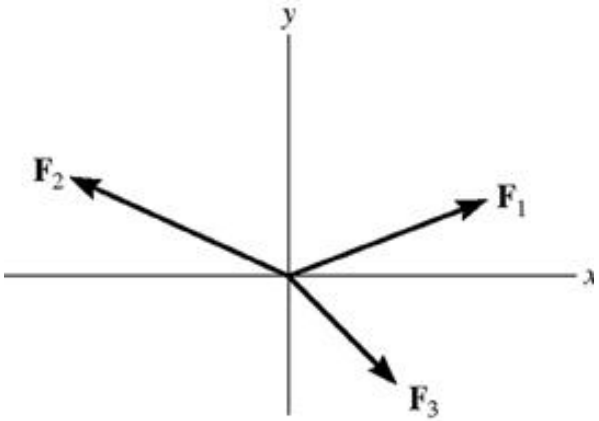
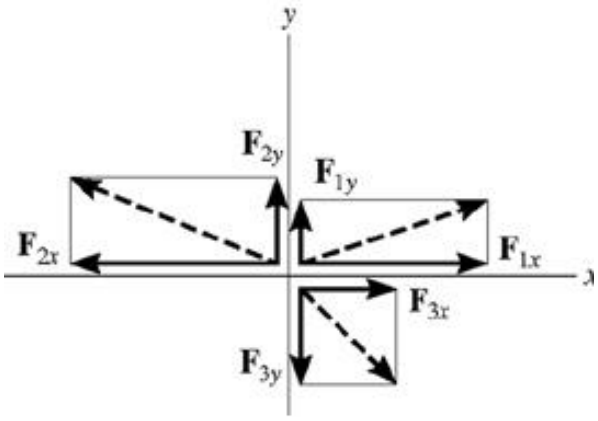
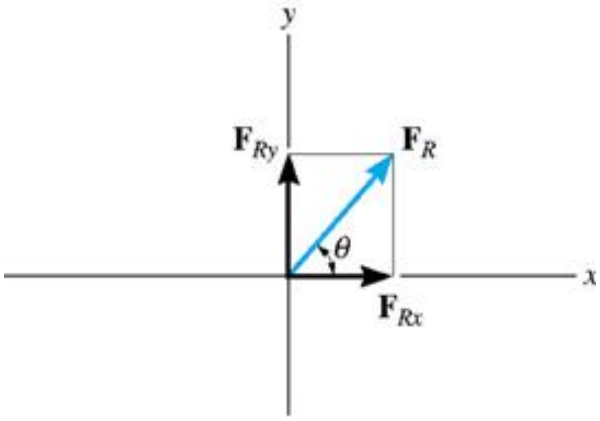
PEF 2308

Aula 2 – Conceitos iniciais


y
x
F2
F1
F3


y
x
F2y
F1y
F2x
F1x
F3x
F3y


y
x
FRy
FR
θ
FRx

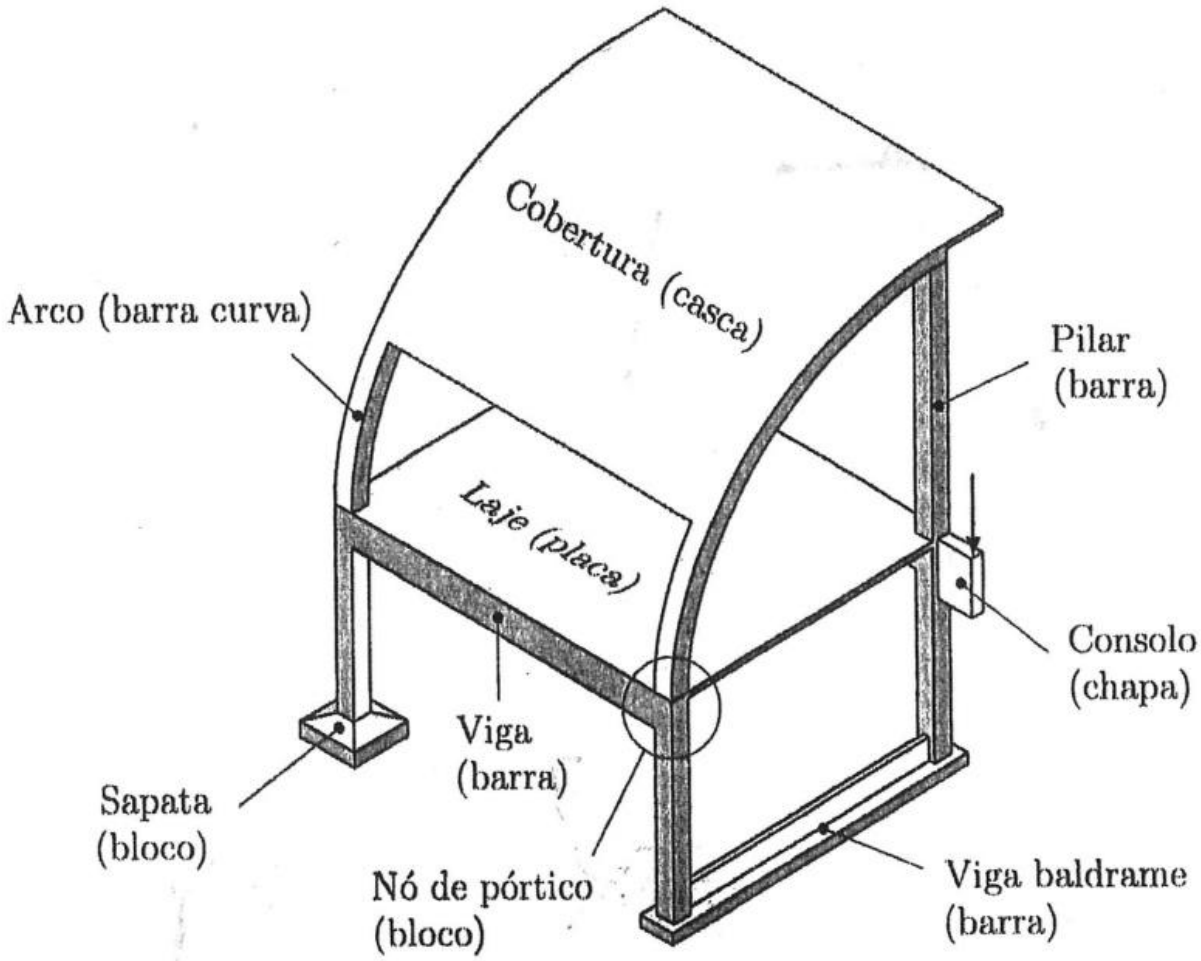
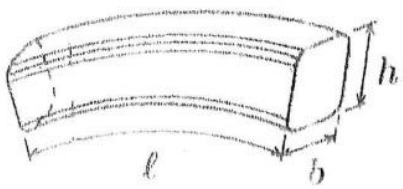
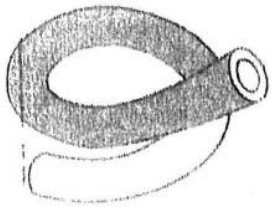


Fig. 1: Elementos estruturais de uma edificação.

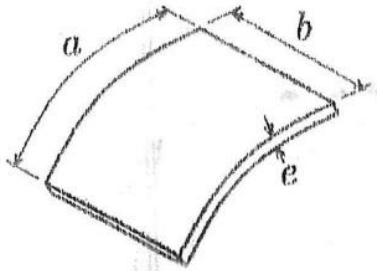
(a) Barra



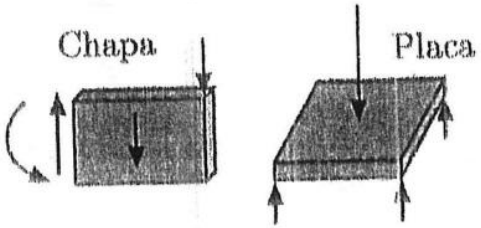
$b, h \ll \ell$



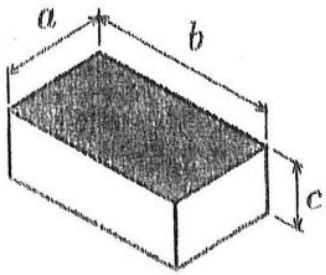
(b) Folha



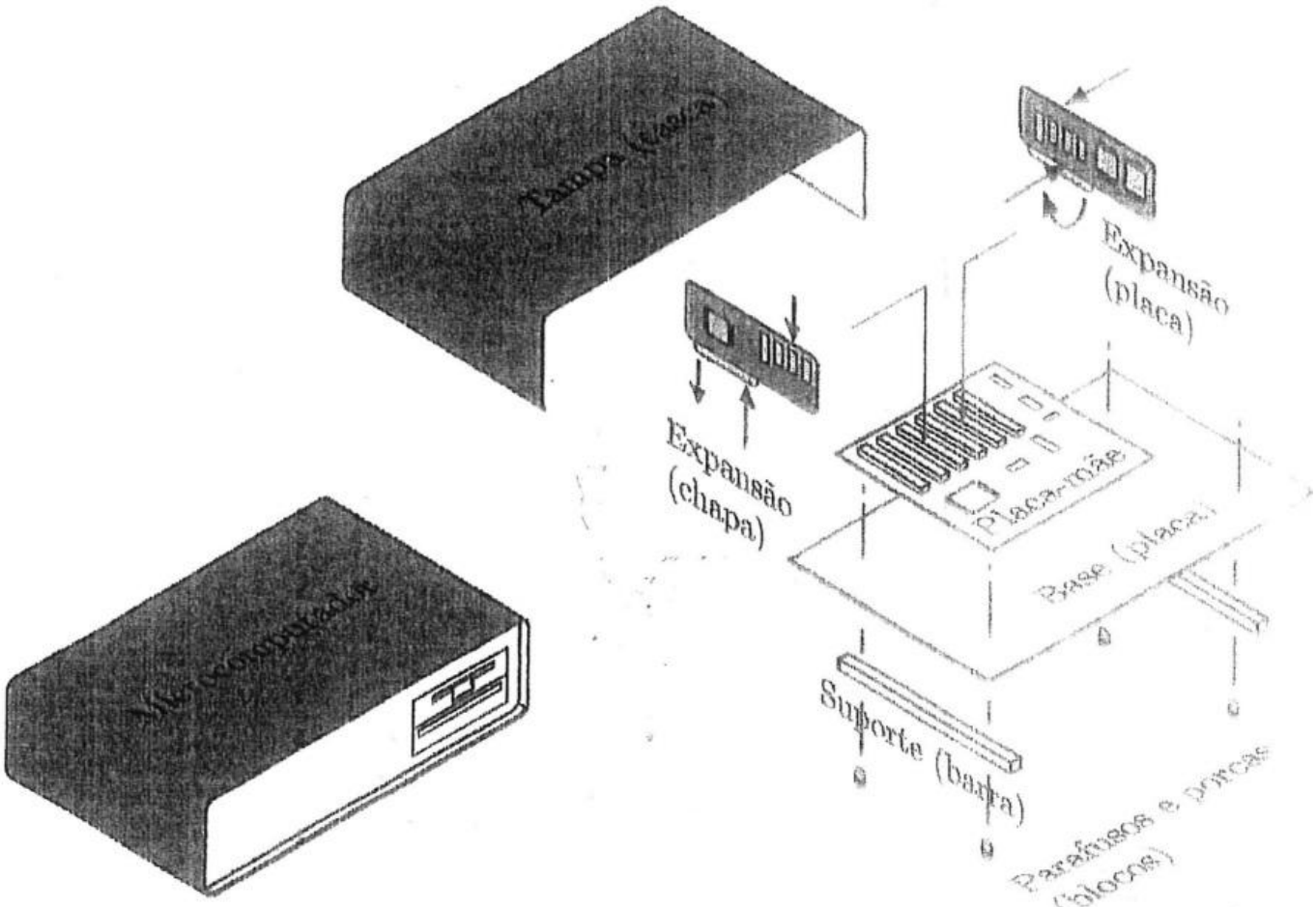
$e \ll a, b$



(c) Bloco



$a \approx b \approx c$


Tampa (lateral)
Expansão
(placa)
Expansão
(chapa)
Placa-mãe
Base (placa)
Suporte (barra)
Parafusos e porcas
(blocos)

Para um corpo em repouso em relação a um sistema inercial, as leis de Euler[3] fornecem:

