Momento Linear, Impulso e Colisões
(Cap. 8)

Definição de momento linear

$$\vec{F} = m \frac{d\vec{v}}{dt}$$

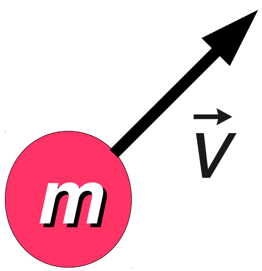
***m* constante:**

$$\vec{F} = \frac{d(m\vec{v})}{dt}$$

Momento linear = quantidade de movimento

Momento linear:

$$\boxed{\vec{p} = m\vec{v}}$$



2ª lei de Newton:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

"A força é igual à taxa de variação da quantidade de movimento"

Impulso

$$\vec{J} = \int_{\Delta t} \vec{F}\, d t$$

Impulso de *F* no intervalo $\Delta t = t_2 - t_1$

$$\vec{F} = \vec{F}(t)$$

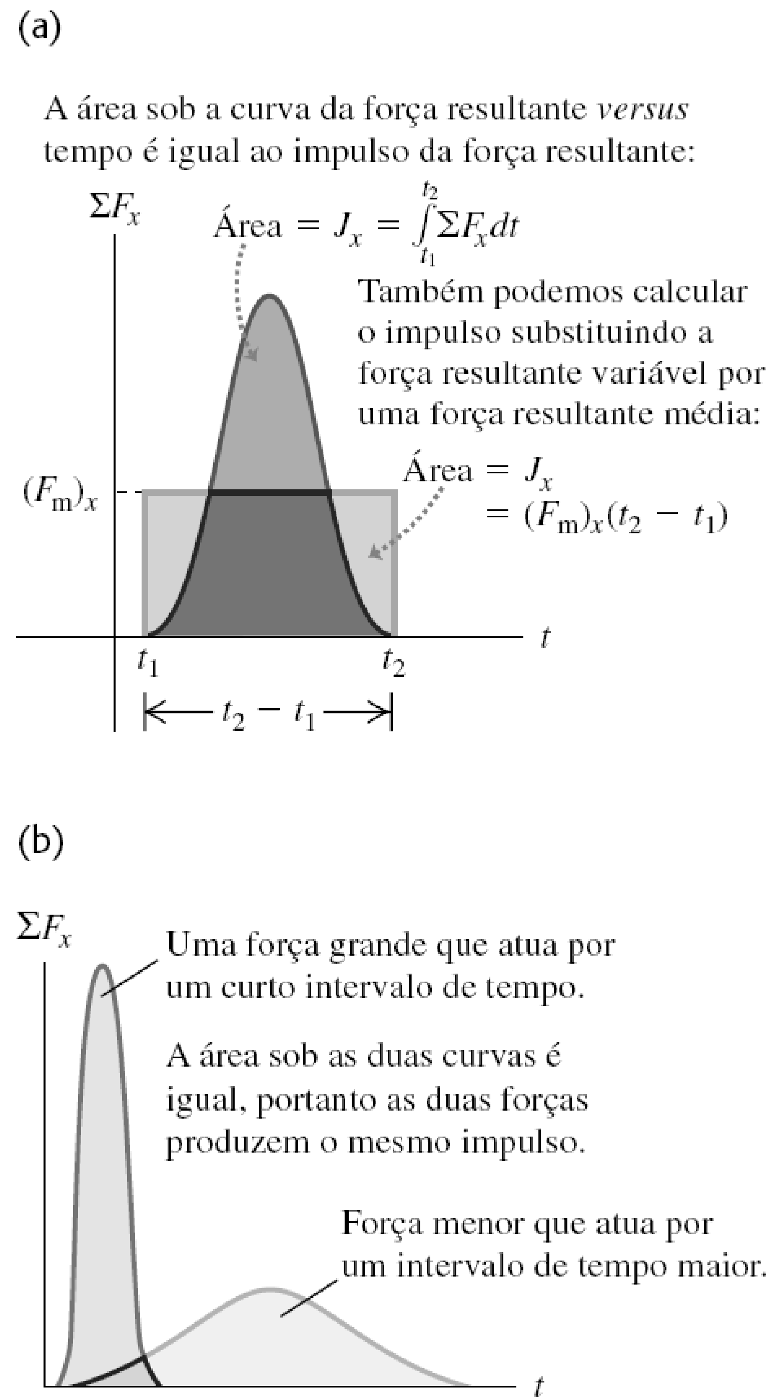
$$(se\ \vec{F} = cte \Rightarrow \vec{J} = \vec{F}(t_2 - t_1) = \vec{F}\Delta t)$$

2ª lei: $\vec{F} = \dfrac{d\vec{p}}{d t}$ $\qquad \vec{F}\, dt = d\vec{p}$

$$\int_{\Delta t} \vec{F}\, d t = \int_{\Delta t} d\vec{p} = \vec{p}_2 - \vec{p}_1 = \Delta \vec{p}$$

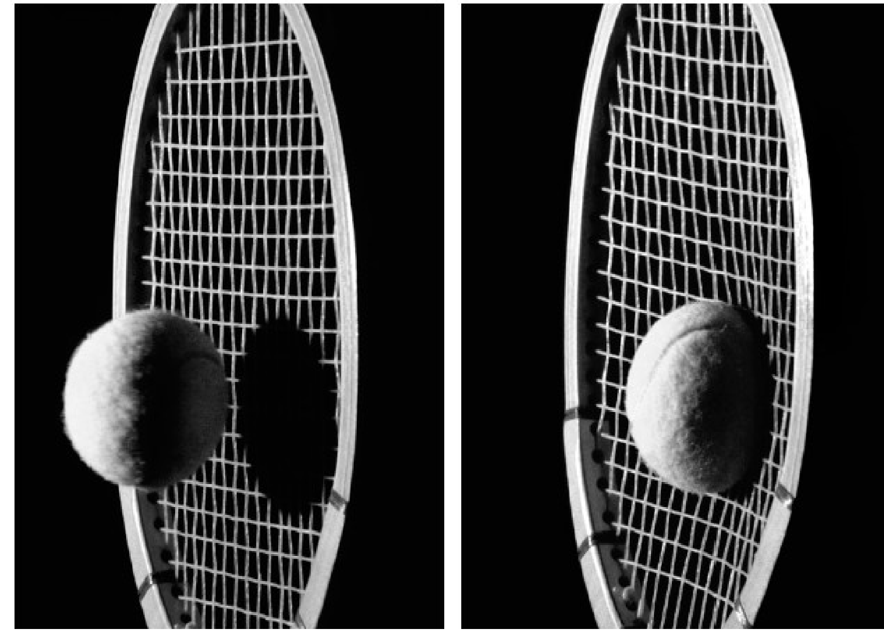
$$\vec{J} = \Delta \vec{p}$$

Teorema do Impulso-Momento Linear



8.3 O significado da área sob um gráfico ΣF_x *versus* t.

Momento linear x energia cinética



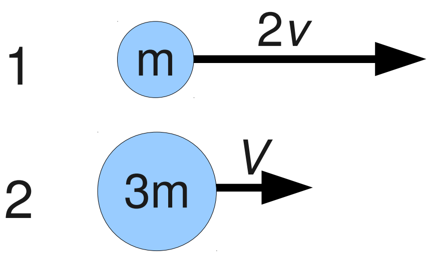
Momento linear – Vetor – Impulso

$$p = m\vec{v} \qquad d\vec{J} = d\vec{p} = \vec{F}\,dt$$

Energia cinética – Escalar - Trabalho

$$K = \frac{1}{2} m v^2 \qquad dW = dK = \vec{F} \,.\, d\vec{r}$$

Ex.: Comparação entre objetos de massas e velocidades diferentes



$$p_1 = 2mv \qquad K_1 = 2mv^2$$

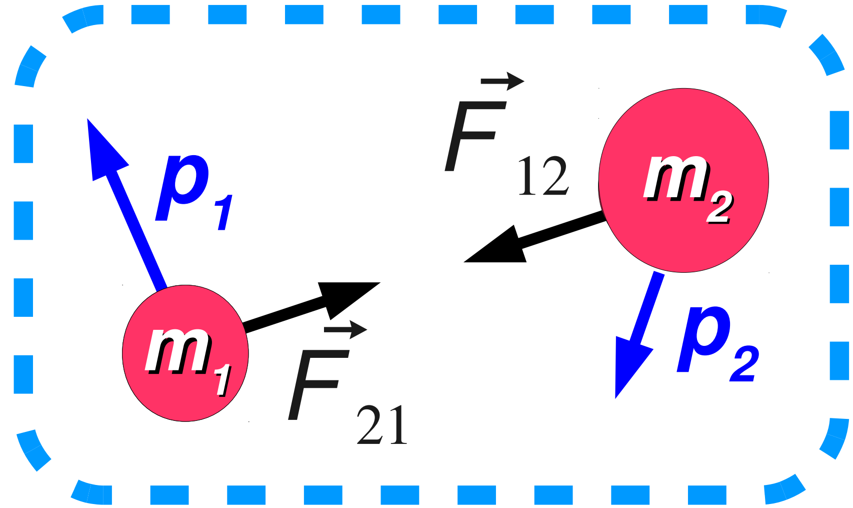
$$p_2 = 3mv \qquad K_2 = \frac{3}{2} m v^2$$

$$p_1 < p_2 \qquad K_1 > K_2$$

Conservação do momento linear total

Sistema ISOLADO

(auxência de forças externas)



m.l. total: $\vec{P} = \vec{p}_1 + \vec{p}_2$

$$\frac{d\vec{P}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \vec{F}_{21} + \vec{F}_{12}$$

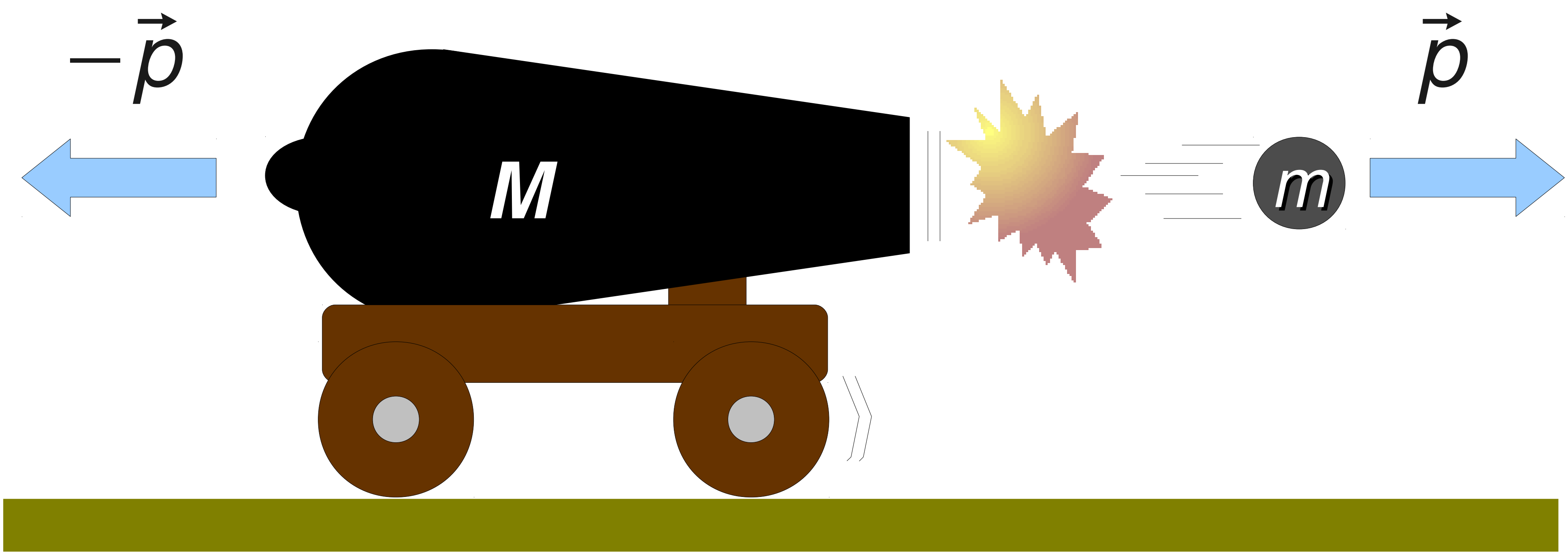
Forças internas

3ª lei: $\vec{F}_{21} = -\vec{F}_{12}$

$$\vec{F}_{12} + \vec{F}_{21} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0$$

$$\boxed{\vec{P} = \vec{p}_1 + \vec{p}_2 = cte}$$

Conservação do momento linear total



$$\vec{P}=0 \quad \vec{p}=m\vec{v}=-M\vec{V} \quad \vec{V}=-\frac{m}{M}\vec{v} \quad \frac{V}{v}=\frac{m}{M}$$

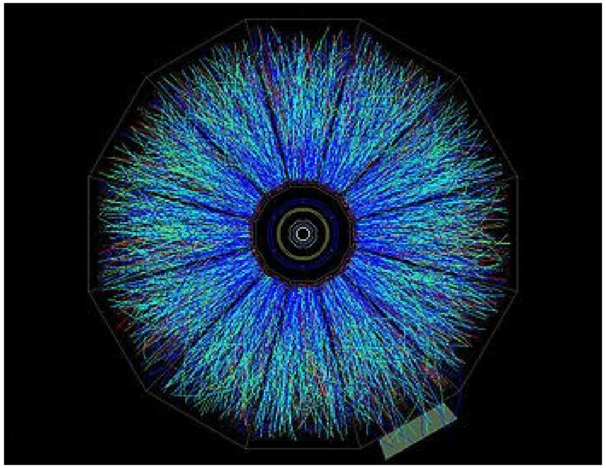
Conservação do momento linear total

- generalização para *n* partículas

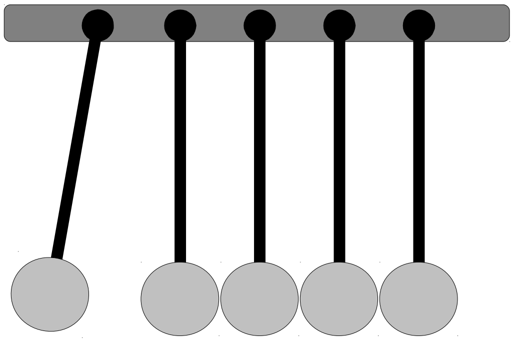
$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n$$

$$\frac{d\vec{P}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{\vec{p}_2}{dt} + \ldots = \sum_{i \neq 1}^{n} \vec{F}_{1\mathrm{i}} + \sum_{i \neq 2}^{n} \vec{F}_{2\mathrm{i}} + \ldots = \sum_{i}^{n} \sum_{j\,(i \neq j)}^{n} \vec{F}_{ij}$$

