DAY	TIME	LECTURE
Monday 05/11	14.00 -14.45	Historical Framework - A Global Dynamics Perspective in the Nonlinear Analysis of Systems/Structures
	15.00 -15.45	Achieving Load Carrying Capacity: Theoretical and Practical Stability
	16.00 -16.45	Dynamical Integrity: Concepts and Tools_1
Wednesday 07/11	14.00 -14.45	Dynamical Integrity: Concepts and Tools_2
	15.00 -15.45	Global Dynamics of Engineering Systems
	16.00 -16.45	Dynamical integrity: Interpreting/Predicting Experimental Response
Monday 12/11	14.00 -14.45	Techniques for Control of Chaos
	15.00 -15.45	A Unified Framework for Controlling Global Dynamics
	16.00 -16.45	Response of Uncontrolled/Controlled Systems in Macro- and Micro-mechanics
Wednesday 14/11	14.00 -14.45	A Noncontact AFM: (a) Nonlinear Dynamics and Feedback Control (b) Global Effects of a Locally-tailored Control
	15.00 -15.45	**Exploiting Global Dynamics to Control AFM Robustness**
	16.00 -16.45	Dynamical Integrity as a Novel Paradigm for Safe/Aware Design

12.2 – Exploiting Global Dynamics to Control AFM Robustness

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Coworkers: V. Settimi, S. Lenci

OUTLINE

1. A CONTROL PROCEDURE OF GLOBAL EVENTS

   ANALYTICAL CONTROL OF HOMOCLINIC BIFURCATION OF HILLTOP SADDLE

   IDENTIFYING THE BIFURCATION ACTUALLY TRIGGERING EROSION

2. NUMERICAL CONTROL PROCEDURE

   VALIDATION

   CONTROLLING HOMOCLINIC BIFURCATION OF IN-WELL SADDLE

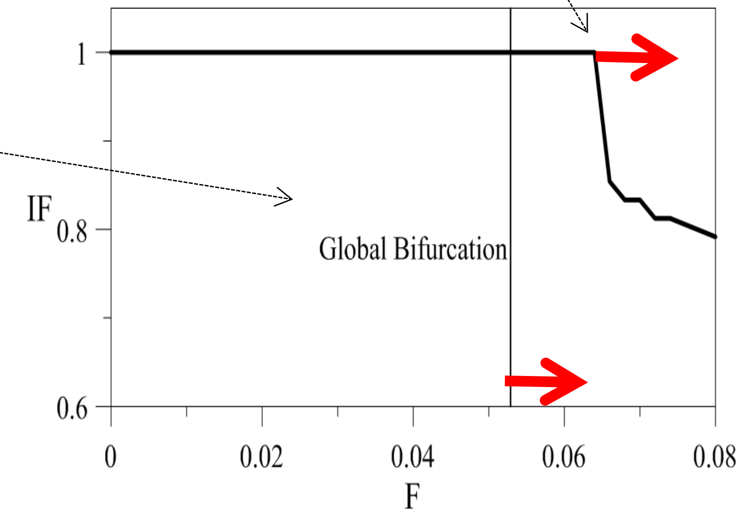
3. SUMMARY AND COMMENTS

A GLOBALLY-TAILORED CONTROL

FOR PRACTICAL PURPOSES: **PREVENTING** or **DELAYING** safe basin **EROSION (**profile **fall down)**

2. ALTERNATIVE, GLOBALLY ORIENTED, CONTROL:

preventing/delaying **EROSION** by **INCREASING some GLOBAL BIFURCATION THRESHOLD** (homo/heteroclinic tangency of invariant manifolds of saddle points)



PROBLEMS:

Detecting the **SADDLE(S) triggering erosion** and **identifying** its **homo/heteroclinic bifurcation**

- **HILLTOP SADDLE**, as usual, **??** (can be **analytically** detected by Melnikov function)
- DIFFERENT **SECONDARY SADDLE ??** (to be identified **numerically**)

A CONTROL PROCEDURE OF GLOBAL EVENTS

Homo/heteroclinic bifurcations **delayed** by **addition** of **superharmonics** to basic **harmonic** excitation *(Lenci and Rega, 1998, 2004):*

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\alpha_1 x - \alpha_3 x^3 - \dfrac{\Gamma_1}{(1+x)^2} - \rho_1 y + \boxed{\sum_{j=1}^{N} \tilde{U}_j \sin\left(j\omega t + \psi_j\right)} \end{cases}$$

- **Optimal** choice of controlling superharmonics → optimal excitation that **maximizes** the **global bifurcation load**

- Different types of control, **one-side** (only **one** bifurcation **controlled**) and **global** (**more** bifurcations **controlled**) dependent on **governing global mechanism**

