Eletromagnetismo I

Prof. Luis Abramo - 2° Semestre 2015

Preparo: Diego Oliveira

Aula 9

Aplicações da Lei de Ampère e Biot-Savart

Magnetostática:

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

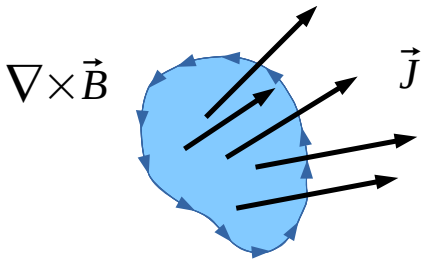
Lei de Biot-Savart:

$$\vec{B} = \frac{\mu_0}{4\pi} \int dV' \, \vec{J}(\vec{r}\,') \times \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int dV' \, I \, d\vec{\ell}\,' \times \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3}$$

Teorema de Stokes aplicado à lei de Ampère

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad \Rightarrow \quad \int d\vec{S} \cdot \nabla \times \vec{B} = \mu_0 \underbrace{\int d\vec{S} \cdot \vec{J}}_{\substack{\text{corrente que} \\ \text{passa pela área}}}$$

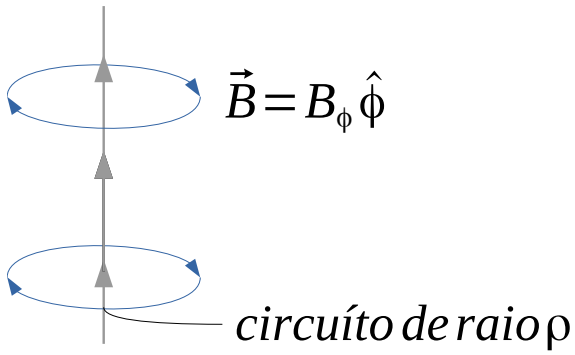


$$\boxed{\oint d\vec{\ell} \cdot \vec{B} = \mu_0 I \, (\text{Área})}$$ Lei de Ampère Integral

Dependendo da circunstância, usamos uma ou outra dessas leis! Ou mesmo:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int dV' \, \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \quad \longrightarrow \quad \vec{B} = \nabla \times \vec{A}$$

Ex 1: Fio infinito: Nesse caso, é possível calcular $\vec{B}$ tanto por Biot-Savart, quanto pela Lei de Ampère. Mas é muito mas fácil (devido à simetria) usar a lei de Ampère:

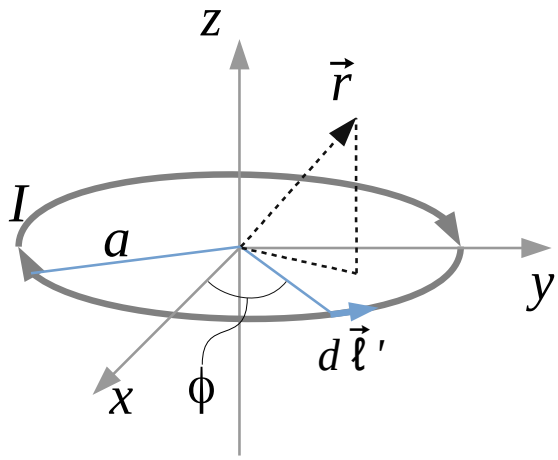


$$d\vec{\ell} = \rho\, d\phi \hat{e}_\phi$$

$$\oint d\vec{\ell} \cdot \vec{B} = \int_0^{2\pi} \left(d\phi \rho \hat{e}_\phi\right) \cdot \vec{B} = \int_0^{2\pi} \left(d\phi \rho \hat{e}_\phi\right) \cdot \left(B_\rho \hat{e}_\rho + B_\phi \hat{e}_\phi\right) = 2\pi\rho B = \mu_0 I$$

$$\therefore \boxed{\vec{B} = \frac{\mu_0 I}{2\pi\rho} \hat{e}_\phi}$$

Ex 2: Laço circular: Nesse caso não há simetria suficiente para utilizar a Lei de Ampère diretamente, então recorremos a Biot-Savart.



