Planejamento de Rotas - Parte III
Incerteza

SSC5955

Slides adaptados de Masahiro Ono - MIT

Planejamento de Rotas com Incertezas

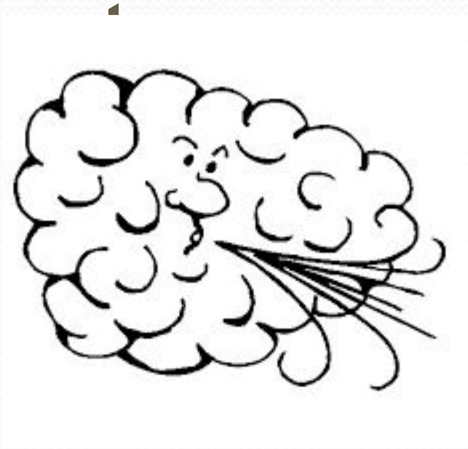
Planejamento de Rotas com Incertezas

Riscos possíveis

Riscos inaceitáveis

Planejamento de Rotas com Incertezas

Incertezas



Robust Model Predictive Control[1]

- Receding horizon planner reagem a incertezas **depois** que alguma coisa errada ocorre.

- Podemos tomar precauções antes de alguma coisa errada ocorrer?

Incertezas nos modelos

- Modelo determinístico e discreto:

$$x_{t+1} = Ax_t + Bu_t$$

- Modelo multiplicativo de incerteza

$$x_{t+1} = (A + \Delta A)x_t + Bu_t$$

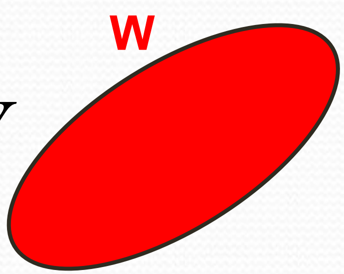
- Modelo aditivo de incerteza

$$x_{t+1} = Ax_t + Bu_t + w_t$$

Incertezas nos modelos

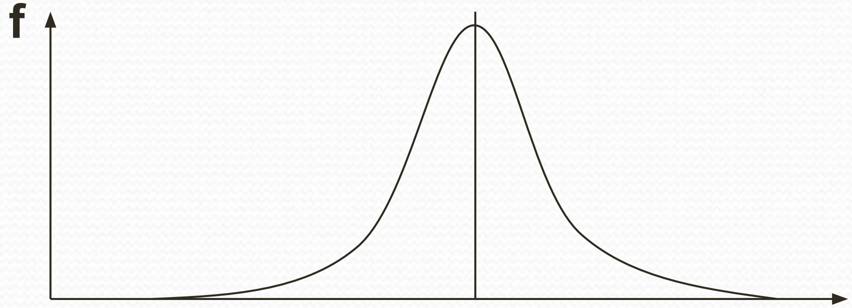
- Incerteza limitada

$$w_t \in W$$



- Incerteza estocástica

$$w_t \sim f(w)$$



f: função de distribuição de probabilidade

Incerteza Estocástica

- Otimizando penalizando probabilidade de falha

$$\min_{\mathbf{U}} J(\bar{\mathbf{X}}, \mathbf{U}) + p f(\mathbf{U})$$

Probabilidade de falha

- Otimizando com restrições de probabilidade (chance-contraints).

$$\min_{\mathbf{U}} J(\bar{\mathbf{X}}, \mathbf{U})$$

$$s.t. \quad f(\mathbf{U}) \leq \Delta$$

Limite de risco

Probabilidade de falha

Problema

Definição: Dado um sistema com uma dinâmica discreta no tempo e estocástica, encontrar uma sequência de controles que minimize uma função de custo enquanto limita a probabilidade de violar restrições, dentro de um horizonte de planejamento finito, dentro de um dado limite superior (chance constraint)[2].

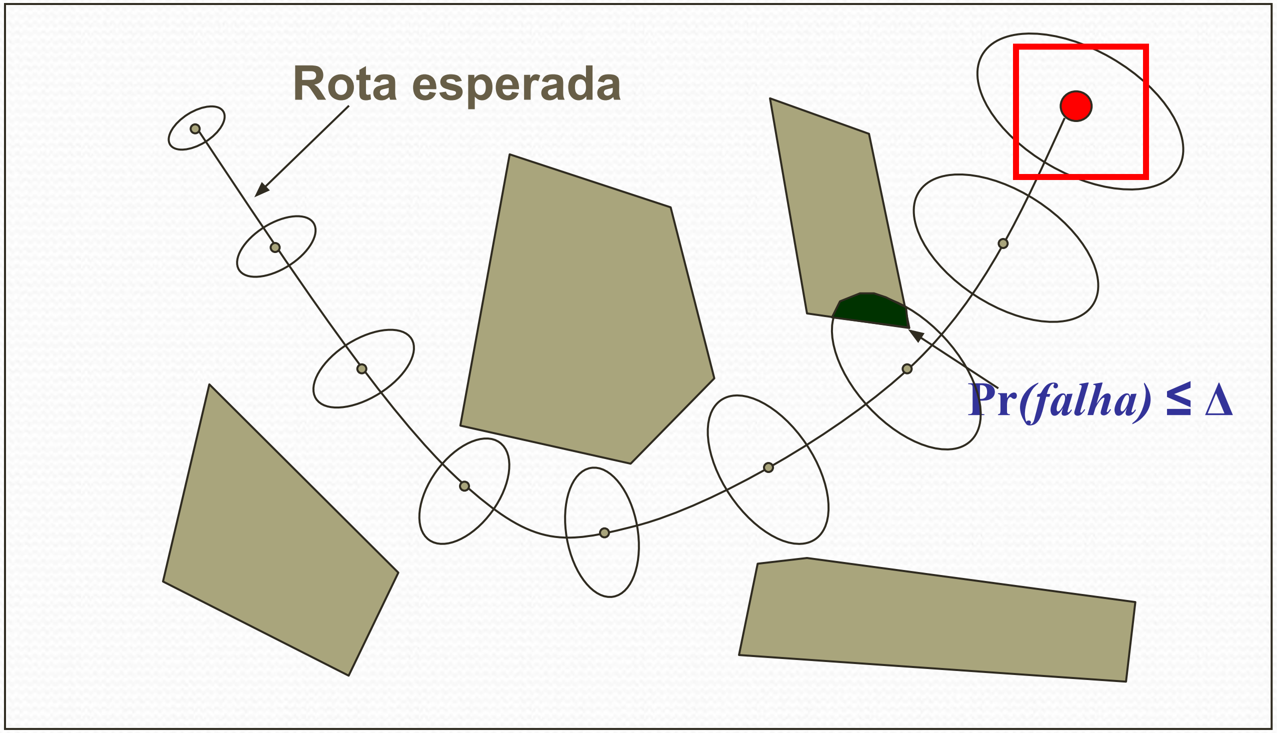
Problema

- Exemplo: Planejar uma trajetória que minimize consumo de combustível enquanto limita a 0.1% a probabilidade de atingir um obstáculo ou adentrar uma região de risco.

Planejamento de Rota Robusto

- Definir uma rota ótima até o destino tal que

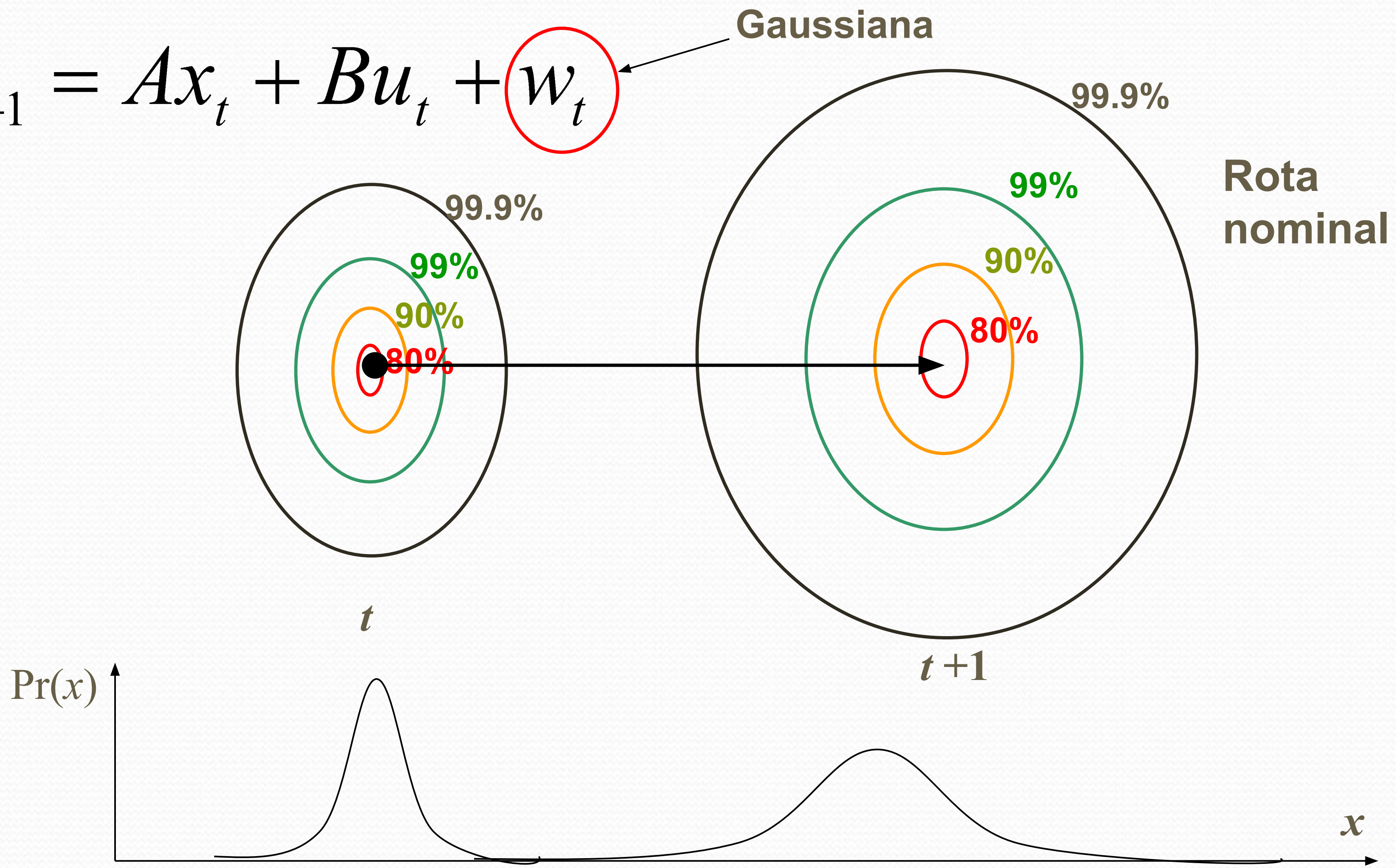
  Pr(falha) ≤ Δ."
- Uso das chance contraints.