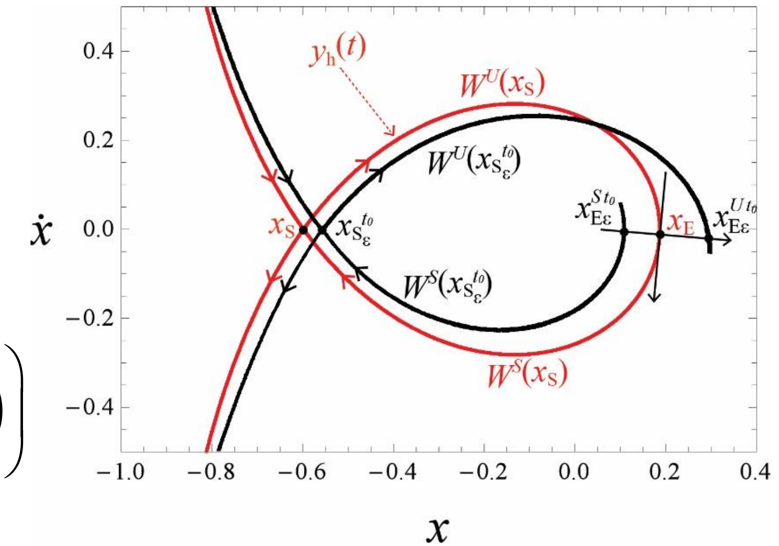
- Still detectable through **Melnikov** function (if **hilltop saddle**)

- **Performance** of control measured by the **gain**, ratio between critical amplitudes of **control** (harmonic + superharmonics) and **reference** (harmonic) excitation

$$G = \frac{U_{1,cr}}{U_{1,cr}^{h}}$$

MELNIKOV METHOD:

Perturbative technique to compute distance between the perturbed stable and unstable manifolds



PERTURBED SYSTEM

$$\mathbf{x} = \mathbf{f} + \varepsilon \mathbf{g} = \begin{cases} \dot{x} = y \\ \dot{y} = -\alpha_1 x - \alpha_3 x^3 - \dfrac{\Gamma_1}{(1+x)^2} + \varepsilon\left(-\rho_1 y + x \sum_{j=1}^{N} \tilde{U}_j \sin\left(j\omega t + \psi_j\right)\right) \end{cases}$$

MELNIKOV FUNCTION = manifolds **distance** to first order

$$M(t_0) = \mathbf{f} \wedge \mathbf{g} = \int_{-\infty}^{+\infty} y_h(t)\left(-\rho_1 y_h(t) + x_h(t) \sum_{j=1}^{N} \tilde{U}_j \sin\left(j\omega(t+t_0) + \psi_j\right)\right) dt$$

$$y_h(t) = \text{homoclinic horbit} = \frac{dx_h}{dt} = \pm\sqrt{2\left(V(x_s) - V(x)\right)}, \quad V(x) = \alpha_1 \frac{x^2}{2} + \alpha_3 \frac{x^4}{4} - \frac{\Gamma_1}{1+x}$$

MELNIKOV METHOD:

Perturbative technique to compute distance between the perturbed stable and unstable manifolds

MELNIKOV FUNCTION = manifolds **distance** to first order

$$M(t_0) = \mathbf{f} \wedge \mathbf{g} = \int_{-\infty}^{+\infty} y_h(t)\left(-\rho_1 y_h(t) + x_h(t)\sum_{j=1}^{N} \tilde{U}_j \sin\left(j\omega(t+t_0)+\psi_j\right)\right)dt$$

$$= -2\rho_1 I_1 + 2I_2(j\omega)\sum_{j=1}^{N} \tilde{U}_j \cos\left(j\omega t_0 + \psi_j\right)$$

$$= -2\rho_1 I_1 + 2\tilde{U}_1 I_2(\omega) h(\omega t_0)$$

$$y_h(t) = \text{homoclinic horbit} = \frac{dx_h}{dt} = \pm\sqrt{2\left(V(x_s) - V(x)\right)}, \quad V(x) = \alpha_2 \frac{x^2}{2} + \alpha_3 \frac{x^4}{4} - \frac{\Gamma_1}{1+x}$$

$$I_1 = \int_{x_E}^{x_S} y_h(x)dx < 0, \quad I_2(j\omega) = \int_{x_E}^{x_S} x \sin\left(j\omega \int_{x_E}^{x} \frac{dr}{\sqrt{2\left(V(x_s) - V(r)\right)}}\right) d\,x$$

$$h(\omega t_0) = \sum_{j=1}^{N} h_j \cos\left(j\omega t_0 + \psi_j\right), \quad h_j = \frac{\tilde{U}_j I_2(j\omega)}{\tilde{U}_1 I_2(\omega)}$$

MANIFOLDS INTERSECTION $\rightarrow$ Simple zero of the **Melnikov** function

$$M\left(t_0\right)=0 \quad \Rightarrow \quad -2\rho_1 I_1 + 2\tilde{U}_1 I_2\left(\omega\right) h\left(\omega t_0\right)=0$$

CRITICAL AMPLITUDE $$\boxed{\tilde{U}_{1,cr}=\frac{\tilde{U}^h_{1,cr}}{M}}$$

Critical amplitude with harmonic forcing: $\tilde{U}^h_{1,cr}=\dfrac{\rho_1 I_1}{\left|I_2\left(\omega\right)\right|}$

$$M=\max_{m\in[0,2\pi]}\left\{-h\left(m\right)\right\}=-\min_{m\in[0,2\pi]}\left\{h\left(m\right)\right\}$$

MANIFOLDS INTERSECTION $\rightarrow$ Simple zero of the **Melnikov** function

$$M(t_0)=0 \quad \Rightarrow \quad -2\rho_1 I_1 + 2\tilde{U}_1 I_2(\omega) h(\omega t_0) = 0$$

