A Multi-objective Approach of Differential Evolution Optimization Applied to Electromagnetic Problems

Gustavo C. Tenaglia , Luiz Lebensztajn

A Multi-objective Approach of Differential Evolution Optimization Applied to Electromagnetic Problems

Gustavo C. Tenaglia – Majoring in Electrical Engineering

Extracurricular Activity that introduces undergraduate students to “scientific world”.

Subject: **optimization on electromagnetic problems.**

Coordinated by: **Prof. Luiz Lebensztajn** – Ph.D in Electrical Engineering

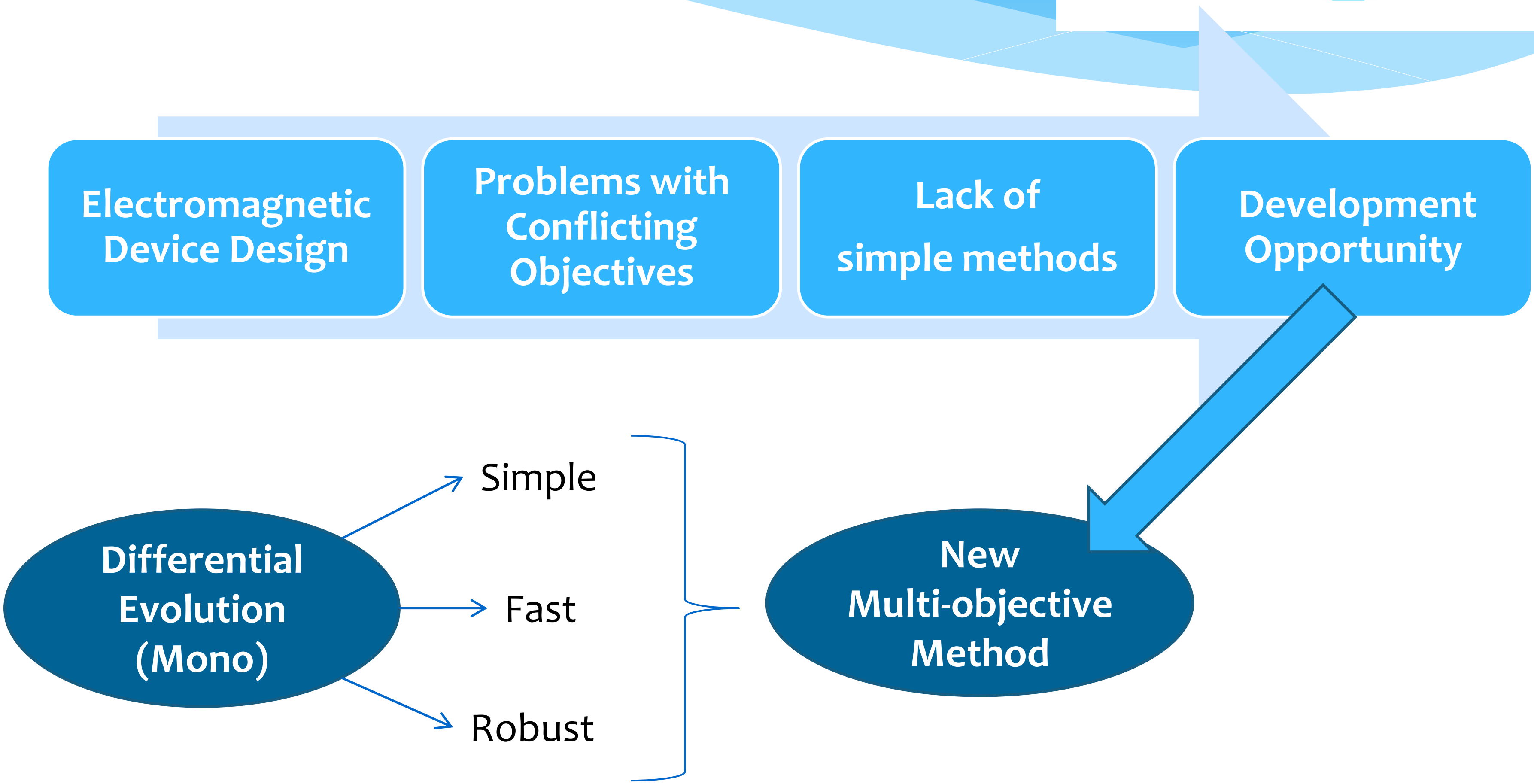
Where it was developed?

Sponsored by:

Content

- Introduction
- The Proposed Algorithm - *MultiDE*
- Problems and Results
  - A) Analytical Problems (High dimension)
  - B) Device Design – Brushless DC motor (with constrains)
  - C) Device Design – Team 25 (with no constrains)
- Conclusion

Introduction



Introduction

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Electromagnetic Device Design → Problems with Conflicting Objectives → Lack of simple methods → Development Opportunity

New Multi-objective Method → Based on DE* → *"MultiDE"*

* DE = Differential Evolution

Introduction

Electromagnetic Device Design

Problems with Conflicting Objectives

Lack of simple methods

Development Opportunity

Why not?

New Multi-objective Method

Based on DE*

"MultiDE"

Simple?

Fast?

Robust?

* DE = Differential Evolution

Introduction

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Electromagnetic Device Design → Problems with Conflicting Objectives → Lack of simple methods → Development Opportunity

Based on DE* → "MultiDE" → Necessity of Pareto Treatment

* DE = Differential Evolution

Introduction

Electromagnetic Device Design

Problems with Conflicting Objectives

Lack of simple methods

Development Opportunity

Necessity of Pareto Treatment

SPEA*

*SPEA = Strength Pareto Evolutionary Algorithms (Zittler and Theile)

Introduction

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Electromagnetic Device Design → Problems with Conflicting Objectives → Lack of simple methods → Development Opportunity

SPEA*

- External Pareto File
- Classify Elements with a Strength
- Reduce Pareto Set with a Cluster

*SPEA = Strength Pareto Evolutionary Algorithms (Zittler and Theile)

The Proposed Algorithm

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MultiDE: Step by Step

As original DE, it was designed to be SIMPLE.

The Proposed Algorithm

Facilities:

Low number of parameters

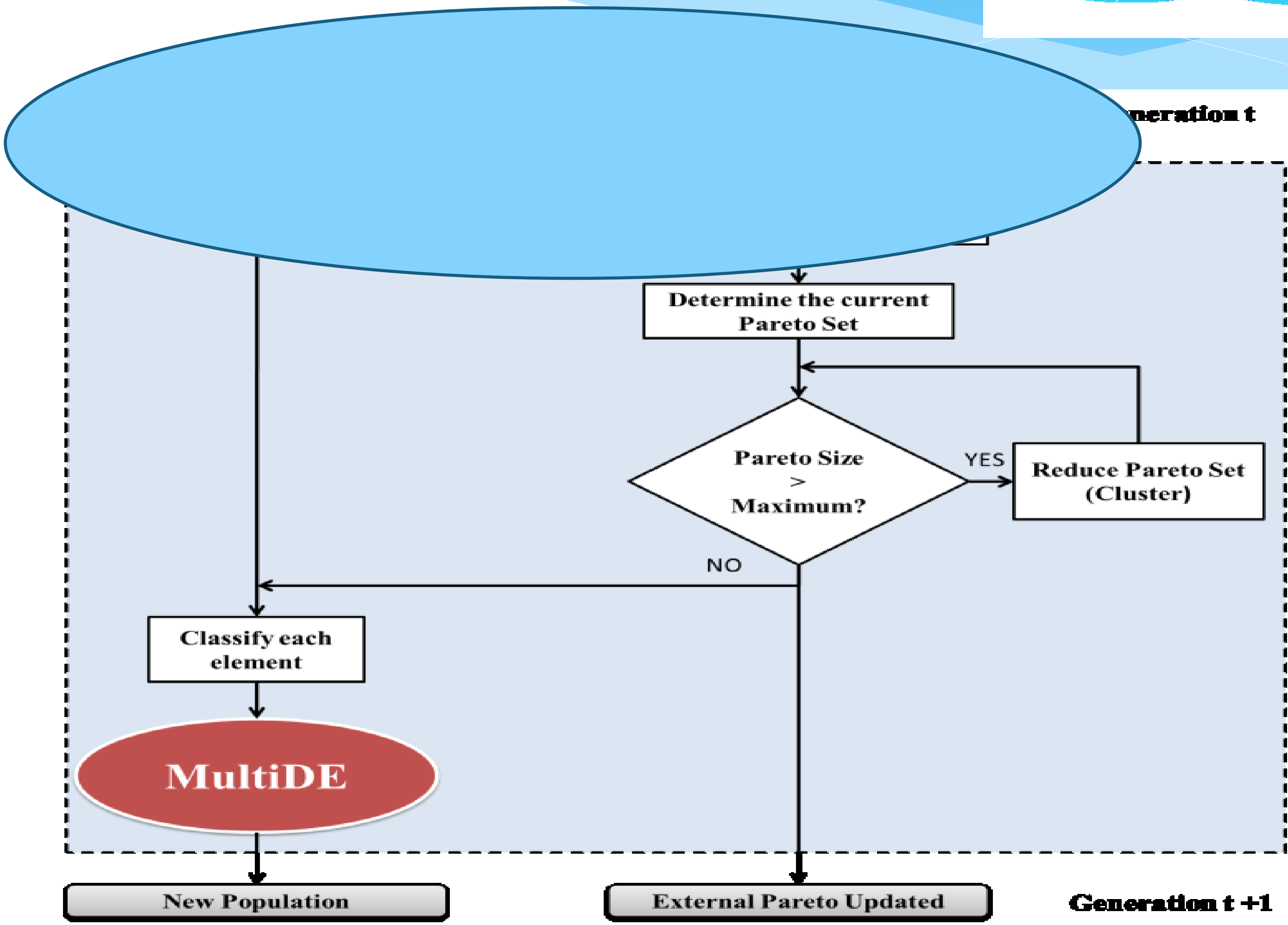
- Mutation Factor – **MF** ∈ ℝ [0; 0.5] (as in original DE)
- Crossover Rate – **CR** ∈ ℝ [0;1] (as in original DE)
- Population Size – **NP** ∈ ℕ (as in original DE)
- Maximum size of the Pareto Set – **MaxP** ∈ ℕ (new parameter)