$$\frac{d\vec{P}}{dt} = \sum_{i<j} (\vec{F}_{ij} + \vec{F}_{ji}) = 0$$

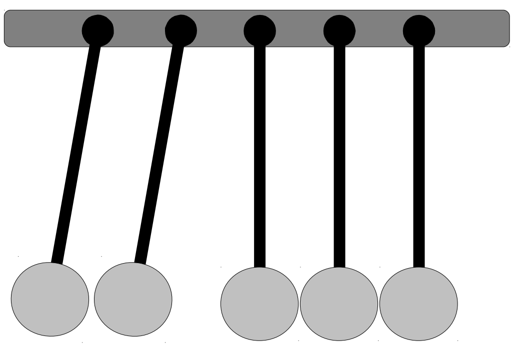


Colisões em 1D – colisões elásticas

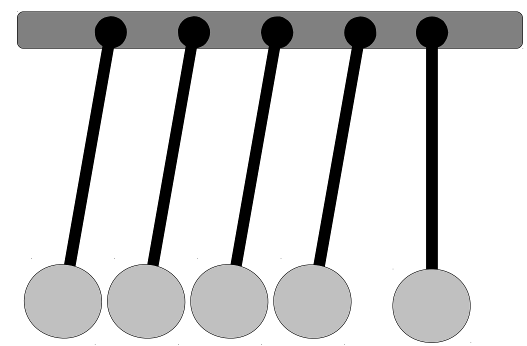


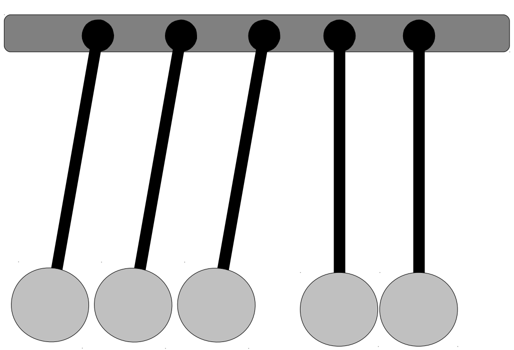
ANTES

$$P = m\,v$$



$$P = 2\,m\,v$$

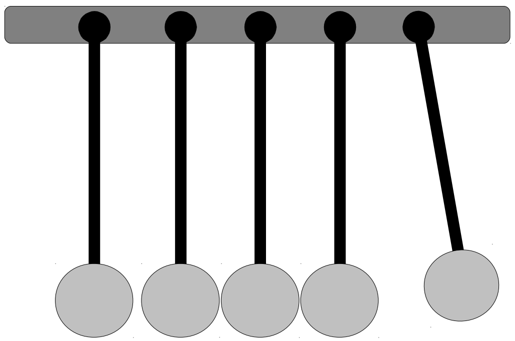




$$P = 3\,m\,v$$

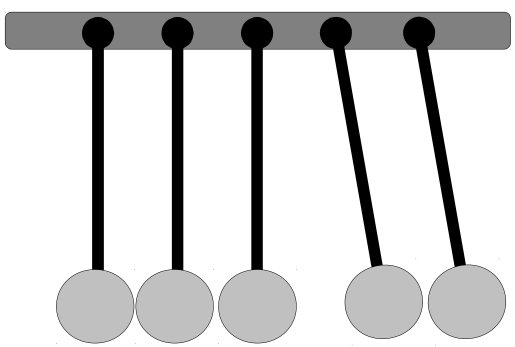
$$P = 4\,m\,v$$

Colisões em 1D – colisões elásticas

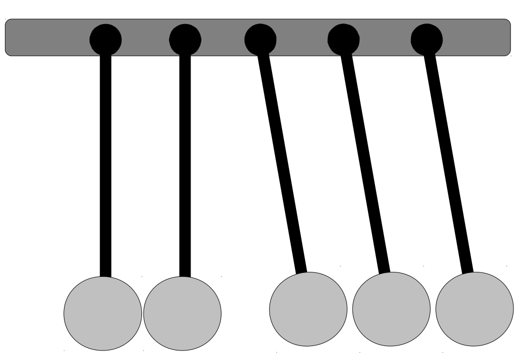


DEPOIS

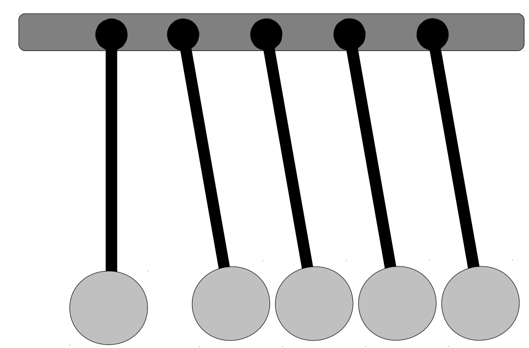
$$P = m\,v$$



$$P = 2\,m\,v$$



$$P = 3\,m\,v$$



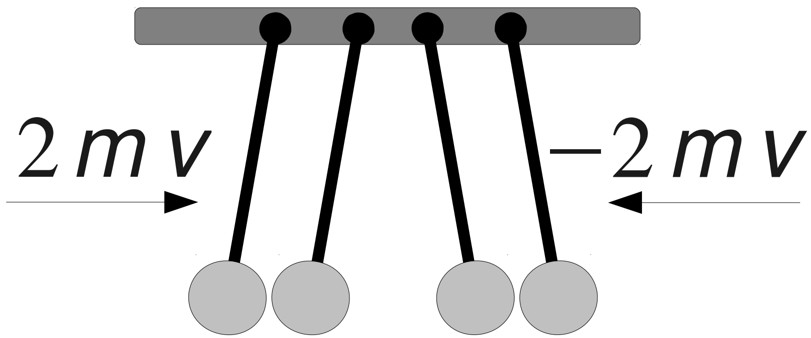
$$P = 4\,m\,v$$

Colisões em 1D

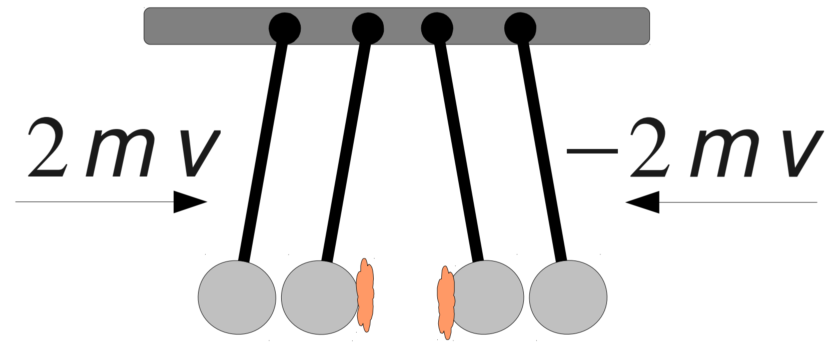
$$P=0$$

ANTES

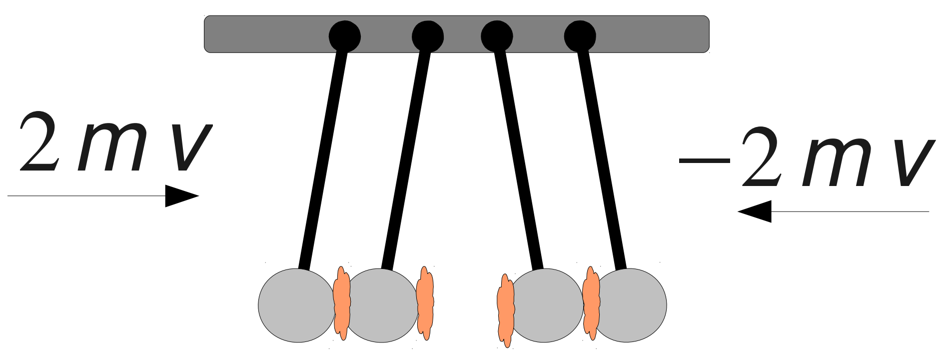
Elástica



Inelástica



Totalmente inelástica

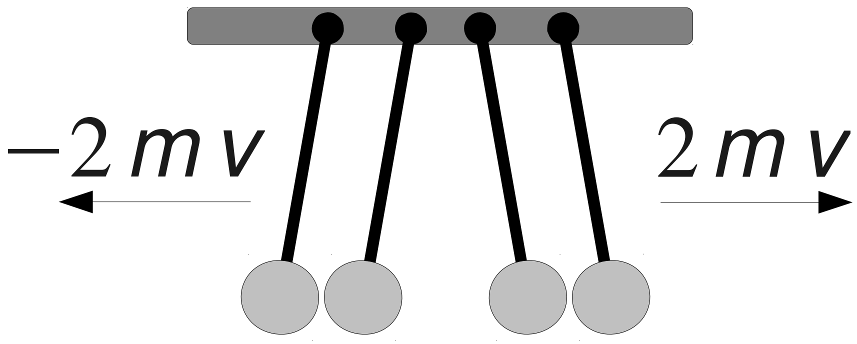


Colisões em 1D

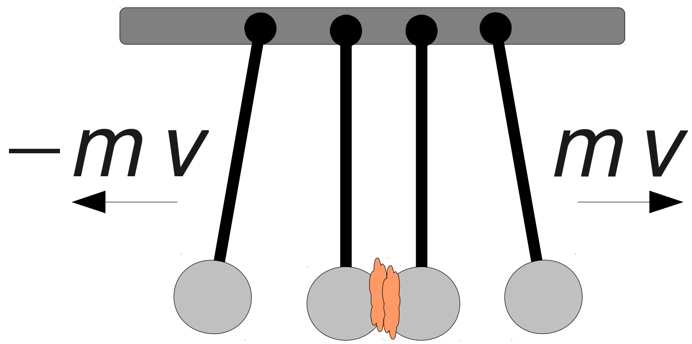
$P = 0$

DEPOIS

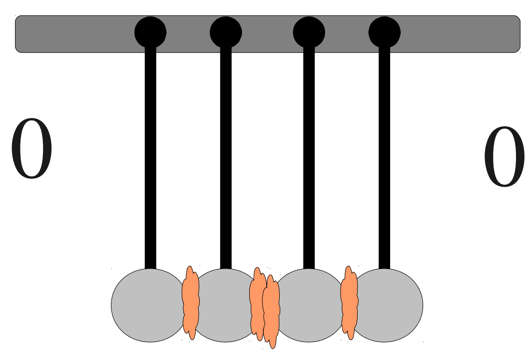
Elástica



Inelástica

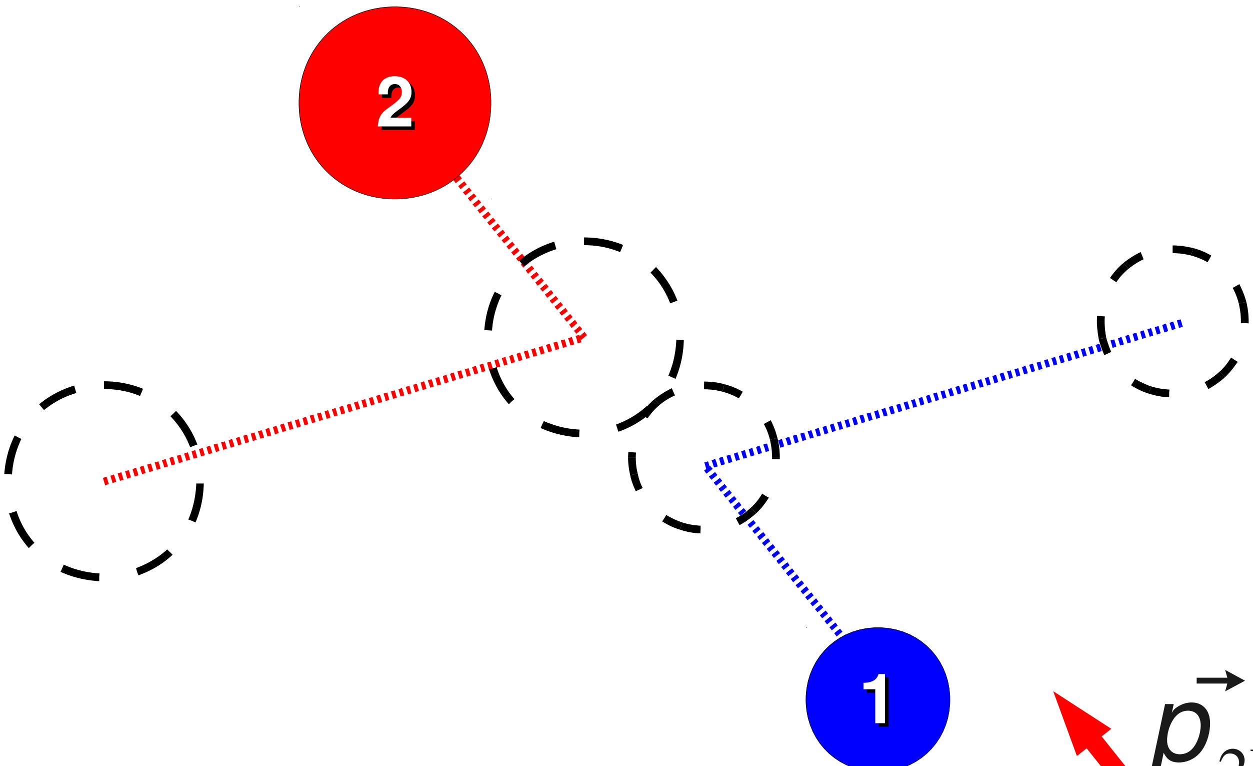


Totalmente inelástica



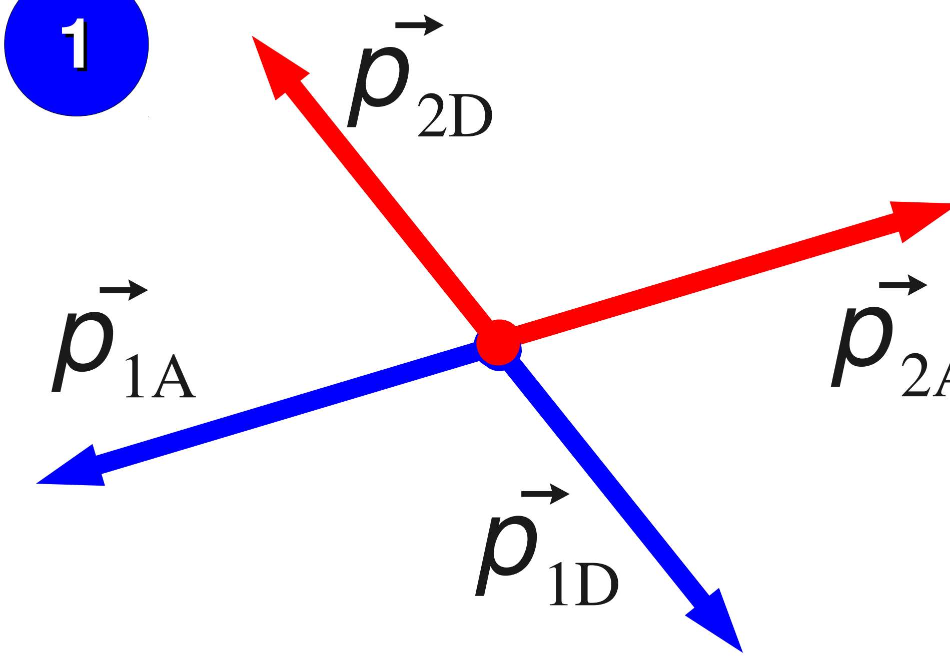
Colisão em 2D (elástica)