CRITICAL AMPLITUDE

$$\tilde{U}_{1,cr} = \frac{\tilde{U}^h_{1,cr}}{M}$$

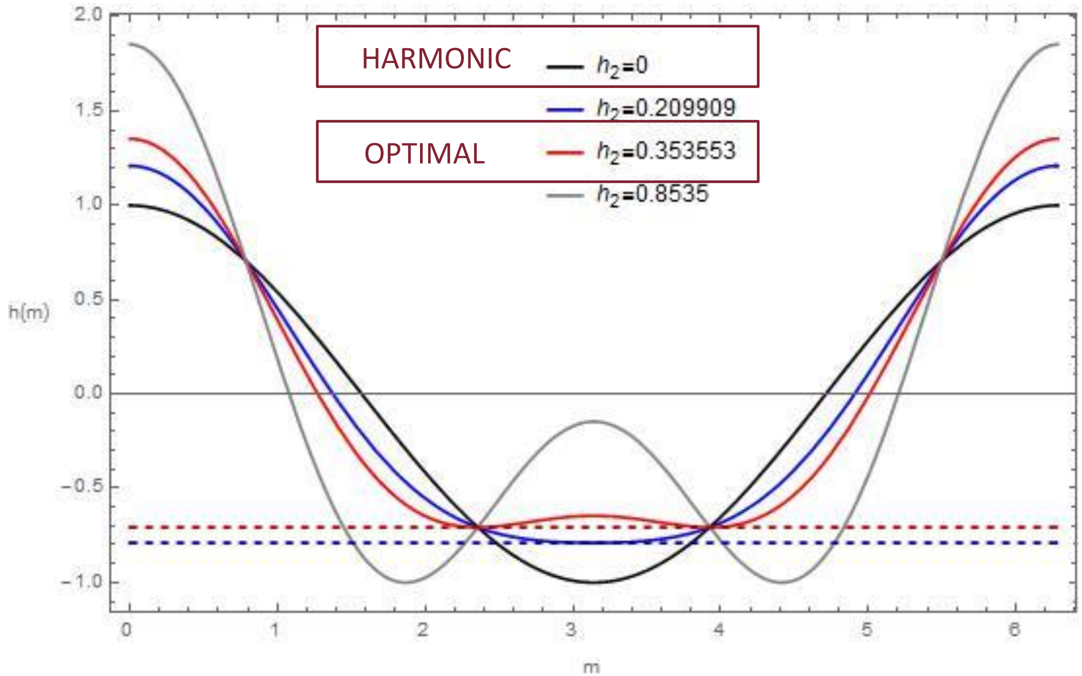
Critical amplitude with harmonic forcing: $\tilde{U}^h_{1,cr} = \dfrac{\rho_1 I_1}{|I_2(\omega)|}$

$$M = \max_{m\in[0,2\pi]}\{-h(m)\} = -\min_{m\in[0,2\pi]}\{h(m)\}$$

WITH 1 SUPERHARMONIC:

$\boldsymbol{h(m)}$ FOR VARYING h_2 COEFFICIENTS

the larger the minimum of *h*(*m*), the larger the distance

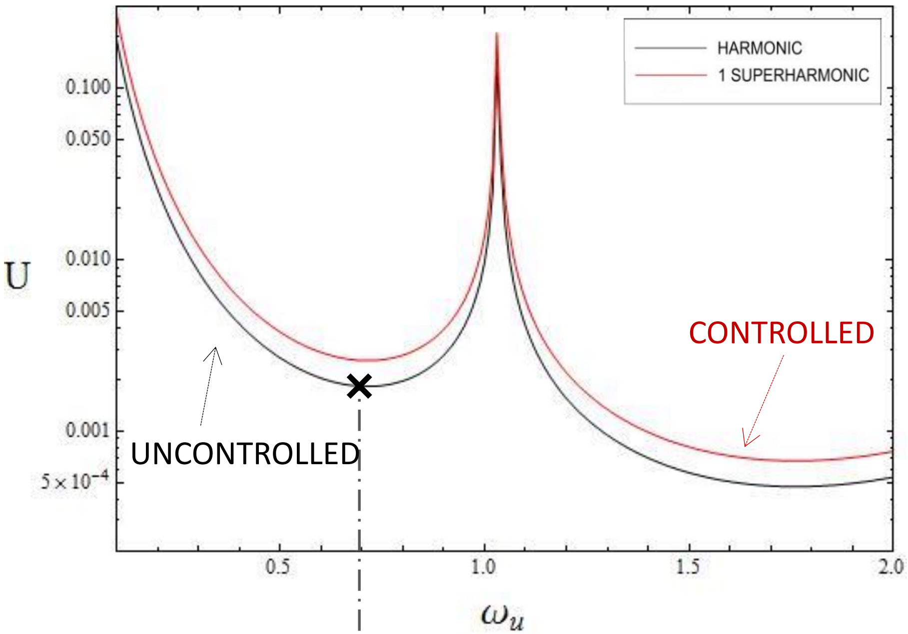


$$h(m) = \sum_{j=1}^{N} h_j \cos(jm + \psi_j),$$

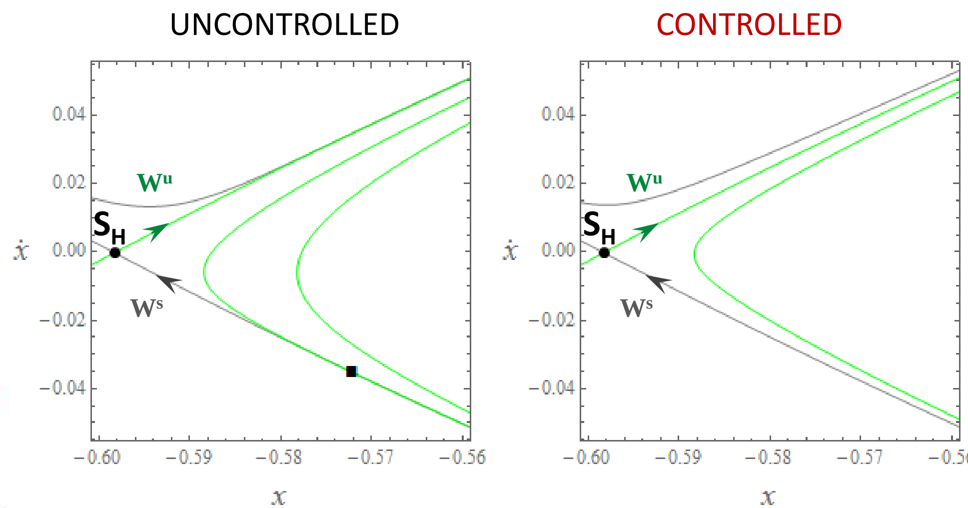
$$h_j = \frac{\tilde{U}_j I_2(j\omega)}{\tilde{U}_1 I_2(\omega)}$$

CONTROLLED VS UNCONTROLLED

WITH 1 SUPERHARMONIC:



NUMERICAL VERIFICATION

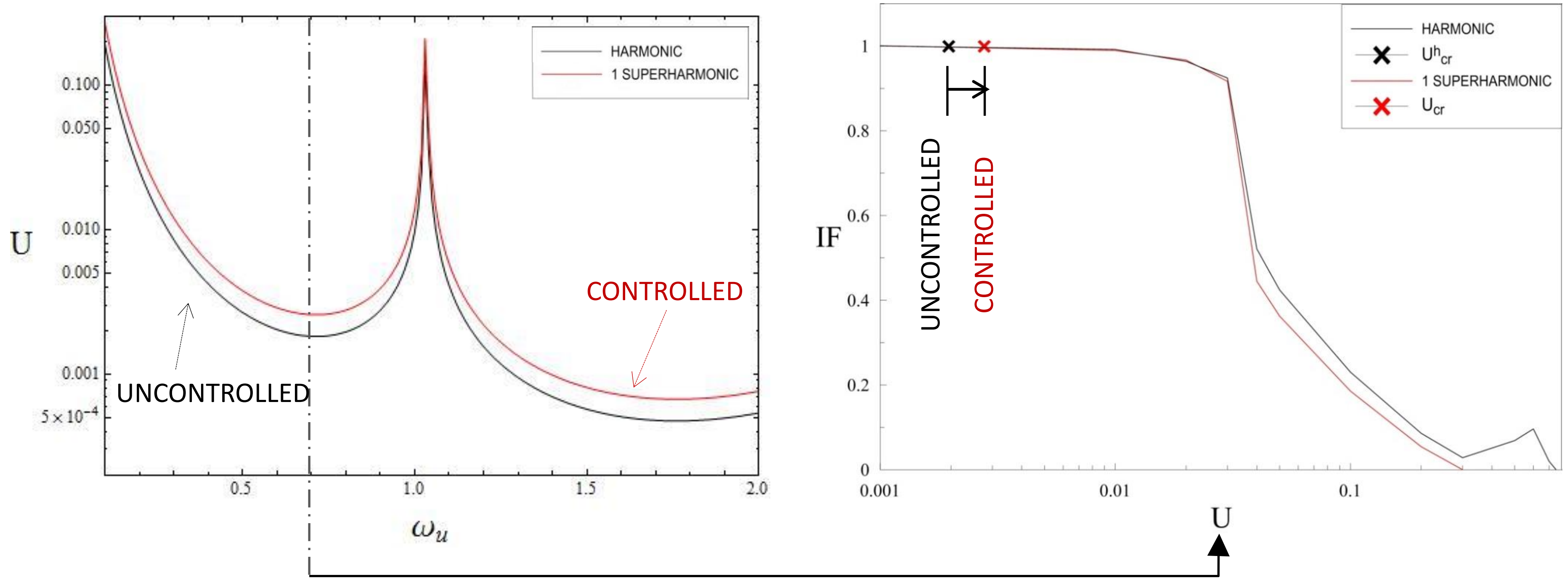


INCREASE of critical amplitude ……and…… **SEPARATION of MANIFOLDS**

HOMOCLINIC BIFURCATION OF HILLTOP SADDLE - 4 -

CONTROLLED VS UNCONTROLLED

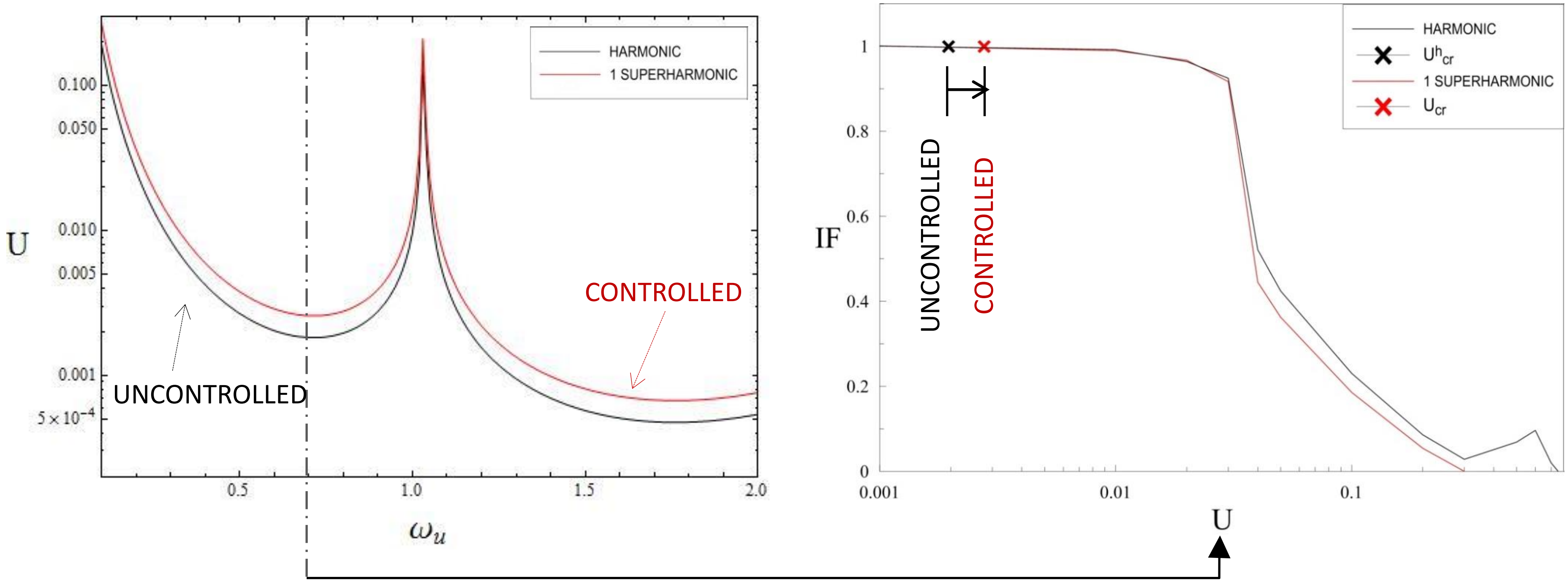
WITH 1 SUPERHARMONIC:



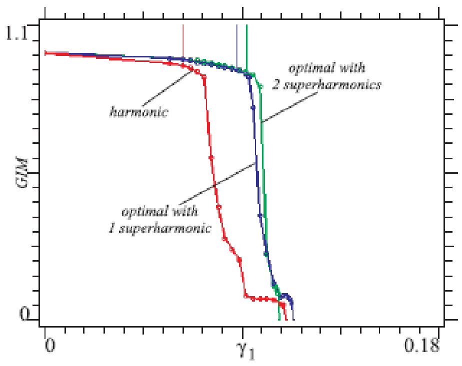
…… **but** ….. **NO improvement of erosion profile!!**

CONTROLLED VS UNCONTROLLED

WITH 1 SUPERHARMONIC:

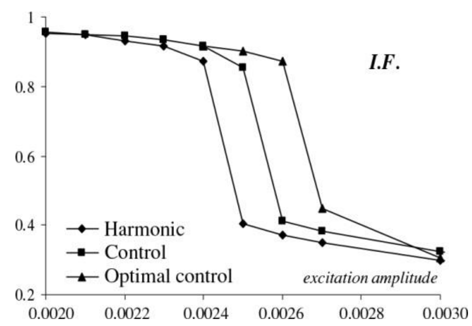


...... **<u>but</u>** ..... **NO improvement of erosion profile!!**



HELMHOLTZ
(Lenci, Rega, 2003)

MEMS
(Lenci, Rega 2006)



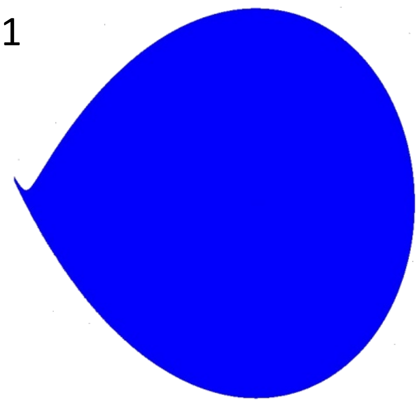
DIFFERENTLY
from other
ESCAPE oscillators

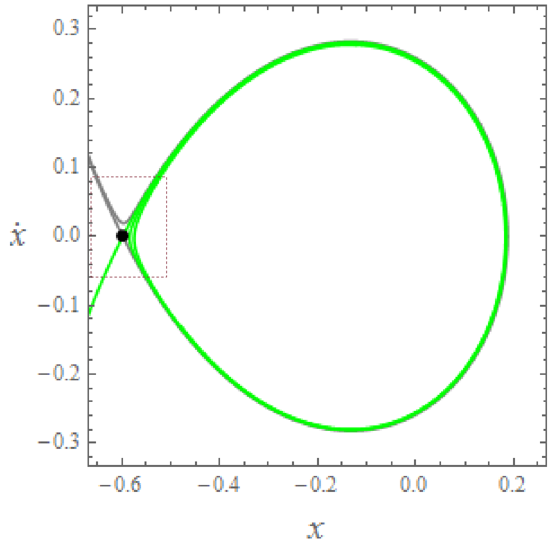
Why no improvement of erosion profiles? ANALYSIS OF **BASINS EVOLUTION**