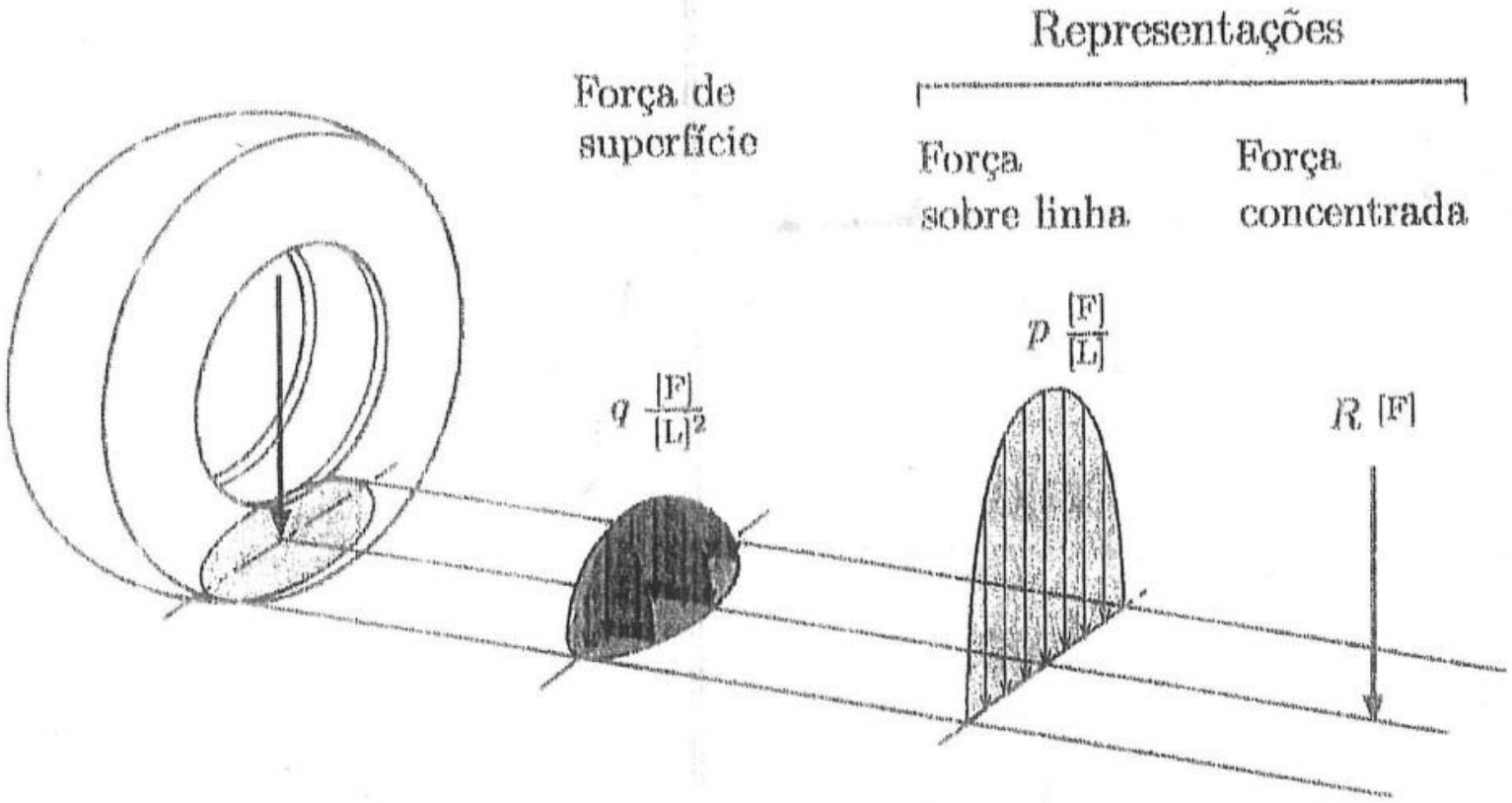
$$\sum_{i=1}^{n_F} \boldsymbol{F}_i = \boldsymbol{0}, \qquad \sum_{j=1}^{n_M} \boldsymbol{M}_{\mathrm{O}\,j} = \boldsymbol{0}, \tag{2.1}$$

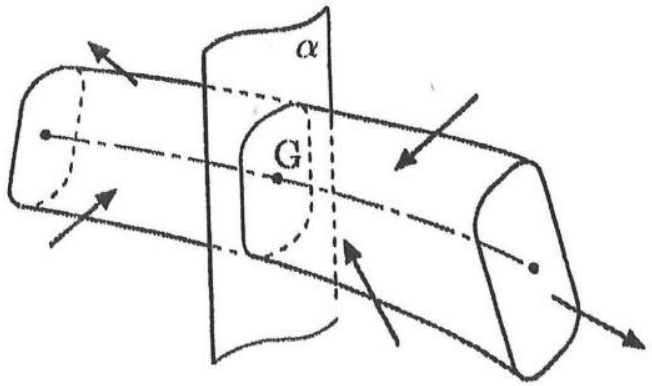
correspondendo ao equilíbrio de n_F forças $\boldsymbol{F}_i$ e n_M momentos $\boldsymbol{M}_j$ em relação a um polo arbitrário O. Reescrevendo a equação acima empregando as componentes de força e momento em relação a três eixos ortogonais x, y e z passando por O, obtemos

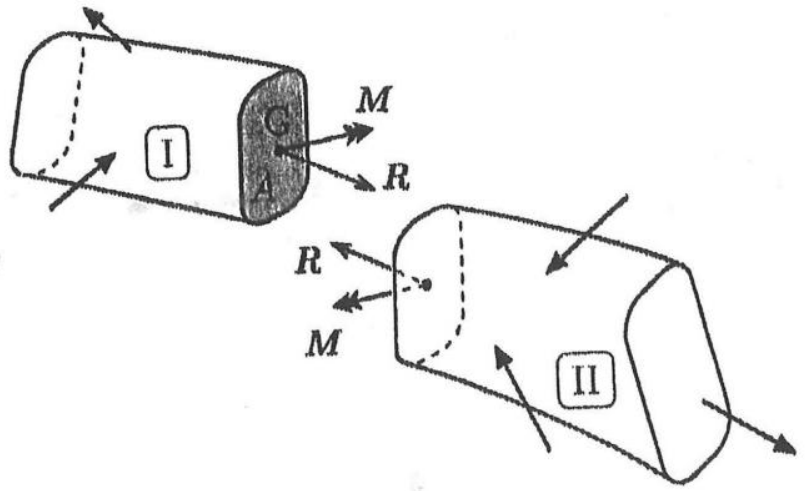
$$\begin{aligned} \sum F_x &= 0, & \sum M_{\mathrm{O}x} &= 0, \\ \sum F_y &= 0, & \sum M_{\mathrm{O}y} &= 0, \\ \sum F_z &= 0, & \sum M_{\mathrm{O}z} &= 0, \end{aligned} \tag{2.2}$$

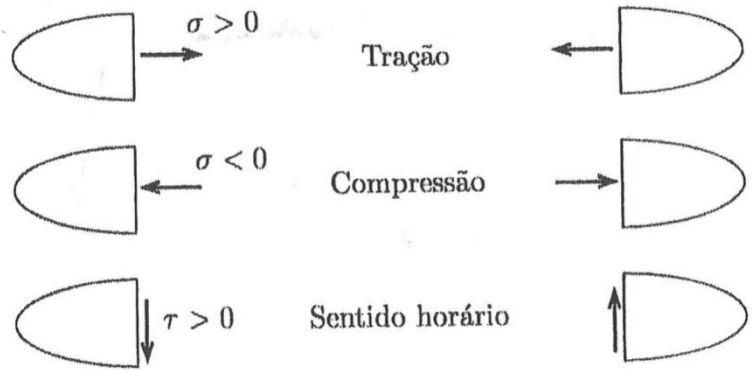
onde os índices foram omitidos. Para um sistema de forças coplanares em que as forças e momentos atuam no plano definido pelos eixos x e y, restam apenas três equações não-identicamente nulas:

$$\begin{aligned} \sum F_x &= 0, \\ \sum F_y &= 0, \end{aligned} \qquad \sum M_{\mathrm{O}z} = 0. \tag{2.3}$$


Representações
Força de superfície
Força sobre linha
Força concentrada
$q \frac{[F]}{[L]^2}$
$p \frac{[F]}{[L]}$
R [F]

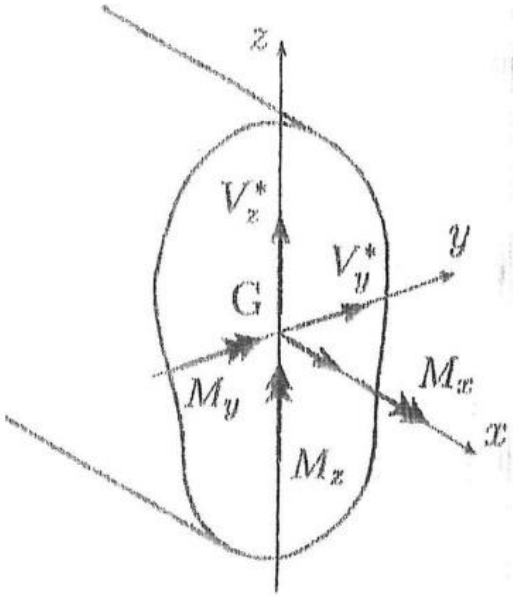

α
G

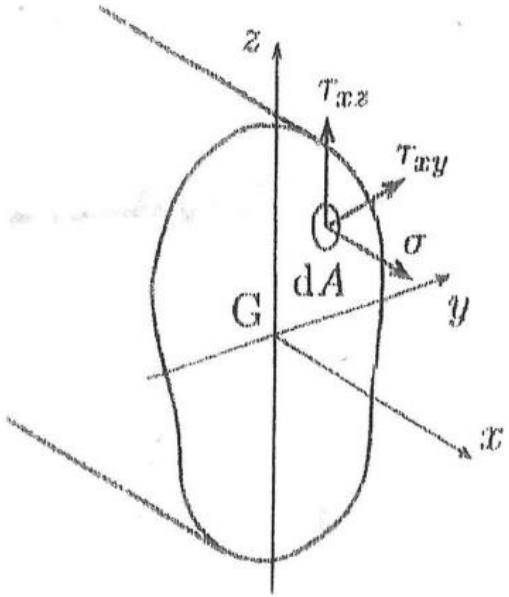

I
G
M
R
R
M
II



$$\mathrm{d}N = \sigma\,\mathrm{d}A, \qquad \mathrm{d}V_y^* = \tau_{xy}\,\mathrm{d}A, \qquad \mathrm{d}V_z^* = \tau_{xz}\,\mathrm{d}A,$$

$$\mathrm{d}M_y = \sigma\,z\,\mathrm{d}A, \qquad \mathrm{d}M_z = -\sigma\,y\,\mathrm{d}A, \qquad \mathrm{d}M_x = (\tau_{xz}\,y - \tau_{xy}\,z)\,\mathrm{d}A$$





Esforços solicitantes e componentes de tensão na seção transversal.

$$\mathrm{d}N = \sigma\,\mathrm{d}A, \qquad \mathrm{d}V_y^* = \tau_{xy}\,\mathrm{d}A, \qquad \mathrm{d}V_z^* = \tau_{xz}\,\mathrm{d}A,$$

$$\mathrm{d}M_y = \sigma\,z\,\mathrm{d}A, \qquad \mathrm{d}M_z = -\sigma\,y\,\mathrm{d}A, \qquad \mathrm{d}M_x = (\tau_{xz}\,y - \tau_{xy}\,z)\,\mathrm{d}A$$

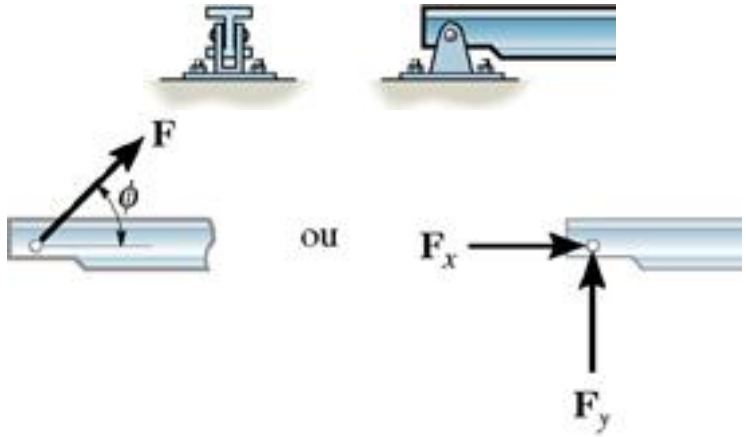
$$N = \int_A \sigma\,\mathrm{d}A, \qquad V_y^* = \int_A \tau_{xy}\,\mathrm{d}A, \qquad V_z^* = \int_A \tau_{xz}\,\mathrm{d}A,$$

$$M_y = \int_A \sigma\,z\,\mathrm{d}A, \qquad M_z = -\int_A \sigma\,y\,\mathrm{d}A, \qquad M_x = \int_A (\tau_{xz}\,y - \tau_{xy}\,z)\,\mathrm{d}A.$$

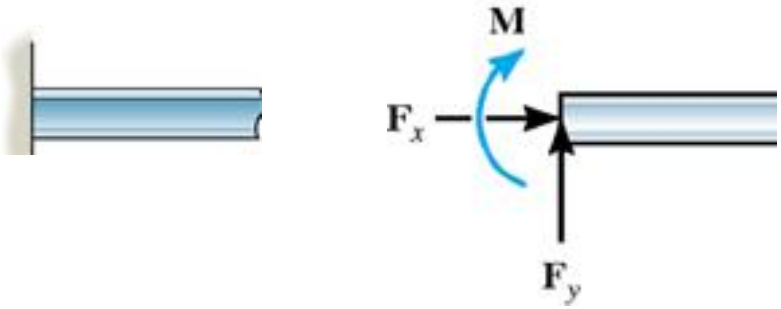
APOIOS NO PLANO

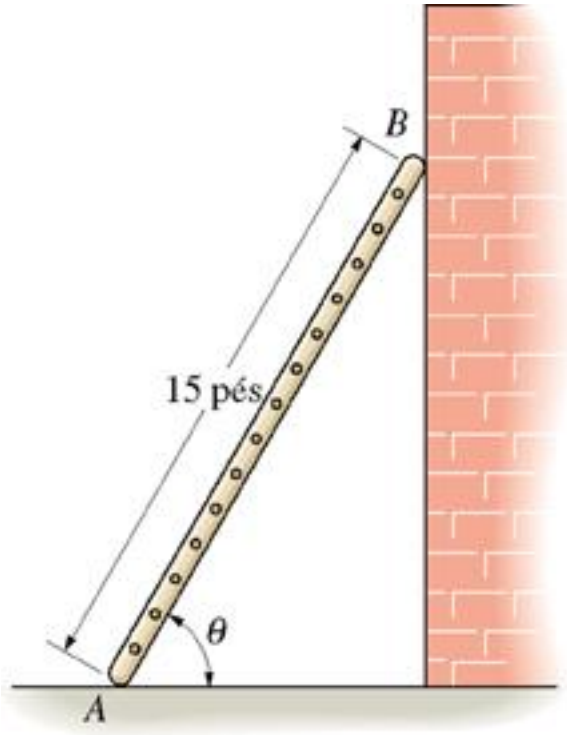
ARTICULAÇÃO MÓVEL:

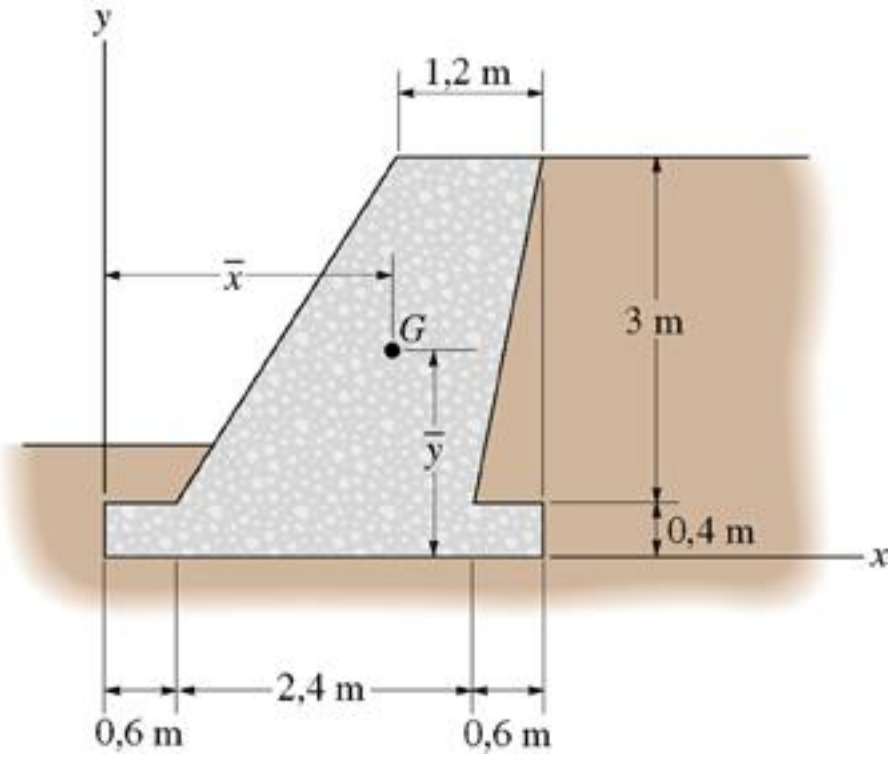
ARTICULAÇÃO FIXA:

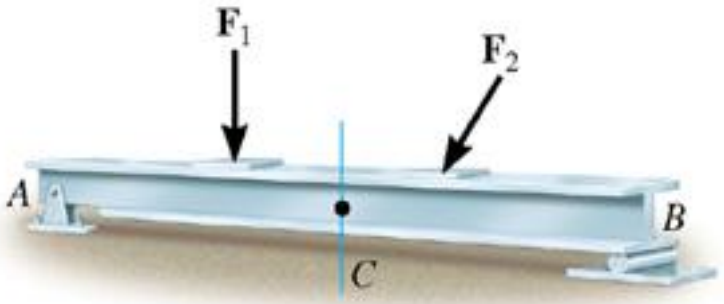


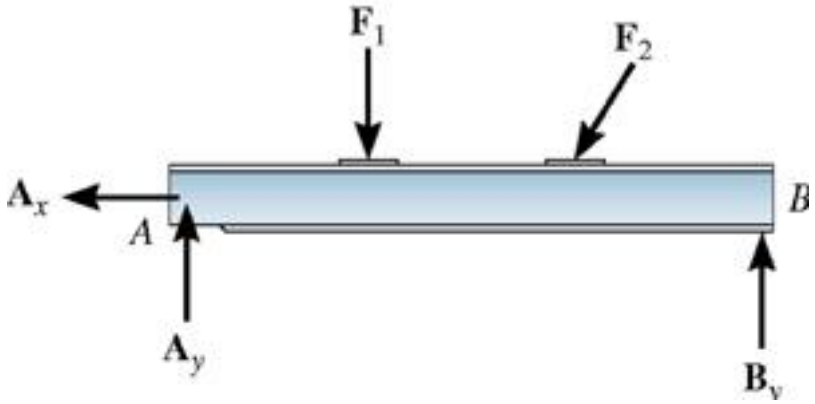
ENGASTAMENTO:

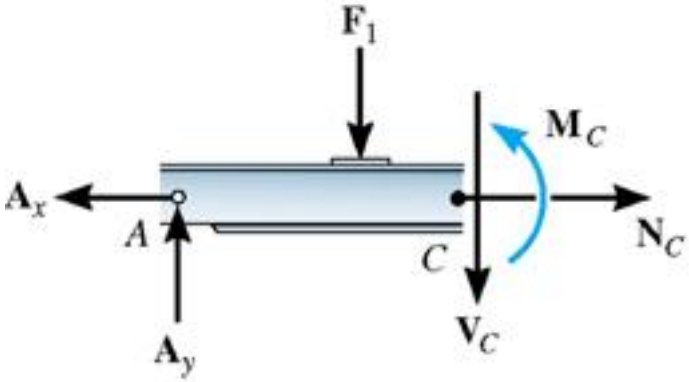


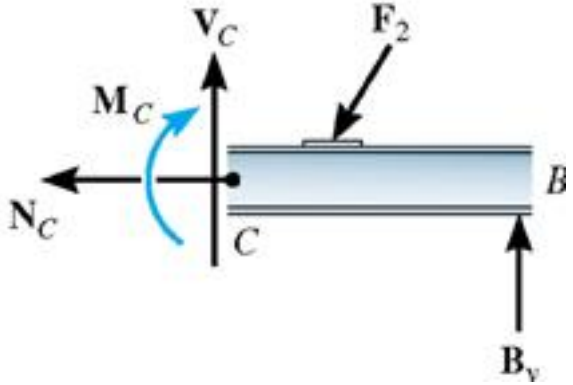

B
15 pés
θ
A

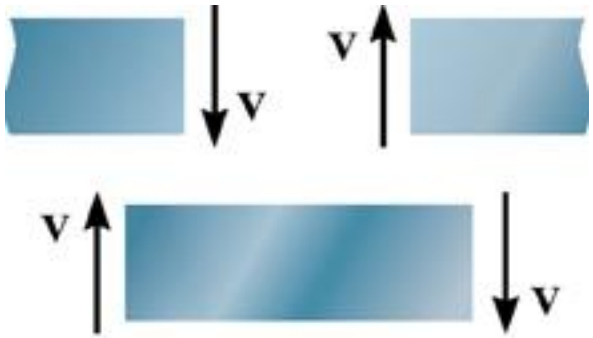

y
1,2 m
$\bar{x}$
G
3 m
$\bar{y}$
0,4 m
x
2,4 m
0,6 m
0,6 m


F1
F2
A
B
C


F1
F2
Ax
A
B
Ay
By


F1
MC
Ax
A
C
NC
Ay
VC


VC
F2
MC
B
NC
C
By

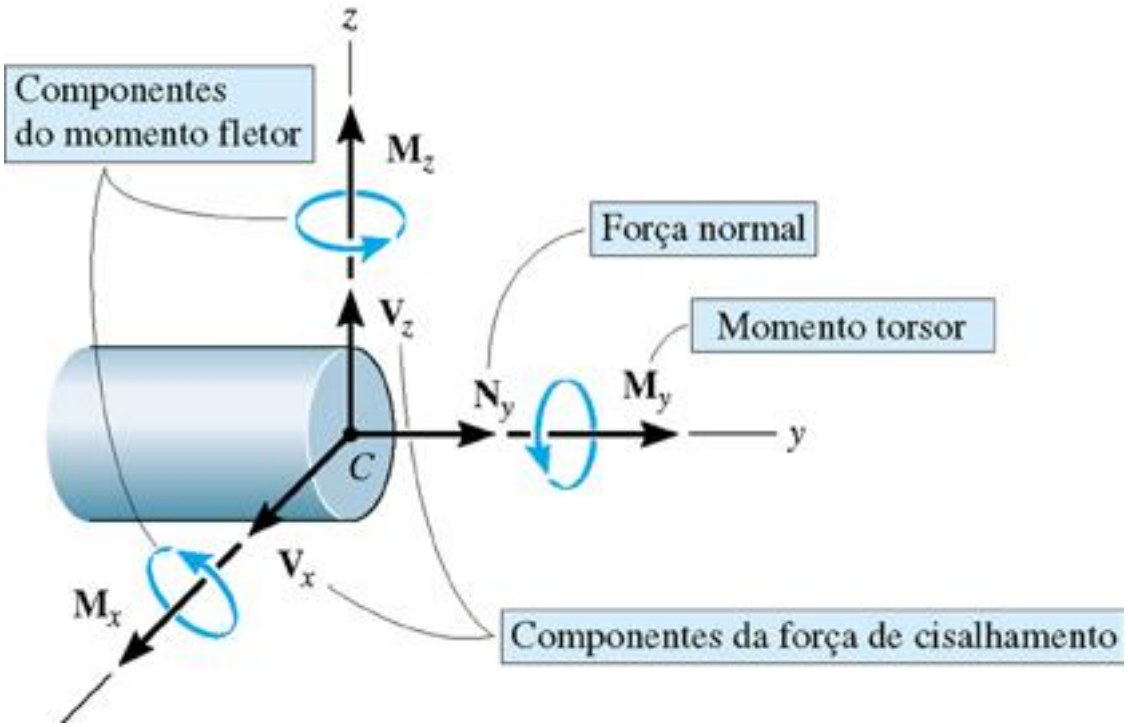


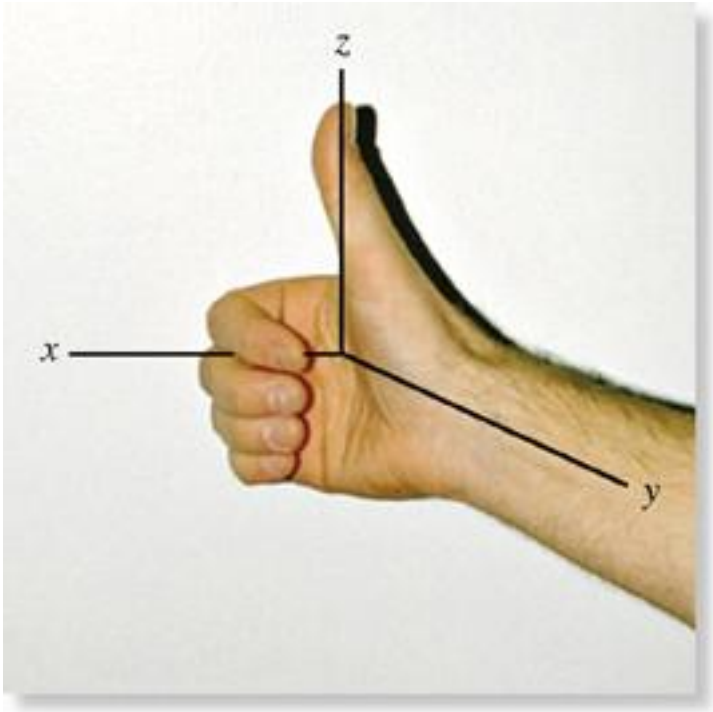
Força de cisalhamento positiva

Momento fletor positivo

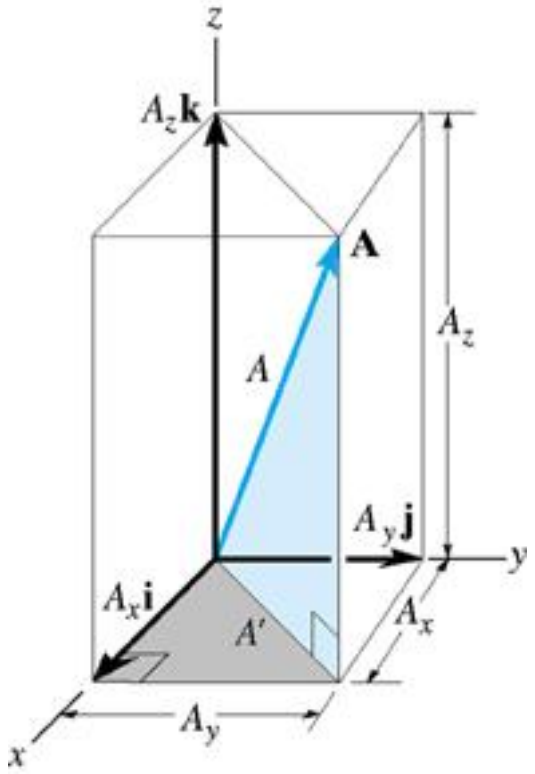
Convenção de sinais para a viga

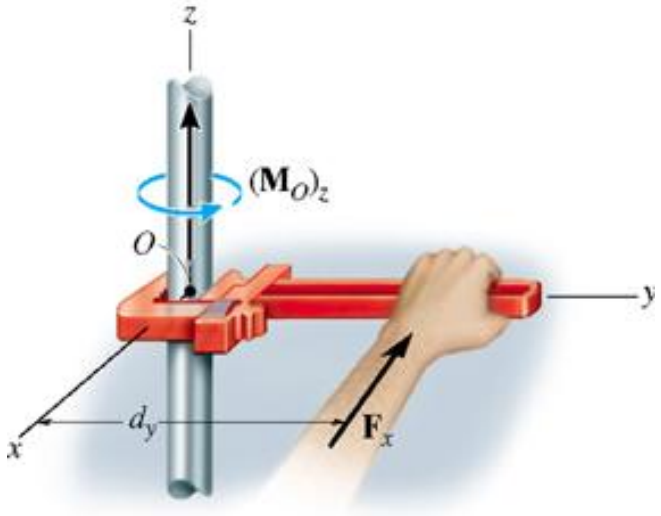

Strip
Products

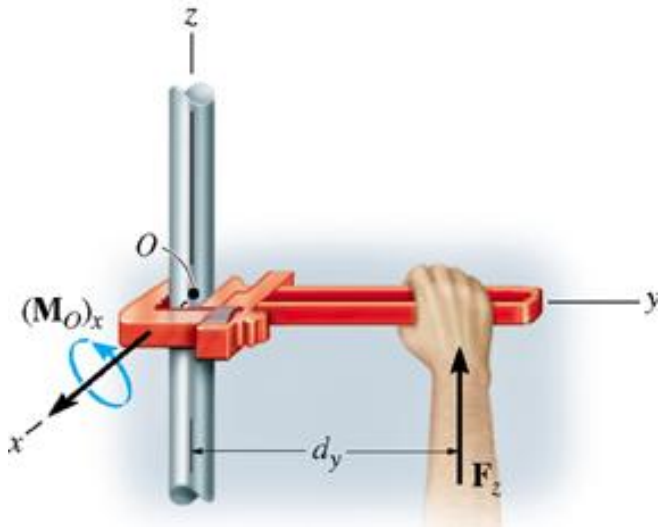

z
Componentes
do momento fletor
M_z
Força normal
V_z
Momento torsor
N_y
M_y
y
C
V_x
M_x
Componentes da força de cisalhamento