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

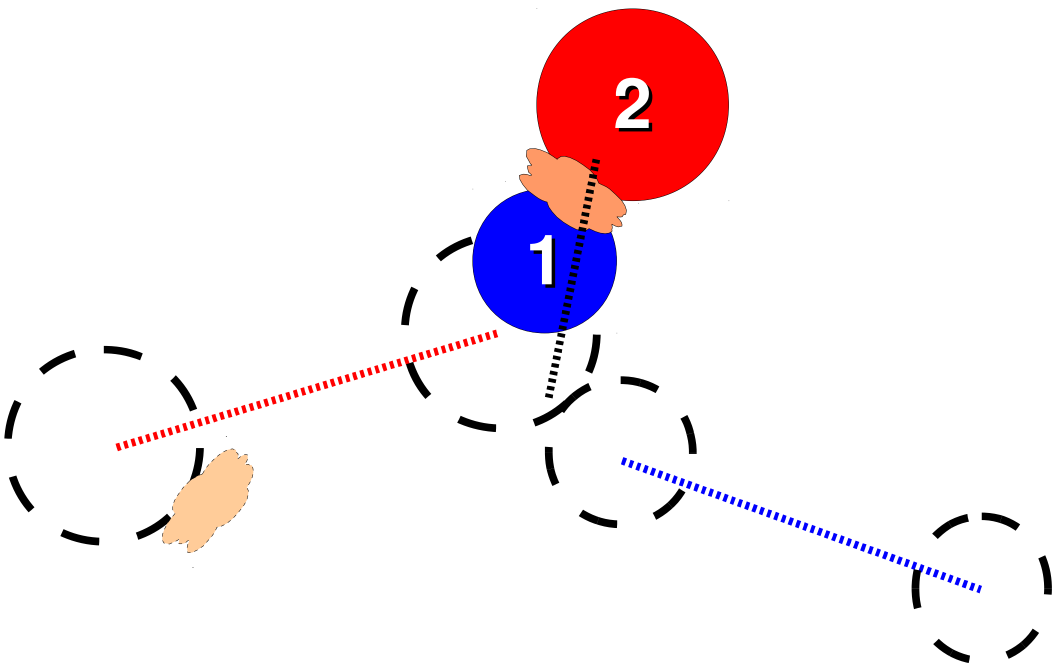


exemplo:

$$\vec{P}_A = 0 \Rightarrow \vec{P}_D = 0$$



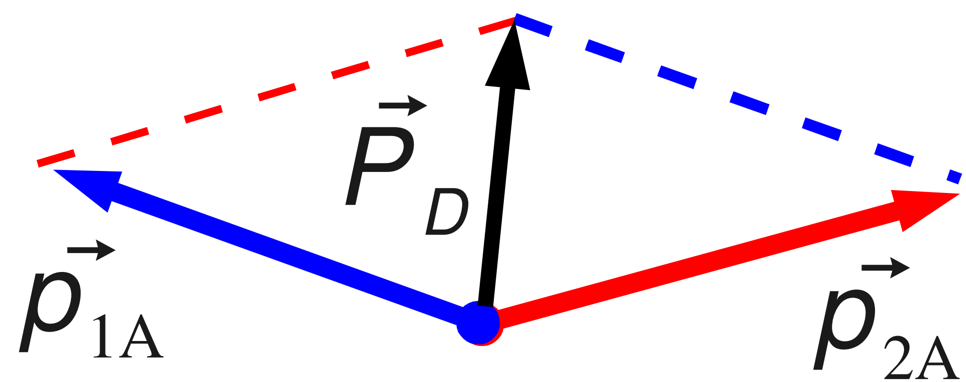
Colisão em 2D (totalmente inelástica)



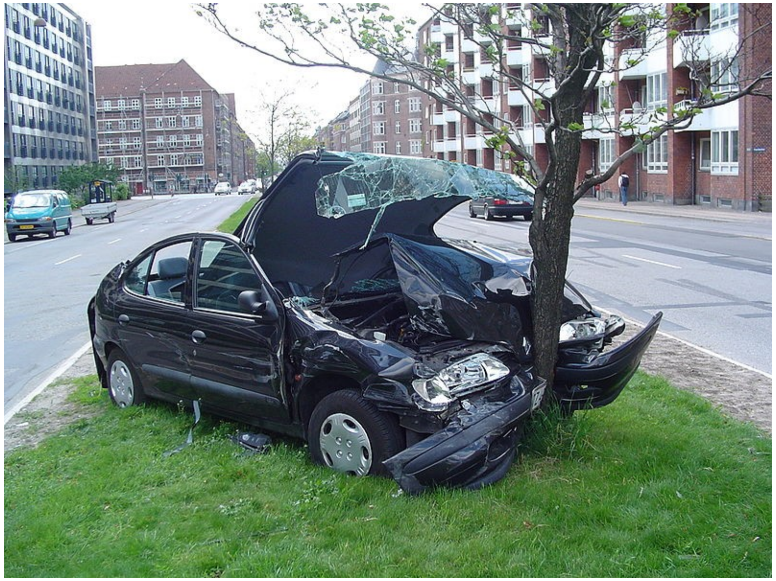
$$\vec{P}_A = \vec{p}_{1\mathrm{A}} + \vec{p}_{2\mathrm{A}}$$

$$\vec{P}_D = \vec{P}_A \quad \text{(Cons. m.l.)}$$

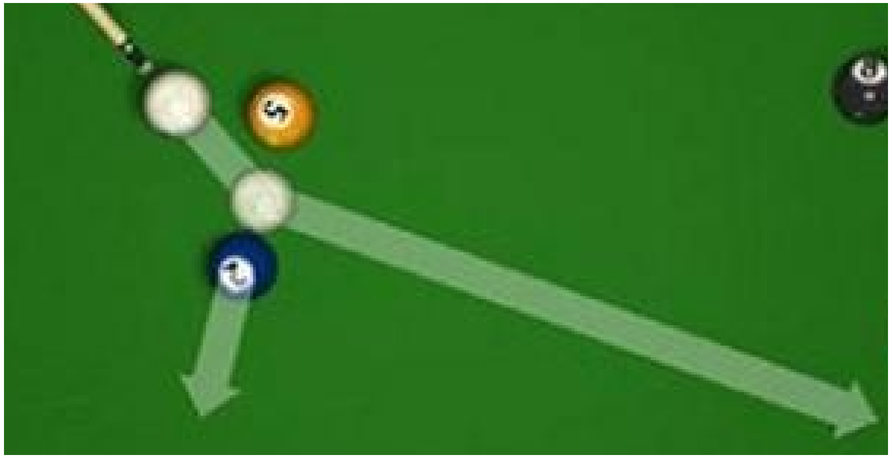
$$\vec{P}_D = \vec{p}_{1\mathrm{A}} + \vec{p}_{2\mathrm{A}}$$



Colisões inelásticas



Colisão elástica – conservação de momento e energia cinética



2 corpos; 2D: $v_i^2 = v_{ix}^2 + v_{iy}^2$

$$P_{Ax} = P_{Dx} \Rightarrow m_1 v_{1Ax} + m_2 v_{2Ax} = m_1 v_{1Dx} + m_2 v_{2Dx}$$

$$P_{Ay} = P_{Dy} \Rightarrow m_1 v_{1Ay} + m_2 v_{2Ay} = m_1 v_{1Dy} + m_2 v_{2Dy}$$

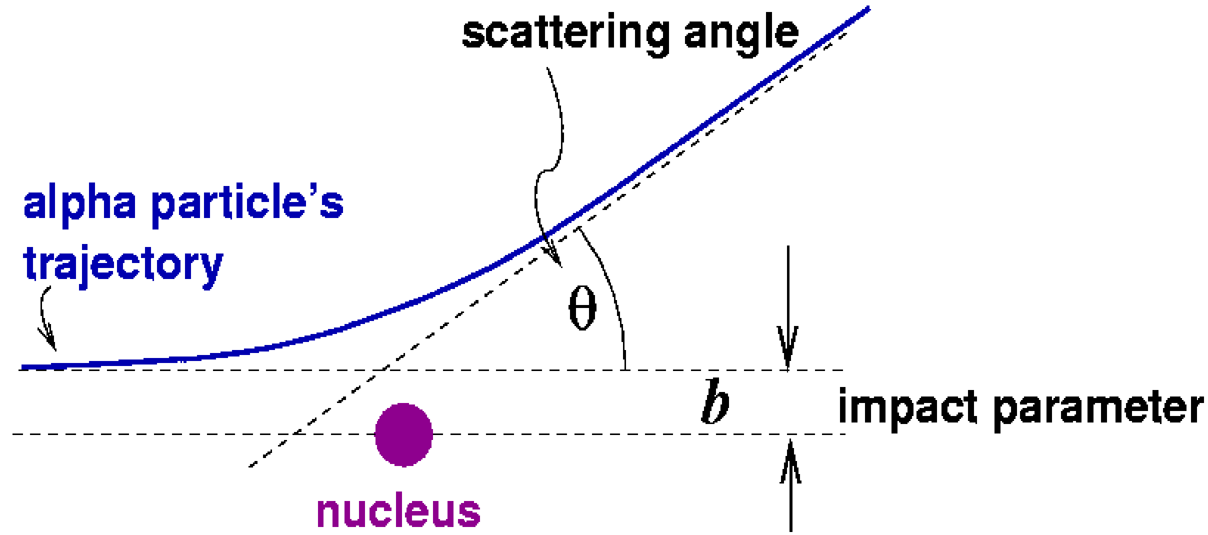
$$K_A = K_D \Rightarrow \frac{1}{2} m_1 v_{1A}^2 + \frac{1}{2} m_2 v_{2A}^2 = \frac{1}{2} m_1 v_{1D}^2 + \frac{1}{2} m_2 v_{2D}^2$$

Dadas as cond. iniciais: $v_{1Ax}, v_{1Ay}, v_{2Ax}, v_{2Ay}$ conhecidas, sistema de 3 equações e 4 incógnitas: $v_{D1x}, v_{D1y}, v_{D2x}, v_{D2y}$

Precisa mais uma equação (ex. Ângulo de espalhamento).

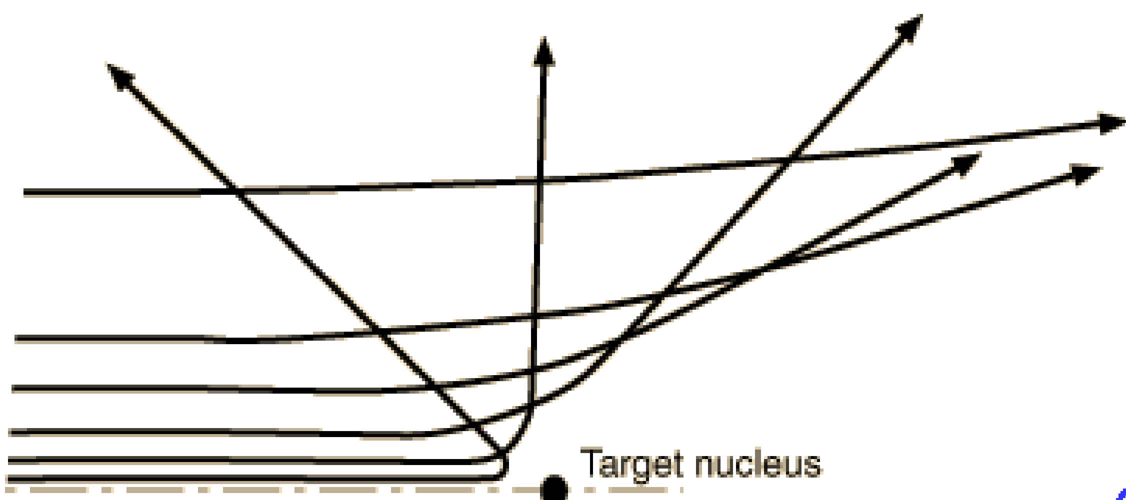
Colisão elástica – espalhamento Rutherford

$$\vec{F}(\vec{r}) = \frac{c}{r^2}\hat{r}$$

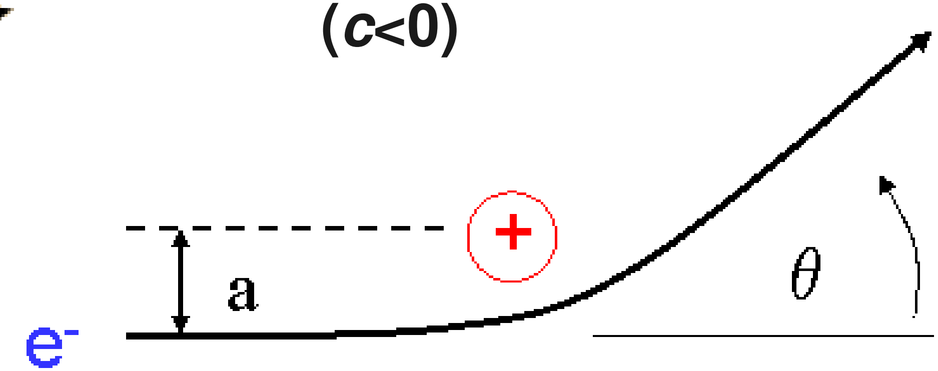


Função deflexão

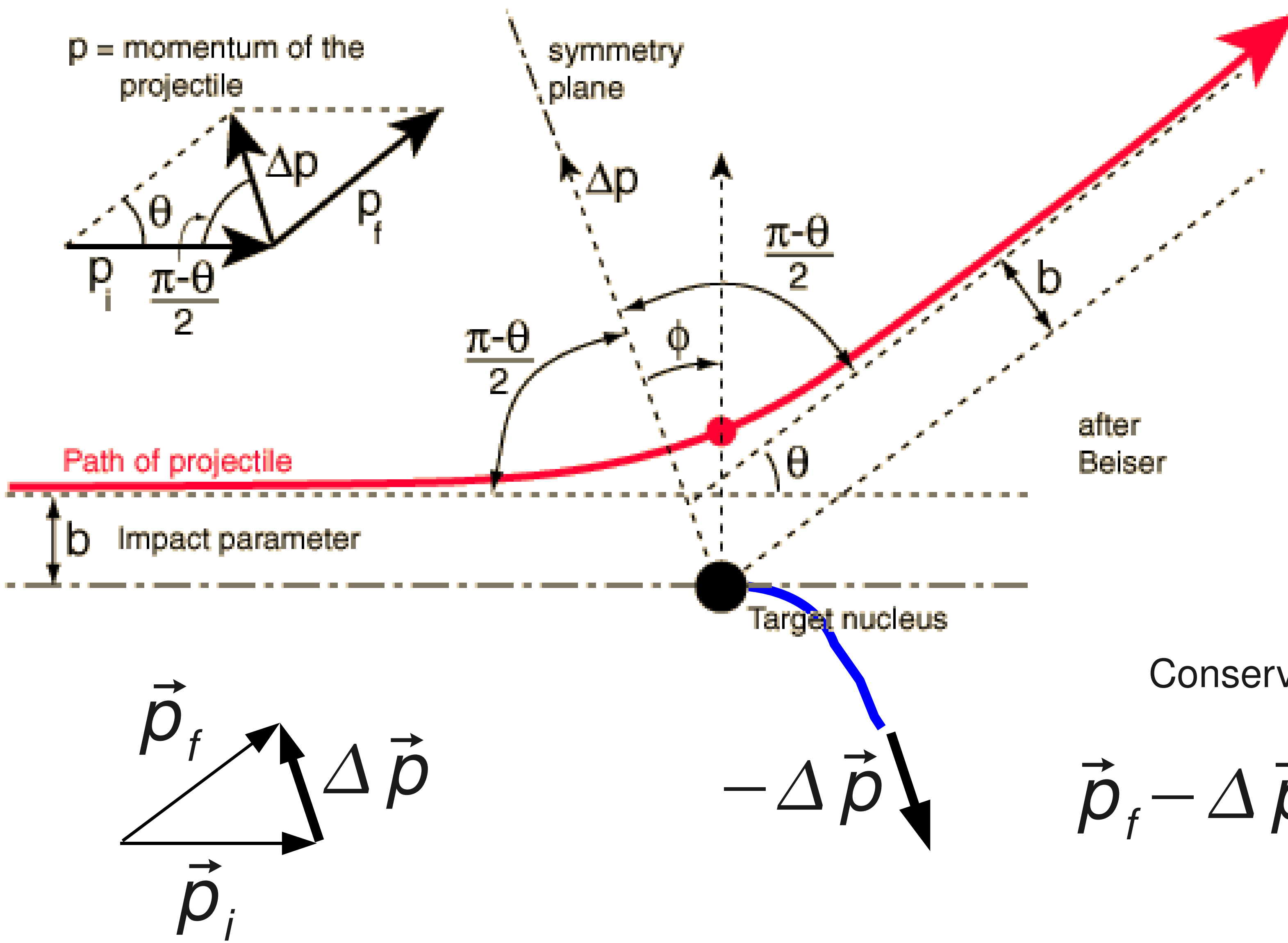
$$\Rightarrow \theta = \theta(b)$$



Força atrativa
(c<0)



