SEM0551 Fenômenos de Transporte

EQUAÇÕES DE TRANSPORTE DE MASSA, QUANTIDADE DE MOVIMENTO E ENERGIA DE UM FLUIDO

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Universidade de São Paulo

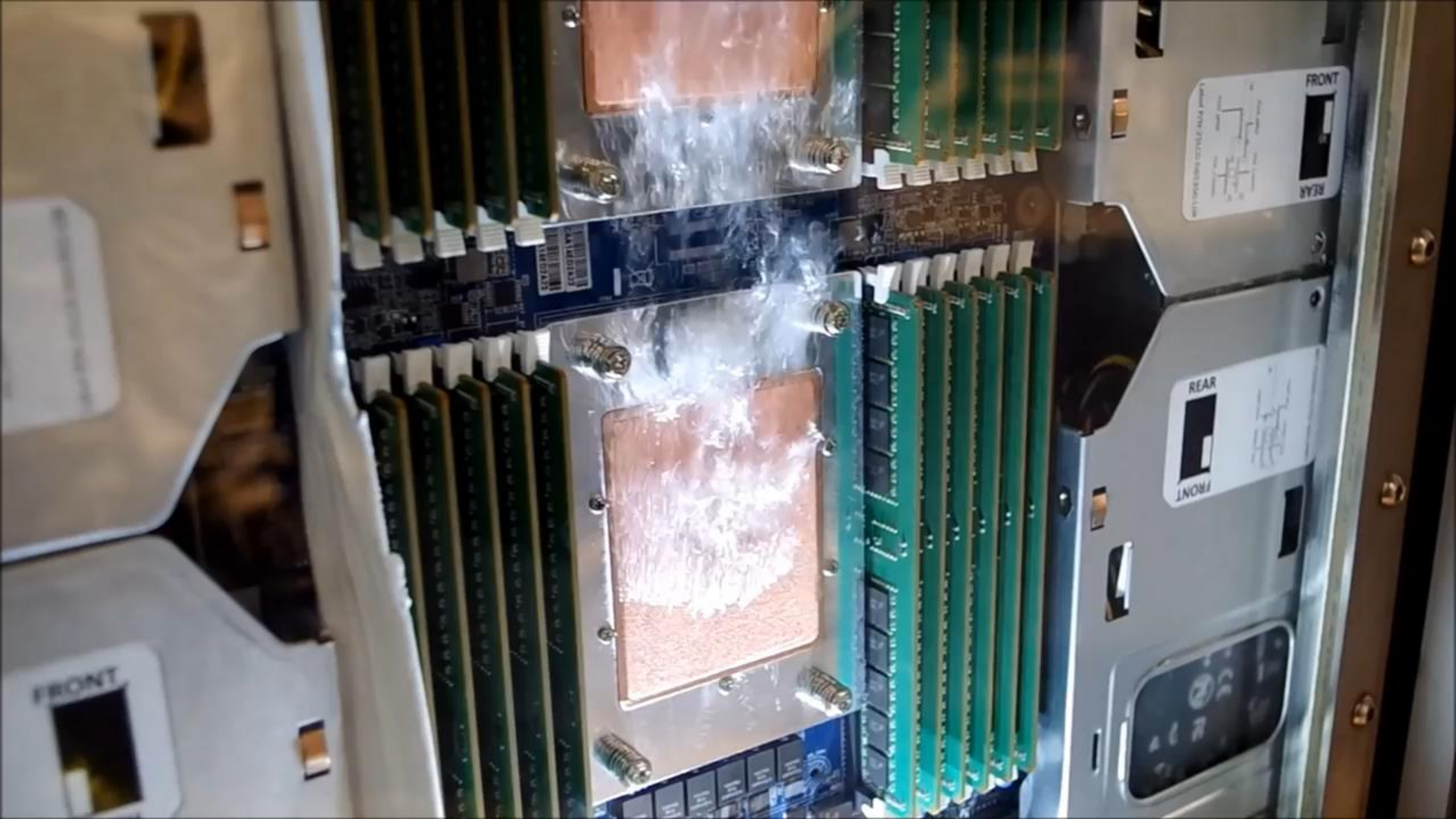
Preâmbulo: mecanismos de transferência de calor por convecção...

Condução

Calor → <u>Convecção</u> → Acoplamento entre dois fenômenos: escoamento de um fluido e transferência de calor

transporte de energia térmica

Radiação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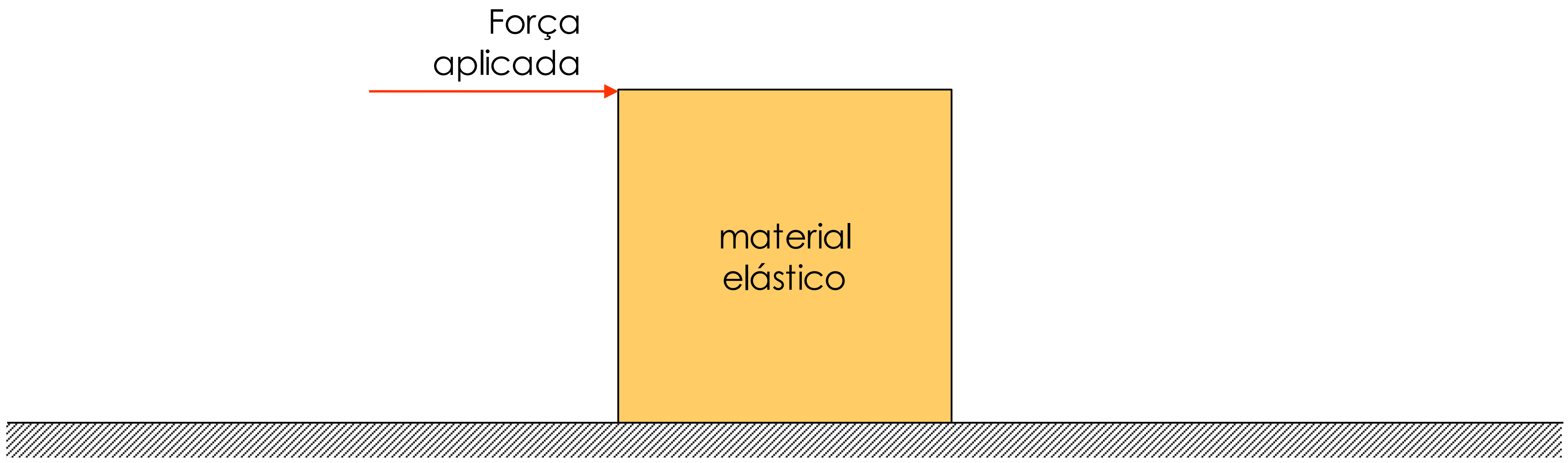
Escoamento de materiais elásticos e fluidos...



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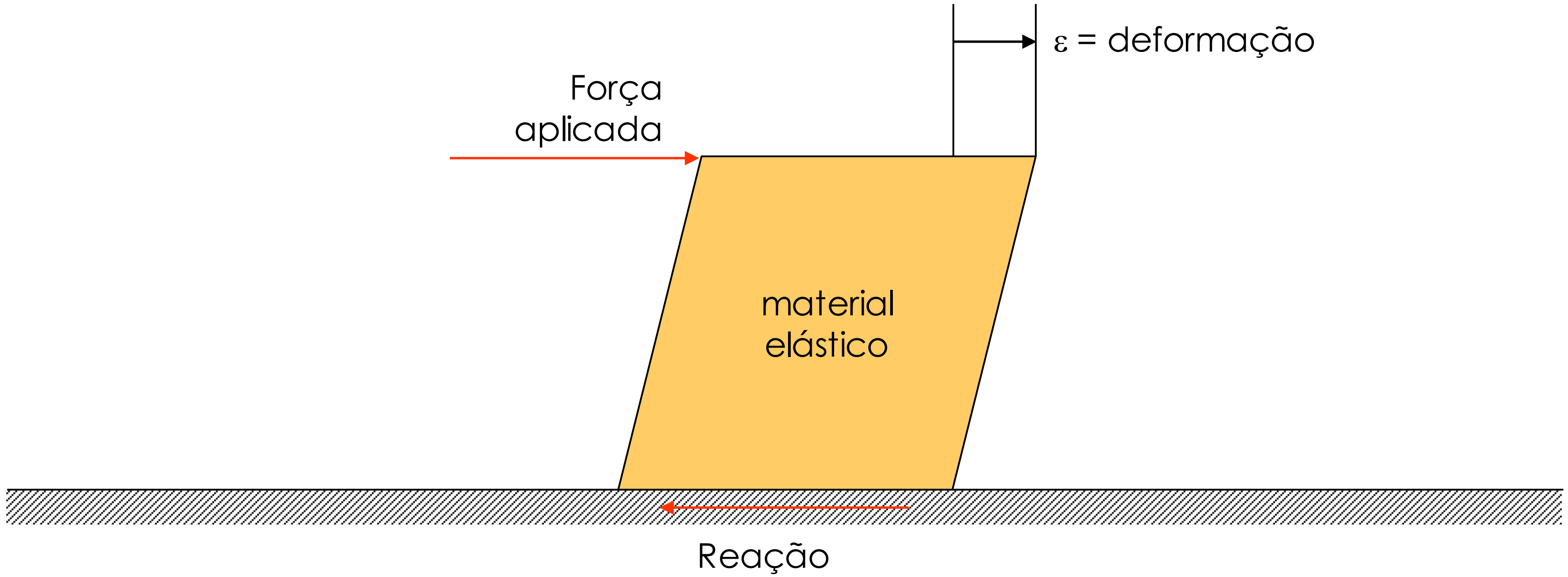

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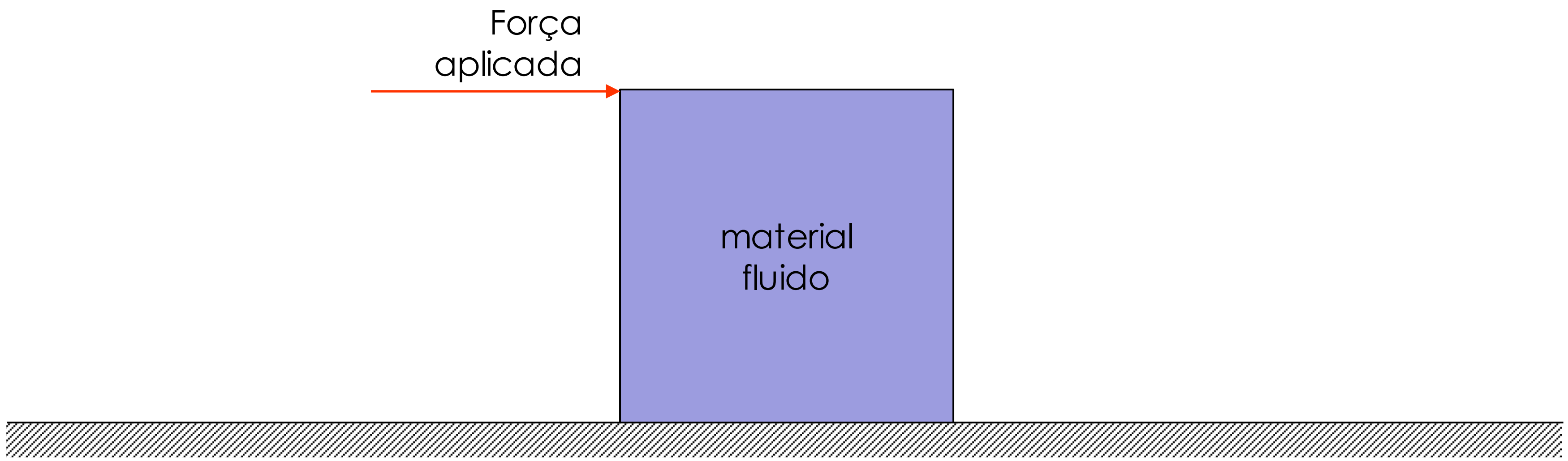

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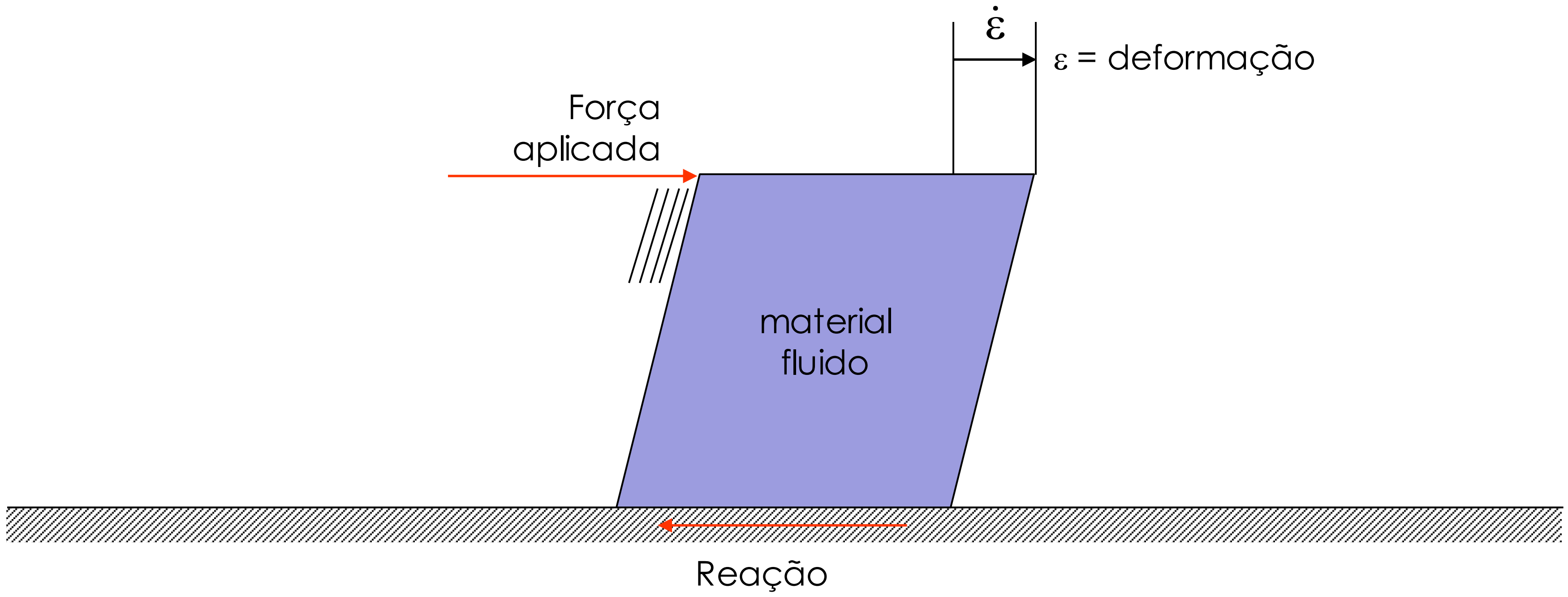

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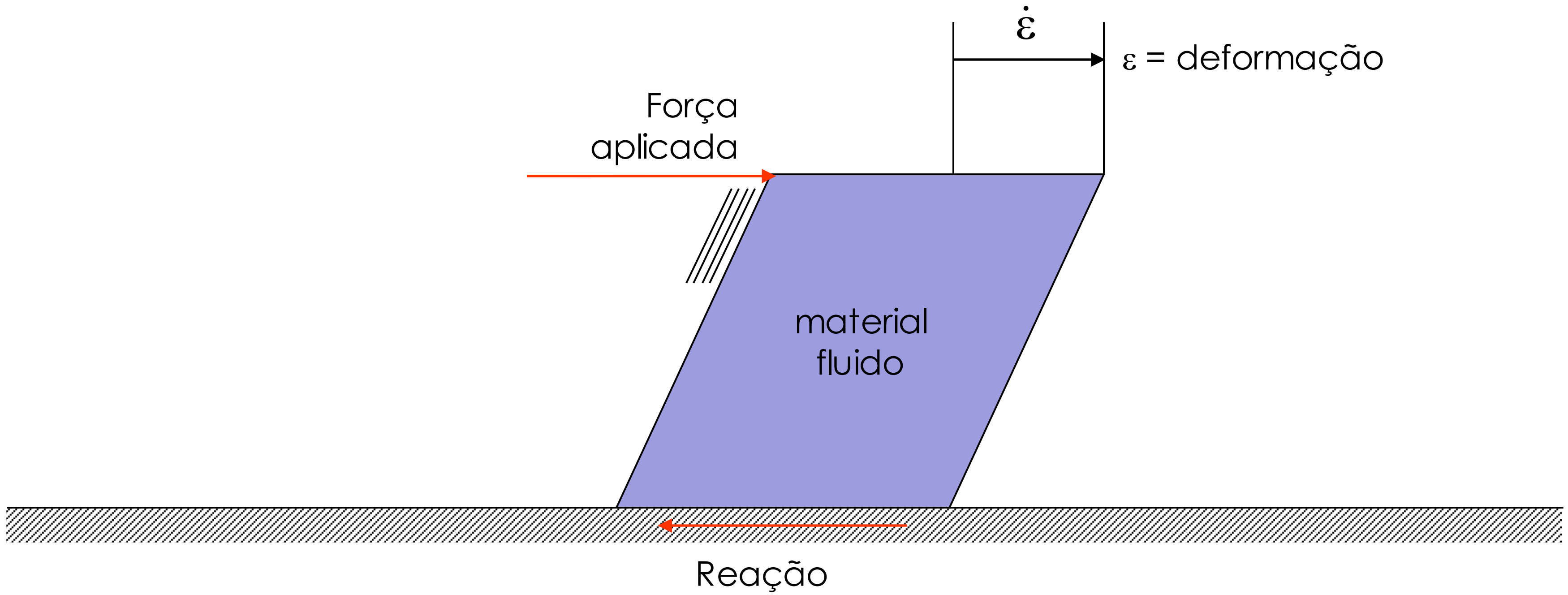

$$\varepsilon = \dot{\varepsilon} \cdot t$$

deformação — taxa de deformação

Escoamento de materiais elásticos e fluidos...