Easy to be coded and modified (by using Matlab, for example)

The Proposed Algorithm

1. **The current Population and the External Pareto set are put together.**

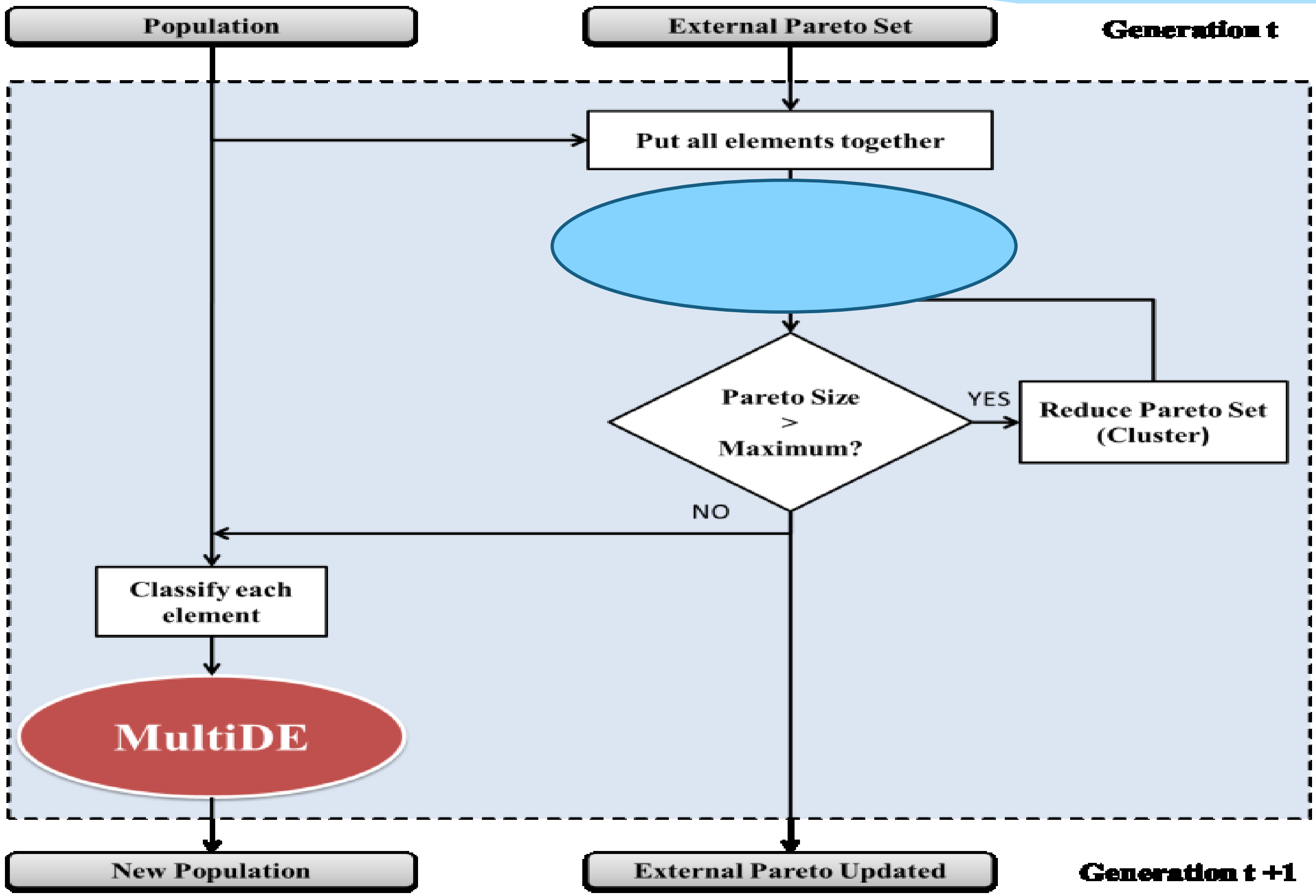
The Proposed Algorithm



The Proposed Algorithm

1. The current Population and the External Pareto set are put together;
2. **Non dominated solutions form the current Pareto Set;**