Espalhamento Rutherford – momento transferido

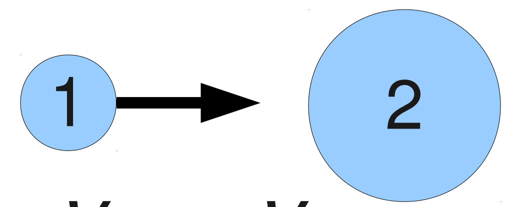


Conservação

$$\vec{p}_f - \Delta\vec{p} = \vec{p}_i$$

Colisão elástica em 1D (x)

ex. corpo (2) inicialmente em repouso:



$$v_{1A} = v_{1Ax}$$

$$P_A = P_D \Rightarrow m_1 v_{1Ax} = m_1 v_{1Dx} + m_2 v_{2Dx}$$

$$K_A = K_D \Rightarrow \frac{1}{2} m_1 v_{1Ax}^2 = \frac{1}{2} m_1 v_{1Dx}^2 + \frac{1}{2} m_2 v_{2Dx}^2$$

Condição inicial: v_{1Ax} conhecida $(v_{2Ax} = 0)$
Sistema de 2 equações e 2 incógnitas: v_{1Dx}, v_{2Dx}

Solução (vide livro, pg. 263. Obs: notação diferente...):

$$v_{1Dx} = \frac{m_1 - m_2}{m_1 + m_2} v_{1Ax} \qquad v_{2Dx} = \frac{2\, m_1}{m_1 + m_2} v_{1Ax}$$

Situações:
m_1=m_2
m_1<m_2
m_1>m_2

Colisão totalmente inelástica em 1D

ex. corpo (2) inicialmente em repouso:

Conhecidas as massas, e dada a altura *y* que o pêndulo sobe, calcular a velocidade da bala v_1

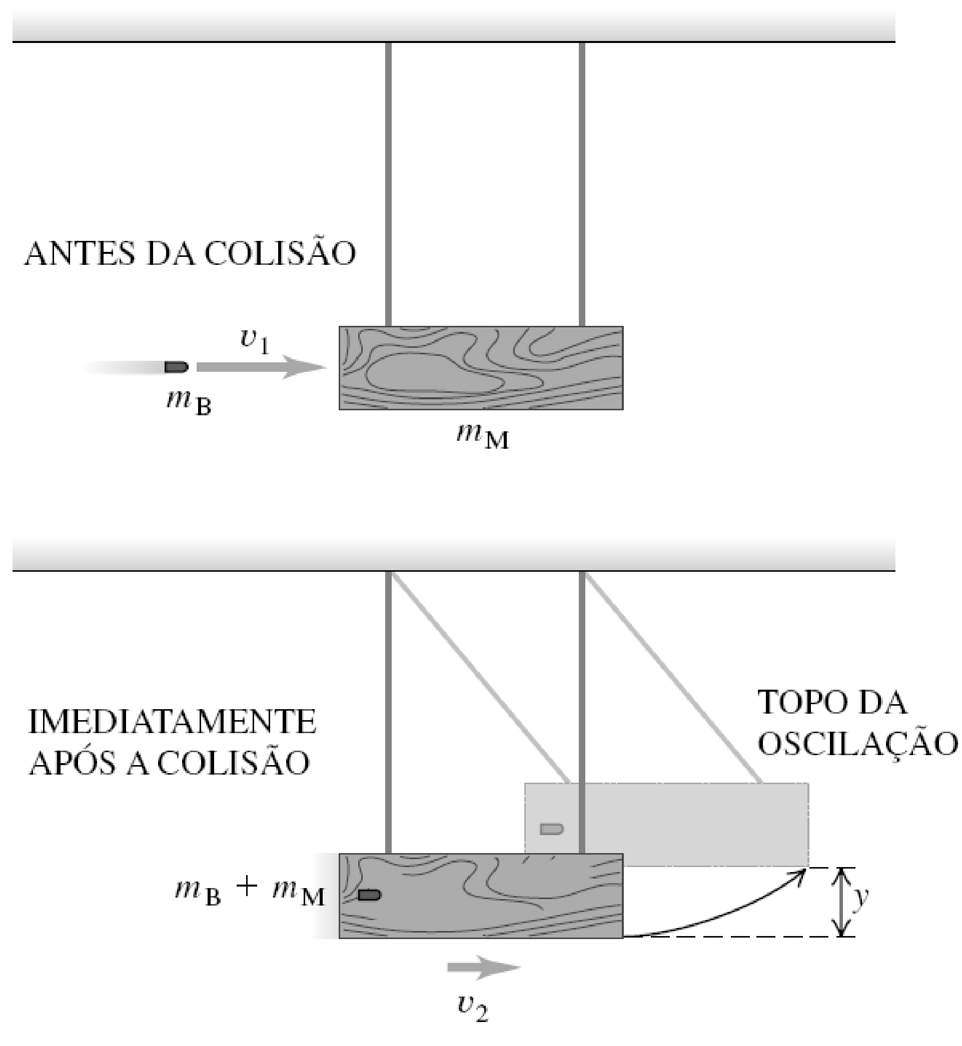


Figura 8.18 Um pêndulo balístico.

Colisão totalmente inelástica em 1D

ex. corpo (2) inicialmente em repouso:

Conhecidas as massas, e dada a altura *y* que o pêndulo sobe, calcular a velocidade da bala v_1

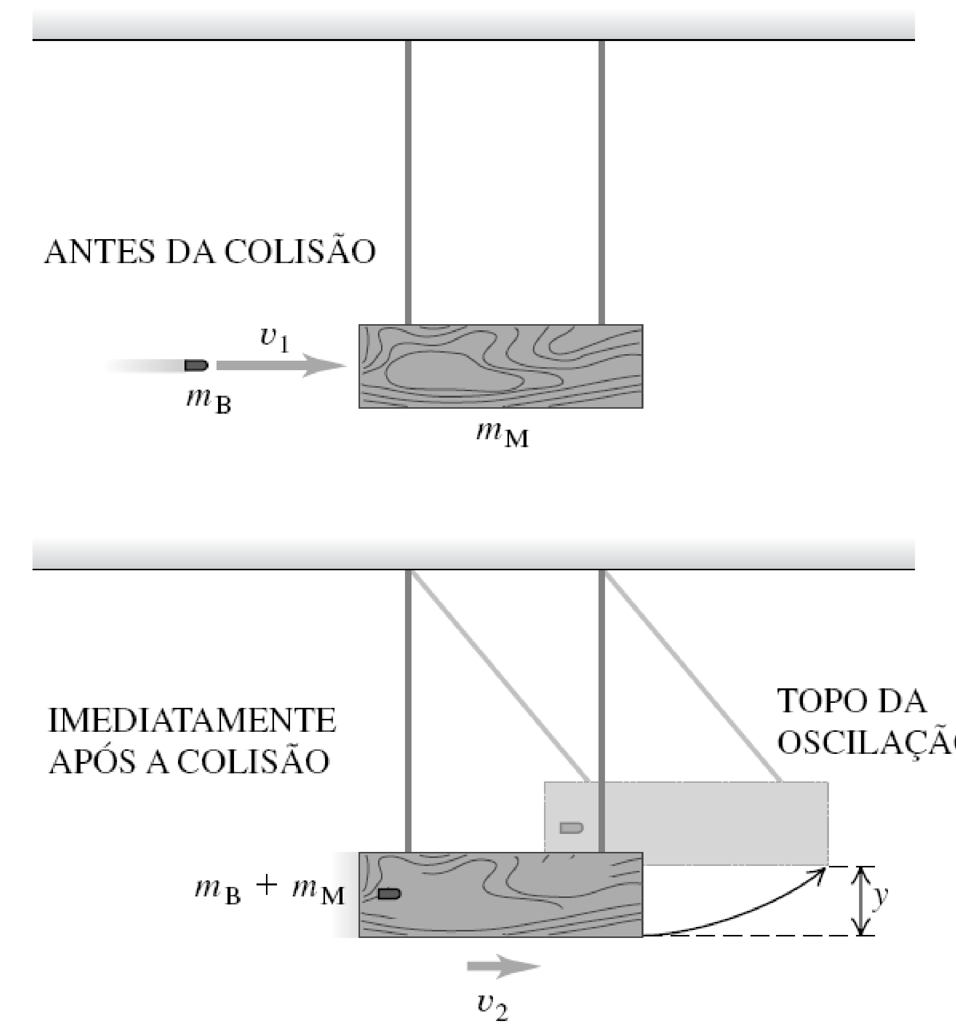


Figura 8.18 Um pêndulo balístico.

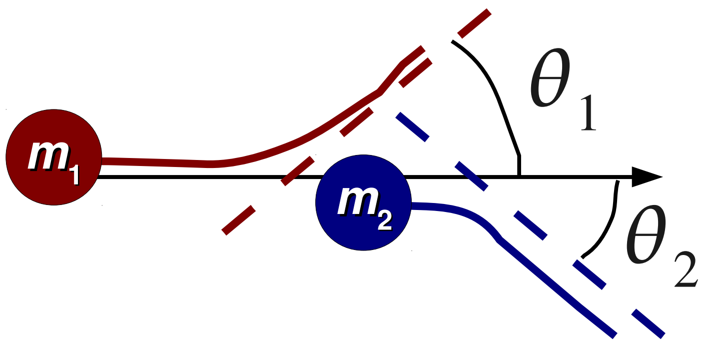
$$v_2 = \sqrt{(2 g y)}$$

$$m_B v_1 = (m_B + m_M) v_2$$

$$v_1 = \frac{m_B + m_M}{m_B} \sqrt{(2 g y)}$$

Colisão elástica em 2D

ex.: “alvo” inicialmente em repouso



Cons. Mom. lin.

$$\vec{p}_{1i} = \vec{p}_{1f} + \vec{p}_{2f}$$

Cons. En. cin.

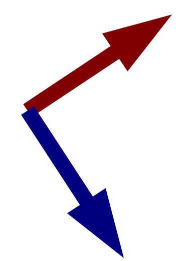
$$\frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2}$$

Caso de massas iguais *$m_1 = m_2$*: $p_{1i}^2 = p_{1f}^2 + p_{2f}^2$

$$p_{1i}^2 = (\vec{p}_{1f} + \vec{p}_{2f}) \cdot (\vec{p}_{1f} + \vec{p}_{2f})$$

$$p_{1i}^2 = p_{1f}^2 + p_{2f}^2 + 2\,\vec{p}_{1f} \cdot \vec{p}_{2f}$$

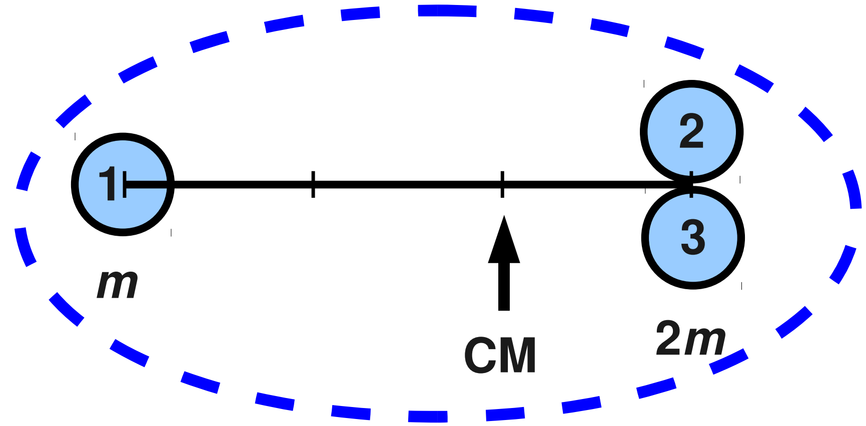
$$\Rightarrow \vec{p}_{1f} \cdot \vec{p}_{2f} = 0$$