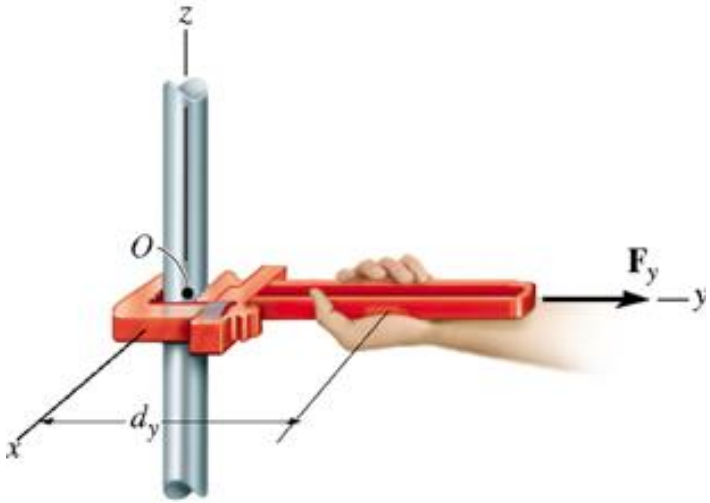


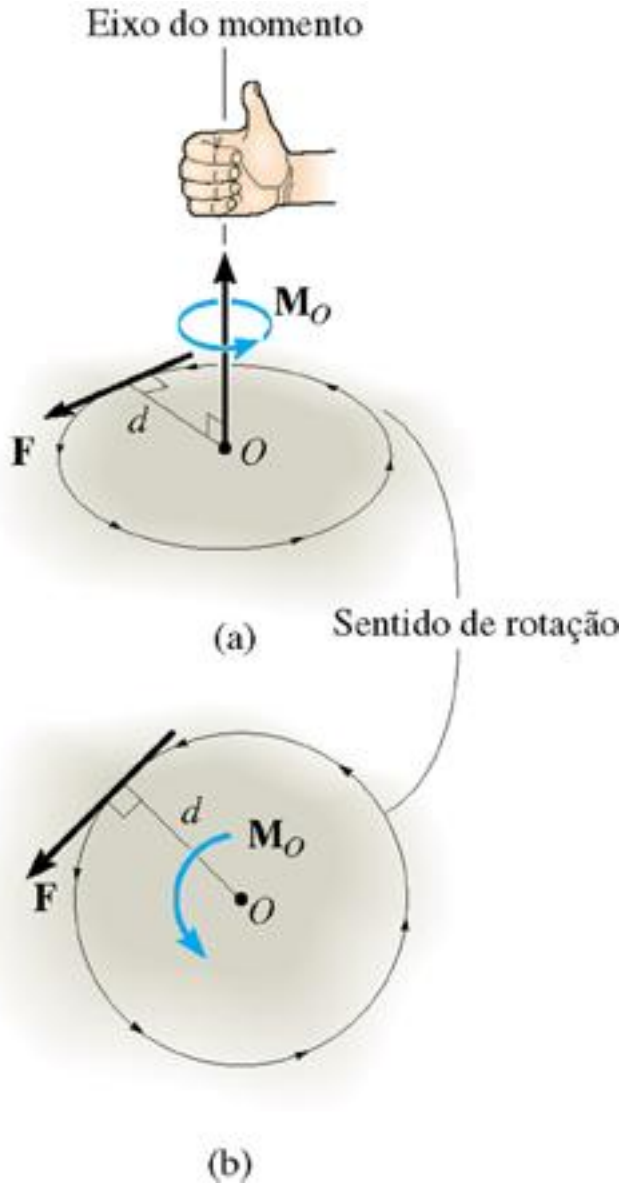
Sistema de coordenadas da mão direita

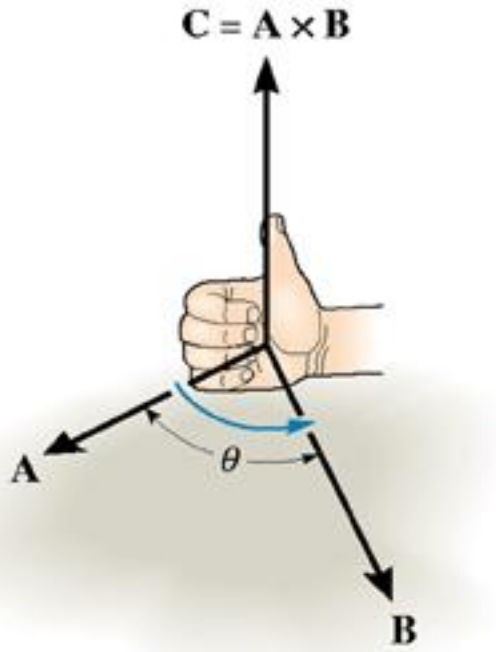


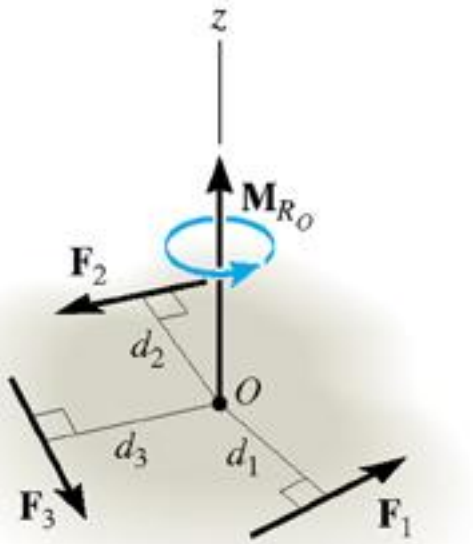

z
$(\mathbf{M}_O)_z$
O
y
x
d_y
$\mathbf{F}_x$

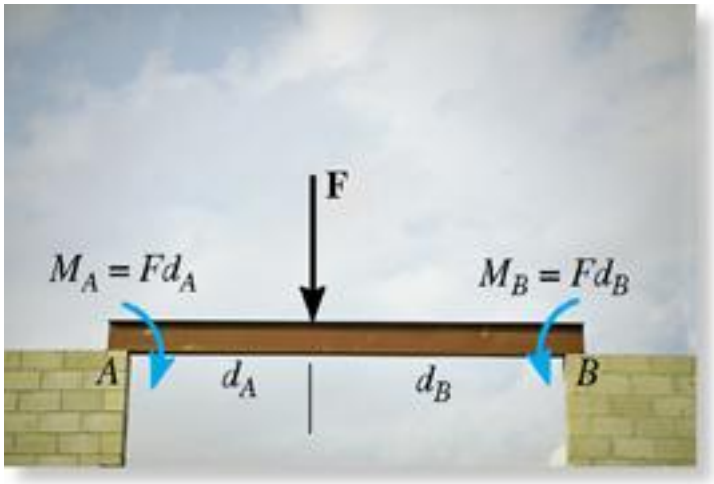

z
O
$(\mathbf{M}_O)_x$
y
x
d_y
$\mathbf{F}_z$

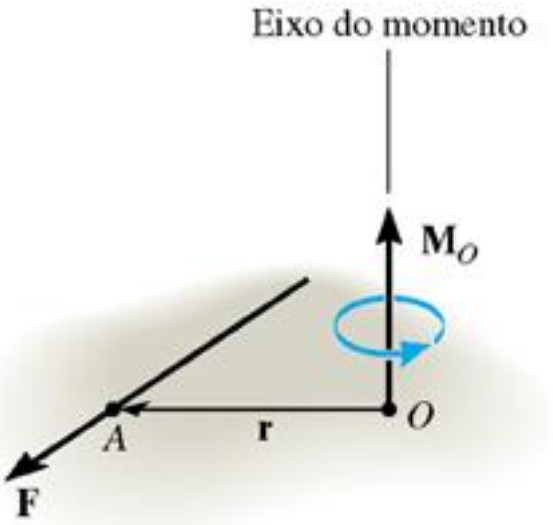

z
O
$\mathbf{F}_y$
y
x
d_y

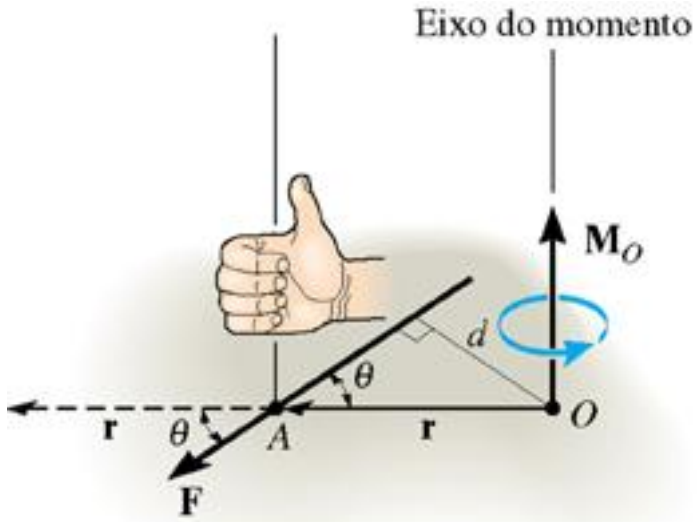

Eixo do momento
$\mathbf{M}_O$
F
d
O
(a)
Sentido de rotação
d
$\mathbf{M}_O$
F
O
(b)