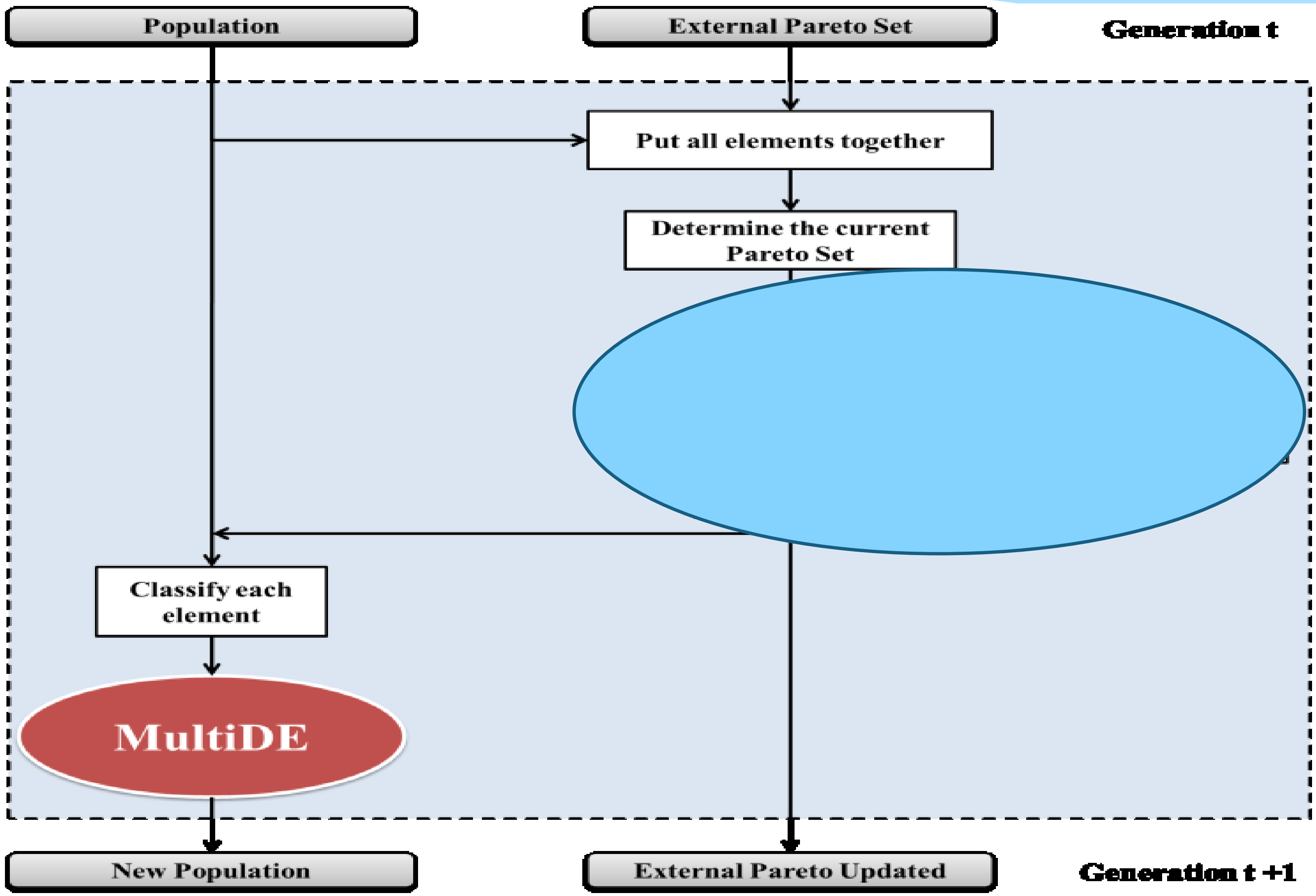
The Proposed Algorithm



The Proposed Algorithm

1. The current Population and the External Pareto set are put together;
2. Non dominated solutions form the current Pareto Set;
3. **If the number of elements on the Pareto Set is greater than MaxP, then a reduction is done (cluster), as proposed in SPEA.**

The Proposed Algorithm



The Proposed Algorithm

1. The current Population and the External Pareto set are put together;
2. Non dominated solutions form the current Pareto Set;
3. If the number of elements on the Pareto Set is greater than MaxP, then a reduction is done (cluster), as proposed in SPEA;
4. **Both Pareto Set and current Population are placed in the same file. Each element is classified and sorted by its strength (S):**

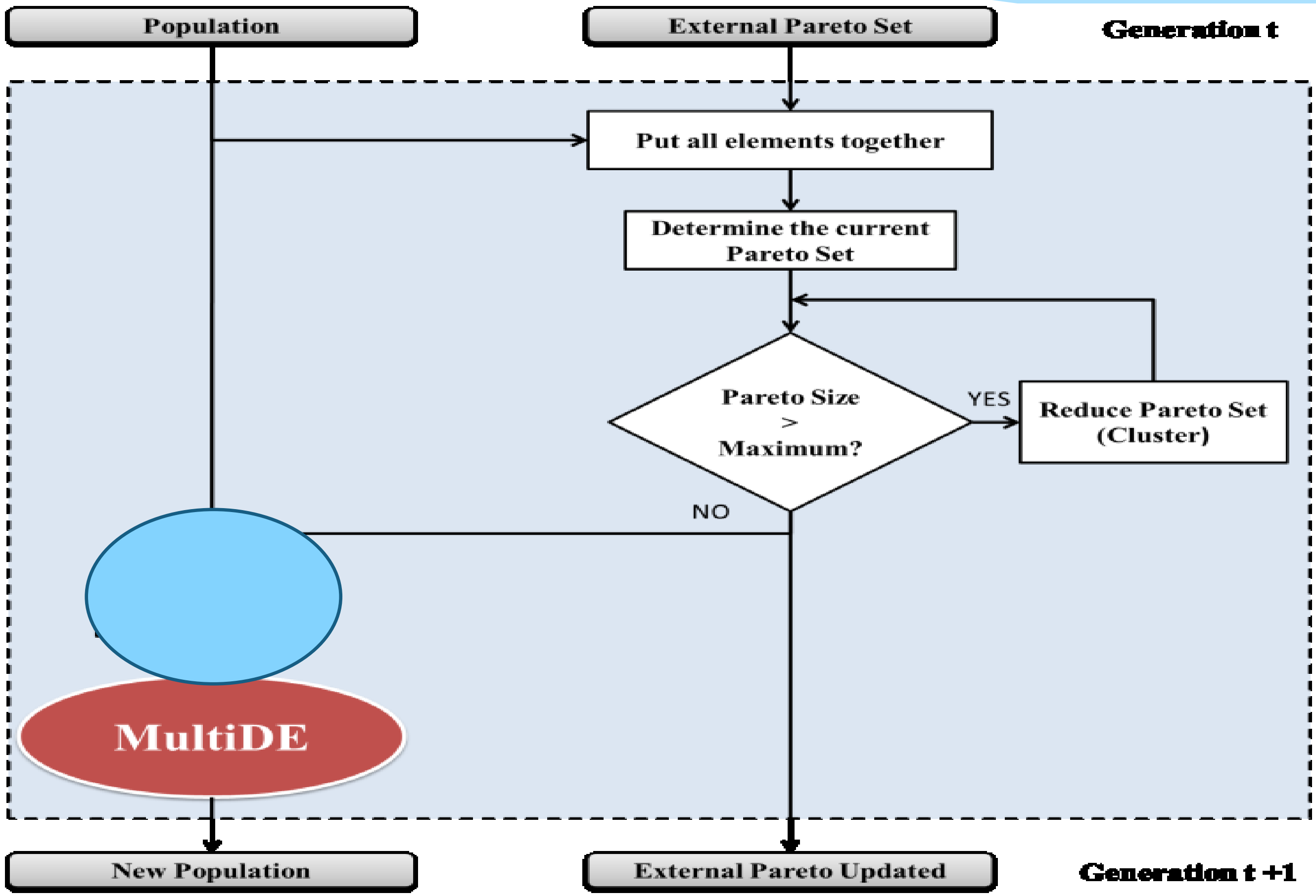
   - ✓ **Strength of <u>non-dominated</u> elements:**

     $S_i = \frac{d_i}{NP+1}$, **where d_i the number of elements that this solution covers.**

   - ✓ **Strength of <u>dominated</u> elements:**

     Is the number of elements that covers it;

The Proposed Algorithm



The Proposed Algorithm

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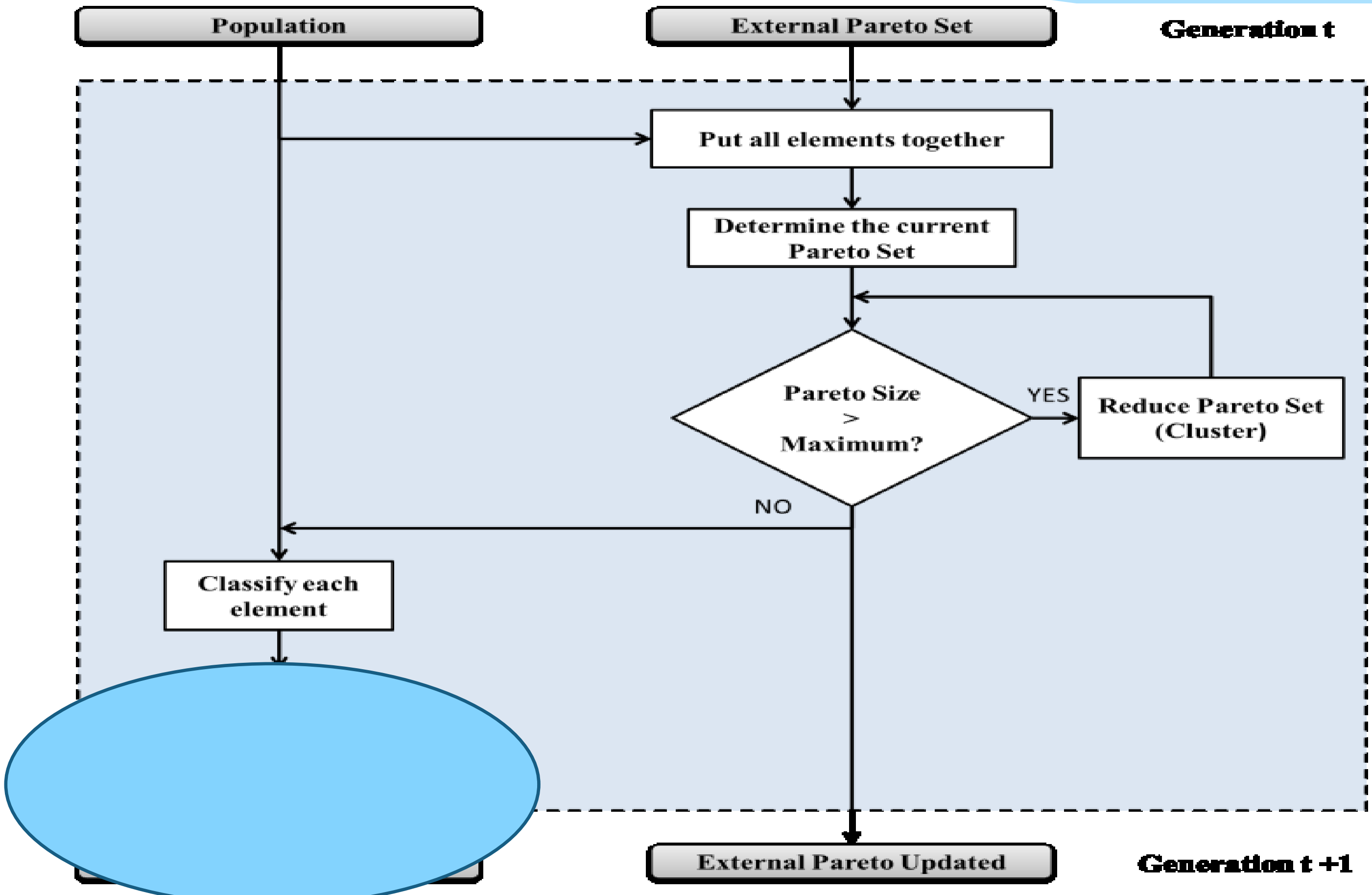
1. The current Population and the External Pareto set are put together;
2. Current Pareto Set is formed by non-dominated solutions;
3. If the number of elements on the Pareto Set is greater than MaxP;
   a reduction is done (cluster) as proposed in SPEA;
4. Both Pareto Set and current Population are placed in the same file. Each
   element is classified and sorted by its strength (S);
5. **Elements are ranked according its Strength and *MultiDE* is finally applied, creating a new population to the next generation.**

