Hilbert-Huang Transform

PEF5737 - Non-linear dynamics and stability

Prof. Dr. Carlos E. N. Mazzilli
Prof. Dr. Guilherme Rosa Franzini

Outline

1. Objetivos
2. Introduction
   - Fourier Transform
3. Examples of application in FIV problems
   - Flow around a cylinder
   - AM/FM - Vortex-induced vibrations data
   - Concomitant VIV and top-motion excitation
   - Response transitions - flexible cylinder VIV
4. Final remarks

Objetivos

- To provide a brief presentation of the Hilbert-Huang Tranform (HHT), its motivation and possible gains associated with its application;
- To show some case studies.

Outline

- Commonly adopted for spectral analyses of digital data (discrete-time signals): Fast Fourier Transform algorithm - FFT;

Fourier Transform

- Commonly adopted for spectral analyses of digital data (discrete-time signals): Fast Fourier Transform algorithm - FFT;
- Decomposes the signal on a space defined by harmonic functions ($\sin(\omega t)$, $\cos(\omega t)$, $\sin(2\omega t)$, $\cos(2\omega t)$,...). The frequencies do not depend on time;

Fourier Transform

- Commonly adopted for spectral analyses of digital data (discrete-time signals): Fast Fourier Transform algorithm - FFT;
- Decomposes the signal on a space defined by harmonic functions ($\sin(\omega t)$, $\cos(\omega t)$, $\sin(2\omega t)$, $\cos(2\omega t)$,...). The frequencies do not depend on time;
- Proper for stationary data arisen from linear systems;

Fourier Transform

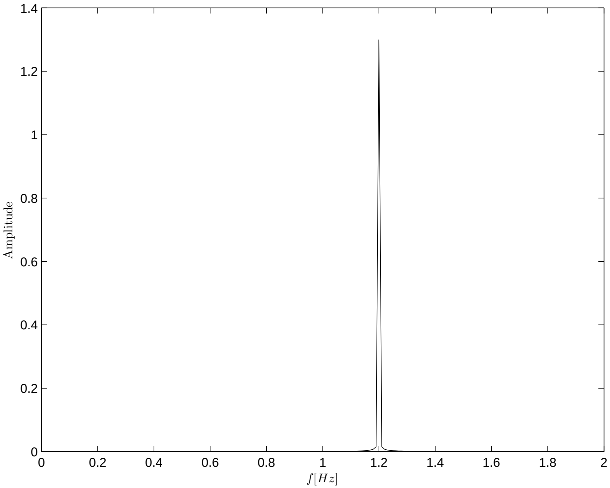
- Commonly adopted for spectral analyses of digital data (discrete-time signals): Fast Fourier Transform algorithm - FFT;
- Decomposes the signal on a space defined by harmonic functions ($\sin(\omega t)$, $\cos(\omega t)$, $\sin(2\omega t)$, $\cos(2\omega t)$,...). The frequencies do not depend on time;
- Proper for stationary data arisen from linear systems;
- Represents a time-domain signal $x(t)$ by means of a set of amplitudes (or energy) that depends of discrete frequencies.

Practical example

Let $x(t)$ being a signal of length (sample size) N. Such a signal is sampled with a constant sampling frequency f_s. The amplitude spectrum can be calculated using the following MATLAB® script:

- $Xs = fft(x);$
- $freq = [0 : N - 1]fs/N;$
- $freq = freq(1 : fix(N/2));$
- $Xs = Xs(1 : fix(N/2));$
- $Amp = 2 * abs(Xs)/N;$
- plot(freq,Amp)

$x(t) = 1.3\sin(2\pi 1.2 t)$



Limitations

- Linearity (superposition of effects) is intrinsically adopted;

Limitations

- Linearity (superposition of effects) is intrinsically adopted;
- The signal must be free of time-dependent effects such as, for examples, jumps and frequency/amplitude modulations;

Limitations

- Linearity (superposition of effects) is intrinsically adopted;
- The signal must be free of time-dependent effects such as, for examples, jumps and frequency/amplitude modulations;
- Other hypothesis (see, for example, the book by Bendat & Pierson).