Modelo Estocástico

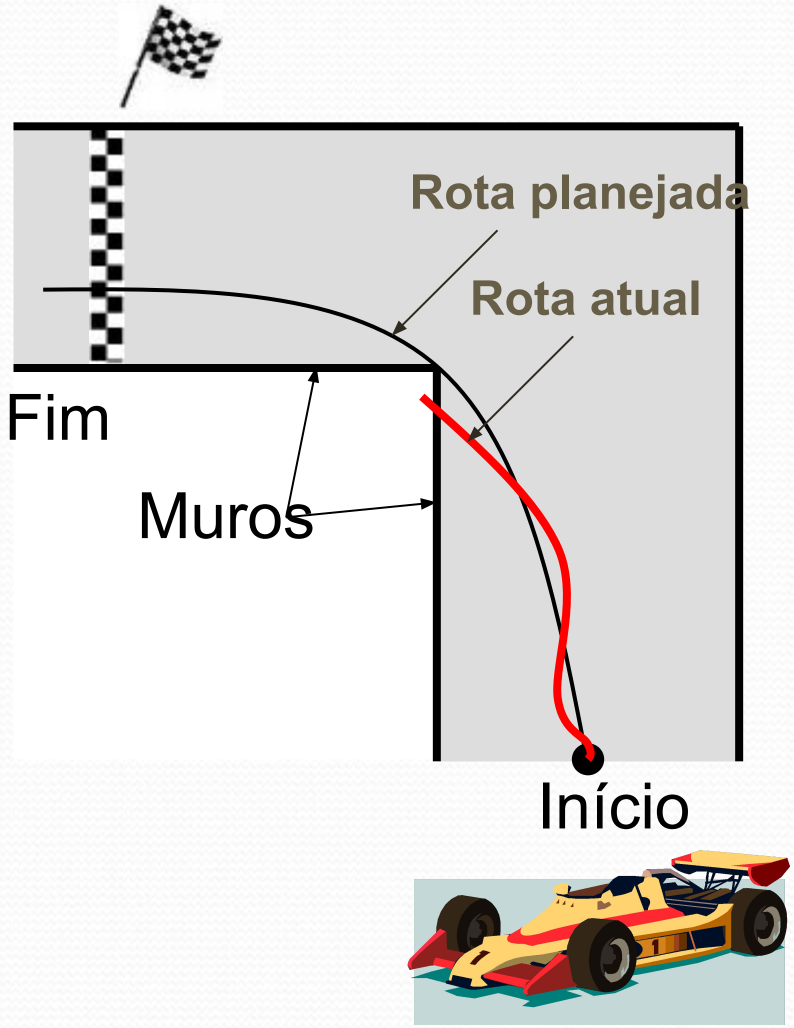
- Stochastic Plant Model - SPM

$$x_{t+1} = Ax_t + Bu_t + w_t$$



Exemplo

- **Planejamento rota em uma corrida de carro**



- Impossível garantir 100% de segurança
- Corredores querem reduzir chance de acidentes P(acidente) < 0.1%
  - **Chance constraint**

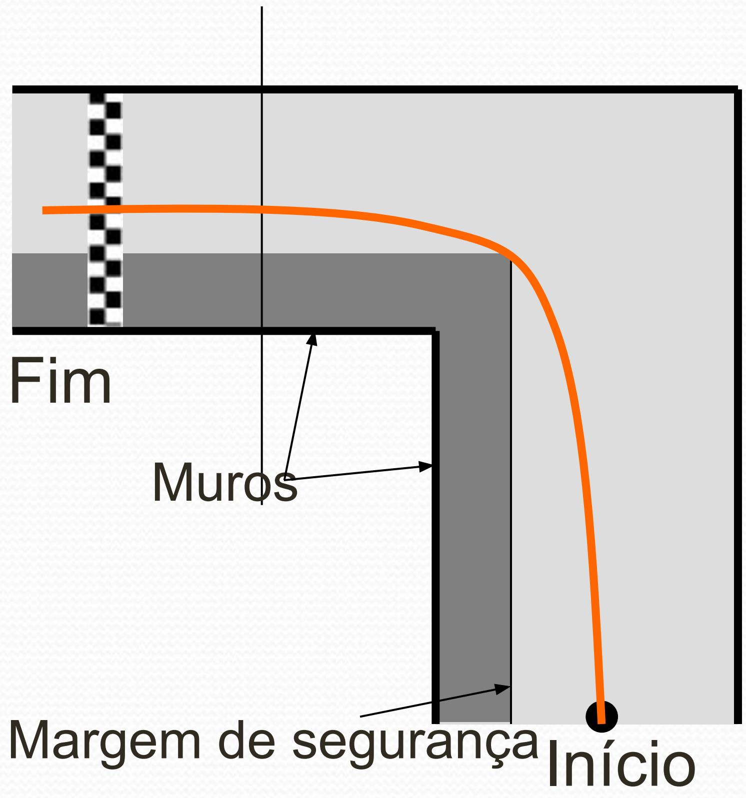
Exemplo

- **Planejamento rota em uma corrida de carro**

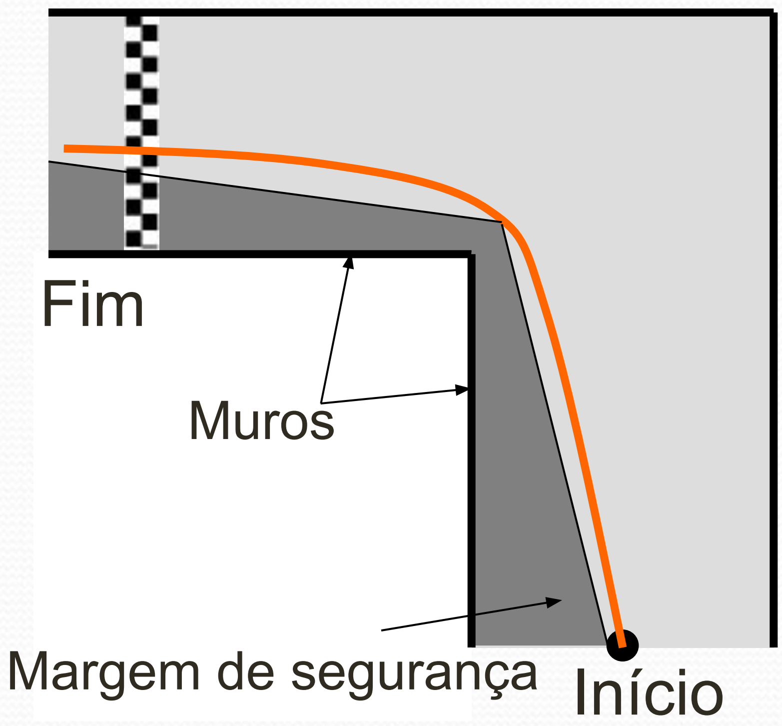
- Impossível garantir 100% de segurança
- Corredores querem reduzir chance de acidentes P(acidente) < 0.1%
  - **Chance constraint**
- Solução: Ajustar uma margem de segurança que garanta o limite de risco

Ajustando a margem de segurança

(a) Uniforme



(b) Não uniforme

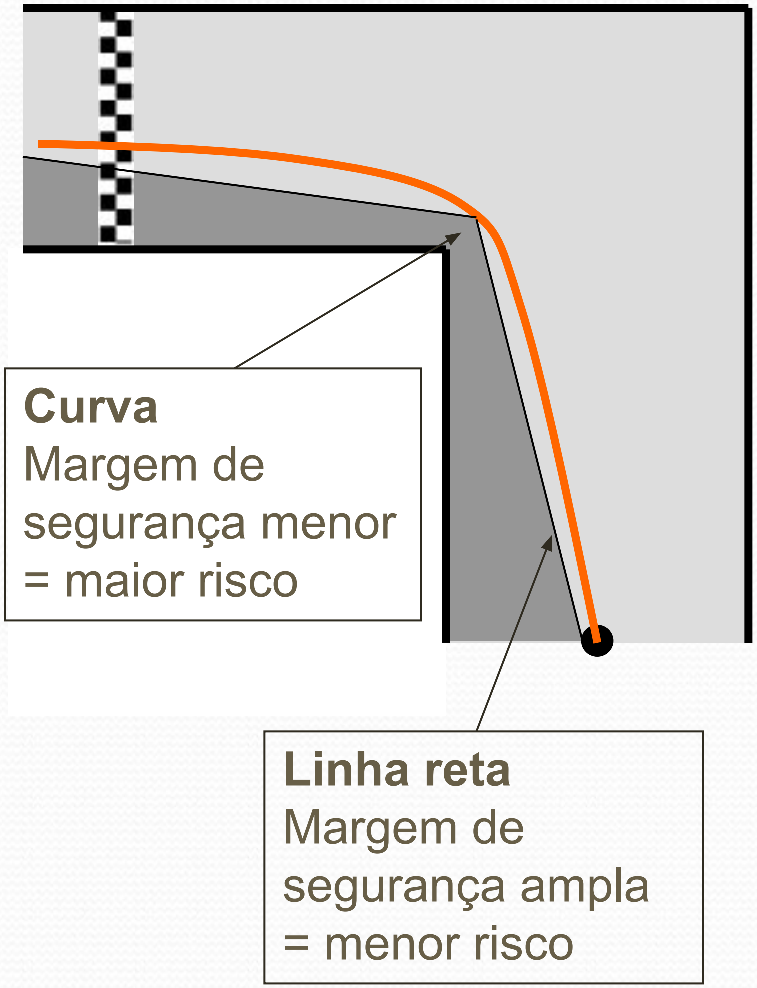


Alocação de Risco

- Estratégia: Incorrer em um risco maior na curva para obter ganho de tempo ao invés de assumir o mesmo risco durante toda a rota.