The Proposed Algorithm



The Proposed Algorithm

The process keeps running until at least one of the stop criteria is attained, for example: number of evaluations, deviation between runs or time processing.

"MultiDE"

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* **The main idea of MultiDE was based on the original method (DE). First NP solutions will create the next population:**

1) A difference between two random vectors ($X_{rand_1}(t)$ and $X_{rand_2}(t)$) is weighted by MF.
2) This vector is added to one of the first NP solutions.
3) A new mutated element ($X_j(t+1)$) is created:

$$X_j(t+1) = X_j(t) + MF.\left(X_{rand_1}(t) - X_{rand_2}(t)\right)$$

Where j={1,2,3... NP}

Is importante to notice that MaxP first solutions belongs to the current Pareto Set

"MultiDE"

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* **In order to increase diversity, the mutated vector suffers a "Crossover".**
  On this step, $X_{j,i}(t+1)$ is mixed with $X_j(t)$ as follows:

$$X_{j,i}(t+1) = \begin{cases} X_{j,i}(t+1) & if \quad (randb(k) \leq CR) \\ X_{j,i}(t) & if \quad (randb(k) > CR) \end{cases}$$

i ={ 1, … , Number of variables}

j = {1, … , NP}

- *randb(k)* is a real random number between 0 and 1.
- This number is compared with CR for every *i*th position of vector of variables.
- If *randb(k)* is lower than CR, the *i*th from $X_{j,i}(t+1)$ is maintained, otherwise the *i*th Element of the solution will take this position.

Problems and Results

***MultiDE* was tried on these three situations:**

A) Test functions (Deb1, Deb2 and Deb3)

B) The Brushless DC Wheel Motor Problem (Brisset and Brochet)

C) The Optimization of the Die Press mold (TEAM 25)

Problems and Results

***MultiDE* was tried on these three situations:**

A) **Test functions (Deb1, Deb2 and Deb3)**

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A) Deb functions

Those function are benchmarks proposed by Deb to try optimization methods.

- ✓ They are high dimensional functions (D = 30).
- ✓ Deb1, Deb2 and Deb3 : solved by using MultiDE and **gamultiobj**[1] ("MultiGA")
- ✓ The Pareto Frontier of each problem is:
  - Deb1: convex
  - Deb2: non-convex
  - Deb3: discrete

[1]Genetic Algorithms (multi-objective) available on Matlab

A) Deb functions

- The main formulation to any “Deb problem” is represented as follows:

$$\begin{cases} min\left[f_1(x), f_2(x)\right] \\ x \in \mathbb{R} = \{x_1, \dots, x_m\} \ and \quad 0 \leq x_m \leq 1 \\ f_1(x) = x_1 \\ f_2(x) = g(x_2, \dots, x_m) \cdot h\left(f_1(x), g(x_2, \dots, x_m)\right) \end{cases}$$

To Deb1, Deb2 and Deb3, $g(x_2, \dots, x_m)$ is also the same:

$$g(x_2, \dots, x_{30}) = 1 + 9 \cdot \sum_{i=2}^{30} \frac{x_i}{29}$$

A) Deb functions

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Particularities on each "Deb" are represented below:

$$h(f_1, g)_{Deb1} = 1 - \sqrt{f_1/g}$$

$$h(f_1, g)_{Deb2} = 1 - (f_1/g)^2$$

$$h(f_1, g)_{Deb3} = 1 - \sqrt{f_1/g} - (f_1/g) \cdot sin(10\pi f_1)$$

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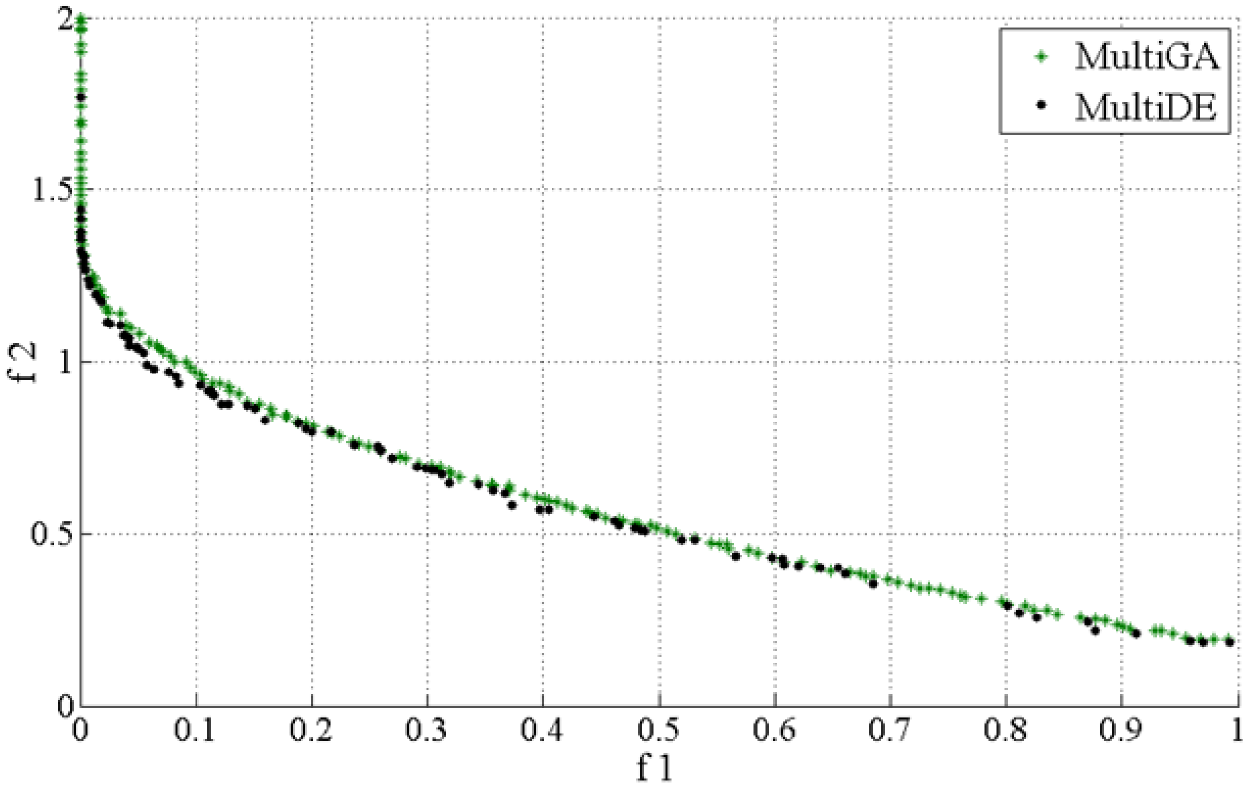
Deb functions:

Results

function: Deb1

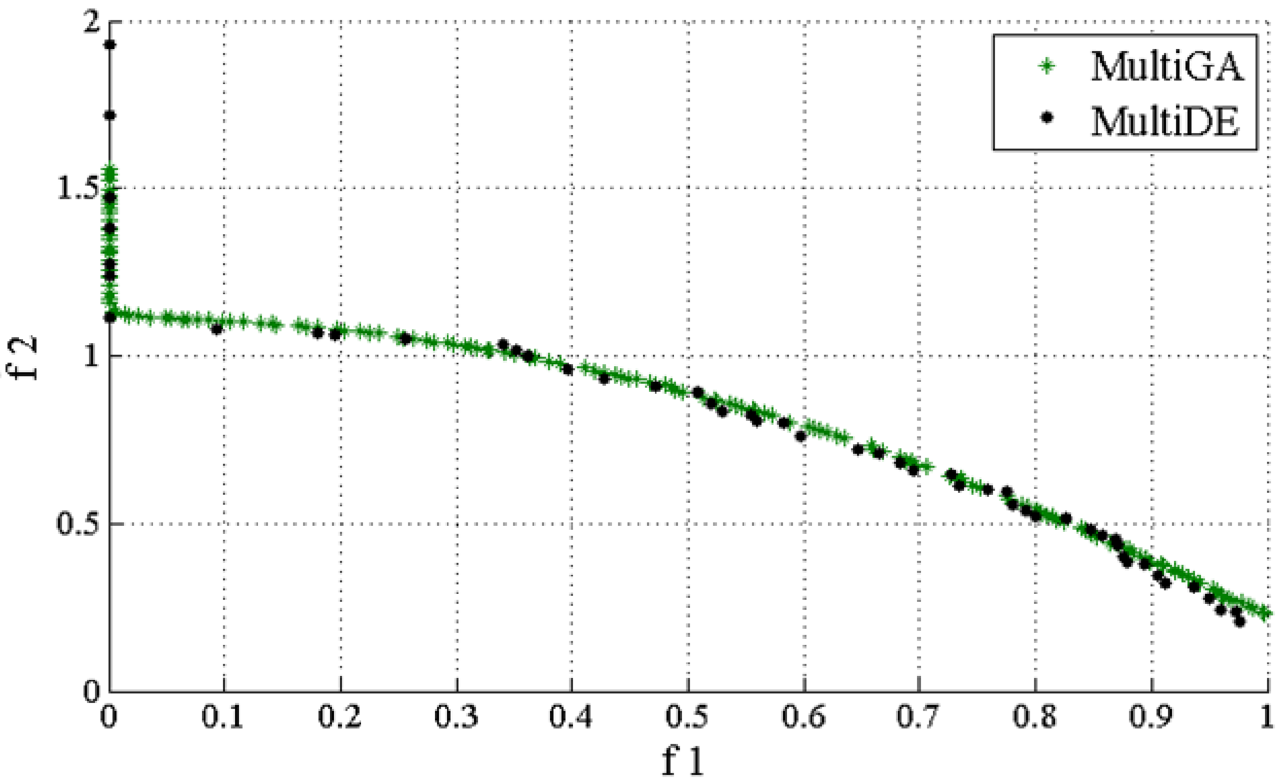


Common Parameters:
NP = 400
MF = 0.15
CR = 0.95

MaxP = 100 (*multiDE*)

Stop Criteria:
Nº of Iterations = 3000

function: Deb2

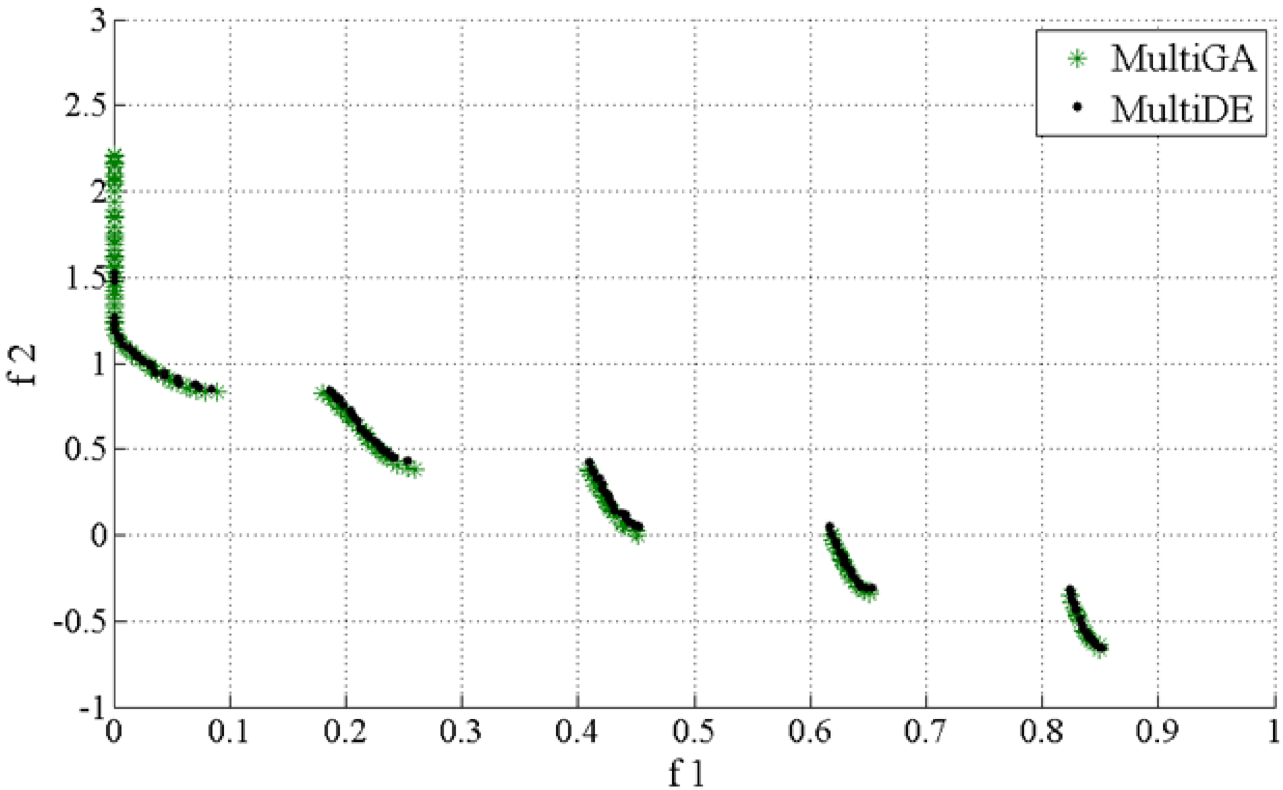


Common Parameters:
NP = 400
MF = 0.15
CR = 0.95

MaxP = 100 (*multiDE*)

Stop Criteria:
Nº of Iterations = 3000

function: Deb3



Common Parameters:
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Analytical Problems

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By applying similar conditions as:

1. The same number of elements on the Initial Population (NP)
2. Similar crossover (CR) and mutation (MF) rates
3. The same number of evaluations (3000)

Analytical Problems

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Analytical Problems

By applying similar conditions as:

1. The same number of elements on the Initial Population (NP)
2. Similar crossover (CR) and mutation (MF) rates
3. The same number of evaluations (3000)

MultiDE* and *MultiGA found similar Pareto Frontiers!

To the "Device Design Problems" analysis

just MultiDE results are shown

Problems and Results

A) Test Function – Deb

B) **The Brushless DC Wheel Motor Problem**

C) The Optimization of the Die Press mold (TEAM 25)

B) The Brushless DC Wheel Motor Problem

Problem was proposed by S. Brisset and P. Brochet[1]:

"The analytical model is composed of 78 non-linear equations.
The electric, magnetic, and thermal phenomena are taken into account[2]."

The goal of this design problems is:

Find a set of designs that:

Minimize the motor mass **and** Maximize its efficiency (respecting *constraints*).

[1] "Analytical model for the optimal design of a brushless DC wheel motor" COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, Vol. 24 No 3, pp.829 - 848, 2005.