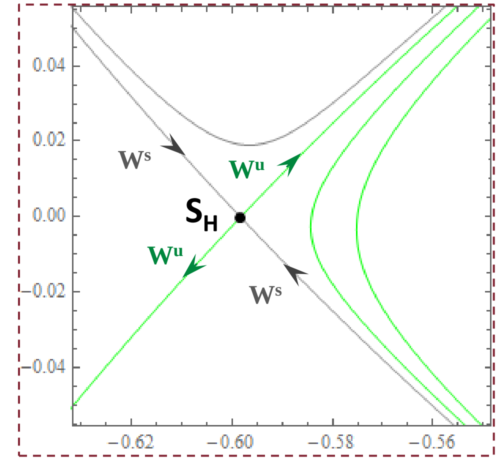
$\omega = 0.7$

UNCONTROLLED SYSTEM: MELNIKOV FUNCTION → HOMOCLINIC BIFURCATION at **U=0.001823**

BEFORE U = 0.001



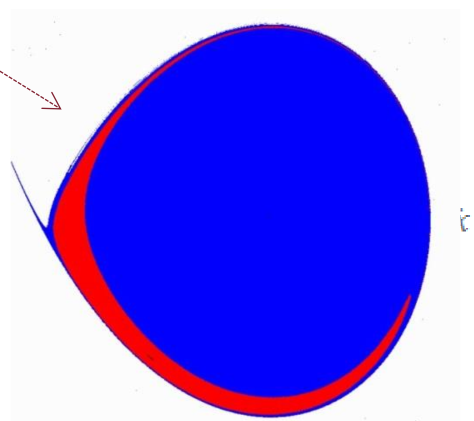


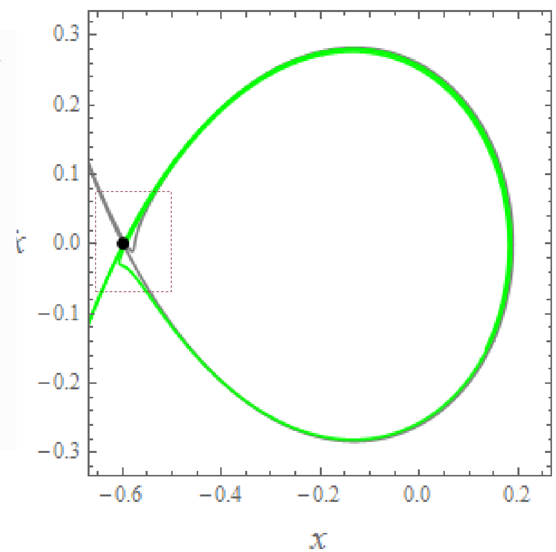


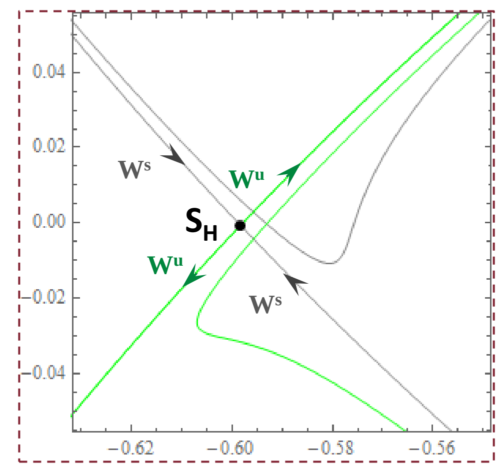
AFTER U = 0.005

Just very minor fractalization of outer boundary

NO EROSION of potential well **!!!**







IDENTIFYING THE EVENT ACTUALLY TRIGGERING EROSION - 1 -

$\omega = 0.7$

Accurate numerical investigation of the global bifurcation scenario

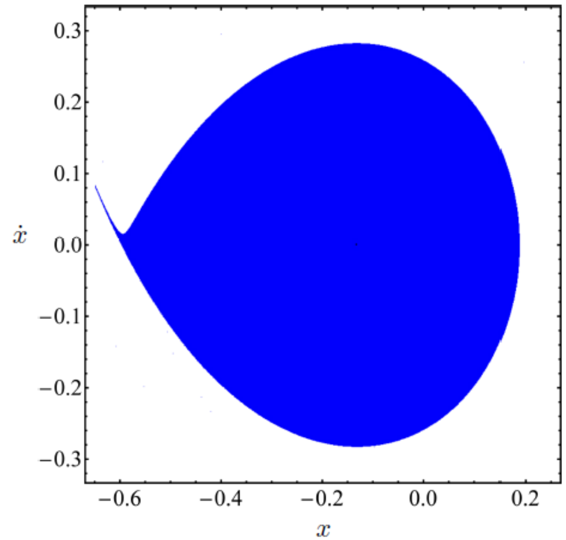
	Harmonic Amplitude	Global Event	Saddle Involved
i.	0.001697	Onset of in-well saddle	S_1
ii.	0.001823	Homoclinic bif. $W_r^s(S_H) \cap W_r^u(S_H)$	S_H
iii.	0.006375	Homoclinic bif. $W_l^u(S_1) \cap W_l^s(S_1)$	S_1
iv.	0.006676	Homoclinic bif. $W_l^u(S_1) \cap W_r^s(S_1)$	S_1
v.	0.006676	Heteroclinic bif. $W_l^u(S_1) \cap W_r^s(S_H)$	S_1, S_H