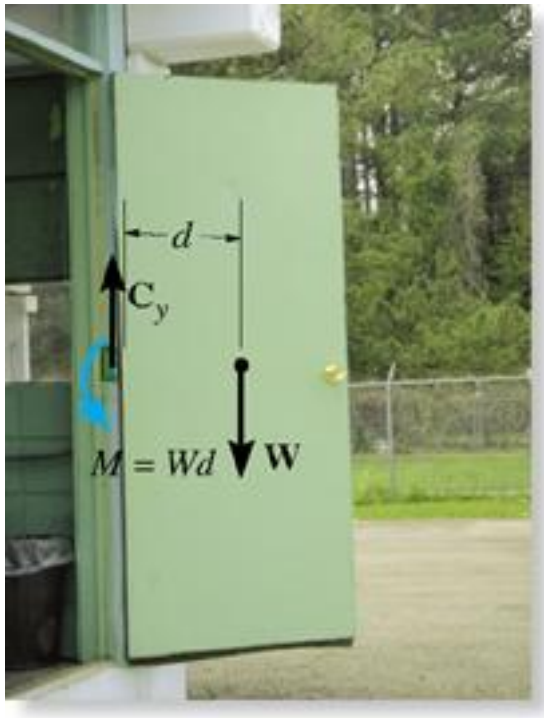

C = A × B
A
θ
B

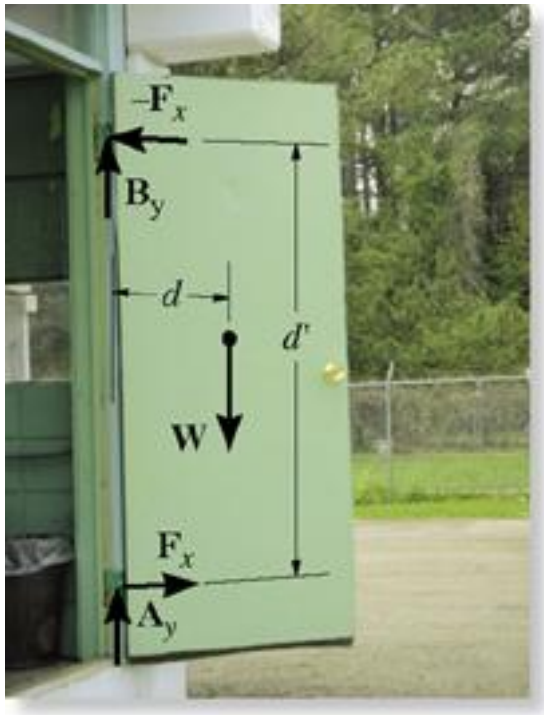

z
$\mathbf{M}_{R_O}$
$\mathbf{F}_2$
d_2
O
d_3
d_1
$\mathbf{F}_3$
$\mathbf{F}_1$

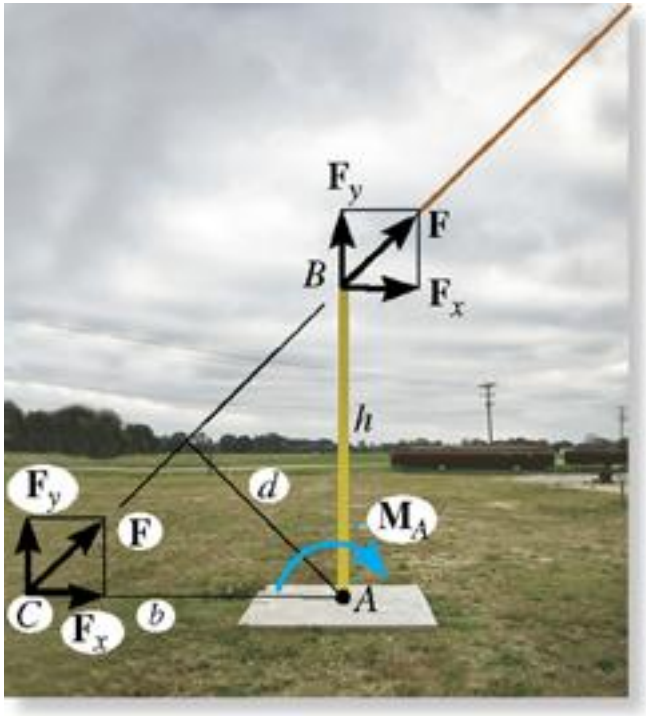

$\mathbf{F}$
$M_A = Fd_A$
$M_B = Fd_B$
A
d_A
d_B
B

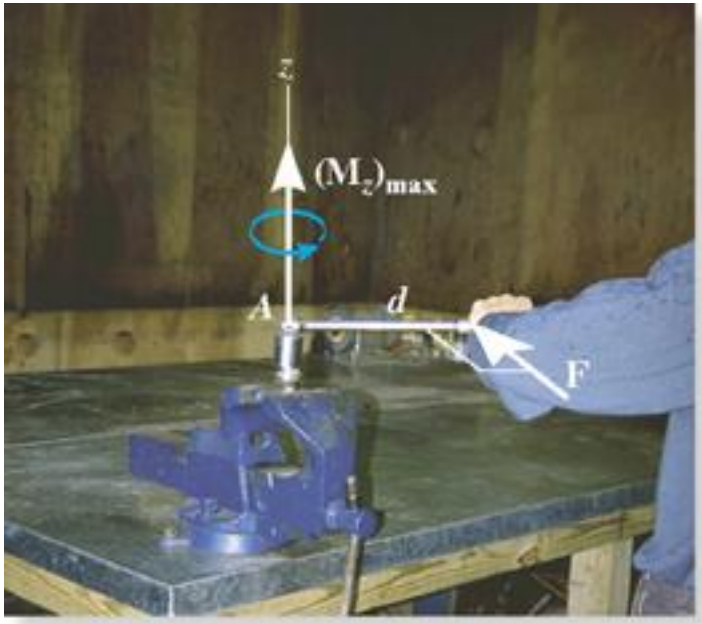

Eixo do momento
$\mathbf{M}_O$
A
r
O
F

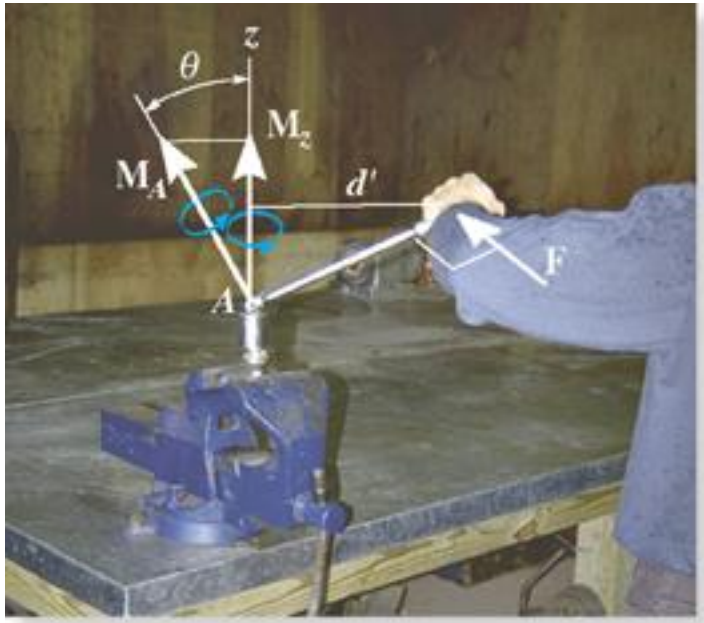

Eixo do momento
$\mathbf{M}_O$
d
θ
r
θ
A
r
O
F

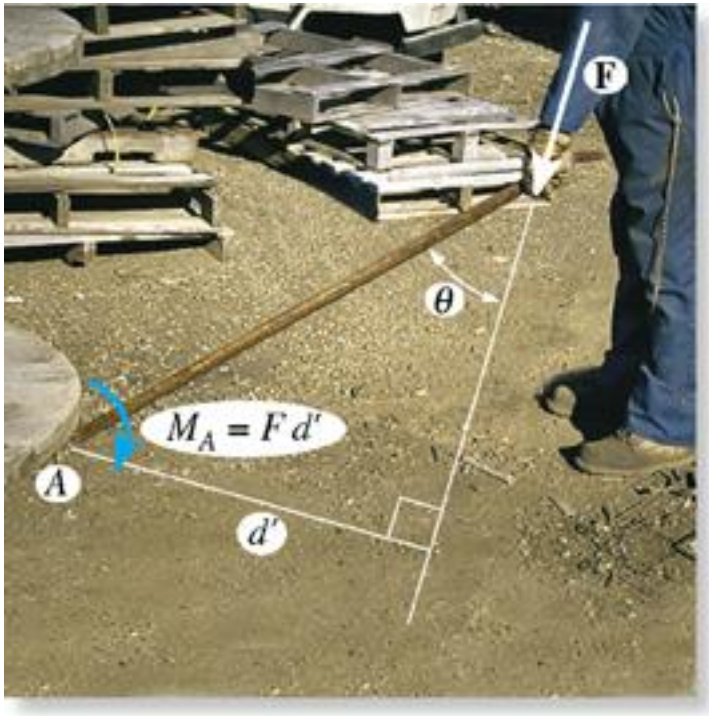

d
C_y
$M = Wd$
W


$-F_x$
B_y
d
d'
W
F_x
A_y

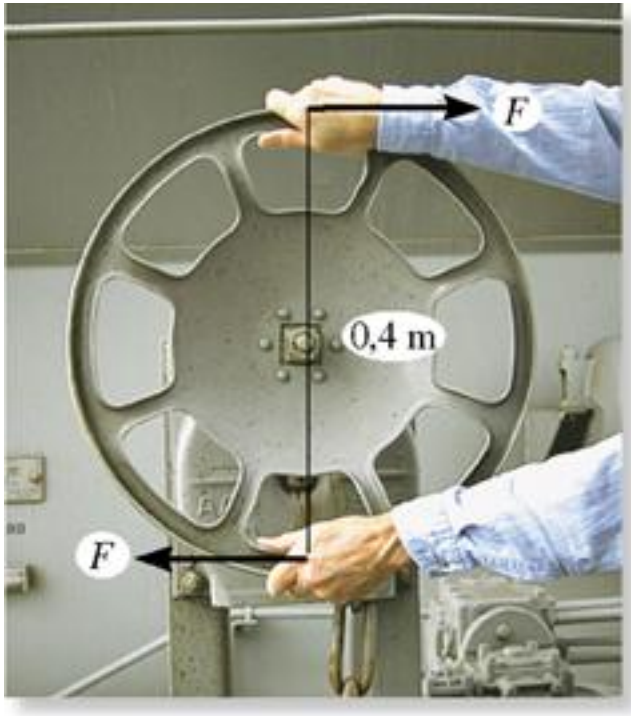

$\mathbf{F}_y$
$\mathbf{F}$
B
$\mathbf{F}_x$
h
$\mathbf{F}_y$
d
$\mathbf{F}$
$\mathbf{M}_A$
C
$\mathbf{F}_x$
b
A

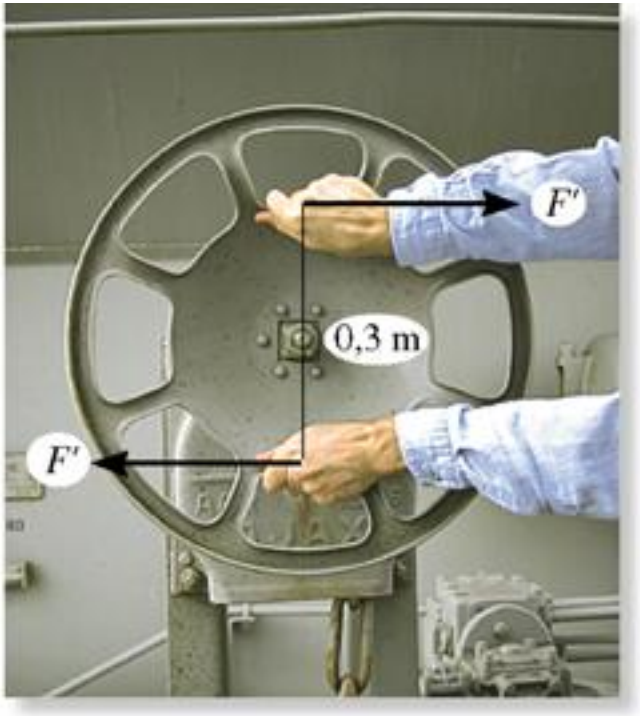

z
$(M_z)_{max}$
A
d
F

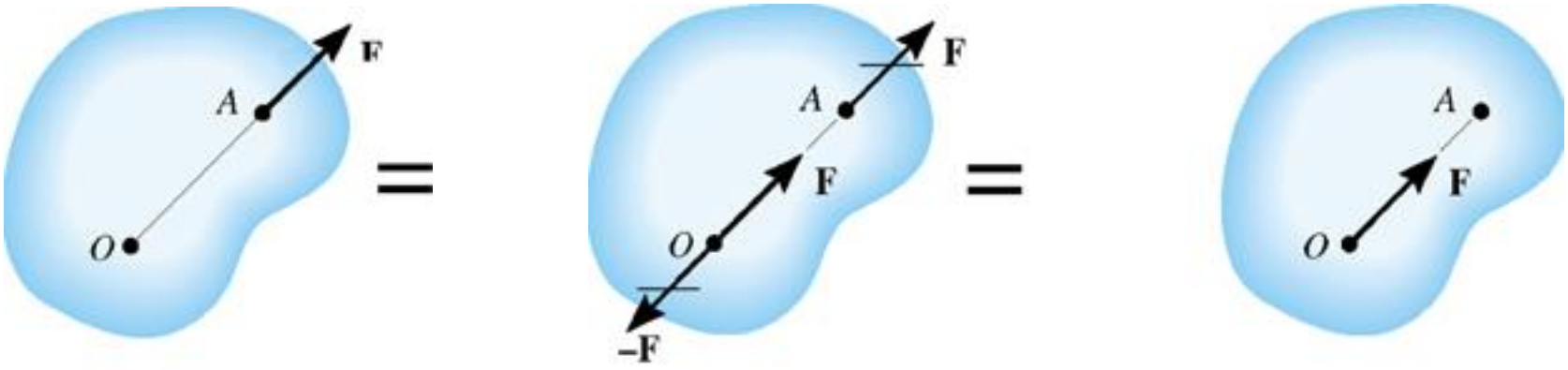

z
θ
M_z
M_A
d'
A
F

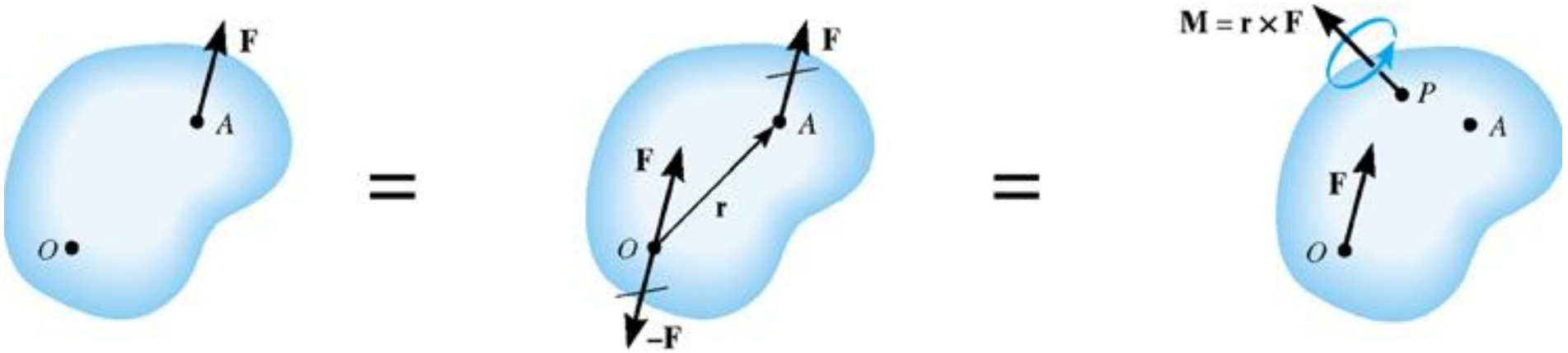

F
θ
A
$M_A = F\,d'$
d'