- *Alocação de risco*
  - Incorre em risco quando isso leva a uma recompensa relevante, enquanto economiza o risco quando a recompensa for menor.

Margem de segurança não uniforme



Chance-Constraints e Model Control Predictive (MPC)

$$\min_{\mathbf{U}} J(\mathbf{X},\mathbf{U}) \quad s.t.$$

Função Objetivo: custo, por exemplo, consumo de combustível

$$\forall_{0 \le t \le T-1} \; x_{t+1} = Ax_t + Bu_t$$

Equações de dinâmica discreta

$$\bigwedge_{t=0}^{T} \bigwedge_{i=0}^{N} h_t^{iT} x_t \le g_t^i$$

Restrições

$$\mathbf{X} = \left[x_0 \; \cdots \; x_T\right]^T$$

Vetor de estados, por exemplo, posição do veículo

$$\mathbf{U} = \left[u_0 \; \cdots \; u_{T-1}\right]^T$$

Controles aplicados

Chance-Constraints e Model Control Predictive (MPC)

$$\min_{\mathbf{U}} J(\mathbf{X},\mathbf{U})$$ Função Objetivo: custo, por exemplo, consumo de combustível

$$s.t.$$

$$\forall_{0\le t\le T-1} \; x_{t+1} = Ax_t + Bu_t$$ **Equações de dinâmica discreta**

$$w_t \sim N(0,\Sigma_t)$$

$$x_0 \sim N(\bar{x}_0,\Sigma_{x,0})$$

$$\bigwedge_{t=0}^{T} \bigwedge_{i=0}^{N} h_t^{iT} x_t \le g_t^i$$ **Restrições**

Chance-Constraints e Model Control Predictive (MPC)

$$\min_{\mathbf{U}} J(\mathbf{X}, \mathbf{U}) \quad s.t.$$

Função Objetivo: custo, por exemplo, consumo de combustível

$$\forall_{0 \le t \le T-1} \; x_{t+1} = Ax_t + Bu_t$$

Equações de dinâmica discreta

$$w_t \sim N(0, \Sigma_t)$$

$$x_0 \sim N(\bar{x}_0, \Sigma_{x,0})$$

$$\bigwedge_{t=0}^{T} \bigwedge_{i=0}^{N} h_t^{iT} x_t \le g_t^i$$

Restrições

Chance-Constraints e Model Control Predictive (MPC)

$$\min_{\mathbf{U}} J(\mathbf{X},\mathbf{U}) \quad s.t.$$

Função Objetivo: custo, por exemplo, consumo de combustível

$$\underset{0\le t\le T-1}{\forall} x_{t+1} = Ax_t + Bu_t$$

Equações de dinâmica discreta

$$w_t \sim N(0,\Sigma_t)$$

$$x_0 \sim N(\bar{x}_0,\Sigma_{x,0})$$

Limite superior na probabilidade de falha = *Limite de risco*

$$\Pr\left[\bigwedge_{t=0}^{T}\bigwedge_{i=0}^{N} h_t^{iT} x_t \le g_t^i\right] \ge 1-\Delta$$

Chance constraints

Modelando Chance Constraints

$$\Pr\left[\bigwedge_{t=0}^{T}\bigwedge_{i=0}^{N} h_t^{iT} x_t \le g_t^i\right] \ge 1-\Delta$$

Desigualdade Booleana

$$\Pr[A \cup B] \le \Pr[A] + \Pr[B]$$

Modelando Chance Constraints

$$\Pr\left[\bigwedge_{t=0}^{T}\bigwedge_{i=0}^{N} h_t^{iT} x_t \le g_t^i\right] \ge 1 - \Delta$$

Limite superior na probabilidade de violar qualquer restrição dentro do horizonte de planejamento

$$\bigwedge_{t}\bigwedge_{i}\left(\Pr\left[h_t^{iT} x_t \le g_t^i\right] \ge 1 - \delta_t^i\right)$$

$$\sum_{t,i} \delta_t^i \le \Delta$$

Limite superior na probabilidade de violar a *i-ésima* restrição no tempo *t*

Risk allocation: $\boldsymbol{\delta} = \left[\delta_1^1, \delta_1^2 \square\ \delta_T^N\right]$

Modelando Chance Constraints

Risk allocation optimization

$$\min_{\boldsymbol{\delta}} \min_{\mathbf{U}} J(\mathbf{X},\mathbf{U})$$

$s.t.$

$$\forall_{0\le t\le T-1} \quad x_{t+1} = Ax_t + Bu_t$$

Equações de dinâmica discreta

$$w_t \sim N(0,\Sigma_t)$$

$$x_0 \sim N(\bar{x}_0,\Sigma_{x,0})$$

$$\bigwedge_t \bigwedge_i \Pr\left[h_t^{iT} x_t \le g_t^i\right] \ge 1-\delta_t^i$$

$$\sum_{t,i} \delta_t^i \le \Delta$$

Chance constraints

Modelando Chance Constraints

Variável aleatória

$$\Pr\left[h_t^{iT} x_t \le g_t^i\right] \ge 1 - \delta_t^i$$

Restrição de probabilidade (chance constraint)

Variável determinística

$$\Leftrightarrow h_t^{iT} \bar{x}_t \le g_t^i - m_t^i\left(\delta_t^i\right)$$

Restrição determinística

"Aperto" na restrição - Constraint tightening

onde,

$$m_t^i\left(\delta_t^i\right) = -\sqrt{2 h_t^{iT} \Sigma_{x,t} h_t^i}\,\mathrm{erf}^{-1}\left(2\delta_t^i - 1\right)$$

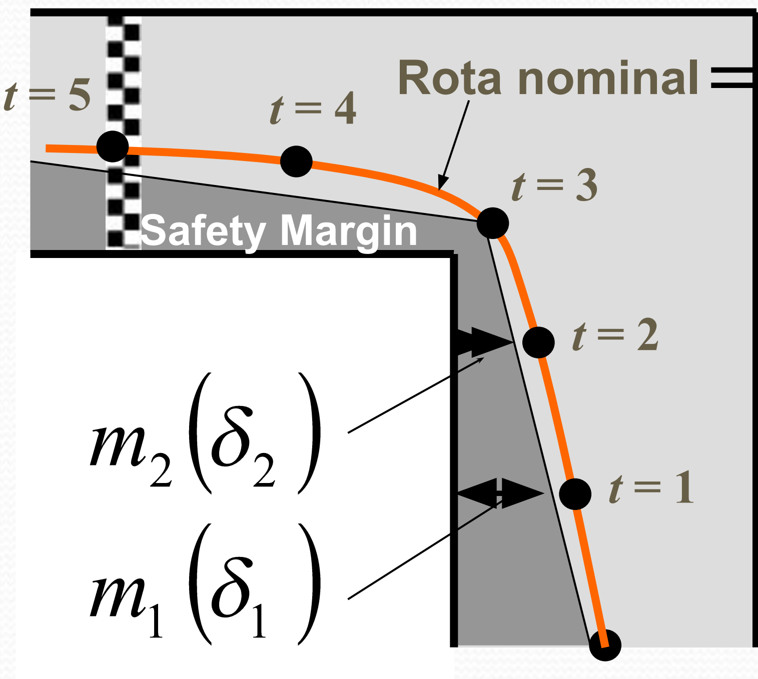
Inversa da função de distribuição acumulada

$$x_t \sim N(\bar{x}_t, \Sigma_{x,t})$$

$$\Sigma_{x,t} = \sum_{n=0}^{t-1} A^n \Sigma_w (A^n)^T + \Sigma_{x,0}$$

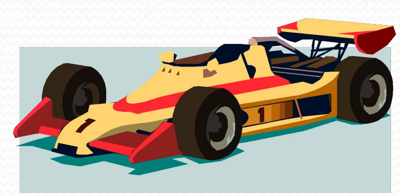
Modelando Chance Constraints

$$h_t^{iT}\left(\overline{x}_t\right) \le g_t^i - m_t^i\left(\delta_t^i\right)$$



= Média do estado do veículo

Margem de segurança



Modelando Chance Constraints

$$\min_{\boldsymbol{\delta}} \min_{\mathbf{U}} J(\mathbf{X}, \mathbf{U})$$

$s.t.$

$$\forall_{0 \le t \le T-1} \; x_{t+1} = Ax_t + Bu_t \quad \forall_{0 \le t \le T-1} \; \bar{x}_{t+1} = A\bar{x}_t + Bu_t$$

$$w_t \sim N(0, \Sigma_t)$$

$$x_0 \sim N(\bar{x}_0, \Sigma_{x,0})$$

Convexa?

$$h_t^{iT} \bar{x}_t \le g_t^i - m_t^i\left(\delta_t^i\right)$$

$$\sum_{t,k} \delta_t^i \le \Delta$$

Mixed Integer Non Linear Programming (MINLP) Formulation

- **Objective Function**

$$\min g(\cdot) \qquad (1)$$

- **Dynamic Equation**

$$\mu_t = A^t \hat{x}_0 + \sum_{s=0}^{t-1} A^{t-s-1} B u_s \qquad \forall(t) \qquad (2)$$

- **Goal Position**

$$\mu_T = x_{goal} \qquad (3)$$

MINLP Formulation

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}(\delta_{jt}) + b_{ji} - a_{ji}^{T}\mu_t \leq M(1 - Z_{jti}) \qquad \forall(j,t,i) \qquad (4)$$

$$c_{t,i}(\delta_{jt}) = \boldsymbol{err}^{-1}(\mathbf{1} - \mathbf{2}\boldsymbol{\delta}_{jt})\sqrt{a_{ji}^{T}\sum\nolimits_{\boldsymbol{t}} a_{ji}} \quad \forall(j,t,i) \qquad (5)$$

