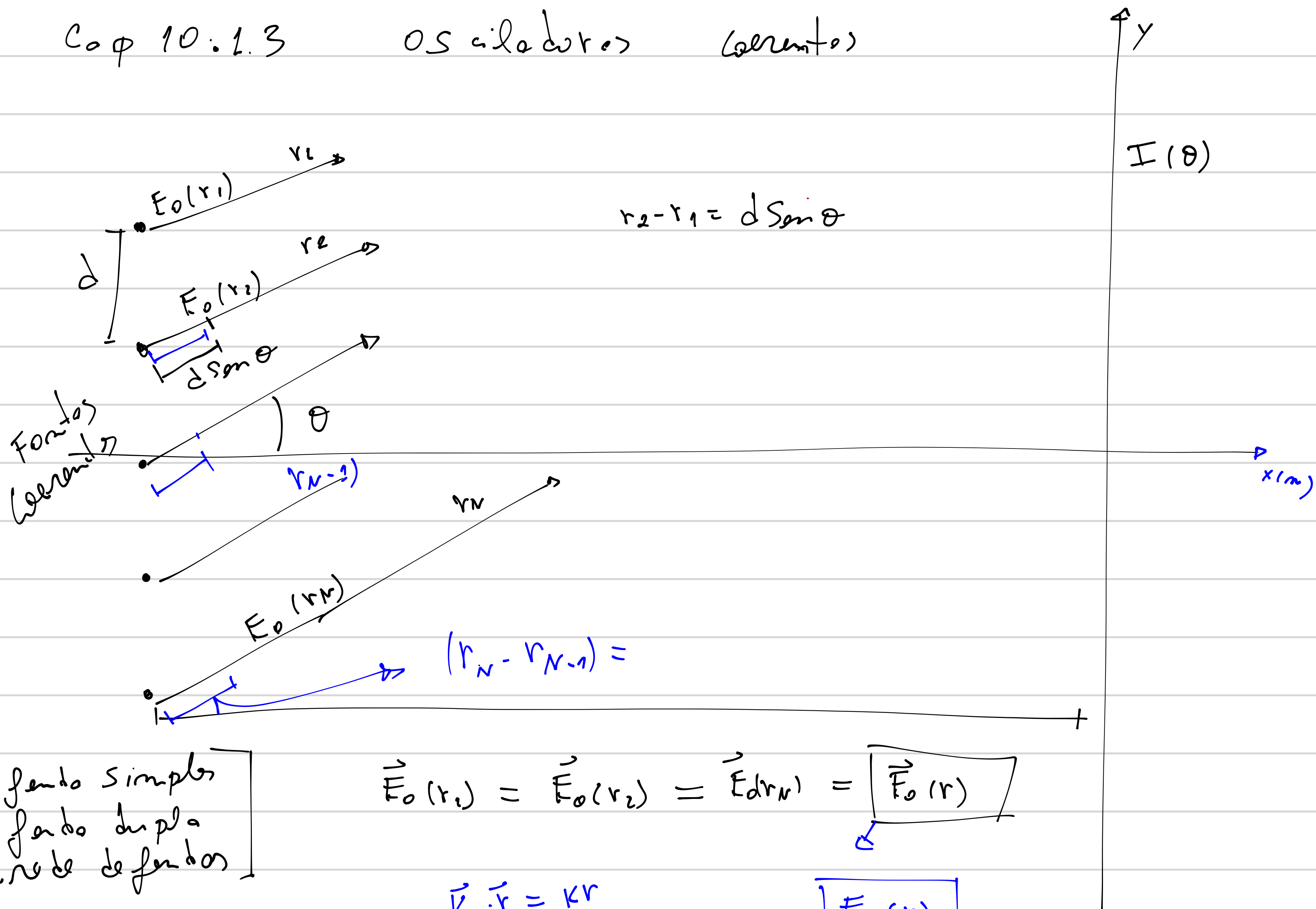
Cap 10.1.3 Osciladores coerentes



$r_2 - r_1 = d\,\text{sen}\,\theta$

fonte simples
fonte dupla
rede de fontes

$\vec{E}_0(r_1) = \vec{E}_0(r_2) = \vec{E}_0(r_N) = \boxed{\vec{E}_0(r)}$

$\vec{k}\cdot\vec{r} = kr$ $\qquad \boxed{E_0(r)}$

E_T no anteparo

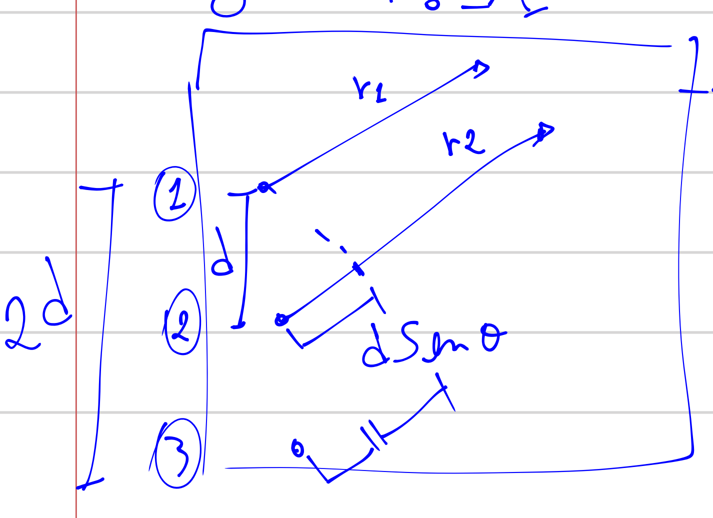
$$E = \underbrace{E_0(r)}\,\underbrace{e^{i(\vec{k}\cdot\vec{r}_1 - \omega t)}}_{\text{oscilador 1}} + \underbrace{E_0(r)}\,e^{i(\vec{k}\cdot\vec{r}_2 - \omega t)} + \dots \underbrace{E_0(r)}\,e^{i(\vec{k}\cdot\vec{r}_N - \omega t)}$$

→ amplitude das N fontes são aproximadamente iguais

$$E = E_0(r)\,e^{-i\omega t}\cdot e^{ikr_1}\left[1 + \frac{e^{ikr_2}}{e^{ikr_1}} + \dots + \frac{e^{ikr_N}}{e^{ikr_1}}\right]$$

$$\left[1 + e^{ik(r_2 - r_1)} + \dots + e^{ik(r_N - r_1)}\right]$$

$$\delta = k_0 \Lambda$$



$$\Lambda = n\,.\,d\,\text{sen}\,\theta$$

$$\delta = k_0 n d\,\text{sen}\,\theta = \frac{2\pi}{\frac{\lambda_0}{n}}\,d\,\text{sen}\,\theta$$

$$\delta = \frac{2\pi}{\lambda}\,d\,\text{sen}\,\theta = k\,d\,\text{sen}\,\theta$$

$$\delta_1 = K(r_2 - r_1) = \delta = K d \, \mathrm{Sen}\,\theta$$

$$\delta_2 = K(r_3 - r_1) = K(2d)\, \mathrm{Sen}\,\theta = 2\delta$$

$$\delta_N = K(r_N - r_1) = K(Nd)\, \mathrm{Sen}\,\theta = N\delta$$

$$E = E_0(r)\, e^{-i\omega t} \cdot e^{iKr_1} \left[1 + e^{i\delta} + e^{i2\delta} + \dots + e^{iN\delta} \right]$$

