Cap. 7. Sobreposição de ondas

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

$\psi_1, \psi_2 \rightarrow$ soluções

$\boxed{\psi_R = \psi_1 + \psi_2}$ $\rightarrow$ também é solução

$$\boxed{\psi_R = \sum_{i=1}^{N} \psi_i}$$

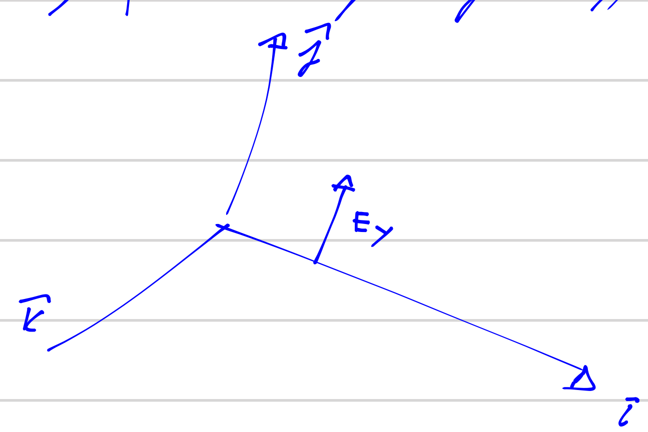
$\vec{E}(\vec{r},t) = E_x\,\hat{i} + E_y\,\hat{j} + E_z\,\hat{k}$

caso particular (E escalar)

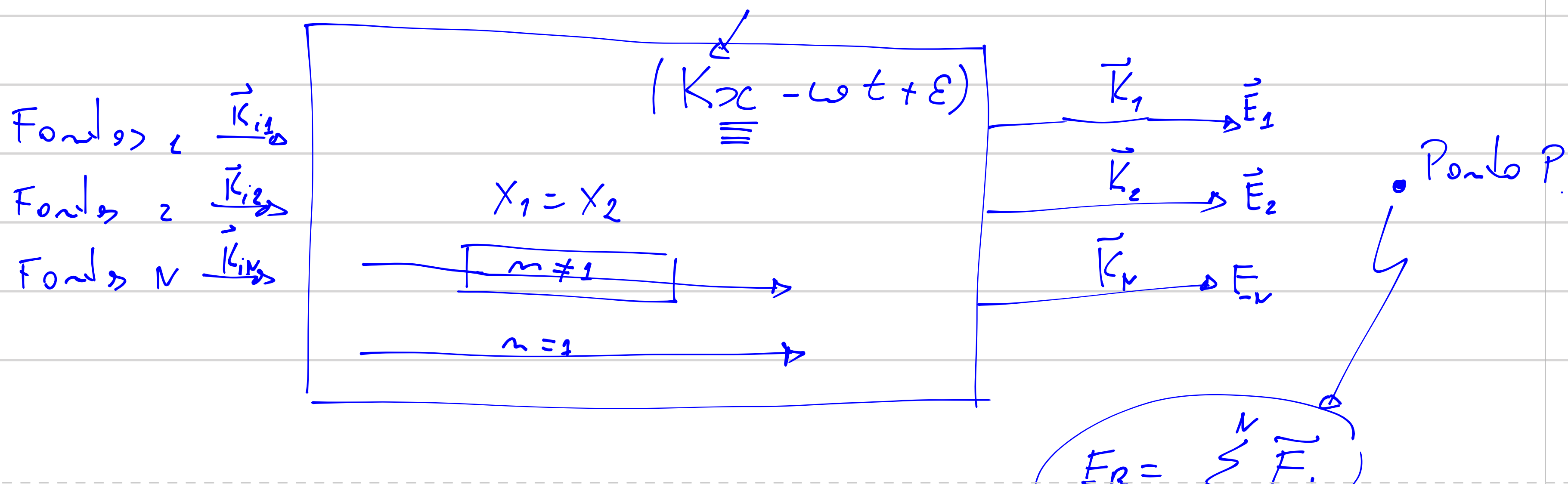
$E \Rightarrow$ uma única direção, por exemplo $\hat{j}$ (E_y)

se propagando em $\hat{i}$

$\boxed{E = E(x,t)}$



Polarização
Difração
Interferência



$i=1$

E_1 e E_2 $\qquad E(x,t) = E_0 \operatorname{Sen}(\omega t \underbrace{-kx-\varepsilon})$

$$E(x,t) = E_0 \operatorname{Sen}(\omega t + \alpha)$$

$$\begin{cases} E_1 = E_{01} \operatorname{Sen}(\omega t + \alpha_1) \\ E_2 = E_{02} \operatorname{Sen}(\omega t + \alpha_2) \end{cases}$$