A focus of interest: Signals AM/FM

- Wavelets appears as an alternative of time-frequency domain data. Linearity is supposed in this technique.

A focus of interest: Signals AM/FM

- Wavelets appears as an alternative of time-frequency domain data. Linearity is supposed in this technique.
- Hilbert Transform: Represents a signal as $x(t) = a(t)e^{j\phi(t)}$. The instantaneous frequency is given by $\omega(t) = \dot{\phi}(t)$. Locally, the signal must have zero mean;

A focus of interest: Signals AM/FM

- Wavelets appears as an alternative of time-frequency domain data. Linearity is supposed in this technique.
- Hilbert Transform: Represents a signal as $x(t) = a(t)e^{j\phi(t)}$. The instantaneous frequency is given by $\omega(t) = \dot{\phi}(t)$. Locally, the signal must have zero mean;
- The seek for an alternative time-frequency domain analysis proper for non-stationary signals arisen from non-linear systems is justified: Hilbert-Huang Transform.

- Introduced in Huang et al (2008): Proper for dealing with non-stationary data arisen from non-linear systems;

Hilbert-Huang Transform

- Introduced in Huang et al (2008): Proper for dealing with non-stationary data arisen from non-linear systems;
- Makes use of an empirical approach for data decomposition: *Empirical mode decomposition - EMD*

Hilbert-Huang Transform

- Introduced in Huang et al (2008): Proper for dealing with non-stationary data arisen from non-linear systems;
- Makes use of an empirical approach for data decomposition: *Empirical mode decomposition - EMD*
- EMD scheme results in a number of Intrinsic Mode Functions (IMFs) $\rightarrow$ Each IMF has null averaged value and a particular time-scale;

Hilbert-Huang Transform

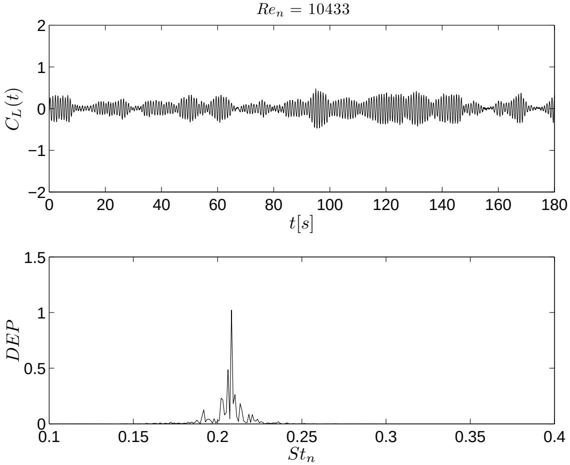
- Introduced in Huang et al (2008): Proper for dealing with non-stationary data arisen from non-linear systems;
- Makes use of an empirical approach for data decomposition: *Empirical mode decomposition - EMD*
- EMD scheme results in a number of Intrinsic Mode Functions (IMFs) $\rightarrow$ Each IMF has null averaged value and a particular time-scale;
- Hilbert-Huang's spectrum: Application of the Hilbert Transform to each IMF.

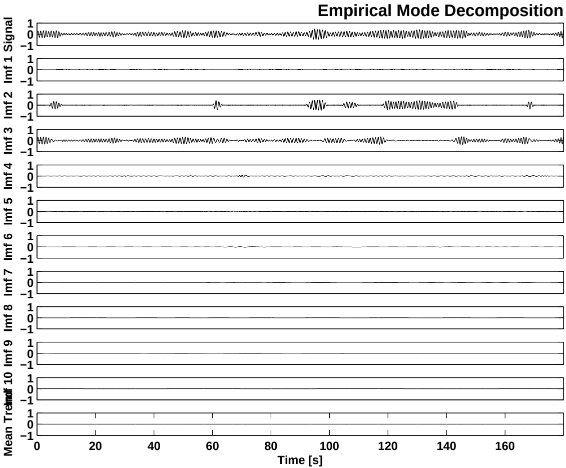
IMFs

- Difference between the number of *extrema* and zero-crossing must be, at most, 1;
- For each instant, the average between the local maximum and minimum must be null.