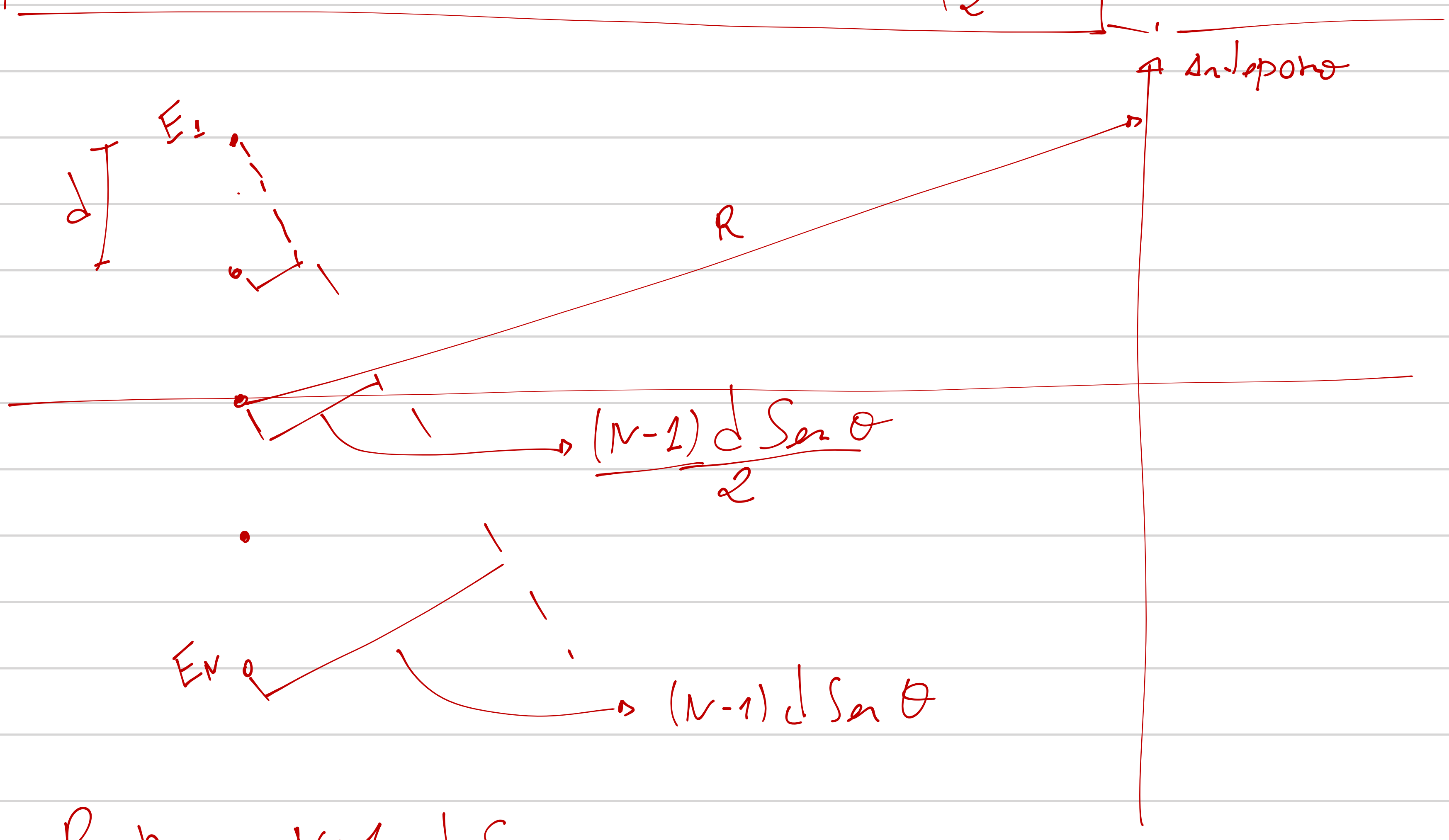
$$1 + (e^{i\delta})^1 + (e^{i\delta})^2 + \dots + (e^{i\delta})^N = \left[\frac{e^{iN\delta} - 1}{e^{i\delta} - 1} \right]$$

$$\boxed{\mathrm{Sen}\, a = \frac{e^{ia} - e^{-ia}}{2i}}$$

$$\Downarrow$$

$$\left[e^{i\frac{\delta}{2}(N-1)} \right] \cdot \frac{\mathrm{Sen}\left(\frac{N\delta}{2}\right)}{\mathrm{Sen}\left(\frac{\delta}{2}\right)}$$

$$\boxed{E = E_0(r)\, e^{-i\omega t}\, e^{iKr_1}\, e^{i\frac{\delta}{2}(N-1)} \cdot \frac{\mathrm{Sen}\left(\frac{N\delta}{2}\right)}{\mathrm{Sen}\left(\frac{\delta}{2}\right)}}$$



$$R - r_1 = \frac{N-1}{2}\, d \, \mathrm{Sen}\,\theta$$

$$R = \left(\frac{N-1}{2}\right) d \, \mathrm{Sen}\,\theta + r_1$$

$$\delta = K d \, \mathrm{Sen}\,\theta$$

$$E = E_0(r)\, e^{-i\omega t} \cdot \underbrace{e^{ikr_1} \cdot e^{i\frac{kd\,\mathrm{sen}\,\theta}{2}\cdot(N-1)}}_{e^{ik\left(r_1 + \frac{N-1}{2} d\,\mathrm{sen}\,\theta\right)} \;=\; e^{ikR}} \cdot \left[\frac{\mathrm{sen}\left(\frac{N\delta}{2}\right)}{\mathrm{sen}\left(\frac{\delta}{2}\right)}\right]$$

$$\boxed{E = E_0(r)\, e^{i(kR-\omega t)} \cdot \frac{\mathrm{sen}\left(\frac{N\delta}{2}\right)}{\mathrm{sen}\left(\frac{\delta}{2}\right)}}$$

$$I \propto \frac{1}{2} E E^*$$

$$\boxed{I = I_0 \frac{\mathrm{sen}^2\left(\frac{N\delta}{2}\right)}{\mathrm{sen}^2\left(\frac{\delta}{2}\right)}}$$