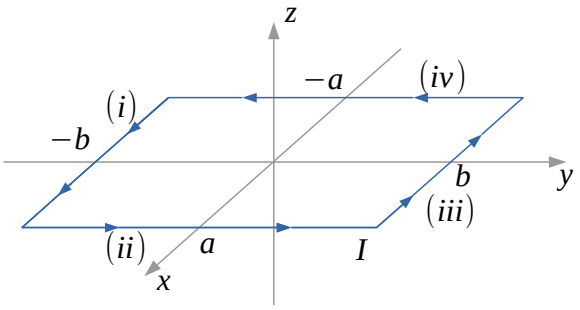
$$d\vec{\ell}' = a d\phi' \hat{e}'_\phi,$$

$$\left.\begin{array}{l} \vec{r} = z\hat{e}_z \\ \\ \vec{r}' = a\hat{e}_\rho \end{array}\right\} \Rightarrow \begin{array}{c} \vec{R} \equiv \vec{r} - \vec{r}' = -a\hat{e}_\rho + z\hat{e}_z, \\ |\vec{R}| = \sqrt{a^2 + z^2} \end{array}$$

$$\begin{aligned} \vec{B} &= \frac{\mu_0 I}{4\pi} \int_0^{2\pi} d\phi'\, a\hat{e}_\phi \times \frac{-a\hat{e}_\rho + z\hat{e}_z}{(a^2+z^2)^(3/2)} \\ &= \frac{\mu_0 I}{4\pi} \int_0^{2\pi} d\phi' \left[a\hat{e}_\phi \times \hat{e}_\rho + z\hat{e}_\phi \times \hat{e}_z \right] \\ &= \frac{\mu_0 I}{4\pi} \frac{a}{(a^2+b^2)^(3/2)} \left[2\pi a \hat{e}_z + \vec{0}\right] \end{aligned}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2} \frac{a^2}{(a^2+z^2)^{3/2}} \hat{e}_z}$$

Ex 3: Laço Retangular



$d\vec{\ell}$:

$$
\begin{array}{llll}
i) & y=-b, & d\vec{\ell}=\hat{e}_x dx & (-a\le x\le a)\\
ii) & y=a, & d\vec{\ell}=\hat{e}_y dy & (-b\le y\le b)\\
iii) & y=-b, & d\vec{\ell}=-\hat{e}_x dx & (-a\le x\le a)\\
iv) & y=-a, & d\vec{\ell}=-\hat{e}_y dy & (-b\le y\le b)
\end{array}
$$

Atenção aos sinais! Em (iii), 0 $''dx''$ é negativo por que x vai de a a $-a$, isto é,

$$\int_{(iii)} dx(\ldots) = -\int_{-a}^{a} dx(\ldots)$$

Definindo $\vec{R}=\vec{r}-\vec{r}'=(-x')\hat{e}_x+(-y')\vec{y}+z\hat{e}_z$, encontramos que:

$$
\begin{aligned}
\vec{B}=\frac{\mu_0 I}{4\pi}\Bigg[&\int_{-a}^{a} dx'\hat{e}_x \frac{-x'\hat{e}_x-y'\hat{e}_y+z\hat{e}_z}{R^3}\Bigg|_{y=-b} + \int_{-b}^{b} dy'\hat{e}_y \frac{-x'\hat{e}_x-y'\hat{e}_y+z\hat{e}_z}{R^3}\Bigg|_{x=a}\\
&-\int_{-a}^{a} dx'\hat{e}_x \frac{-x'\hat{e}_x-y'\hat{e}_y+z\hat{e}_z}{R^3}\Bigg|_{y=b} - \int_{-b}^{b} dy'\hat{e}_y \frac{-x'\hat{e}_x-y'\hat{e}_y+z\hat{e}_z}{R^3}\Bigg|_{x=-a}\Bigg]
\end{aligned}
$$

Os termos $\hat{e}_x\times\hat{e}_z=-\hat{e}_y$ e $\hat{e}_y\times\hat{e}_z=\hat{e}_x$ se cancelam (verifiquem!), e temos então:

$$\vec{B}=\frac{\mu_0 I}{4\pi}\left[2\int_{-a}^{a}\frac{b\hat{e}_z}{(x'^2+b^2+z^2)^{3/2}}+2\int_{-b}^{b}dy'\frac{a\hat{e}_z}{(a^2+y'^2+z^2)^{3/2}}\right]$$