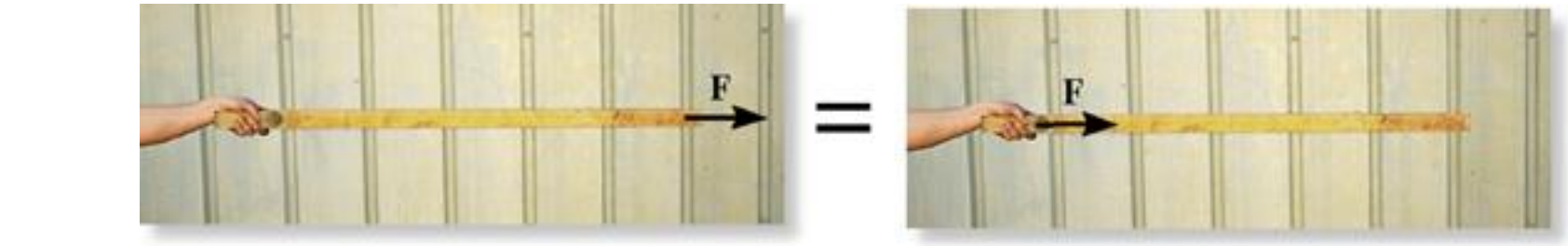

F
d
A
$(M_A)_{max} = F\,d$

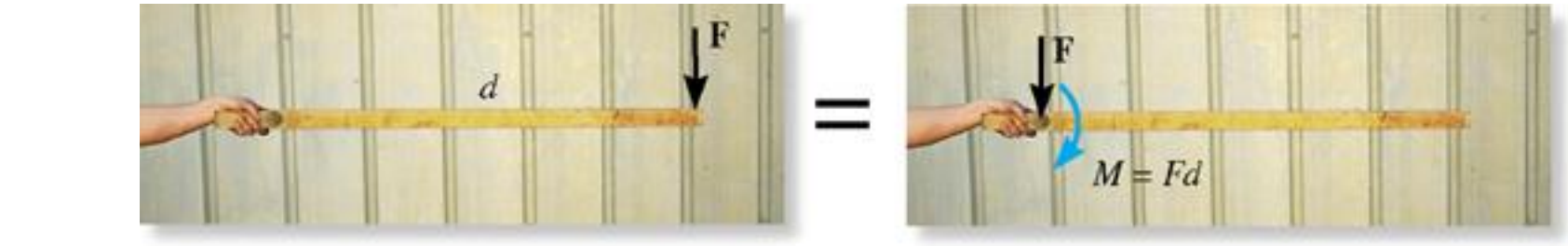

F
0,4 m
F

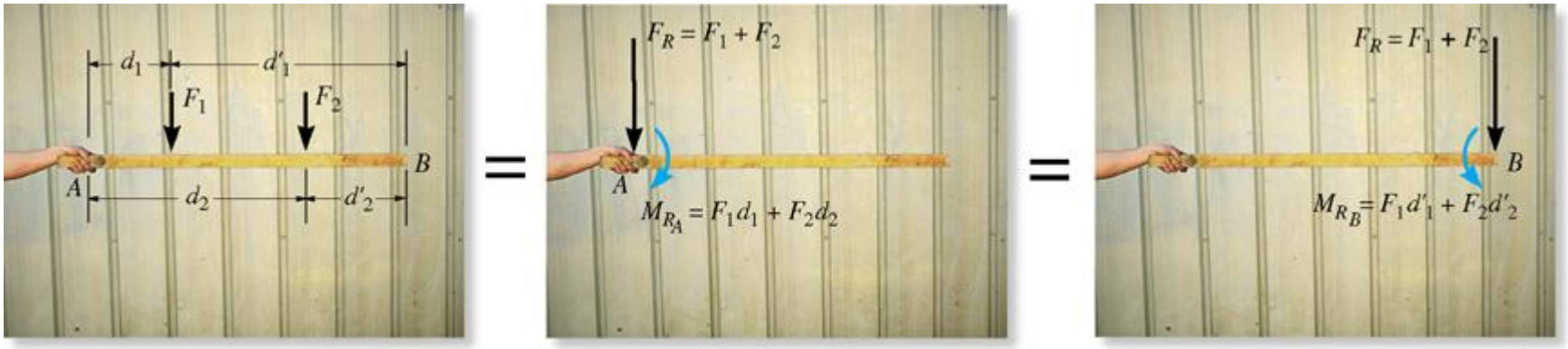

F'
0,3 m
F'

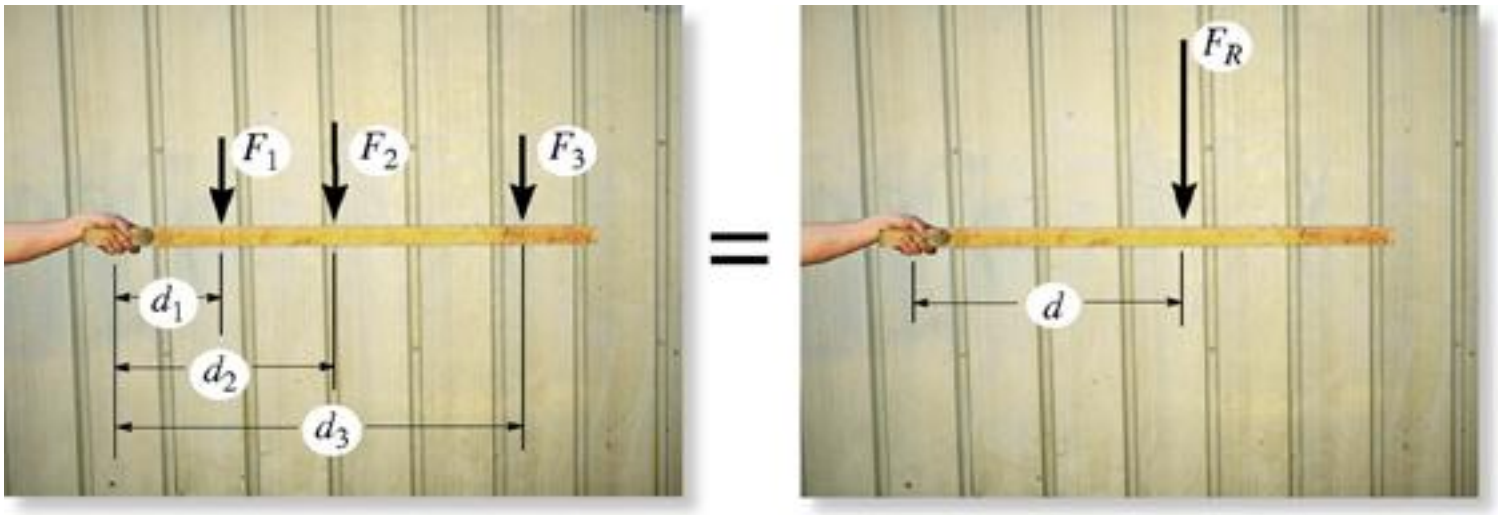

F
A
O
=
F
A
F
O
–F
=
A
F
O


F
A
O
=
F
A
F
r
O
–F
=
M = r × F
P
A
F
O

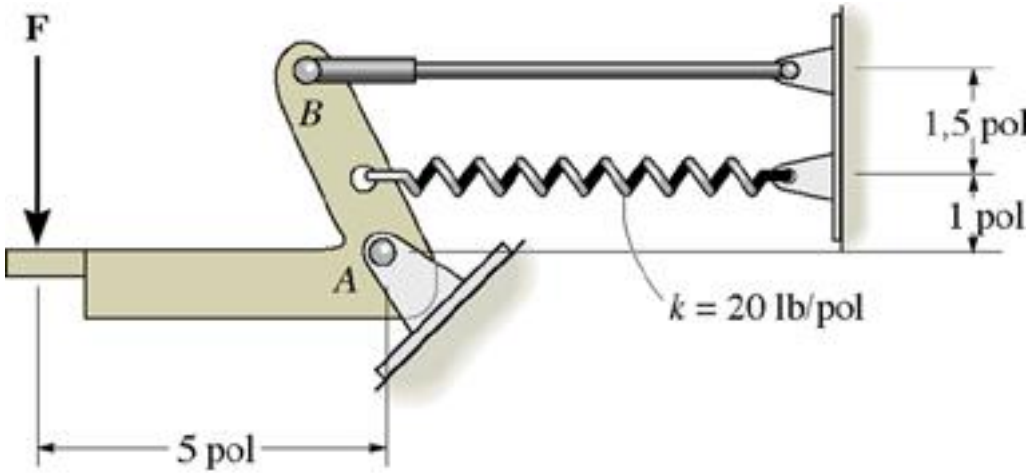

F
=
F

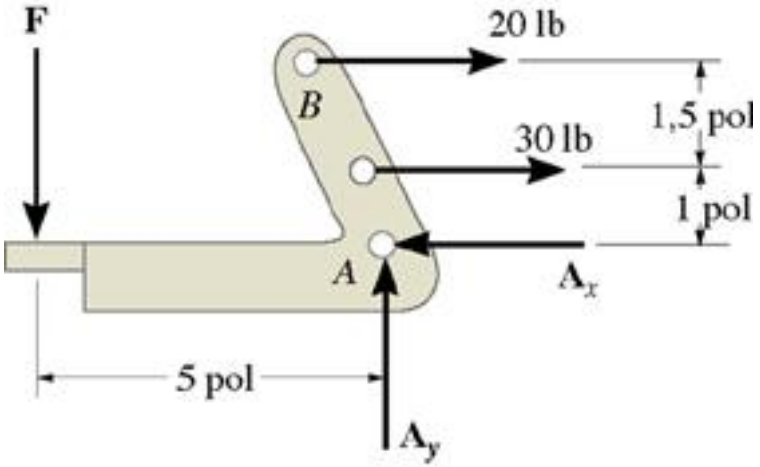

d
F
=
F
M = Fd

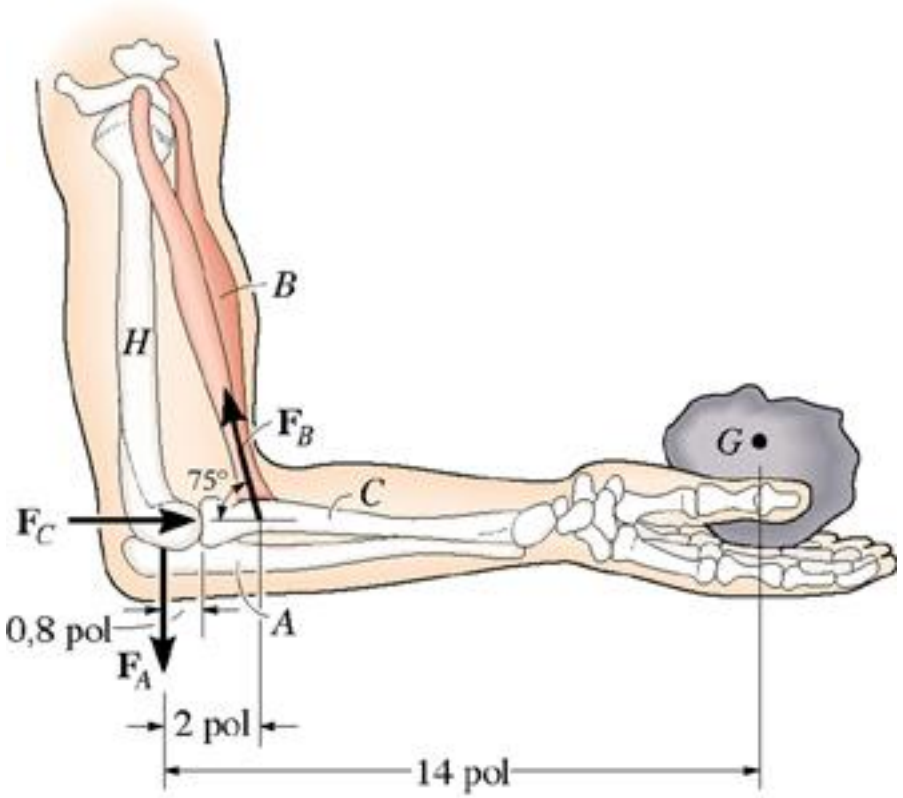

d1
d'1
F1
F2
A
B
d2
d'2
=
FR = F1 + F2
A
MRA = F1d1 + F2d2
=
FR = F1 + F2
B
MRB = F1d'1 + F2d'2