(perfil) Intensidade no anteparo para diferentes número de fontes

Exemplo $N=2$ (fenda dupla, mas muito estreita)

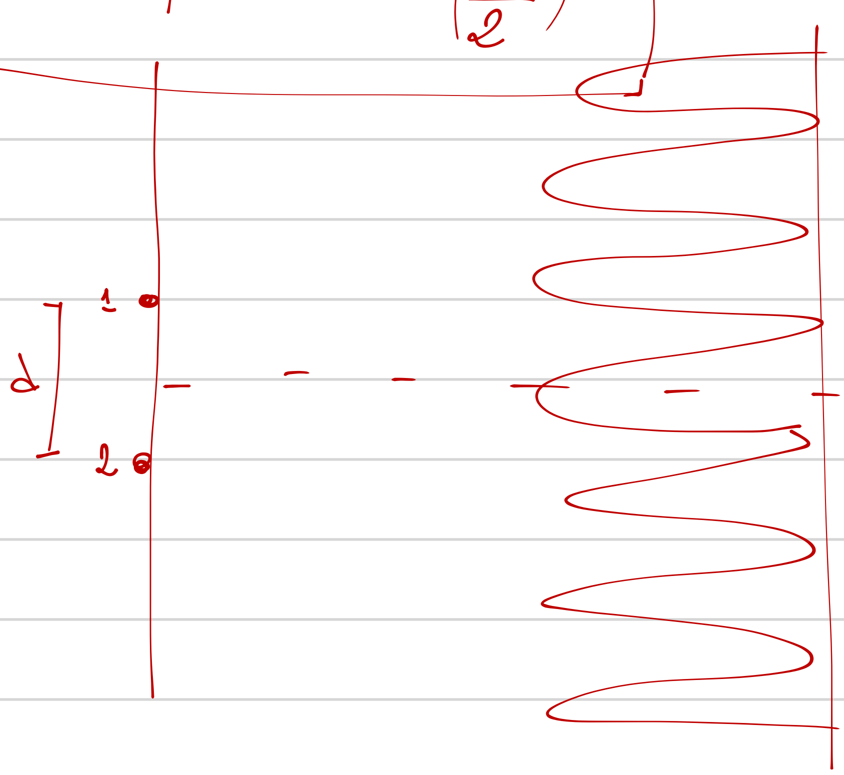
$$\mathrm{sen}\, 2x = 2\, \mathrm{sen}\, x \cdot \cos x$$

$$I = I_0 \frac{\mathrm{sen}^2\left[2\cdot\frac{\delta}{2}\right]}{\mathrm{sen}^2\left(\frac{\delta}{2}\right)} = \frac{\left[2\cdot \cancel{\mathrm{sen}\left(\frac{\delta}{2}\right)} \cdot \cos\left(\frac{\delta}{2}\right)\right]^2}{\cancel{\mathrm{sen}^2\left(\frac{\delta}{2}\right)}}$$