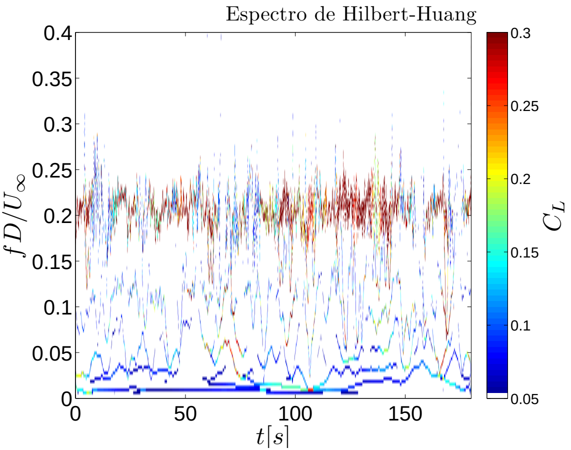
Outline

Flow around a cylinder




Empirical Mode Decomposition
Signal
Imf 1
Imf 2
Imf 3
Imf 4
Imf 5
Imf 6
Imf 7
Imf 8
Imf 9
Imf 10
Mean Trend
1
0
−1
0
20
40
60
80
100
120
140
160
Time [s]

Flow around a cylinder

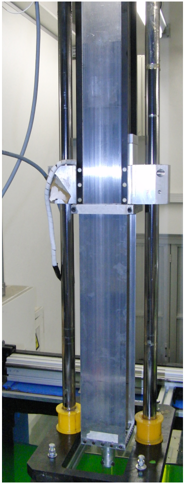


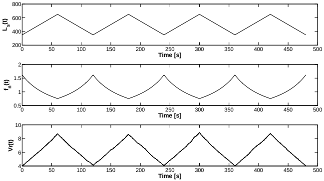
Outline

AM/FM VIV

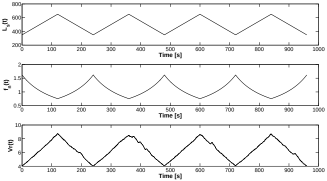
- Vortex-induced vibrations (VIV): Resonant phenomenon that can be modeled by using a non-linear oscillator coupled to an elastic one;
- Large structural responses within the interval $3 < U_r = U/f_n D < 12$;
- Classical experimental investigations: The reduced velocity U_r is varied by increasing the free-stream velocity U_∞;
- Non-usual experiment (Franzini et al (2008,2010): U_∞ is kept constant and U_r is varied by changing the natural frequency.