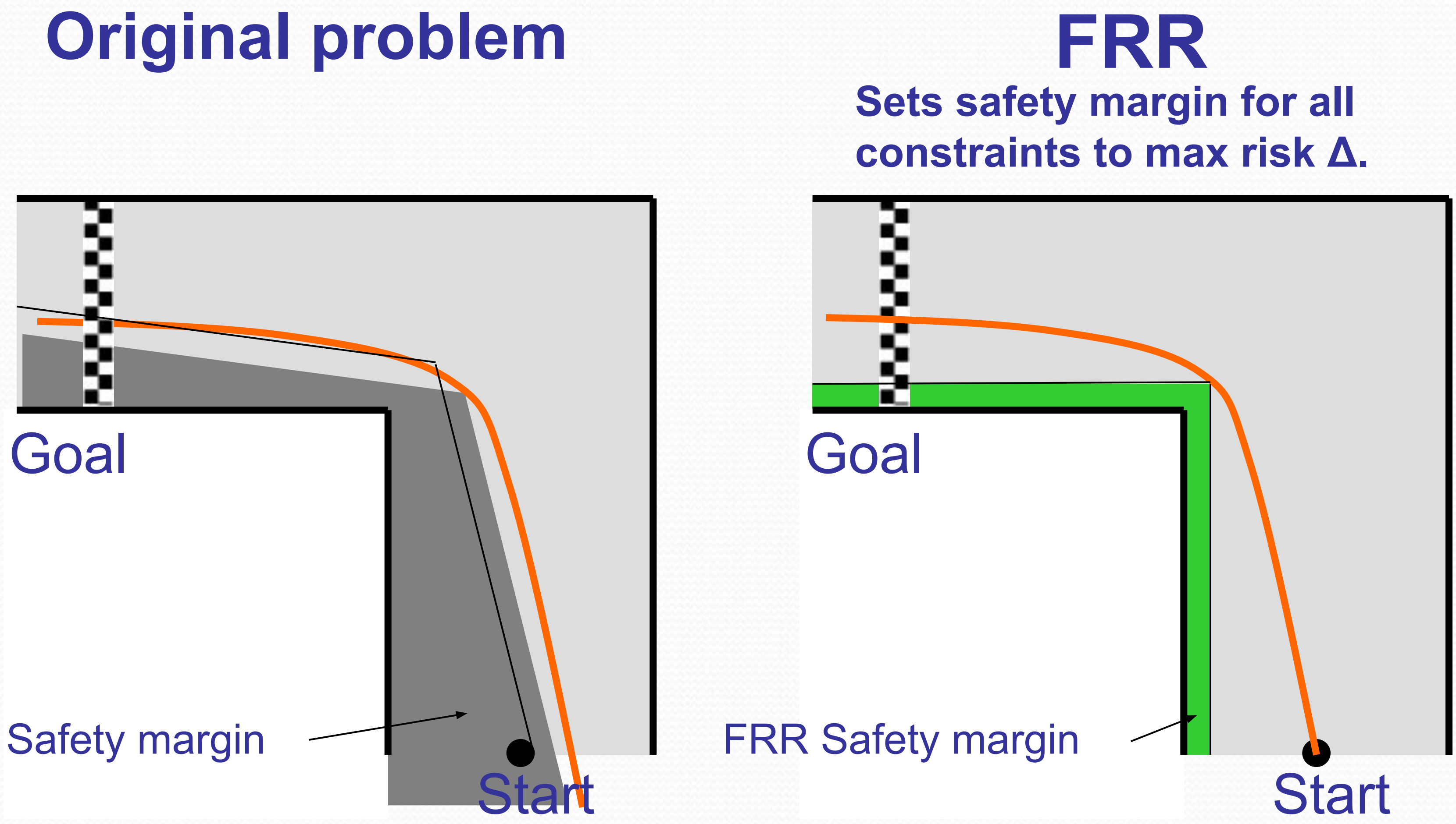
$$\sum_{j}\sum_{t}\delta_{jt} \leq \Delta \qquad (6)$$

$$\sum_{i \in G_j} Z_{jti} \geq 1 \qquad \forall(j,t) \qquad (7)$$

MINLP Formulation

Como lidar com isso????

Fixed Risk Relaxation: Intuição



- Results in an **infeasible** solution to the original problem.
- Gives **lower bound** for the cost of the original problem.

Fixed Risk Relaxation (FRR)

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}\left(\delta_{jt}\right) + b_{ji} - a_{ji}^{T}\mu_t \leq M\left(1 - Z_{jti}\right) \qquad \forall(j,t,i) \qquad (4)$$

$$c_{t,i}\left(\delta_{jt}\right) = \boldsymbol{err}^{-\mathbf{1}}(\mathbf{1} - \mathbf{2}\boldsymbol{\delta}_{jt}) \sqrt{a_{ji}^{T} \sum\nolimits_{\boldsymbol{t}} a_{ji}} \qquad \forall(j,t,i) \qquad (5)$$

$$\sum_{j} \sum_{t} \delta_{jt} \leq \Delta \qquad (6)$$

Fixed Risk Relaxation (FRR)

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}(\Delta) + b_{ji} - a_{ji}^{T}\mu_t \leq M\left(1 - Z_{jti}\right) \quad \forall (j,t,i) \qquad (4)$$

$$c_{t,i}(\Delta) = \boldsymbol{err}^{-\mathbf{1}}(\mathbf{1} - \mathbf{2}\Delta)\sqrt{a_{ji}^{T}\sum\nolimits_{\boldsymbol{t}} a_{ji}} \quad \forall (j,t,i) \qquad (5)$$

~~$\sum_{j}\sum_{t}\delta_{jt} \leq \Delta$ (6)~~

$$\delta_{jt} = \Delta$$

Fixed Risk Tightening (FRT)

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}(\delta_{jt}) + b_{ji} - a_{ji}^T \mu_t \leq M(1 - Z_{jti}) \quad \forall (j,t,i) \qquad (4)$$

$$c_{t,i}(\delta_{jt}) = \boldsymbol{err}^{-1}(\mathbf{1} - \mathbf{2}\boldsymbol{\delta}_{jt}) \sqrt{a_{ji}^T \sum\nolimits_{t} a_{ji}} \quad \forall (j,t,i) \qquad (5)$$

$$\sum_j \sum_t \delta_{jt} \leq \Delta \qquad (6)$$

Fixed Risk Tightening (FRT)

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}\left(\frac{\Delta}{\boldsymbol{T.J}}\right) + b_{ji} - a_{ji}^{T}\mu_t \leq M\left(1 - Z_{jti}\right) \qquad \forall(j,t,i) \qquad (4)$$

$$c_{t,i}\left(\frac{\Delta}{\boldsymbol{T.J}}\right) = \boldsymbol{err^{-1}}\left(\boldsymbol{1} - \boldsymbol{2}\,\frac{\Delta}{\boldsymbol{T.J}}\right)\sqrt{a_{ji}^{T}\sum\nolimits_{\boldsymbol{t}} a_{ji}} \quad \forall(j,t,i) \qquad (5)$$

$$\sum_{j}\sum_{t}\delta_{jt} \leq \Delta \qquad (6)$$

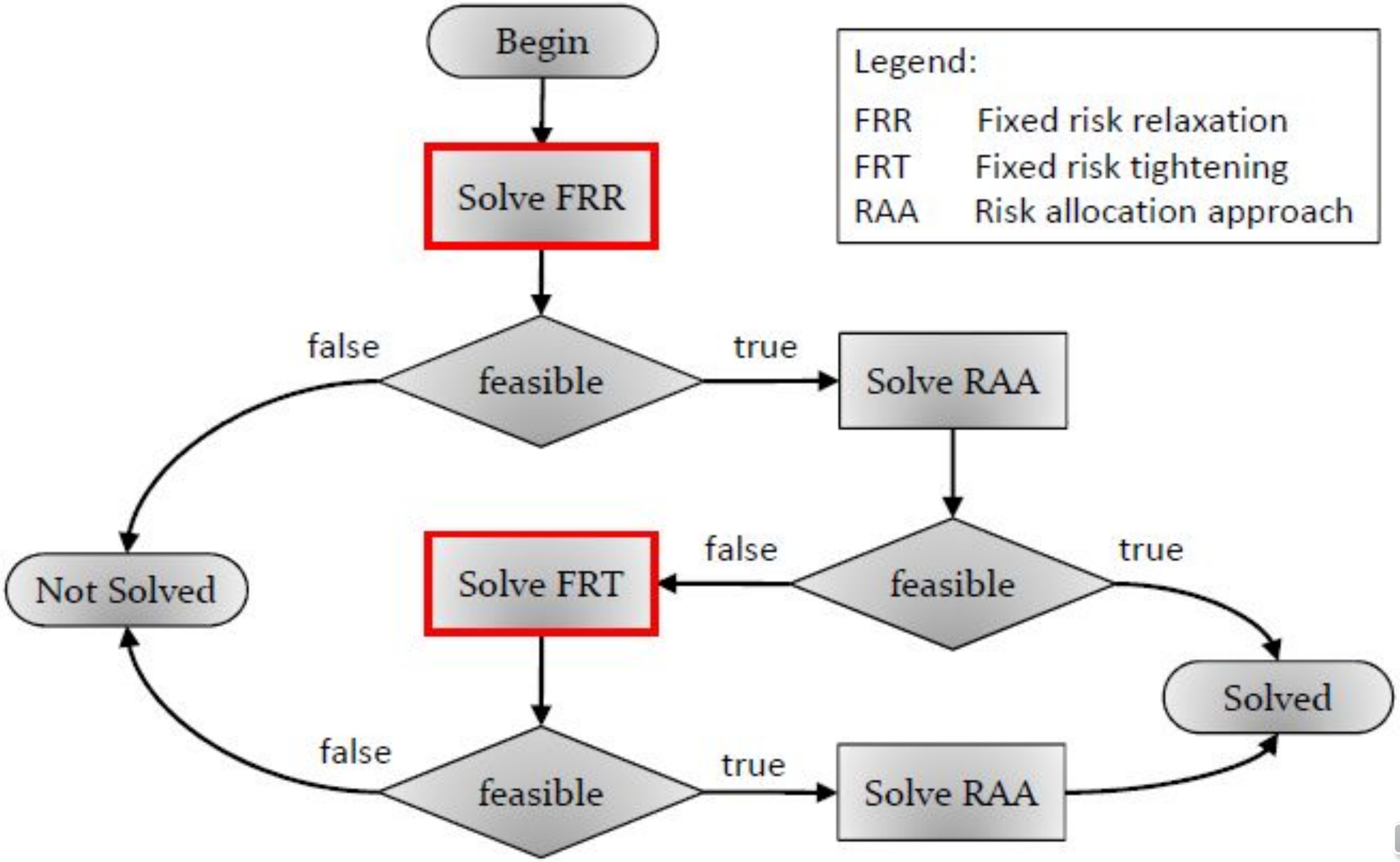
$$\delta_{jt} = \frac{\Delta}{\boldsymbol{T.J}}$$

MINLP Formulation

- Lidando com restrição (5) no FRR e FRT:

  Calcular previamente alocação de risco e fornecer como entrada para os modelos FRR e FRT.