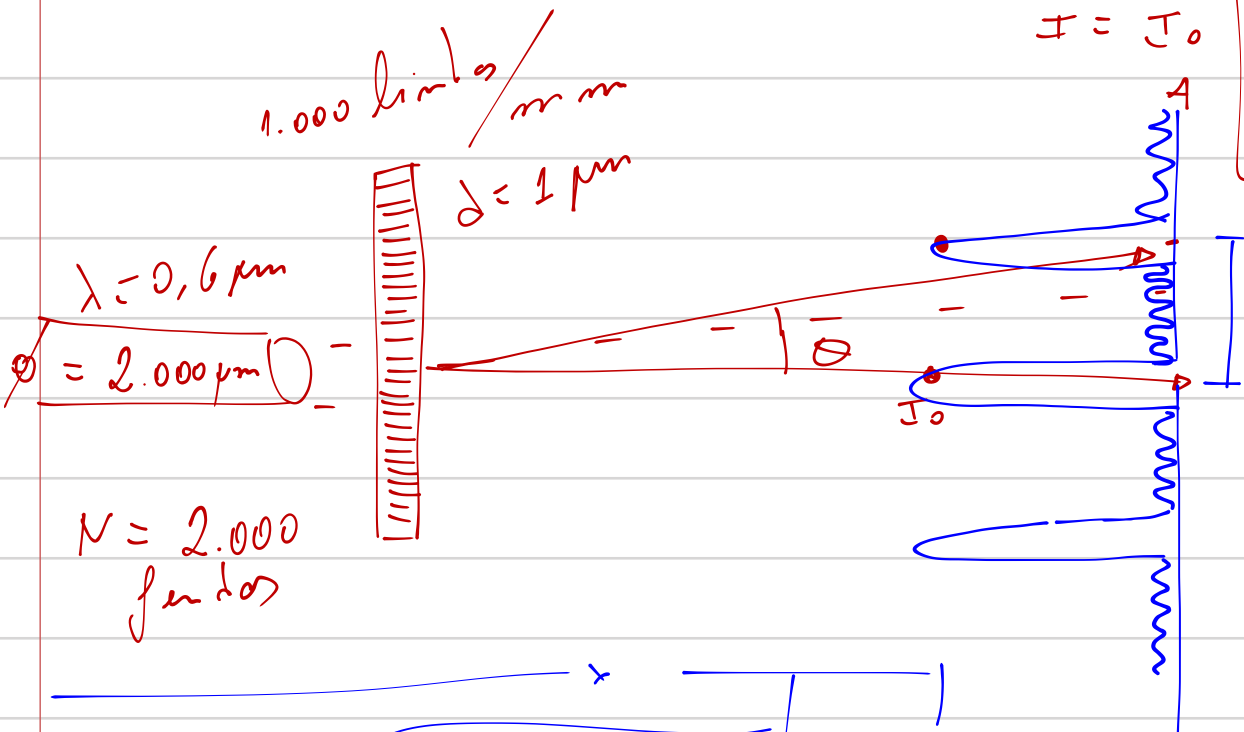
$$\boxed{I = 4 I_0 \cos^2\left(\frac{\delta}{2}\right)}$$

$\delta = k d\, \mathrm{sen}\,\theta$

$d =$ separação entre as fendas



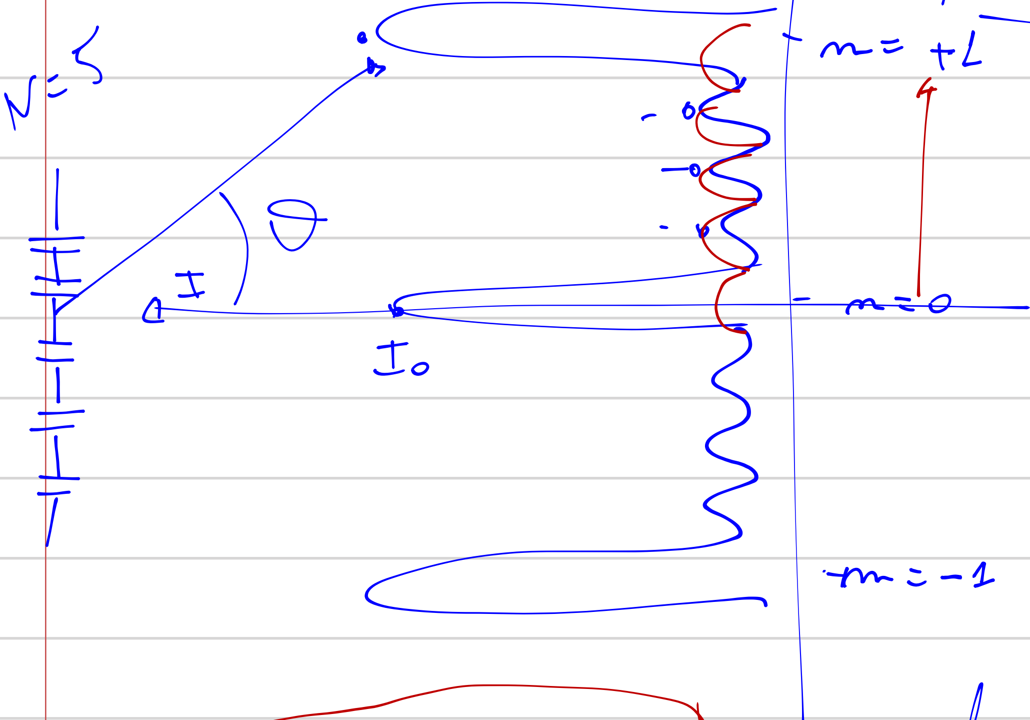
Exemplo: Rede de fendas ou de Difração



$$I = I_0 \left[\frac{\sin^2\left(2000\,\frac{\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)} \right]$$