$$\boxed{\vec{p}_{1f} \perp \vec{p}_{2f}}$$

$$\theta_1 + \theta_2 = \frac{\pi}{2} \qquad (90^\circ)$$

Centro de massa



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}$$

Centro de massa

Sólidos de formas simétricas

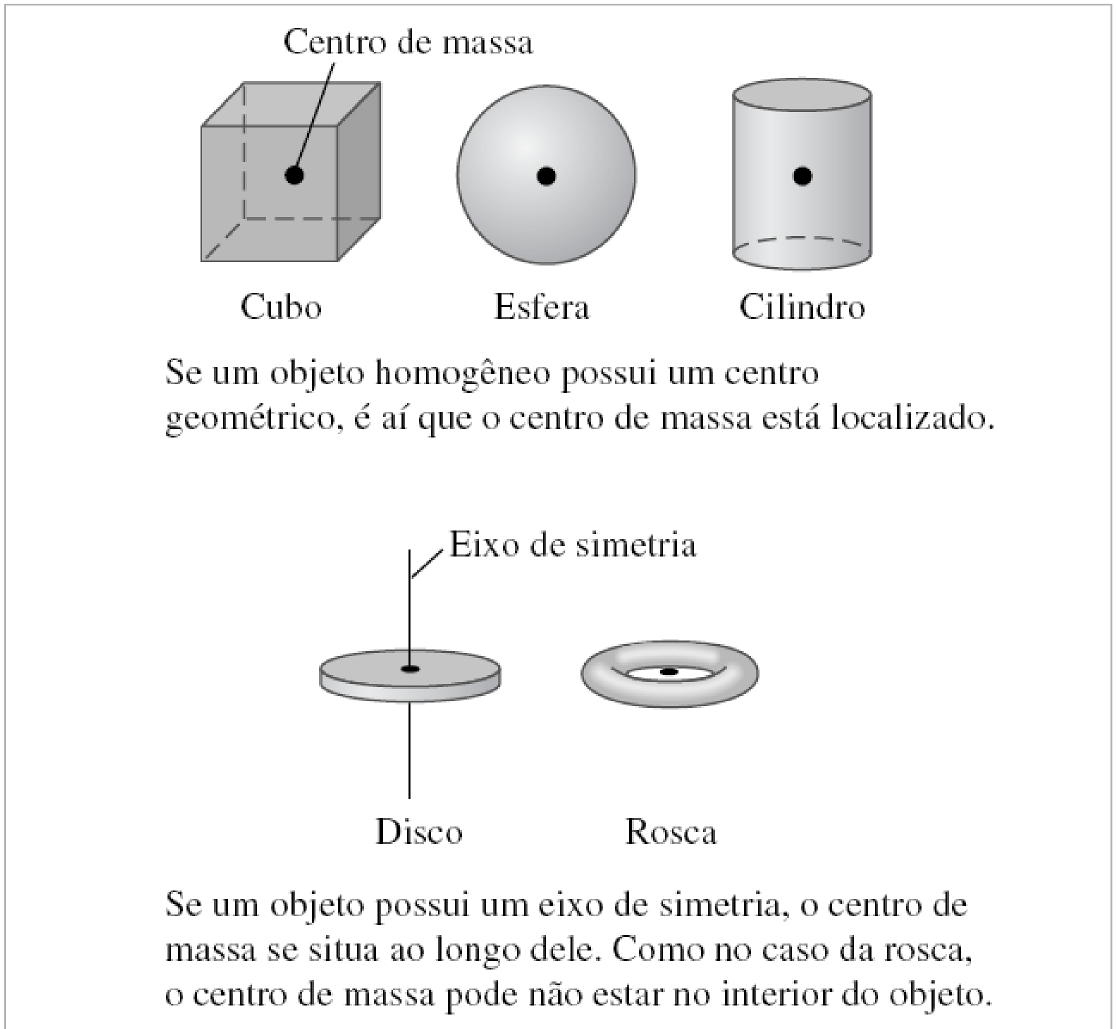
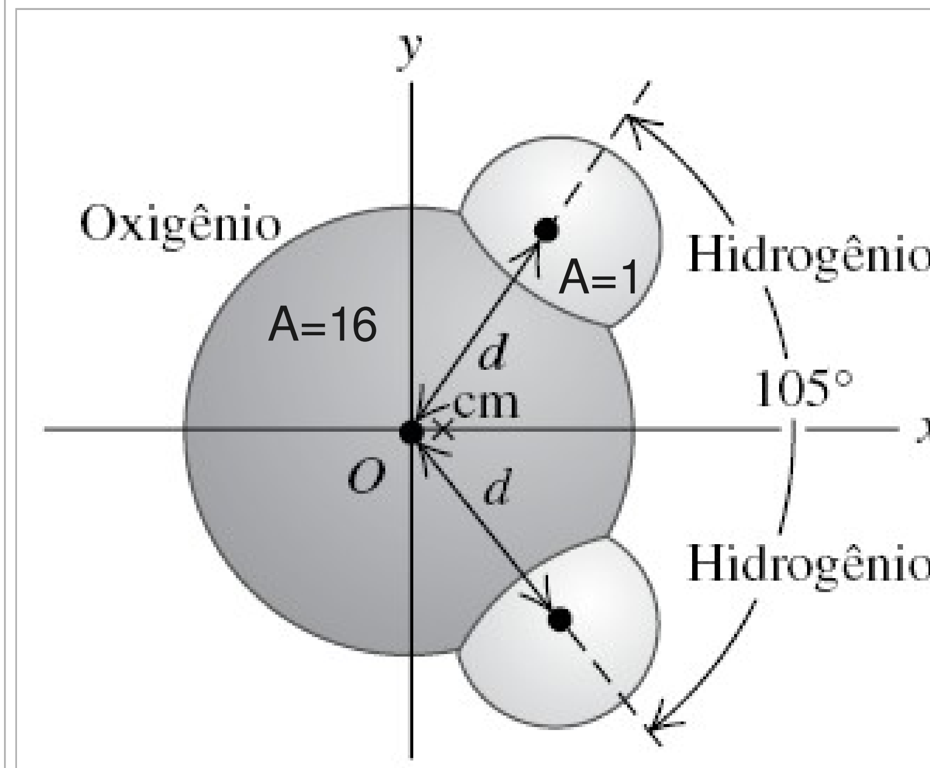


Figura 8.28 Localização do centro de massa de um objeto simétrico.

CM da molécila de água

X_{cm} = 0.068 d (d≈1Å)



Movimento do centro de massa

$$\vec{v}_{cm} = \frac{d\,\vec{r}_{cm}}{d\,t}$$

$$v_{cm\,x} = \frac{m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$v_{cm\,y} = \ldots$$

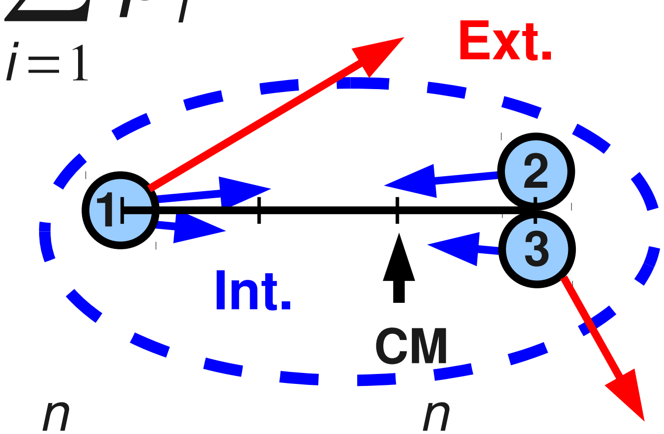
$$v_{cm\,z} = \ldots$$

$$M = m_1 + m_2 + m_3 + \ldots$$

$$M\,\vec{v}_{cm} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \ldots = \sum_{i=1}^{n} \vec{p}_i = \vec{P}$$

Aceleração do CM e ação de forças externas

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots = \sum_{i=1}^{n} \vec{p}_i = \vec{P}$$



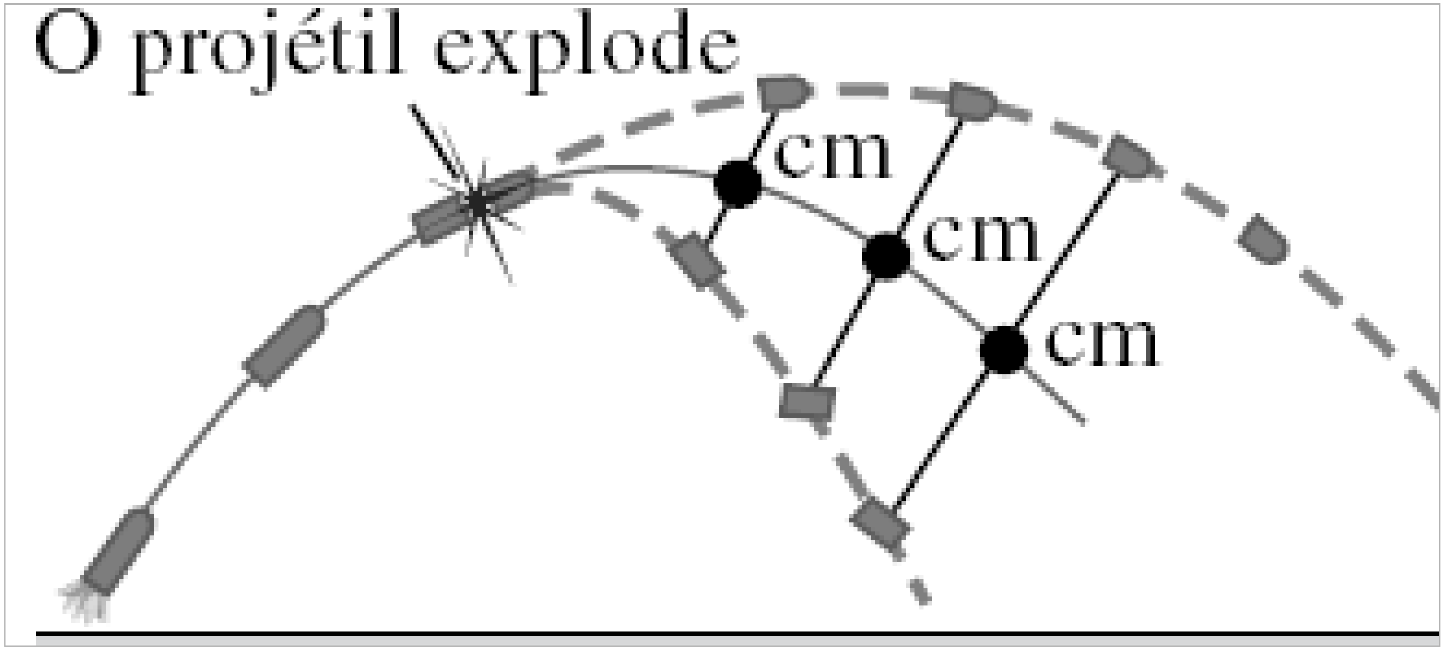
$$\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} \qquad M\vec{a}_{cm} = \frac{d\vec{P}}{dt}$$

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots = \sum_{i=1}^{n} m_i\vec{a}_1 = \sum_{i=1}^{n} \vec{F}_i$$

$$\sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} \vec{F}_i(\mathrm{Ext}) + \sum_{i=1}^{n} \vec{F}_i(\mathrm{Int}) \qquad \sum_{i=1}^{n} \vec{F}_i(\mathrm{Int}) = 0$$

$$\boxed{\sum_{i=1}^{n} \vec{F}_i(\mathrm{Ext}) = M\vec{a}_{cm} = \frac{d\vec{P}}{dt}}$$

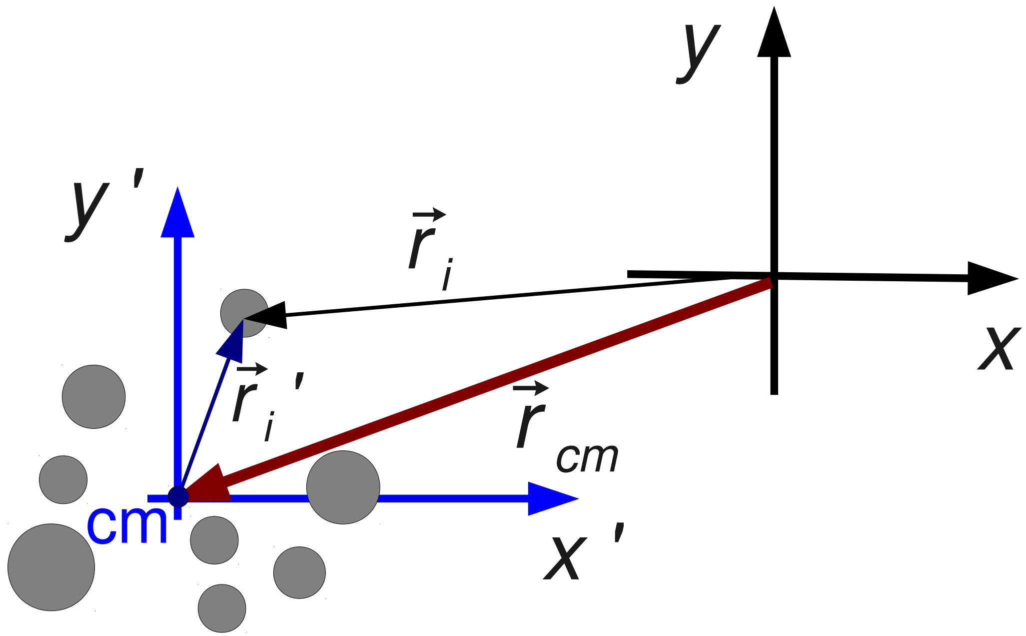
Movimento do CM



Challenger

Fogos de artifício

O Referencial do CM



$$\vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}$$

$$\sum_{i=1}^{n} m_i \vec{r}_{cm} - \sum_{i=1}^{n} m_i \vec{r}_i = 0$$

$$\vec{r}_i\,' = \vec{r}_i - \vec{r}_{cm}$$

$$\sum_{i=1}^{n} m_i (\vec{r}_i - \vec{r}_{cm}) = 0 \Rightarrow \boxed{\sum_{i=1}^{n} m_i \vec{r}_i\,' = 0}$$

"Na ausência de força resultante externa, o ref. do CM é inercial"

$$\sum_{i=1}^{n} \vec{F}_i(\mathrm{Ext}) = M \vec{a}_{cm}$$

$$\sum_{i=1}^{n} \vec{F}_i(\mathrm{Ext}) = 0 \Leftrightarrow \vec{a}_{cm} = 0$$

$$\vec{P} = M \vec{V}_{cm} = cte$$

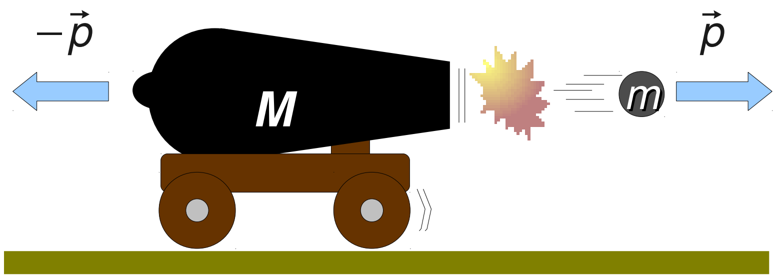
Sistema isolado → Mom. Lin. constante

Física no referencial do CM

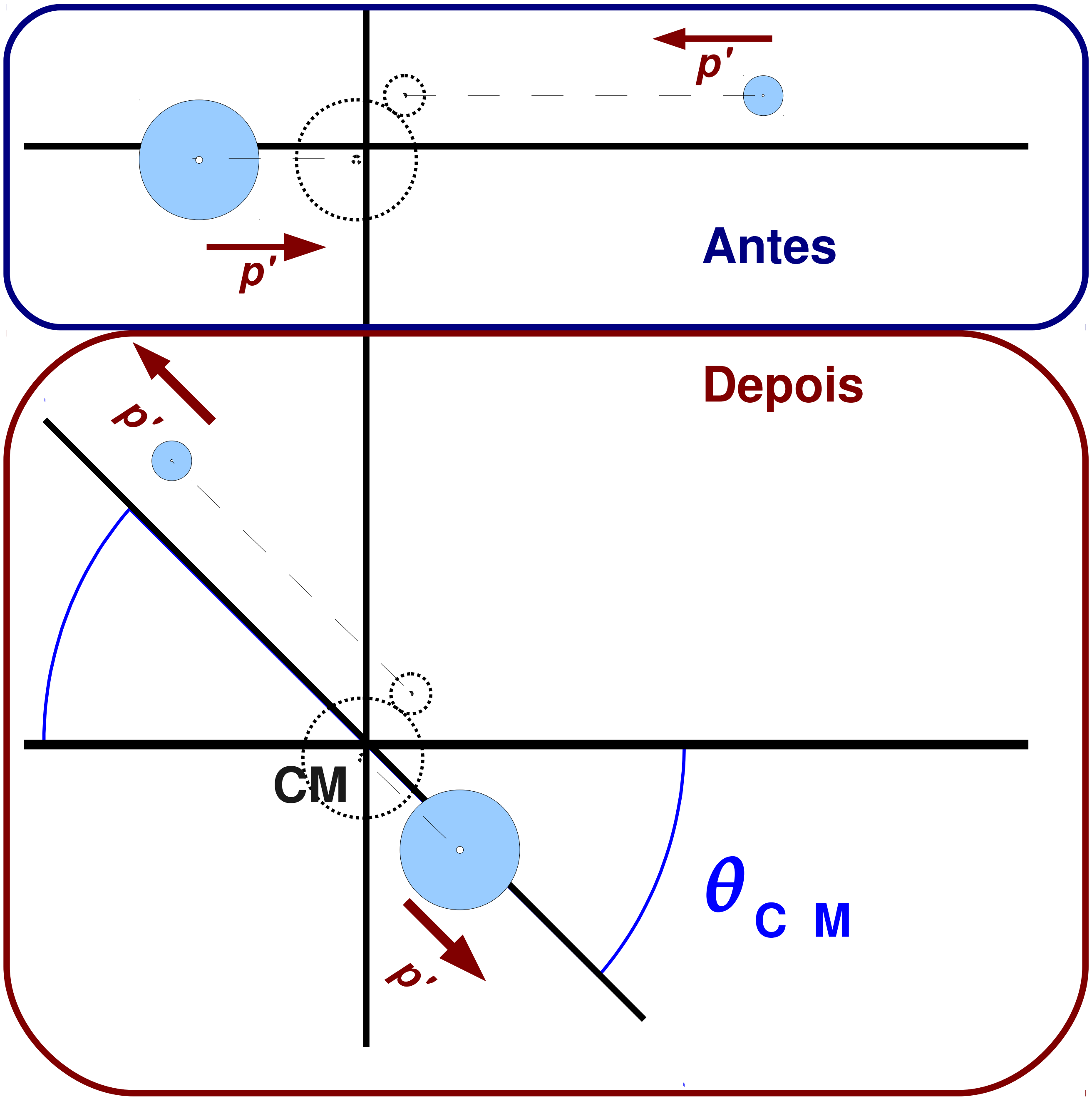
$$\sum_{i=1}^{n} m_i \vec{r}_i{}' = 0 \qquad \frac{d}{dt} \rightarrow \ldots \qquad \sum_{i=1}^{n} m_i \vec{v}_i{}' = \sum_{i=1}^{n} \vec{p}_i{}' = \vec{P}' = 0$$