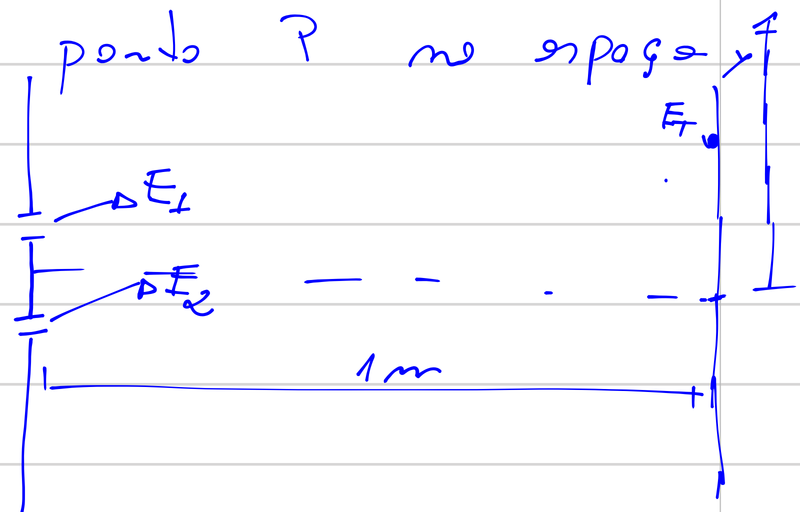
$\omega_1 = \omega_2 = \omega \Rightarrow$ é a mesma fonte $\Rightarrow$ Experimento com frequência única

$\alpha_1 \Rightarrow K_1, X_1, \varepsilon_1$

$\alpha_2 \Rightarrow K_2, X_2, \varepsilon_2$

E_T = Campo total em algum ponto P no espaço

$$E_T = E_1 + E_2$$



$\operatorname{Sen}(a+b) = \operatorname{Sen} a \operatorname{Cos} b + \operatorname{Cos} a \operatorname{Sen} b$

$E_T = E_{01} \operatorname{Sen}(\omega t + \alpha_1) + E_{02} \operatorname{Sen}(\omega t + \alpha_2)$

$= E_{01}[\operatorname{Sen}\omega t \cdot \operatorname{Cos}\alpha_1 + \operatorname{Cos}\omega t \cdot \operatorname{Sen}\alpha_1] +$

$+ E_{02}[\operatorname{Sen}\omega t \cdot \operatorname{Cos}\alpha_2 + \operatorname{Cos}\omega t \operatorname{Sen}\alpha_2]$

$$E_T = \operatorname{Sen}\omega t \underbrace{[E_{01}\operatorname{Cos}\alpha_1 + E_{02}\operatorname{Cos}\alpha_2]}_{E_0 \operatorname{Cos}\alpha} + \operatorname{Cos}\omega t \underbrace{[E_{01}\operatorname{Sen}\alpha_1 + E_{02}\operatorname{Sen}\alpha_2]}_{E_0 \operatorname{Sen}\alpha} =$$

$$\boxed{E_T = E_0 \operatorname{Cos}\alpha \cdot \operatorname{Sen}\omega t + E_0 \operatorname{Sen}\alpha \cdot \operatorname{Cos}\omega t}$$

$\operatorname{Sen}(a+b) = \operatorname{Sen} a \operatorname{Cos} b + \operatorname{Cos} a \operatorname{Sen} b$

$b = \alpha \qquad a = \omega t$

$$\boxed{E_T = E_0 \operatorname{Sen}(\omega t + \alpha)}$$

$$\div \quad \frac{E_0 \operatorname{Sen}\alpha}{E_0 \operatorname{Cos}\alpha} = \boxed{\operatorname{tg}\alpha = \frac{E_{01}\operatorname{Sen}\alpha_1 + E_{02}\operatorname{Sen}\alpha_2}{E_{01}\operatorname{Cos}\alpha_1 + E_{02}\operatorname{Cos}\alpha_2}}$$

ainda: $(E_0 \operatorname{Cos}\alpha)^2 = (E_{01}\operatorname{Cos}\alpha_1 + E_{02}\operatorname{Cos}\alpha_2)^2$