AM/FM-VIV

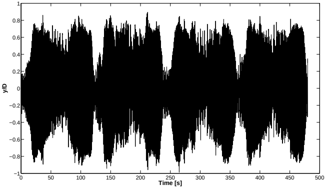




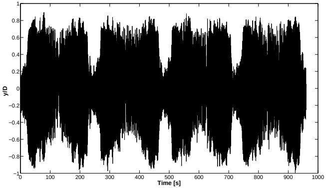
(a) Clamp speed $5mm/s$



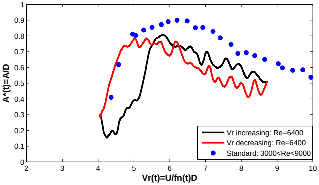
(b) Clamp speed $25mm/s$



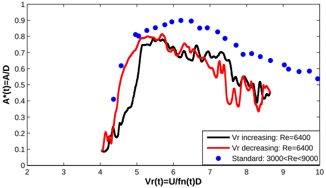
(a) Clamp speed $5 mm/s$



(b) Clamp speed $25 mm/s$

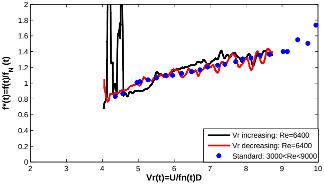


(a) Clamp speed 5mm/s

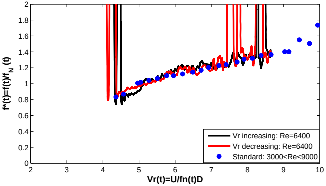


(b) Clamp speed 25mm/s

AM/FM-VIV



(a) Clamp speed $5 mm/s$



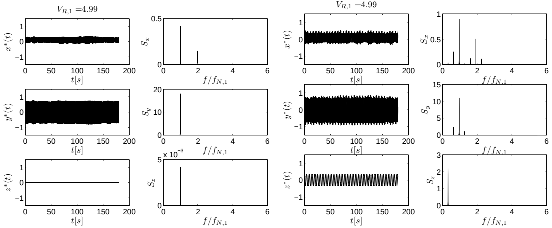
(b) Clamp speed $25 mm/s$

Outline

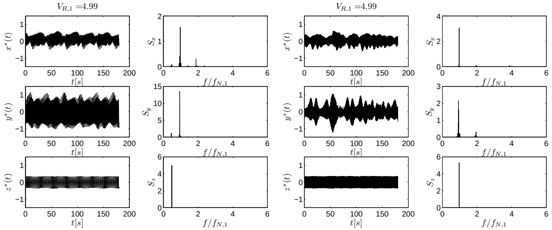
Concomitant VIV and top-motion excitation

Franzini et al (2015). Concomitant flexible cylinder VIV and top-motion excitation.

Fourier Transform (PSD) analysis



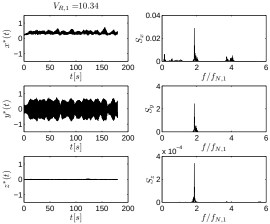
(a) No top motion (b) 1 : 3



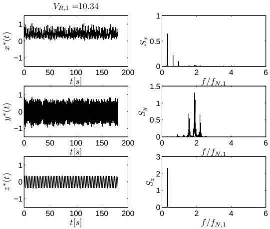
(c) 1 : 2 (d) 1 : 1

$U_{r,1} = 10.34$

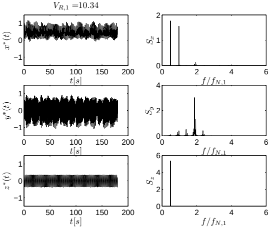
Fourier Transform (PSD) analysis



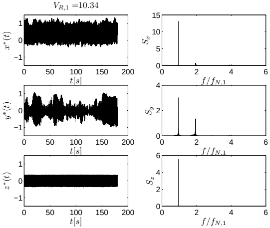
(a) No top motion



(b) 1 : 3



(c) 1 : 2



(d) 1 : 1

PSD analysis

- No top motion cases: Synchronization with the natural frequencies of the model;
- Top motion cases: Marked presence of subharmonic components $\rightarrow$ Richer spectral distribution;
- Presence of sum and difference frequencies: Can be explained under the linear dynamics scope;

PSD analysis

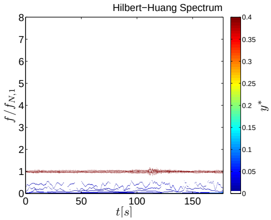
- No top motion cases: Synchronization with the natural frequencies of the model;
- Top motion cases: Marked presence of subharmonic components$\rightarrow$ Richer spectral distribution;
- Presence of sum and difference frequencies: Can be explained under the linear dynamics scope;
- $k_q(t) = \bar{k} + \Delta k_q \cos \Omega_t t$ (time dependent stiffness), $q = Q \cos \Omega_{N,1} t$ (modal displacement), F_q (forcing term);

PEFUSP

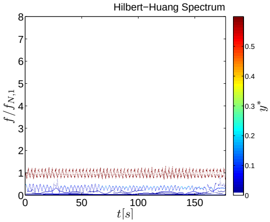
- No top motion cases: Synchronization with the natural frequencies of the model;
- Top motion cases: Marked presence of subharmonic components$\rightarrow$ Richer spectral distribution;
- Presence of sum and difference frequencies: Can be explained under the linear dynamics scope;
- $k_q(t) = \bar{k} + \Delta k_q \cos \Omega_t t$ (time dependent stiffness), $q = Q \cos \Omega_{N,1} t$ (modal displacement), F_q (forcing term);
- Equation of motion $m_q \ddot{q} + c\dot{q} + \bar{k}q = F_q(t) - \frac{\Delta k Q}{2}[\cos(\Omega_{N,1} + \Omega_t)t + \cos(\Omega_{N,1} - \Omega_t)t]$