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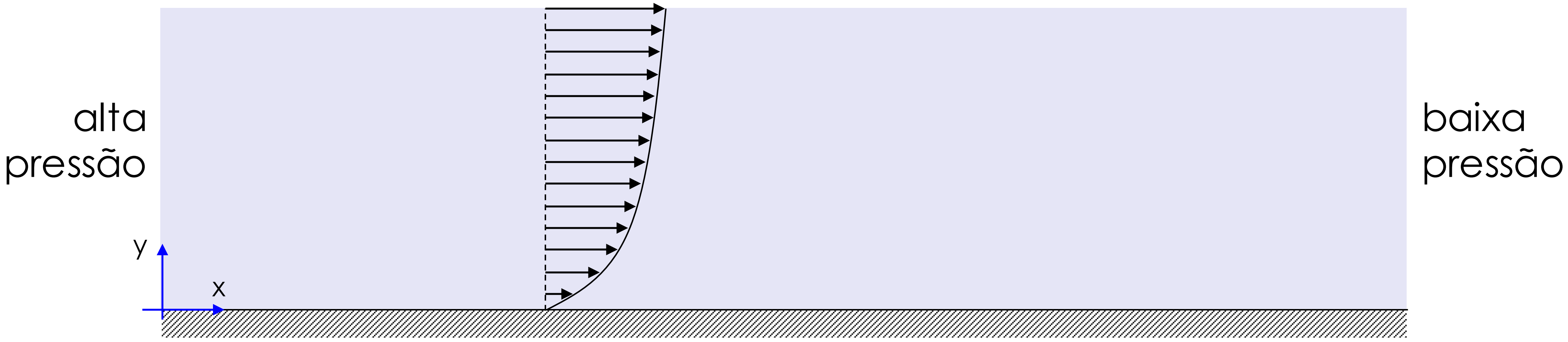

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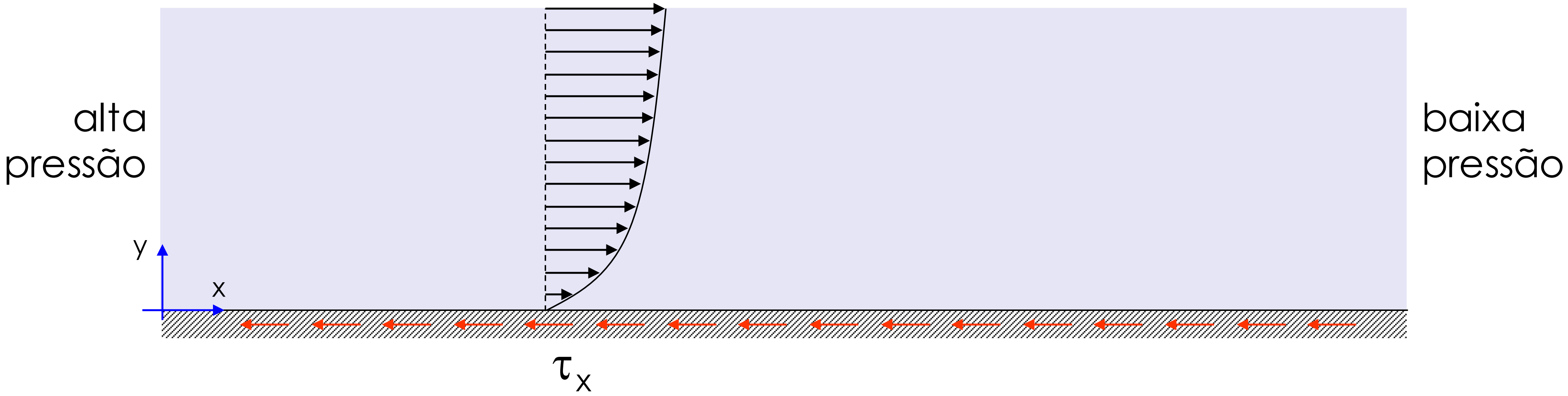

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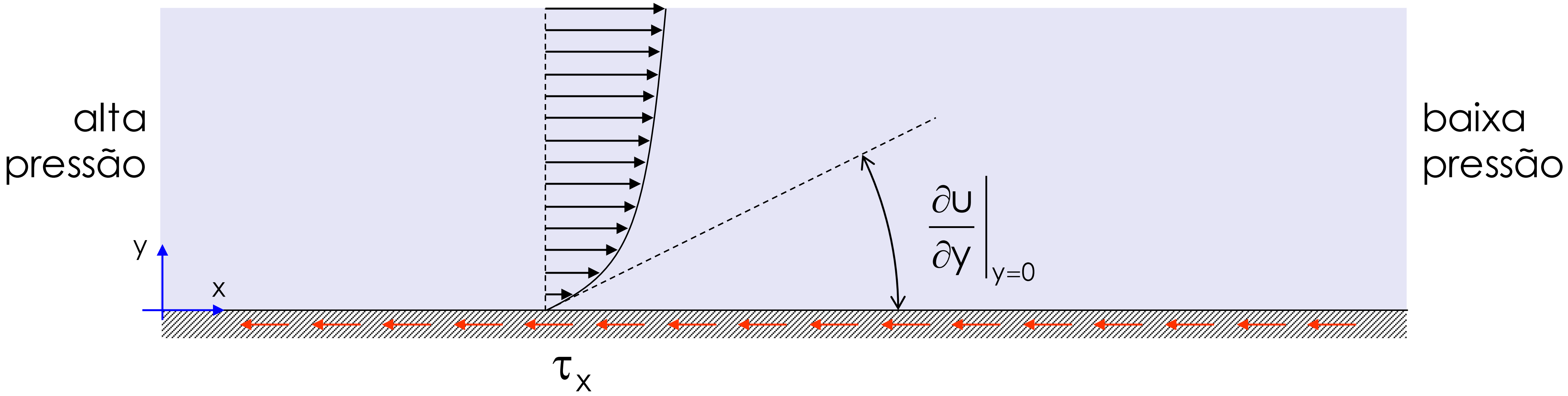

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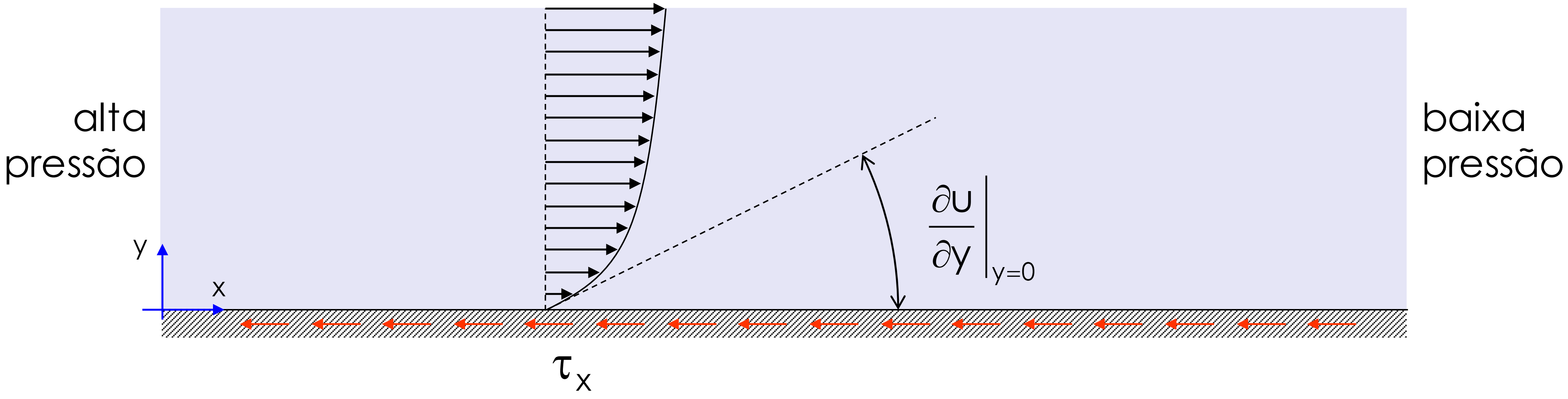
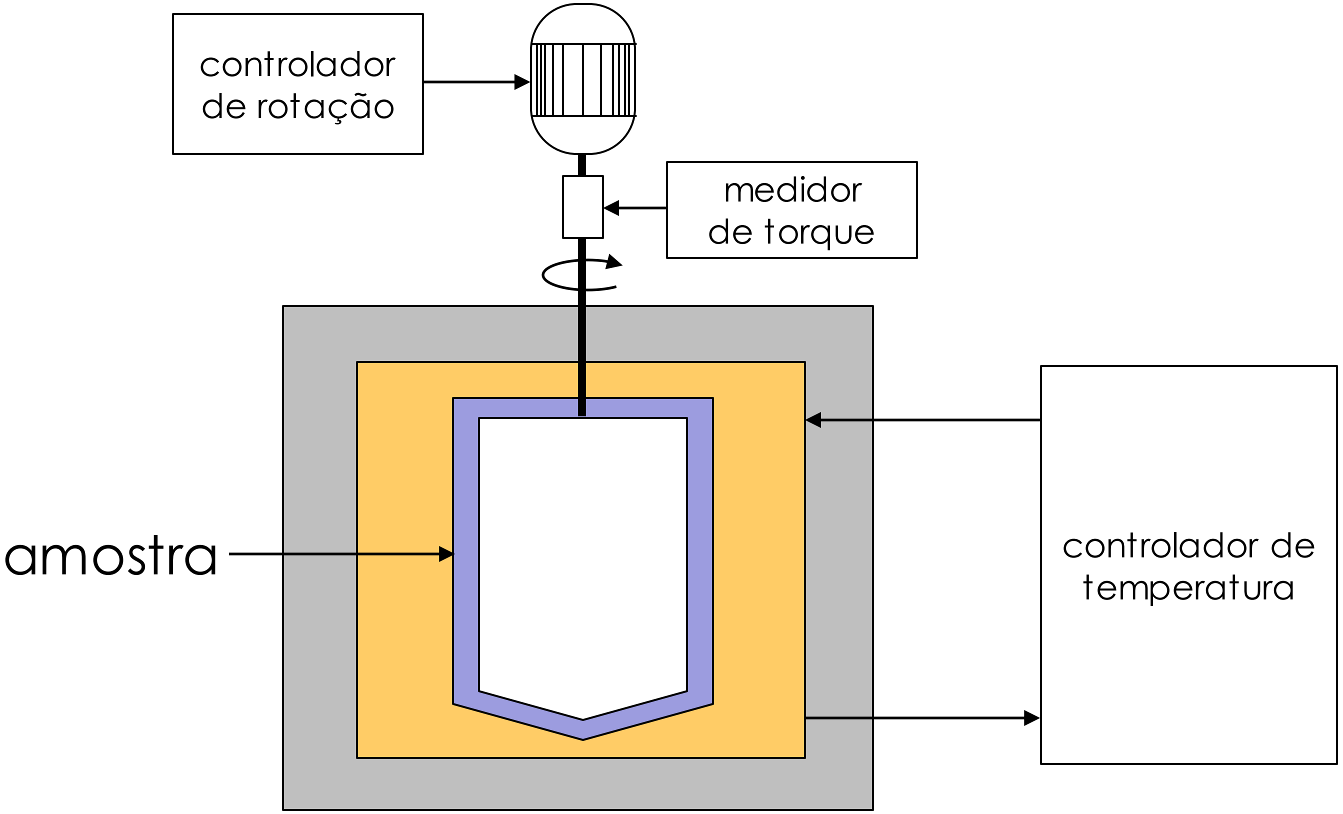
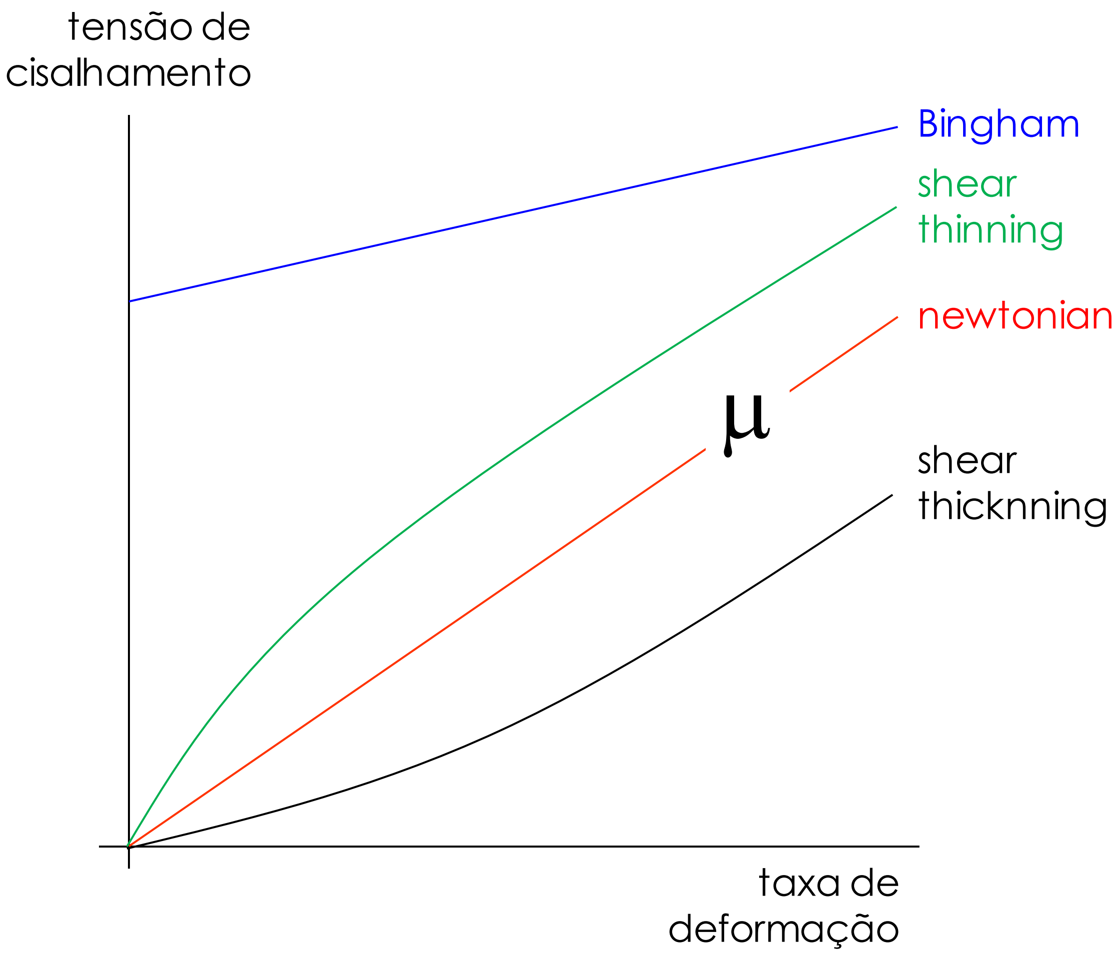


$$\tau_x \overset{hyp}{=} \mu \cdot \left.\frac{\partial u}{\partial y}\right|_{y=0} \Leftarrow$$

Fluido newtoniano
(modelo reológico)

Reômetro rotativo para medição da viscosidade...





$$\tilde{T} = f[\tilde{D}]$$

Newtoniano: água, óleo, glicerina e gases submetidos a taxas de deformação moderadas