"O momento linear total no ref. do CM (caso inercial) é zero"

Exemplos simples:



Colisão elástica no ref. do CM

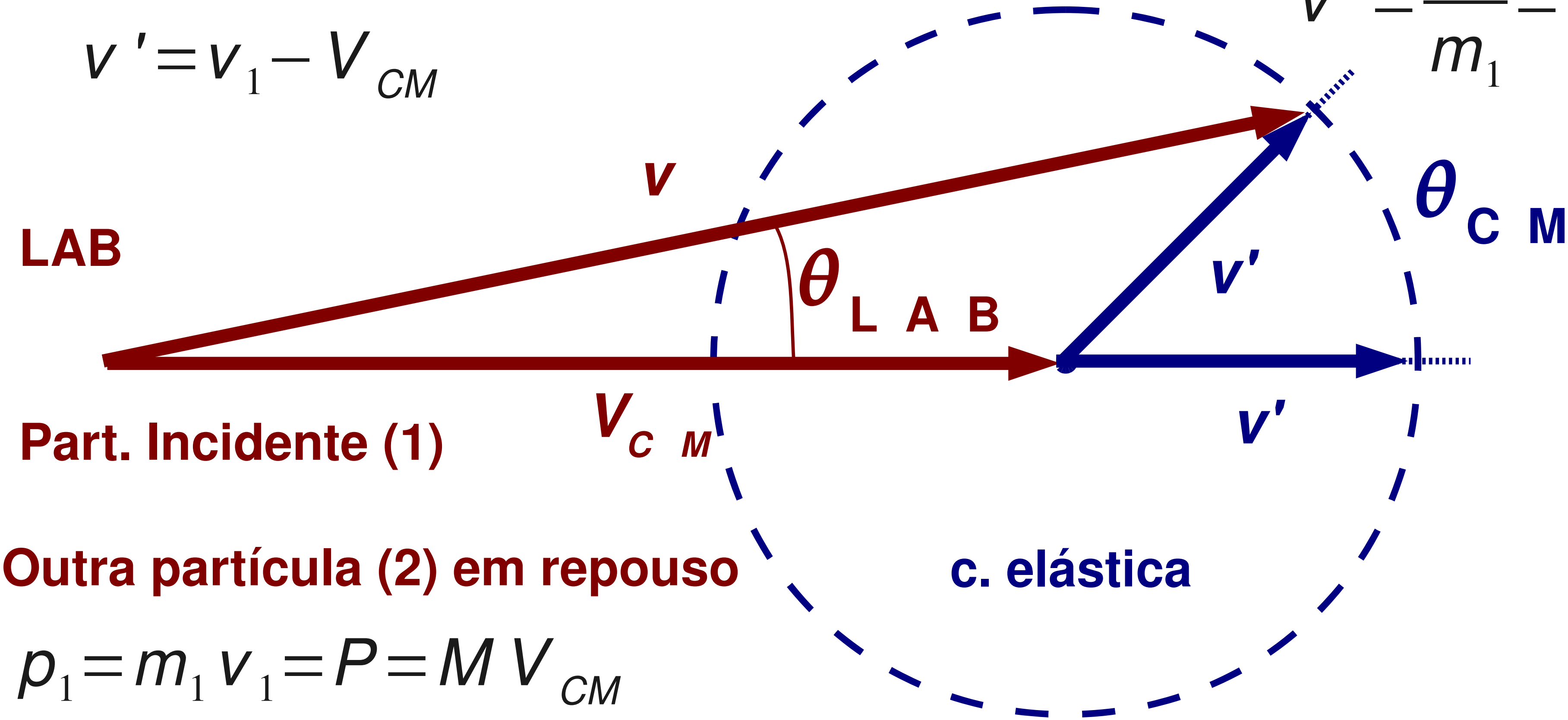


Conversão ref. do Lab. ↔ ref. do CM

$$\vec{v} = \vec{V}_{CM} + \vec{v}\,'$$

$$v\,' = v_1 - V_{CM}$$

$$v\,' = \frac{p\,'}{m_1} = cte$$



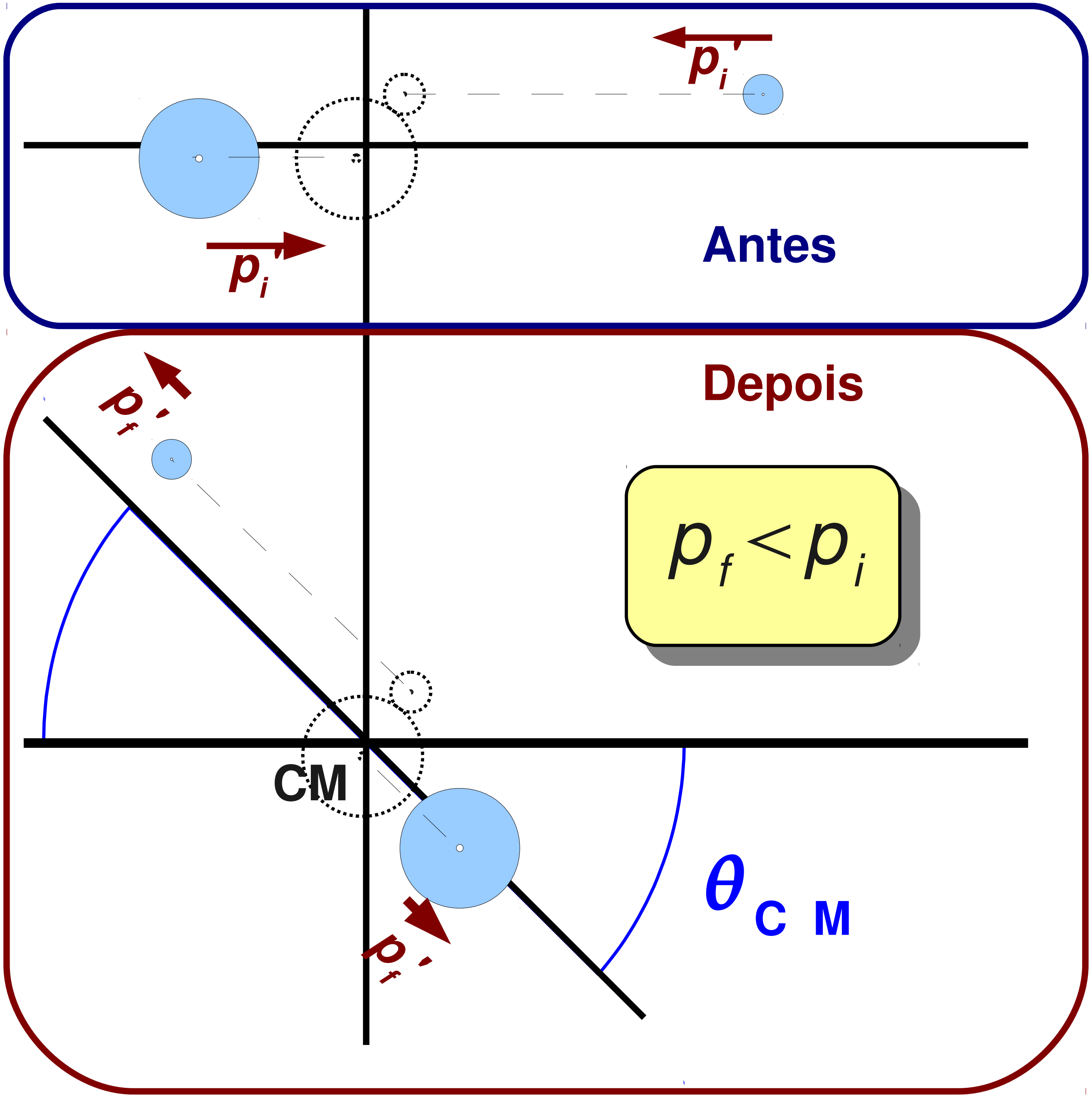
$$p_1 = m_1 v_1 = P = M\, V_{CM}$$

$$V_{CM} = \frac{m_1}{M} v_1 \qquad (M = m_1 + m_2)$$

$$v\,' = v_1 \left(1 - \frac{m_1}{M}\right) = v_1 \frac{m_2}{M}$$

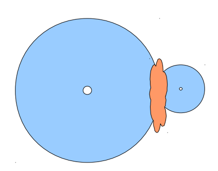
$$\sin\theta_{LAB}(Máx.) = \frac{v\,'}{V_{CM}} = \frac{m_2}{m_1}$$

Colisão inelástica no ref. do CM



$$p_f < p_i$$

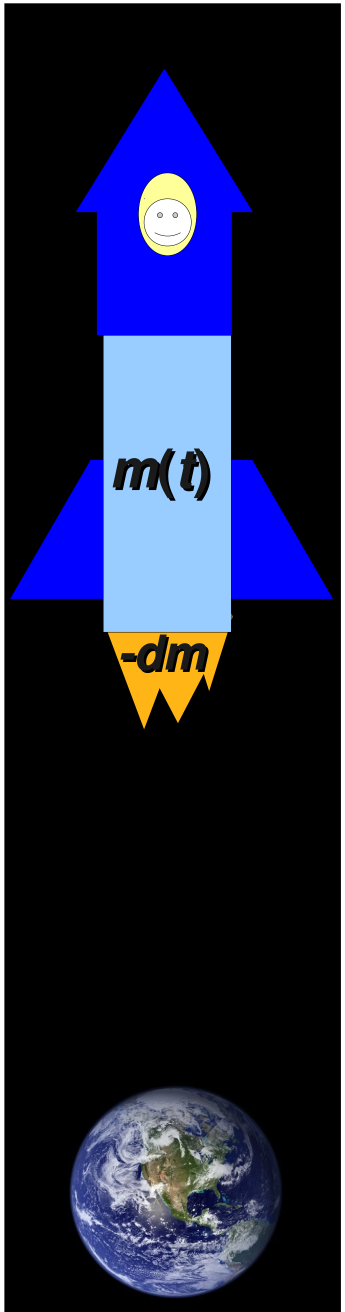
Totalmente inelástica



“$p_f = 0$”

Rotação?

Massa “variável” - Propulsão de foguetes



No ref. da terra: $v_1 = v$ **(foguete)** $v_2 = -v_e + v$ **(comb.)**

Vel. de exaustão dos gases

$dm < 0$

Antes de queimar -*dm*:

$$p(t) = m(t)\,v(t) = m\,v$$

Depois de queimar -*dm*:

$$p(t+dt) = (m+dm)(v+dv) \approx m\,v + m\,dv + dm\,v$$

Variação: $dp = p(t+dt) - p(t) = m\,dv + dm\,v$

Cons. de Mom. Lin. total: $dp - dm\,v_2 = 0$

$$m\,dv + v\,dm = dm\,v_2 = -dm\,v_e + dm\,v$$

$$m\,dv = -dm\,v_e$$

$$\boxed{F = m\frac{dv}{dt} = -\frac{dm}{dt}v_e}$$

Força de propulsão, ou *empuxo*, do foguete

Velocidade do foguete

$$se\ \vec{F}_{ext}=0 \qquad \vec{F}=m\frac{d\vec{v}}{dt}=-\frac{dm}{dt}(-\vec{v}_e) \qquad d\vec{v}=\frac{dm}{m}\vec{v}_e$$

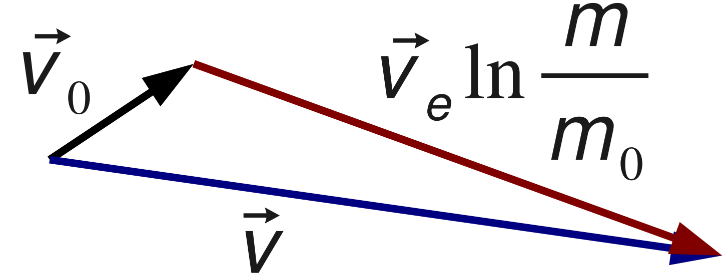
Caso co-linear: $$v-v_0=\int_{v_0}^{v} dv'=-v_e\int_{m_0}^{m}\frac{dm'}{m'}=-v_e\ln\frac{m}{m_0}$$

$$\boxed{v=v_0+v_e\ln\frac{m_0}{m}}$$

$$e^{\frac{v-v_0}{v_e}}=\frac{m_0}{m}$$

Caso não co-linear, mas com $\vec{v}_e\ cte$

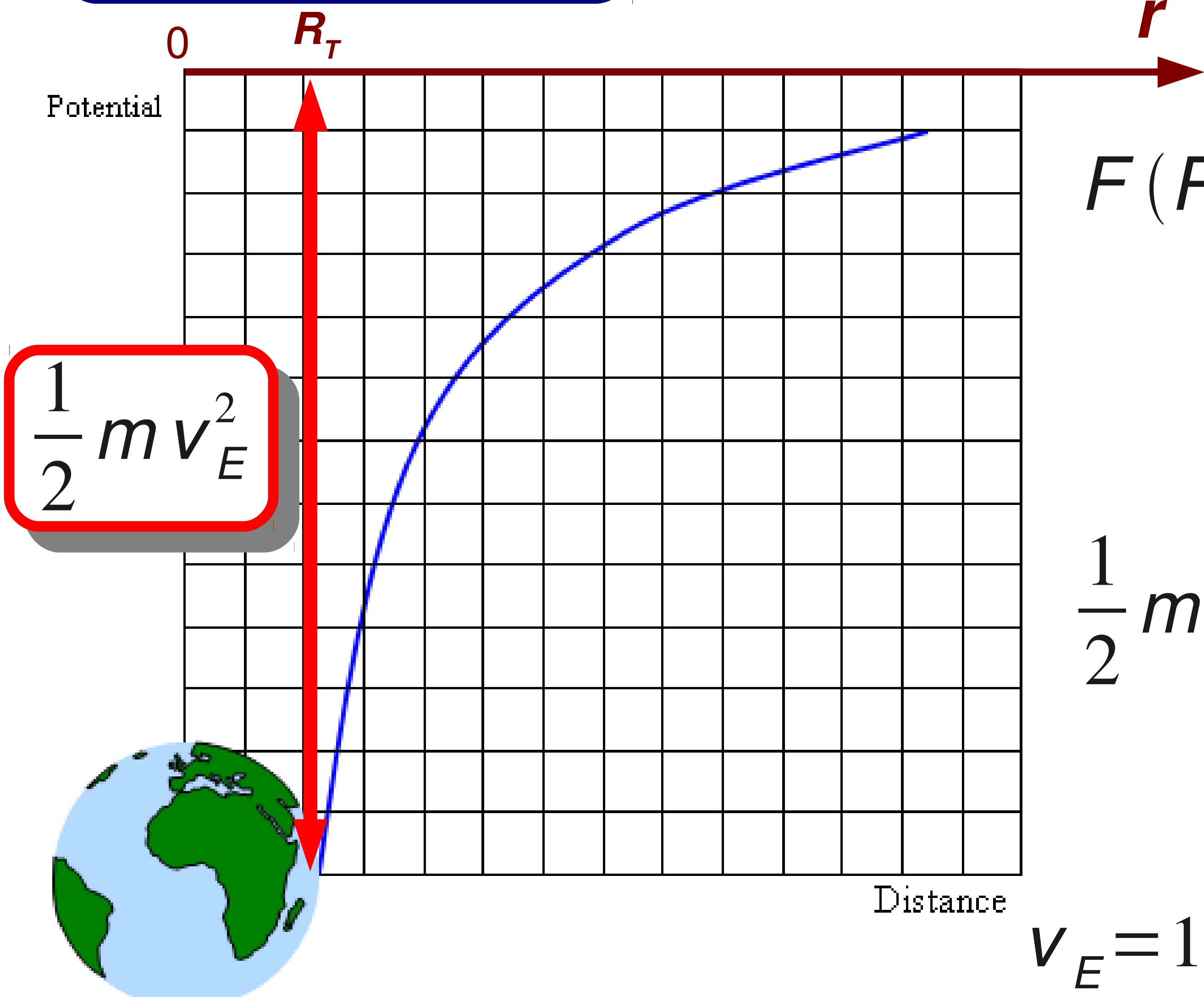
$$\boxed{\vec{v}=\vec{v}_0+\vec{v}_e\ln\frac{m}{m_0}}$$