$U_{r,1} = 4.99$

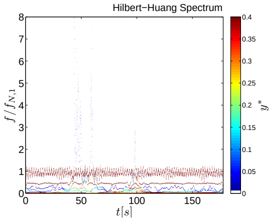
HHT Analysis



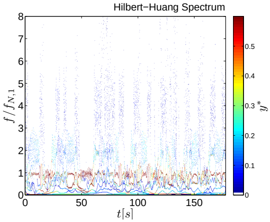
(a) No top motion



(b) 1 : 3

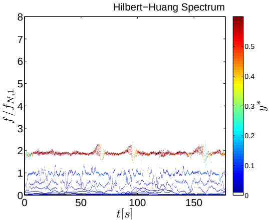


(c) 1 : 2

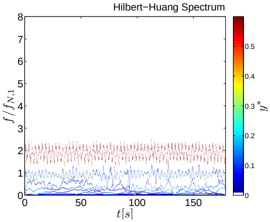


(d) 1 : 1

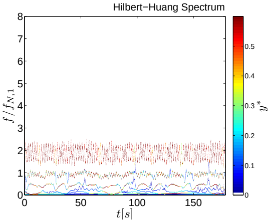
HHT Analysis



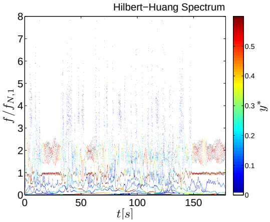
(e) No top motion



(f) 1 : 3



(g) 1 : 2



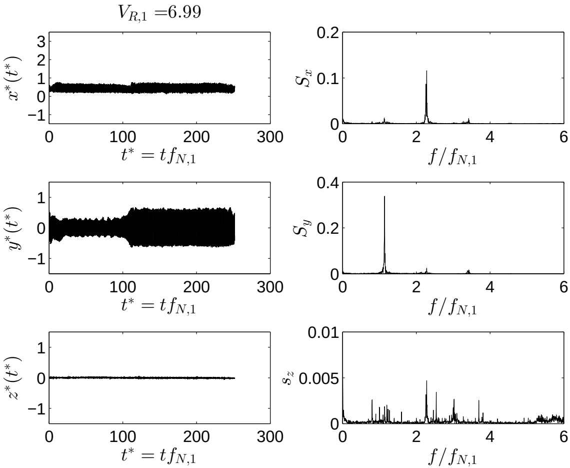
(h) 1 : 1

HHT Analysis

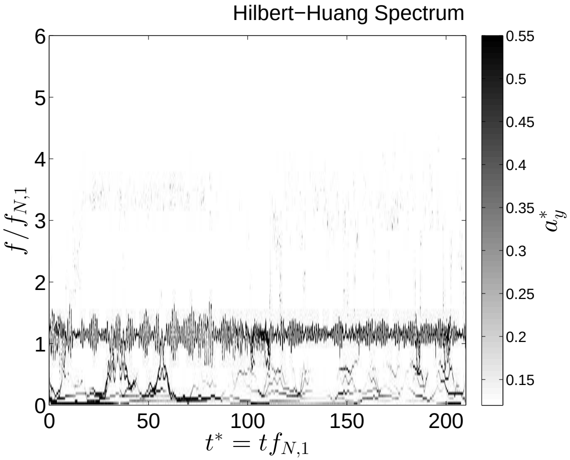
- HHT $\rightarrow$ Time-frequency domain spectral analysis, introduced by Huang et al (1998);
- HHT $\rightarrow$ Can be used aiming at identifying events in the time series;
- HHT $\rightarrow$ **Sum and difference components are replaced by a continuous trace of frequency, which varies with f_t;** $\Rightarrow$ Information not given by the PSD.

