$268.93.$$

At an interest rate of 5%, you should select (a) because it has the higher present discounted value.

If the interest rate is 15%, the present discounted values of the two income streams would be:

$$PDV_a = (\$150)(1.15)^{-1} + (\$150)(1.15)^{-2}, \text{ or}$$

$$PDV_a = \$130.43 + \$113.42 = \$243.85, \text{ and}$$

$$PDV_b = (\$130)(1.15)^{-1} + (\$160)(1.15)^{-2}, \text{ or}$$

$$PDV_b = \$113.04 + \$120.98 = \$234.02.$$

You should still select (a).

3. Suppose the interest rate is 10%. What is the value of a coupon bond that pays \$80 per year for each of the next five years and then makes a principal repayment of \$1000 in the sixth year? Repeat for an interest rate of 15%.

The value of the bond is the present discounted value, *PDV*, of the stream of payments made by the bond over the next six years. The *PDV* of each payment is:

$$PDV = \frac{FV}{(1+R)^t},$$

where R is the interest rate, equal to 10% (i.e., 0.10), and t is the number of years in the future. The value of all coupon payments over five years is therefore

$$PDV = \frac{80}{(1+R)} + \frac{8}{(1+R)^2} + \frac{80}{(1+R)^3} + \frac{80}{(1+R)^4} + \frac{80}{(1+R)^5}, \text{ or}$$

$$PDV = 80\left(\frac{1}{1.1} + \frac{1}{1.21} + \frac{1}{1.331} + \frac{1}{1.4641} + \frac{1}{1.61051}\right) = \$303.26.$$

The present value of the final payment of \$1000 in the sixth year is

$$PDV = \frac{\$1000}{1.1^6} = \frac{\$1000}{1.771} = \$564.47.$$

Thus the present value of the bond is \$303.26 + \$564.47 = \$867.73 when the interest rate is 5%.

With an interest rate of 15%, the value of the bond is

$$PDV = 80(0.870 + 0.756 + 0.658 + 0.572 + 0.497) + (1000)(0.432), \text{ or}$$

$$PDV = \$268.17 + \$432.33 = \$700.50.$$

As the interest rate increases, the value of the bond decreases.

4. A bond has two years to mature. It makes a coupon payment of \$100 after one year and both a coupon payment of \$100 and a principal repayment of \$1000 after two years. The bond is selling for \$966. What is its effective yield?

Determine the interest rate that will yield a present value of \$966 for an income stream of \$100 after one year and \$1100 after two years. Find R such that

$$966 = (100)(1 + R)^{-1} + (1100)(1 + R)^{-2}.$$

Algebraic manipulation yields

$$966(1 + R)^2 = 100(1 + R) + 1100, \text{ or}$$

$$966 + 1932R + 966R^2 - 100 - 100R - 1100 = 0, \text{ or}$$

$$966R^2 + 1832R - 234 = 0.$$

Use the quadratic formula to solve for R:

$$R = 0.12 \text{ or } -2.017.$$

Since −2.017 does not make economic sense, the effective yield is 12%.

5. **Equation (15.5) (page 572) shows the net present value of an investment in an electric motor factory. Half of the $10 million cost is paid initially and the other half after a year. The factory is expected to lose money during its first two years of operation. If the discount rate is 4%, what is the *NPV*? Is the investment worthwhile?**

   Using $R = 0.04$, Equation (15.5) becomes

   $$NPV = -5 - \frac{5}{(1.04)} - \frac{1}{(1.04)^2} - \frac{0.5}{(1.04)^3} + \frac{0.96}{(1.04)^4} + \frac{0.96}{(1.04)^5} + \cdots + \frac{0.96}{(1.04)^{20}} + \frac{1}{(1.04)^{20}}.$$

   Calculating the *NPV* we find:

   $NPV = -5 - 4.808 - 0.925 - 0.445 + 0.821 + 0.789 + 0.759 + 0.730 + 0.701 + 0.674 + 0.649 + 0.624 + 0.600 + 0.577 + 0.554 + 0.533 + 0.513 + 0.493 + 0.474 + 0.456 + 0.438 + 0.456 = -0.338.$

   The investment loses $338,000 and is not worthwhile. However, were the discount rate 3%, the *NPV* = $866,000, and the investment would be worth undertaking.

6. **The market interest rate is 5% and is expected to stay at that level. Consumers can borrow and lend all they want at this rate. Explain your choice in each of the following situations:**

   a. **Would you prefer a $500 gift today or a $540 gift next year?**

      The present value of $500 today is $500. The present value of $540 next year is

      $$\frac{\$540.00}{1.05} = \$514.29.$$

      Therefore, you should prefer the $540 next year.

   b. **Would you prefer a $100 gift now or a $500 loan without interest for four years?**

      If you take the $500 loan, you can invest it for the four years and then pay back the $500. The future value of the $500 is

      $$500(1.05)^4 = \$607.75.$$

      After you pay back the $500 you will have $107.75 left to keep. The future value of the $100 gift is

      $$100(1.05)^4 = \$121.55.$$

      You should take the $100 gift because its future value is larger.

   c. **Would you prefer a $350 rebate on an $8000 car or one year of financing for the full price of the car at 0% interest?**

      The interest rate is 0%, which is 5% less than the current market rate. You save $400 = (0.05)($8000) one year from now. The present value of this $400 is

      $$\frac{\$400}{1.05} = \$380.95.$$

      This is greater than $350. Therefore, choose the financing.

d. You have just won a million-dollar lottery and will receive $50,000 a year for the next 20 years. How much is this worth to you today?

If you get the first year's payment immediately (as is usually the case), the *PDV* of the payments of $50,000 per year for 20 years is less than two-thirds of $1 million.

$$PDV = 50{,}000 + \frac{50{,}000}{1.05} + \frac{50{,}000}{(1.05)^2} + \frac{50{,}000}{(1.05)^3} + \cdots + \frac{50{,}000}{(1.05)^{18}} + \frac{50{,}000}{(1.05)^{19}} = \$654{,}266.04.$$

e. You win the "honest million" jackpot. You can have $1 million today or $60,000 per year for eternity (a right that can be passed on to your heirs). Which do you prefer?

The present discounted value of the $60,000 perpetuity is $60,000/0.05 = $1,200,000, which makes it advisable take the $60,000 per year payment. Note that it takes 32 years before the *PDV* of the perpetuity exceeds $1,000,000, so if you don't expect to live that long, you should take the perpetuity only if you care about your heirs.

f. In the past, adult children had to pay taxes on gifts over $10,000 from their parents, but parents could make interest-free loans to their children. Why did some people call this policy unfair? To whom were the rules unfair?

Any gift of $N from parent to child could be made without taxation by lending the child $\frac{\$N(1+r)}{r}$. For example, to avoid taxes on a $20,000 gift, the parent would lend the child $420,000. With that money, the child could earn $21,000 in interest at the 5% interest rate during the next year. At the end of the year the child would repay the $420,000 loan (interest-free) and keep the $21,000 in interest. The present discounted value of the $21,000 received one year in the future is $21,000/1.05 = $20,000. So the child gets a "gift" with a present value of $20,000 without paying any taxes. People of more moderate incomes would find these rules unfair: they might be able to afford to give the child $20,000 directly (which would not be tax free) but would not have the resources to lend the child $420,000.

7. Ralph is trying to decide whether to go to graduate school. If he spends two years in graduate school, paying $15,000 tuition each year, he will get a job that will pay $60,000 per year for the rest of his working life. If he does not go to school, he will go into the work force immediately. He will then make $30,000 per year for the next three years, $45,000 for the following three years, and $60,000 per year every year after that. If the interest rate is 10%, is graduate school a good financial investment?

After the sixth year, Ralph's income will be the same with or without the graduate school education, so we can ignore all income after the first six years. With graduate school, the present value of income for the next six years (assuming all payments occur at the end of the year) is

$$-\frac{\$15{,}000}{(1.1)^1} - \frac{\$15{,}000}{(1.1)^2} + \frac{\$60{,}000}{(1.1)^3} + \frac{\$60{,}000}{(1.1)^4} + \frac{\$60{,}000}{(1.1)^5} + \frac{\$60{,}000}{(1.1)^6} = \$131{,}150.35.$$

Without graduate school, the present value of income for the next six years is

$$\frac{\$30{,}000}{(1.1)^1} + \frac{\$30{,}000}{(1.1)^2} + \frac{\$30{,}000}{(1.1)^3} + \frac{\$45{,}000}{(1.1)^4} + \frac{\$45{,}000}{(1.1)^5} + \frac{\$45{,}000}{(1.1)^6} = \$158{,}683.95.$$

The payoff from graduate school is not large enough to justify the foregone income and tuition expense while Ralph is in school; he should therefore not go to school.

8. Suppose your uncle gave you an oil well like the one described in Section 15.8. (Marginal production cost is constant at $50.) The price of oil is currently $80 but is controlled by a cartel that accounts for a large fraction of total production. Should you produce and sell all your oil now or wait to produce? Explain your answer.

If a cartel accounts for a large fraction of total production, today's price minus marginal cost, $P^t - MC$ will rise at a rate less than the rate of interest. This is because the cartel will choose output such that *marginal revenue* minus *MC* rises at the rate of interest. Since price exceeds marginal revenue, $P^t - MC$ will rise at a rate less than the rate of interest. So, to maximize net present value, all your oil should be sold today, and your profits should be invested at the rate of interest.

9. You are planning to invest in fine wine. Each case costs $100, and you know from experience that the value of a case of wine held for t years is $100t^{1/2}$. One hundred cases of wine are available for sale, and the interest rate is 10%.

a. How many cases should you buy, how long should you wait to sell them, and how much money will you receive at the time of their sale?

One way to get the answer is to compare holding a case of wine to putting your $100 in the bank. The bank pays interest of 10%, while the wine increases in value at the rate of

$$\frac{\frac{d(value)}{dt}}{value} = \frac{50t^{-0.5}}{100t^{0.5}} = \frac{1}{2t}.$$

As long as $t < 5$, the return on wine is greater than or equal to 10%. When $t > 5$, the return on wine drops below 10%. Therefore, $t = 5$ is the time to switch your wealth from wine to the bank. Since each case is a good investment, you should buy all 100 cases.

Another way to get the answer is to use an advanced formula for calculating the *PDV*. This formula assumes continuous compounding rather than annual compounding. If you buy a case and sell it after *t* years, you pay $100 now and receive $100t^{0.5}$ when it is sold. The *NPV* of this investment is

$$NPV = -100 + e^{-rt}(100\ t^{0.5}) = -100 + e^{-0.1t}(100\ \ t^{0.5})$$

If you do buy a case, you will want to choose *t* to maximize the *NPV*. This implies differentiating with respect to *t* to obtain the necessary condition that

$$\frac{dNPV}{dt} = (e^{-0.1t})(50\ \ t^{-0.5}) - (0.1e^{-0.1t})(100\ \ t^{0.5}) = 0.$$

Multiply both sides of the first order condition by $e^{0.1t}$ to obtain

$$50\ \ t^{-0.5} - 10\ \ t^{0.5} = 0, \text{ or } t = 5.$$

If you held the case for five years, the *NPV* would be

$$-100 + e^{(-0.1)(5)}(100)(5^{0.5}) = \$35.62.$$

[Note that you get a slightly different *NPV* if you use annual discounting: $NPV = -100 + 223.61/(1.1)^5 = 38.84$.]

Therefore, you should buy a case and hold it for five years. The value of the case at the time of sale is $(\$100)(5^{0.5}) = \$223.61$. Again, you should buy all 100 cases.

b. Suppose that at the time of purchase, someone offers you $130 per case immediately. Should you take the offer?

You just bought the wine and are offered $130 for resale, so you would make an immediate profit of $30. However, if you hold the wine for five years, the *NPV* of your profit is $35.62 as shown in part a. Therefore, the *NPV* if you sell immediately rather than hold for five years is $30 − 35.62 = −$5.62, so you should not sell. You are better off holding the wine for five years rather than selling it for a quick profit.

c. How would your answers change if the interest rate were only 5%?

If the interest rate changes from 10% to 5%, the *NPV* calculation is

$$NPV = -100 + (e^{-0.05t})(100t^{0.5}).$$

As before, we maximize this expression:

$$\frac{dNPV}{dt} = (e^{-0.05t})(50\ t^{-0.5}) - (0.05e^{-0.05t})(100\ t^{0.5}) = 0.$$

By multiplying both sides of the first order condition by $e^{0.05t}$, it becomes

$$50t^{-0.5} - 5t^{0.5} = 0,$$

or $t = 10$. If we hold the case 10 years, *NPV* is

$$-100 + e^{(-0.05)(10)}(100)(10^{0.5}) = \$91.80.$$

With a lower interest rate, it pays to hold onto the wine longer before selling it, because the value of the wine is increasing at the same rate as before. Again, you should buy all 100 cases.

Now if someone offers you $130 immediately, you should definitely not sell. By selling you make a quick $30 profit per case, but by holding for 10 years the present value of your profit is $91.80 per case.

10. Reexamine the capital investment decision in the disposable diaper industry (Example 15.4) from the point of view of an incumbent firm. If P&G or Kimberly-Clark were to expand capacity by building three new plants, they would not need to spend $60 million on R&D before startup. How does this advantage affect the *NPV* calculations in Table 15.5 (page 577)? Is the investment profitable at a discount rate of 12%?

If the only change in the cash flow for an incumbent firm is the absence of an initial pre-2015 $60 million R&D expenditure, then the *NPV* calculations in Table 15.5 simply increase by $60 million for each discount rate:

Discount Rate:	0.05	0.10	0.15
NPV:	140.5	43.1	−15.1

To determine whether the investment is profitable at a discount rate of 12%, we must recalculate the *NPV*. At 12%,

$$NPV = -60 - \frac{93.4}{1.12} - \frac{56.6}{(1.12)^2} + \frac{40}{(1.12)^3} + \frac{40}{(1.12)^4} + \frac{40}{(1.12)^5} + \frac{40}{(1.12)^6} + \frac{40}{(1.12)^7} + \frac{40}{(1.12)^8}$$
$$+ \frac{40}{(1.12)^9} + \frac{40}{(1.12)^{10}} + \frac{40}{(1.12)^{11}} + \frac{40}{(1.12)^{12}} + \frac{40}{(1.12)^{13}} + \frac{40}{(1.12)^{14}} + \frac{40}{(1.12)^{15}} = \$16.3 \text{ million.}$$

Thus, the incumbent would find it profitable to expand capacity.

11. Suppose you can buy a new Toyota Corolla for $20,000 and sell it for $12,000 after six years. Alternatively, you can lease the car for $300 per month for three years and return it at the end of the three years. For simplification, assume that lease payments are made yearly instead of monthly – i.e., that they are $3600 per year for each of three years.

a. If the interest rate, *r*, is 4%, is it better to lease or buy the car?

Compute the *NPV* of each option. The *NPV* of buying the car is:

$$-20{,}000+\frac{12{,}000}{1.04^6}=-10{,}516.23.$$

The *NPV* of leasing the car, assuming you have to pay at the beginning of each year, is:

$$-3600-\frac{3600}{1.04}-\frac{3600}{1.04^2}=-10{,}389.94.$$

In this case, you are better off leasing the car because the *NPV* is higher (i.e., less negative).

b. Which is better if the interest rate is 12%?

Again, compute the *NPV* of each option. The *NPV* of buying the car is:

$$-20{,}000+\frac{12{,}000}{1.12^6}=-13{,}920.43.$$

The *NPV* of leasing the car is:

$$-3600-\frac{3600}{1.12}-\frac{3600}{1.12^2}=-9684.18.$$

You are still better off leasing the car because the *NPV* is higher.

c. At what interest rate would you be indifferent between buying and leasing the car?

You are indifferent between buying and leasing if the two *NPV*'s are equal or:

$$-20{,}000+\frac{12{,}000}{(1+r)^6}=-3600-\frac{3600}{(1+r)}-\frac{3600}{(1+r)^2}.$$

In this case, you need to solve for *r*. The easiest way to do this is to use a spreadsheet and calculate the two *NPV*'s for different values of *r*. Observe first that the interest rate will be something less than 4% given that at 4% the best option was to lease, and as the interest rate rose to 12% leasing became even a better option. The interest rate will be in the neighborhood of 3.8% as shown in the table below.

r	*NPV* Buy	*NPV* Lease
0.03	–9,950.19	–10,488.49
0.037	–10,350.41	–10,419.24
0.038	–10,406.06	–10,409.45
0.039	–10,461.33	–10,399.68
0.04	–10,516.23	–10,389.94

11 (alternate) Suppose you can buy a new Toyota Corolla for \$20,000 and sell it for \$12,000 after six years. Alternatively, you can lease the car for \$300 per month for three years and return it at the end of the three years. Then you can lease another Corolla for another three years for the same monthly payment. For simplification, assume that lease payments are made yearly instead of monthly—i.e., that they are \$3600 per year for each of three years. *Compare leasing the car for two consecutive three-year periods against owning the car for six years.*

a. If the interest rate, *r*, is 4%, is it better to lease or buy the car?

The *NPV* of owning the car is

$$-20{,}000+\frac{12{,}000}{1.04^6}=-10{,}516.23.$$

The *NPV* of leasing the car for six years is

$$-3600-\frac{3600}{1.04}-\frac{3600}{(1.04)^2}-\frac{3600}{(1.04)^3}-\frac{3600}{(1.04)^4}-\frac{3600}{(1.04)^5}=-19{,}626.56.$$

You should buy the car because the *NPV* is much higher (less negative).

b. Which is better if the interest rate is 12%?

The *NPV* of owning the car is

$$-20{,}000+\frac{12{,}000}{1.12^6}=-13{,}920.43.$$

The *NPV* of leasing the car is now

$$-3600-\frac{3600}{1.12}-\frac{3600}{(1.12)^2}-\frac{3600}{(1.12)^3}-\frac{3600}{(1.12)^4}-\frac{3600}{(1.12)^5}=-16{,}577.19.$$

You are still better off buying the car than leasing it.

c. At what interest rate would you be indifferent between buying and leasing the car?

You are indifferent between buying and leasing if the two *NPV*'s are equal:

$$-20{,}000+\frac{12{,}000}{(1+r)^6}=-3600-\frac{3600}{1+r}-\frac{3600}{(1+r)^2}-\frac{3600}{(1+r)^3}-\frac{3600}{(1+r)^4}-\frac{3600}{(1+r)^5}$$

In this case, you need to solve for *r*. The easiest way to do this is to use a spreadsheet and calculate the two *NPV*'s for different values of *r*. Observe first that the interest rate will be something greater than 12% given that at 4% the best option was to buy, and as the interest rate rose to 12% buying was still better but by a smaller margin. The interest rate that equates these two *NPV*'s is almost exactly 16.6% as the spreadsheet results below reveal.

r	***NPV* Buy**	***NPV* Lease**
0.16	−15,074.69	−15,387.46
0.165	−15,200.17	−15,251.27
0.166	−15,224.82	−15,224.35
0.167	−15,249.32	−15,197.52
0.17	−15,321.94	−15,117.65

12. A consumer faces the following decision: She can buy a computer for \$1000 and \$10 per month for Internet access for three years, or she can receive a \$400 rebate on the computer (so that its cost is \$600) but agree to pay \$25 per month for three years for Internet access. For simplification, assume that the consumer pays the access fees yearly (i.e., \$10 per month = \$120 per year).

a. What should the consumer do if the interest rate is 3%?

Calculate the *NPV* in each case. Assuming that the Internet fees are paid at the beginning of each year, the *NPV* of the first option is

$$-1000-120-\frac{120}{1.03}-\frac{120}{1.03^2}=-1349.62.$$

The *NPV* of the second option with the rebate is

$$-600-300-\frac{300}{1.03}-\frac{300}{1.03^2}=-1474.04.$$

In this case, the first option gives a higher (less negative) *NPV*, so the consumer should pay the \$1000 now and pay \$10 per month for Internet access.

b. What if the interest rate is 17%?

Repeat the calculations in part a using 17%. The *NPV* of the first option is

$$-1000-120-\frac{120}{1.17}-\frac{120}{1.17^2}=-1310.23.$$

The *NPV* of the second option with the rebate is

$$-600-300-\frac{300}{1.17}-\frac{300}{1.17^2}=-1375.56.$$

In this case, the first option still gives a higher *NPV*, so the consumer should pay the \$1000 now and pay \$10 per month for Internet access.

c. At what interest rate will the consumer be indifferent between the two options?

