

FORMULÁRIO: P1

$$\frac{dy}{dx} + p(x)y(x) = q(x) \quad y(x) = e^{-\int p(u)du} \left[\int e^{\int p(u)du} q(s)ds + C \right]$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\vec{r} = r\hat{r}, \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}, \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

$$\vec{r} = \rho\hat{\rho} + z\hat{k}, \quad \vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k}, \quad \vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{\phi} + \ddot{z}\hat{k}$$

$$\vec{r} = r\hat{r}, \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\hat{\phi}, \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\theta} + (r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\hat{\phi}$$

$$\vec{\nabla} = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi} \quad \vec{\nabla} = \frac{\partial}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial}{\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla} \wedge \vec{B} = \frac{1}{r\sin\theta} \left[\frac{\partial}{\partial\theta}(\sin\theta B_\phi) - \frac{\partial B_\theta}{\partial\phi} \right] \hat{r} + \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial B_r}{\partial\phi} - \frac{\partial(rB_\phi)}{\partial r} \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial(rB_\theta)}{\partial r} - \frac{\partial B_r}{\partial\theta} \right] \hat{\phi}$$

$$\vec{\nabla} \wedge \vec{A} = \left(\frac{1}{\rho} \frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho} \right) \hat{\phi} + \left(\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi} \right) \hat{k}$$

$$\vec{F}(\vec{r}) = -\frac{GM_T m}{r^2} \hat{r} \quad V(r) = -\int_{r_0}^r F(r') dr' \quad V_{\text{efetivo}} = \frac{L^2}{2mr^2} + V(r) \\ \vec{F} = -\vec{\nabla} V$$

$$\int_{r_0}^r \frac{dr}{(E - V(r) - \frac{L^2}{2mr^2})^{1/2}} = \sqrt{\frac{2}{m}} t \quad mr^2\dot{\theta} = L \quad \vec{L} = \vec{r} \wedge \vec{p}$$

$$\int_{r_0}^r \frac{L}{mr^2} \frac{dr}{\left(E - V(r) - \frac{L^2}{2mr^2} \right)^{1/2}} = \sqrt{\frac{2}{m}} (\theta - \theta_0)$$

$$\frac{1}{r} = B + A \cos\theta \text{ onde } B > A \text{ elipse com } B = \frac{1}{a(1-\epsilon^2)} \text{ e } A = \epsilon B$$

$$B = A \text{ parábola com } B = A = \frac{1}{a}$$

$$B < A \text{ hipérbole com } \begin{cases} 0 < B < A \text{ (ramo +)} \\ -A < B < 0 \text{ (ramo -)} \end{cases} \quad B = \frac{\pm 1(\text{ramo} \pm)}{a(\epsilon^2 - 1)} \text{ e } A = \pm \epsilon B(\text{ramo} \pm)$$

$$\frac{d^2 u}{d\theta^2} + u = -\frac{m}{L^2 u^2} F\left(\frac{1}{u}\right), \quad u = 1/r \quad \left(\frac{du}{d\theta} \right)^2 + u^2 = \frac{2m}{L^2} \left[E - V\left(\frac{1}{u}\right) \right]$$

$$E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + V(r)$$

FORMULÁRIO: P2

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\vec{r} = r\hat{r}, \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}, \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

$$\vec{r} = \rho\hat{\rho} + z\hat{k}, \quad \vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k}, \quad \vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{\phi} + \ddot{z}\hat{k}$$

$$\vec{r} = r\hat{r}, \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\hat{\phi}, \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\theta} + (r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\hat{\phi}$$

$$\vec{\nabla} = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi} \qquad \vec{\nabla} = \frac{\partial}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial}{\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla} \wedge \vec{B} = \frac{1}{r\sin\theta} \left[\frac{\partial}{\partial\theta}(\sin\theta B_\phi) - \frac{\partial B_\theta}{\partial\phi} \right] \hat{r} + \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial B_r}{\partial\phi} - \frac{\partial(rB_\phi)}{\partial r} \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial(rB_\theta)}{\partial r} - \frac{\partial B_r}{\partial\theta} \right] \hat{\phi}$$

$$\vec{\nabla} \wedge \vec{A} = \left(\frac{1}{\rho} \frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho} \right) \hat{\phi} + \left(\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi} \right) \hat{k}$$

$$\left(\frac{d\vec{A}}{dt} \right)_s = \left(\frac{d\vec{A}}{dt} \right)_{s'} + \vec{\omega} \wedge \vec{A}$$

$$m \left(\frac{d^2\vec{r}}{dt^2} \right)_{s'} = m \left(\frac{d^2\vec{r}}{dt^2} \right)_s - 2m\vec{\omega} \wedge \vec{v}_{s'} - m\vec{\omega} \wedge (\vec{\omega} \wedge \vec{r}) - m\vec{\dot{\omega}} \wedge \vec{r}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k}, \quad L = T - V, \quad H = \sum_{k=1}^f p_k \dot{q}_k - L$$

$$\dot{q}_k = \frac{\partial H}{\partial p_k} \quad \dot{p}_k = -\frac{\partial H}{\partial q_k} \quad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t} \quad \text{onde, } k = 1, 2, \dots, f$$