$\delta = k d \sin\theta$

$(0 - 2\pi)$



$d \sin\theta = m\lambda$

$(N - 2) =$ max locais $= 3$

$N = 3 + 2$

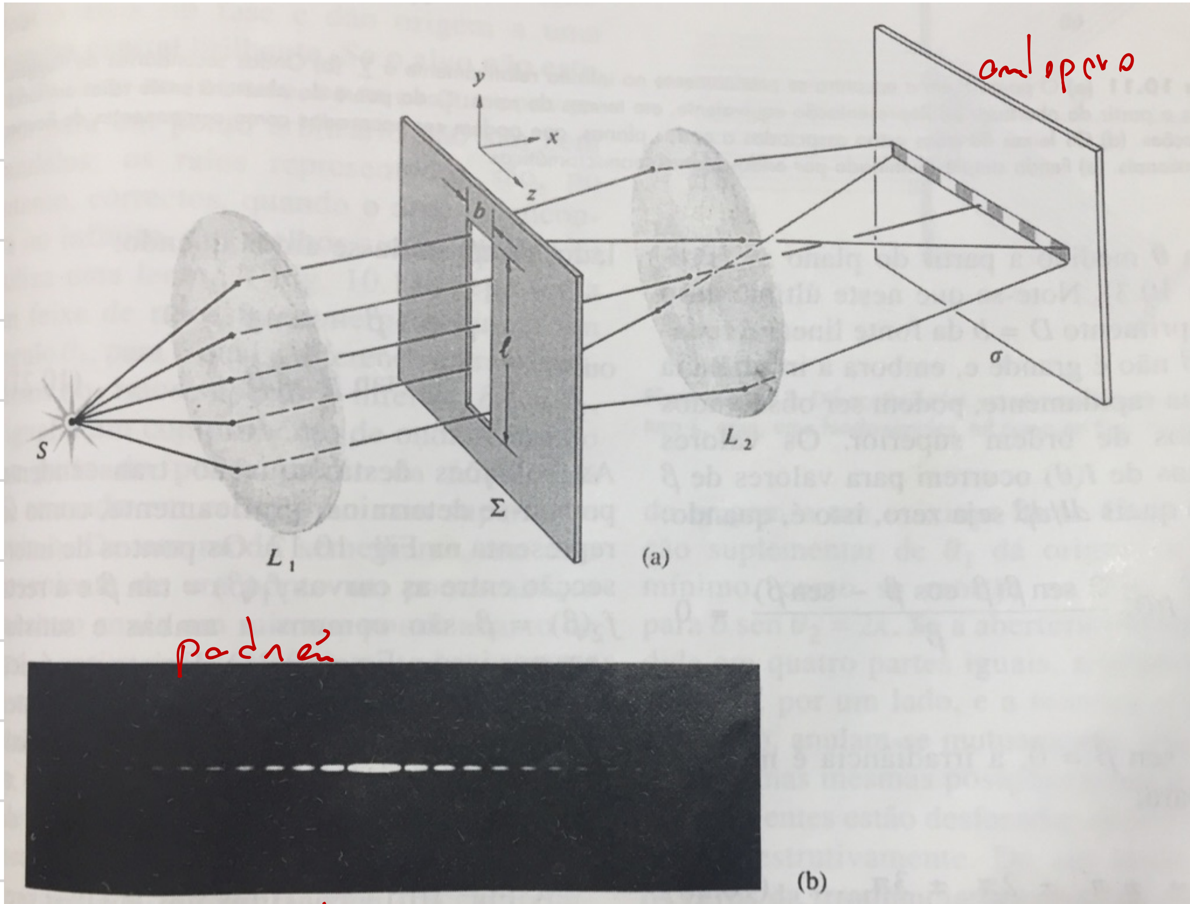
$\boxed{N = 5 \text{ fendas}}$

$$I = I_0 \frac{\sin^2\left(N\,\frac{\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)} \rightarrow 5 \text{ ciclos}$$