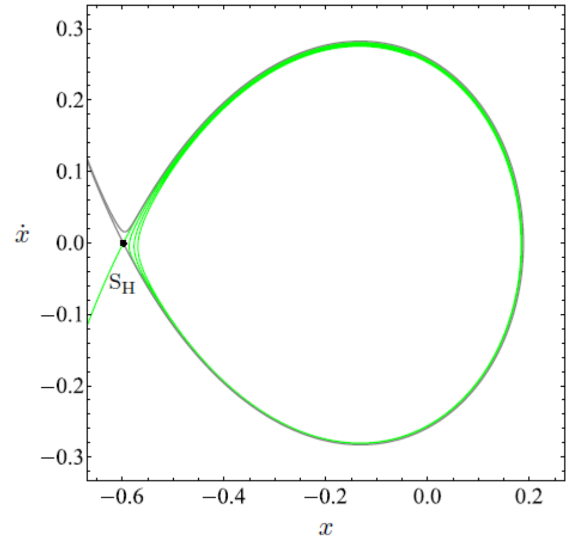
ω = 0.7

i. U=0.001697 - ONSET OF IN-WELL SADDLE S_1

BEFORE

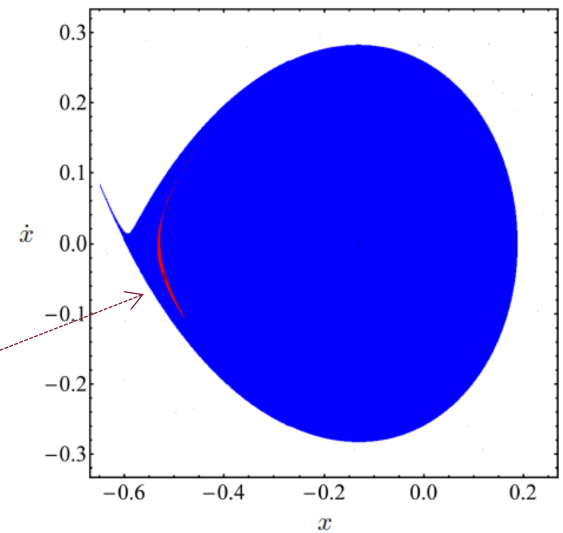
U=0.0015

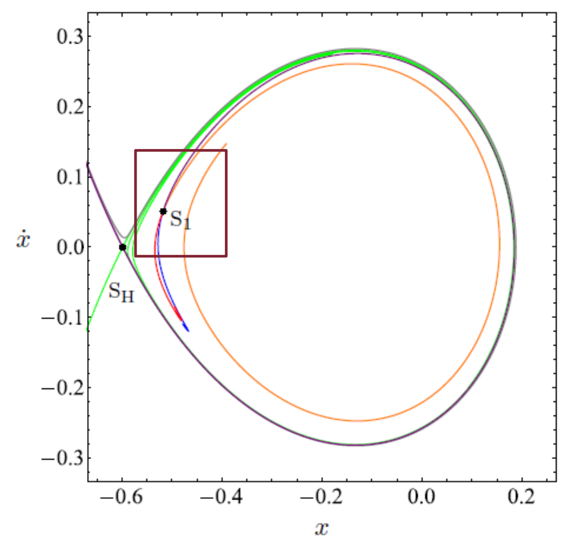




AFTER

U=0.0018





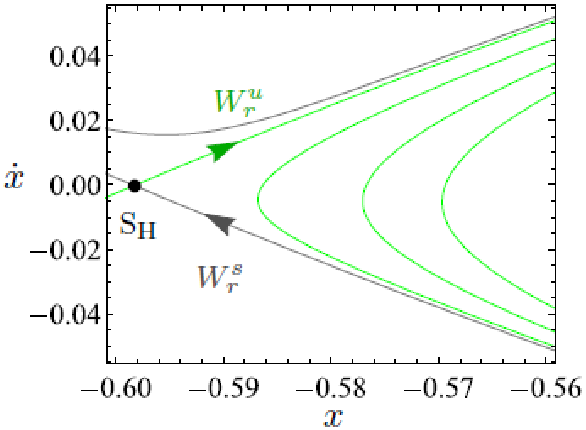
Related to the arise of the competing P1 solution basin (red)