The consumer is indifferent between the two options if the *NPV* of each option is the same. Therefore, set the *NPV*'s equal to each other and solve for r:

$$-1000-120-\frac{120}{1+r}-\frac{120}{(1+r)^2}=-600-300-\frac{300}{1+r}-\frac{300}{(1+r)^2}$$

$$220=\frac{180}{1+r}+\frac{180}{(1+r)^2}$$

$$220(1+r)^2=180(1+r)+180$$

$$220r^2+260r-140=0.$$

Using the quadratic formula to solve for the interest rate r results in a value of approximately $r = 40.2\%$. So the consumer would have to have an extremely high interest rate to make the rebate worthwhile.

PART FOUR

Information, Market Failure, and the Role of Government

Chapter 16
General Equilibrium and Economic Efficiency

■ Teaching Notes

This chapter extends the analysis of many of the earlier chapters in the text. Section 16.1 covers general equilibrium analysis and extends supply/demand analysis to situations where more than one market is involved and there is feedback between the markets. Section 16.2 uses the Edgeworth box diagram to explore efficiency in consumption, which extends the analysis of Chapter 4. Section 16.3 covers different views of equity and examines the relationship between equity and efficiency. While students often find this material interesting and thought provoking, it can be skipped without interrupting the flow of the rest of the chapter if time is short. Section 16.4 introduces the production possibilities frontier to examine efficiency in production, which harks back to Chapter 7. Section 16.5 discusses gains from trade and comparative advantage. Since these topics are covered in other economics courses, you could also omit this section if time is limited. Section 16.6 is a nice summary of Sections 16.2 and 16.4 and the efficiency of competitive markets. Section 16.7 discusses types of market failure and serves as a bridge to the last two chapters of the book.

The distinction between partial and general equilibrium is readily accepted by students, but they might find the graphical analysis of Figure 16.1 intimidating. Although this is not a complete discussion of general equilibrium, students can learn to appreciate the limitations of partial equilibrium analysis and the need to consider interactions among markets. Stress the importance of using general equilibrium analysis for economy-wide policies; e.g., raising the minimum wage or requiring the use of ethanol in gasoline.

To provide a context for the discussion of exchange economies, you might start by discussing two children trading cookies and potato chips at lunchtime. For a more serious example, see the famous article by R.A. Radford, “The Economic Organization of a POW Camp,” *Economica*, November 1945, 12:48, 189–201. Since students sometimes find the definition of Pareto optimality confusing, try expressing it in different ways; e.g., “an allocation is Pareto-efficient if every reallocation that makes someone better off also makes someone else worse off,” or “an allocation is *not* Pareto-efficient if you can make someone better off without making someone else worse off.” Explain why movements toward the contract curve are Pareto-improving while movements along the contract curve are not Pareto-improving. Point out that all competitive equilibria are Pareto-efficient but not all Pareto-efficient points are competitive equilibria given a particular initial allocation. Emphasize that the competitive equilibrium depends on the initial allocation.

You can use the Edgeworth box diagram developed in Section 16.2 to show the distinction between efficiency and equity in Section 16.3. For example, a point on the contract curve near one corner might be less preferred because of equity considerations than a point off the curve but nearer to the middle of the box. This conflict introduces the problem of defining equity and incorporating it into an economic analysis. Table 16.2 presents four definitions of equity. After introducing them, you could ask the class to vote on which definition is closest to their concept of equity. Then ask the students to defend their choices, which should lead to an interesting discussion.

Section 16.4 defines input efficiency and the production possibilities frontier (*PPF*). Students may ask whose indifference curve is shown in Figure 16.10. Rather than being the indifference curve for a single consumer, this curve is sometimes called a community indifference curve which shows combinations of food and clothing that keep each consumer's utility constant.

Section 16.5 covers the important topic of gains from trade. Students, however, often come to the course believing that free trade has cost many U.S. workers their jobs. Thus, you may want to discuss protectionism and its costs using Example 16.4 for some data. It is also a good idea to make the point that there are often disruptions caused by free trade, and there are real costs to workers who cannot easily move from one industry to another. These costs are not captured in the simple static models in the chapter whose results should be thought of as long-run consequences of trade.

■ Questions for Review

1. **Why can feedback effects make a general equilibrium analysis substantially different from a partial equilibrium analysis?**

   A partial equilibrium analysis focuses on the interaction of supply and demand in *one* market. It ignores the effect that shifts in supply and demand in that market might have on the markets for complements, substitutes, and inputs. A general equilibrium analysis takes feedback effects into account, where a price or quantity adjustment in one market can cause a price or quantity adjustment in related markets. Ignoring these feedback effects can lead to inaccurate forecasts of the full effect of changes in one market. An initial shift in demand in one market, for example, can cause a shift in demand in a related market, which can then cause a second shift in demand in the first market. A partial equilibrium analysis will stop at the initial shift whereas a general equilibrium analysis will continue on and on, incorporating possible shifts in demand in related markets and the ensuing feedback effects on the first market.

2. **In the Edgeworth box diagram, explain how one point can simultaneously represent the market baskets owned by two consumers.**

   The Edgeworth box diagram allows us to represent the distribution of two goods between two individuals. The box is formed by inverting the indifference curves of one individual and superimposing these on the indifference curves of another individual. The sides of the box represent the total amounts of the two goods available to the consumers. For example, the height of the vertical axis represents the total amount of, say, clothing that is available. For any point in the diagram, the vertical distance between the point and the bottom of the box is the amount of clothing that one consumer has, and the vertical distance between the point and the top of the box is the amount of clothing owned by the other consumer. Likewise, the horizontal distance between the point and the left side of the box represents the amount of the other good, say food, belonging to the first consumer and the distance to the right side of the box is the amount of food the other consumer has. Therefore, each point in the box represents a different allocation of the two goods between the two individuals.

3. **In the analysis of exchange using the Edgeworth box diagram, explain why both consumers' marginal rates of substitution are equal at every point on the contract curve.**

   The contract curve in an Edgeworth box diagram is the set of points where the indifference curves of the two individuals are tangent. We know that the marginal rate of substitution is equal to the (negative) slope of the indifference curves. Also, when two curves are tangent at a point, their slopes are equal. Thus, by defining the contract curve as a set of indifference curve tangencies, the marginal rates of substitution between the two goods are equal for the two individuals at every point on the contract curve, assuming convex indifference curves.

4. "Because all points on a contract curve are efficient, they are all equally desirable from a social point of view." Do you agree with this statement? Explain.

If society is only concerned with efficiency and not with equity, then all points on the contract curve are equally desirable. Since it is impossible to make comparisons of utility between individuals, economics focuses on efficiency. But most societies are also concerned with equity (i.e., whether the final allocation is fair, however that might be defined). In this case, all points on the contract curve are not equally desirable.

5. How does the utility possibilities frontier relate to the contract curve?

The utility possibilities frontier plots the utility levels of the two consumers for all points on the contract curve. One consumer's utility is plotted on the vertical axis and the other consumer's utility is shown on the horizontal axis. While points that lie between the origin and the utility possibilities frontier are feasible, they are not efficient because they are not on the frontier, and trades are possible that will make one or both individuals better off without making anyone worse off. Points outside the frontier are not feasible unless the individuals are given greater amounts of one or both goods.

6. In the Edgeworth production box diagram, what conditions must hold for an allocation to be on the production contract curve? Why is a competitive equilibrium on the contract curve?

When constructing an Edgeworth box for the production of two goods with two inputs, each point in the box represents an allocation of the two inputs between the two production processes. With production, each point can be ordered according to the total output. These points lie on isoquants instead of on indifference curves. Since each point simultaneously represents the allocation of inputs to two production processes, it lies on two isoquants, one for each production process. The production contract curve represents all combinations of inputs that are technically efficient. Thus there would be no way to increase the output of one good without decreasing the output of the other good.

A competitive equilibrium is one point on the production-contract curve. It is the intersection of the production-contract curve and a line passing through the initial allocation with a slope equal to the ratio of input prices. (The ratio of prices dictates the rates at which inputs can be traded in the market.) For a competitive equilibrium to hold, each producer must use inputs so that the slopes of the isoquants are equal to one another (assuming convex isoquants) and also equal to the ratio of the prices of the two inputs. Therefore, the competitive equilibrium is efficient in production.

7. How is the production possibilities frontier related to the production contract curve?

We can graph the quantities of each good produced (each point in the Edgeworth box) on a graph, where the vertical axis represents the output of one good and the horizontal axis represents the output of the other good. The production contract curve is represented in this graph as the production possibilities frontier. It is analogous to the utility possibilities frontier for consumers. Points inside the frontier are feasible but inefficient; points outside the frontier are infeasible and attainable only when more inputs become available or technological change increases productivity. Points on the production possibilities frontier are the same as those on the production contract curve. The difference is that the production contract curve measures inputs on the axes and the production possibilities frontier measures outputs on the axes.

8. What is the marginal rate of transformation (*MRT*)? Explain why the *MRT* of one good for another is equal to the ratio of the marginal costs of producing the two goods.

The marginal rate of transformation, *MRT*, is equal to the absolute value of the slope of the production possibilities frontier, *PPF*, and measures how much of one output must be given up to produce one more unit of the other output. The total cost of all inputs is the same at each point on the *PPF* because the amount of each input is fixed. Therefore, when we move down along the frontier, the cost of producing one output is reduced by the same amount as the cost of producing the other output is increased. Suppose the *MRT* is 4, then we must give up 4 units of the output on the vertical axis to get one more unit of the output on the horizontal axis.

This means that the total cost of producing the 4 units is the same as the total cost of producing the one unit, so the marginal cost of the good on the horizontal axis is 4 times the marginal cost of the good on the vertical axis.

9. Explain why goods will not be distributed efficiently among consumers if the *MRT* is not equal to the consumers' marginal rate of substitution.

If the marginal rate of transformation, *MRT*, is not equal to the marginal rate of substitution, *MRS*, we could reallocate inputs in producing output to leave the consumers and producers better off. If the *MRS* is greater than the *MRT*, then consumers are willing to pay more for another unit of a good than it will cost the producers to produce it. Both consumers and producers can therefore be made better off by arranging a trade somewhere between what consumers are willing to pay and what the producers have to pay to produce the extra unit. Note also that when $MRT \neq MRS$, the ratio of marginal costs will not be equal to the ratio of prices. This means that one good is being sold at a price below marginal cost and one good is being sold at a price above marginal cost. We should increase the output of the good whose price is above marginal cost and reduce the output of the other good whose price is below marginal cost.

10. Why can free trade between two countries make consumers of both countries better off?

Free trade between two countries expands each country's effective production possibilities frontier and allows each country to consume at a point above its original production possibilities frontier. Since each country has a comparative advantage in the production of some good or service, trade allows each country to specialize in the area where it has this advantage. It trades these outputs for those more cheaply produced in another country. Therefore, specialization benefits consumers in both countries.

11. If Country *A* has an absolute advantage in the production of two goods compared to Country *B*, then it is not in Country *A*'s best interest to trade with Country *B*. True or false? Explain.

This statement is false. A country can have an absolute advantage in the production of all goods, but it still will have a comparative advantage only in the production of some goods. Suppose Country *A* requires 4 units of labor to produce good 1 and 8 units of labor to produce good 2, whereas Country *B* requires 8 units and 12 units respectively. Country *A* can produce both goods more cheaply, so it has an absolute advantage in the production of both goods. However, trade is based on comparative advantage, which depends on how much of one good must be given up for one more unit of the other good. Here, Country *A* must give up 2 units of good 1 for another unit of good 2 whereas Country *B* must give up only 1.5 units of good 1 for another unit of good 2. Country *B* therefore has a comparative advantage in producing good 2. Likewise, Country *A* has a comparative advantage in producing good 1 and should trade good 1 to Country *B* for good 2.

12. Do you agree or disagree with each of the following statements? Explain.

a. If it is possible to exchange 3 pounds of cheese for 2 bottles of wine, then the price of cheese is 2/3 the price of wine.

This is a true statement. If 3 pounds of cheese can be exchanged for 2 bottles of wine, then cheese must have a cost that is 2/3 that of wine and the price of cheese will be 2/3 the price of wine.

b. A country can only gain from trade if it can produce a good at a lower absolute cost than its trading partner.

This statement is false. Trade is based on comparative advantage and not absolute advantage. A country can be absolutely worse at producing all goods, but will nonetheless be comparatively better at producing one or more goods and will gain by producing and trading these for other goods.

c. If there are constant marginal and average costs of production, then it is in a country's best interest to specialize completely in the production of some goods but to import others.

This is a true statement as long as the production costs differ across counties. If Country *A* has to always give up 2 units of good 1 for another unit of good 2 and Country *B* always has to give up 3 units of good 1 for another unit of good 2, then Country *A* should produce enough of good 2 to satisfy demand in both countries. Likewise, Country *B* should produce enough of good 1 to satisfy demand in both countries (its cost is 1/3 vs. 1/2 for Country *A*). Note that in reality, the marginal and average costs will tend to rise after a while as more resources are devoted to any given industry, so marginal and average costs are unlikely to remain constant regardless of output.

d. Assuming that labor is the only input, if the opportunity cost of producing a yard of cloth is 3 bushels of wheat per yard, then wheat must require 3 times as much labor per unit produced as cloth.

This statement is false. If the country must give up 3 bushels of wheat to produce another yard of cloth then the same labor resources that are producing the 3 bushels of wheat are required to produce the one yard of cloth. Therefore the yard of cloth requires three times as much labor as a bushel of wheat.

13. What are the four major sources of market failure? Explain briefly why each prevents the competitive market from operating efficiently.

The four major sources of market failure are market power, incomplete information, externalities, and public goods. We know from the study of market structures that *market power* leads to situations where price does not equal marginal cost. In these situations, the producer is producing too little. Consumers would be made better off if the good were produced under a competitive market structure, thereby lowering price until price were equal to marginal cost. *Incomplete information* implies that prices do not reflect either the marginal cost of production or the change in utility from changes in consumption. Either too much or too little (at the extreme, none) is produced and consumed. *Externalities* occur when a consumption or production activity influences other consumption or production activities, and these effects are not reflected in market prices. *Public goods* are goods that can be consumed at prices below marginal cost (at the extreme, freely) because consumers cannot be excluded. In these four cases, prices do not send the proper signals to either producers or consumers to increase or decrease production or consumption. Thus the market mechanism cannot equate social marginal costs with social marginal benefits.

■ Exercises

1. Suppose gold (*G*) and silver (*S*) are substitutes for each other because both serve as hedges against inflation. Suppose also that the supplies of both are fixed in the short run ($Q_G = 75$ and $Q_S = 300$) and that the demands for gold and silver are given by the following equations:

$$P_G = 975 - Q_G + 0.5P_S \qquad \text{and} \qquad P_S = 600 - Q_S + 0.5P_G.$$

a. What are the equilibrium prices of gold and silver?

In the short run, the quantity of gold, Q_G, is fixed at 75. Substitute Q_G into the demand equation for gold:

$$P_G = 975 - 75 + 0.5P_S = 900 + 0.5P_S.$$

In the short run, the quantity of silver, Q_S, is fixed at 300. Substituting Q_S into the demand equation for silver:

$$P_S = 600 - 300 + 0.5P_G = 300 + 0.5P_G.$$

Since we now have two equations and two unknowns, substitute the price of gold into the silver demand function and solve for the price of silver:

$$P_S = 300 + (0.5)(900 + 0.5P_S), \text{ or } P_S = \$1000.$$

Now substitute the price of silver into the demand function for gold:

$$P_G = 900 + (0.5)(1000) = \$1400.$$

b. What if a new discovery of gold doubles the quantity supplied to 150. How will this discovery affect the prices of both gold and silver?

When the quantity of gold increases by 75 units from 75 to 150, both prices fall. To see this, resolve the system of equations:

$$P_G = 975 - 150 + 0.5P_S, \text{ or } P_G = 825 + (0.5)(300 + 0.5P_G), \text{ or } P_G = \$1300.$$

The price of silver is equal to: $P_S = 300 + (0.5)(1300) = \$950$.

2. Using general equilibrium analysis, and taking into account feedback effects, analyze the following:

a. The likely effects of outbreaks of disease on chicken farms on the markets for chicken and pork.

If consumers are worried about the quality of the chicken, then they may choose to consume pork instead. This will shift the demand curve for chicken down and to the left and the demand curve for pork up and to the right. Feedback effects will partially offset the shift in the chicken demand curve, because some people may switch back to chicken when the price of pork rises. This will shift the demand curve for chicken back to the right by some amount, raising the price of chicken and shifting the demand curve for pork a bit further to the right. Overall, we would expect the price of chicken to drop, but not by as much as if there were no feedback effects. The price of pork will increase, perhaps by more than the increase in the absence of feedback effects.

The other possibility is that the outbreaks of disease cause a reduction in supply of chicken. This would increase the price of chicken, which would then lead to an increase in demand for pork. The price of pork would increase in response, which would boost the demand for chicken by some (probably small) amount, raising the price of chicken again. This would increase the demand for pork some more, and so on. Ultimately these effects will peter out, but the final increases in prices will be greater than if there had been no feedback effects.

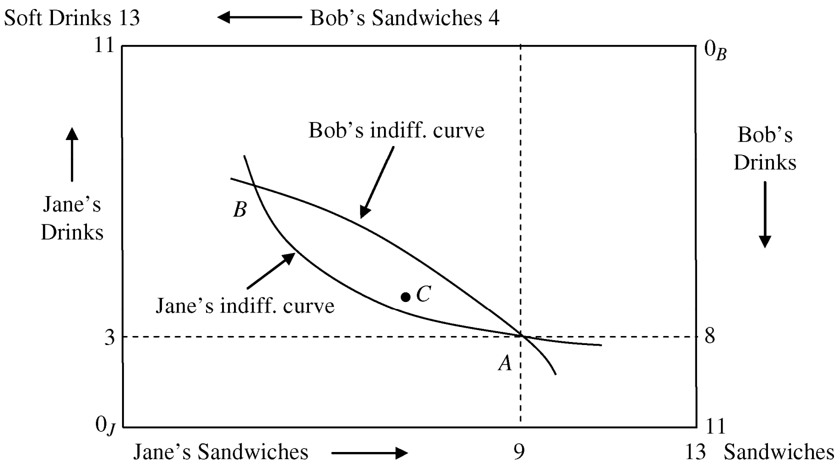
b. The effects of increased taxes on airline tickets on travel to major tourist destinations such as Florida and California and on the hotel rooms in those destinations.

The increase in tax raises the price of airline tickets, making it more costly to fly. The resulting increase in ticket prices would reduce the quantity of airline tickets sold. This in turn would reduce the demand for hotel rooms by out-of-town visitors, causing the demand curve for hotel rooms to shift down and to the left and reducing the price of hotel rooms. For the feedback effects, the lower price for hotel rooms may encourage some consumers to travel more, in which case the demand for airline tickets would shift back up and to the right, partially offsetting the initial decline in quantity demanded. The increase in airline demand would increase the demand for hotel rooms, causing hotel room prices to increase somewhat. In the end, we would still expect reduced quantities of both airline ticket sales and hotel rooms, higher airline ticket prices (due to the tax) and reduced prices for hotel rooms.

3. Jane has 3 liters of soft drinks and 9 sandwiches. Bob, on the other hand, has 8 liters of soft drinks and 4 sandwiches. With these endowments, Jane's marginal rate of substitution (*MRS*) of soft drinks for sandwiches is 4 and Bob's *MRS* is equal to 2. Draw an Edgeworth box diagram to show whether this allocation of resources is efficient. If it is, explain why. If it is not, what exchanges will make both parties better off?

Given that $MRS_{Bob} \neq MRS_{Jane}$, the current allocation of resources is inefficient. Jane and Bob could trade to make one of them better off without making the other worse off. Although we do not know the exact shape of Jane and Bob's indifference curves, we do know the slope of both indifference curves *at the current allocation*, because we know that $MRS_{Jane} = 4$ and $MRS_{Bob} = 2$. At the current allocation (point *A* in the diagram), Jane is willing to trade 4 sandwiches for 1 drink, or she will give up 1 drink in exchange for 4 sandwiches. Bob is willing to trade 2 sandwiches for 1 drink, or he will give up 1 drink in exchange for 2 sandwiches. Jane will give 4 sandwiches for 1 drink while Bob is willing to accept only 2 sandwiches in exchange for 1 drink.