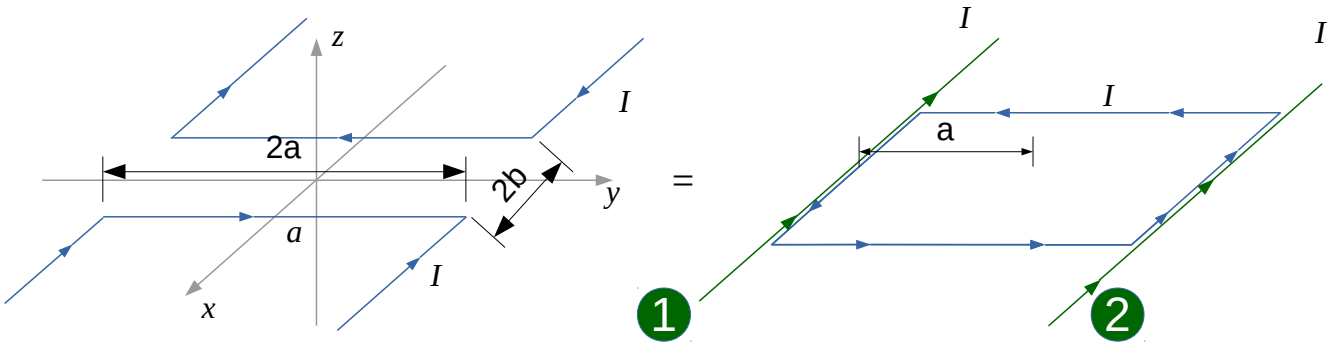
Agora, note que

$$\int_{-a}^{a} dx'\frac{1}{(x'^2+c^2)^{3/2}}=\frac{2a}{c^2(a^2+c^2)^{1/2}}$$

$$\vec{B}=\frac{\mu_0 I}{2\pi}\hat{e}_z\left[\frac{2ba}{(b^2+z^2)\sqrt{a^2+b^2+z^2}}+\frac{2ab}{(a^2+z^2)\sqrt{a^2+b^2+z^2}}\right]$$

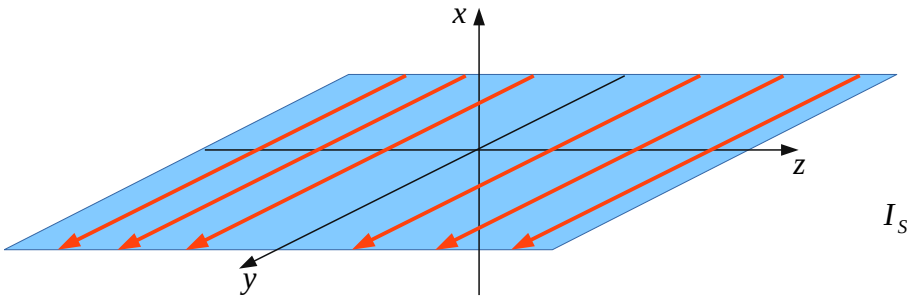
$$\boxed{\vec{B}=\frac{\mu_0 I}{\pi}\frac{ab}{\sqrt{a^2+b^2+z^2}}\left(\frac{1}{a^2+z^2}+\frac{1}{b^2+z^2}\right)\hat{e}_z}$$

Ex 4:



Note que, em $x = y = 0$, e para qualquer z, os campos magnéticos dos fios ① e ② se somam
Por tanto em $x = y = 0$, temos

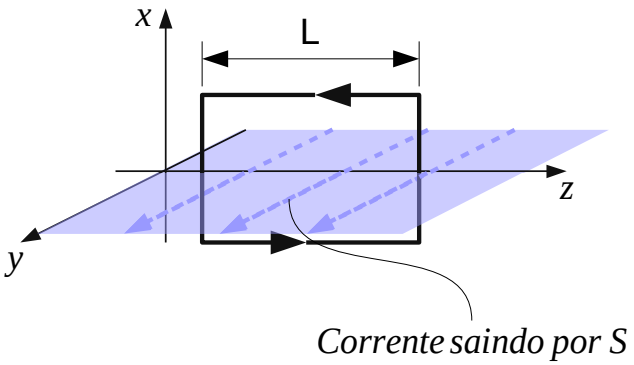
Ex 5: Plano infinito, com corrente superficial $\vec{K} = K_0 \hat{e}_y$, com $K_0 =$ constante.



$$\vec{J} = K_0 \delta(x)\, \hat{e}_y$$

$$I_S = \int d\vec{S} \cdot \vec{J} \begin{cases} 0 & se\, \vec{S} \perp \vec{J} \\ K_0 L & se\, \vec{S} \,//\, \vec{J} \end{cases}$$

Poderíamos utilizar Biot-Savart com $\vec{J} = K_0 \delta(x) \hat{e}_y$, mas nesse caso é muito mais simples usar a Lei de Ampère, pois $\vec{B} = B_y \hat{e}_y$ (simetria).

$$-2B_z L = \mu_0 I_S = \mu_0 K_0 L$$

$$\Rightarrow B_z = \begin{cases} -\mu_0 K_0/2. & x > 0 \\ \mu_0 K_0/2. & x < 0 \end{cases}$$