Customized Solution Approach (CSA)



Dá pra fazer melhor!!!

....

Obstáculos e pontos de passagem

$$x_t \in \mathbb{I}_j \quad \Leftrightarrow \quad \bigwedge_{i \in H_j^{\mathbb{I}}} h_i^T \mathbf{x}_t \leq g_i \qquad (4.6)$$

$$x_t \in \mathbb{O}_j \quad \Leftrightarrow \quad \bigvee_{i \in H_j^{\mathbb{O}}} h_i^T \mathbf{x}_t \geq g_i \qquad (4.7)$$

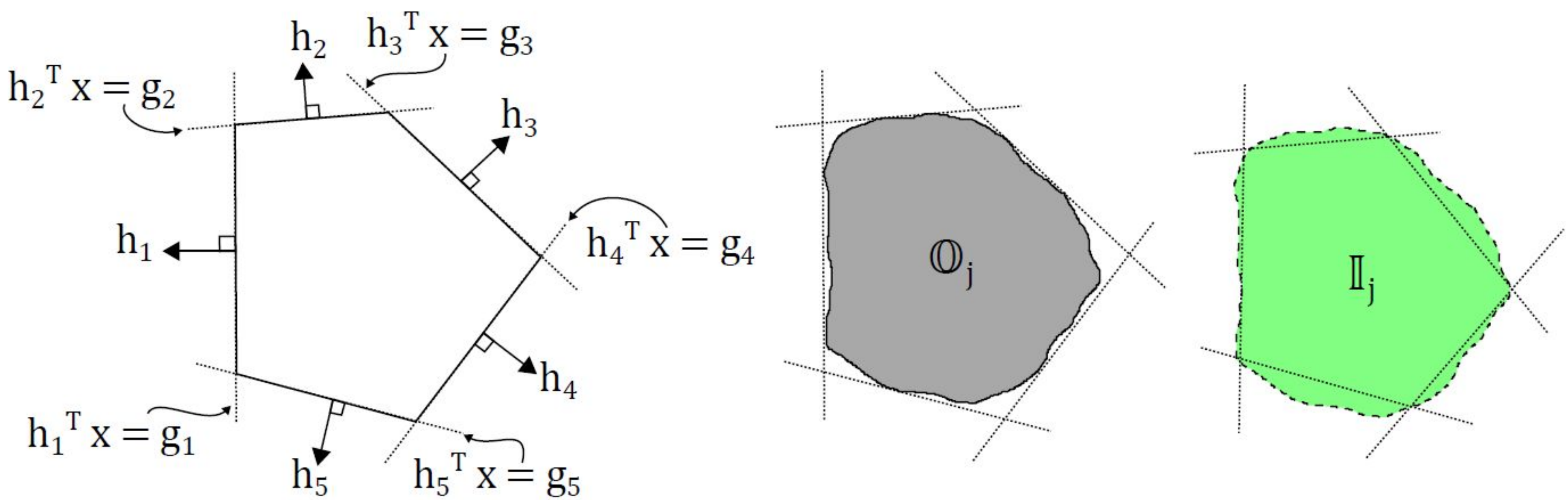


Figura 4.2: Região convexa para modelar obstáculos $\mathbb{O}_j$ e pontos de passagem $\mathbb{I}_j$

Dinâmica determística para o CCPP

$$\mathbf{x}_{t+1} = A\mathbf{x}_t + B\mathbf{u}_t + \omega_t \qquad \forall(t)$$

$$\bar{x}_{t+1} = A\bar{x}_t + B\bar{u}_t \qquad \forall(t)$$

$$\Sigma_{t+1} = A\,\Sigma_t A^T + \Sigma_{w_t} \qquad \forall(t)$$

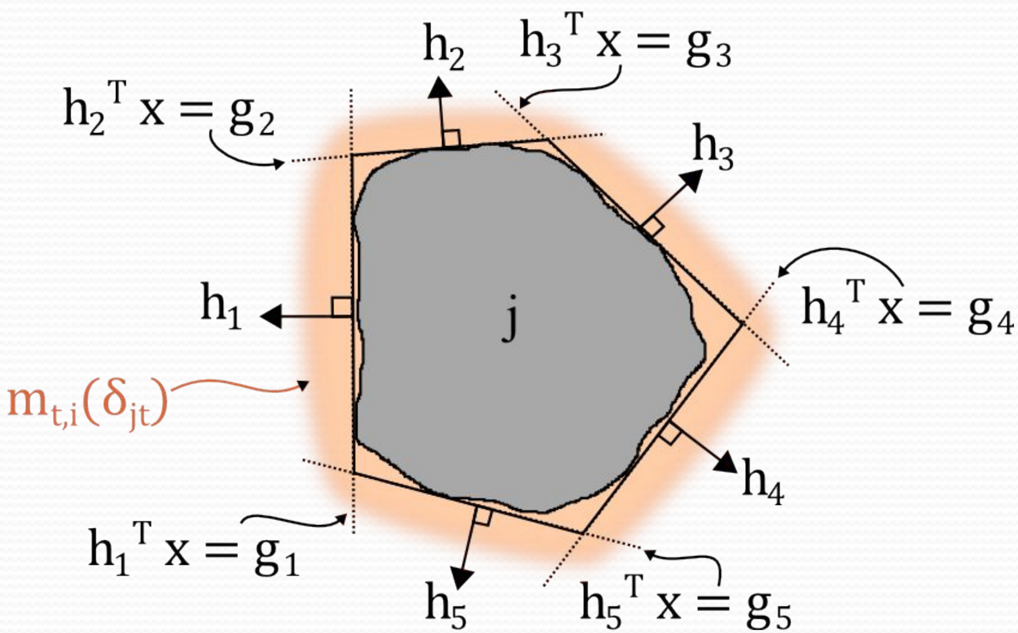
$$\mathbf{x}_t \sim \mathcal{N}(\bar{\mathbf{x}}_t, \Sigma_t)$$

Chance constraints

$$x_t \in \mathbb{I}_j \Leftrightarrow \bigwedge_{i \in H_j^{\mathbb{I}}} h_i^T \mathbf{x}_t \leq g_i \Rightarrow \bigwedge_{i \in H_j^{\mathbb{I}}} [g_i - h_i^T \bar{x}_t \geq m_{t,i}(\delta_{jt})]$$

$$x_t \in \mathbb{O}_j \Leftrightarrow \bigvee_{i \in H_j^{\mathbb{O}}} h_i^T \mathbf{x}_t \geq g_i \Rightarrow \bigvee_{i \in H_j^{\mathbb{O}}} [h_i^T \bar{x}_t - g_i \geq m_{t,i}(\delta_{jt})]$$

$$m_{t,i}(\xi) = erf^{-1}(1 - 2\xi)\sqrt{2 h_i^T \Sigma_t h_i} \Rightarrow 0 < \xi \leq \Delta \leq 0.5$$



Chance constraints

$$Pr\left[\left(\bigwedge_{j\in\mathbb{I}}\bigwedge_{t\in\mathcal{T}(\mathbb{I}_j)}\mathbf{x}_t\in\mathbb{I}_j\right)\wedge\left(\bigwedge_{j\in\mathbb{O}}\bigwedge_{t\in\mathcal{T}(\mathbb{O}_j)}\mathbf{x}_t\in\mathbb{O}_j\right)\right]\geq 1-\Delta$$

$$\bigwedge_{i\in H_j^{\mathbb{I}}}\left[g_i-h_i^T\,\bar{x}_t\geq m_{t,i}(\delta_{jt})\right]$$

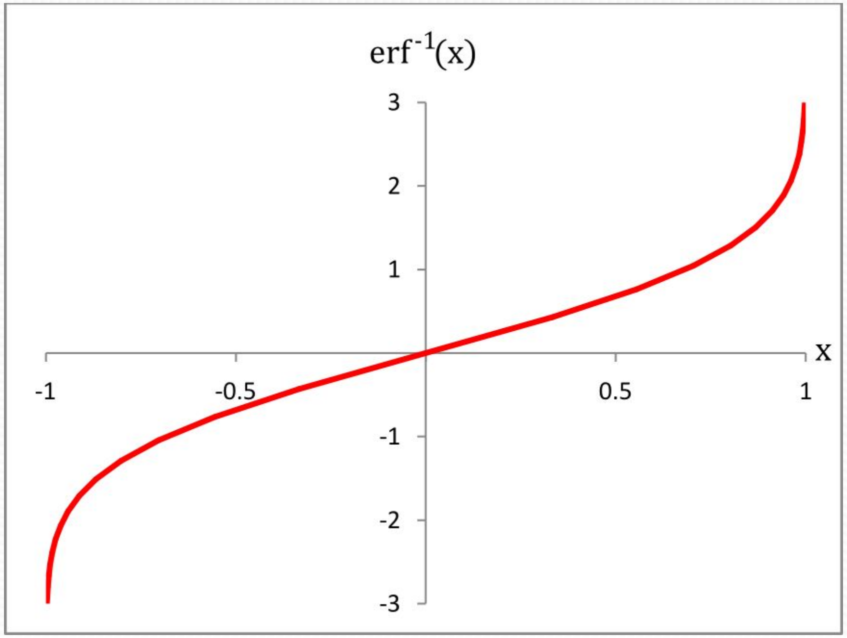
$$\bigvee_{i\in H_j^{\mathbb{O}}}\left[h_i^T\,\bar{x}_t-g_i\geq m_{t,i}(\delta_{jt})\right]$$