Any exchange that leaves both parties inside the lens-shaped area between points *A* and *B* will make both better off. For example, if Jane gives Bob 3 sandwiches for 1 drink, they will be at point *C*. Jane is better off because she was willing to give up 4 sandwiches but only had to give up 3. Bob is better off because he was willing to accept 2 sandwiches and actually received 3. Jane ends up with 4 drinks and 6 sandwiches and Bob ends up with 7 drinks and 7 sandwiches, and both are better off than at point *A*.



4. Jennifer and Drew consume orange juice and coffee. Jennifer's *MRS* of orange juice for coffee is 1 and Drew's *MRS* of orange juice for coffee is 3. If the price of orange juice is \$2 and the price of coffee is \$3, which market is in excess demand? What do you expect to happen to the prices of the two goods?

Jennifer is willing to trade 1 coffee for 1 orange juice. Drew is willing to trade 3 coffees for 1 orange juice. In the market, it is possible to trade 2/3 of a coffee for an orange juice. Both will find it optimal to trade coffee in exchange for orange juice since they are willing to give up more for orange juice than they have to. There is an excess demand for orange juice and an excess supply of coffee. The price of coffee will decrease and the price of orange juice will rise. Notice also that at the given rates of *MRS* and prices, both Jennifer and Drew have a higher marginal utility per dollar for orange juice as compared to coffee.

5. Fill in the missing information in the following tables. For each table, use the information provided to identify a possible trade. Then identify the final allocation and a possible value for the *MRS* at the efficient solution. (*Note*: There is more than one correct answer.) Illustrate your results using Edgeworth Box diagrams.

a. Norman's *MRS* of food for clothing is 1 and Gina's *MRS* of food for clothing is 4:

Individual	Initial Allocation	Trade	Final Allocation
Norman	**6*F*, 2*C***	1*F* for 3*C*	5*F*, 5*C*
Gina	**1*F*, 8*C***	3*C* for 1*F*	2*F*, 5*C*

Gina will give 4 clothing for 1 food while Norman is willing to accept 1 clothing for 1 food. If they settle on 2 or 3 units of clothing for one unit of food they will both be better off. Let's say they settle on 3 units of clothing for 1 unit of food. Gina will give up 3 units of clothing and receive 1 unit of food so her final allocation is 2*F* and 5*C*. Norman will give up 1 food and gain 3 clothing so his final allocation is 5*F* and 5*C*. Gina's *MRS* will decrease and Norman's will increase, so given they must be equal in the end, it will be somewhere between 1 and 4, in absolute value terms. So a possible value for both individual's *MRS* at the efficient solution is 2.5.

b. Michael's *MRS* of food for clothing is 1/2 and Kelly's *MRS* of food for clothing is 3.

Individual	Initial Allocation	Trade	Final Allocation
Michael	**10*F*, 3*C***	3*F* for 3*C*	7*F*, 6*C*
Kelly	**5*F*, 15*C***	3*C* for 3*F*	8*F*, 12*C*

Michael will give 1/2 clothing for 1 food (or 2 food for 1 clothing) while Kelly is willing to trade 3 clothing for 1 food (i.e., she will accept 1/3 food for 1 clothing). If they settle on a trading rate of 1 unit of food for 1 unit of clothing they will both be better off. Suppose Michael gives up 3 units of food and receives 3 units of clothing, so his final allocation is 7*F* and 6*C*. Kelly will thus give up 3 units of clothing and gain 3 units of food, so her final allocation is 8*F* and 12*C*. Kelly's *MRS* will decrease and Michael's will increase, so given they must be equal in equilibrium, their *MRS* value will be somewhere between 3 and 1/2. So a possible value for each person's *MRS* is 2 at the efficient solution.

6. In the analysis of an exchange between two people, suppose both people have identical preferences. Will the contract curve be a straight line? Explain. Can you think of a counterexample?

Given that the contract curve intersects the origin for each individual, a straight line contract curve would be a diagonal line running from one origin to the other. The slope of this line is $\frac{Y}{X}$, where Y is the total amount of the good on the vertical axis and X is the total amount of the good on the horizontal axis. (x_1, y_1) are the amounts of the two goods allocated to one individual and $(x_2, y_2) = (X - x_1, Y - y_1)$ are the amounts of the two goods allocated to the other individual. A straight line contract curve would have the equation

$$y_1 = \left(\frac{Y}{X}\right)x_1.$$

We need to show that when the marginal rates of substitution for the two individuals are equal ($MRS^1 = MRS^2$), the point lies on this linear contract curve.

For example, consider the utility function $U = x_i^2 y_i$. Then

$$MRS^i = \frac{MU_x^i}{MU_y^i} = \frac{2x_i y_i}{x_i^2} = \frac{2y_i}{x_i}.$$

If MRS^1 equals MRS^2, then

$$\left(\frac{2y_1}{x_1}\right) = \left(\frac{2y_2}{x_2}\right).$$

Is this point on the contract curve? Yes, because

$$x_2 = X - x_1 \text{ and } y_2 = Y - y_1, \text{ so}$$

$$2\left(\frac{y_1}{x_1}\right) = 2\left(\frac{Y - y_1}{X - x_1}\right).$$

This means that

$$\frac{y_1(X - x_1)}{x_1} = Y - y_1, \quad \text{or} \quad \frac{y_1 X - y_1 x_1}{x_1} = Y - y_1, \text{ and}$$

$$\frac{y_1 X}{x_1} - y_1 = Y - y_1, \quad \text{or} \quad \frac{y_1 X}{x_1} = Y, \quad \text{or} \quad y_1 = \left(\frac{Y}{X}\right) x_1.$$

With this utility function we find $MRS^1 = MRS^2$, and the contract curve is a straight line. This should be the case for utility functions with strictly convex indifference curves. However, if the consumers' preferences are such that the goods are perfect complements or perfect substitutes, the contract curve is not necessarily well defined. For example, with perfect substitutes the indifference curves are straight lines, so every point is a point of tangency between indifference curves, and thus there is no unique contract curve. With perfect complements, there may be a "thick" contract path, not a single line.

7. Give an example of conditions when the production possibilities frontier might not be concave.

The production possibilities frontier is concave if at least one of the production functions exhibits decreasing returns to scale. If both production functions exhibit constant returns to scale, then the production possibilities frontier is a straight line. If both production functions exhibit increasing returns to scale, then the production function is convex. The following numerical examples can be used to illustrate this concept. Assume that $L = 5$ is the fixed labor input, and X and Y are the two goods. The first example is the decreasing returns to scale case, the second example is the constant returns to scale case, and the third example is the increasing returns to scale case. Note further that it is not necessary that both products have identical production functions.

Product *X*		**Product *Y***		***PPF***	
L	***X***	***L***	***Y***	***X***	***Y***
0	0	0	0	0	30
1	10	1	10	10	28
2	18	2	18	18	24
3	24	3	24	24	18
4	28	4	28	28	10
5	30	5	30	30	0

Product *X*		**Product *Y***		***PPF***	
L	***X***	***L***	***Y***	***X***	***Y***
0	0	0	0	0	50
1	10	1	10	10	40
2	20	2	20	20	30
3	30	3	30	30	20
4	40	4	40	40	10
5	50	5	50	50	0

Product *X*		**Product *Y***		***PPF***	
L	***X***	***L***	***Y***	***X***	***Y***
0	0	0	0	0	80
1	10	1	10	10	58
2	22	2	22	22	38
3	38	3	38	38	22
4	58	4	58	58	10
5	80	5	80	80	0

8. A monopsonist buys labor for less than the competitive wage. What type of inefficiency will this use of monopsony power cause? How would your answer change if the monopsonist in the labor market were also a monopolist in the output market?

When market power exists, the market will not allocate resources efficiently. The wage paid by a monopsonist is below the competitive wage, so too little labor is used in the production process and hence too little output is produced. If the firm is also a monopolist in the output market, the firm's MRP_L curve, which is its demand curve for labor, lies below the MRP_L for a competitive firm. Thus, the monopolist's demand for labor is less than for a competitive firm. The result is that even less labor is hired and less output produced. So having monopoly power in the output market exacerbates the inefficiencies in both the labor market and the output market.

9. The Acme Corporation produces *x* and *y* units of goods Alpha and Beta, respectively.

a. Use a production possibility frontier to explain how the willingness to produce more or less Alpha depends on the marginal rate of transformation of Alpha for Beta.

The production possibilities frontier shows all efficient combinations of Alpha and Beta. The marginal rate of transformation of Alpha for Beta is the absolute value of the slope of the production possibilities frontier. The slope measures the marginal cost of producing one good relative to the marginal cost of producing the other. To increase *x*, the units of Alpha, Acme must release inputs in the production of Beta and redirect them to producing Alpha. The rate at which it can efficiently substitute away from Beta to Alpha is given by the marginal rate of transformation.

b. Consider two cases of production extremes: (i) Acme produces zero units of Alpha initially, or (ii) Acme produces zero units of Beta initially. If Acme always tries to stay on its production possibility frontier, describe the initial positions of cases (i) and (ii). What happens as the Acme Corporation begins to produce *both* goods?

In situation (i), Acme produces no units of Alpha and the maximum possible amount of Beta. In situation (ii), it produces the maximum possible amount of Alpha and no units of Beta. Consider what happens in case (i) as Acme starts producing some Alpha along with Beta. Assuming the usual concave shape for the production possibility frontier, Acme will initially enjoy large increases in Alpha production without giving up much Beta. As it produces more and more of Alpha, however, it will have to give up larger and larger increments of Beta because the *MRT* increases (assuming Alpha is plotted on the vertical axis). The same reasoning applies in case (ii) with the roles of Alpha and Beta reversed.

10. In the context of our analysis of the Edgeworth production box, suppose that a new invention changes a constant-returns-to-scale food production process into one that exhibits sharply increasing returns. How does this change affect the production contract curve?

In an Edgeworth production box, the production contract curve is made up of the points of tangency between the isoquants of the two production processes. A change from a constant-returns-to-scale production process to a sharply-increasing-returns-to-scale production process does not necessarily imply a change in the shape of the isoquants. One can simply redefine the quantities associated with each isoquant such that proportional increases in inputs yield greater-than-proportional increases in outputs. Under this scenario, the marginal rate of technical substitution would not change. Thus, there would be no change in the production contract curve.

However, if the change to a sharply-increasing-returns-to-scale technology involved a change in the trade-off between the two inputs (a change in the shape of the isoquants), then the production contract curve would change.

For example, suppose the original production functions for food and the other good (call it clothing) were both $Q = LK$ with $MRTS = \frac{K}{L}$, then the shape of the isoquants would not change if the new production function for food were $Q = L^2K^2$ with $MRTS = \frac{K}{L}$. In both cases the production contract curve would be linear. The shape of the production contract curve would become nonlinear, however, if the new production function for food were $Q = L^2K$ with $MRTS = 2\left(\frac{K}{L}\right)$, assuming the production function for clothing was unchanged.

11. Suppose that Country *A* and Country *B* both produce wine and cheese. Country *A* has 800 units of available labor, while Country *B* has 600 units. Prior to trade, Country *A* consumes 40 pounds of cheese and 8 bottles of wine, and Country *B* consumes 30 pounds of cheese and 10 bottles of wine.

	Country *A*	**Country *B***
Labor per pound cheese	**10**	**10**
Labor per bottle wine	**50**	**30**

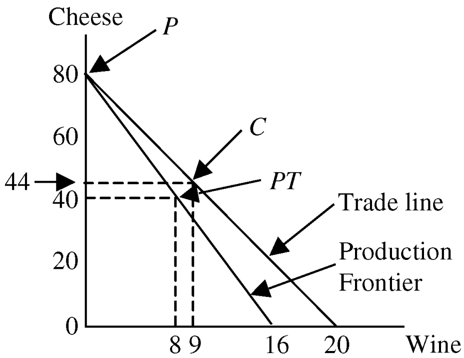
a. Which country has a comparative advantage in the production of each good? Explain.

To produce another bottle of wine, Country *A* needs 50 units of labor, and must therefore produce 5 fewer pounds of cheese. The opportunity cost of a bottle of wine is therefore 5 pounds of cheese. For Country *B* the opportunity cost of a bottle of wine is 3 pounds of cheese. Since

Country *B* has a lower opportunity cost, it has the comparative advantage and should produce the wine and Country *A* should produce the cheese. Country *A* has the comparative advantage in producing cheese because the opportunity cost of cheese in *A* is 1/5 of a bottle of wine while in Country *B* it is 1/3 of a bottle of wine.

b. Determine the production possibilities curve for each country, both graphically and algebraically. (Label the pre-trade production point *PT* and the post-trade production point *P*.)

Country *A*'s production frontier is given by $10C + 50W = 800$, or $C = 80 - 5W$, and Country *B*'s production frontier is $10C + 30W = 600$, or $C = 60 - 3W$. The slope of the frontier for Country *A* is -5, which is the opportunity cost of wine in terms of cheese. Therefore, in Country *A* the price of wine is 5 and in Country *B* the price of wine is 3. Country *A*'s production possibilities curve is the steeper line in the diagram. If Country *A* uses all its labor to make cheese, it can produce 80 pounds of cheese; if it uses all its labor to make wine, it can make 16 bottles of wine.



Before trade, Country *A* produces the combination of cheese and wine at point *PT* (40*C* and 8*W*). After trade, the price of cheese will settle somewhere between 3 and 5. Country *A* will produce only cheese and Country *B* will produce only wine, and they will trade with each other for the other good. Country *A* will produce 80 pounds of cheese at point *P*. Each can consume at a point on the trade line that lies above and outside the production frontier. If the terms of trade result in a wine price of 4, Country *A* will be able to consume combinations of cheese and wine along the shifted-out line (the trade line) in the diagram.

c. Given that 36 pounds of cheese and 9 bottles of wine are traded, label the post-trade consumption point *C*.

See the graph for Country *A* above. Before trade the country consumed and produced at point *PT*, with 40 pounds of cheese and 8 bottles of wine. After trade, Country *A* will completely specialize in the production of cheese and will produce at point *P*. Given the quantities traded, Country *A* will consume $80 - 36 = 44$ pounds of cheese and $0 + 9 = 9$ bottles of wine. This is point *C* on the graph. The graph for Country *B* is similar except that Country *B* will produce only wine. Country *B*'s trade line will be the same as the one for Country *A*, and Country *B* will consume $0 + 36 = 36$ pounds of cheese and $20 - 9 = 11$ bottles of wine.

d. Prove that both countries have gained from trade.

Both countries have gained from trade because they can now both consume more of both goods than they could before trade. Graphically we can see this by noticing that the trade line lies to the right of the production frontier. After trade, each country can consume on the trade line and can consume more of both goods. Numerically, Country *A* consumes 4 more pounds of cheese and 1 more bottle of wine after trade as compared to pre-trade, and Country *B* consumes 6 more pounds of cheese and 1 more bottle of wine.

e. What is the slope of the price line at which trade occurs?

The slope is −4 because 36 pounds of cheese were traded for 9 bottles of wine, so 4 pounds of cheese were given up for every bottle of wine. Therefore, the price of each bottle of wine is 4 (in terms of cheese).

12. Suppose a bakery has 16 employees to be designated as bread bakers (*B*) and cake bakers (*C*), so that $B + C = 16$. Draw the production possibilities frontier for bread (*y*) and cakes (*x*) for the following production functions:

a. $\boldsymbol{y = 2B^{0.5}}$ **and** $\boldsymbol{x = C^{0.5}}$

There are two ways to do this. First is to calculate a few points on the production possibilities frontier (*PPF*) and then draw the curve. Because there are 16 employees, $B = 16 - C$. So, for any value of C we can calculate the amount of cake produced by C employees and the amount of bread produced by $16 - C$ employees using the production functions for x and y. The table below does this for selected values of C.

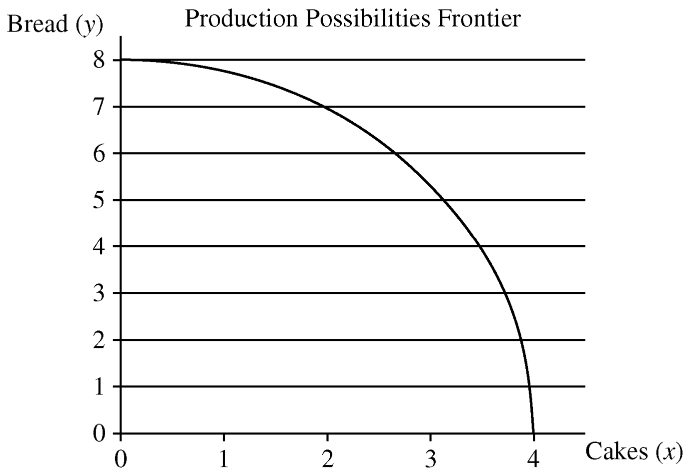
Cake Bakers (*C*)	Cake (*x*)	Bread (*y*)
0	0	8.00
4	2.00	6.93
8	2.83	5.66
12	3.46	4.00
16	4.00	0

A more elegant method is to solve for the *PPF* function.

Since $B = 16 - C$, we can write $\boldsymbol{y = 2(16 - C)^{0.5}}$. And since $\boldsymbol{x = C^{0.5}}$, it follows that $\boldsymbol{C = x^2}$.

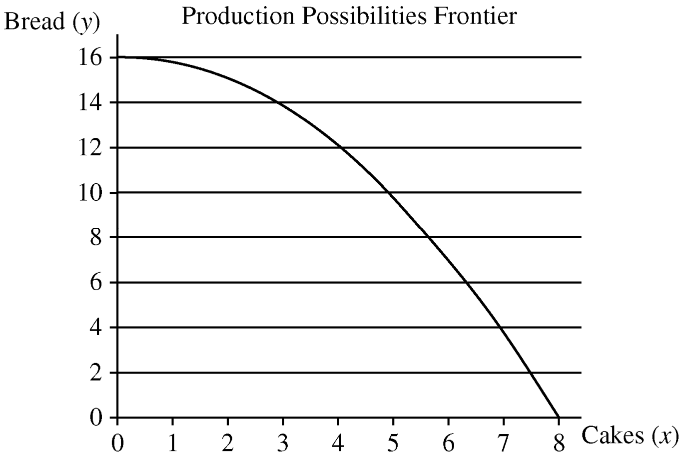
Substituting for C in the expression for y, we get $\boldsymbol{y = 2(16 - x^2)^{0.5}}$, which can also be written as $\boldsymbol{y = 2\sqrt{16 - x^2}}$. This is the equation for the *PPF*. You can check that you get the same set of x and y values in the table above using this equation.

The plot of the *PPF* is given in the diagram below. It has the usual concave shape.



b. $\boldsymbol{y = B}$ **and** $\boldsymbol{x = 2C^{.5}}$

Since $B = 16 - C$, we can write the equation for y as $y = 16 - C$. Now solve x's production function for C, which yields $\boldsymbol{C = \left(\frac{x}{2}\right)^2}$. Substitute into the expression for y to get the function for the *PPF*: $\boldsymbol{y = 16 - \left(\frac{x}{2}\right)^2}$. This *PPF* is plotted below. Note that even though the production function for bread (y) is linear, the *PPF* is concave because the production function for cake (x) has diminishing returns to scale.



Chapter 17
Markets With Asymmetric Information

■ Teaching Notes

This chapter explores different situations in which one party knows more than the other; i.e., there is asymmetric information. Section 17.1 discusses the famous "lemons" problem where the seller has more information than the buyer, and the cases of insurance and credit markets where the buyer has more information. The issue of adverse selection is highlighted. Section 17.2 discusses market signaling as a mechanism to deal with the problem of asymmetric information. Section 17.3 discusses moral hazard where one party has more information about his or her behavior than does the other party. Section 17.4 discusses the principal-agent problem, and Section 17.5 extends the analysis to the case of an integrated firm. Both sections address the issue of differing goals between owners and managers. Section 17.6 examines efficiency wage theory.

You might introduce asymmetric information by reminding students that virtually every topic covered in the course has assumed perfect information. For example, except for Chapter 5 and sections of Chapter 15, we have assumed perfect knowledge of the future (no uncertainty). In models of uncertainty, consumers and producers play "games against nature." In models of asymmetric information, they are playing games against each other.