$(E_0 \operatorname{Sen}\alpha)^2 = (E_{01}\operatorname{Sen}\alpha_1 + E_{02}\operatorname{Sen}\alpha_2)^2$

$$\operatorname{Cos}(a-b) = \operatorname{Cos} a \operatorname{Cos} b + \operatorname{Sen} a \operatorname{Sen} b$$

$$\boxed{E_0^2 = E_{01}^2 + E_{02}^2 + \underbrace{2E_{01}E_{02}\cos(\alpha_1 - \alpha_2)}_{}}$$

→ termo de interferência

$$\boxed{\delta \equiv \alpha_1 - \alpha_2}$$

$\delta = 0, \pm 2\pi, \pm 4\pi,$ etc — amplitude máxima

$\delta = \pm\pi; \pm 3\pi, \pm 5\pi,$ etc — amplitude mínima

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$$\alpha = Kx - \varepsilon \qquad \alpha_1 = K_1 x_1 - \varepsilon_1$$
$$\alpha_2 = K_2 x_2 - \varepsilon_2$$

$$\delta = (K_1x_1 - \varepsilon_1) - (K_2x_2 - \varepsilon_2)$$
$$= (K_1x_1 - K_2x_2) + (\varepsilon_2 - \varepsilon_1)$$

$$K = \frac{2\pi}{\lambda}$$

$$n = \frac{c}{v} = \frac{\lambda_0 f}{\lambda f} = \frac{\lambda_0}{\lambda}$$

$$\frac{n}{\lambda_0} = \frac{1}{\lambda}$$

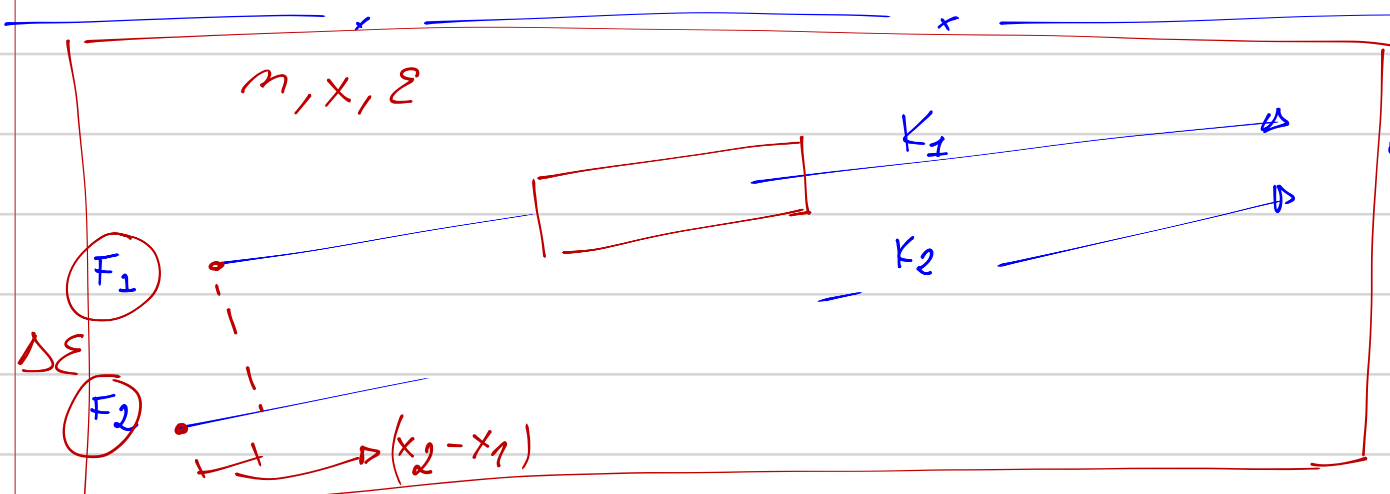
$$K = \frac{2\pi n}{\lambda_0} \qquad \begin{cases} K_1 = \dfrac{2\pi n_1}{\lambda_0} \\ K_2 = \dfrac{2\pi n_2}{\lambda_0} \end{cases}$$

$$\boxed{\delta = \frac{2\pi}{\lambda_0}\left[\underbrace{n_1x_1} - \underbrace{n_2x_2}\right] + (\varepsilon_2 - \varepsilon_1)}$$

→ São os caminhos ópticos dos raios $\vec{K}_1$ e $\vec{K}_2$

$(n_1x_1 - n_2x_2) \Rightarrow$ diferença de caminho óptico

$\Lambda = n_1x_1 - n_2x_2$ " " percurso óptico



• ponto "p"