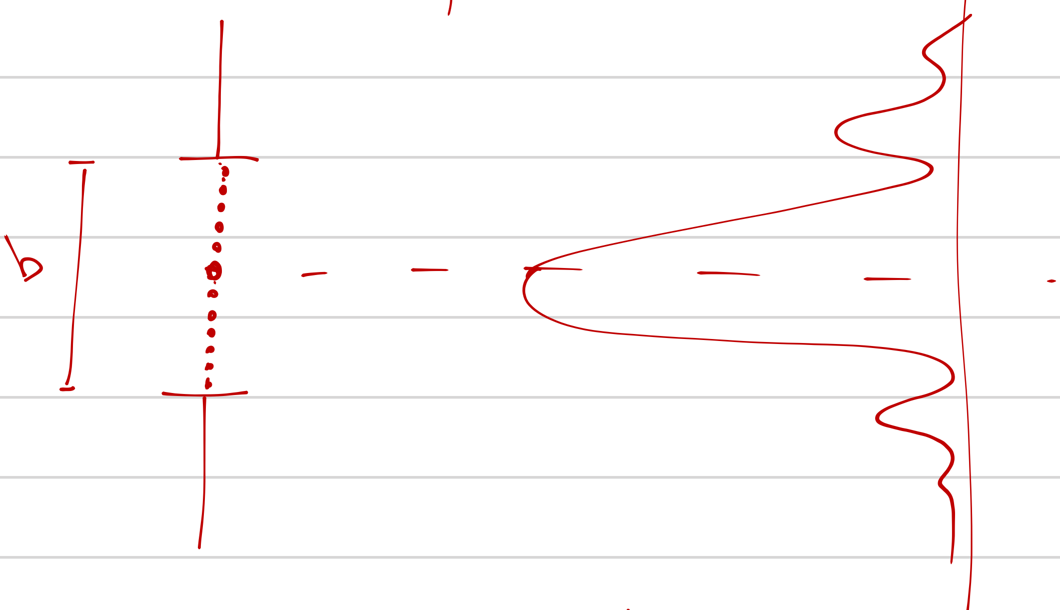
$0 - 2\pi$ 1 ciclo

$\delta = k d \sin\theta$

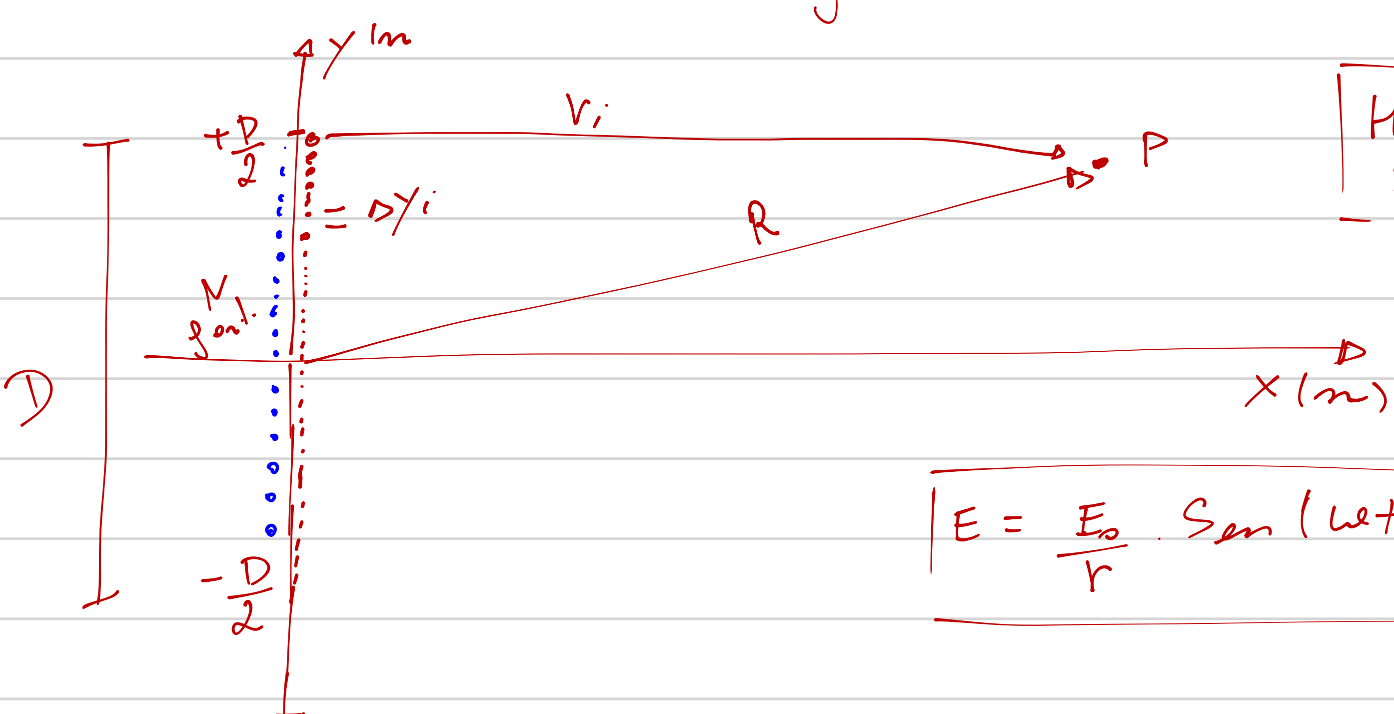
Para uma fenda apenas



Vista superior



Considere uma fonte linear de osciladores



$\boxed{\text{Há } N \text{ fontes sobre } D}$

$$\boxed{E = \frac{E_0}{r} \cdot \operatorname{Sen}(\omega t - kr)}$$

E_i = de um segmento de fontes Δy_i

$$E_i = \frac{\mathcal{E}_0}{r_i} \operatorname{Sen}(\omega t - kr) \cdot \left[\Delta y_i \left(\frac{N}{D}\right)\right]$$

Exemplo $E_D = \left[\frac{\varepsilon_0}{r_i}\sin(\omega t - kr)\right]\left[\frac{\cancel{D}\cdot N}{\cancel{D}}\right]$

Se somarmos sobre uma região do interior

$$E = \sum_{i=1}^{M \text{ pontos}} \frac{\varepsilon_0 N}{D r_i}\sin(\omega t - kr)\,\Delta y_i$$

→ E_L = Intensidade da fonte por unidade de comprimento

$$\boxed{E = E_L \int_{-D/2}^{+D/2} \frac{\sin(\omega t - kr)}{r}\,dy}$$