Many students are likely to have bought or sold a used car and will therefore find the lemons model interesting. You could start your presentation by asking the sellers of used cars how they determined their asking price and whether they thought they received a fair price when they sold their vehicle. Emphasize the intuition of the model before presenting Figure 17.1. If they have understood the model, they should realize why they might have gotten less for their car than they thought it was worth. You can ask how a seller can convince a buyer that his used car is of high quality. See if students can come up with strategies such as encouraging the buyer to have the car inspected by a mechanic. You could also ask why some car ads indicate that the used car is a "one-owner" car. If they have trouble with that question, ask whether it would matter to them as potential buyers if they found out a car had been sold a number of times before.

The market for insurance is also one with which most students are familiar. Although car insurance is required in many states, liability limits may vary from policy to policy. Discuss how risk-averse individuals will want to purchase policies with higher limits but so will people who think they are more likely to have accidents. Ask how insurance companies determine the riskiness of insuring a particular driver. If you have used the example of buying a house in Chapter 15, you may extend it here by considering how bankers determine whether borrowers will default on their home loans.

When discussing market signaling, point out the dual function of education (as training and as a signal of higher productivity). The "Simple Model of Job Market Signaling," is presented in Section 17.2. When you cover Figure 17.2, explain how educational degrees lead to discontinuities, and stress the relationship between degrees, guarantees, and warranties of educational quality.

Moral hazard is an easy concept to illustrate with examples, but it is important to draw a clear distinction between adverse selection and moral hazard. For example, consider someone who buys homeowner's insurance. The adverse selection problem is that a greater proportion of people who believe their homes are likely to suffer damage will want to buy insurance than people who think it is unlikely their homes will suffer damage. Thus insurance companies will find that the homes they insure are more likely than average to be damaged. The moral hazard problem is that once people have homeowner's insurance, they are less vigilant in protecting their homes from damage. This is true regardless of whether the home was originally more or less likely to suffer damage. The ownership of insurance itself changes the probability of damage because it changes people's behavior.

The principal-agent problem is presented in the context of the relationship between employer and employee and between managers and owners. It can be generalized to the relationship between a regulator and a regulated firm and to the relationship between voters and elected officials.

In discussing the problems of monitoring agents, you can introduce the concept of transactions costs if you have not done so previously. The most interesting topic of this section (I think) is how to design contracts to provide the proper incentives for agents to perform in the interest of the principal. The starred Section 17.5 extends this topic to managerial incentives in an integrated firm; that is, a firm consisting of several divisions, each with its own managers. The model can be applied to government contracts, e.g., defense contracts, for a discussion of cost-plus contracting.

The shirking model of efficiency wages is conceptually difficult. After discussing efficiency in Chapter 16, students might wonder what is so efficient about paying workers a wage that is greater than the value of their marginal product. Stress the role of asymmetric information here: firms have imperfect information about individual worker productivity. You might find it helpful to read the references in Footnote 20. While Yellen's article is concise, Stiglitz's is more general, discussing shirking on page 20 and the relationship between efficiency wage theory and unemployment on pages 33–37.

■ Questions for Review

1. **Why can asymmetric information between buyers and sellers lead to market failure when a market is otherwise perfectly competitive?**

   Asymmetric information leads to market failure because the transaction price does not reflect either the marginal benefit to the buyer or the marginal cost of the seller. The competitive market fails to achieve an output with a price equal to marginal cost. In some extreme cases, if there is no mechanism to reduce the problem of asymmetric information, the market collapses completely. For example, in the used car case buyers do not know for sure if they will be getting a high or low quality car, and as a result they tend to be willing to pay less for a car than high quality owners are willing to accept. As a result, not many high quality cars will be offered for sale and this leads to market failure.

2. **If the used car market is a "lemons" market, how would you expect the repair record of used cars that are sold to compare with the repair record of those not sold?**

   In the market for used cars, the seller has a better idea of the quality of the used car than does the buyer. The repair record of a used car is one indicator of its quality. We would expect that, at the margin, cars with good repair records would be kept while cars with poor repair records would be sold. Thus, you would expect repair records of used cars that are sold to be worse than those of used cars not sold, i.e., kept by owners.

3. **Explain the difference between adverse selection and moral hazard in insurance markets. Can one exist without the other?**

   In insurance markets, both adverse selection and moral hazard exist. Adverse selection refers to the self-selection of individuals who purchase insurance policies. That is, people who are less risky than average will, at the margin, choose not to insure, while people more risky than the population as a whole will choose to insure. As a result, the insurance company is left with a riskier pool of policy holders. The problem of moral hazard occurs after the insurance is purchased. Once insurance is purchased, less risky individuals might engage in behavior characteristic of more risky individuals. If policy holders are fully insured, they have little incentive to avoid risky situations.

An insurance firm may reduce adverse selection, without reducing moral hazard, and vice versa. Collecting information such as a medical history to determine the riskiness of a *potential* customer helps insurance companies reduce adverse selection. Insurance companies also reevaluate premiums (sometimes canceling policies) when many claims are made, thereby reducing moral hazard. Copayments also reduce moral hazard by creating a disincentive for policyholders to engage in risky behavior.

4. Describe several ways in which sellers can convince buyers that their products are of high quality. Which methods apply to the following products: Maytag washing machines, Burger King hamburgers, large diamonds?

Some sellers signal the quality of their products to buyers through (1) investment in a good reputation, (2) the standardization of products, (3) certification (e.g., the use of educational degrees in the labor market), (4) guarantees, and (5) warranties. Maytag signals the high quality of its washing machines by offering one of the best warranties in the market. Burger King relies on the standardization of its hamburgers, e.g., the Whopper. The sale of a large diamond is accompanied by a certificate that verifies the weight and shape of the stone and discloses any flaws.

5. Why might a seller find it advantageous to signal the quality of a product? How are guarantees and warranties a form of market signaling?

Firms producing high-quality products would like to charge higher prices, but to do this successfully, potential consumers must be made aware of the quality differences among brands. One method of providing product quality information is through guarantees (i.e., the promise to return what has been given in exchange if the product is defective) and warranties (i.e., the promise to repair or replace if defective). Since low-quality producers are unlikely to offer costly signaling devices, consumers can correctly view a guarantee or an extensive warranty as a signal of high quality, thus confirming the effectiveness of these measures as signaling devices.

6. Joe earned a high grade point average during his four years of college. Is this achievement a strong signal to Joe's future employer that he will be a highly productive worker? Why or why not?

Yes, for the most part a high grade point average is a strong signal to the employer that the employee will perform at an above-average level. Regardless of what he actually learned, it indicates that Joe is able to outperform the majority of students. On the other hand, Joe could have padded his schedule with easy classes, and/or classes taught by easy professors.

7. Why might managers be able to achieve objectives other than profit maximization, which is the goal of the firm's shareholders?

It is difficult and costly for shareholders (the firm's owners) to constantly monitor the actions of the firm's managers, so managers' behavior is never scrutinized 100% of the time. Therefore, managers have some leeway to pursue their own objectives and not just profit maximization.

8. How can the principal-agent model be used to explain why public enterprises, such as post offices, might pursue goals other than profit maximization?

Managers of public enterprises can be expected to act in much the same way as managers of private enterprises, in terms of having an interest in power and other perks, in addition to profit maximization. The problem of overseeing a public enterprise is one of asymmetric information. The manager (agent) is more familiar with the cost structure of the enterprise and the benefits to the customers than the principal, an elected or appointed official, who must elicit cost information controlled by the manager. The costs of eliciting and verifying the information, as well as independently gathering information on the benefits provided by the public enterprise, can be more than the difference between the agency's potential net returns ("profits") and realized returns. This difference provides room for slack, which can be distributed to the management as personal benefits, to the agency's workers as greater-than-efficient job security, or to the agency's customers in the form of greater-than-efficient provision of goods or services.

9. Why are bonus and profit-sharing payment schemes likely to resolve principal-agent problems, whereas a fixed-wage payment will not?

With a fixed wage, the agent-employee has no incentive to maximize productivity. If the agent-employee is hired at a fixed wage equal to the marginal revenue product of the average employee, there is no incentive to work harder than the least productive worker. Bonus and profit-sharing schemes involve a lower fixed wage than fixed-wage schemes, but they include a bonus wage. The bonus can be tied to the profitability of the firm, to the output of the individual employee, or to that of the group in which the employee works. These schemes provide a greater incentive for agents to maximize the objective function of the principal.

10. What is an efficiency wage? Why is it profitable for the firm to pay it when workers have better information about their productivity than firms do?

An efficiency wage, in the context of the shirking model, is the wage at which no shirking occurs. If employers cannot monitor employees' productivity, then employees may shirk (work less productively), which will affect the firm's output and profits. It therefore pays the firm to offer workers a higher-than-market wage, thus reducing the workers' incentive to shirk, because they know that if they are fired and end up working for another firm, their wage will fall. Firms may also pay efficiency wages in order to reduce turnover among employees. If employees are paid a higher wage, all else the same, they will tend to be happier at their jobs and less likely to leave and find new jobs. High turnover rates can be costly for the firm in terms of having to continually train new employees.

■ Exercises

1. Many consumers view a well-known brand name as a signal of quality and will pay more for a brand-name product (e.g., Bayer aspirin instead of generic aspirin, Birds Eye frozen vegetables instead of the supermarket's own brand). Can a brand name provide a useful signal of quality? Why or why not?

A brand name can provide a useful signal of quality for several reasons. First, when information asymmetry is a problem, one solution is to create a "brand-name" product. Standardization of the product produces a reputation for a given level of quality that is signaled by the brand name. Second, if the development of a brand-name reputation is costly (i.e., advertising, warranties, etc.), the brand name is a signal of higher quality. Finally, pioneer products, by virtue of their "first-mover" status, enjoy consumer loyalty if the products are of acceptable quality. The uncertainty surrounding newer products inhibits defection from the pioneering brand-name product.

2. Gary is a recent college graduate. After six months at his new job, he has finally saved enough to buy his first car.

a. Gary knows very little about the differences between makes and models. How could he use market signals, reputation, or standardization to make comparisons?

Gary's problem is one of asymmetric information. As a buyer of a first car, he will be negotiating with sellers who know more about cars than he does. His first choice is to decide between a new or used car. If he buys a used car, he must choose between a professional used-car dealer and an individual seller. Each of these three types of sellers (the new-car dealer, the used-car dealer, and the individual seller) uses different market signals to convey quality information about their products.

The new-car dealer, working with the manufacturer (and relying on the manufacturer's reputation) can offer standard and extended warranties that guarantee the car will perform as advertised.

Because few used cars carry a manufacturer's warranty, and because the used-car dealer is not intimately familiar with the condition of the cars on his or her lot (because of their wide variety and disparate previous usage), it is not in his or her self-interest to offer extensive warranties. The used-car dealer, therefore, must rely on reputation, particularly on a reputation of offering "good values." Since the individual seller neither offers warranties nor relies on reputation, purchasing from such a seller could make it advisable to seek additional information from an independent mechanic or from reading the used-car recommendations in *Consumer Reports*. Given his lack of experience, Gary should gather as much information about these market signals, reputation, and standardization as he can afford.

b. **You are a loan officer in a bank. After selecting a car, Gary comes to you seeking a loan. Because he has only recently graduated, he does not have a long credit history. Nonetheless, the bank has a long history of financing cars for recent college graduates. Is this information useful in Gary's case? If so, how?**

The bank's problem in lending money to Gary is also one of asymmetric information. Gary has a much better idea than the bank about the quality of the car and his ability to pay back the loan. While the bank can learn about the car through the reputation of the manufacturer and through inspection (if it is a used car), the bank has little information on Gary's ability to handle credit. Therefore, the bank must infer information about Gary's credit worthiness from easily available information, such as his recent graduation from college, how much he borrowed while in school, and the similarity of his educational and credit profile to that of college graduates currently holding car loans from the bank. If recent graduates have built a good reputation for paying off their loans, Gary can use this reputation to his advantage, but poor repayment patterns by this group will lessen his chances of obtaining a car loan from this bank.

3. **A major university bans the assignment of D or F grades. It defends its action by claiming that students tend to perform above average when they are free from the pressures of flunking out. The university states that it wants all its students to get A's and B's. If the goal is to raise overall grades to the B level or above, is this a good policy? Discuss this policy with respect to the problem of moral hazard.**

By eliminating the lowest grades, the university creates a moral hazard problem similar to that which is found in insurance markets. Since they are protected from receiving a low grade, some students will have little incentive to work at above-average levels. The policy only addresses the pressures facing below-average students, i.e., those who flunk out. Average and above-average students do not face the pressure of failing. For these students, the pressure of earning top grades (instead of learning a subject well) remains. Their problems are not addressed by this policy. Therefore, the policy creates a moral hazard problem primarily for the below-average students who are its intended beneficiaries.

4. **Professor Jones has just been hired by the economics department at a major university. The president of the Board of Regents has stated that the university is committed to providing top-quality education for undergraduates. Two months into the semester, Jones fails to show up for his classes. It seems he is devoting all his time to research rather than to teaching. Jones argues**

that his research will bring prestige to the department and the university. Should he be allowed to continue exclusively with research? Discuss with reference to the principal-agent problem.

The Board of Regents and its president can be thought of as the principals of the university, while faculty are the agents. The dual purpose of most universities is teaching students and producing research, and most faculty are hired to perform both tasks.

The problem is that teaching effort can be easily monitored (particularly if Jones does not show up for class), while the benefits of establishing a prestigious research reputation are uncertain and are realized only over time. While the quantity of research is easy to see, determining research quality is more difficult. The university should not simply take Jones' word regarding the benefits of his research and allow him to continue exclusively with his research without altering his payment scheme. One alternative would be to tell Jones that he does not have to teach if he is willing to accept a lower salary. On the other hand, the university could offer Jones a bonus if, due to his research reputation, he is able to bring prestige to the department and university, particularly if this results in a lucrative grant or donations to the university.

5. **Faced with a reputation for producing automobiles with poor repair records, a number of American companies have offered extensive guarantees to car purchasers (e.g., a seven-year warranty on all parts and labor associated with mechanical problems).**

   a. **In light of your knowledge of the lemons market, why is this a reasonable policy?**

   At one time, American companies enjoyed a reputation for producing high-quality cars. More recently, faced with competition especially from Japanese car manufacturers, their products appeared to customers to be of lower quality. As this reputation spread, customers were less willing to pay high prices for American cars. To reverse this trend, American companies invested in quality control, improving the repair records of their products. Consumers, however, still considered American cars to be of lower quality (lemons, in some sense), and would not buy them, American companies were forced to signal the improved quality of their products to their customers. One way of providing this information is through improved warranties that directly address the issue of poor repair records. This was a reasonable reaction to the "lemons" problem that they faced.

   b. **Is the policy likely to create a moral hazard problem? Explain.**

   Moral hazard occurs when the insured party (here, the owner of an American automobile with an extensive warranty) can influence the probability of the event that triggers payment (the repair of the automobile). The coverage of all parts and labor associated with mechanical problems reduces the incentive to maintain the automobile. Hence, a moral hazard problem is created by the offer of extensive warranties. To avoid this problem, manufacturers often stipulate that their warranties will not be honored unless the owner has all recommended routine maintenance done and has proof that the maintenance was performed.

6. **To promote competition and consumer welfare, the Federal Trade Commission requires firms to advertise truthfully. How does truth in advertising promote competition? Why would a market be less competitive if firms advertised deceptively?**

Truth in advertising promotes competition by providing the information necessary for consumers to make optimal decisions. Competitive forces function properly only if consumers are aware of all prices (and qualities), so comparisons may be made. In the absence of truthful advertising, buyers are unable to make these comparisons because goods priced identically can be of different quality. Hence there will be a tendency for buyers to stick with proven products, reducing competition between existing firms and discouraging entry. Note that monopoly rents may result when consumers stick with proven products.

7. An insurance company is considering issuing three types of fire insurance policies: (i) complete insurance coverage, (ii) complete coverage above and beyond a $10,000 deductible, and (iii) 90% coverage of all losses. Which policy is more likely to create moral hazard problems?

Moral hazard problems arise with fire insurance when the insured party can influence the probability of a fire and the magnitude of loss from a fire. The property owner can engage in behavior that reduces the probability of a fire by, for example, inspecting and replacing faulty wiring, and by making sure oily rags and other flammable items are not stored indoors. The magnitude of losses can be reduced by the installation of fire extinguishers and warning systems like smoke alarms and by the storage of valuables away from areas where fires are likely to start.

After purchasing complete insurance, the insured has little incentive to reduce either the probability or the magnitude of the loss, and the moral hazard problem will be worst with this coverage. In order to compare a $10,000 deductible to 90% coverage, we would need information on the value of the potential loss. Both policies reduce the moral hazard problem posed by complete coverage. If the property is worth less (more) than $100,000, the total loss to the owner will be less (more) with 90% coverage than with the $10,000 deductible. So, for example, if the value of the property is above $100,000, the owner is more likely to engage in fire prevention efforts under the policy that offers 90% coverage than under the one that offers the $10,000 deductible.

8. You have seen how asymmetric information can reduce the average quality of products sold in a market, as low-quality products drive out high-quality products. For those markets in which asymmetric information is prevalent, would you agree or disagree with each of the following? Explain briefly:

a. The government should subsidize *Consumer Reports*.

Asymmetric information implies unequal access to information by either buyers or sellers, a problem that leads to inefficient markets or market collapse. Encouraging the gathering and publishing of information can be advantageous in general because it helps consumers make better decisions and promotes honesty on the part of sellers. It is not clear, however, that subsidizing *Consumer Reports* would be appropriate. Currently, people who find the information useful pay for it, and those who do not want the information (perhaps because they can judge the quality of products themselves) do not have to pay for it. If *Consumer Reports* were subsidized, everyone would pay for it. Also, if the government subsidized it, the government might require it to use particular quality tests or treat U.S. manufacturers differently from foreign manufacturers, compromising the magazine's objectivity. Note, though, that the government does provide an indirect subsidy to the publication; it has granted Consumers Union (the publisher of *Consumer Reports*) nonprofit status.

b. The government should impose quality standards—e.g., firms should not be allowed to sell low-quality items.

This is a bad idea. First of all, some people prefer low-quality goods if they are sufficiently inexpensive. Banning low-quality goods would reduce consumers' choices and reduce utility. Secondly, after imposing quality standards, the government would have to monitor the quality of all items sold, and this would be very costly.

c. The producer of a high-quality good will probably want to offer an extensive warranty.

Agree. This option provides the least-cost solution to the problems of asymmetric information. It allows the producer to distinguish its product from low-quality goods because it is more costly for the low-quality producer to offer an extensive warranty than for the high-quality producer to offer one.

d. The government should require *all* firms to offer extensive warranties.

Disagree. By requiring *all* firms to offer extensive warranties, the government would drive most low-quality products out of the market, because it would be too costly for low-quality producers to service such warranties. The effect is similar to a ban on low-quality products as in part b. Such a requirement would also negate the market signaling value of warranties offered by producers of high-quality goods. Thus, to the extent that some low-quality goods were still being sold, high-quality producers would have a more difficult time signaling the quality of their products.

9. Two used car dealerships compete side by side on a main road. The first, Harry's Cars, always sells high-quality cars that it carefully inspects and, if necessary, services. On average, it costs Harry's $8000 to buy and service each car that it sells. The second dealership, Lew's Motors, always sells lower-quality cars. On average, it costs Lew's only $5000 for each car that it sells. If consumers knew the quality of the used cars they were buying, they would pay $10,000 on average for Harry's cars and only $7000 on average for Lew's cars.

Without more information, consumers do not know the quality of each dealership's cars. In this case, they would figure that they have a 50-50 chance of ending up with a high-quality car, and are thus willing to pay $8500 for a car.

Harry has an idea: He will offer a bumper-to-bumper warranty for all cars he sells. He knows that a warranty lasting *Y* years will cost $500*Y* on average, and he also knows that if Lew tries to offer the same warranty, it will cost Lew $1000*Y* on average.

a. Suppose Harry offers a one-year warranty on all of the cars he sells.

i. What is Lew's profit if he does not offer a one-year warranty? If he does offer a one-year warranty?

Without offering the warranty, consumers will know that Lew's cars are of low quality, so Lew would make a profit of $2000 per car ($7000 – 5000). If he were to offer the warranty, each car would cost Lew $6000 (including repairs under warranty), but as consumers would then not be able to determine the quality of the cars (since both dealers would offer the same warranty) they will be willing to pay only $8500 for all cars, and Lew's will make $2500 per car ($8500 – 6000).

ii. What is Harry's profit if Lew does not offer a one-year warranty? If he does offer a one-year warranty?