Velocidade de escape

$$\boxed{U(r) = -\frac{G\,m\,m_T}{r}}$$

$$F(r) = -\frac{dU}{dr} = -\frac{G\,m\,m_T}{r^2}$$



$$F(R_T) = -\frac{G\,m\,m_T}{R_T^2} = -mg$$

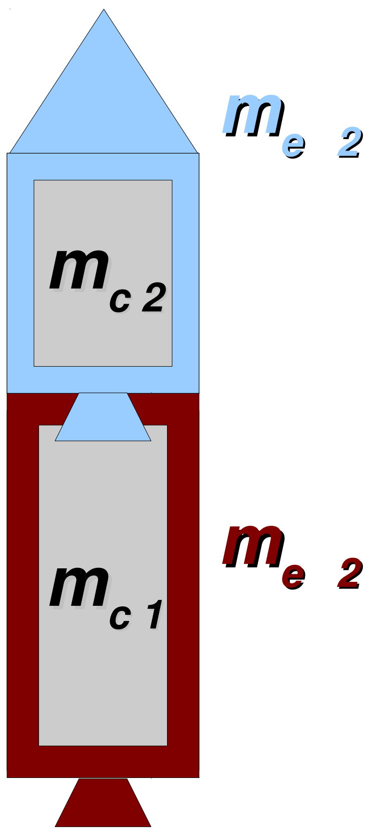
$$F(R_T) = \frac{G\,m_T}{R_T^2} = g$$

$$\frac{1}{2} m v_E^2 = -U(R_T) = mg\,R_T$$

$$v_E = \sqrt{(2\,g\,R_T)}$$

$$v_E = 11.2\ \mathrm{km/s} = 40300\ \mathrm{km/h}$$

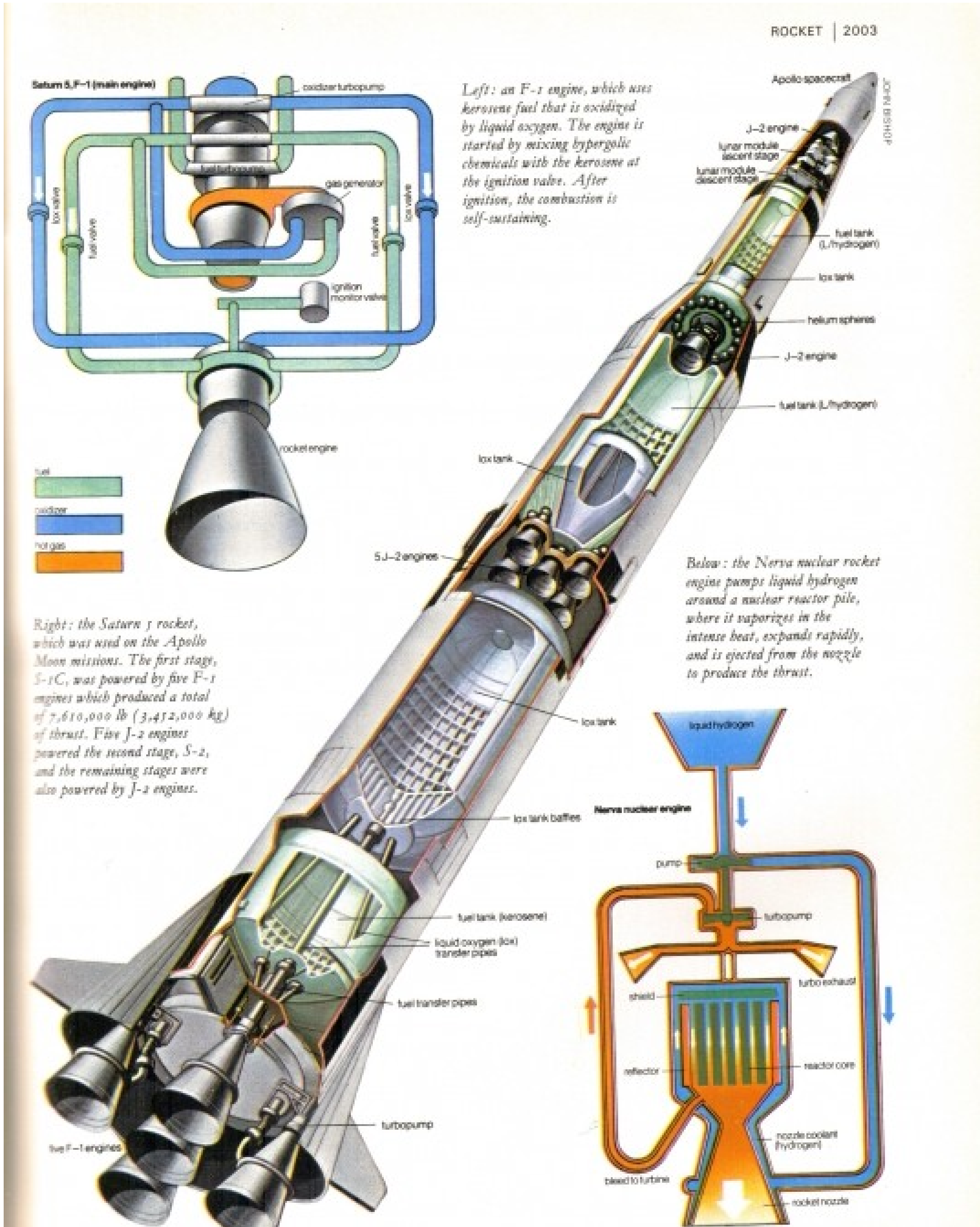
Foguetes de vários estágios



$$v = v_0 + v_e \ln \frac{m_0}{m}$$

$$v_e \approx 10.000 - 13.000\,\mathrm{km/h}$$

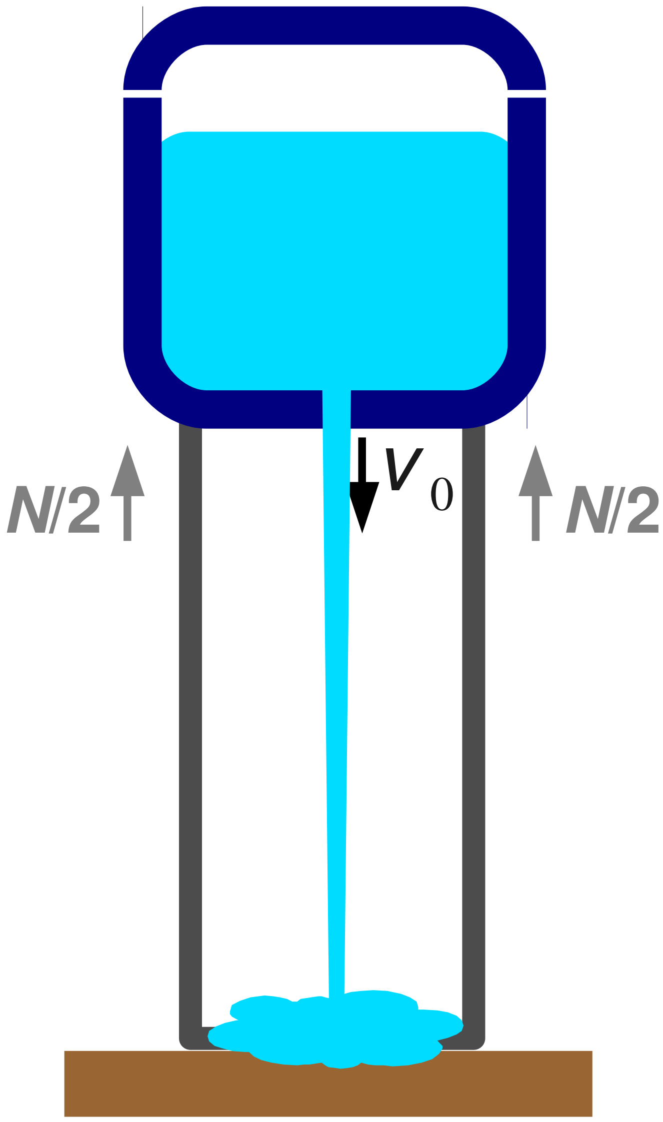
Foguetes de vários estágios



Atlantis

Saturn 5 (Apollo)

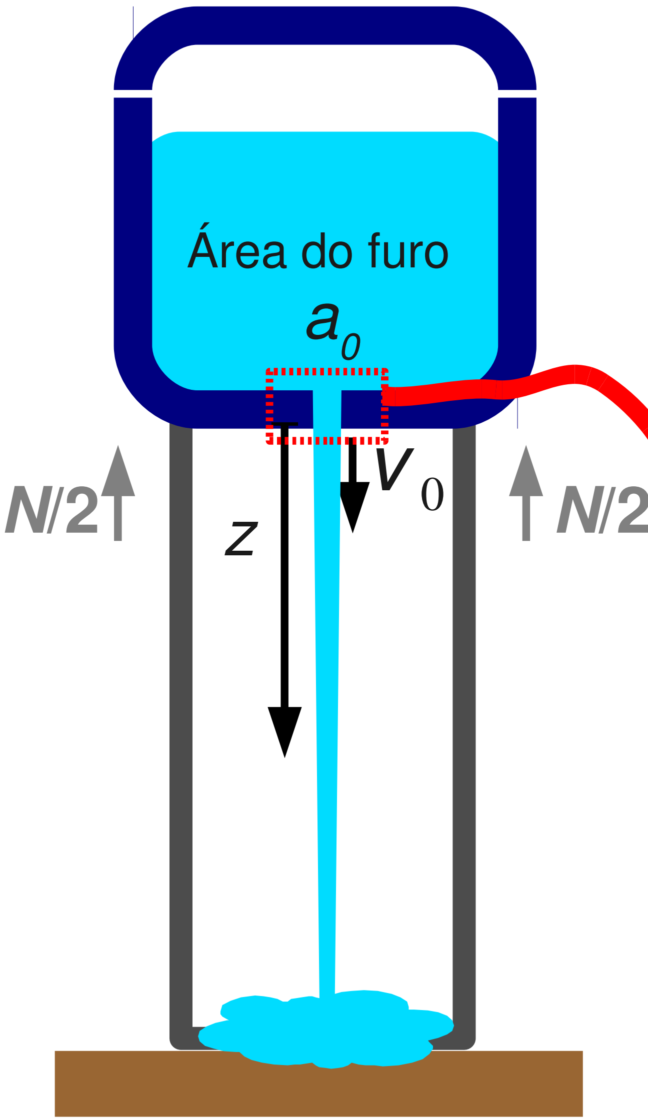
Outras situações de massa variável



“Peso” de caixa d'água vazando
(na verdade quero saber *N*)

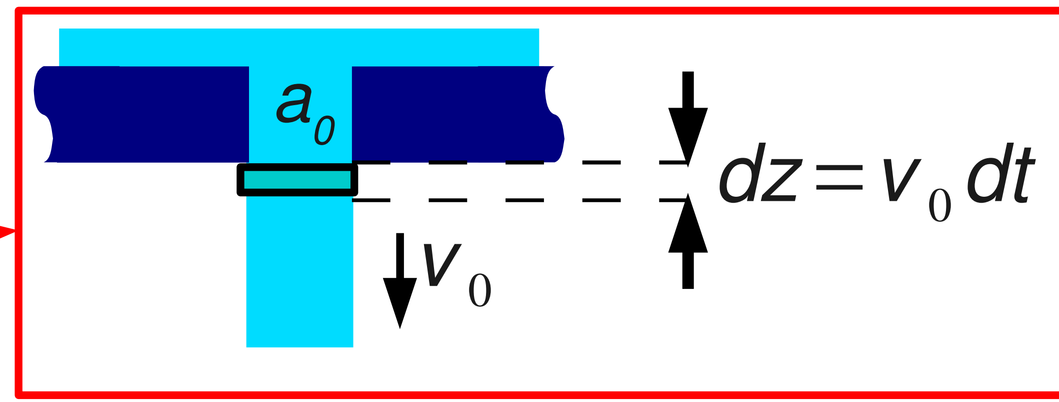
$$N = mg + \frac{dm}{dt} v_0 \qquad \left(\frac{dm}{dt} < 0\right)$$