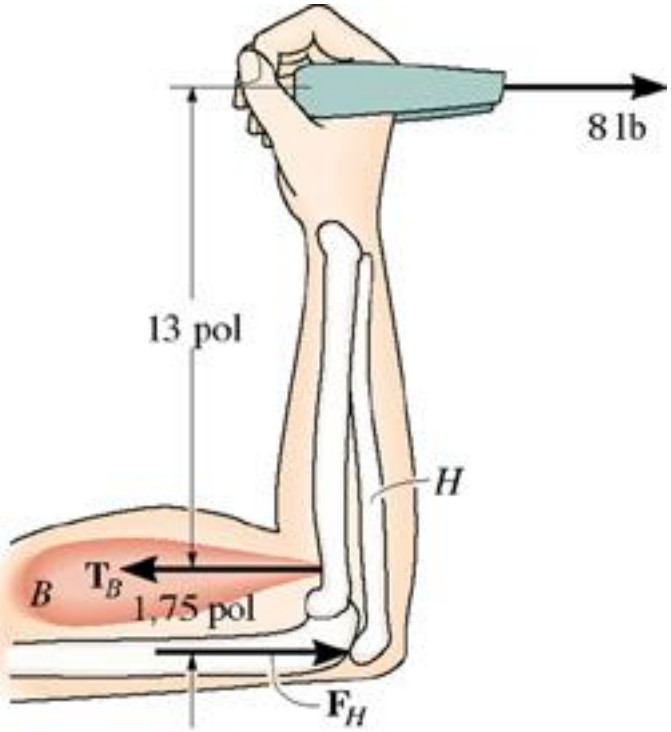

F_1
F_2
F_3
d_1
d_2
d_3
=
F_R
d

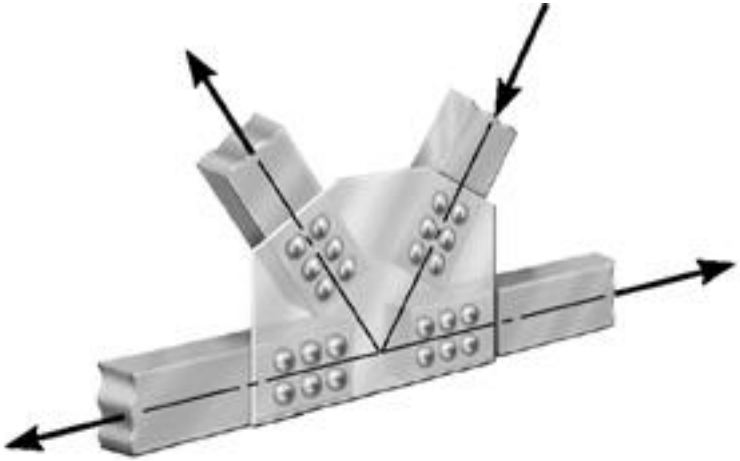

B
A

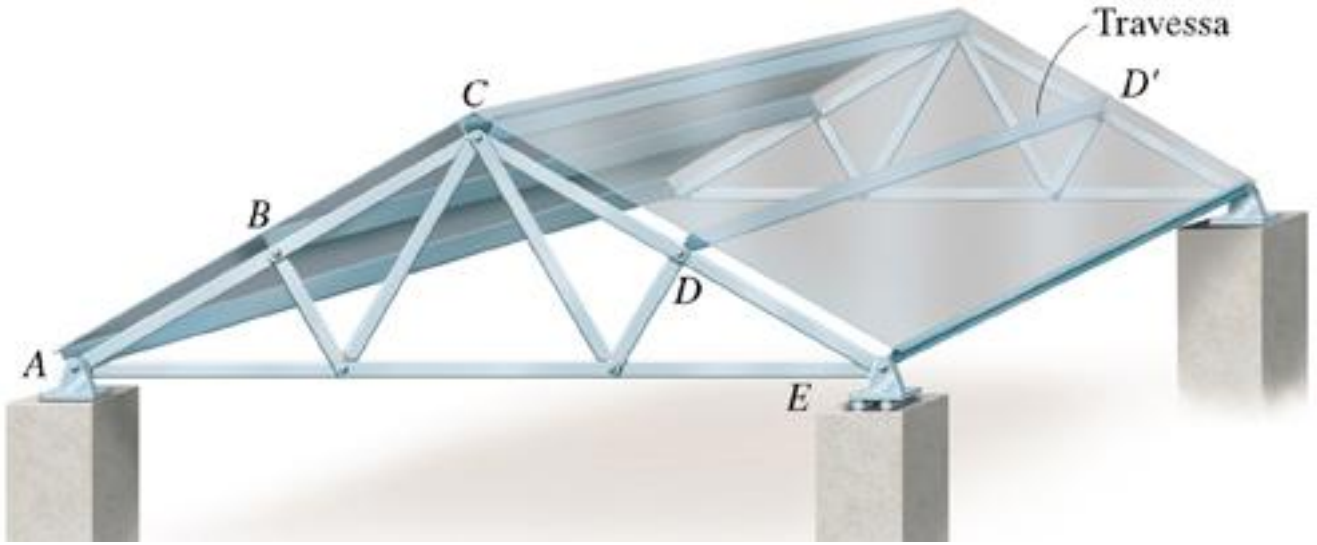

F
B
A
1,5 pol
1 pol
k = 20 lb/pol
5 pol

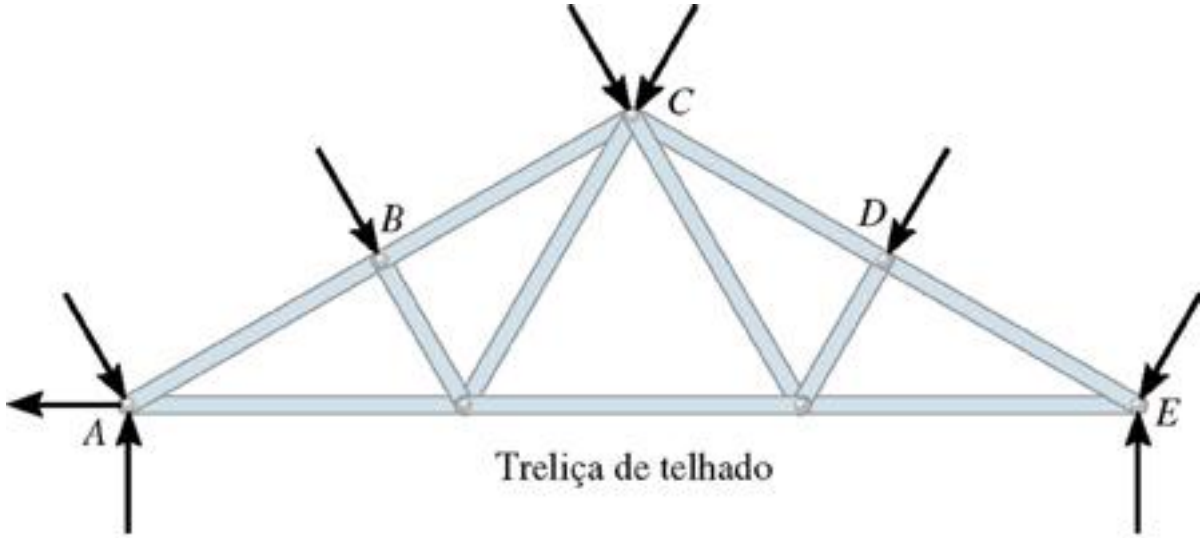

F
20 lb
B
1,5 pol
30 lb
1 pol
A
A_x
5 pol
A_y


B
H
F_B
G
75°
C
F_C
0,8 pol
A
F_A
2 pol
14 pol

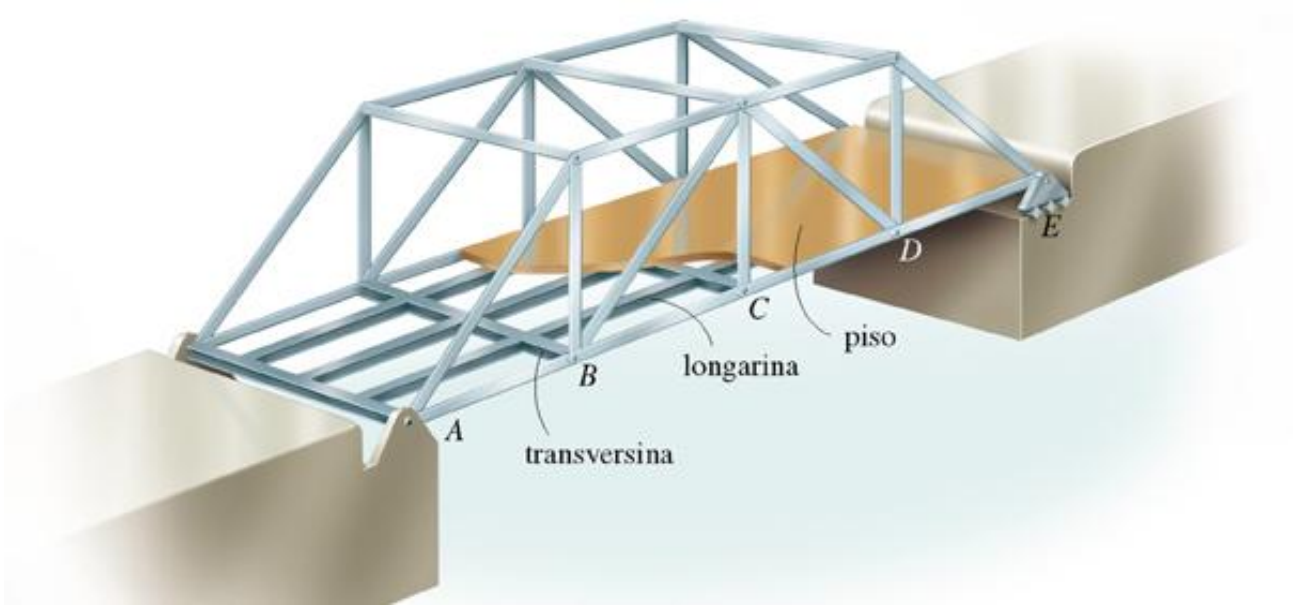

8 lb
13 pol
H
B
T_B
1,75 pol
F_H

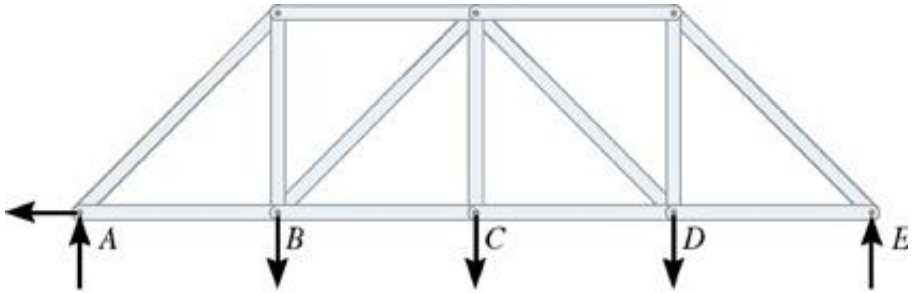




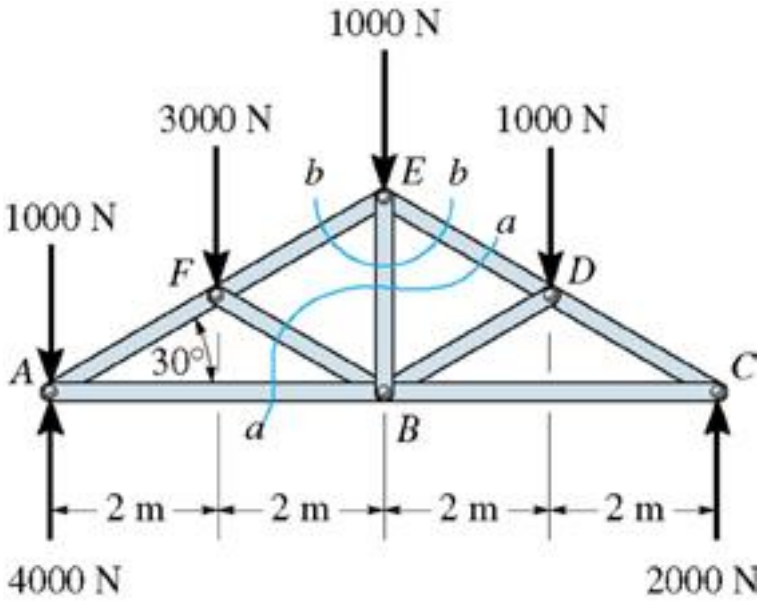


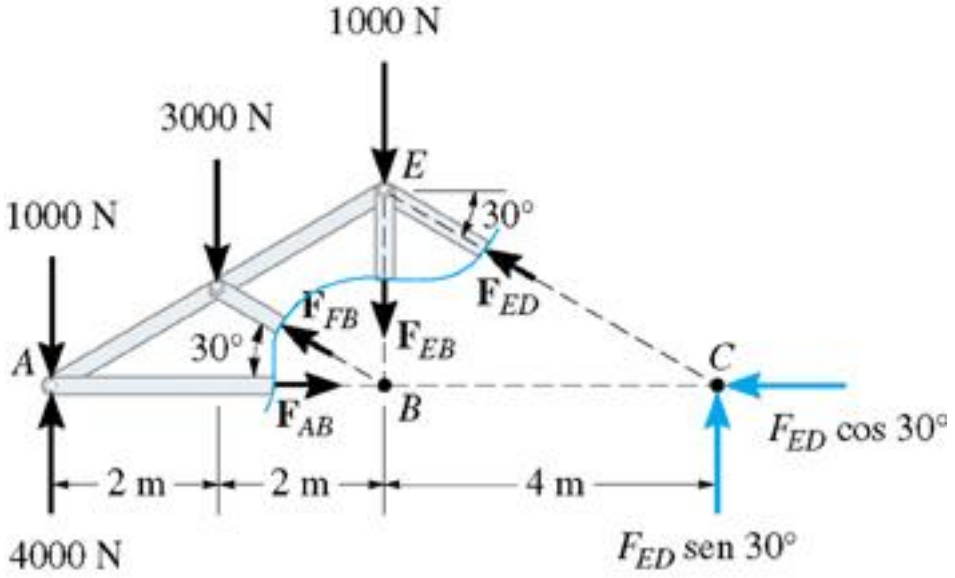
Treliça de telhado

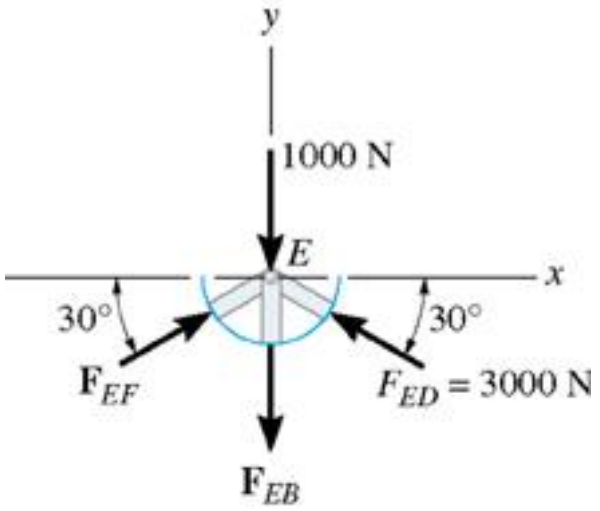


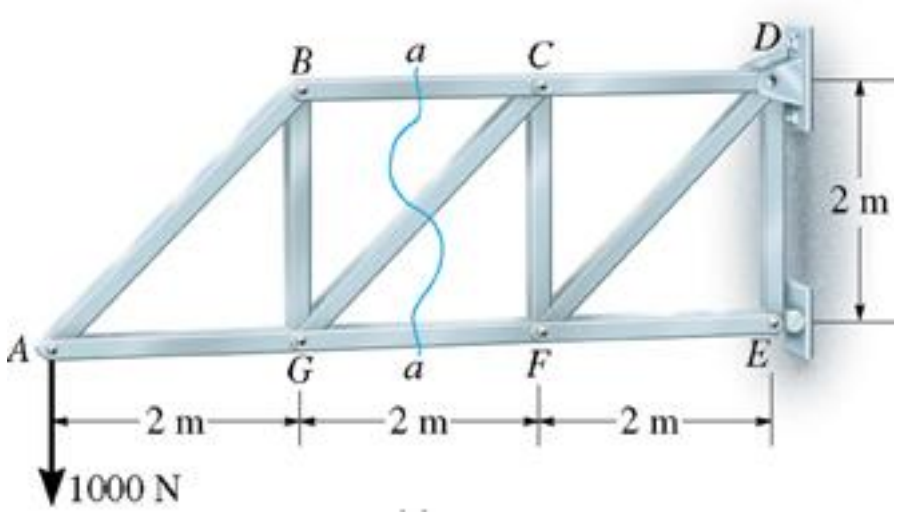


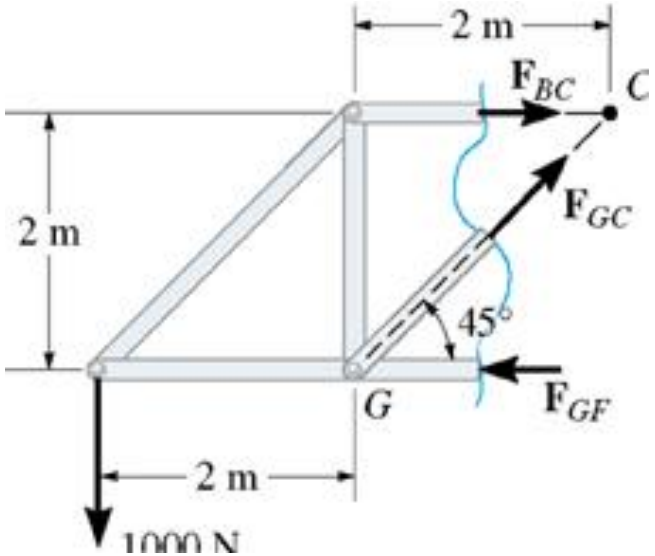
Treliça da Ponte

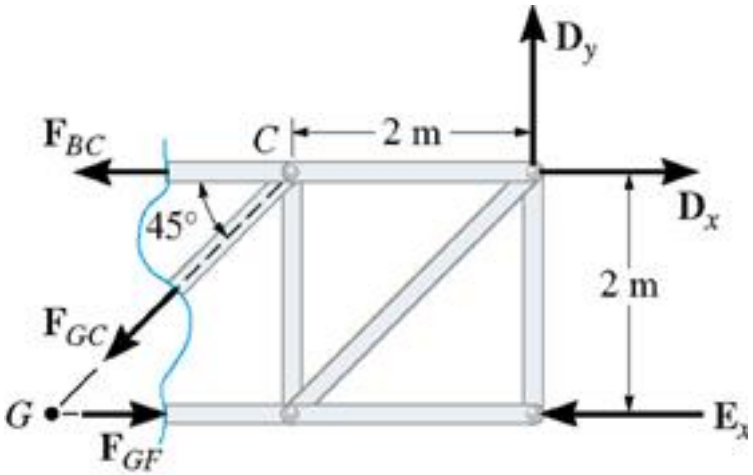

1000 N
3000 N
1000 N
1000 N
b
E
b
a
F
D
30°
A
C
a
B
2 m
2 m
2 m
2 m
4000 N
2000 N

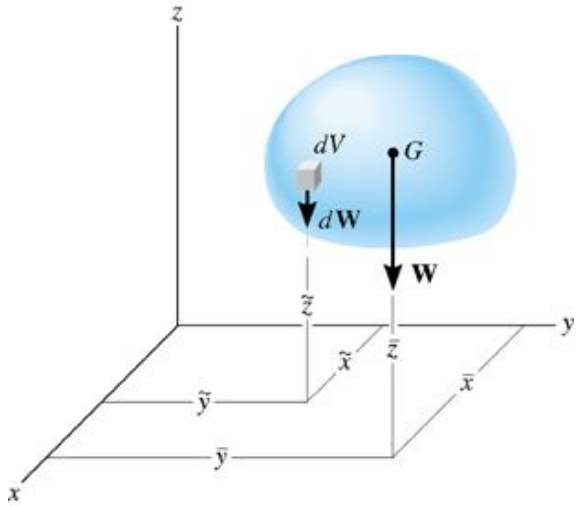

1000 N
3000 N
E
1000 N
30°
$\mathbf{F}_{ED}$
$\mathbf{F}_{FB}$
$\mathbf{F}_{EB}$
30°
A
C
B
$\mathbf{F}_{AB}$
F_{ED} cos 30°