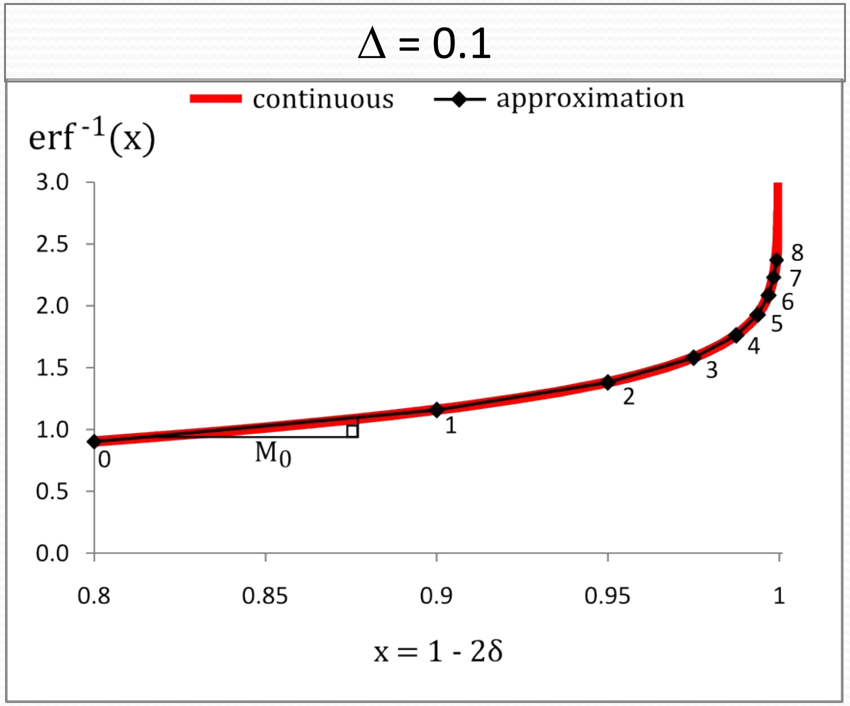
$$\sum_t\sum_j\delta_{jt}\leq\Delta$$

$$m_{t,i}(\xi)=erf^{-1}(1-2\xi)\sqrt{2h_i^T\Sigma_t h_i}$$

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

Aproximando a função inversa do erro





$$1 - 2\delta_{jt} = \bar{x}_0 + \sum_{n=0}^{N-1} \lambda_{njt}$$

$$erf^{-1}(x) := \bar{y}_0 + \sum_{n=0}^{N-1} M_n \lambda_{njt}$$

$$0 \leq \lambda_{njt} \leq [\bar{x}_{n+1} - \bar{x}_n]$$

$$\lambda_{n+1,jt} > 0 \Rightarrow \lambda_{njt} = [\bar{x}_{n+1} - \bar{x}_n]$$

$$\bar{x}_n = 1 - 2\left(\frac{\Delta}{2^n}\right) \quad n = 0, \dots, N$$

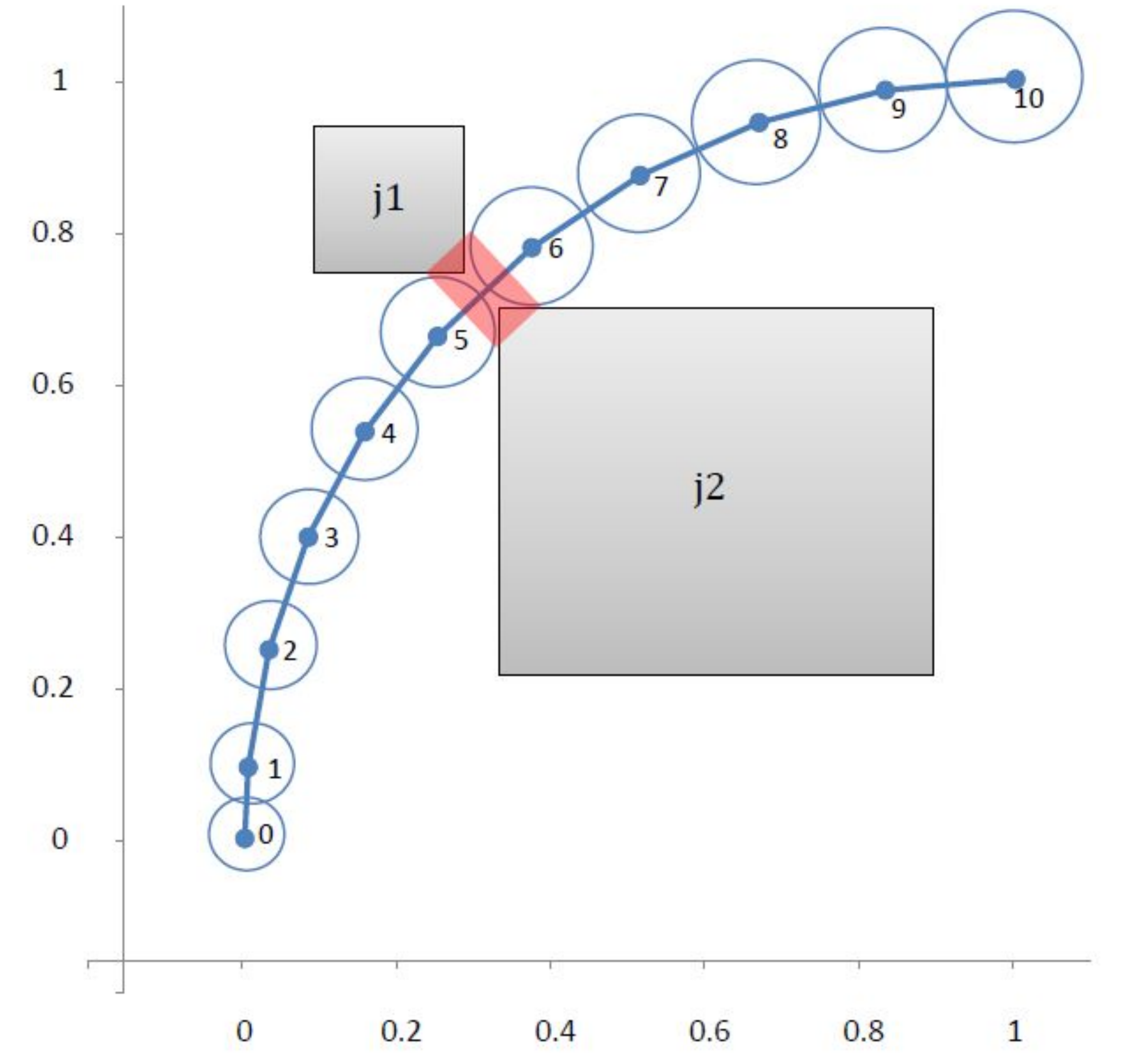
$$\bar{y}_n = erf^{-1}(\bar{x}_n) \quad n = 0, \dots, N$$

$$M_n = \frac{\bar{y}_{n+1} - \bar{y}_n}{\bar{x}_{n+1} - \bar{x}_n} \quad n = 0, \dots, N - 1$$

Nasa: Houston, we have a problem!!!!

….

Underestimated Risk

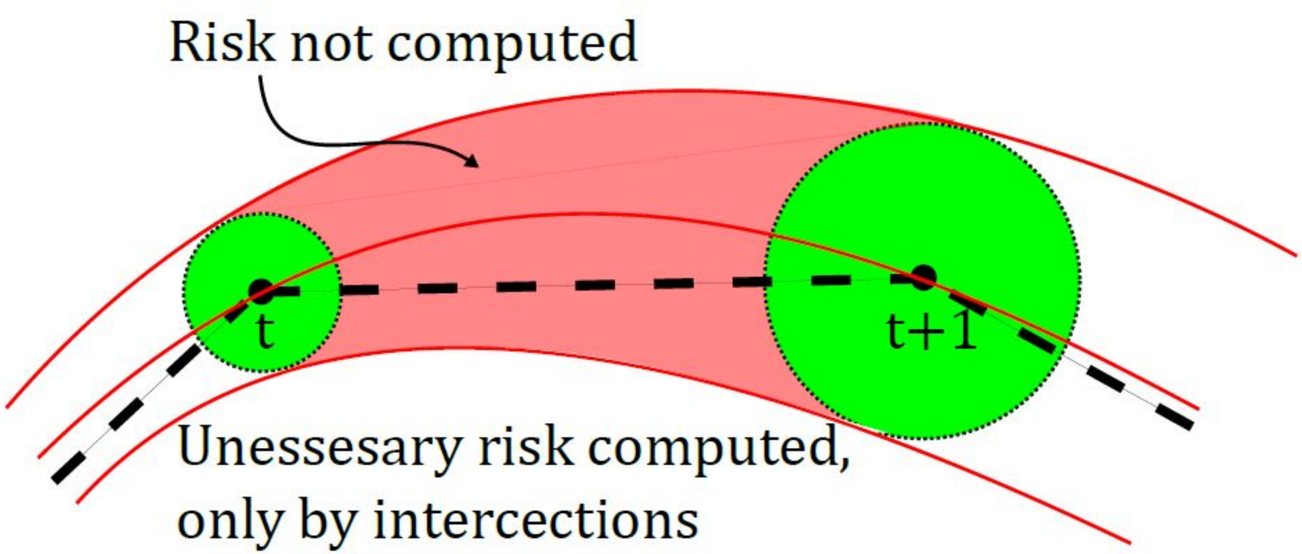


Estados do Veículo (EV)

$$a_i^T \bar{\mathbf{x}}_t \geq b_i + c_{i,t}(\delta_{jt}) + M(Z_{ti} - 1) \qquad \forall (t, j, i \in H_j)$$

$$\sum_{i \in H_j} Z_{ti} \geq 1 \qquad \forall (t, j)$$

$$\sum_j \sum_t \delta_{jt} \leq \Delta$$



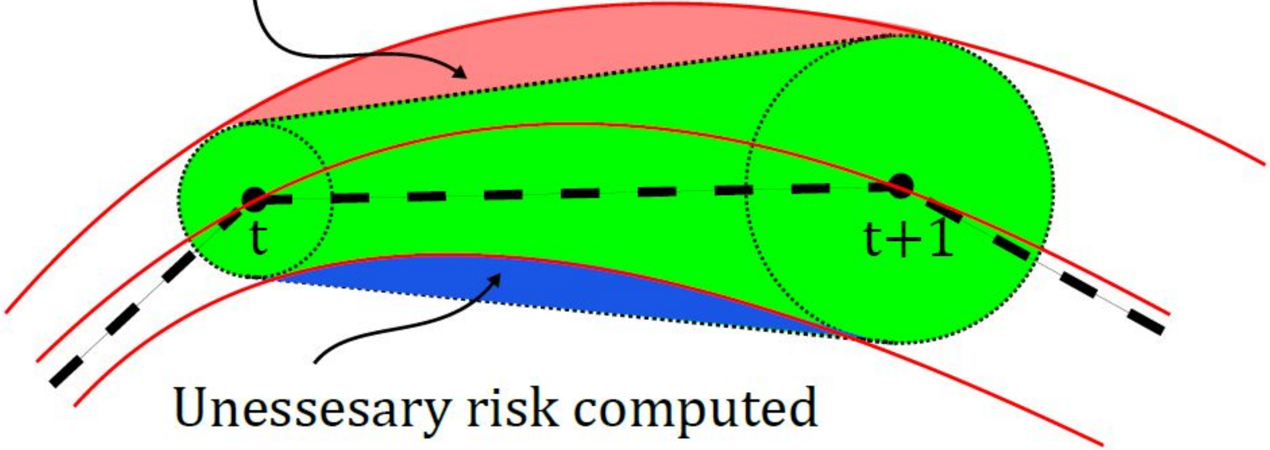
(a) Estados do veículo (EV)