[2] http://l2ep.univ-lille1.fr/come/benchmark-wheel-motor/Math.htm

The Brushless DC Wheel Motor Problem

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* Stator Diameter (D_s)……………………………………
* Magnetic Induction in the air gap (B_e)………………
* Current Density in the Conductors (δ)………………
* Magnetic Induction in the teeth (B_d) ………………
* Magnetic induction in the stator back iron (B_{cs})……

Parameters (Bounds)		
Ds [mm]	150	300
Be [T]	0.50	0.76
δ[A/mm²]	2.0	5.0
Bd [T]	0.9	1.8
Bcs [T]	100	100
	Lower	**Upper**

B) The Brushless DC Wheel Motor Problem

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* Outer Diameter (D_{ext})…………………………………………….
* Inner Diameter (D_{int})…………………………………………….
* Current on phases that doesn't demagnetize the magnets (I_{max})……
* Temperature of the magnets (T_a)……………………………………
* Determinant that calculate the slot height must be positive…………

Constraints	
Dext	<340mm
Dint	>76mm
Imax	>125A
Ta	<120°C
Det(Ds,δ,Bd,Bs)	>0

The Brushless DC Wheel Motor Problem: **Results**

B) The Brushless DC Wheel Motor Problem

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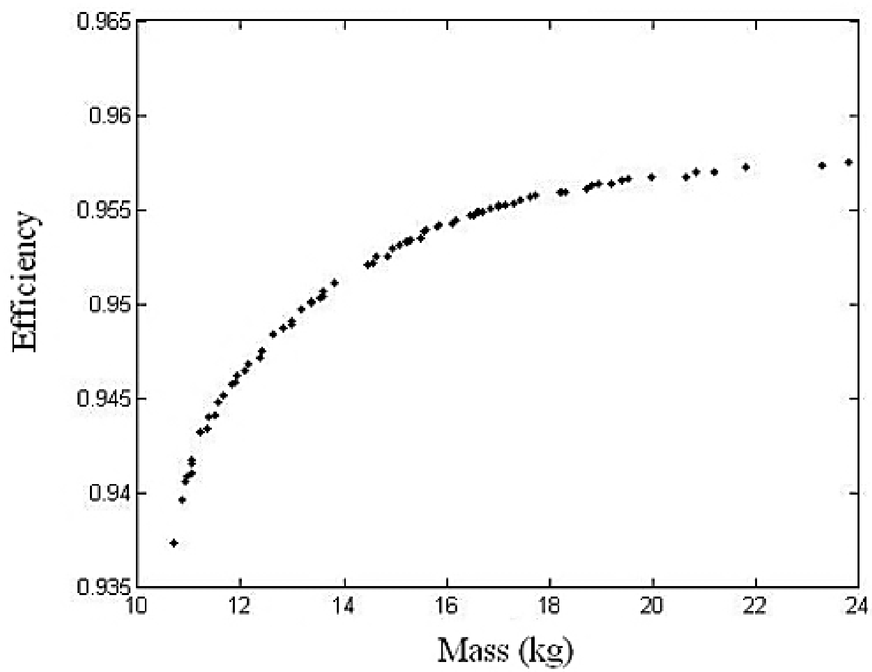
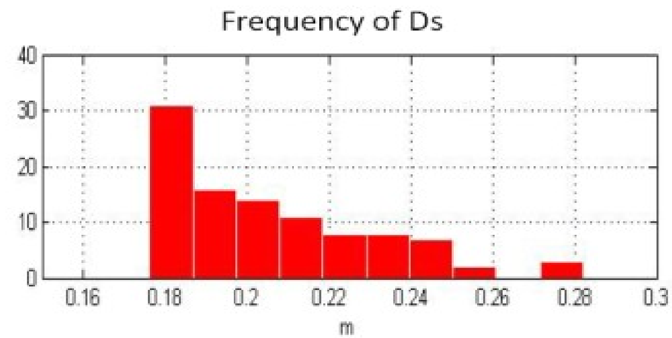
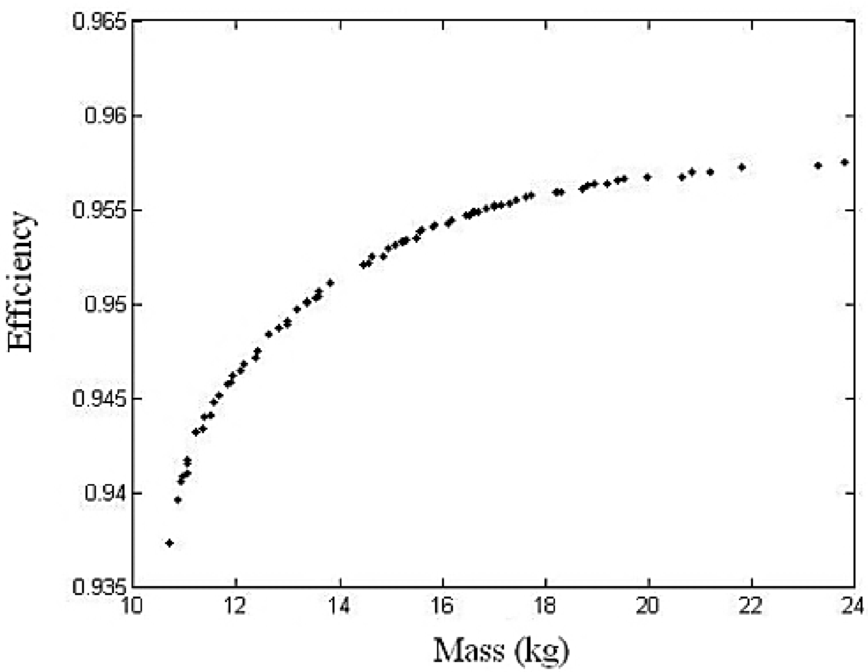
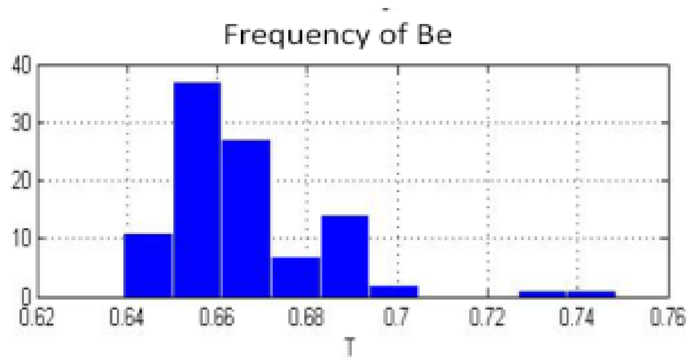




Stator Diameter (Ds)

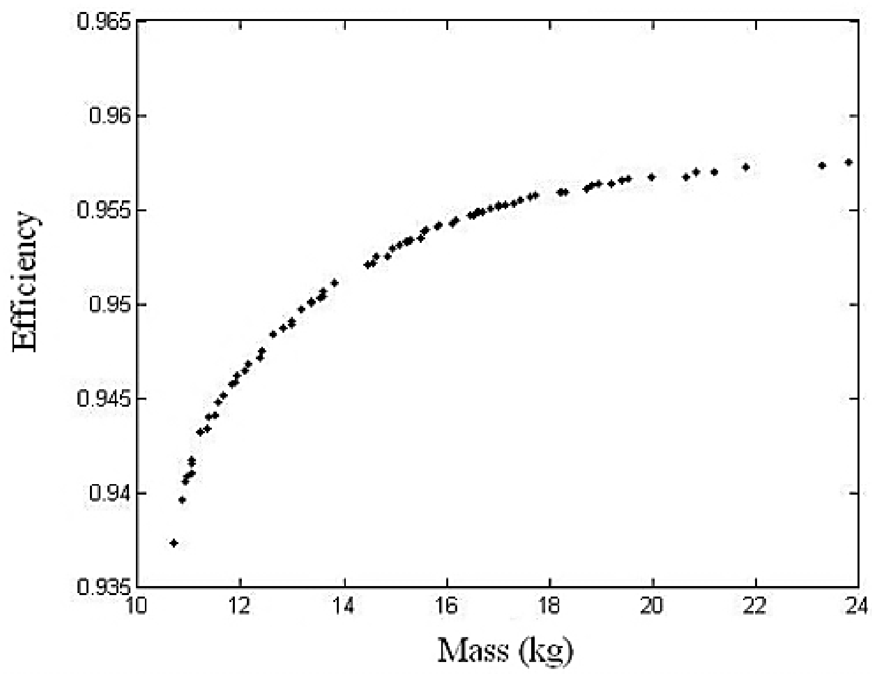
B) The Brushless DC Wheel Motor Problem

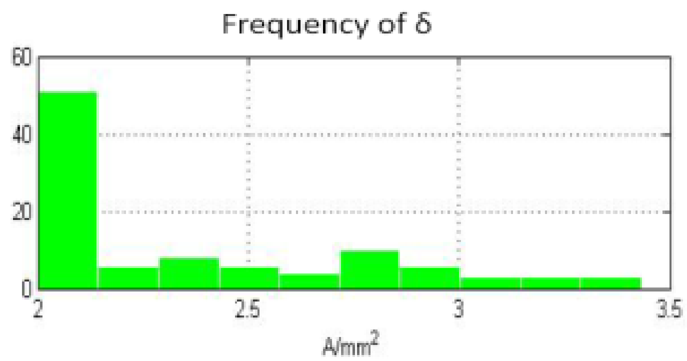




Magnetic Induction in the air gap (Be)