2 m
2 m
4 m
4000 N
F_{ED} sen 30°

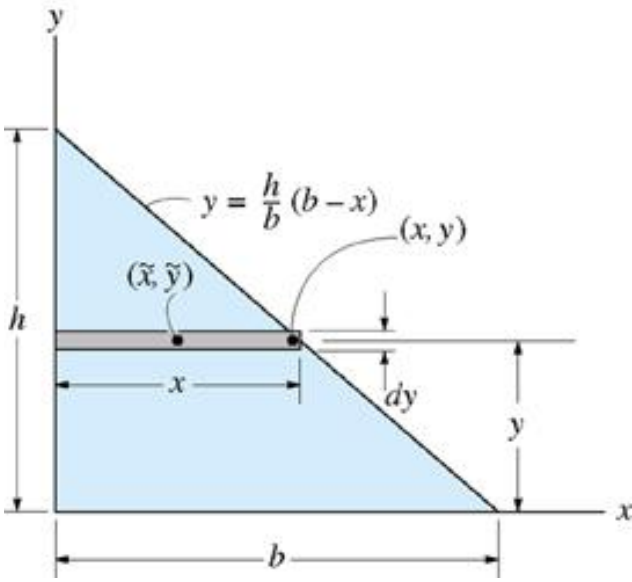

y
1000 N
E
x
30°
30°
$\mathbf{F}_{EF}$
F_{ED} = 3000 N
$\mathbf{F}_{EB}$

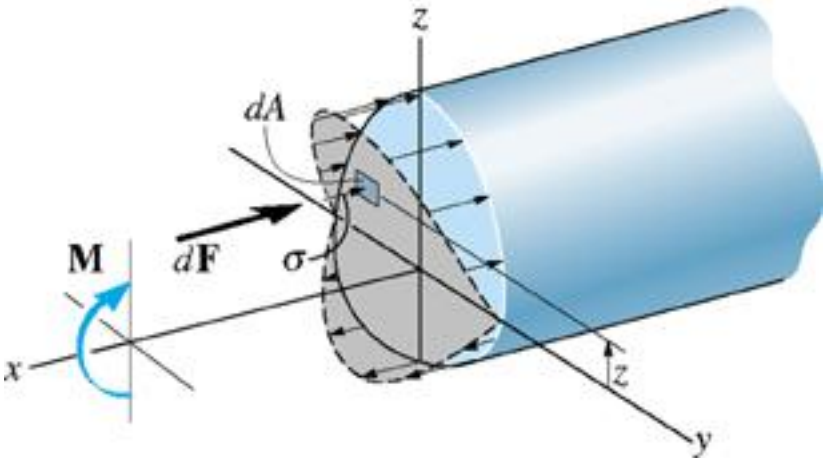

B
a
C
D
2 m
A
G
a
F
E
2 m
2 m
2 m
1000 N

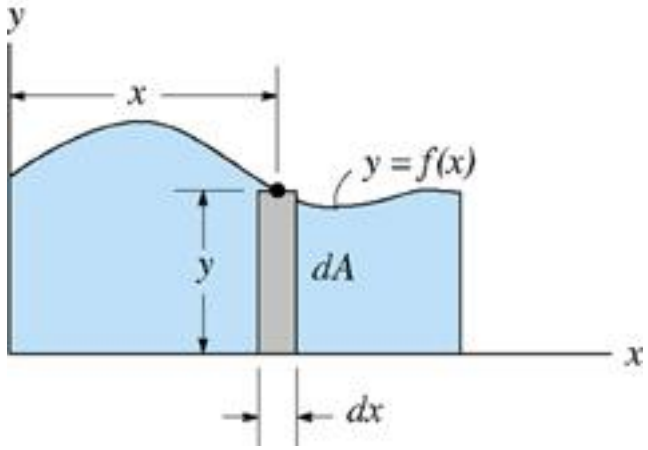

2 m
F_BC
C
F_GC
2 m
45°
G
F_GF
2 m
1000 N

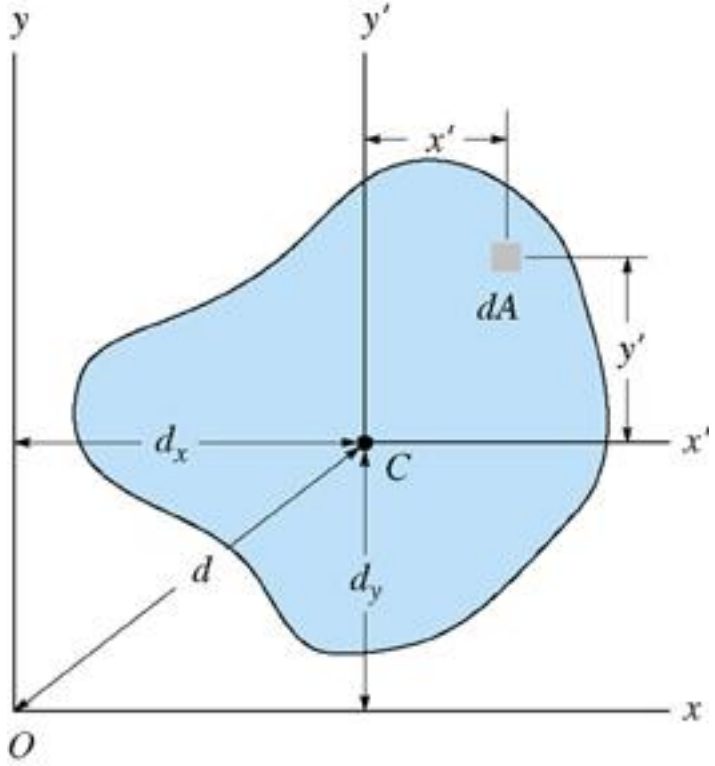

D_y
F_BC
C
2 m
D_x
45°
2 m
F_GC
G
E_x
F_GF

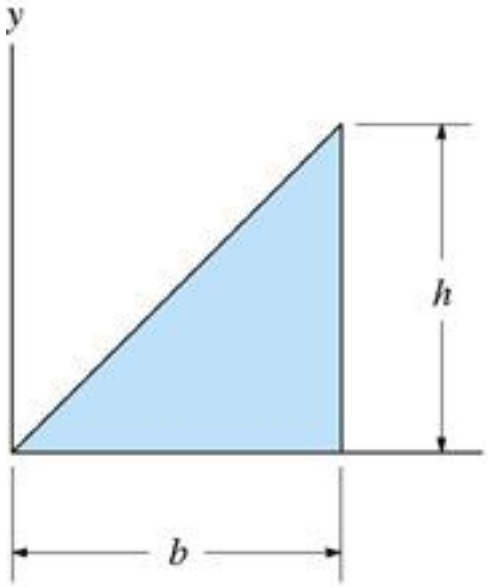

z
dV
G
dW
W
y
x
$\bar{z}$
$\tilde{x}$
$\bar{x}$
$\tilde{y}$
$\bar{y}$

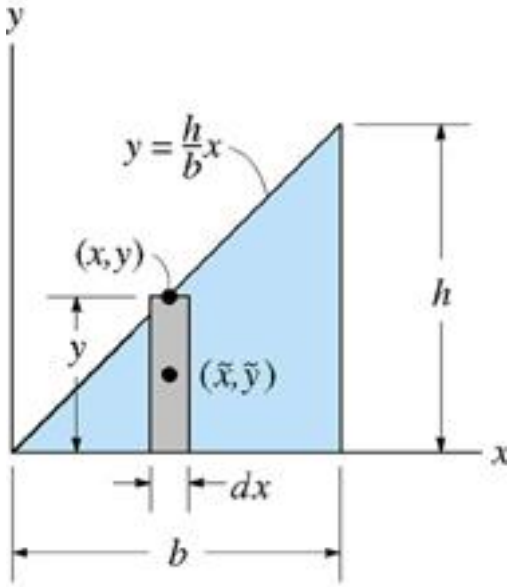

y
$y = \frac{h}{b}(b - x)$
(x, y)
$(\tilde{x}, \tilde{y})$
h
x
dy
y
x
b

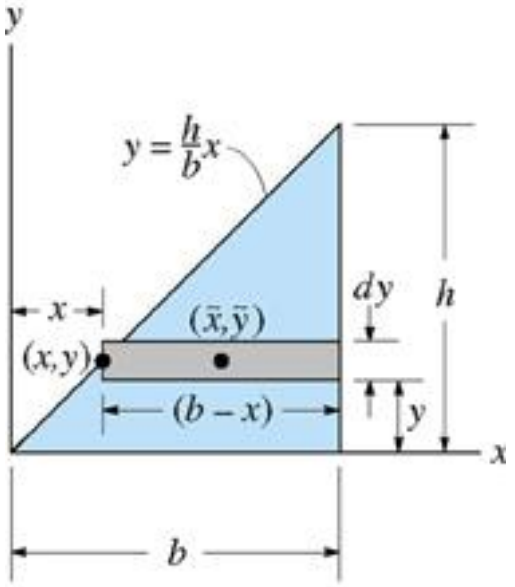

z
dA
M
dF
σ
x
z
y

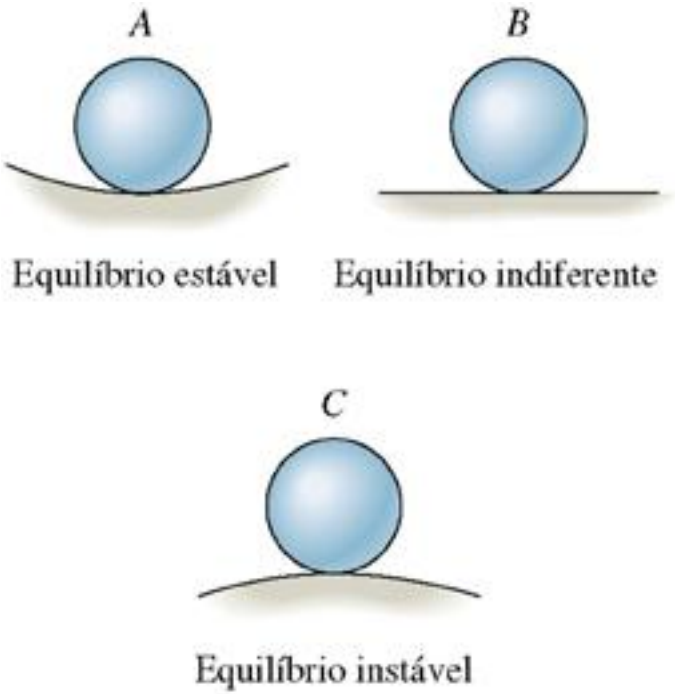

y
x
y = f(x)
y
dA
x
dx


y
y'
x'
dA
y'
d_x
C
x'
d
d_y
x
O


y
h
b


y
$y = \frac{h}{b}x$
(x,y)
y
$(\tilde{x},\tilde{y})$
h
x
dx
b


y
$y = \frac{h}{b}x$
x
(x,y)
$(\bar{x},\bar{y})$
dy
h
(b – x)
y
x
b


A
B
Equilíbrio estável
Equilíbrio indiferente
C
Equilíbrio instável