Região Convexa (RC)

$$a_i^T \bar{\mathbf{x}}_t \geq b_i + c_{i,t}(\delta_{jt}) + M(Z_{ti} - 1) \qquad \forall (t, j, i \in H_j)$$

$$\bigvee_{i \in H_j} (Z_{t,i} \wedge Z_{t+1,i}) = 1 \qquad \forall (t, j)$$

$$\sum_j \sum_t \delta_{jt} \leq \Delta$$



(b) Região Convexa (RC), Anexo A

Lemma: For all obstacles j and time steps t, to encode obstacle avoidance, we need to ensure that $\exists i \in G_j$ such that $Z_{j,t,i} \wedge Z_{j,t-1,i} = 1$. This occurs, if and only if constraints (14)–(18) hold. Mathematically, we have

$$\bigvee_{i \in G_j} (Z_{j,t,i} \wedge Z_{j,t-1,i}) = 1 \quad \Leftrightarrow \quad \text{(14)–(18) hold}$$

$$\sum_{i \in G_j} p_{j,t,i} \geq 1 \tag{14}$$

$$p_{j,t,i} \geq Z_{j,t,i} + Z_{j,t-1,i} - 1 \tag{15}$$

$$p_{j,t,i} \leq Z_{j,t,i} \tag{16}$$

$$p_{j,t,i} \leq Z_{j,t-1,i} \tag{17}$$

$$p_{j,t,i} \geq 0. \tag{18}$$

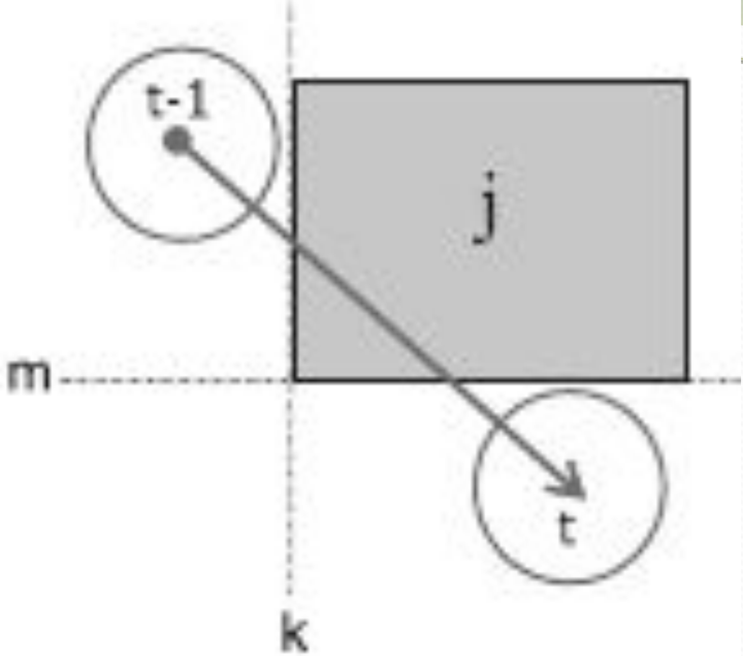
Proof: If $\exists i \in G_j$ such that $(Z_{j,t,i} \wedge Z_{j,t-1,i}) = 1$

$$\Leftrightarrow \sum_{i \in G_j} (Z_{j,t,i} \wedge Z_{j,t-1,i}) \geq 1 \Leftrightarrow \sum_{i \in G_j} p_{j,t,i} \geq 1$$

where $p_{j,t,i} = Z_{j,t,i} \wedge Z_{j,t-1,i}$. Thus, we can have

$$p_{j,t,i} = Z_{j,t,i} \wedge Z_{j,t-1,i} \Leftrightarrow$$

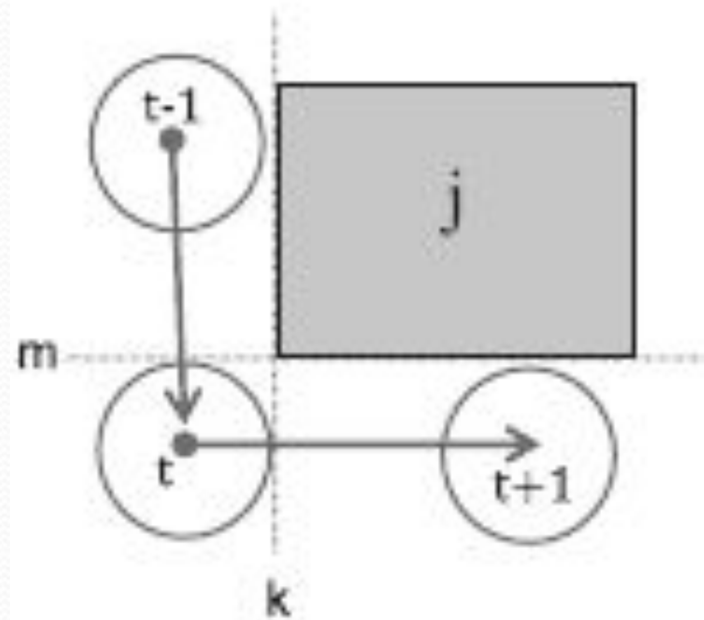
$$\left\{ \begin{array}{c} p_{j,t,i} \geq Z_{j,t,i} + Z_{j,t-1,i} - 1 \quad \forall i \in G_j, \forall j, (t > 0) \\ p_{j,t,i} \leq Z_{j,t,i} \quad \forall i \in G_j, \forall j, (t > 0) \\ p_{j,t,i} \leq Z_{j,t-1,i} \quad \forall i \in G_j, \forall j, (t > 0) \end{array} \right\}.$$



$$\left\{ \begin{array}{c} p_{j,t,k} = (Z_{j,t,k} \wedge Z_{j,t-1,k}) = (0 \wedge 1) = 0 \\ \vee \\ p_{j,t,m} = (Z_{j,t,m} \wedge Z_{j,t-1,m}) = (1 \wedge 0) = 0 \\ \Rightarrow \\ \sum_{i \in G_j} p_{j,t,i} = p_{j,t,k} + p_{j,t,m} = 0 + 0 = \mathbf{0} \geq \mathbf{1} \\ \textbf{infeasible} \end{array} \right\}$$

$$
\left\{
\begin{array}{c}
p_{j,t,k} = (Z_{j,t,k} \wedge Z_{j,t-1,k}) = (1 \wedge 1) = 1 \\
\vee \\
p_{j,t,m} = (Z_{j,t,m} \wedge Z_{j,t-1,m}) = (1 \wedge 0) = 0 \\
\Rightarrow \\
\sum_{i \in G_j} p_{j,t,i} = p_{j,t,k} + p_{j,t,m} = \\
1 + 0 = 1 \geq 1 \\
\textbf{feasible}
\end{array}
\right\}
$$

$$
\left\{
\begin{array}{c}
p_{j,t+1,k} = (Z_{j,t+1,k} \wedge Z_{j,t,k}) = (0 \wedge 1) = 0 \\
\vee \\
p_{j,t+1,m} = (Z_{j,t+1,m} \wedge Z_{j,t,m}) = (1 \wedge 1) = 1 \\
\Rightarrow \\
\sum_{i \in G_j} p_{j,t+1,i} = p_{j,t+1,k} + p_{j,t+1,m} = \\
0 + 1 = 1 \geq 1 \\
\textbf{feasible}
\end{array}
\right\}
$$



Formulação Final

Dinâmica:

$$\mu_t = A^t \hat{x}_0 + \sum_{s=0}^{t-1} A^{t-s-1} B u_s$$

Posição Inicial:

$$\mu_T = \hat{x}_{\text{goal}}.$$

Formulação Final

Obstacle avoidance:

$$\sum_{i \in G_j} p_{j,t,i} \geq 1$$

$$p_{j,t,i} \geq Z_{j,t,i} + Z_{j,t-1,i} - 1$$

$$p_{j,t,i} \leq Z_{j,t,i}$$

$$p_{j,t,i} \leq Z_{j,t-1,i}$$

$$p_{j,t,i} \geq 0.$$

Formulação Final

Risk Allocation:

$$c_{i,t}(\delta_{jt}) + b_i - a_i^T \mu_t \leq \bar{M}(1 - Z_{jti})$$

$$\Rightarrow \left(\bar{y}_0 + \sum_{n=0}^{N-1} M_n \lambda_{njt} \right) \gamma_{it} + b_i - a_i^T \mu_t \leq \bar{M}(1 - Z_{jti})$$

$$\sum_j \sum_t \delta_{jt} \leq \Delta \Rightarrow \sum_j \sum_t (1 - \bar{x}_0 - \sum_{n=0}^{N-1} \lambda_{njt}) \leq 2 \cdot \Delta.$$