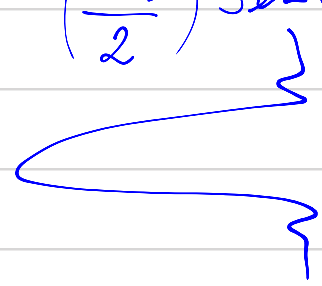
Considere $r = \underline{R - y\sin\theta} + \cancel{\left(\frac{y^2}{2R}\right)\cos^2\theta} + \dots$

$$E = \frac{E_L}{R}\int_{-D/2}^{+D/2}\sin[\omega t - k(R - y\sin\theta)]\,dy$$

$$E = \frac{E_L D}{R}\,\frac{\sin[(kD/2)\sin\theta]}{(kD/2)\sin\theta}\cdot\sin(\omega t - kR)$$

D = largura da fenda

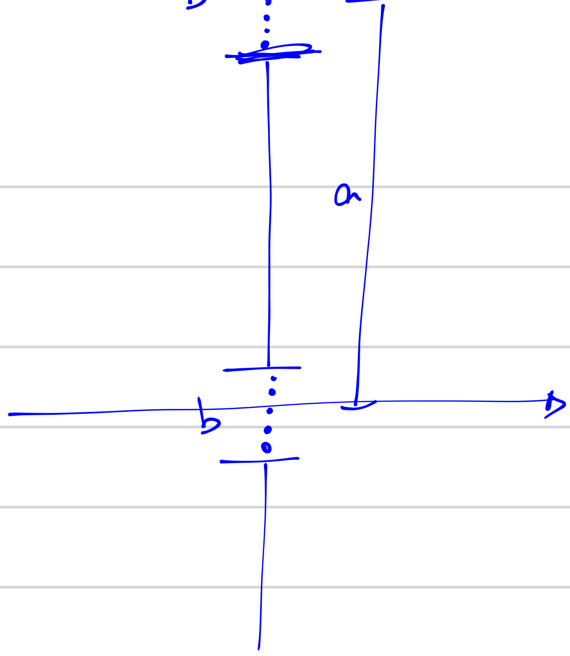
$\beta = \left(\frac{kD}{2}\right)\sin\theta$



$$\boxed{I = \left(\frac{E_L D}{R}\right)^2\frac{\sin^2\beta}{\beta^2}\cdot\frac{1}{2}}$$

→ Este é o perfil da fenda simples

Tarefa: Deduza a expressão da Intensidade no anteparo p/ uma fenda dupla



$$\boxed{I = 4 I_0 \cos^2\alpha \cdot \frac{\sin^2\beta}{\beta}}$$

$$\alpha = \frac{ka}{\lambda}\sin\theta$$

$$\beta = \frac{kb}{\lambda}\sin\theta$$

$$I \longrightarrow E = \int_{-b/2}^{b/2} [\quad] \, dy + \int_{-\frac{b}{2}+a}^{+\frac{b}{2}+a} [\quad] \, dy$$