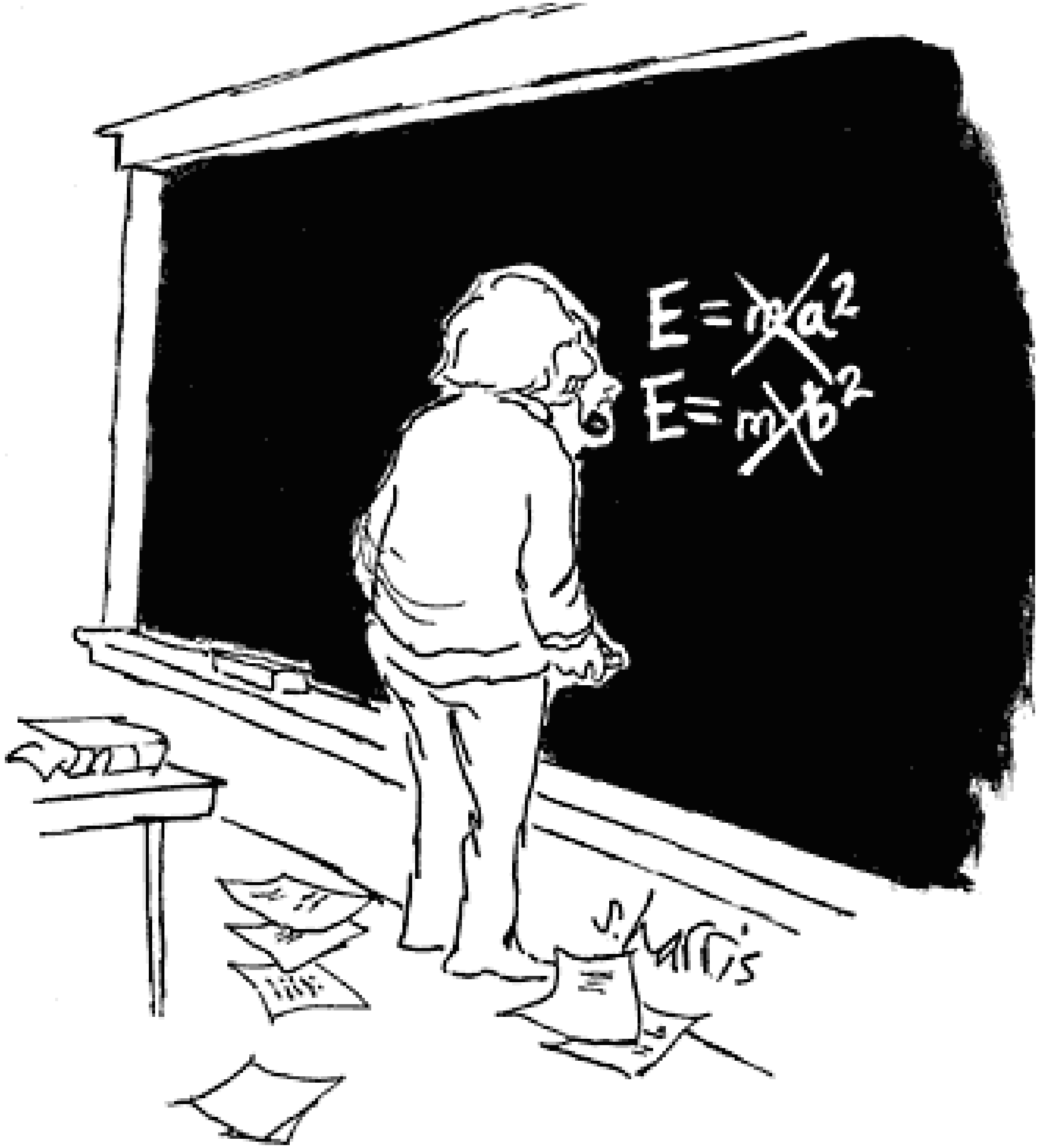
Bingham: passam a escoar acima de uma determinada tensão de cisalhamento (tinta e dentifrício)

Pseudoplástico: (shear-thinning) escoam mais facilmente sob altas tensões de cisalhamento

Dilatante: (shear thickenning) tornam-se progressivamente mais resistentes na medida em que o cisalhamento aumenta (fluidos de acoplamento, água e amido, etc.)

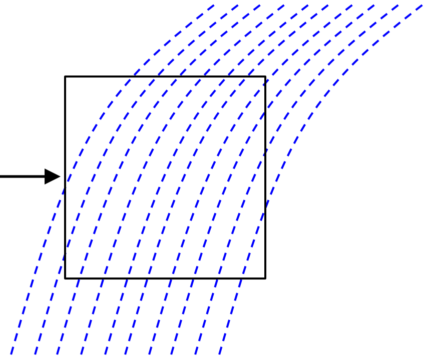
Banzé no Oeste (1974)

Equações governantes do movimento de um fluido...


E = ma2
E = mb2
S. Harris

Equações governantes do movimento de um fluido...

Abordagem euleriana (volume de controle):



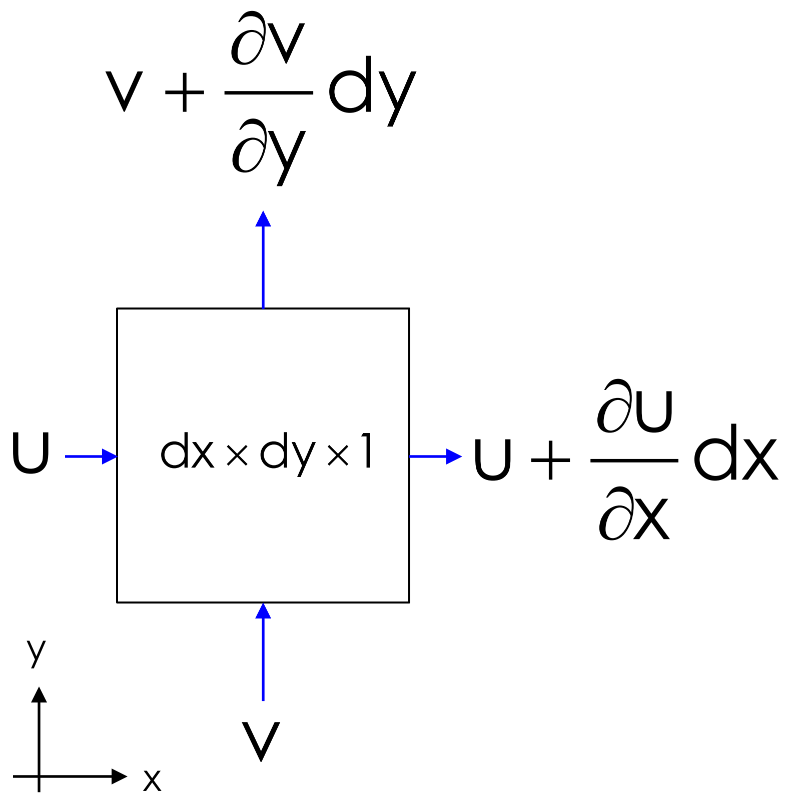
campo de escoamento

1) Inventário de massa
2) Inventário de quantidade de movimento
3) Inventário de energia

Expressão geral dos inventários:

(fluxo líquido de  entrando...) - (fluxo líquido de  saindo...) = (taxa de variação de  no VC)

Inventário de massa... (incompressível e regime permanente)



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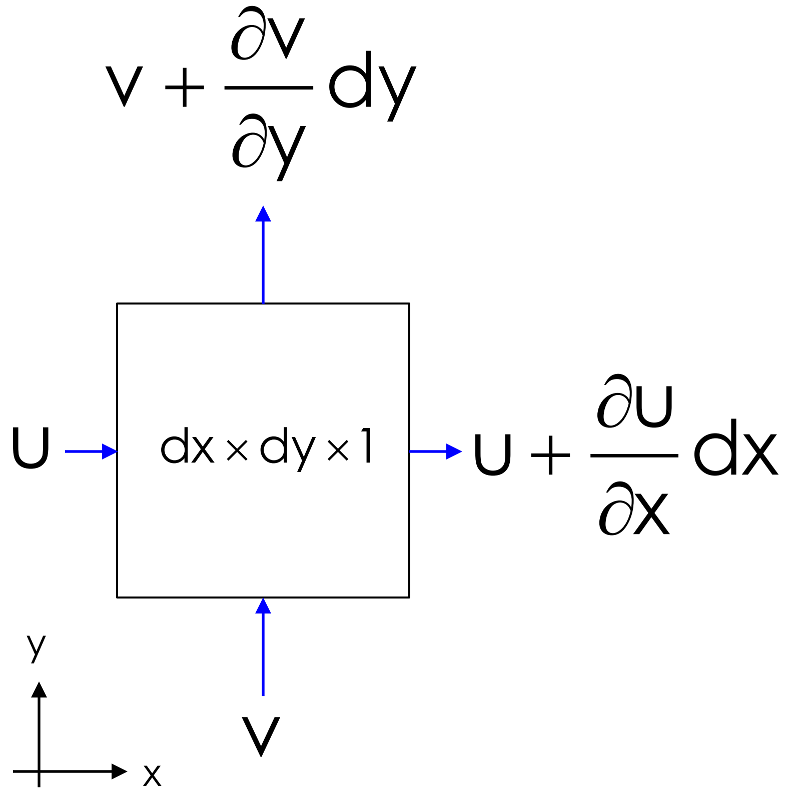
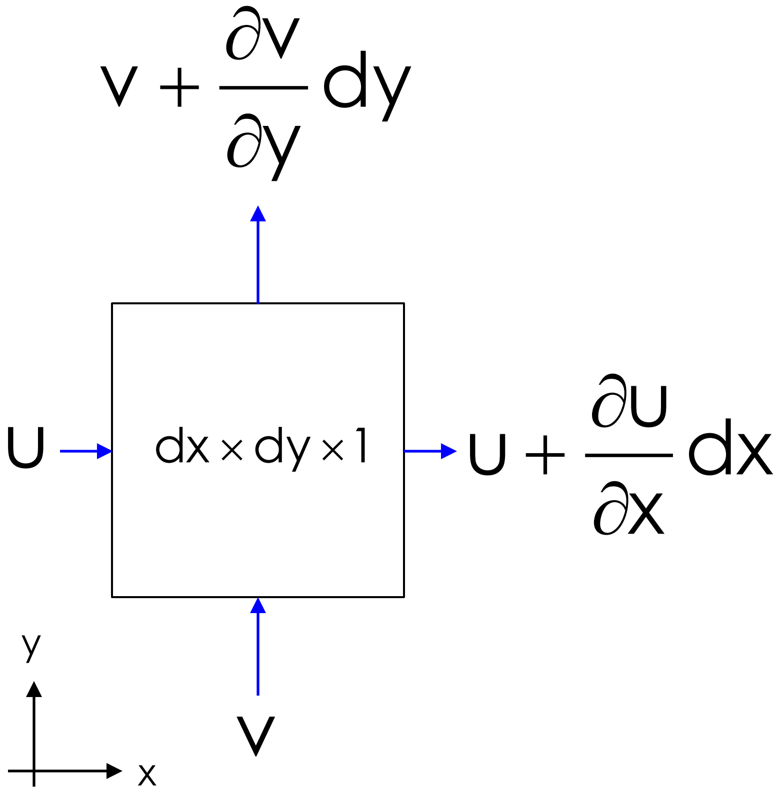


$$\rho u \cdot dy + \rho v \cdot dx +$$

$$-\rho\left(u + \frac{\partial u}{\partial x} dx\right) \cdot dy - \rho\left(v + \frac{\partial v}{\partial y} dy\right) \cdot dx = 0$$

Inventário de massa... (incompressível e regime permanente)



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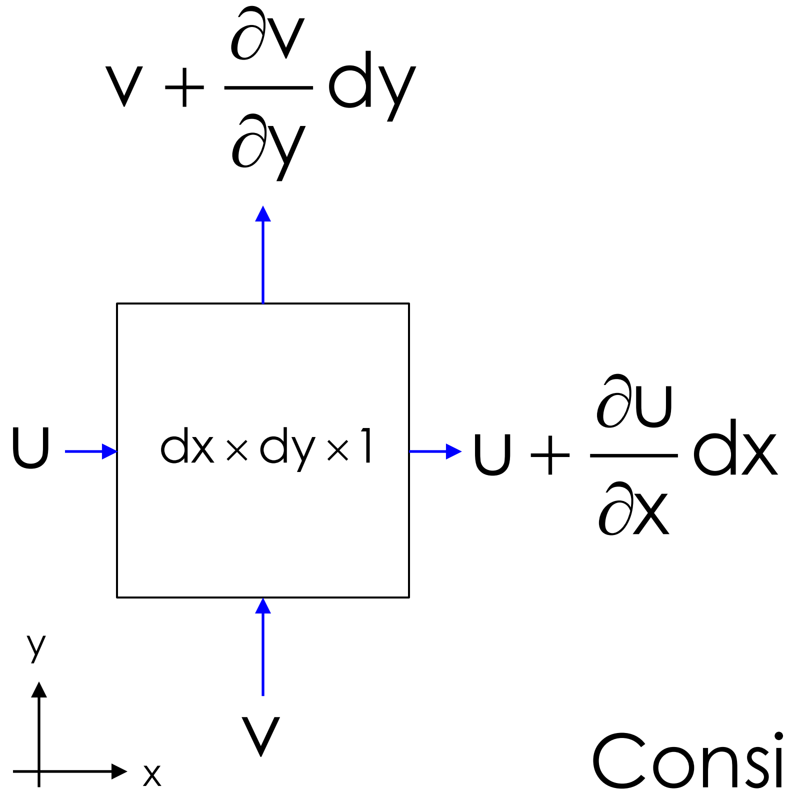

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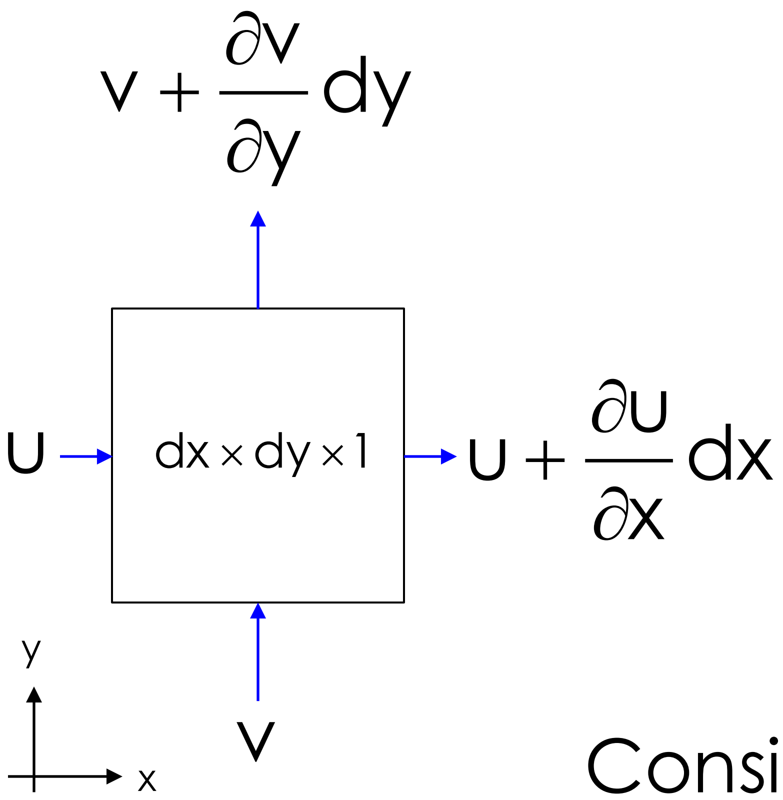
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Considerando 3D...