If Lew does not offer a one-year warranty then Harry's can buy its cars for $8000, sell the cars for $10,000, and make a profit of $1500 per car after the $500 warranty cost. If Lew does offer a one-year warranty then Harry's will be able to sell its cars for only $8500, and the company will not make any profit.

iii. Will Lew's match Harry's one-year warranty?

Lew's will match Harry's warranty because, if it does, its profit increases from $2000 to $2500 per car.

iv. Is it a good idea for Harry to offer a one-year warranty?

No. Harry should not offer the one-year warranty unless he thinks that Lew will act irrationally and not offer a one-year warranty. Given that Lew will match the warranty, Harry is better off not offering the warranty.

b. What if Harry offers a two-year warranty? Will this offer generate a credible signal of quality? What about a three-year warranty?

If Harry offers a two-year warranty, each car will cost \$9000. Harry will earn \$1000 per car, as consumers will recognize the higher quality of its cars. Lew's will not offer a two-year warranty, because if they do, they will earn a profit of only \$1500 per car, which is less than the \$2000 they would earn without offering the warranty. So the two-year warranty is a credible signal.

With a three-year warranty Harry would be making \$500 per car, the same that he would have made had it not signaled the higher quality of its cars with a warranty. Therefore, Harry would not offer a three-year warranty.

c. If you were advising Harry, how long a warranty would you urge him to offer? Explain why.

Harry needs to offer a warranty of sufficient length so that Lew's will not find it profitable to match the warranty, and such that Harry's profit is at least as high as it is without offering a warranty. Let t denote the number of years of the warranty, then Lew's will offer a warranty according to the following inequality:

$$7000 - 5000 \leq 8500 - 5000 - 1000t, \text{ or } t \leq 1.5.$$

Therefore, Harry should offer a 1.5-year warranty on his cars. Lew will not find it profitable to match this warranty, and Harry's profit will be \$10,000 – 8000 – 500(1.5) = \$1250.

10. As chairman of the board of ASP Industries, you estimate that your annual profit is given by the table below. Profit (Π) is conditional upon market demand and the effort of your new CEO. The probabilities of each demand condition occurring are also shown in the table.

Market Demand	**Low Demand**	**Medium Demand**	**High Demand**
Market Probabilities	**0.30**	**0.40**	**0.30**
Low Effort	**Π = \$ 5 million**	**Π = \$10 million**	**Π = \$15 million**
High Effort	**Π = \$10 million**	**Π = \$15 million**	**Π = \$17 million**

You must design a compensation package for the CEO that will maximize the firm's expected profit. While the firm is risk neutral, the CEO is risk averse. The CEO's utility function is

Utility = $W^{0.5}$ when making low effort

Utility = $W^{0.5} - 100$ when making high effort

where W is the CEO's income. (The –100 is the "utility cost" to the CEO of making a high effort.) You know the CEO's utility function, and both you and the CEO know all of the information in the preceding table. You do *not* know the level of the CEO's effort at time of compensation or the exact state of demand. You do see the firm's profit, however.

Of the three alternative compensation packages below, which do you as chairman of ASP Industries prefer? Why?

Package 1: Pay the CEO a flat salary of $575,000 per year

Package 2: Pay the CEO a fixed 6% of yearly firm profits

Package 3: Pay the CEO a flat salary of $500,000 per year and then 50% of any firm profits *above* $15 million

The issue here is how to get your CEO to make high effort but not give away too much in profits. For each package, first calculate whether the executive will make high or low effort. Then calculate firm profits under each effort level to determine which package maximizes your profits. The CEO's expected utility under the three packages:

Package 1: The CEO will give low effort to maximize utility:

Low Effort: $E(U) = (\$575{,}000)^{0.5} = 758.29$

High Effort: $E(U) = (\$575{,}000)^{0.5} - 100 = 658.29$.

Package 2: The CEO will give high effort to maximize utility:

Low Effort: $E(U) = 0.3(0.06 \times 5{,}000{,}000)^{0.5} + 0.4(0.06 \times 10{,}000{,}000)^{0.5} + 0.3(0.06 \times 15{,}000{,}000)^{0.5} = 758.76$

High Effort: $E(U) = 0.3[(0.06 \times 10{,}000{,}000)^{0.5} - 100] + 0.4[(0.06 \times 15{,}000{,}000)^{0.5} - 100] + 0.3[(0.06 \times 17{,}000{,}000)^{0.5} - 100] = 814.84$

Package 3: The CEO will give high effort to maximize utility:

Low Effort: $E(U) = 0.3(500{,}000)^{0.5} + 0.4(500{,}000)^{0.5} + 0.3(500{,}000)^{0.5} = 707.11$

High Effort: $E(U) = 0.3[(500{,}000)^{0.5} - 100] + 0.4[(500{,}000)^{0.5} - 100] + 0.3[(1{,}500{,}000)^{0.5} - 100] = 762.40$

The firm's expected profit with each effort level before deducting the CEO's expected compensation is:

Low Effort: $0.30 \times \$5\text{m} + 0.40 \times \$10\text{m} + 0.30 \times \$15\text{m} = \$10\text{m}$

High Effort: $0.30 \times \$10\text{m} + 0.40 \times \$15\text{m} + 0.30 \times \$17\text{m} = \$14.1\text{m}$

Now calculate the firm's expected profit under each package net of expected CEO compensation:

Package 1: Low Effort: $E(\Pi) = \$10\text{m} - \$0.575\text{m} = \$9.425$ million

Package 2: High Effort: $E(\Pi) = \$14.1 - (0.30 \times \$0.6\text{m} + 0.40 \times \$0.9\text{m} + 0.30 \times \$1.02\text{m}) = \$13.254\text{m}$

Package 3: High Effort: $E(\Pi) = \$14.1\text{m} - (0.30 \times \$0.5\text{m} + 0.40 \times \$0.5\text{m} + 0.30 \times \$1.5\text{m}) = \$13.3\text{m}$

To maximize the expected profits of ASP Industries, you should recommend compensation Package 3 that uses a flat salary and then a large bonus when the firm does exceptionally well and makes $17 million. This package is best because it maximizes ASP's expected profit net of compensation – here at a value of $13.3 million.

Notice that if you gave only a huge bonus when the firm did exceptionally well, the CEO's risk aversion might lead him or her to make low effort, or more likely, leave the company to work elsewhere. The flat salary offsets the disincentive effects of a risky but motivating package. This is the usual form of executive compensation. Notice too that compensation is tied to firm profitability.

11. A firm's short-run revenue is given by $R = 10e - e^2$, where e is the level of effort by a typical worker (all workers are assumed to be identical). A worker chooses his level of effort to maximize wage less effort $w - e$ (the per-unit cost of effort is assumed to be 1). Determine the level of effort and the level of profit (revenue less wage paid) for each of the following wage arrangements. Explain why these different principal-agent relationships generate different outcomes.

a. $w = 2$ for $e \geq 1$; otherwise $w = 0$.

There is no incentive for the worker to provide an effort that exceeds 1, as the wage received by the worker will be 2 if the worker provides one unit of effort but will not increase if the worker provides more effort.

The profit for the firm will be revenue minus the wages paid to the worker:

$$\pi = (10)(1) - 1^2 - 2 = \$7.$$

In this principal-agent relationship there is no incentive for the worker to increase his or her effort, as the wage is not related to the revenues of the firm.

b. $w = R/2$.

The worker will maximize the wage net of the effort required to obtain that wage; that is, the worker will maximize:

$$w - e = \frac{10e - e^2}{2} - e, \quad \text{or} \quad 4e - 0.5e^2.$$

To find the maximum effort that the worker is willing to put forth, take the first derivative with respect to effort, set it equal to zero, and solve for effort:

$$\frac{d(4e - 0.5e^2)}{de} = 4 - e = 0, \quad \text{or} \quad e = 4.$$

The wage the worker will receive will be

$$w = \frac{R}{2} = \frac{10(4) - 4^2}{2} = 12.$$

The profits for the firm will be

$$\pi = ((10)(4) - 4^2) - 12 = \$12.$$

With this principal-agent relationship, the wage that the individual worker receives is related to the revenue of the firm. Therefore, we see greater effort on the part of the worker, and as a result, greater profits for the firm.

c. $w = R - 12.5$.

Again, the worker will maximize the wage net of the effort required to obtain that wage; that is, the worker will maximize:

$$w - e = (10e - e^2) - 12.50 - e, \quad \text{or} \quad 9e - e^2 - 12.50.$$

To find the maximum effort that the worker is willing to put forth, take the first derivative with respect to effort, set it equal to zero, and solve for effort:

$$\frac{d(9e - e^2 - 12.50)}{de} = 9 - 2e = 0, \quad \text{or} \quad e = 4.5.$$

The wage the worker will receive is

$$w = R - 12.50 = ((10)(4.5) - 4.5^2) - 12.5 = 12.25.$$

The profits for the firm will be

$$\pi = ((10)(4.5) - 4.5^2) - 12.25 = \$12.50.$$

With this principal-agent relationship, the wage of the worker is more directly related to the performance of the firm than in either a or b. Therefore, compared to the first two wage arrangements, the worker is willing to supply more effort resulting in higher profits for the firm.

12. UNIVERSAL SAVINGS & LOAN has \$1000 to lend. Risk-free loans will be paid back in full next year with 4% interest. Risky loans have a 20% chance of defaulting (paying back nothing) and an 80% chance of paying back in full with 30% interest.

a. How much profit can the lending institution expect to earn? Show that the expected profits are the same whether the lending institution makes risky or risk-free loans.

UNIVERSAL SAVINGS & LOAN earns \$40 in interest on a risk-free loan and \$300 on a risky loan that is paid back. Therefore, expected profits are

Risk-free: $E(\pi) = \$40$ and
Risky: $E(\pi) = 0.80(300) + 0.20(-1000) = \$40$.

b. Now suppose that the lending institution knows that the government will "bail out" UNIVERSAL if there is a default (paying back the original \$1000). What type of loans will the lending institution choose to make? What is the expected cost to the government?

Now the expected profit on risky loans is $E(\pi) = 0.80(300) + 0.20(0) = \$240$.

UNIVERSAL is much better off making risky loans because the expected profit of \$240 is much greater than the \$40 expected profit from risk-free loans.

The cost to the government is \$1000 for each loan that defaults, so the expected cost is $0.80(0) + 0.20(1000) = \$200$ per loan.

c. Suppose that the lending institution doesn't know for sure that there will be a bail out, but one will occur with probability *P*. For what values of *P* will the lending institution make risky loans?

UNIVERSAL's expected profit on risky loans is \$40 with a bail out and \$240 with a bail out. If the probability of a bail out is P, then the probability of no bail out is $1 - P$. Therefore UNIVERSAL's expected profit on risky loans is $240P + 40(1 - P) = 40 + 200P$.

As long as the expected profit on risky loans is greater than on risk-free loans, UNIVERSAL will want to make risky loans. So as long as $40 + 200P > 40$, UNIVERSAL will want to make risky loans. This is true as long as $P > 0$. So as long as there is even a tiny chance (like $P = 0.01$) that the government will bail out the lending institution, UNIVERSAL will make risky loans.

Chapter 18
Externalities and Public Goods

■ Teaching Notes

This chapter discusses the remaining types of market failure that were introduced at the end of Chapter 16 and were not covered in Chapter 17. Section 18.1 defines the concept of externalities, both positive and negative. Section 18.2 discusses methods of correcting for the market failure that arises in the presence of externalities. These two sections give a good self-contained overview of externalities and possible remedies. If you have limited time, try to cover at least these sections.

Section 18.3 considers stock externalities where the social cost is due to the accumulated stock of a pollutant. Global warming is an example that will garner student interest. If you want to pursue the global warming example (Example 18.5), you might consider assigning Exercise 8, but do so only if students know how to calculate net present values. If you do assign Exercise 8, you probably ought to give students the exact net benefit values for all 101 years. These are provided with the answer for that exercise in the following pages. Without the exact values, you will not be able to solve for the requested discount rate.

Section 18.4 deals with property rights and the Coase theorem. Section 18.5 discusses common property resources such as fisheries, and Section 18.6 covers public goods. Section 18.7 offers a brief discussion of determining the optimal level of a public good. Overall the chapter provides a good overview of some very interesting problems. Any instructor who has the time and desire to expand upon the presentation in the chapter can find a wealth of information by consulting an environmental or resource economics textbook. There are an abundance of examples related to pollution and natural resource issues that you could talk about. Check your local newspaper for ideas.

The production and consumption of many goods involve the creation of externalities. Stress the divergence between social and private costs, and the difference between the private (competitive) equilibrium and the socially optimal (efficient) equilibrium. Although private competitive markets produce too much pollution, it is critical to make sure students understand that the optimal amount of pollution is *not* zero. In fact, it is interesting to ask students to define what zero pollution would mean (it's not entirely clear) and what life would be like without pollution (no cars, trucks, man-made fertilizer, computers, cell phones, etc.).

You can use students' knowledge of consumer and producer surplus to explore the welfare gain of moving from the competitive to the efficient equilibrium. Exercise 9 presents the classic beekeeper/apple-orchard problem, originally popularized in James E. Meade, "External Economies and Diseconomies in a Competitive Situation," *Economic Journal*, March 1952, 62:245, 54–67. Empirical research on this example has shown that beekeepers and orchard owners have solved many of their problems: see Steven N.S. Cheung, "The Fable of the Bees: An Economic Investigation," *Journal of Law and Economics*, April 1973, 16:1, 11–33.

One of the main themes of the law and economics literature since 1969 is the application of Coase's insight on the assignment of property rights. The original article is clear and can be understood by students. Stress the problems posed by transactions costs. For a lively debate, ask students whether non-smokers should be granted the right to smokeless air in public places (see Exercise 5). For an extended discussion of the Coase Theorem at the undergraduate level, see A. Mitchell Polinsky, Chapters 3–6, *An Introduction to Law & Economics*, Aspen Publishers, 3rd edition, 2003.

The last two sections of the chapter focus on public goods and private choice. Point out the similarities and differences between public goods and other activities with externalities. Since students confuse nonrival and nonexclusive goods, create a table similar to the following and give examples to fill in the cells:

	Exclusive	**Nonexclusive**
Rival	Most Goods	Congested Local Roads
Nonrival	Cable TV Programs	Public Goods

When determining the amount of a public good the government should provide, some students will not understand why we add individual demand curves vertically rather than horizontally. Stress that by summing horizontally we are finding the total quantity supplied/demanded at any given price. By summing vertically we are finding the total willingness to pay for a given quantity. The coverage of public choice is a limited introduction to the subject, but you can easily expand on this material. A logical extension of this chapter is an introduction to cost-benefit analysis.

■ Questions for Review

1. **Which of the following describes an externality and which does not? Explain the difference.**

   a. **A policy of restricted coffee exports in Brazil causes the U.S. price of coffee to rise—an increase which in turn also causes the price of tea to rise.**

   Externalities cause inefficiencies because the price of the good does not reflect the true social value of the good. A policy of restricting coffee exports in Brazil causes the U.S. price of coffee to rise because supply is reduced. As the price of coffee rises, consumers switch to tea, thereby increasing the demand for tea, and hence increasing the price of tea. These are market effects, not externalities.

   b. **An advertising blimp distracts a motorist who then hits a telephone pole.**

   The advertising blimp is producing information. However, its method of supplying this information can be distracting for some consumers, such as those who happen to be driving. The blimp is creating a negative externality that influences drivers' safety. Since the price the advertising firm charges its client does not incorporate the externality of distracting drivers, too much of this type of advertising is produced from the point of view of society as a whole.

2. **Compare and contrast the following three mechanisms for treating pollution externalities when the costs and benefits of abatement are uncertain: (a) an emissions fee, (b) an emissions standard, and (c) a system of transferable emissions permits.**

   The choice between an emissions fee and an emissions standard depends on the marginal cost and marginal benefit of reducing pollution. First, suppose small changes in abatement yield large benefits while adding little to cost. In this case, if an emissions fee is set too low because of uncertainty, the firm will produce far too many emissions, so a standard is better. However, if small changes in abatement yield little benefit while adding greatly to cost, the cost of reducing emissions is high. In this case, fees should be used because setting a standard too high (due to uncertainty) yields little benefit but increases costs way beyond the efficient level.

A system of transferable emissions permits combines the features of fees and standards to reduce pollution. Under this system, a standard is set and fees are used to transfer permits to firms that value them the most (i.e., firms with high abatement costs). However, because of uncertainty, the total number of permits can be incorrectly chosen. Too few permits will reduce emissions to inefficiently low levels and create excess demand for the permits, increasing their price and inefficiently diverting resources to owners of the permits.

Typically, pollution control agencies implement one of the three mechanisms, measure the results, reassess the success of their choice, then reset new levels of fees or standards or select a new policy tool.

3. **When do externalities require government intervention? When is such intervention unlikely to be necessary?**

   Economic efficiency can be achieved without government intervention when the externality affects a small number of people so that bargaining costs are small. As the Coase theorem tells us, the resulting outcome will be efficient in this case regardless of how property rights are specified. When these conditions are not met, government intervention is often required.

4. **Consider a market in which a firm has monopoly power. Suppose in addition that the firm produces under the presence of either a positive or a negative externality. Does the externality necessarily lead to a greater misallocation of resources?**

   In the presence of a negative externality, a competitive market produces too much output compared to the socially optimal amount. But a monopolist restricts output, so it is possible that the monopolist will produce an output closer to the socially optimal solution. In the case of a positive externality, competitive firms produce too little output. Because a monopolist produces even less output, the monopolist causes a greater misallocation of resources.

5. **Externalities arise solely because individuals are unaware of the consequences of their actions. Do you agree or disagree? Explain.**

   Disagree. It is not that people are unaware but that they have no economic incentive to consider and account for all of the consequences of their actions. If a firm dumps waste into a river that affects a swimming area downstream, it is generating a negative externality for the people downstream. This action maximizes the firm's profit if the firm incurs no private costs for dumping and is not forced to consider the external costs it is imposing on users of the swimming area. This is true whether the firm is aware of these social costs or not.

6. **To encourage an industry to produce at the socially optimal level, the government should impose a unit tax on output equal to the marginal cost of production. True or false? Explain.**

   This statement is false. While a tax can encourage firms to produce at the socially optimal level, the tax should be set equal to the marginal external cost and not the marginal private cost. Competitive firms will maximize profit by producing at the point where price is equal to marginal cost. When there are external costs involved, the marginal private cost of the firm is too low from society's point of view, and as a result too much output is produced. By setting a tax equal to the additional social cost not being realized by the firm (the marginal external cost) the firm will be encouraged to consider all costs and will reduce output because the tax will increase its overall marginal cost.

7. **George and Stan live next door to each other. George likes to plant flowers in his garden, but every time he does, Stan's dog comes over and digs them up. Stan's dog is causing the damage, so if economic efficiency is to be achieved, it is necessary that Stan pay to put up a fence around his yard to confine the dog. Do you agree or disagree? Explain.**

Disagree. Economic efficiency does not require that Stan pay for the fence; it merely requires that Stan and George resolve the problem so that social welfare (total benefits less total costs) is maximized, regardless of who pays for it. For example, George and Stan could split the cost of a fence, George could pay Stan to get rid of his dog, or Stan could pay George not to plant flowers.

Given typical property rights, it seems likely that George could sue Stan and that a court would require Stan to pay for a fence or get rid of his dog. And it seems fair that Stan should have to do this, but it is not required for economic efficiency.

8. An emissions fee is paid to the government, whereas an injurer who is sued and held liable pays damages directly to the party harmed by an externality. What differences in the behavior of victims might you expect to arise under these two arrangements?

When victims can receive the damages directly, they are more likely to file a claim, initiate a suit, and try to overstate their damages. When victims do not receive the damages directly, they are less likely to report violations and are less likely to overstate their damages. In theory, emissions fees paid to the government equal the damage inflicted on others and hence move firms toward the socially optimal level of production. But since the fees are paid to the government rather than to the individuals who were injured, the affected individuals are less likely to file a complaint than they would if they received compensation for the damages directly.