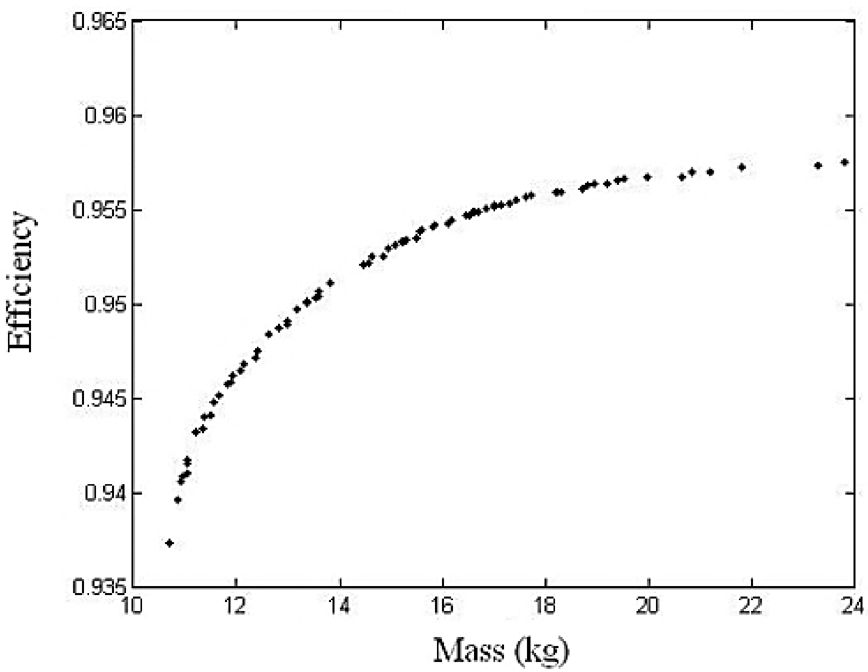
B) The Brushless DC Wheel Motor Problem

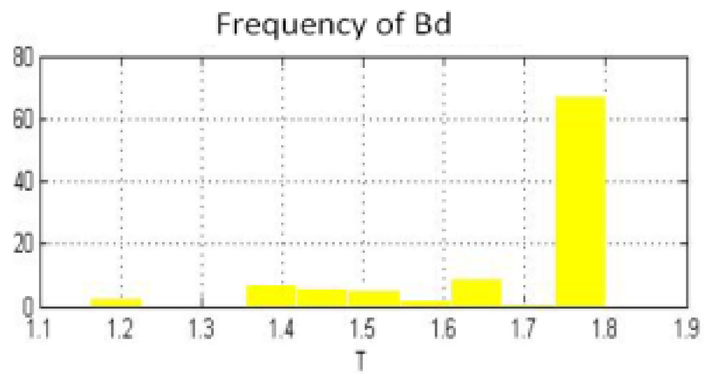




Current Density in the Conductors (δ)

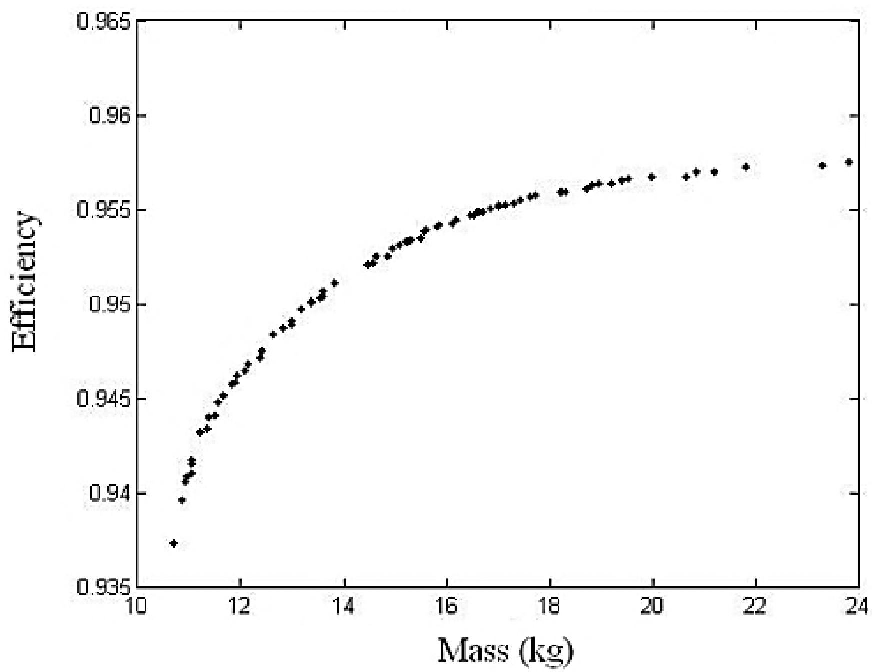
B) The Brushless DC Wheel Motor Problem

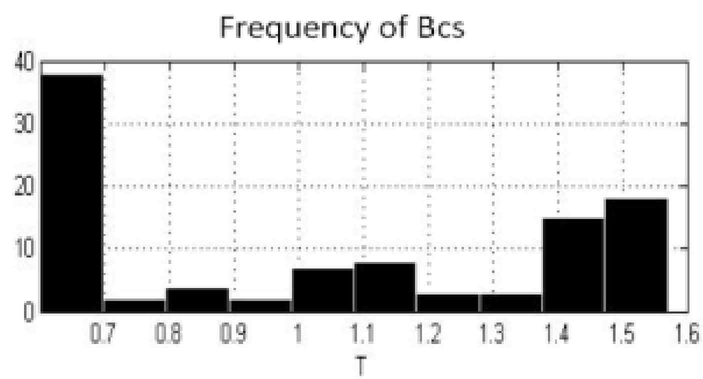




Magnetic Induction in the teeth (Bd)

B) The Brushless DC Wheel Motor Problem





Magnetic induction in the stator back iron (Bcs)

Problems and Results

A) Test Function – Deb

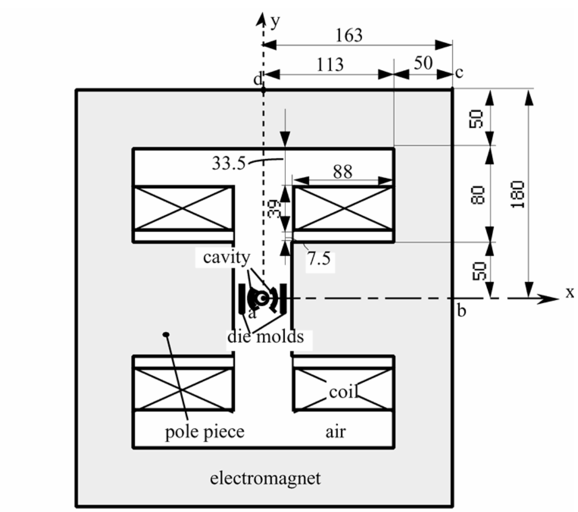
B) The Brushless DC Wheel Motor Problem

C) **The Optimization of the Die Press mold (TEAM 25)**

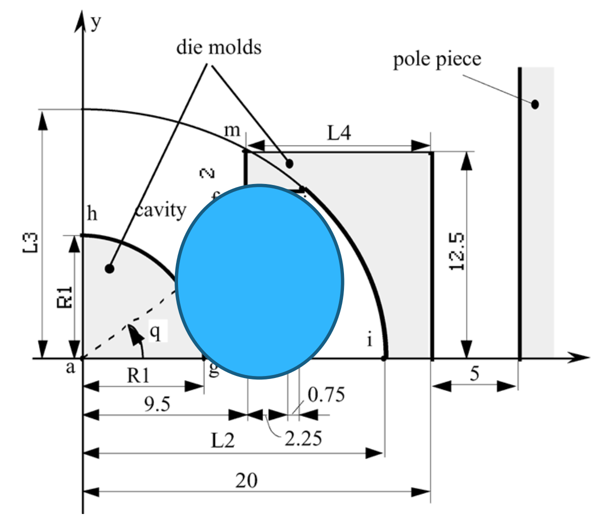
C) The Optimization of the Die Press Mold (TEAM 25)

The goal of TEAM workshop problem 25 is:

Optimize the shape of a die press mold to obtain <u>a radial magnetic induction</u> distribution on a specified cavity.