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \vec{\nabla} \cdot \vec{U} = 0$$

Equação da continuidade, independente de coordenadas

Inventário de massa... (incompressível e regime permanente)



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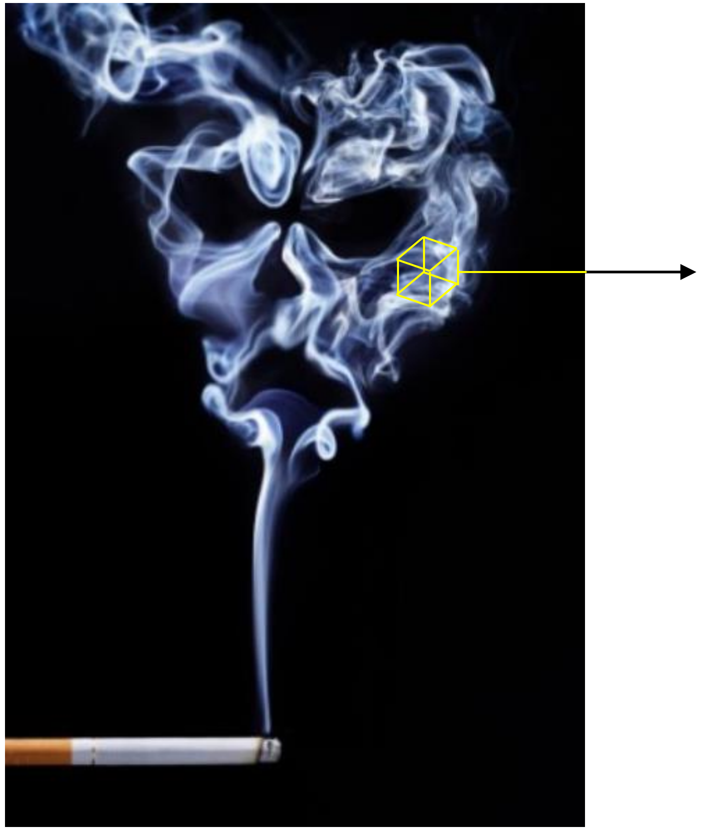
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

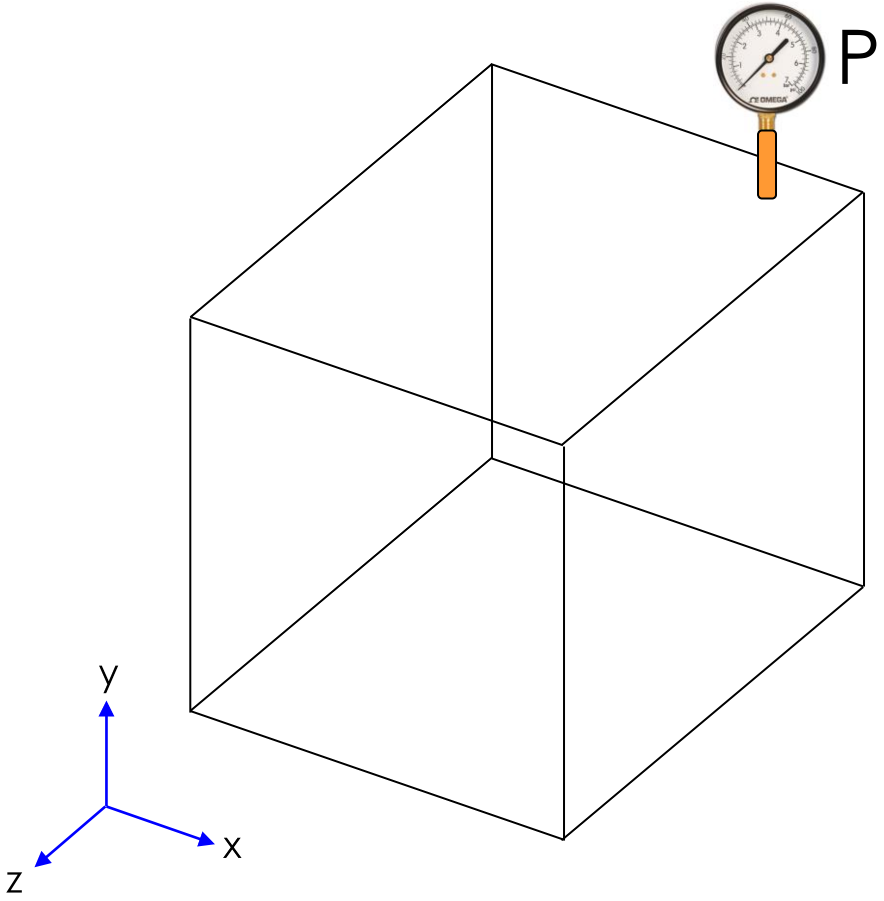
Considerando 3D, compressível e transiente...

$$\Rightarrow \quad \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{U}) = 0$$

Equação da continuidade, independente de coordenadas

Inventário de quantidade de movimento...





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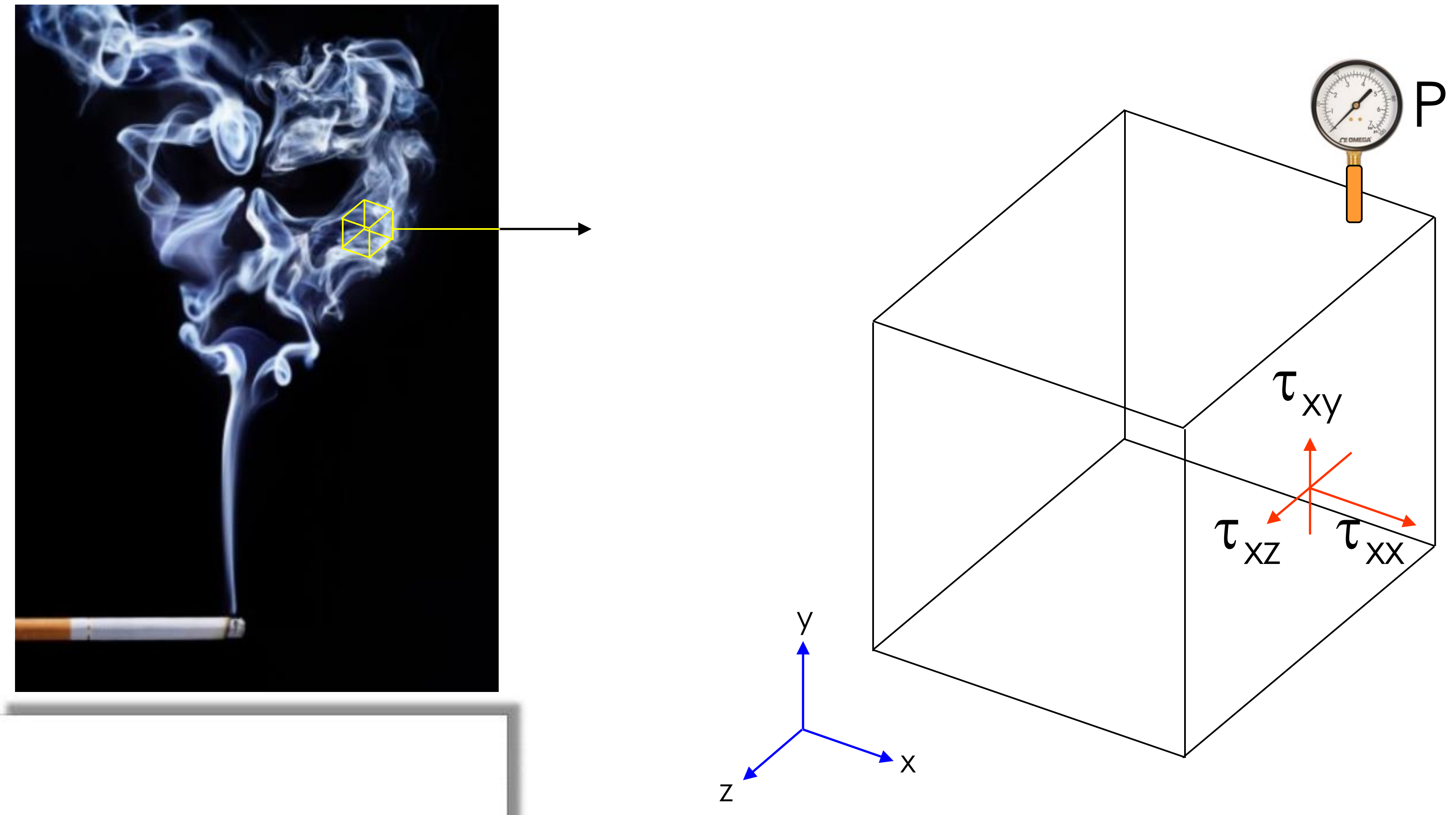

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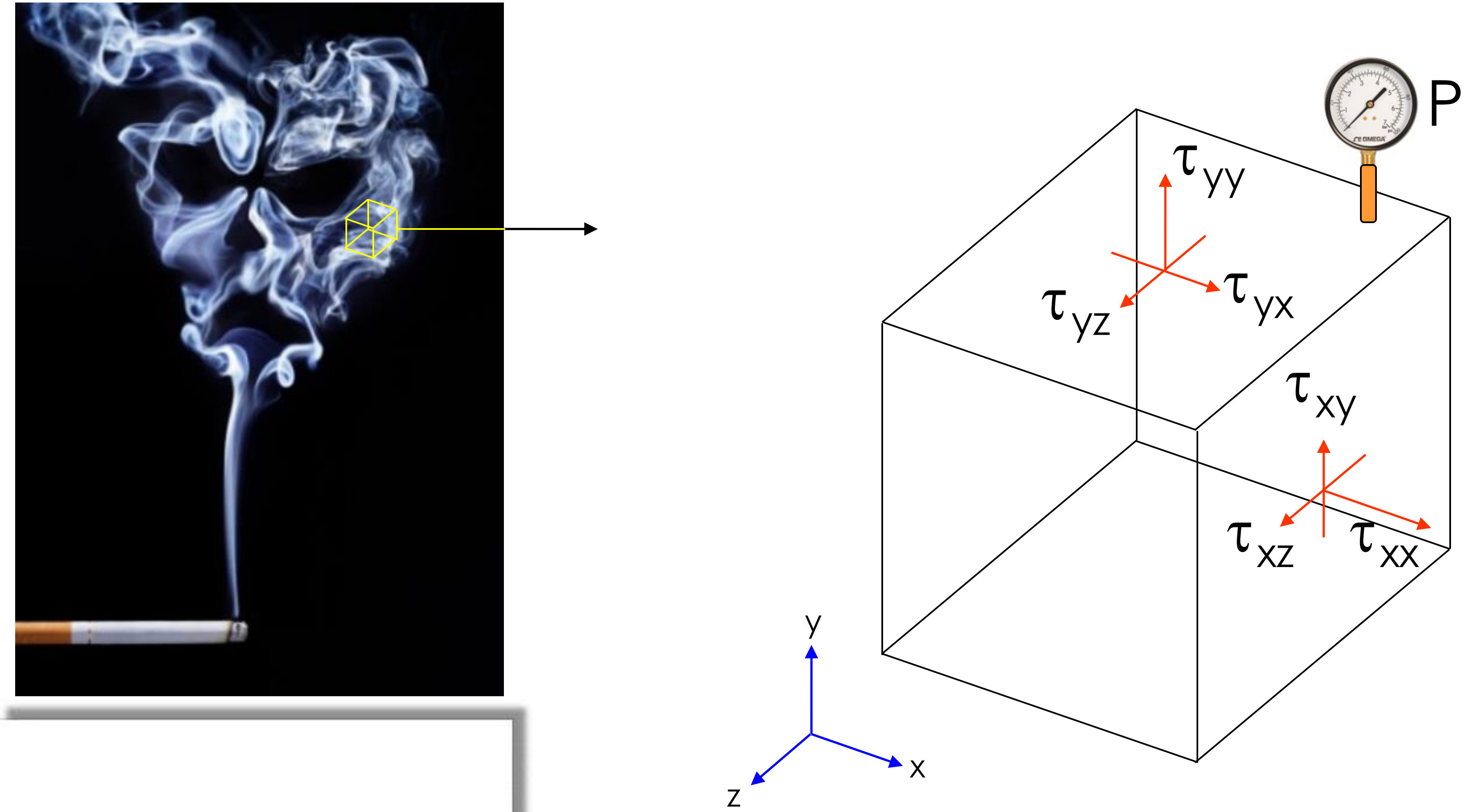

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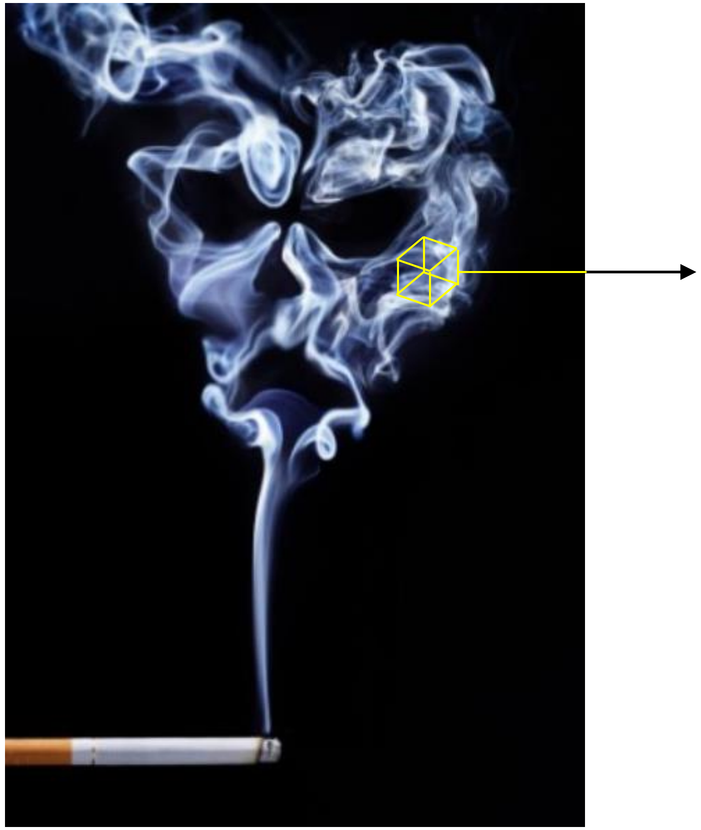

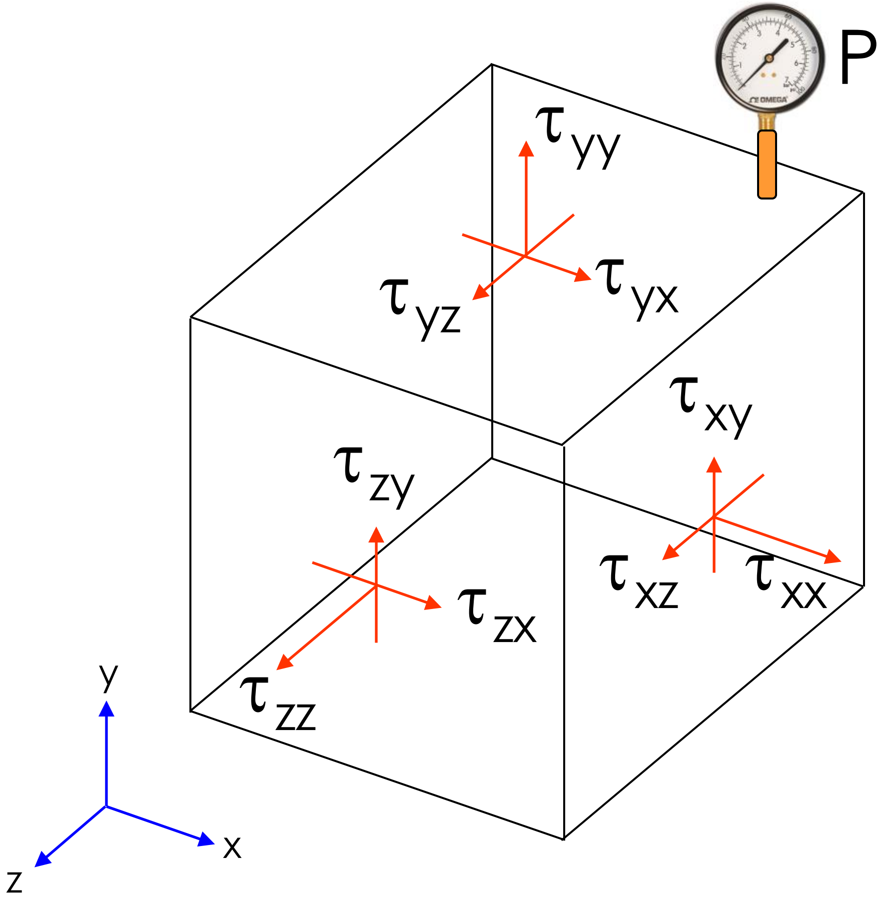

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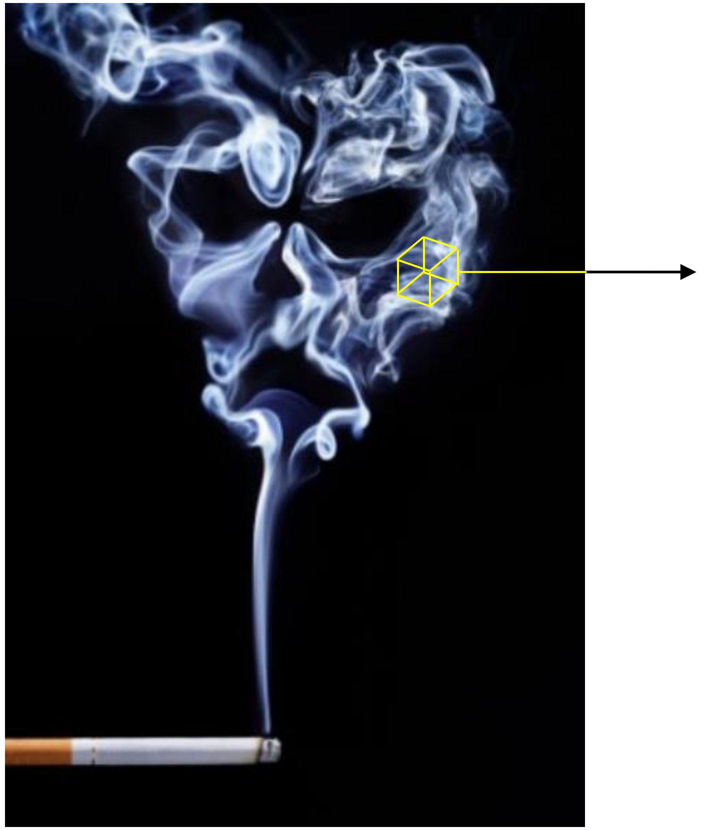