9. Why does free access to a common property resource generate an inefficient outcome?

Free access to a resource means that the marginal cost to the user is less than the marginal social cost, because each user has no incentive to consider how his use of the resource will affect the use of the resource by others. The use of a common property resource by a person or firm reduces others' use of it. For example, the use of water by one consumer restricts its use by another. Since private marginal cost is below social marginal cost, too much of the resource is consumed by the individual user, creating an inefficient outcome. Each individual using the common property resource considers only his own actions and does not consider how all of the users collectively are affecting the resource.

10. Public goods are both nonrival and nonexclusive. Explain each of these terms and show clearly how they differ from each other.

A good is *nonrival* if, for any level of production, the marginal cost of providing the good to an additional individual is zero (although the cost to produce an additional unit could be greater than zero). A good is *nonexclusive* if it is impossible or very expensive to exclude individuals from consuming it once it is available to one individual. Public goods are *nonrival* and *nonexclusive*. Good examples are national defense, a lighthouse, and public television. Some goods are nonrival but exclusive such as a bridge during low traffic periods. One more person can use the bridge without any additional cost to the bridge authority and without imposing costs on other drivers in the form of congestion, but the bridge authority can exclude users by setting up tollbooths. Some goods are nonexclusive but rival. For example, a large lake can be nonexclusive because anyone can use it, but the more people there are fishing, the fewer fish are available to others, so it is rival.

11. A village is located next to 1000 acres of prime grazing land. The village presently owns the land and allows all residents to graze cows freely. Some members of the village council have suggested that the land is being overgrazed. Is this likely to be true? These same members have also suggested that the village should either require grazers to purchase an annual permit or sell off the land to the grazers. Would either of these be a good idea?

It is true that the common land is likely to be overgrazed since each individual will consider only their own private cost and not the total social cost of grazing. The social cost of grazing is likely to be higher than any one individual's private cost because no one individual has an incentive to take into account how his grazing affects the opportunities of others. As a result, conservation efforts by individuals are pointless.

For example, one individual could decide to graze only in certain areas during certain times of the year, while preserving other areas for other times of the year. However, the individual will not do this if the resource is common property as any other grazer can come along and freely disrupt the preservation system that the individual has set up.

Selling annual permits may help, but an annual permit will exclude only those grazers whose total benefits are less than the price of the permit. Anyone who buys the permit will still have the same incentive to overgraze the commons. Selling the land outright is a better solution to the overgrazing problem. If an individual purchases the land she will then have an incentive to consider all of the costs associated with using the land, and as a result will use it in such a way that the resource is preserved, since she alone captures all of the benefits of preserving the resource. Another possibility would be to charge users based on the amount of grazing their cows do. If the grazing fee were set correctly, the efficient amount of grazing could be induced. However, it might be difficult to determine the correct fee, and the village would have to keep track of each resident's grazing and bill him or her accordingly.

12. Public television is funded in part by private donations, even though anyone with a television set can watch for free. Can you explain this phenomenon in light of the free rider problem?

The free rider problem refers to the difficulty of excluding people from consuming a nonexclusive commodity. Non-paying consumers can "free-ride" on commodities provided by paying customers. Public television is funded in part by contributions. Some viewers contribute, but most watch without paying, hoping that someone else will pay so they will not have to. To combat this problem these stations ask consumers to assess their true willingness to pay and ask them to contribute up to this amount. They then attempt to make those people feel good about their actions and make everyone else feel guilty for free riding.

13. Explain why the median voter outcome need not be efficient when majority-rule voting determines the level of public spending.

The median voter is the citizen with the middle preference: half the voting population is more strongly in favor of the issue and half is more strongly opposed. Under majority-rule voting, where each citizen's vote is weighted equally, the preferred spending level on public-goods provision of the *median voter* will win an election against any other alternative. However, majority rule is not necessarily efficient, because it gives each citizen's preferences equal weight. For an efficient outcome, we would need a system that measures and aggregates the willingness to pay of those citizens consuming the public good. Majority rule is not this system. However, as we have seen in previous chapters, majority rule is equitable in the sense that all citizens are treated equally. Thus, we again find a trade-off between equity and efficiency.

14. Would you consider Wikipedia a public good? Does it provide any positive or negative externalities?

Wikipedia (www.wikipedia.org) is a free online encyclopedia that is written and edited primarily by anonymous Internet volunteers who are not paid for doing so. To determine whether it is a public good, recall that public goods are nonrival and nonexclusive. Wikipedia is nonrival because the cost of providing the service to one more user is essentially zero. However, even though it is free, it is not inherently a nonexclusive good because it would be possible to exclude people from consuming the service if Wikipedia required a user fee or some other condition (such as volunteering to write or edit) to use the product. Therefore Wikipedia does not appear to be a public good in the full sense of the term.

An externality occurs when an action by a consumer or producer affects other consumers and/or producers, but is not accounted for in the market price. Since users benefit from the information they glean from Wikipedia but don't have to pay for it, there are positive externalities. There may also be negative externalities to the extent that the information provided by Wikipedia makes it easier for students to plagiarize when writing research papers. This imposes a cost on instructors who must check student work for such behavior.

■ Exercises

1. **A number of firms have located in the western portion of a town after single-family residences took up the eastern portion. Each firm produces the same product and in the process emits noxious fumes that adversely affect the residents of the community.**

   a. **Why is there an externality created by the firms?**

   The noxious fumes emitted by the firms impose costs on the town's residents, and the residents have no control over the quantity of the fumes. Costs may include reduced visibility, difficulty breathing, foul-smelling air, increased health problems, and reduced property values. The firms, however, do not have to pay to release the fumes, so the costs borne by the town's residents are not reflected in the firms' costs or the prices of their products. Thus there is a negative externality created by the firms.

   b. **Do you think that private bargaining can resolve the problem? Explain.**

   If residents anticipated the location of the firms when the eastern part of the town was developed, housing prices would have reflected the disutility of the fumes, and the externality would have been internalized by the housing market in housing prices. In this case there is no problem. If the noxious fumes were not anticipated, private bargaining could resolve the problem of the externality only if there are a relatively small number of parties (both firms and families). Private bargaining would rely on each family's willingness to pay for air quality, but truthful revelation might not be possible. All this will be complicated by the adaptability of the production technology known to the firms and the employment relations between the firms and families. It is unlikely that private bargaining will resolve the problem.

   c. **How might the community determine the efficient level of air quality?**

   The community could determine the economically efficient level of air quality by aggregating the families' willingness to pay and equating it with the marginal cost of pollution reduction. Both steps involve the acquisition of truthful information, which is likely to be quite difficult.

2. **A computer programmer lobbies against copyrighting software, arguing that everyone should benefit from innovative programs written for personal computers and that exposure to a wide variety of computer programs will inspire young programmers to create even more innovative programs. Considering the marginal social benefits possibly gained by this proposal, do you agree with this position?**

   Computer software is an example of a public good. Since it can be costlessly copied, the marginal cost of providing software to an additional user is near zero. Therefore, software is nonrival. (The fixed costs of creating software are high, but the variable costs are low.) Furthermore, it is expensive to exclude consumers from copying and using software because copy protection schemes are available only at high cost or high inconvenience to users. Therefore, software is by and large nonexclusive. As both nonrival and substantially nonexclusive, computer software suffers the problems of public goods provision: the presence of free riders makes it difficult or impossible for markets to provide the efficient level of software. Rather than regulating this market directly, the legal system guarantees property rights to the creators of software. If copyright protection were not enforced, it is likely that the software market would collapse, or that there would be a significant decrease in the quantity of software developed and supplied, which would reduce social benefits. The young programmers would not be inspired to create even more innovative programs because there would be no private reward for doing so. Therefore, you should not agree with the computer programmer.

3. Assume that scientific studies provide you with the following information concerning the benefits and costs of sulfur dioxide emissions:

Benefits of abating (reducing) emissions: $MB = 500 - 20A$

Costs of abating emissions: $MC = 200 + 5A$

where A is the quantity abated in millions of tons and the benefits and costs are given in dollars per ton.

a. What is the socially efficient level of emissions abatement?

To find the socially efficient level of emissions abatement, set marginal benefit equal to marginal cost and solve for A:

$$500 - 20A = 200 + 5A$$

$$A = 12 \text{ million tons.}$$

b. What are the marginal benefit and marginal cost of abatement at the socially efficient level of abatement?

Substitute $A = 12$ into the marginal benefit and marginal cost functions to find the marginal benefit and cost:

$$MB = 500 - 20(12) = 260$$

$$MC = 200 + 5(12) = 260.$$

c. What happens to net social benefits (benefits minus costs) if you abate one million more tons than the efficient level? One million fewer?

Net social benefit is the area under the marginal benefit curve minus the area under the marginal cost curve. At the socially efficient level of abatement this is equal to area $a + b + c + d$ in the figure below, or

$$0.5(500 - 200)(12) = \$1800 \text{ million.}$$

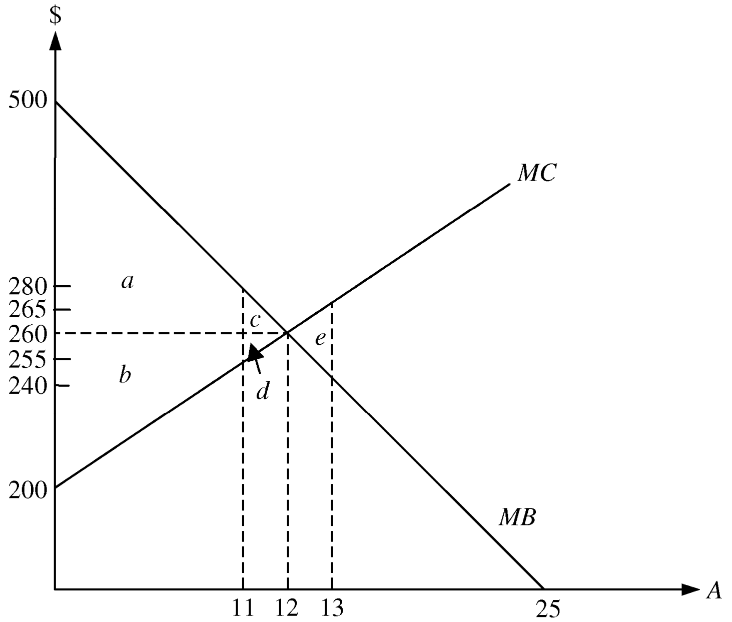
If you abate one million tons too many, the net social benefit is area $a + b + c + d - e$, or

$$1800 - 0.5(265 - 240)(1) = 1800 - 12.5 = \$1787.5 \text{ million.}$$

If you abate 1 million too few tons, then the net social benefit is area $a + b$ or

$$0.5(500 - 280)(11) + (280 - 255)(11) + 0.5(255 - 200)(11) = \$1787.5 \text{ million.}$$

In either case, then, net social benefit falls by 1800 − 1787.5 = $12.5 million.



d. Why is it socially efficient to set marginal benefits equal to marginal costs rather than abating until total benefits equal total costs?

It is socially efficient to set marginal benefit equal to marginal cost rather than total benefit equal to total cost because we want to maximize net benefits, which are total benefits minus total cost. Maximizing total benefits minus total cost means that at the margin, the last unit abated will have an equal cost and benefit. Choosing the point where total benefits are equal to total cost would mean that net benefits equal zero, and would result in too much abatement. This would be analogous to choosing to produce where total revenue was equal to total cost. If total revenue is always equal to total cost by choice, then there will never be any profit. In the case of abatement, the more we abate, the costlier it is. Given that funds tend to be scarce, dollars should be allocated to abatement only so long as the benefit of the last unit of abatement is greater than or equal to the cost of the last unit of abatement.

4. Four firms located at different points on a river dump various quantities of effluent into it. The effluent adversely affects the quality of swimming for homeowners who live downstream. These people can build swimming pools to avoid swimming in the river, and the firms can purchase filters that eliminate harmful chemicals dumped in the river. As a policy advisor for a regional planning organization, how would you compare and contrast the following options for dealing with the harmful effect of the effluent:

a. An equal-rate effluent fee on firms located on the river.

First, one needs to know the value to homeowners of swimming in the river. This information can be difficult to obtain, because homeowners will have an incentive to overstate this value. As an upper bound, if there are no considerations other than swimming, one could use the cost of building swimming pools, either a pool for each homeowner or a public pool for all homeowners. Next, one needs to know the marginal cost of abatement. If the abatement technology is well understood, this information should be readily obtainable. If the abatement technology is not understood, an estimate based on the firms' knowledge must be used.

The choice of a policy tool will depend on the marginal benefits and costs of abatement. If firms are charged an equal-rate effluent fee, the firms will reduce effluent to the point where the marginal cost of abatement is equal to the fee. If this reduction is not high enough to permit swimming, the fee could be increased. Alternatively, revenue from the fees could be used to provide swimming facilities, reducing the need for effluent reduction. If the fee is set equal to the marginal social cost of the pollution, the efficient level of dumping will result.

b. An equal standard per firm on the level of effluent that each can dump.

An equal standard will be efficient only if all the firms have the same marginal abatement costs and the policy maker has complete information regarding the marginal costs and benefits of abatement, so that the efficient level of the standard can be determined. If the marginal costs of abatement differ across firms, an equal standard will result in some firms reducing their effluent by too much and others by too little. Moreover, the standard will not encourage firms to reduce effluent further if new filtering technologies become available.

c. A transferable effluent permit system in which the aggregate level of effluent is fixed and all firms receive identical permits.

A transferable effluent permit system requires the policy maker to determine the efficient total effluent level. Once the permits are distributed and a market develops, firms with higher abatement costs will purchase permits from firms with lower abatement costs. In this way, the effluent level determined by the policy maker will be achieved efficiently. However, unless permits are sold initially rather than given away, no revenue will be generated for the regional organization.

5. Medical research has shown the negative health effects of "secondhand" smoke. Recent social trends point to growing intolerance of smoking in public areas. If you are a smoker and you wish to continue smoking despite tougher anti smoking laws, describe the effect of the following legislative proposals on your behavior. As a result of these programs, do you, the individual smoker, benefit? Does society benefit as a whole?

Since smoking in public areas is similar to polluting the air, the programs proposed here are similar to those examined for air pollution. A bill to lower tar and nicotine levels is similar to an emissions standard, and a tax on cigarettes is similar to an emissions fee. Requiring a smoking permit is similar to a system of emissions permits, assuming that the permits would not be transferable. The individual smoker in all of these programs is being forced to internalize the externality of "secondhand" smoke and will be worse off. Society will be better off if the benefits of a particular proposal outweigh the cost of implementing that proposal. Unfortunately, the benefits of reducing secondhand smoke are uncertain, and assessing those benefits is costly.

a. A bill is proposed that would lower tar and nicotine levels in all cigarettes.

Some smokers might actually smoke more in an effort to maintain a constant level of consumption of nicotine, although the total amount of tar and nicotine released into the air would probably be reduced. The smoker is worse off because he or she will spend more on cigarettes and consume less tar and nicotine. Nonsmokers would be better off because less tar and nicotine would be in the air. It is difficult to know whether society as a whole would be better or worse off.

b. A tax is levied on each pack of cigarettes.

Producers will pay some of the tax and consumers (i.e., smokers) will also pay a portion. Thus the price of cigarettes will increase, and smokers will smoke fewer cigarettes. The extent of the effect of the tax depends on the elasticity of demand for cigarettes. Nonsmokers would be better off because there is less smoking but smokers are worse off, so it is unclear whether society as a whole benefits. Also, some smokers might substitute cigars or pipes for cigarettes, which might actually be worse for nonsmokers.

c. A tax is levied on each pack of cigarettes sold.

It does not matter upon whom the tax is levied, it will be shared between consumers and producers in exactly the same proportions as in part b, so the effects will be the same as in part b.

d. Smokers would be required to carry government-issued smoking permits at all times.

Smoking permits effectively transfer property rights to clean air from smokers to nonsmokers. A major issue with this program would be the high cost of enforcing the permits. The price of the permit would induce some smokers to quit smoking, but it would not raise the marginal cost of smoking. Therefore smokers who bought permits would continue to smoke about the same amount. Again, smokers would be worse off and nonsmokers better off, so it is unclear whether society benefits as a whole.

6. The market for paper in a particular region in the United States is characterized by the following demand and supply curves

$$Q_D = 160{,}000 - 2000P \quad \text{and} \quad Q_S = 40{,}000 + 2000P,$$

where Q_D is the quantity demanded in 100-pound lots, Q_S is the quantity supplied in 100-pound lots, and P is the price per 100-pound lot. Currently there is no attempt to regulate the dumping of effluent into streams and rivers by the paper mills. As a result, dumping is widespread. The marginal external cost (*MEC*) associated with the production of paper is given by the curve $MEC = 0.0006Q_S$.

a. Calculate the output and price of paper if it is produced under competitive conditions and no attempt is made to monitor or regulate the dumping of effluent.

The equilibrium price and output would be where quantity demanded is equal to quantity supplied:

$$160{,}000 - 2000P = 40{,}000 + 2000P$$

$$4000P = 120{,}000$$

$$P = \$30 \text{ per 100-pound lot, and}$$

$$Q = 100{,}000 \text{ lots.}$$

b. Determine the socially efficient price and output of paper.

To find the socially efficient solution, we need to consider the external costs, as given by $MEC = 0.0006Q_S$, as well as the private costs, as given by $Q_S = 40{,}000 + 2000P$. Rewriting the supply curve, the private costs are $P = 0.0005Q_S - 20 = MC$. Therefore,

$$MSC = MC + MEC = 0.0005Q_S - 20 + 0.0006Q_S$$

$$MSC = 0.0011Q_S - 20.$$

Solve the demand curve for price: $P = 80 - 0.0005Q$. This is the marginal benefit curve. Setting marginal social cost equal to marginal benefit,

$$0.0011Q - 20 = 80 - 0.0005Q$$

$$Q = 62{,}500 \text{ lots, and}$$

$$P = \$48.75 \text{ per lot.}$$

c. Explain why the answers you calculated in parts a and b differ.

The equilibrium quantity declined and the equilibrium price rose in part b because the external costs were considered. Ignoring the external costs of paper production results in too much paper being produced and sold at too low a price.

7. In a market for dry cleaning, the inverse market demand function is given by $P = 100 - Q$ and the (private) marginal cost of production for the aggregation of all dry-cleaning firms is given by $MC = 10 + Q$. Finally, the pollution generated by the dry-cleaning process creates external damages given by the marginal external cost curve $MEC = Q$.

a. Calculate the output and price of dry cleaning if it is produced under competitive conditions without regulation.

Set demand equal to supply to find the competitive equilibrium. To do this, set price equal to marginal cost (which is the industry supply function):

$$100 - Q = 10 + Q, \text{ so}$$
$$Q = 45, \text{ and } P = \$55.$$

b. Determine the socially efficient price and output of dry cleaning.

First, calculate the marginal social cost (MSC), which is equal to the marginal external cost plus the private marginal cost. Next, set MSC equal to the market demand function to solve for price and quantity. When all costs are included, the quantity produced will fall and the price will rise:

$$MSC = MC + MEC = (10 + Q) + Q = 10 + 2Q.$$
$$\text{Setting } MSC = MSB\text{: } 10 + 2Q = 100 - Q, \text{ so}$$
$$Q = 30, \text{ and } P = \$70.$$

c. Determine the tax that would result in a competitive market producing the socially efficient output.

If there is a unit tax, t, then the new marginal private cost function is $MC' = (10 + Q) + tQ$. If we now set this new marginal cost function equal to the efficient price of \$70 and substitute 30 for the quantity, we can solve for t:

$$10 + Q + tQ = 70$$
$$30(1 + t) = 60$$
$$1 + t = 2, \text{ and therefore } t = \$1.$$

The tax should be \$1 per unit of output. Note that with $t = 1$, the new private cost function, $(10 + Q) + Q$, is the same as the marginal social cost function.

d. Calculate the output and price of dry cleaning if it is produced under monopolistic conditions without regulation.

The monopolist will set marginal cost equal to marginal revenue. Recall that the marginal revenue curve has a slope that is twice the slope of the demand curve, so $MR = 100 - 2Q = MC = 10 + Q$. Therefore, $Q = 30$ and $P = \$70$, which are the socially efficient levels.

e. Determine the tax that would result in a monopolistic market producing the socially efficient output.

The tax would be zero since the monopolist already produces the socially efficient output in this case.

f. Assuming that no attempt is made to monitor or regulate the pollution, which market structure yields higher social welfare? Discuss.