5 mm < R1 < 9.4 mm
12.6 mm < L2 < 18 mm
4 mm < L4 < 19 mm



C) The Optimization of the Die Press Mold (TEAM 25)

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Original (mono-objective) problem is minimize (function 1):

- $function\ 1 = \sum_{i=1}^{n}\left\{(B_{xip} - B_{xio})^2 + (B_{yip} - B_{yio})^2\right\}$

This problem can be analysed as a multi-objective problem, by introducing two error functions[1]:

- $function\ 2 = max\left(\left|\frac{B_p - B_o}{B_o}\right|\right) \times 100$ (magnitude error)
- $function\ 3 = max\left(\left|\theta_{B_p} - \theta_{B_o}\right|\right)$ (angle error)
- **Optimization problem:** minimize (f1, f2, f3)

p: calculated value
o: specified value

[1] L. Lebensztajn, J.-L. Coulomb, "TEAM Workshop Problem 25: A Multiobjective Analysis", *IEEE Transactions on Magnetics* vol. 40, no. 2, pp. 1042-1045, March 2004.

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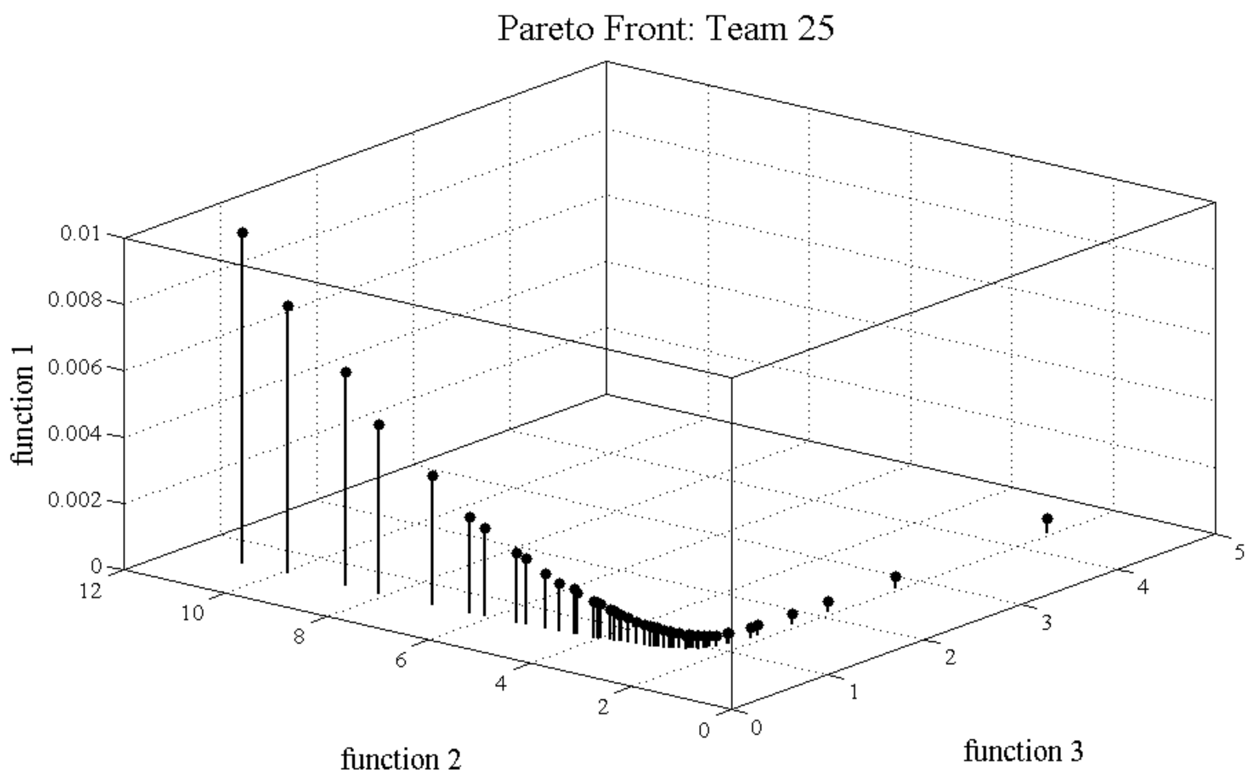
The Optimization of the Die Press Mold (TEAM 25):

Results

C) The Optimization of the Die Press Mold (TEAM 25)



C) The Optimization of the Die Press Mold (TEAM 25)

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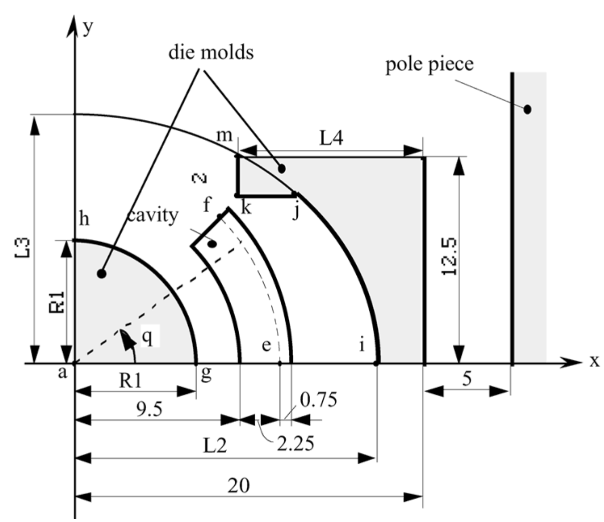
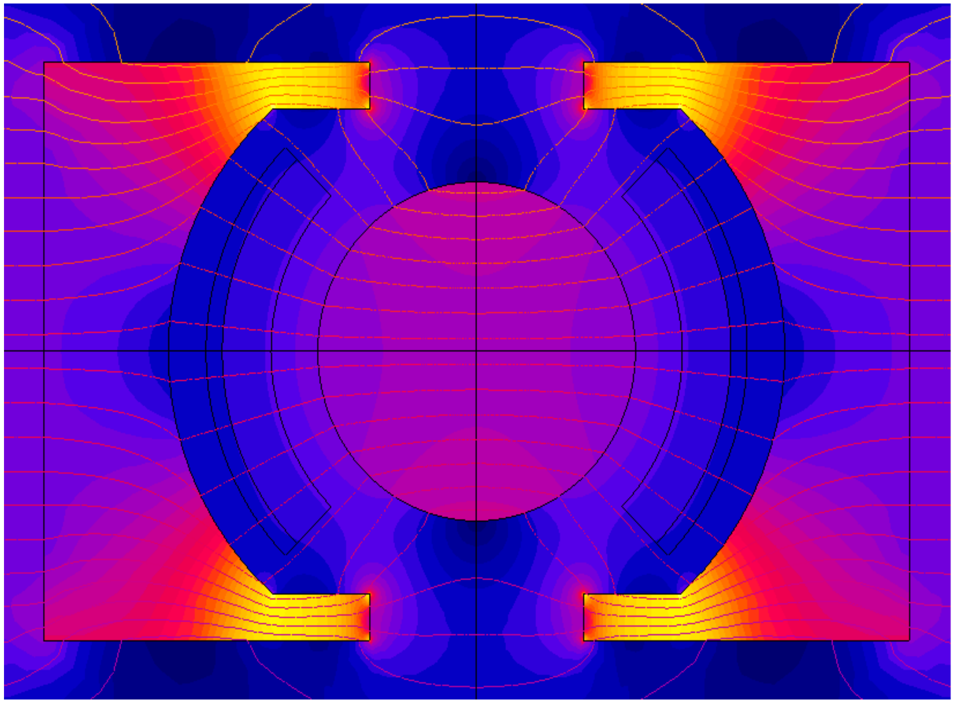
function 1 (T^2)	function 2 (%)	function 3 (°)
0.00028	2.32	1.07
R1 (mm)	**L2 (mm)**	**L4 (mm)**
14	14.4	7.2





Conclusion

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* As a variant of Differential Evolution algorithm the MultiDE attained great results by solving multi-objectives problems.

* It is expected that *MultiDE* can be added to other powerful tools, being an alternative or a complement to the established methods in many applications.