$E_T = E_1 + E_2$

→ outra opção: $E_{(x,t)} = E_0 \cos(Kx - \omega t + \varepsilon)$

→ ainda $\tilde{E}_{(x,t)} = E_0 \, Re\{e^{i(\alpha+\omega t)}\}$

$$\tilde{E}_1(x,t) = E_{01}\, e^{i(\alpha_1+\omega t)}$$

$$\tilde{E}_2(x,t) = E_{02}\, e^{i(\alpha_2+\omega t)}$$

$$E_T = E_1 + E_2 = E_{01}e^{i\alpha_1}\cdot e^{i\omega t} + E_{02}e^{i\alpha_2}\cdot e^{i\omega t}$$

$$= \underbrace{\left(E_{01}e^{i\alpha_1} + E_{02}e^{i\alpha_2}\right)}_{E_0} e^{i\omega t}$$

$$\boxed{E_T = E_0\, e^{i\omega t}}$$

$$\boxed{E_T = \sum_i E_i}$$

ainda $E_0^2 = (\quad)\cdot(\quad)^*$

Desenvolvendo as contas

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}\cdot E_{02}\cos(\alpha_1 - \alpha_2)$$

Fasores

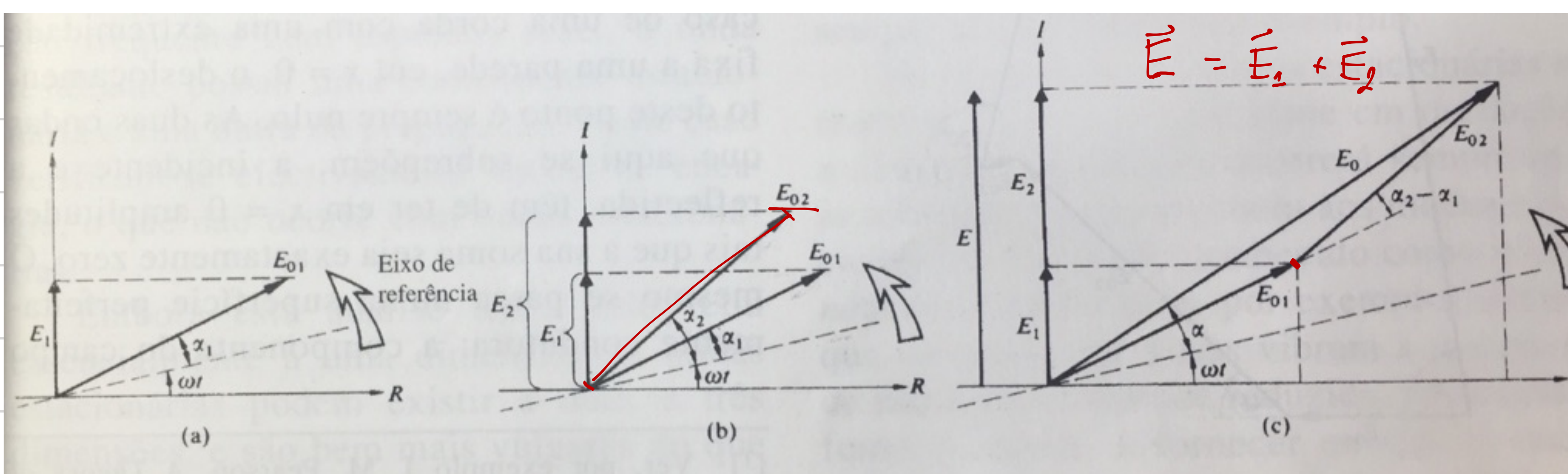
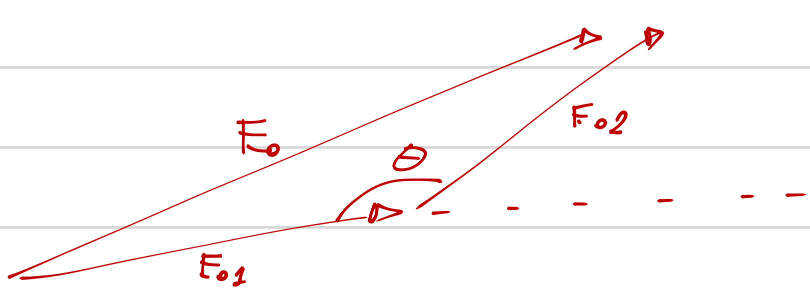


Figura 7.5 Adição de fasores.

$$\theta = \pi - (\alpha_2 - \alpha_1)$$



$$E_0^2 = E_{01}^2 + E_{02}^2 - 2E_{01}E_{02}\cos\theta$$

Desenvolvendo

$$\boxed{E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)}$$