Outras situações de massa variável



"Peso" de caixa d'água vazando

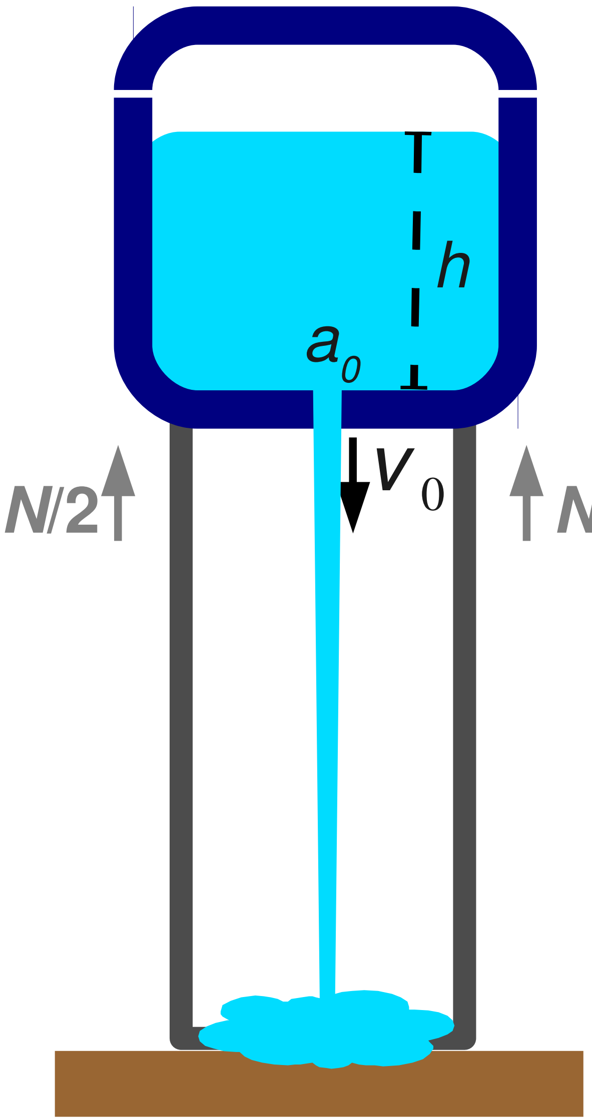
$$N = mg + \frac{dm}{dt} v_0 \qquad \left(\frac{dm}{dt} < 0\right)$$



$$dm = -\rho \underbrace{a_0 dz}_{\text{Elemento de volume}} = -\rho a_0 v_0 dt$$

$$N = mg - \rho a v_0^2$$

Outras situações de massa variável



"Peso" de caixa d'água vazando

$$N = mg + \frac{dm}{dt} v_0 \quad \left(\frac{dm}{dt} < 0\right)$$

$$N = mg - \rho a v_0^2$$

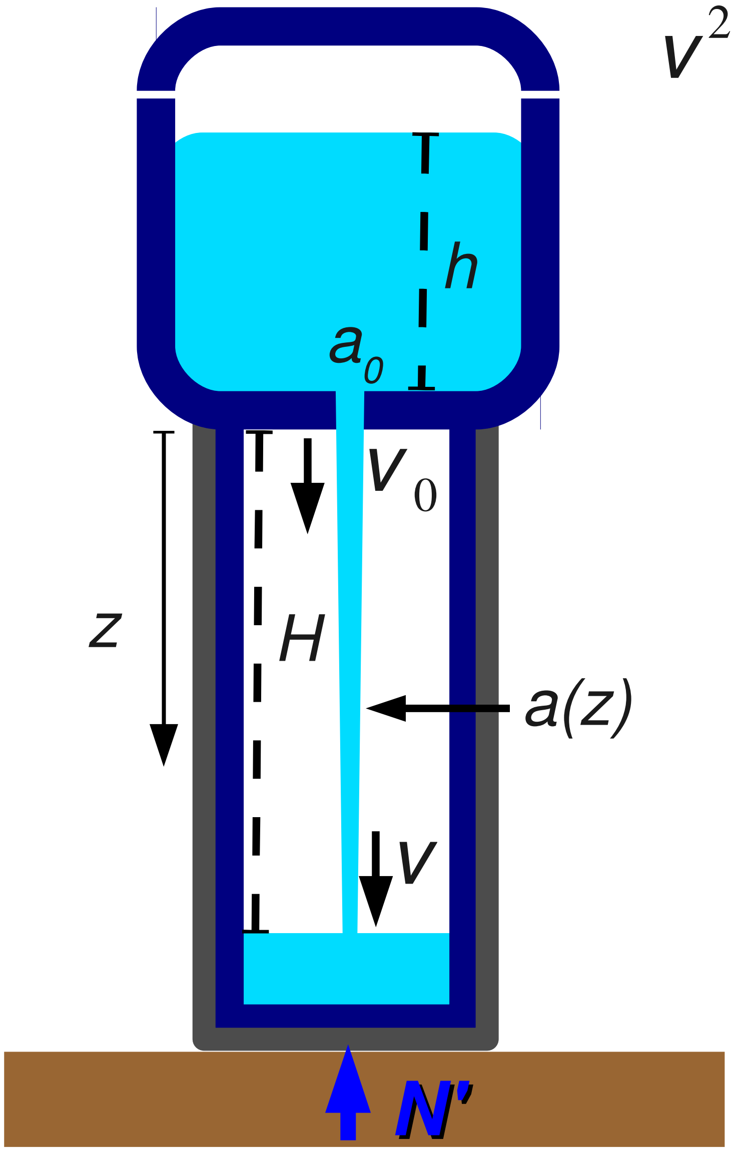
Caixa d'água "conservativa". Princ. Bernoulli.

Pressão (*P*): $\rho g h = P = \frac{1}{2} \rho v_0^2$

$$\rho a v_0^2 = 2 a P \qquad N = mg - 2 a P$$

?

Peso de uma “ampulheta” d'agua



$$v^2 = v_0^2 + 2Hg \Rightarrow (v+v_0)(v-v_0) = 2Hg$$

$$\boxed{N' = mg - \frac{dm}{dt}(v - v_0) - m_{col}\, g}$$

Peso total

Força de *chegada* menos de *saída*

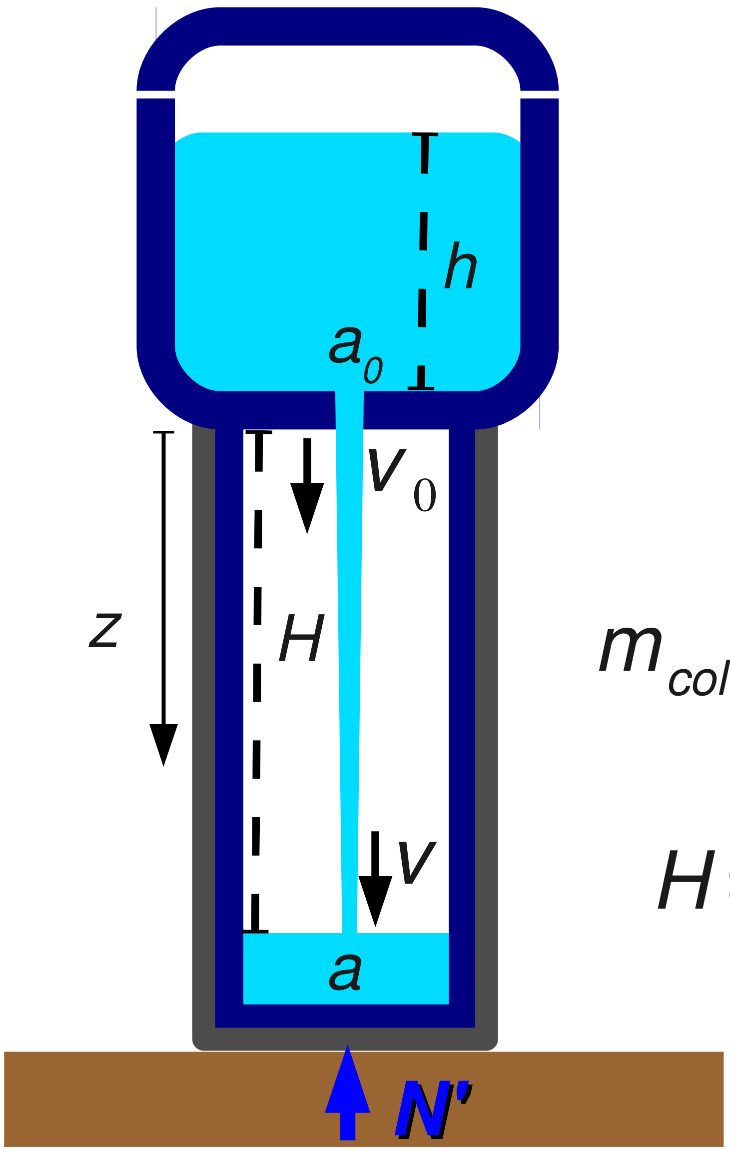
Peso da coluna de água em movimento

Conservação de fluxo na coluna de água:

$$\boxed{-\frac{dm}{dt} = \rho\, a_0 v_0 = \rho\, a(z)\, v(z)}$$

$$\boxed{a(z) = a_0 \frac{v_0}{v(z)}}$$

Peso de uma “ampulheta” d'agua



$$\boxed{N' = mg - \frac{dm}{dt}(v - v_0) - m_{col}\, g}$$

$$\boxed{-\frac{dm}{dt} = \rho\, a_0 v_0} \qquad \boxed{a(z) = a_0 \frac{v_0}{v(z)}}$$

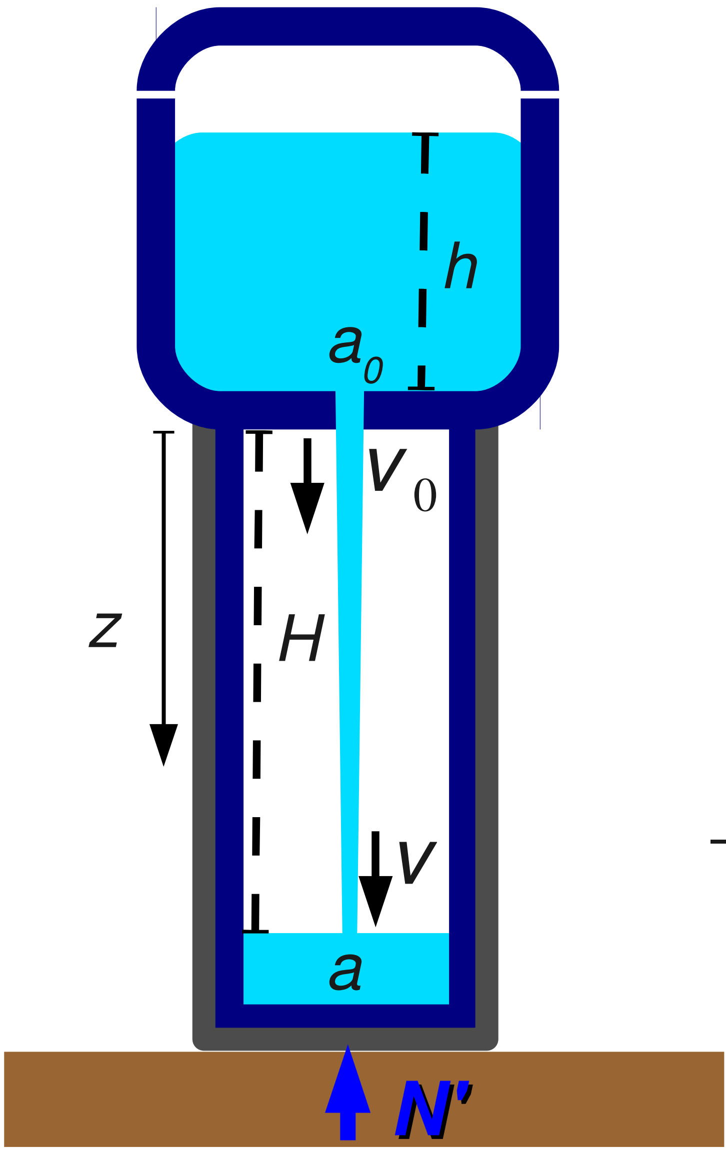
$$m_{col} = \rho \int_0^H a(z)\, dz = \rho \int_0^T a_0 \frac{v_0}{v} v\, dt = \rho\, a_0 v_0 T$$

$$H = \frac{v_0 + v}{2} T \qquad m_{col}\, g = \frac{\rho\, a_0 v_0 2 H g}{(v + v_0)}$$

$$(v + v_0)(v - v_0) = 2Hg$$

Subst. $2Hg$: $\boxed{m_{col}\, g = \rho\, a_0 v_0 (v - v_0)}$

Peso de uma “ampulheta” d'agua



$$\boxed{N' = mg - \frac{dm}{dt}(v - v_0) - m_{col}\, g}$$

$$\boxed{-\frac{dm}{dt} = \rho\, a_0\, v_0}$$

$$\boxed{a(z) = a_0 \frac{v_0}{v(z)}}$$

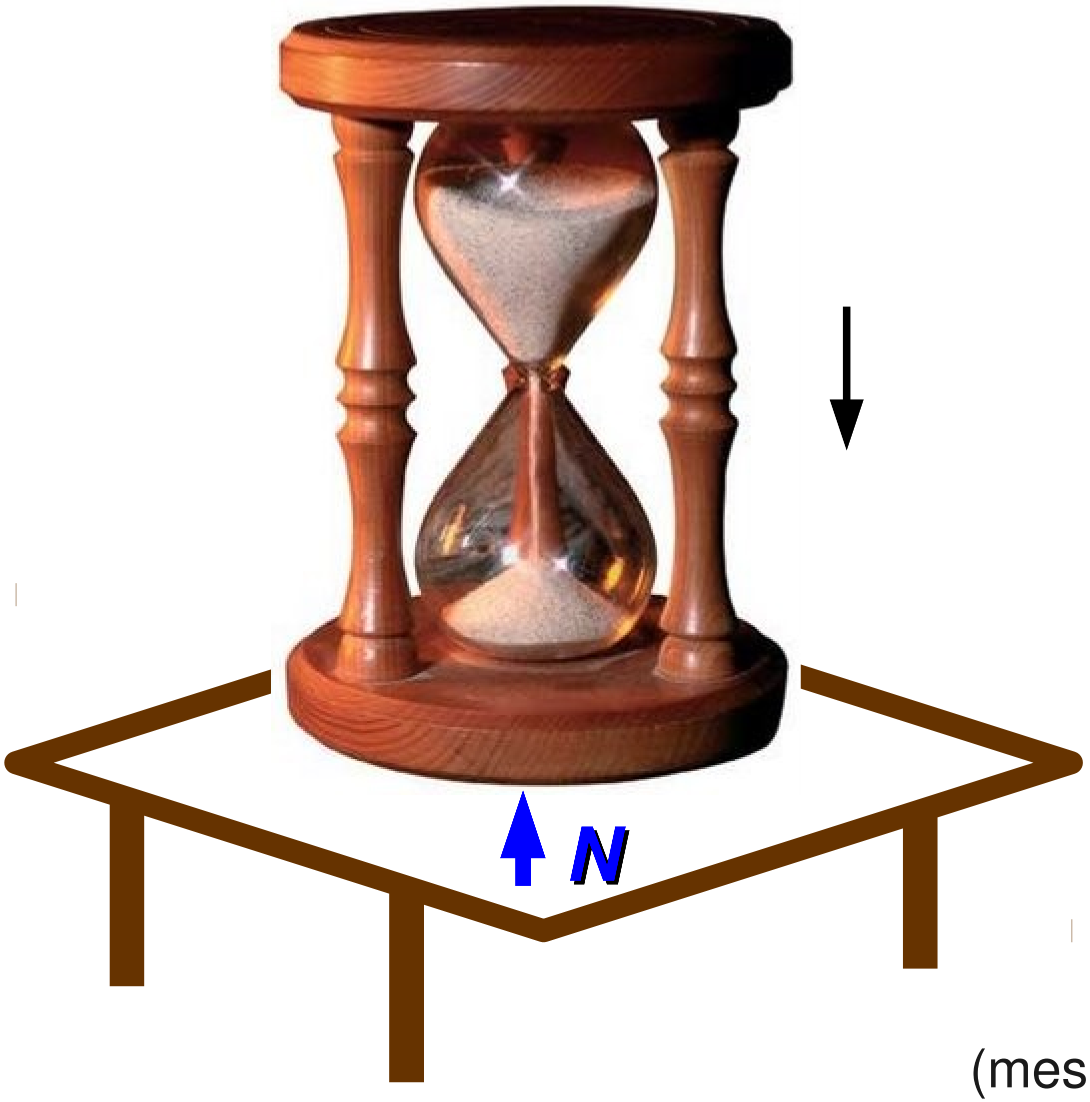
$$\boxed{m_{col}\, g = \rho\, a_0\, v_0 (v - v_0)}$$

$$-\frac{dm}{dt}(v - v_0) = \rho\, a_0\, v_0 (v - v_0) = m_{col}\, g$$

$$N' = mg + m_{col}\, g - m_{col}\, g$$

$$\boxed{N' = mg}\ !$$

Peso de uma ampulheta



$$\vec{P} = \sum m_i v_i \approx cte$$

$$\frac{d\vec{P}}{dt} = \vec{F}_R(\mathrm{ext.}) \approx 0$$

$$F_R(\mathrm{ext.}) = -mg + N$$

$$\boxed{N \approx mg}$$

(mesmo com fluxo de areia não-conservativo)