In this case it is actually the monopolist that yields the higher level of social welfare compared to the competitive market, because the monopolist's profit maximizing price and quantity are the same as the socially efficient solution. Since a monopolist produces less output and sets a higher price than the competitive equilibrium, it may end up producing closer to the social equilibrium when a negative externality is present.

8. Refer back to Example 18.5 on global warming. Table 18.3 (page 683) shows the annual net benefits from a policy that reduces *GHG* emissions by 1% per year. At what discount rate is the *NPV* of this policy just equal to zero?

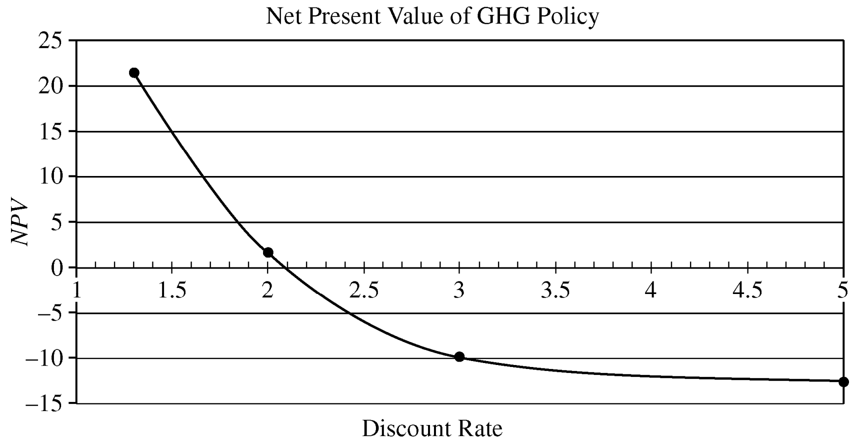
Table 18.3 in the text displays net benefit values at ten-year intervals, so it is not possible to calculate the *NPV* exactly, because the net benefit values change each year. The table below gives the exact net benefit values for each year. Using these numbers, you can compute the *NPV*s given in Example 18.5 and can solve for the discount rate that makes the *NPV* of the policy equal to zero.

Year	Net Benefit	Year	Net Benefit	Year	Net Benefit	Year	Net Benefit
2010	–0.650	2035	–0.813	2060	–1.266	2085	2.139
2011	–0.658	2036	–0.818	2061	–1.199	2086	2.377
2012	–0.665	2037	–0.822	2062	–1.127	2087	2.625
2013	–0.673	2038	–0.825	2063	–1.051	2088	2.884
2014	–0.680	2039	–0.829	2064	–0.970	2089	3.154
2015	–0.688	2040	–0.832	2065	–0.885	2090	3.436
2016	–0.695	2041	–0.834	2066	–0.795	2091	3.730
2017	–0.702	2042	–0.837	2067	–0.700	2092	4.036
2018	–0.710	2043	–0.838	2068	–0.599	2093	4.356
2019	–0.717	2044	–0.840	2069	–0.493	2094	4.689
2020	–0.724	2045	–0.841	2070	–0.382	2095	5.036
2021	–0.731	2046	–0.841	2071	–0.264	2096	5.397
2022	–0.738	2047	–0.841	2072	–0.141	2097	5.774
2023	–0.745	2048	–0.841	2073	–0.011	2098	6.165
2024	–0.751	2049	–0.839	2074	0.126	2099	6.573
2025	–0.758	2050	–0.838	2075	0.269	2100	6.997
2026	–0.764	2051	–0.874	2076	0.420	2101	7.439
2027	–0.770	2052	–0.912	2077	0.578	2102	7.898
2028	–0.777	2053	–0.951	2078	0.743	2103	8.375
2029	–0.782	2054	–0.992	2079	0.916	2104	8.871
2030	–0.788	2055	–1.033	2080	1.098	2105	9.387
2031	–0.794	2056	–1.077	2081	1.288	2106	9.923
2032	–0.799	2057	–1.122	2082	1.487	2107	10.480
2033	–0.804	2058	–1.168	2083	1.695	2108	11.059
2034	–0.809	2059	–1.216	2084	1.912	2109	11.660
						2110	12.284

To find the discount rate that makes $NPV = 0$, set up a spreadsheet with the 101 yearly net benefit values given above. It is easiest to list them in a single column. Make the next column a time variable that starts with a value of zero in 2010 and increases by one each year. The time variable will be 100 in year 2110. Next, choose a cell in your spreadsheet where you list the discount rate, say .013 (i.e., 1.3%). Then, in the column next to the time variable, calculate the present discounted value of each net benefit amount. Do this by writing a formula that references the discount rate that you listed in the separate cell. This way, you will be able to change the discount rate in one cell and all the *PDV* values will change to reflect the new discount rate. Finally, add all the discounted net benefits together to find the *NPV*. If you do this correctly, you will have a $NPV = \$21.3$ trillion when the discount rate is .013. Now try different discount rates until you get an *NPV* of approximately zero. You should find that the discount rate is about .0209, i.e., 2.09%. If you know how to use the solver add-in in Excel, you can use it to find the exact discount rate that makes the *NPV* zero.

If you do not use the exact net benefit values in the table above, there are two other ways to proceed that will give reasonable approximations. First, you could take the net benefit values given in the text and interpolate values for the missing years. Using linear interpolation, you will find a $NPV = 23.0$ when the discount rate equals .013. Using these net benefit values, the discount rate that makes the *NPV* equal zero is .0214, or 2.14%.

Finally, rather than doing any *NPV* calculations, you could simply plot the four different combinations of discount rate and *NPV* values given in the text. Draw a smooth curve through these four points to find the approximate discount rate that drives *NPV* to zero. From the graph below, you can see that it is just shy of 2.1%.



9. **A beekeeper lives adjacent to an apple orchard. The orchard owner benefits from the bees because each hive pollinates about one acre of apple trees. The orchard owner pays nothing for this service, however, because the bees come to the orchard without his having to do anything. Because there are not enough bees to pollinate the entire orchard, the orchard owner must complete the pollination by artificial means, at a cost of \$10 per acre of trees.**

   Beekeeping has a marginal cost $MC = 10 + 5Q$, where Q is the number of beehives. Each hive yields \$40 worth of honey.

   a. **How many beehives will the beekeeper maintain?**

      The beekeeper maintains the number of hives that maximizes profits when marginal revenue is equal to marginal cost. With a constant marginal revenue of \$40 (there is no information that would lead us to believe that the beekeeper has any market power) and a marginal cost of $10 + 5Q$:

$$40 = 10 + 5Q, \text{ or } Q = 6.$$

b. Is this the economically efficient number of hives?

If there are too few bees to pollinate the orchard, the farmer must pay \$10 per acre for artificial pollination. Thus the farmer would be willing to pay up to \$10 to the beekeeper to maintain each additional hive. So the marginal social benefit, *MSB*, of each additional hive is \$50, which is greater than the marginal private benefit of \$40. Assuming that the private marginal cost is equal to the social marginal cost, we set $MSB = MC$ to determine the efficient number of hives:

$$50 = 10 + 5Q, \text{ or } Q = 8.$$

Therefore the beekeeper's private choice of $Q = 6$ is not the socially efficient number of hives.

c. What changes would lead to a more efficient operation?

The most radical change that would lead to more efficient operations would be the merger of the farmer's business with the beekeeper's business. This merger would internalize the positive externality of bee pollination. Short of a merger, the farmer and beekeeper should enter into a contract for pollination services, with the farmer paying \$10 per hive to the beekeeper.

10. There are three groups in a community. Their demand curves for public television in hours of programming, *T*, are given respectively by

$$W_1 = \$200 - T,$$

$$W_2 = \$240 - 2T,$$

$$W_3 = \$320 - 2T.$$

Suppose public television is a pure public good that can be produced at a constant marginal cost of \$200 per hour.

a. What is the efficient number of hours of public television?

The efficient number of hours is the amount *T* such that the sum of the marginal benefits is equal to marginal cost. The demand curves represent the marginal benefits (i.e., willingness to pay) for each group. Therefore, add the demand curves vertically to determine the sum of all marginal benefits: $MSB = W_1 + W_2 + W_3 = 760 - 5T$. Setting this equal to *MC*, $760 - 5T = 200$, so $T = 112$.

You can also see from the table below that $MSB = MC = 200$ at $T = 112$ hours of programming.

	Willingness to Pay			
Hours (T)	**Group 1 (W_1)**	**Group 2 (W_2)**	**Group 3 (W_3)**	**Vertical Sum**
100	100	40	120	260
106	94	28	108	230
112	88	16	96	200
118	82	4	84	170

b. How much public television would a competitive private market provide?

Assume that public TV is not a public good, and that it costs \$200 to produce each hour of programming for each group. To find the number of hours that the private market would provide, add the individual demand curves horizontally. The efficient number of hours is such that the private marginal cost of \$200 is equal to the private marginal benefit for each group. Therefore, price will equal marginal cost of \$200. At a price of \$200, group 1 demands no hours, group 2 demands 20 hours, and group 3 demands 60 hours. So a competitive market would provide 80 hours of programming.

11. Reconsider the common resource problem given in Example 18.7. Suppose that crawfish popularity continues to increase, and that the demand curve shifts from $C = 0.401 - 0.0064F$ to $C = 0.50 - 0.0064F$. How does this shift in demand affect the actual crawfish catch, the efficient catch, and the social cost of common access? (*Hint*: Use the marginal social cost and private cost curves given in the example.)

The relevant information is now the following:

Demand: $C = 0.50 - 0.0064F$

MSC: $C = -5.645 + 0.6509F$.

MPC: $C = -0.357 + 0.0573F$

With an increase in demand, the demand curve for crawfish shifts upward, intersecting the price axis at \$0.50. The private cost curve has a positive slope, so additional effort must be made to increase the catch. Since the social cost curve has a positive slope, the socially efficient catch also increases. Determine the socially efficient catch by setting demand equal to *MSC*:

$$0.50 - 0.0064F = -5.645 + 0.6509F, \text{ or } F^* = 9.35.$$

To determine the price that consumers are willing to pay for this quantity, substitute F^* into the demand equation and solve for C:

$$C = 0.50 - 0.0064(9.35), \text{ or } C = \$0.44.$$

To find the actual crawfish catch, set demand equal to the private marginal cost:

$$0.50 - 0.0064F = -0.357 + 0.0573F, \text{ or } F^{**} = 13.45.$$

To determine the price that consumers are willing to pay for this quantity, substitute F^{**} into the demand equation and solve for C:

$$C = 0.50 - 0.0064(13.45), \text{ or } C = \$0.41.$$

Notice that the marginal social cost of producing 13.45 units is

$$MSC = -5.645 + 0.6509(13.45) = \$3.11.$$

With the increase in demand, the social cost of common access is the area of a triangle with a base of 4.1 million pounds (13.45 – 9.35) and a height of \$2.70 (\$3.11 – 0.41), or $0.5 \times 4.1 \times 2.70 = 5.535$, or \$5,535,000. This is \$3,139,000 more than the social cost of common access with the original demand (which was calculated to be \$2,396,000 in Example 18.7).

12. Georges Bank, a highly productive fishing area off New England, can be divided into two zones in terms of fish population. Zone 1 has the higher population per square mile but is subject to severe diminishing returns to fishing effort. The daily fish catch (in tons) in Zone 1 is

$$F_1 = 200(X_1) - 2(X_1)^2$$

where X_1 is the number of boats fishing there. Zone 2 has fewer fish per mile but is larger, and diminishing returns are less of a problem. Its daily fish catch is

$$F_2 = 100(X_2) - (X_2)^2$$

where X_2 is the number of boats fishing in Zone 2. The marginal fish catch *MFC* in each zone can be represented as

$$MFC_1 = 200 - 4(X_1) \text{ and } MFC_2 = 100 - 2(X_2).$$

There are 100 boats now licensed by the U.S. government to fish in these two zones. The fish are sold at \$100 per ton. Total cost (capital and operating) per boat is constant at \$1000 per day. Answer the following questions about this situation:

a. If the boats are allowed to fish where they want, with no government restriction, how many will fish in each zone? What will be the gross value of the catch?

Without restrictions, the boats will divide themselves so that the average catch (AF_1 and AF_2) for each boat is equal in each zone. (If the average catch in one zone is greater than in the other, boats will leave the zone with the lower catch for the zone with the higher catch.) Solve the following set of equations:

$$AF_1 = AF_2 \text{ and } X_1 + X_2 = 100, \text{ where}$$
$$AF_1 = \frac{200X_1 - 2X_1^2}{X_1} = 200 - 2X_1, \text{ and}$$
$$AF_2 = \frac{100X_2 - X_2^2}{X_2} = 100 - X_2.$$

Therefore, $AF_1 = AF_2$ implies

$$200 - 2X_1 = 100 - X_2,$$
$$200 - 2(100 - X_2) = 100 - X_2, \text{ or } X_2 = \frac{100}{3} \text{ and}$$
$$X_1 = 100 - \left(\frac{100}{3}\right) = \frac{200}{3}.$$

Find the gross catch by substituting the values of X_1 and X_2 into the catch equations:

$$F_1 = (200)\left(\frac{200}{3}\right) - (2)\left(\frac{200}{3}\right)^2 = 13{,}333 - 8889 = 4444, \text{ and}$$
$$F_2 = (100)\left(\frac{100}{3}\right) - \left(\frac{100}{3}\right)^2 = 3333 - 1111 = 2222.$$

The total catch is $F_1 + F_2 = 6666$. At the price of \$100 per ton, the value of the catch is \$666,600. The average catch for each of the 100 boats in the fishing fleet is 66.66 tons.

To determine the profit per boat, subtract total cost from total revenue:

$$\pi = (100)(66.66) - 1000, \text{ or } \pi = \$5666.$$

Total profit for the fleet is \$566,600.

b. If the U.S. government can restrict the number and distribution of the boats, how many should be allocated to each zone? What will be the gross value of the catch? Assume the total number of boats remains at 100.

Assume that the government wishes to maximize the net social value of the fish catch, i.e., the difference between the total social benefit and the total social cost. The government equates the marginal fish catch in both zones, subject to the restriction that the number of boats equals 100:

$$MFC_1 = MFC_2 \text{ and } X_1 + X_2 = 100,$$

Setting $MFC_1 = MFC_2$ implies:

$$200 - 4X_1 = 100 - 2X_2, \text{ or } 200 - 4(100 - X_2) = 100 - 2X_2, \text{ so } X_2 = 50 \text{ and}$$

$$X_1 = 100 - 50 = 50.$$

Find the gross catch by substituting X_1 and X_2 into the catch equations:

$$F_1 = (200)(50) - (2)(50^2) = 10{,}000 - 5000 = 5000 \text{ and}$$

$$F_2 = (100)(50) - 50^2 = 5000 - 2500 = 2500.$$

The total catch is equal to $F_1 + F_2 = 7500$. At the market price of \$100 per ton, the value of the catch is \$750,000. Total profit is \$650,000. Notice that the profits are not evenly divided between boats in the two zones. The average catch in Zone 1 is 100 tons per boat, while the average catch in Zone 2 is 50 tons per boat. Therefore, fishing in Zone 1 yields a higher profit for the owner of the boat.

c. If additional fishermen want to buy boats and join the fishing fleet, should a government wishing to maximize the net value of the catch grant them licenses? Why or why not?

To answer this question, first determine the profit-maximizing number of boats in each zone. Profits in Zone 1 are

$$\pi_1 = 100(200X_1 - 2X_1^2) - 100X_1 \text{ or } \pi_1 = 19{,}000X_1 - 200X_1^2.$$

To determine the optimal number of boats in Zone 1, take the first derivative of the profit function with respect to X_1, set it equal to zero, and solve for X_1:

$$\frac{d\pi_1}{dX_1} = 19{,}000 - 400X_1 = 0, \text{ or } X_1 = 47.5.$$

Substituting X_1 into the profit equation for Zone 1 gives:

$$\pi_1 = 100[200(47.5) - 2(47.5)^2] - 1000(47.5) = \$451{,}250.$$

For Zone 2 follow a similar procedure. Profits in Zone 2 are

$$\pi_2 = 100(100X_2 - X_2^2) - 1000X_2 \text{ or } \pi_2 = 9000X_2 - 100X_2^2.$$

Taking the derivative of the profit function with respect to X_2 gives

$$\frac{d\pi_2}{dX_2} = 9000 - 200X_2 = 0, \text{ or } X_2 = 45.$$

Substituting X_2 into the profit equation for Zone 2 gives:

$$\pi_2 = (100)[100(45) - 45^2) - 1000(45) = \$202{,}500.$$

Total profit from both zones is \$653,750, with 47.5 boats in Zone 1 and 45 boats in Zone 2. Because each additional boat above 92.5 decreases total profit, the government should not grant any more licenses.

Chapter 4 Appendix
Demand Theory—A Mathematical Treatment

■ Exercises

1. Which of the following utility functions are consistent with convex indifference curves and which are not?

a. $U(X,Y) = 2X + 5Y$

b. $U(X,Y) = (XY)^{0.5}$

c. $U(X,Y) = \text{Min}(X,Y)$, where Min is the minimum of the two values of X and Y.

Indifference maps for the three utility functions are presented in Figures 4A.1(a), 4A.1(b), and 4A.1(c). The first is a series of straight lines, the second is a series of hyperbolas, and the third is a series of L-shaped curves. Only the second utility function has strictly convex indifference curves.

To graph the indifference curves which represent the preferences given by $U(X,Y) = 2X + 5Y$, set utility equal to some level, U_0, and solve for Y to get

$$Y = \frac{U_0}{5} - \frac{2}{5}X.$$

Since this is the equation for a straight line, the indifference curves are linear with intercept $\frac{U_0}{5}$ and slope $-\frac{2}{5}$. The graph shows three indifference curves for three different values of U, where $U_0 < U_1 < U_2$.

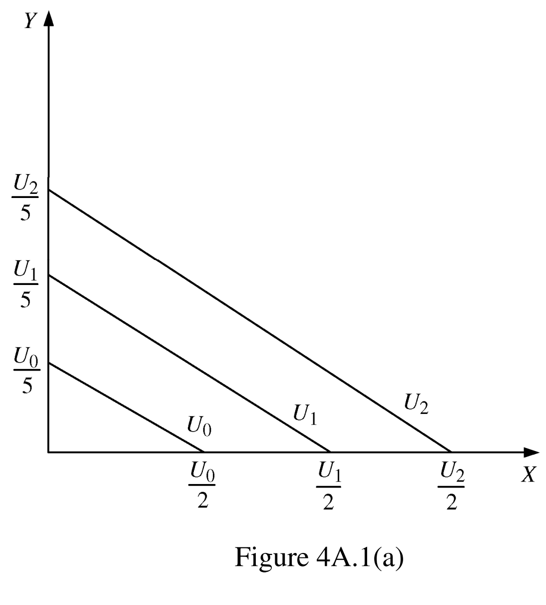


Figure 4A.1(a)

To graph the indifference curves that represent the preferences given by $U(X,Y)=(XY)^{0.5}$, set utility equal a given level U_0 and solve for Y to get

$$Y=\frac{U_0^2}{X}.$$

By plugging in a few values for X and solving for Y, you will be able to graph the indifference curve for utility value U_0, which is illustrated in Figure 4A.1(b), along with the indifference curve for a larger utility value, U_1.

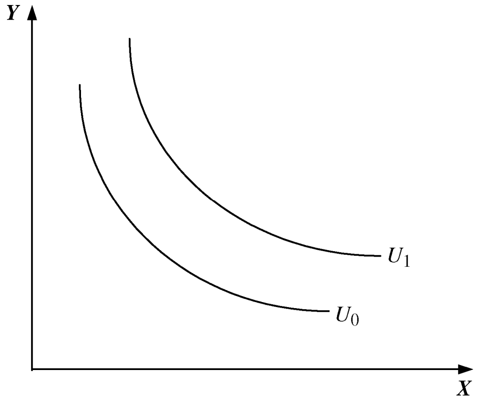


Figure 4A.1(b)

To graph the indifference curves that represent the preferences given by $U(X,Y)=Min(X,Y)$, first note that utility functions of this form result in indifference curves that are L-shaped and represent a complementary relationship between X and Y. In this case, for any given level of utility U_0, the minimum value of X and Y will also be equal to U_0. If X increases but Y does not, utility will not change. If both X and Y change, then utility will change, and we will move to a different indifference curve. See the following table which illustrates how the utility value depends on the amounts of X and Y in the consumption bundle.