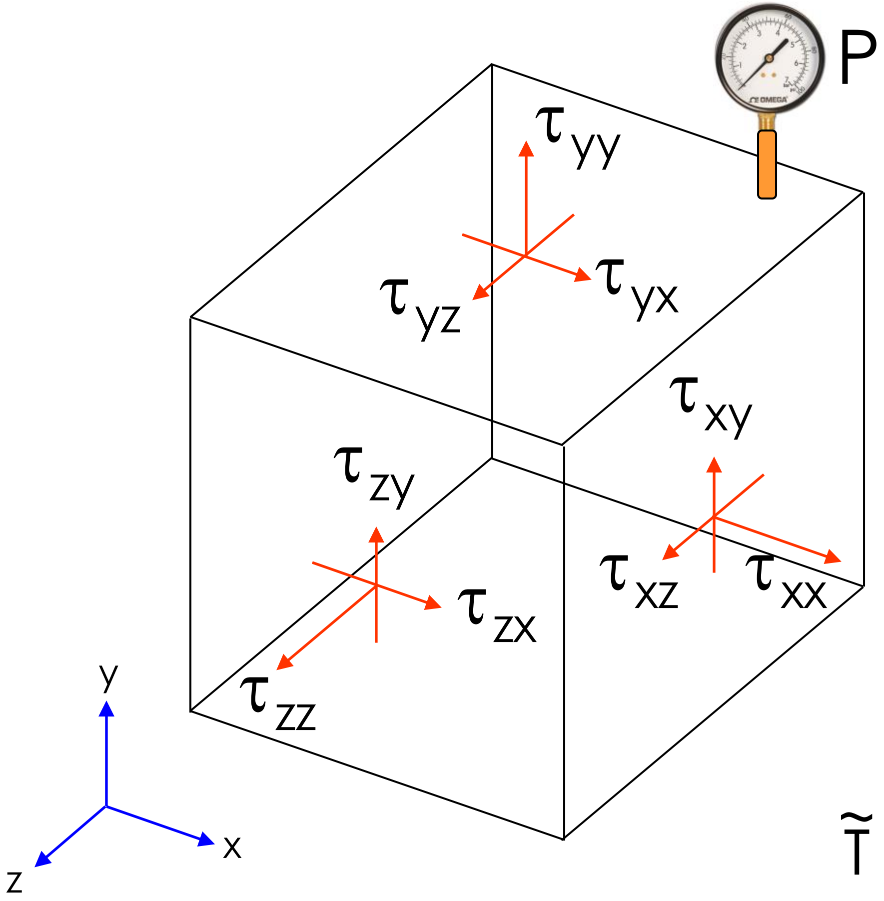

$$\tilde{T} \overset{\text{def}}{=} \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

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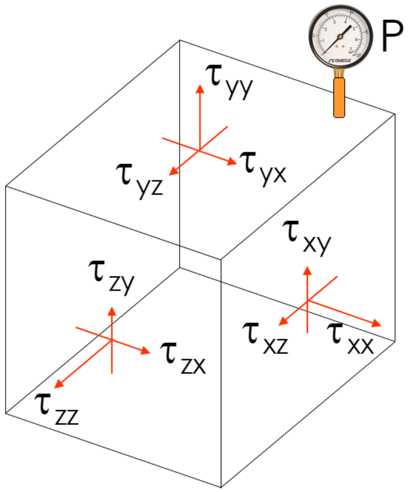

$$\sum F_x = m \cdot a_x$$

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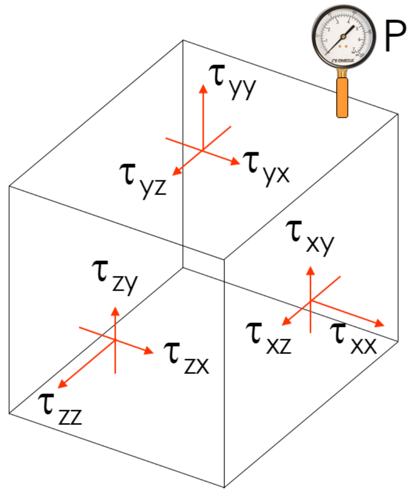

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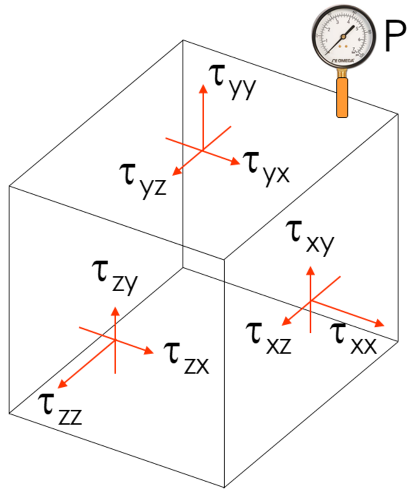

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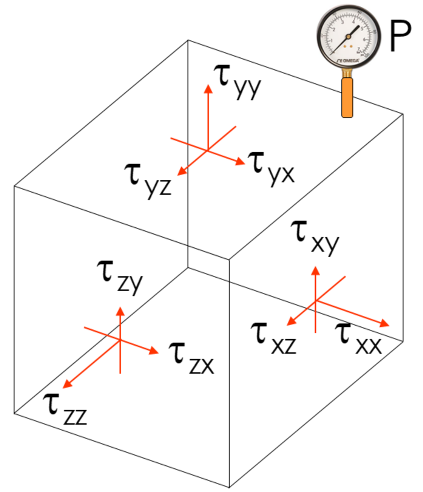

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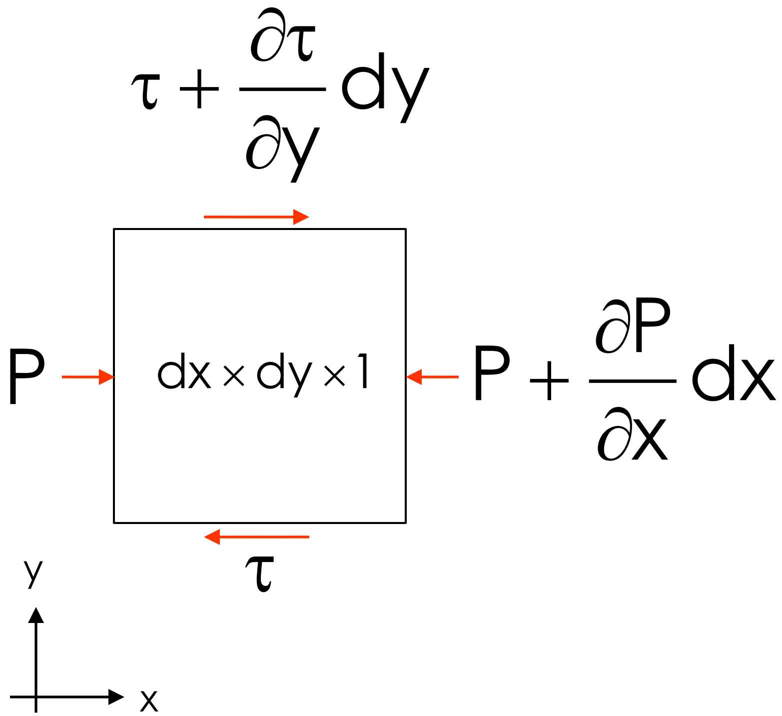

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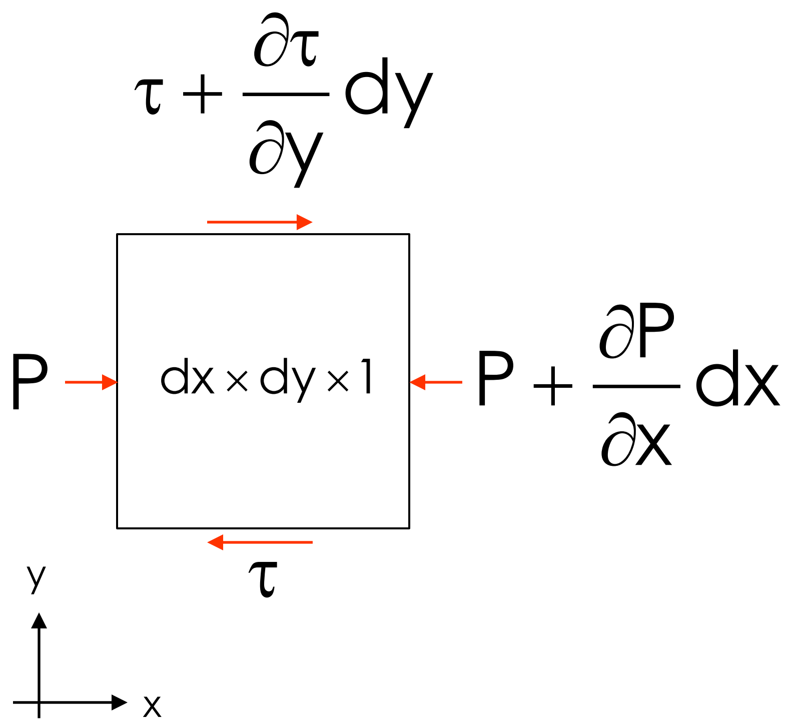

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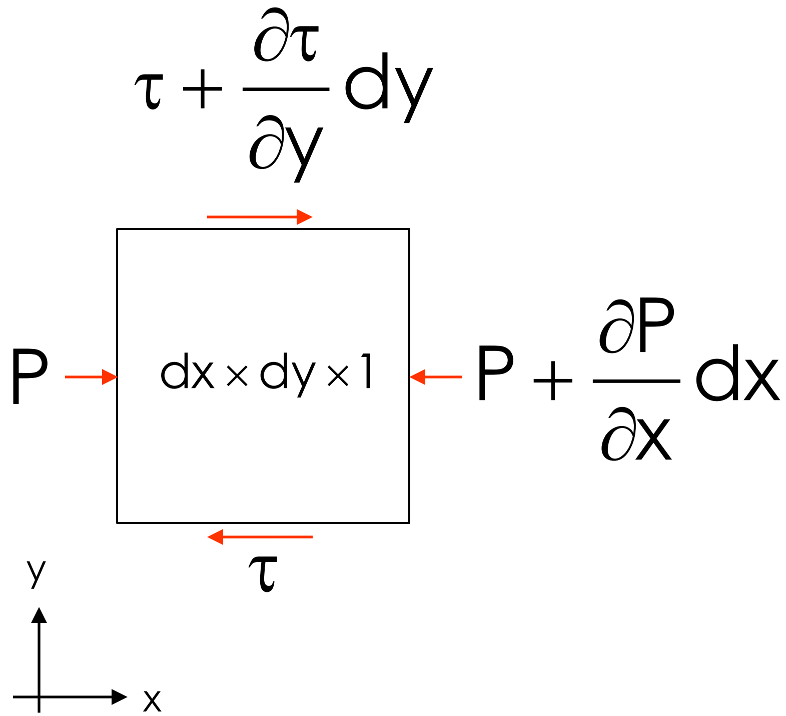

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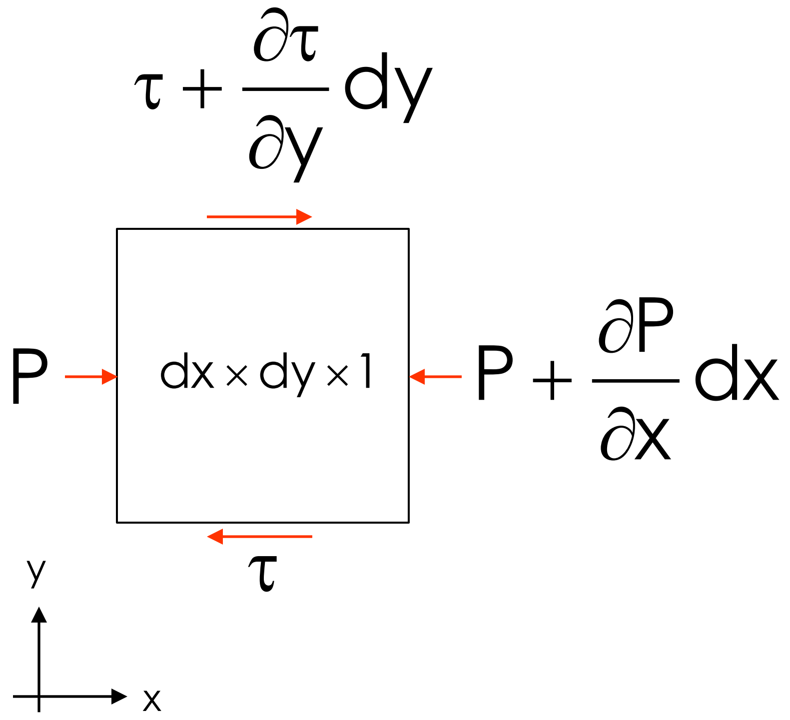

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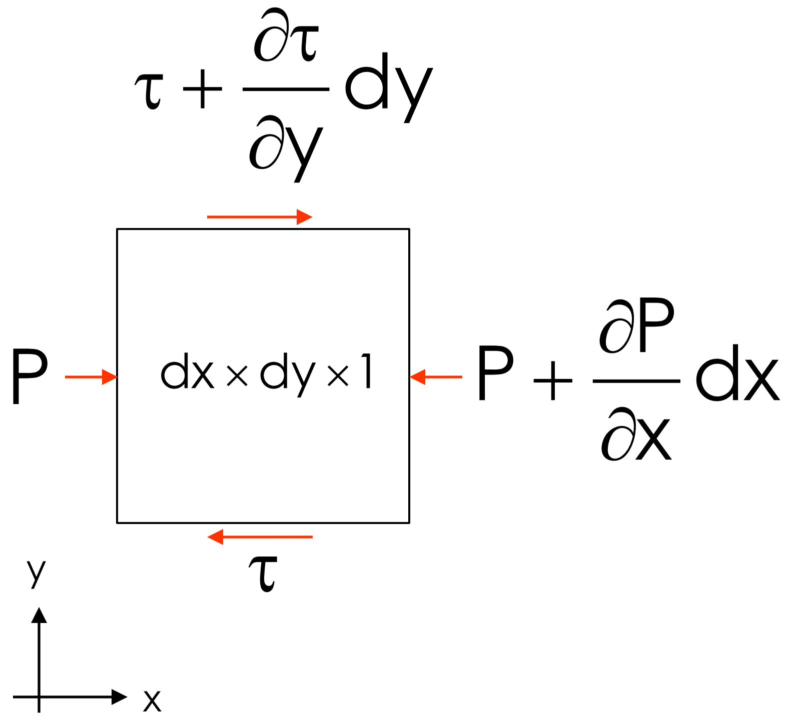

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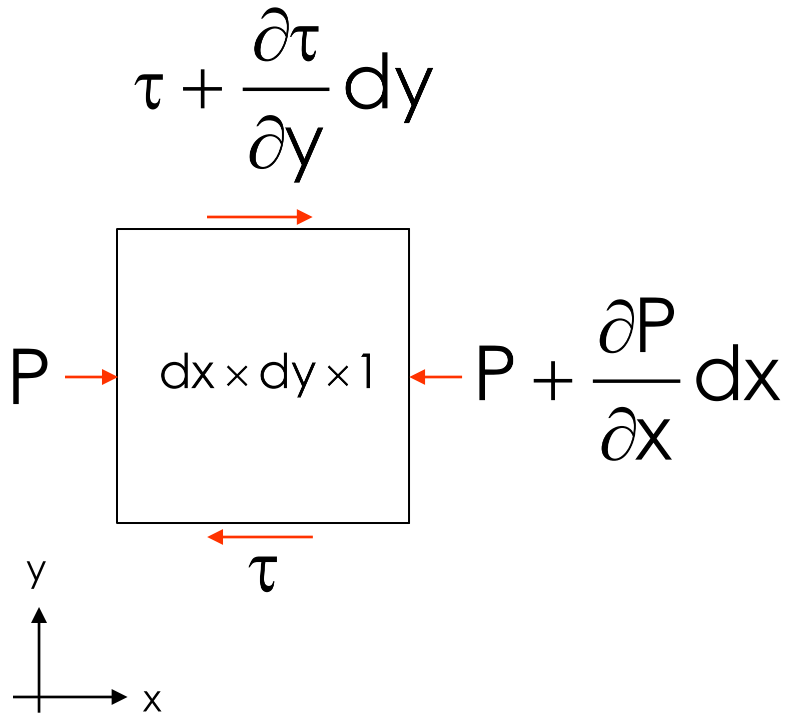

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$$\rho \cdot \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

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$$\underbrace{\rho \cdot \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)}_{\text{aceleração}} = \underbrace{-\frac{\partial P}{\partial x}}_{\text{volume}} + \underbrace{\mu \frac{\partial^2 u}{\partial y^2}}_{\text{superf.}}$$

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$$\rho \cdot \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \qquad \rho \cdot \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2}$$

Coord., transiente... $\rightarrow$ $$\rho \cdot \left(\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \vec{\nabla}\vec{U} \right) = -\vec{\nabla}P + \mu\nabla^2\vec{U} + \sum \vec{F}_{3D}$$

$$\tau = \mu(\nabla U + (\nabla U)^T - \frac{2}{3}\delta(\nabla . U)$$

$$\rho \cdot \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \qquad \rho \cdot \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2}$$

Coord., transiente... → $\rho \cdot \left(\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \vec{\nabla} \vec{U} \right) = -\vec{\nabla} P + \mu \nabla^2 \vec{U} + \sum \vec{F}_{3D}$

Reologia complexa... → $\rho \cdot \left(\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \vec{\nabla} \vec{U} \right) = -\vec{\nabla} P + \vec{\nabla} \cdot \tilde{T} + \sum \vec{F}_{3D}$

$$\tau = \mu(\nabla U + (\nabla U)^T - \frac{2}{3}\delta(\nabla . U)$$

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Coord., transiente... →

$$\rho \cdot \left(\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \vec{\nabla} \vec{U} \right) = -\vec{\nabla} P + \mu \nabla^2 \vec{U} + \sum \vec{F}_{3D}$$