Formulação Final

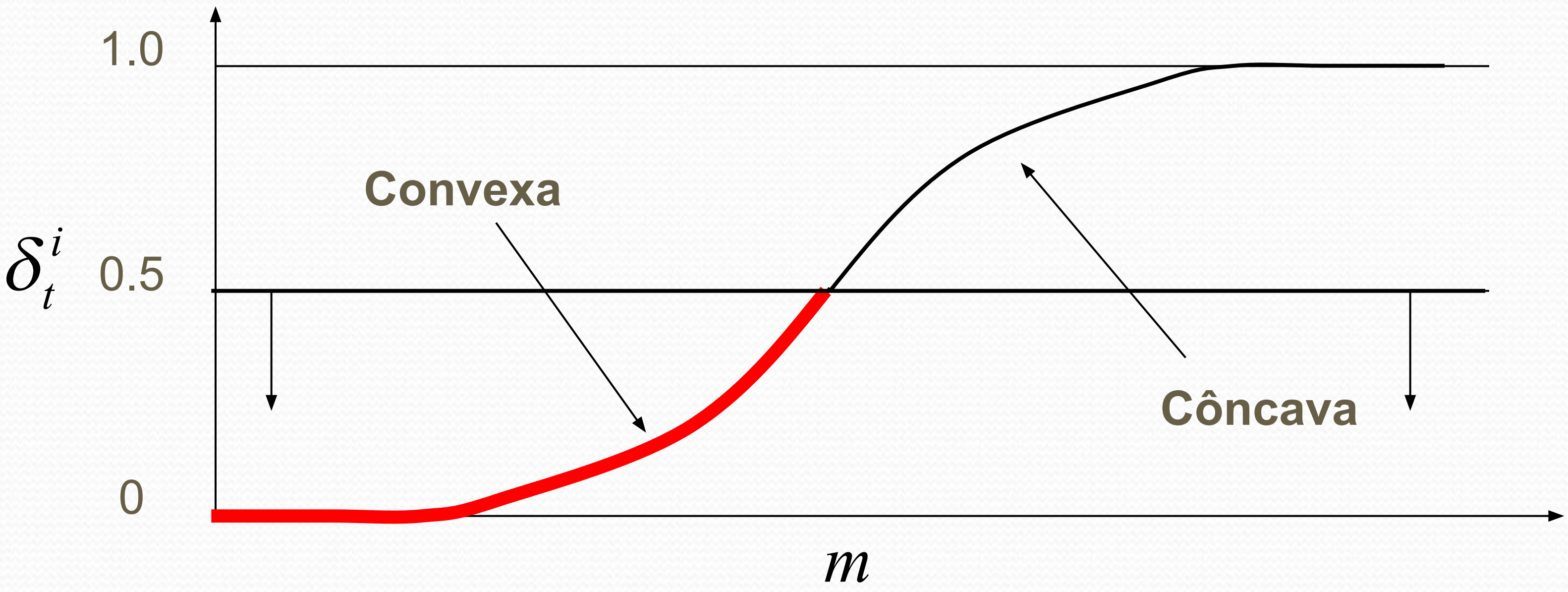
Domínio das variáveis:

$$\mu_t, u_t, \lambda_{njt} \geq 0, Z_{jti} \in \{0, 1\}, p_{jti} \geq 0 .$$

Convexidade das Chance Constraints

$$\Delta \leq 0.5 \Rightarrow \forall t,i \;\; \delta_t^i \leq 0.5$$

$m^i\left(\delta_t^i\right)$ **(função de distribuição acumulada da distribuição Gaussiana)**

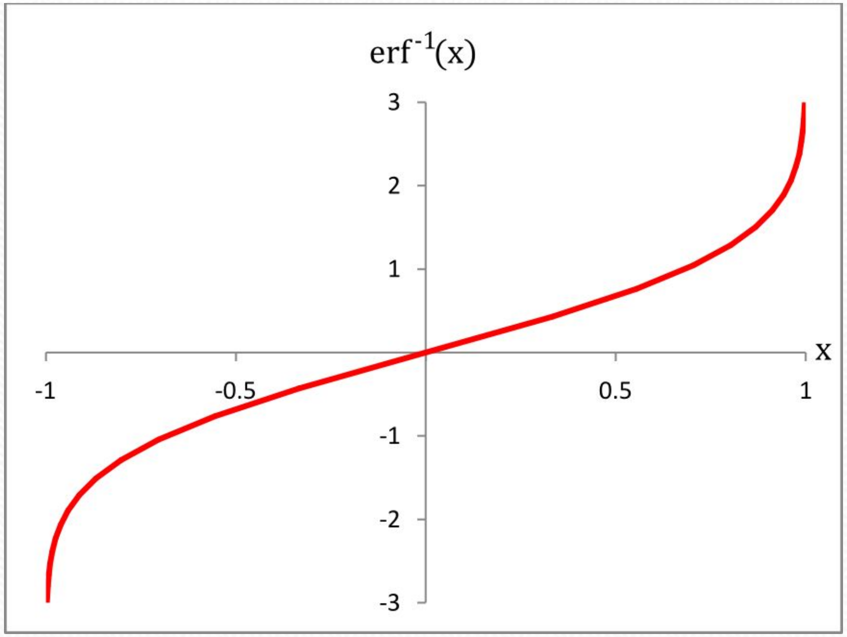


Usuários usualmente não querem incorrer em riscos que levem a uma probabilidade de falha maior que 50%.

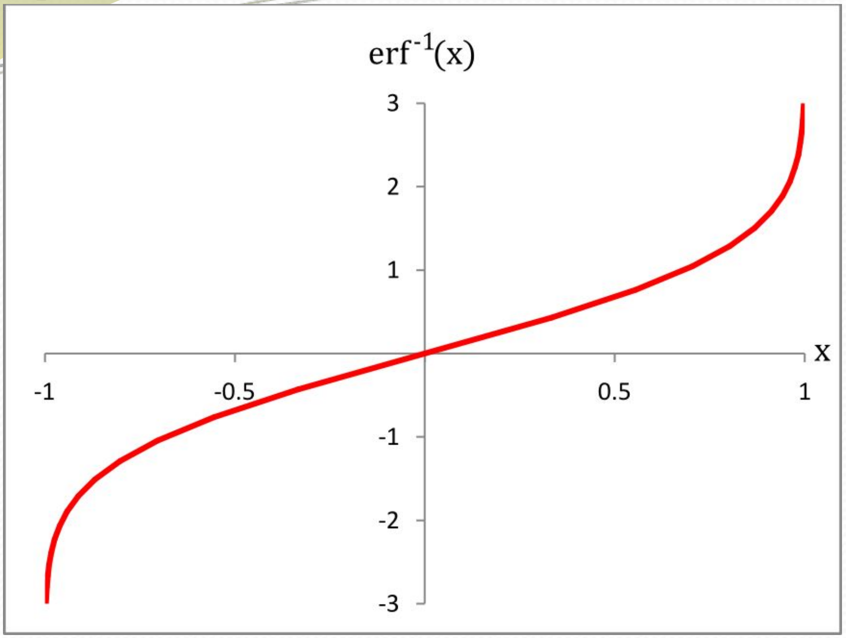
Convexidade das Chance Constraints

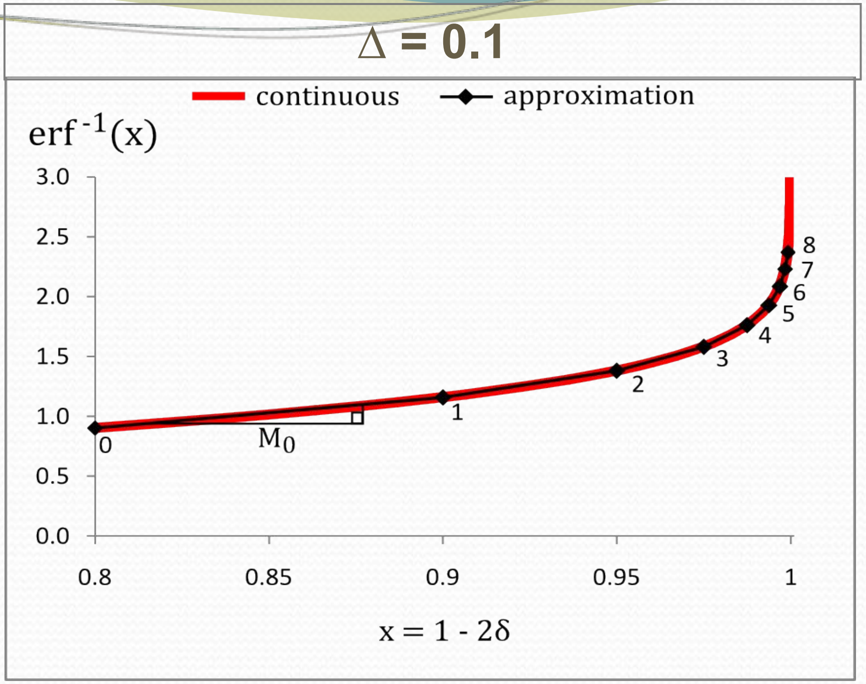
$$m_t^i\left(\delta_t^i\right) = -\sqrt{2 h_t^{iT} \Sigma_{x,t} h_t^i}\,\mathrm{erf}^{-1}\left(2\delta_t^i - 1\right)$$

Inversa da função de distribuição acumulada



Aproximando a função inversa do erro





$$\bar{x}_n = 1 - 2\left(\frac{\Delta}{2^n}\right) \quad n = 0, \ldots, N$$

$$\bar{y}_n = erf^{-1}(\bar{x}_n) \quad n = 0, \ldots, N$$

$$M_n = \frac{\bar{y}_{n+1} - \bar{y}_n}{\bar{x}_{n+1} - \bar{x}_n} \quad n = 0, \ldots, N-1$$

$$1 - 2\delta_{jt} = \bar{x}_0 + \sum_{n=0}^{N-1} \lambda_{njt}$$

$$erf^{-1}(x) := \bar{y}_0 + \sum_{n=0}^{N-1} M_n \lambda_{njt}$$

$$0 \leq \lambda_{njt} \leq [\bar{x}_{n+1} - \bar{x}_n]$$

$$\lambda_{n+1,jt} > 0 \Rightarrow \lambda_{njt} = [\bar{x}_{n+1} - \bar{x}_n]$$

Referências

[1] Ali A. Jalali and Vahid Nadimi, "A Survey on Robust Model Predictive Control from 1999-2006".

[2] Dennis Harald van Hessem. Stochastic inequality constrained closed-loop model predictive control with application to chemical process operation. PhD thesis, Delft University of Technology, 2004