X	***Y***	***U***
10	10	10
10	12	10
12	12	12
12	11	11
8	11	8
8	9	8

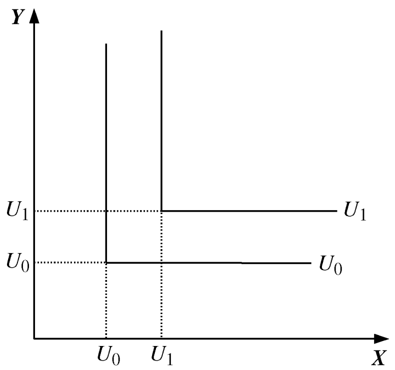


Figure 4A.1(c)

2. Show that the two utility functions given below generate identical demand functions for goods X and Y:

a. $U(X,Y) = \log(X) + \log(Y)$

b. $U(X,Y) = (XY)^{0.5}$

If two utility functions are equivalent, then the demand functions derived from them are identical. Two utility functions are equivalent if you can transform one of them and get the other one. The transformation must be performed by a function that transforms one set of numbers into another set without changing their order. So, for example, the square function could be used, because it does not change the order of numbers that are squared (as long as the numbers are not negative). If w is larger than z, then w^2 is larger than z^2. The logarithm function can also be used as a transformation function, and that is what we use here.

Taking the logarithm of $U(X,Y) = (XY)^{0.5}$ we obtain

$$\log U(X,Y) = 0.5 \log(XY) = 0.5 \, [\log(X) + \log(Y)].$$

Now multiply both sides by 2, which yields the utility function in a.

$$2(\log U(X,Y) = \log(X) + \log(Y).$$

Therefore, the two utility functions are equivalent and will yield identical demand functions. We can also demonstrate this directly by solving for the demand functions in both cases and showing that they are the same.

a. To find the demand functions for X and Y, corresponding to $U(X,Y) = \log(X) + \log(Y)$, we must maximize $U(X,Y)$ subject to the budget constraint. To do this, first write out the Lagrangian function, where λ is the Lagrange multiplier:

$$\Phi = \log(X) + \log(Y) - \lambda(P_X X + P_Y Y - I).$$

Differentiating with respect to X, Y, and λ, and setting the derivatives equal to zero:

$$\frac{\partial \Phi}{\partial X} = \frac{1}{X} - \lambda P_X = 0$$

$$\frac{\partial \Phi}{\partial Y} = \frac{1}{Y} - \lambda P_Y = 0$$

$$\frac{\partial \Phi}{\partial \lambda} = I - P_X X - P_Y Y = 0.$$

The first two conditions imply that $P_X X = \frac{1}{\lambda}$ and $P_Y Y = \frac{1}{\lambda}$.

The third condition implies that $I - \frac{1}{\lambda} - \frac{1}{\lambda} = 0$, or $\lambda = \frac{2}{I}$.

Substituting this expression into $P_X X = \frac{1}{\lambda}$ and $P_Y Y = \frac{1}{\lambda}$ gives the demand functions:

$$X = \left(\frac{I}{2P_X}\right) \text{ and } Y = \left(\frac{I}{2P_Y}\right).$$

Notice that the demand for each good depends only on the price of that good and on income, not on the price of the other good. Also, the consumer spends exactly half her income on each good, regardless of the prices of the goods.

b. To find the demand functions for X and Y, corresponding to $U(X,Y) = (XY)^{0.5} = (X^{0.5})(Y^{0.5})$, first write out the Lagrangian function:

$$\Phi = (X)^{0.5}(Y)^{0.5} - \lambda(P_X X + P_Y Y - I)$$

Differentiating with respect to X, Y, λ and setting the derivatives equal to zero:

$$\frac{\partial \Phi}{\partial X} = 0.5X^{-0.5}Y^{0.5} - \lambda P_X = 0$$

$$\frac{\partial \Phi}{\partial Y} = 0.5X^{0.5}Y^{-0.5} - \lambda P_Y = 0$$

$$\frac{\partial \Phi}{\partial Y} = I - P_X X - P_Y Y = 0$$

Take the first two conditions, move the terms involving λ to the right hand sides, and then divide the first condition by the second. After some algebra, you'll find $\frac{Y}{X} = \frac{P_X}{P_Y}$, or $P_Y Y = P_X X$.

Substitute for $P_Y Y$ in the third condition, which yields $I = 2P_X X$. Therefore, $X = \left(\frac{I}{2P_X}\right)$ and $Y = \left(\frac{I}{2P_Y}\right)$, which are the same demand functions we found for the other utility function.

3. Assume that a utility function is given by Min(X,Y), as in Exercise 1(c). What is the Slutsky equation that decomposes the change in the demand for X in response to a change in its price? What is the income effect? What is the substitution effect?

The full Slutsky equation is $dX/dP_X = \partial X/\partial P_X |_{U=U^*} - X(\partial X/\partial I)$, where the first term on the right represents the substitution effect and the second term represents the income effect. Because there is no substitution effect as price changes with this type of fixed proportions utility function, the substitution effect is zero. Therefore, the Slutsky equation for the fixed proportions utility function is $dX/dP_X = -X(\partial X/\partial I)$. A numerical example will help explain how this works. Suppose the consumer originally purchases 10 units of X, and we know that he would buy 1 more unit if his income increased by \$5 (so that $\partial X/\partial I = 1/\$5 = 0.2$). Using the Slutsky equation, $dX/dP_X = -10(0.2) = -2$. Therefore, if the price of X increased by \$1, the consumer would buy 2 fewer units of X, which would be due solely to the income effect. Conversely, if the price of X decreased by \$1, the consumer would buy 2 more units.

Figure 4A.3 below shows that when the price of X falls, the consumer's budget line pivots out from L_1 to L_2. A parallel shift of the new budget line back to the original indifference curve, U_1, gives us the hypothetical budget line L_3 from which we determine the substitution effect. Because the consumer would purchase the same bundle of X and Y as he did along the original budget line, the substitution effect is zero. The income effect is determined by the shift from budget line L_3 to L_2, which results in an increase in utility from U_1 to U_2 and an increase in consumption of X.

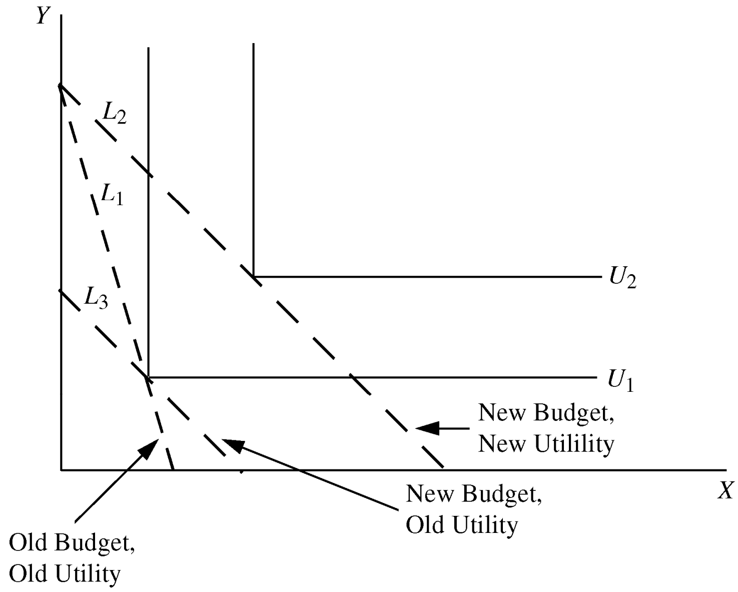


Figure 4A.3

4. Sharon has the following utility function:

$$U(X,Y)=\sqrt{X}+\sqrt{Y}$$

where *X* is her consumption of candy bars, with price P_X = \$1, and *Y* is her consumption of espressos, with P_Y = \$3.

a. Derive Sharon's demand for candy bars and espressos.

Using the Lagrangian method, the Lagrangian equation is

$$\Phi=\sqrt{X}+\sqrt{Y}-\lambda(P_X X+P_Y Y-I).$$

To find the demand functions, we need to maximize the Lagrangian equation with respect to *X*, *Y* and λ, which is the same as maximizing utility subject to the budget constraint. The necessary conditions for a maximum are

(1) $\dfrac{\partial\Phi}{\partial X}=0.5X^{-0.5}-P_X\lambda=0$

(2) $\dfrac{\partial\Phi}{\partial Y}=0.5Y^{-0.5}-P_Y\lambda=0$

(3) $\dfrac{\partial\Phi}{\partial\lambda}=I-P_X X-P_Y Y=0.$

Combining conditions (1) and (2) results in

$$\lambda=\frac{1}{2P_X X^{0.5}}=\frac{1}{2P_Y Y^{0.5}},\text{ so that } P_X X^{0.5}=P_Y Y^{0.5}\text{, and therefore}$$

(4) $X=\left(\dfrac{P_Y^2}{P_X^2}\right)Y.$

Now substitute (4) into (3) and solve for Y. Once you have solved for Y, you can substitute Y back into (4) and solve for X. Note that algebraically there are several ways to solve this type of problem; it does not have to be done exactly as shown here. The demand functions are:

$$Y = \frac{P_X I}{P_Y^2 + P_Y P_X} \text{ or } Y = \frac{I}{12}$$

$$X = \frac{P_Y I}{P_Y^2 + P_Y P_X} \text{ or } X = \frac{3I}{4}.$$

b. Assume that her income I = \$100. How many candy bars and how many espressos will Sharon consume?

Substitute the values for the two prices and income into the demand functions to find that she consumes $X = 75$ candy bars and $Y = 8.33$ espressos.

c. What is the marginal utility of income?

As shown in the appendix, the marginal utility of income equals λ. From part a, $\lambda = \frac{1}{2P_X X^{0.5}} = \frac{1}{2P_Y Y^{0.5}}$. Substitute into either part of the equation to get $\lambda = 0.058$. This is how much Sharon's utility would increase if she had one more dollar to spend.

5. Maurice has the following utility function: $U(X,Y) = 20X + 80Y - X^2 - 2Y^2$, where X is his consumption of CDs, with a price of \$1, and Y is his consumption of movie videos, with a rental price of \$2. He plans to spend \$41 on both forms of entertainment. Determine the number of CDs and video rentals that will maximize Maurice's utility.

Using X as the number of CDs and Y as the number of video rentals, the Lagrangian equation is $\Phi = 20X + 80Y - X^2 - 2Y^2 - \lambda(X + 2Y - 41)$.

To find the optimal consumption of each good, maximize the Lagrangian equation with respect to X, Y and λ, which is the same as maximizing utility subject to the budget constraint. The necessary conditions for a maximum are

(1) $\frac{\partial \Phi}{\partial X} = 20 - 2X - \lambda = 0$

(2) $\frac{\partial \Phi}{\partial Y} = 80 - 4Y - 2\lambda = 0$

(3) $\frac{\partial \Phi}{\partial \lambda} = X + 2Y - 41 = 0.$

Note that in condition (3), both sides have been multiplied by −1. Combining conditions (1) and (2) results in

$\lambda = 20 - 2X = 40 - 2Y$

(4) $2Y = 20 + 2X.$

Now substitute (4) into (3) and solve for X. Once you have solved for X, you can substitute this value back into (4) and solve for Y. Note that algebraically there are several ways to solve this type of problem, and that it does not have to be done exactly as here. The optimal bundle is $X = 7$ CDs and $Y = 17$ movie videos.

Chapter 7 Appendix
Production and Cost Theory— A Mathematical Treatment

■ Exercises

1. Of the following production functions, which exhibit increasing, constant, or decreasing returns to scale?

a. $\boldsymbol{F(K, L) = K^2 L}$

b. $\boldsymbol{F(K, L) = 10K + 5L}$

c. $\boldsymbol{F(K, L) = (KL)^{0.5}}$

Returns to scale refer to the relationship between output and proportional increases in all inputs. This is represented in the following manner (with $\lambda > 1$):

$F(\lambda K, \lambda L) > \lambda F(K, L)$ implies increasing returns to scale;

$F(\lambda K, \lambda L) = \lambda F(K, L)$ implies constant returns to scale; and

$F(\lambda K, \lambda L) < \lambda F(K, L)$ implies decreasing returns to scale.

a. Applying this to $F(K, L) = K^2L$,

$$F(\lambda K, \lambda L) = (\lambda K)^2 (\lambda L) = \lambda^3 K^2 L = \lambda^3 F(K, L).$$

This is greater than $\lambda F(K, L)$; therefore, this production function exhibits increasing returns to scale.

b. Applying the same technique to $F(K, L) = 10K + 5L$,

$$F(\lambda K, \lambda L) = 10\lambda K + 5\lambda L = \lambda F(K, L).$$

This production function exhibits constant returns to scale.

c. Applying the same technique to $F(K, L) = (KL)^{0.5}$,

$$F(\lambda K, \lambda L) = (\lambda K, \lambda L)^{0.5} = (\lambda^2)^{0.5} (KL)^{0.5} = \lambda (KL)^{0.5} = \lambda F(K, L).$$

This production function exhibits constant returns to scale.

2. The production function for a product is given by $\boldsymbol{q = 100KL}$. If the price of capital is \$120 per day and the price of labor \$30 per day, what is the minimum cost of producing 1000 units of output?

The cost-minimizing combination of capital and labor is the one where

$$MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}.$$

The marginal product of labor is $\frac{\partial q}{\partial L} = 100K$. The marginal product of capital is $\frac{\partial q}{\partial K} = 100L$. Therefore, the marginal rate of technical substitution is

$$\frac{100K}{100L} = \frac{K}{L}.$$

To determine the optimal capital-labor ratio set the marginal rate of technical substitution equal to the ratio of the wage rate to the rental rate of capital:

$$\frac{K}{L} = \frac{30}{120}, \text{ or } L = 4K.$$

Substitute for L in the production function and solve for K when output is 1000 units:

$$1000 = (100)(K)(4K), \text{ or } K = 1.58.$$

Because L equals $4K$ this means L equals $4(1.58) = 6.32$.

With these levels of the two inputs, total cost is:

$$TC = wL + rK, \text{ or}$$

$$TC = (30)(6.32) + (120)(1.58) = \$379.20.$$

3. **Suppose a production function is given by $F(K, L) = KL^2$; the price of capital is \$10 and the price of labor \$15. What combination of labor and capital minimizes the cost of producing any given output?**

   The cost-minimizing combination of capital and labor is the one where

$$MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}.$$

   The marginal product of labor is $\frac{\partial q}{\partial L} = 2KL$. The marginal product of capital is $\frac{\partial q}{\partial K} = L^2$.

   Set the marginal rate of technical substitution equal to the input price ratio to determine the optimal capital-labor ratio:

$$\frac{2KL}{L^2} = \frac{15}{10}, \text{ or } K = 0.75L.$$

   Therefore, the capital-labor ratio (K/L) should be 0.75 to minimize the cost of producing any given output. Note that the optimal ratio does not depend on the amount of output q. This form of production function (the Cobb-Douglas) has that characteristic, but not all production functions do.

4. **Suppose the process of producing lightweight parkas by Polly's Parkas is described by the function**

$$\mathbf{q = 10K^{0.8}(L - 40)^{0.2}}$$

   where q is the number of parkas produced, K the number of computerized stitching-machine hours, and L the number of person-hours of labor. In addition to capital and labor, \$10 worth of raw materials is used in the production of each parka.

   We are given the production function: $q = F(K, L) = 10K^{0.8}(L - 40)^{0.2}$

We also know that the cost of production, in addition to the cost of capital and labor, includes \$10 of raw material per unit of output. This yields the following total cost function:

$$TC(q) = wL + rK + 10q$$

a. By minimizing cost subject to the production function, derive the cost-minimizing demands for *K* and *L* as a function of output (*q*), wage rates (*w*), and rental rates on machines (*r*). Use these results to derive the total cost function: that is, costs as a function of *q*, *r*, *w*, and the constant \$10 per unit materials cost.

We need to find the combinations of K and L that will minimize this cost function for any given level of output q and factor prices r and w. To do this, set up the Lagrangian:

$$\Phi = wL + rK + 10q - \lambda[10K^{0.8}(L-40)^{0.2} - q]$$

Differentiating with respect to K, L, and λ, and setting the derivatives equal to zero:

(1) $\dfrac{\partial \Phi}{\partial K} = r - 10\lambda(0.8)K^{-0.2}(L-40)^{0.2} = 0$

(2) $\dfrac{\partial \Phi}{\partial L} = w - 10\lambda K^{0.8}(0.2)(L-40)^{-.08} = 0$

(3) $\dfrac{\partial \Phi}{\partial L} = 10K^{0.8}(L-40)^{0.2} - q = 0.$

Note that (3) has been multiplied by −1. The first 2 equations imply:

$$r = 10\lambda(.8)K^{-0.2}(L-40)^{0.2} \text{ and } w = 10\lambda K^{0.8}(0.2)(L-40)^{-0.8}.$$

or

$$\frac{r}{w} = \frac{4(L-40)}{K}.$$

This further implies:

$$K = \frac{4w(L-40)}{r} \quad \text{and} \quad L - 40 = \frac{rK}{4w}.$$

Substituting the above equations for K and $(L-40)$ into equation (3) yields solutions for K and L:

$$q = 10\left(\frac{4w}{r}\right)^{0.8}(L-40)^{0.8}(L-40)^{0.2} \quad \text{and} \quad q = 10K^{0.8}\left(\frac{rK}{4w}\right)^{2}.$$

or

$$L = \frac{r^{0.8}q}{30.3w^{0.8}} + 40 \text{ and } K = \frac{w^{0.2}q}{7.6r^{0.2}}.$$

We can now obtain the total cost function in terms of only r, w, and q by substituting these cost-minimizing values for K and L into the total cost function:

$$TC(q) = wL + rK + 10q$$

$$TC(q) = \frac{wr^{0.8}q}{30.3w^{0.8}} + 40w + \frac{rw^{0.2}q}{7.6r^{0.2}} + 10q$$

$$TC(q) = \frac{w^{0.2}r^{0.8}q}{30.3} + 40w + \frac{r^{0.8}w^{0.2}q}{7.6} + 10q.$$

b. This process requires skilled workers, who earn \$32 per hour. The rental rate on the machines used in the process is \$64 per hour. At these factor prices, what are total costs as a function of *q*? Does this technology exhibit decreasing, constant, or increasing returns to scale?

Given the values $w = 32$ and $r = 64$, the total cost function becomes:

$$TC(q) = 19.2q + 1280.$$

The average cost function is then given by

$$AC(q) = 19.2 + 1280/q.$$

So average cost falls as output increases. To find returns to scale, choose an input combination and find the level of output, and then double all inputs and compare the new and old output levels. Assume $K = 50$ and $L = 60$. Then $q_1 = 10(50)^{0.8}(60 - 40)^{0.2} = 416.3$. When $K = 100$ and $L = 120$, $q_2 = 10(100)^{0.8}(120 - 40)^{0.2} = 956.4$. Since $q_2/q_1 > 2$, the production function exhibits increasing returns to scale.

c. Polly's Parkas plans to produce 2000 parkas per week. At the factor prices given above, how many workers should the firm hire (at 40 hours per week) and how many machines should it rent (at 40 machine-hours per week)? What are the marginal and average costs at this level of production?

Given $q = 2000$ per week, we can calculate the required amount of inputs K and L using the formulas derived in part a:

$$L = \frac{r^{0.8}}{30.3w^{0.8}} + 40 \text{ and } K = \frac{w^{0.2}q}{7.6r^{0.2}}.$$

Thus $L = 154.9$ worker hours and $K = 229.1$ machine hours. Assuming a 40-hour week, $L = 154.9/40 = 3.87$ workers per week, and $K = 229.1/40 = 5.73$ machines per week. Polly's Parkas should hire 4 workers and rent 6 machines per week, assuming she cannot hire fractional workers and machines.

We know that the total cost and average cost functions are given by:

$$TC(q) = 19.2q + 1280$$

$$AC(q) = 19.2 + 1280/q,$$

so the marginal cost function is

$$MC(q) = d\,TC(q)/d\,q = 19.2.$$

Marginal costs are constant at \$19.20 per parka and average costs are 19.2 + 1280/2000 or \$19.84 per parka.