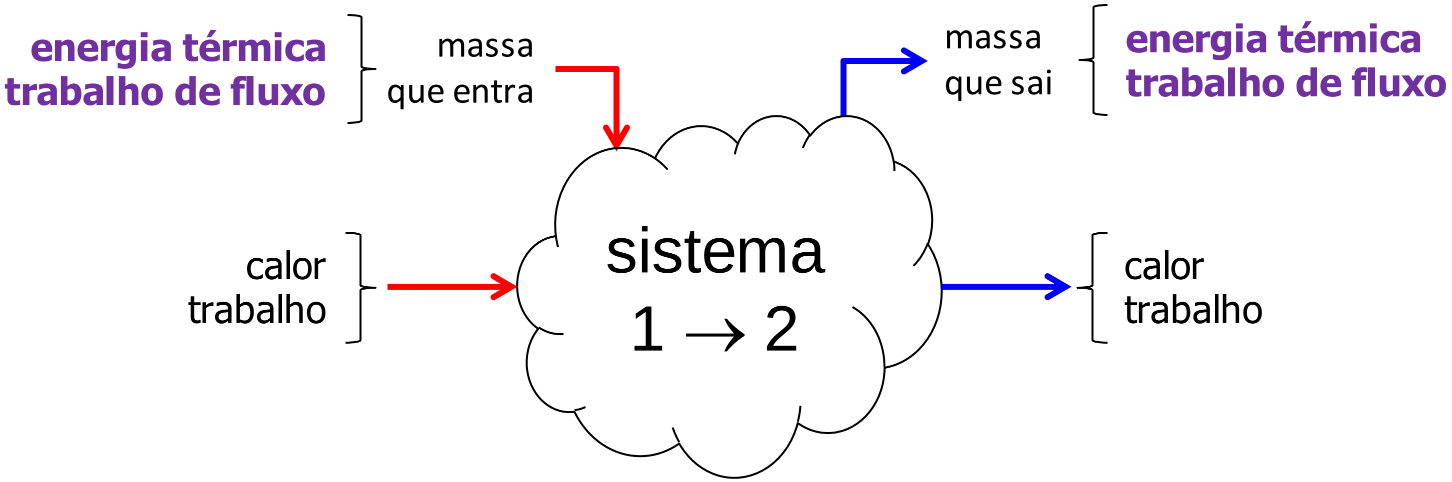
Reologia complexa... →

$$\rho \cdot \left(\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \vec{\nabla} \vec{U} \right) = -\vec{\nabla} P + \vec{\nabla} \cdot \tilde{T} + \sum \vec{F}_{3D}$$

- $\frac{\partial \vec{U}}{\partial t}$ → aceleração transiente
- $\vec{U} \cdot \vec{\nabla} \vec{U}$ → aceleração convectiva
- $-\vec{\nabla} P$ → gradiente de pressão
- $\vec{\nabla} \cdot \tilde{T}$ → div. do tensor de tensões
- $\sum \vec{F}_{3D}$ → forças de campo

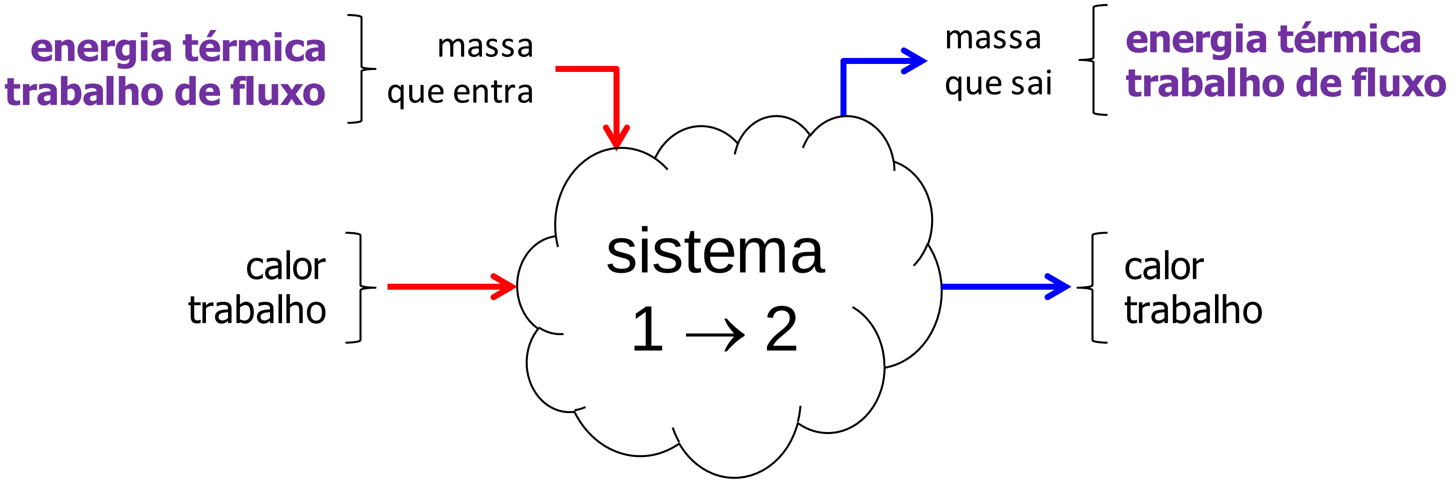
$$\tau = \mu(\nabla U + (\nabla U)^T - \frac{2}{3}\delta(\nabla . U)$$

Inventário de energia...



$$\dot{Q} - \dot{W} = \sum_{sai} \dot{m}_k \cdot \left(h_k + g z_k + \mathbb{V}_k^2 / 2\right) - \sum_{entra} \dot{m}_k \cdot \left(h_k + g z_k + \mathbb{V}_k^2 / 2\right)$$

Inventário de energia...



entrada de calor

saída de trabalho

energia carreada pelo fluxo de massa

$$\dot{Q} - \dot{W} = \sum_{sai} \dot{m}_k \cdot \left(h_k + g z_k + V_k^2 / 2\right) - \sum_{entra} \dot{m}_k \cdot \left(h_k + g z_k + V_k^2 / 2\right)$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{calor} - \left[\dot{E}_{sai} - \dot{E}_{ent}\right]_{trabalho} + \underbrace{\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{massa}} = \frac{dE}{dt}$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{massa,x} = (\rho u h)dxdy - \left[(\rho u h)dxdy + \frac{\partial(\rho u h)}{\partial x}dxdy\right] = -\frac{\partial(\rho u h)}{\partial x}dxdy$$

$$h = C_P \cdot T \rightarrow$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{massa,x} = -\rho C_P\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dxdy \qquad \left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{massa,y} = -\rho C_P\left(v\frac{\partial T}{\partial y} + T\frac{\partial v}{\partial y}\right)dxdy$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{massa} = -\rho C_P\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dxdy$$

Obs.: a equação da continuidade foi considerada...

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{calor} - \left[\dot{E}_{sai} - \dot{E}_{ent}\right]_{trabalho} + \left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{massa} = \frac{dE}{dt}$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{calor,x} = Q_x - \left[Q_x + \frac{\partial Q_x}{\partial x}dx\right] = -\frac{\partial Q_x}{\partial x}dx$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{calor,x} = -\frac{\partial}{\partial x}\left(-k\frac{\partial^2 T}{\partial x^2}dy\right)dx = k\frac{\partial T}{\partial x}dxdy$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{calor,y} = -\frac{\partial}{\partial y}\left(-k\frac{\partial T}{\partial y}dx\right)dy = k\frac{\partial^2 T}{\partial y^2}dxdy$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{calor} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)dxdy$$

$$\left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{calor} - \left[\dot{E}_{sai} - \dot{E}_{ent}\right]_{trabalho} + \left[\dot{E}_{ent} - \dot{E}_{sai}\right]_{massa} = \frac{dE}{dt}$$

trabalho das forças de campo...

trabalho das forças viscosas (força × velocidade)...

$$\tilde{T} : \tilde{D}$$

produto escalar do tensor de tensões pelo tensor de taxas de deformação...

trabalho de fluxo

$u \leftarrow h$

$$\rho C_P\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$\rho C_P\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tilde{T} : \tilde{D}$$

$$\rho C_P\left(\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T\right) = \vec{\nabla} \cdot (k \vec{\nabla} T) + \tilde{T} : \tilde{D}$$

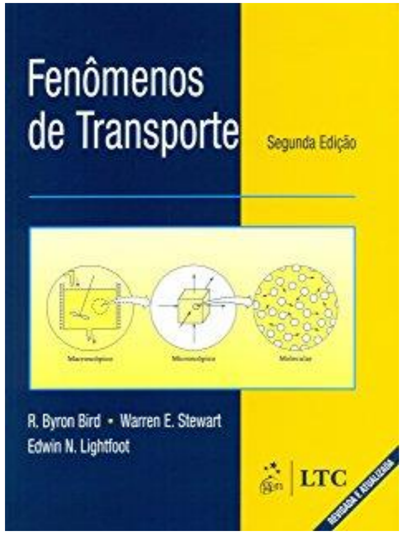
Função de dissipação viscosa:

$$\tilde{T} : \tilde{D} \overset{\text{mod}}{=} \mu \Phi(\vec{U})$$

"Força × deslocamente" (energia mecânica transformada em energia térmica devido à ação da viscosidade)

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 - \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2$$

Obs.: a dedução pode ser encontrada em...



Obs.: significativo a altas velocidades...

Equações governantes:

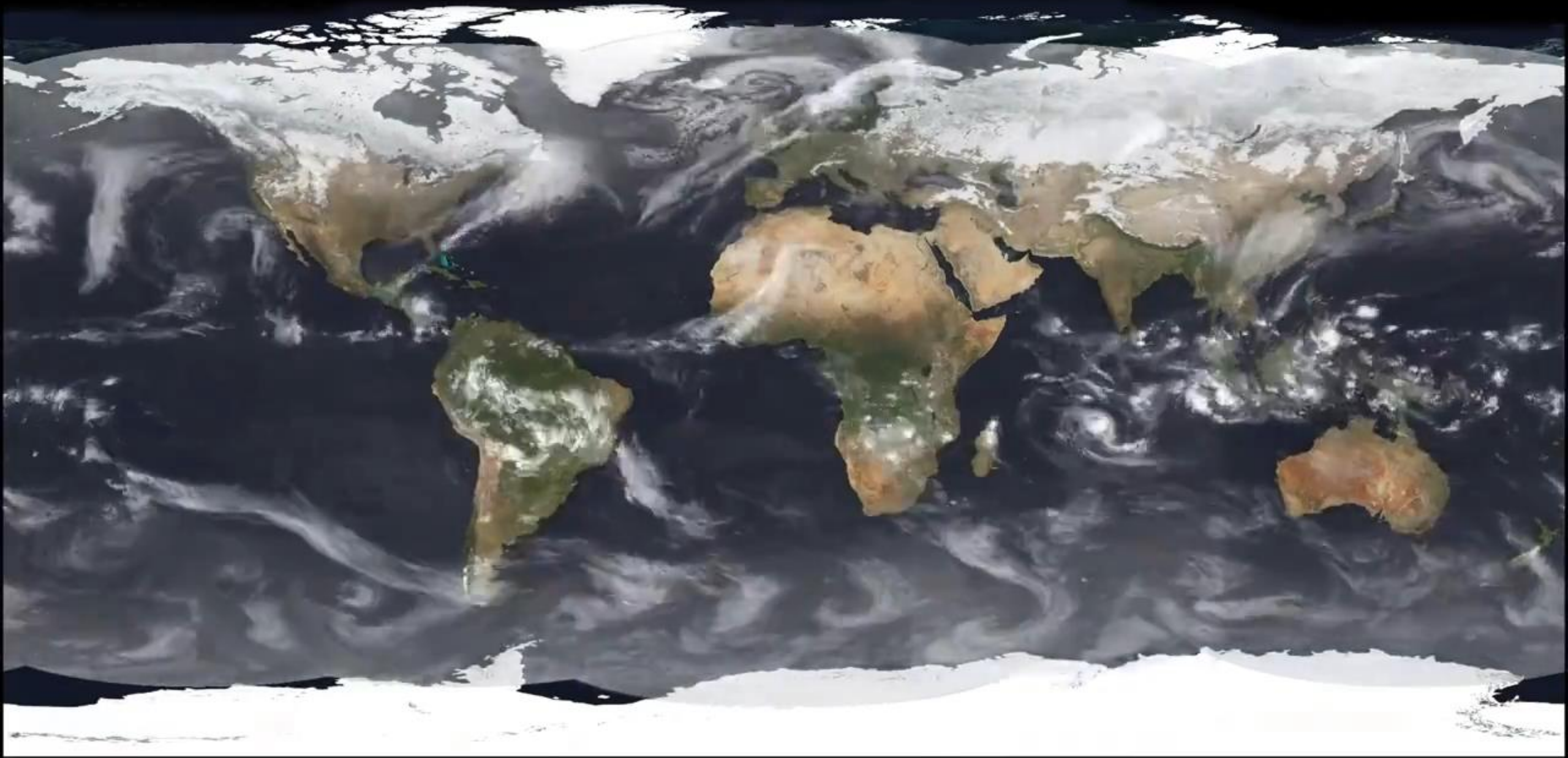
Continuidade (massa) $\rightarrow$ $$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{U}) = 0$$

Q. de movimento (Navier-Stokes) $\rightarrow$ $$\rho \cdot \left(\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \vec{\nabla} \vec{U} \right) = -\vec{\nabla} P + \vec{\nabla} \cdot \tilde{T} + \sum \vec{F}_{3D}$$

Energia (1^{ra} lei) $\rightarrow$ $$\rho C_P \left(\frac{\partial T}{\partial t} + \vec{U} \cdot \vec{\nabla} T \right) = \vec{\nabla} \cdot (k \vec{\nabla} T) + \tilde{T} : \tilde{D}$$

Escalas microscópicas (Kolmogorov) a escalas sinóticas...