Outline

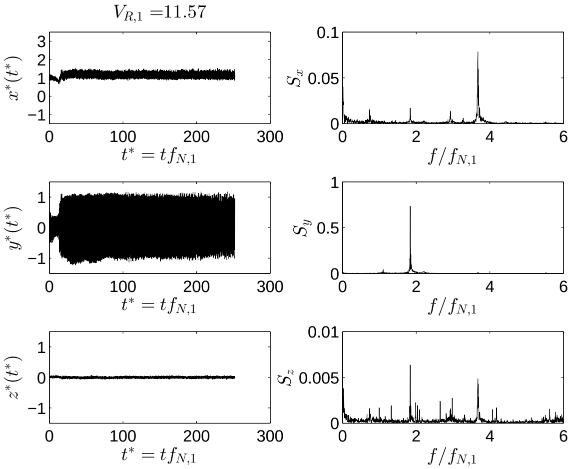
Jumps in the response



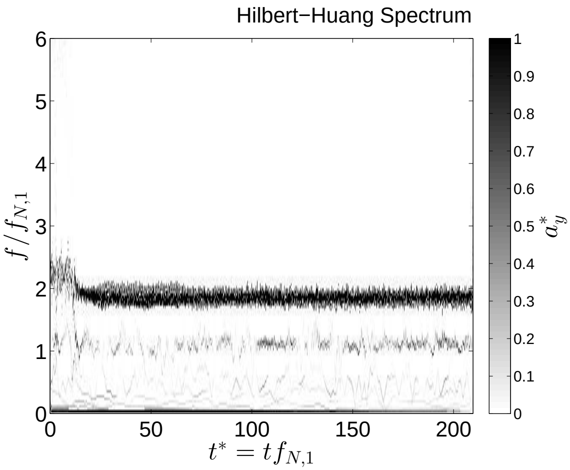
Jumps in the response



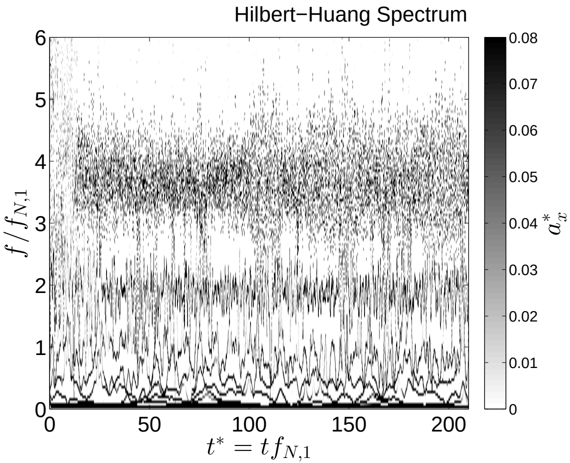
Jumps in the response



Jumps in the response



Jumps in the response



Final remarks

- Fourier Transform (FT): Proper for stationary data arisen from linear systems;
- Hilbert-Huang Transform: Also proper for non-stationary signals arisen from non-linear systems;
- : HHT: Applies the empirical mode decomposition for generating a set of intrinsic mode functions;
- HHT: Applies the Hilbert Transform on each IMF.

Final remarks

- Joint use of FT and HHT $\rightarrow$ Complimentary results on the same problem.

Bibliography

G. R. Franzini, R. T. Gonçalves, C. P. Pesce, A. L. C. Fujarra, C. E. N. Mazzilli, J. R. Meneghini, and P. Mendes. Vortex-induced vibration experiments with a long semi-immersed flexible cylinder under tension modulation: Fourier transform and hilbert-huang spectral analyses. Journal of the Brazilian Society of Mechanical Sciences and Engineering, Online, 2014.

G. R. Franzini, A. A. P. Pereira, A. L. C. Fujarra, and C. P. Pesce. Experiments on VIV under frequency modulation and at constant Reynolds number. In Proceedings of OMAE 08 , 27th Int. Conference on Offshore Mechanics and Arctic Engineering, 2008.

G. R. Franzini, C. P. Pesce, R. T. Gonçalves, A. L. C. Fujarra, and J. R. Meneghini. An experimental investigation on frequency modulated VIV in a water channel. In Proceedings of the 7th Bluff-Bodies Wakes and Vortex-Induced Vibrations Conference, 2010.

Bibliography

Huang, N. E., Shen, Z., Long, S. R., Wu, M. C., Shih, H. H., Zheng, Q., Yen, N., Tung, C. C., Liu, H. H. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Royal Society London, 1998.