FORMULÁRIO

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\vec{r} = r\hat{r}, \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}, \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} \quad (\text{polares})$$

$$\vec{r} = \rho\hat{\rho} + z\hat{k}, \quad \vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k}, \quad \vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{\phi} + \ddot{z}\hat{k} \quad (\text{cilíndricas})$$

$$\vec{r} = r\hat{r}, \quad \vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\hat{\phi}, \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\theta} + (r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\hat{\phi} \quad (\text{esféricas})$$

$$\vec{\nabla} = \frac{\partial}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{k} \quad \vec{\nabla} = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla} \wedge \vec{A} = \left(\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) \hat{\phi} + \left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right) \hat{k}$$

$$\vec{\nabla} \wedge \vec{B} = \frac{1}{r\sin\theta} \left[\frac{\partial}{\partial \theta} (\sin\theta B_\phi) - \frac{\partial B_\theta}{\partial \phi} \right] \hat{r} + \frac{1}{r} \left[\frac{1}{\sin\theta} \frac{\partial B_r}{\partial \phi} - \frac{\partial(r B_\phi)}{\partial r} \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial(r B_\theta)}{\partial r} - \frac{\partial B_r}{\partial \theta} \right] \hat{\phi}$$

$$\vec{F}(\vec{r}) = -\frac{GM_T m}{r^2} \hat{r} \quad V(r) = -\int_{r_0}^r F(r') dr' \quad V_{\text{efetivo}} = \frac{L^2}{2mr^2} + V(r) \\ \vec{F} = -\vec{\nabla} V$$

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$$\int_{r_0}^r \frac{L}{mr^2} \frac{dr}{\left(E - V(r) - \frac{L^2}{2mr^2} \right)^{1/2}} = \sqrt{\frac{2}{m}} (\theta - \theta_0)$$

$$\frac{1}{r} = B + A \cos \theta \quad \text{onde } B > A \quad \text{elipse com } B = \frac{1}{a(1-\epsilon^2)} \quad \text{e } A = \epsilon B$$

$$B = A \quad \text{parábola com } B = A = \frac{1}{a}$$

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$$\frac{d^2 u}{d\theta^2} + u = -\frac{m}{L^2 u^2} F\left(\frac{1}{u}\right), \quad u=1/r \quad \left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2m}{L^2} \left[E - V\left(\frac{1}{u}\right) \right]$$

$$E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + V(r)$$

$$\left(\frac{d\vec{A}}{dt}\right)_s = \left(\frac{d\vec{A}}{dt}\right)_{s'} + \vec{\omega} \wedge \vec{A}$$

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_s = m\left(\frac{d^2\vec{r}}{dt^2}\right)_{s'} + 2m\vec{\omega} \wedge \vec{v}' + m\vec{\omega} \wedge (\vec{\omega} \wedge \vec{r}) + m\vec{\dot{\omega}} \wedge \vec{r}$$

$$Q_k = \sum_{i=1}^N F_{ix} \frac{\partial x_i}{\partial q_k} + F_{iy} \frac{\partial y_i}{\partial q_k} + F_{iz} \frac{\partial z_i}{\partial q_k} \quad Q_k = -\frac{\partial V}{\partial q_k} \quad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} = Q_k$$

$$\left(\frac{\partial T}{\partial \dot{q}_k}\right)=p_k$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=0 \quad \text{onde } L=T-V$$

$$T=\sum_{k=1}^{3N}\sum_{l=1}^{3N}\frac{1}{2}A_{kl}\dot{q}_k\dot{q}_l+\sum_{k=1}^{3N}B_k\dot{q}_k+T_0$$

$$A_{kl}=\sum_{i=1}^N m_i \left(\frac{\partial x_i}{\partial q_k} \frac{\partial x_i}{\partial q_l} + \frac{\partial y_i}{\partial q_k} \frac{\partial y_i}{\partial q_l} + \frac{\partial z_i}{\partial q_k} \frac{\partial z_i}{\partial q_l} \right)$$

$$B_k=\sum_{i=1}^N m_i \left(\frac{\partial x_i}{\partial q_k} \frac{\partial x_i}{\partial t} + \frac{\partial y_i}{\partial q_k} \frac{\partial y_i}{\partial t} + \frac{\partial z_i}{\partial q_k} \frac{\partial z_i}{\partial t} \right)$$

$$T_0=\sum_{i=1}^N\frac{1}{2}m_i\left[\left(\frac{\partial x_i}{\partial t}\right)^2+\left(\frac{\partial y_i}{\partial t}\right)^2+\left(\frac{\partial z_i}{\partial t}\right)^2\right]$$

$$H=\sum_{k=1}^f p_k \dot{q}_k - L$$

$$\dot{q}_k=\frac{\partial H}{\partial p_k}$$

$$\dot{p}_k=-\frac{\partial H}{\partial q_k}$$

$$\frac{\partial H}{\partial t}=-\frac{\partial L}{\partial t} \quad \text{com } k=1, 2, \dots, f$$