$\Rightarrow$



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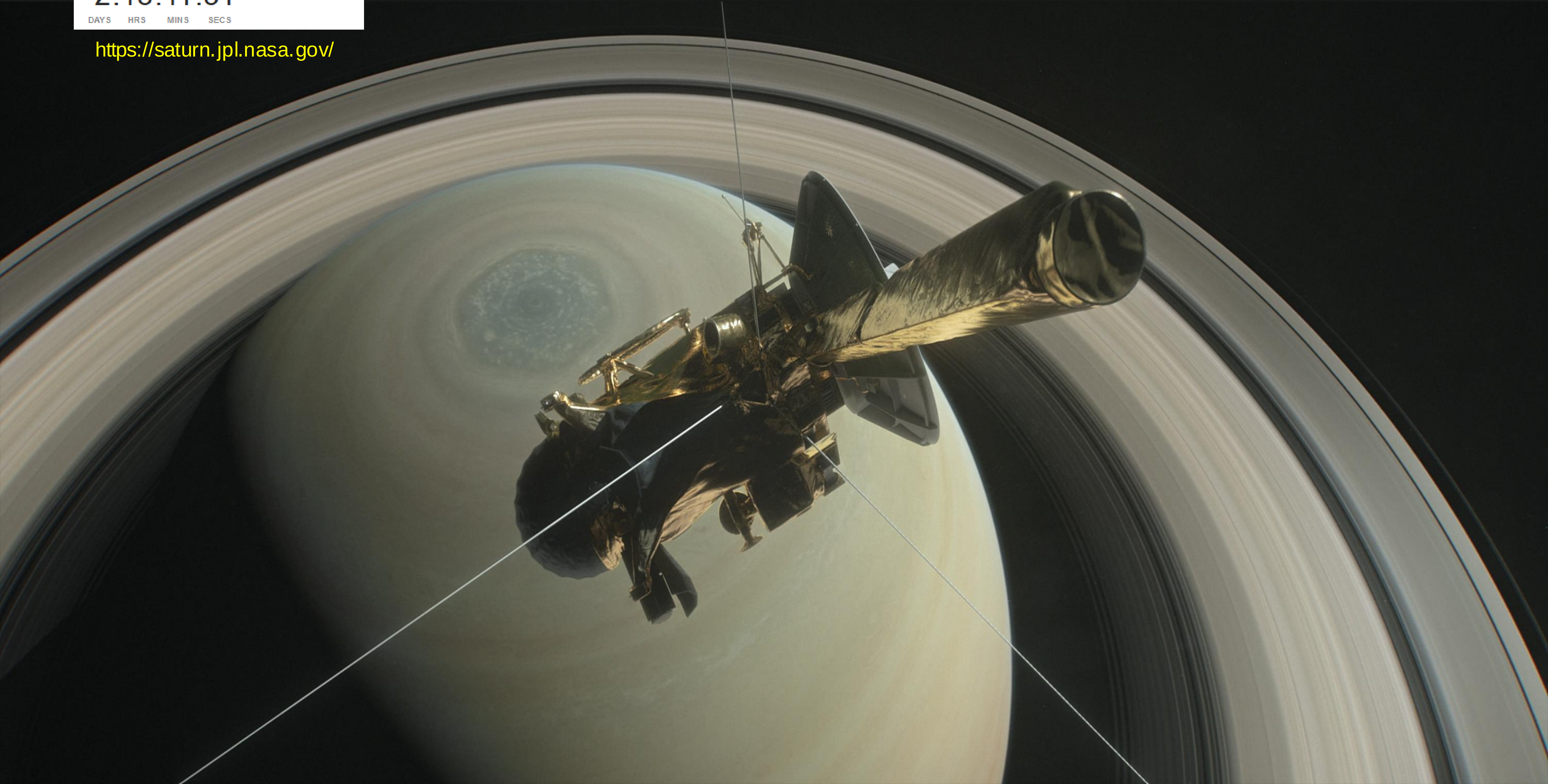
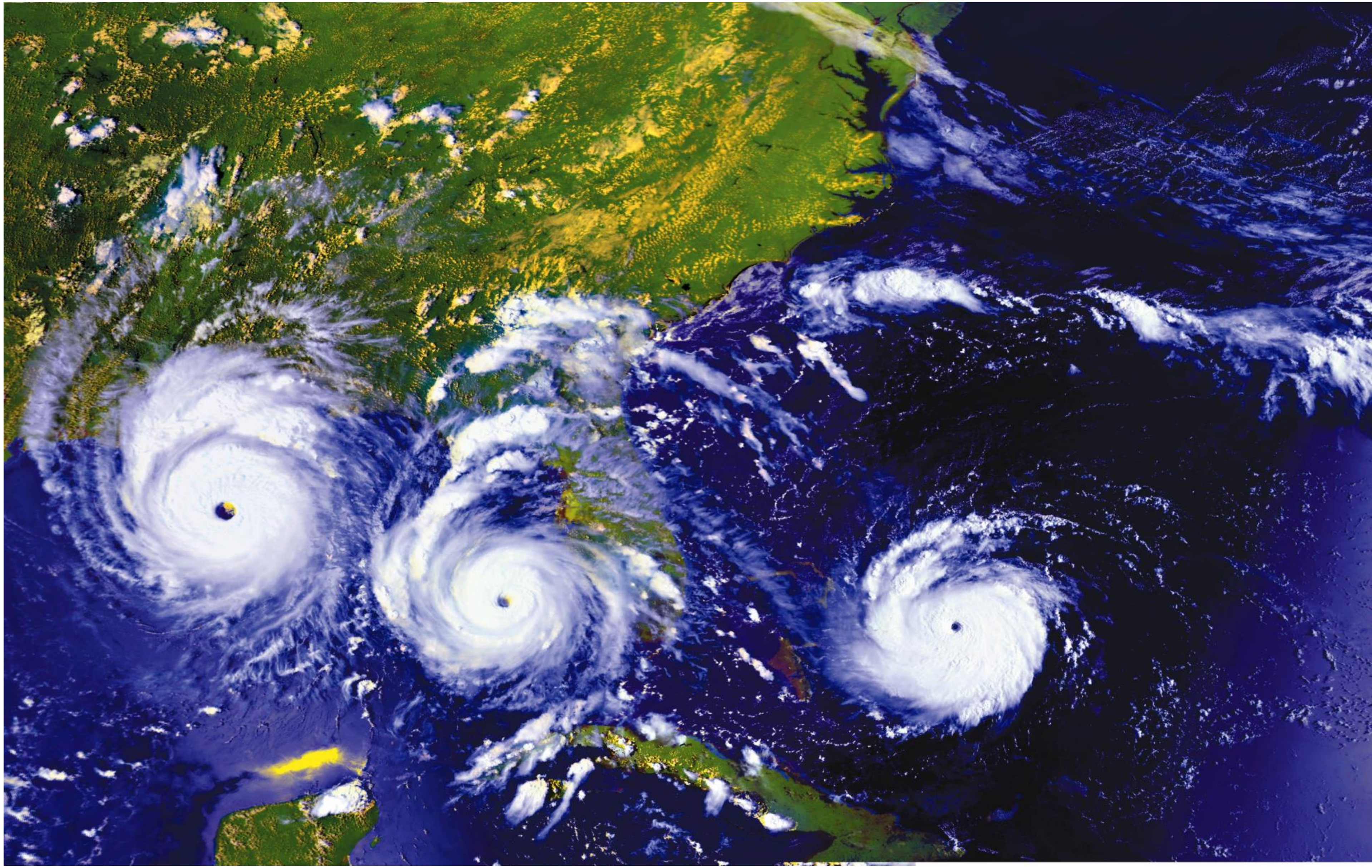
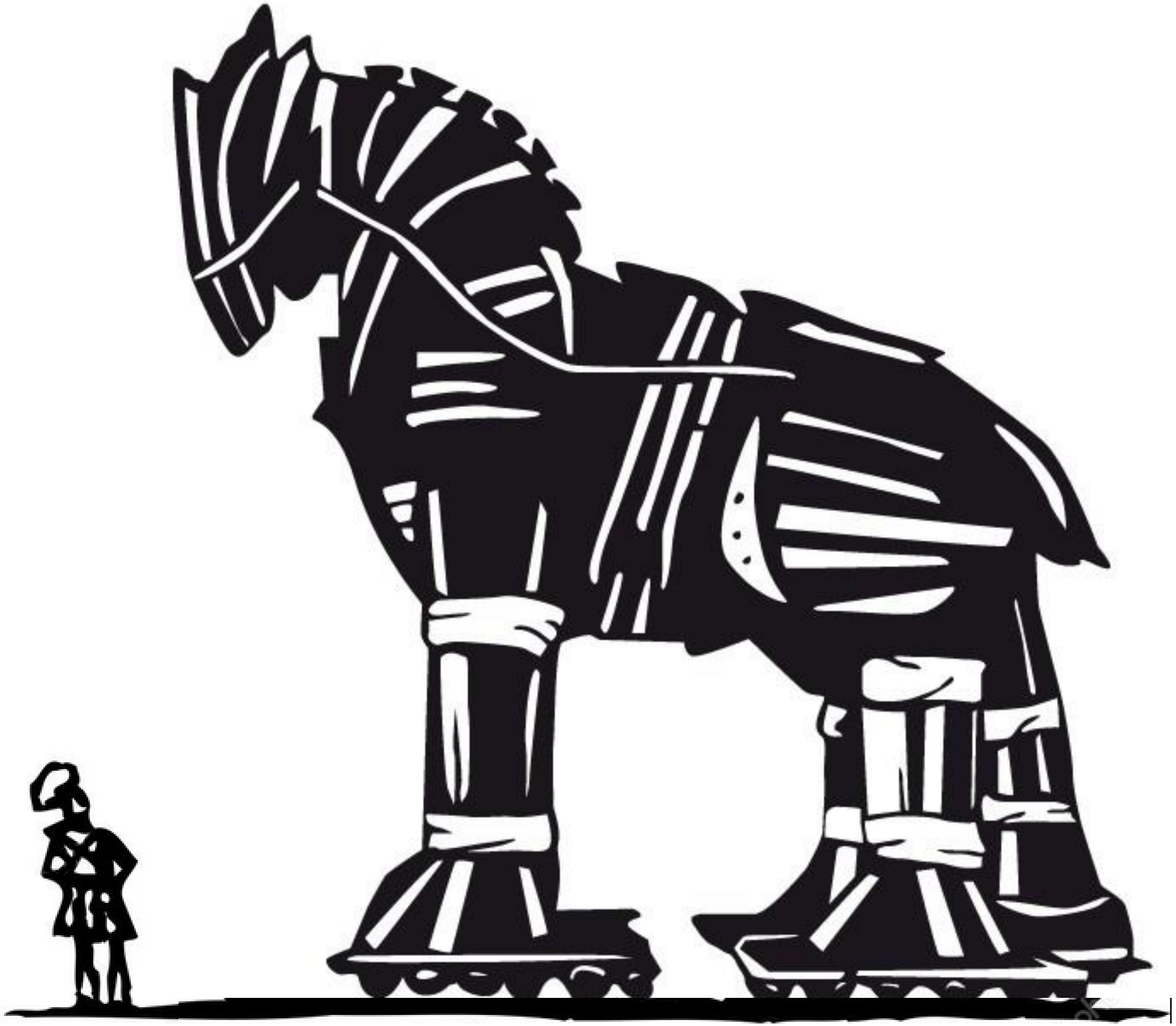


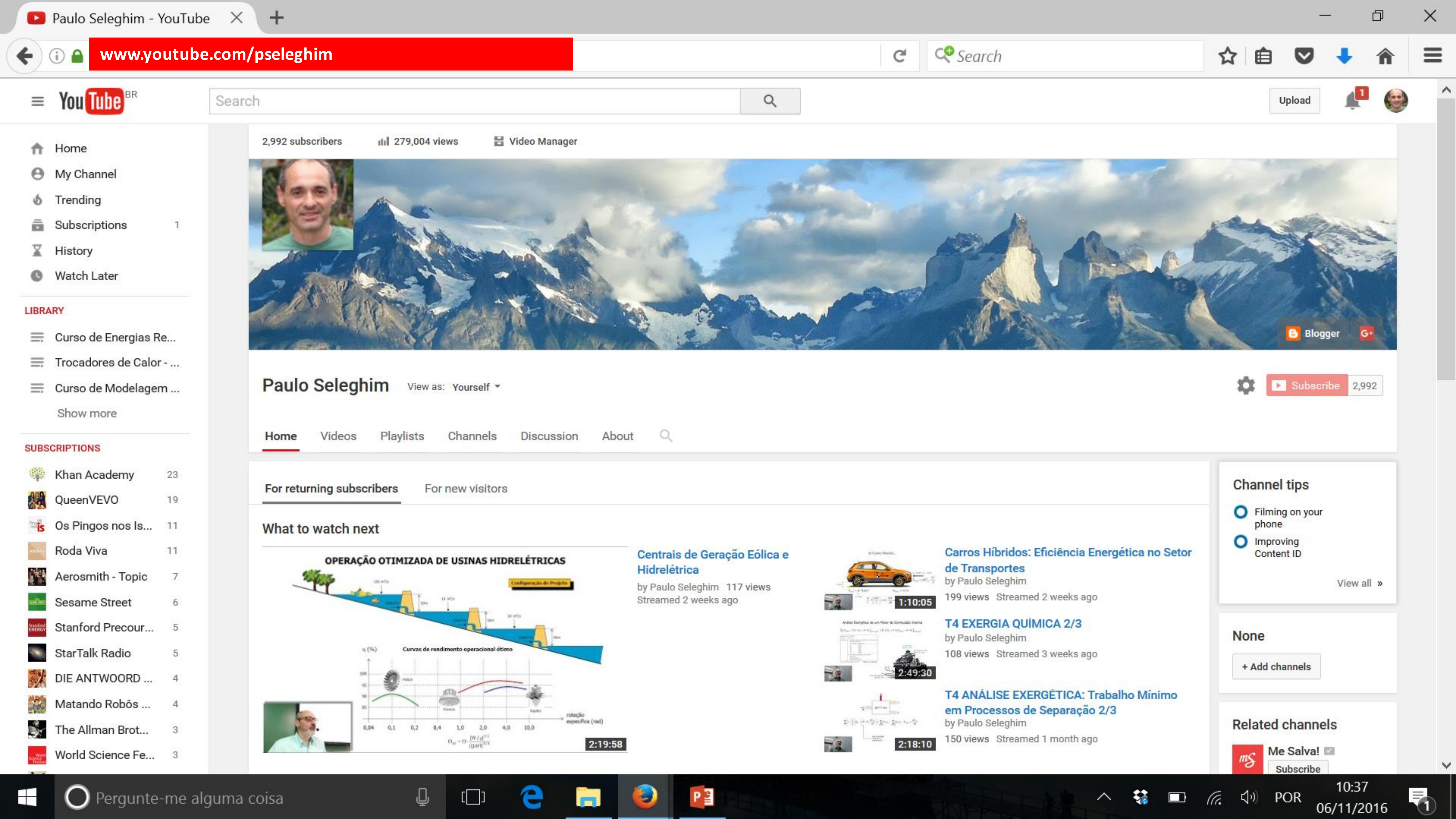
Por que os furacões no hemisfério norte giram no sentido anti-horário e no hemisfério sul giram no sentido horário ?



Furacão Andrew 1992

Próxima aula...





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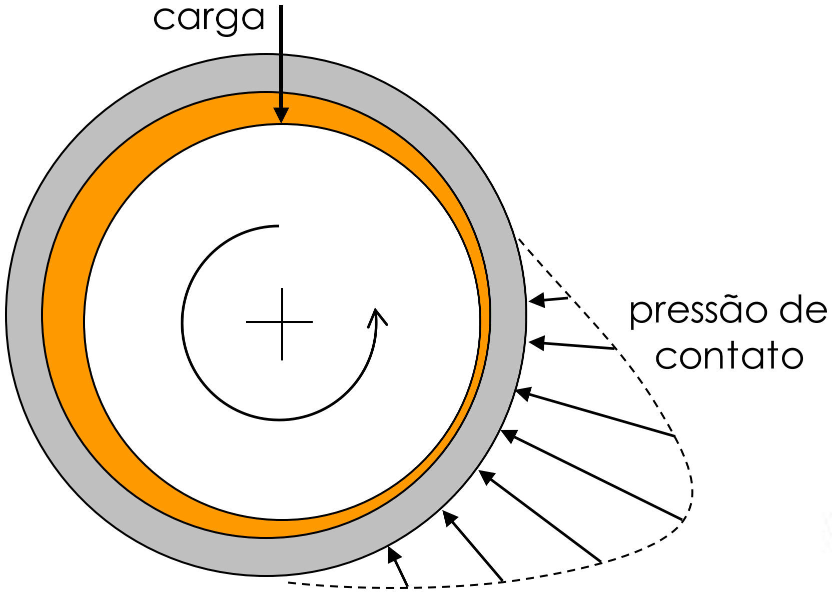
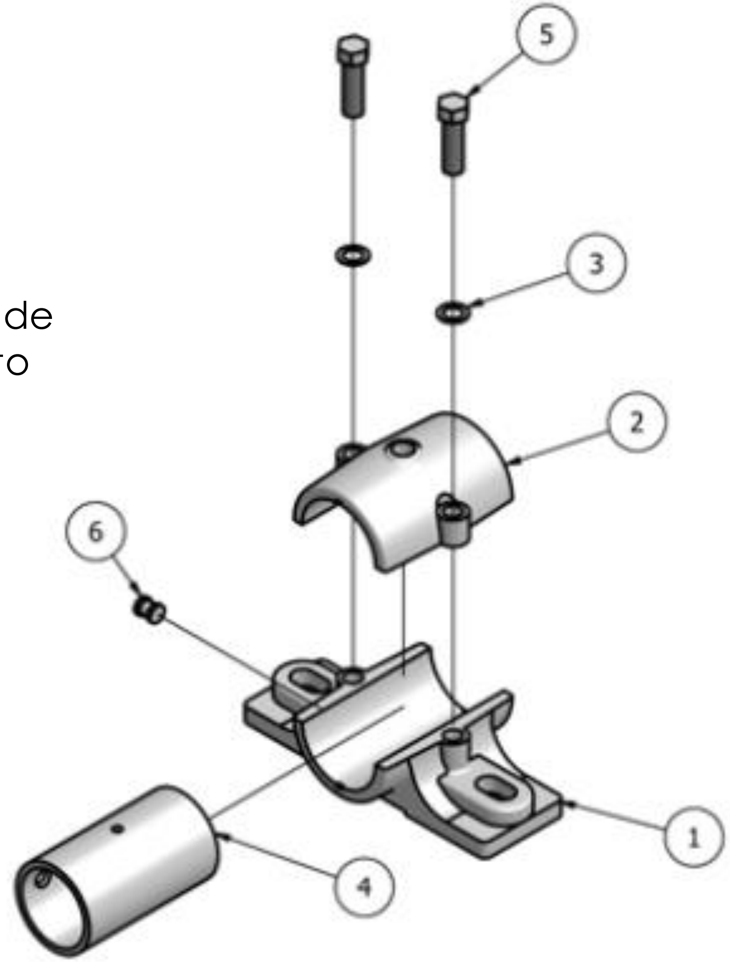
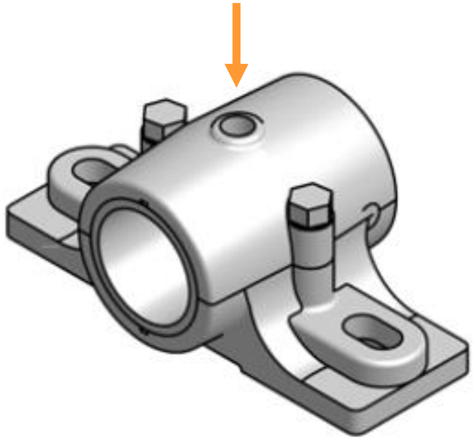
Exemplo (Çg 6-1): o escoamento de óleo em um mancal de escorregamento pode ser aproximado cf. mostrado na figura abaixo (Couette). A distância entre as placas é de 2mm e sua velocidade relativa é de 12m/s, sendo que, em ambas, a temperatura é mantida em 20°C. Nestas condições calcule a) os campos de velocidade e temperatura e b) a máxima temperatura e o fluxo de calor do óleo para as placas.





$$k = 0.145 \text{ W/m} \cdot \text{K}$$
$$\mu = 0.800 \text{N} \cdot \text{s/m}^2$$

propriedades do óleo @ 20°C



JOURNAL BEARING ASSEMBLED
SCALE 1 / 2

$$\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

PARTS LIST			
ITEM	QTY	PART NUMBER	MATERIAL
1	1	JOURNAL BEARING BOTTOM	MALLEABLE IRON
2	1	JOURNAL BEARING TOP	MALLEABLE IRON
3	2	LOCKWASHER	MALLEABLE IRON
4	1	BUSHING	MALLEABLE IRON
5	2	CAP SCREW	MALLEABLE IRON
6	1	BEARING	MALLEABLE IRON

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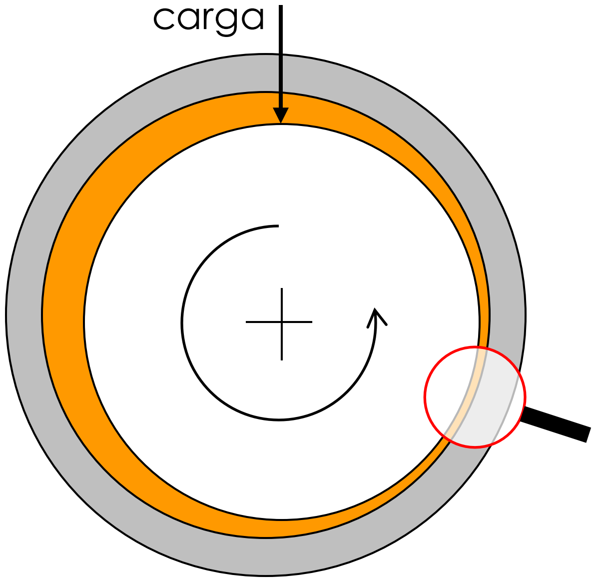

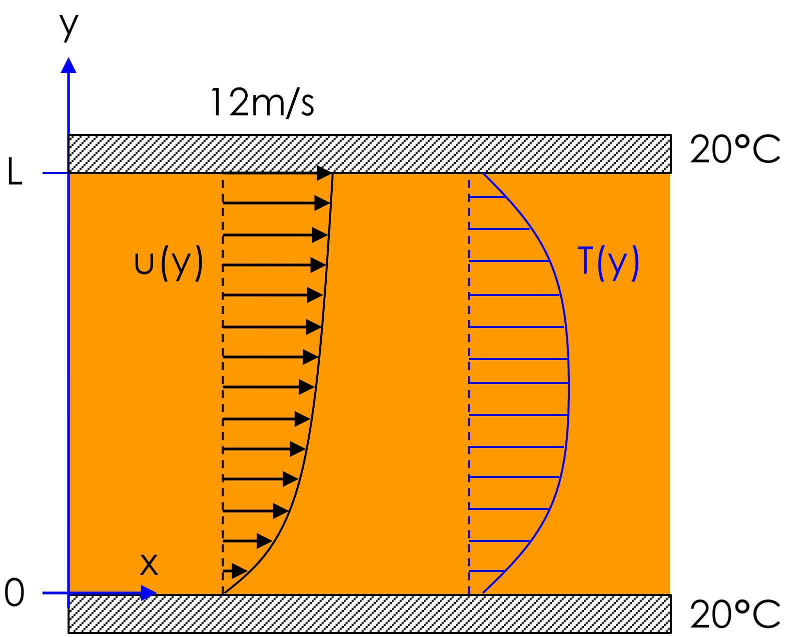

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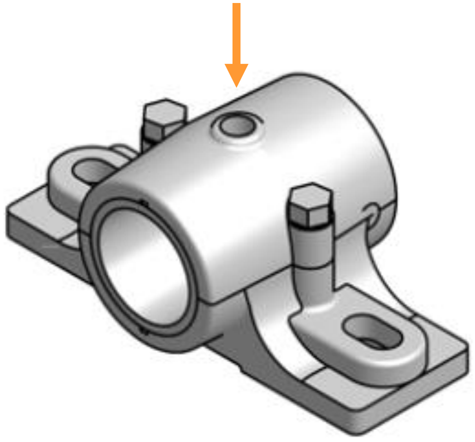
propriedades do óleo @ 20°C



JOURNAL BEARING ASSEMBLED
SCALE 1 / 2

$$\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

PARTS LIST			
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Balanço de massa (continuidade):

$$\vec{\nabla} \cdot \vec{U} = 0 \rightarrow \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \left(u, v, w \right) = 0$$

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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \rightarrow \frac{\partial u}{\partial x} = 0 \rightarrow u = u(y)$$

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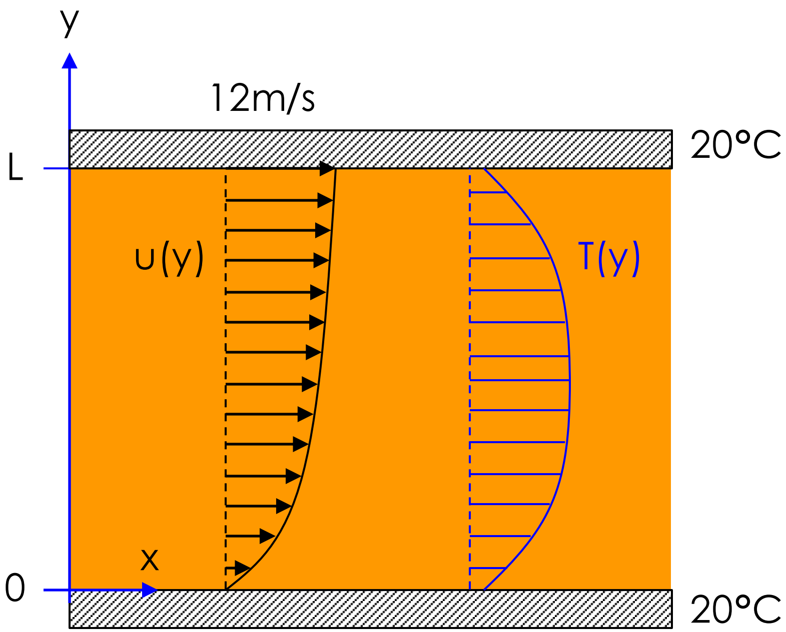
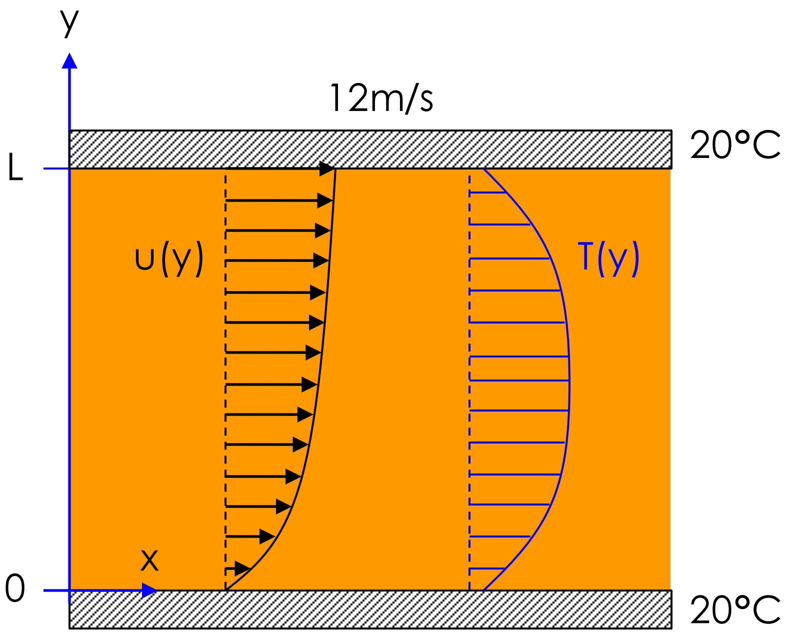
$$\rho \cdot \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \rightarrow \frac{d^2 u}{dy^2} = 0$$

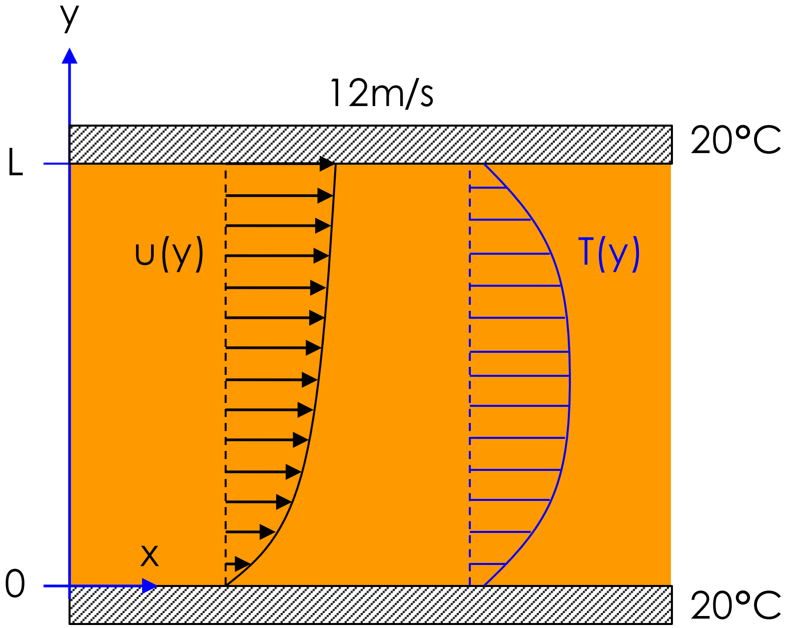


$$\frac{d^2u}{dy^2} = 0 \rightarrow u(y) = C_1 \cdot y + C_2$$



$$\frac{d^2u}{dy^2} = 0 \rightarrow u(y) = C_1 \cdot y + C_2$$

$$u(0) = 0 \text{ e } u(L) = \forall \rightarrow \qquad u(y) = \frac{\forall}{L} \cdot y$$



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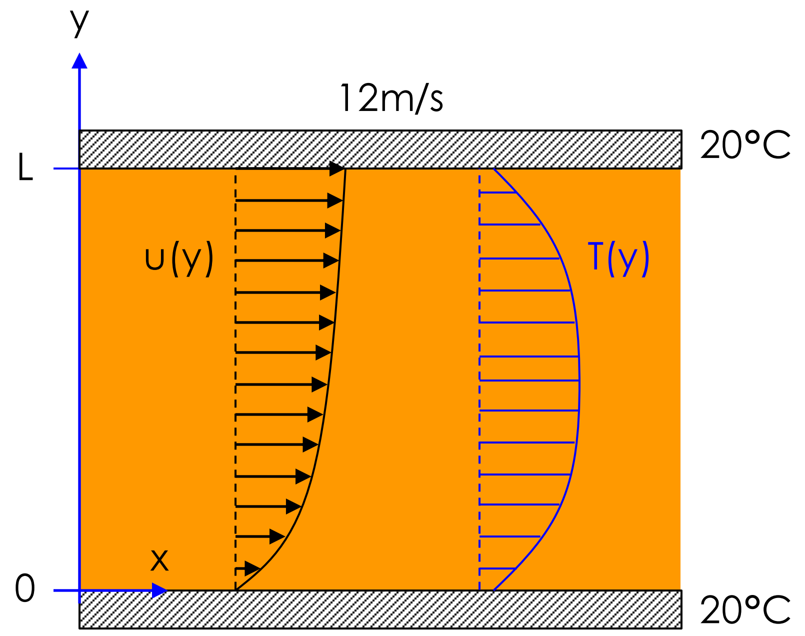
$$\rho C_P \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tilde{T} : \tilde{D}$$



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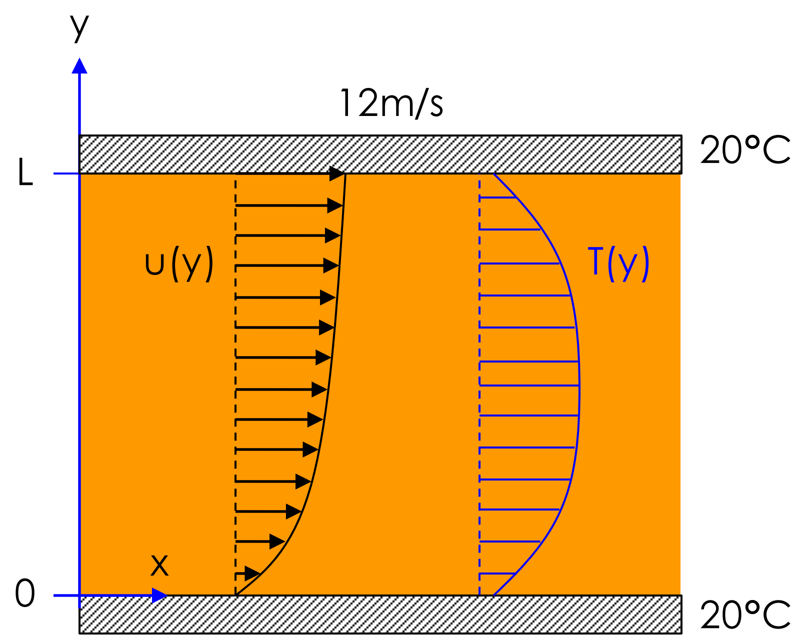
$$k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 = 0$$



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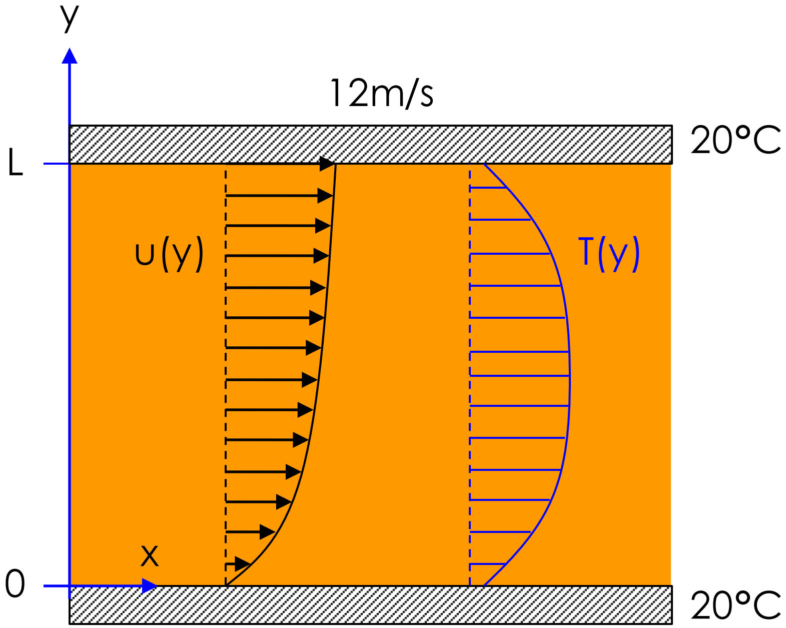

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Balanço de energia:

$$\rho C_P \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tilde{T} : \tilde{D}$$

$$k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 = 0 \rightarrow k \frac{\partial^2 T}{\partial y^2} = -\mu \left(\frac{\forall}{L} \right)^2$$

$$k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2=0 \rightarrow k\frac{\partial^2 T}{\partial y^2}=-\mu\left(\frac{V}{L}\right)^2$$

→ parabólico em y

$$k\frac{\partial^2 T}{\partial x^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 = 0 \rightarrow k\frac{\partial^2 T}{\partial x^2} = -\mu\left(\frac{\forall}{L}\right)^2$$ → parabólico em y

$$T(0) = T(L) = T_0 \rightarrow \qquad T(y) = T_0 + \frac{\mu\forall^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

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$$y=L/2\rightarrow \qquad T_{max}=T_0+\frac{\mu\forall^2}{2k}\left(\frac{L/2}{L}-\frac{(L/2)^2}{L^2}\right)=T_0+\frac{\mu\forall^2}{8k}$$

$$k\frac{\partial^2 T}{\partial x^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 = 0 \rightarrow k\frac{\partial^2 T}{\partial x^2} = -\mu\left(\frac{\mathcal{V}}{L}\right)^2 \quad \rightarrow \text{parabólico em y}$$

$$T(0) = T(L) = T_0 \rightarrow \qquad T(y) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

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$$T_{max} = 20 + \frac{(0.8\,Ns/m^2)(12\,m/s)^2}{8(0.145\,W/m/{}^\circ C)} = 119^\circ C$$

$$T(y) = T_0 + \frac{\mu \forall^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

$$q_0 = -k\frac{dT}{dy}\bigg|_{y=0} = ... = -\frac{\mu \forall^2}{2L}$$

$$q_0 = -\frac{(0.8 Ns/m^2)(12 m/s)^2}{2(0.002 m)} = -28.800 kW/m^2$$

Observação: as propriedades termofísicas foram avaliadas @ 20°C... T_m = (119+20)/2= 69.5 °C...