$\omega = 0.7$

ii. U=0.001823 - HOMOLINIC BIFURCATION OF THE HILLTOP SADDLE S_H

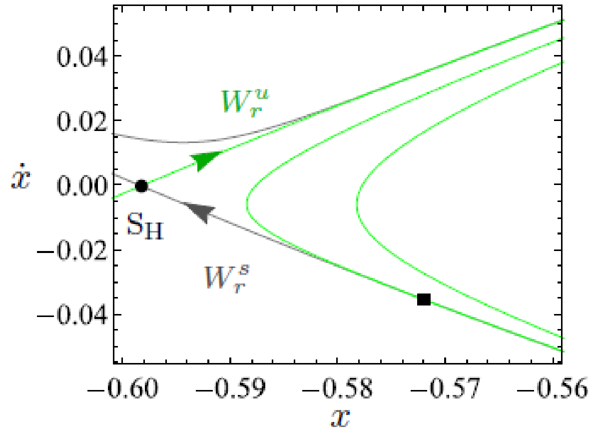
BEFORE

U=0.0015



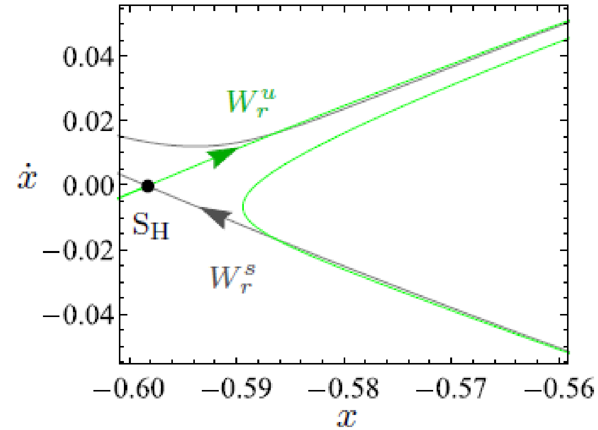
TANGENCY

U=0.001823



AFTER

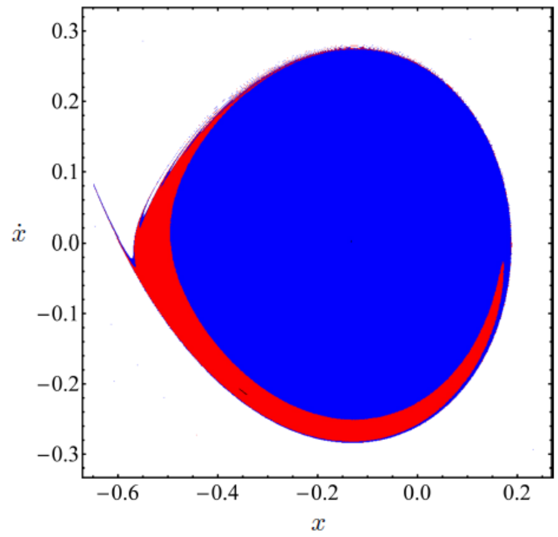
U=0.002

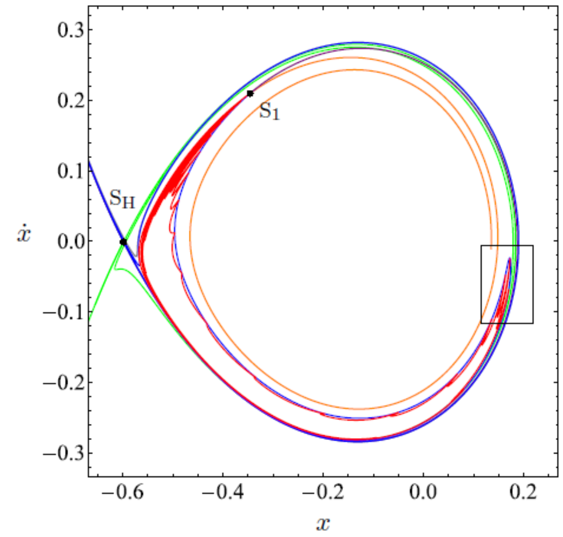


ALREADY CONTROLLED THROUGH ANALYTICAL PROCEDURE (MELNIKOV METHOD)

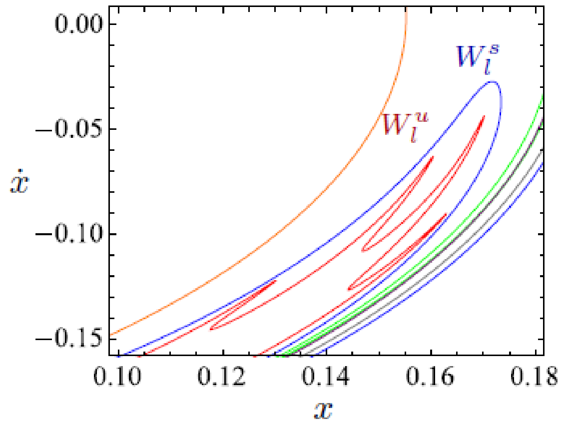
ω = 0.7

iii. U=0.006375 - HOMOCLINIC BIFURCATION OF THE IN-WELL SADDLE S_1

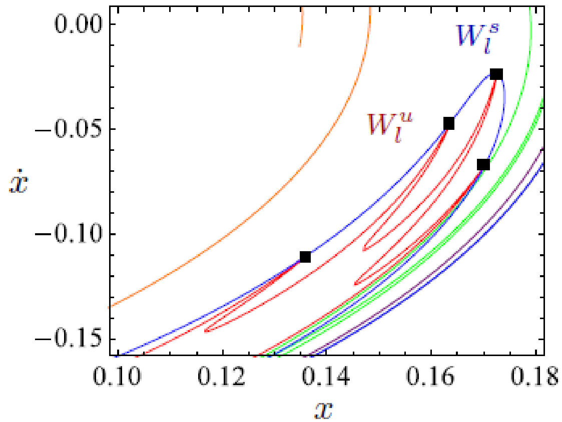




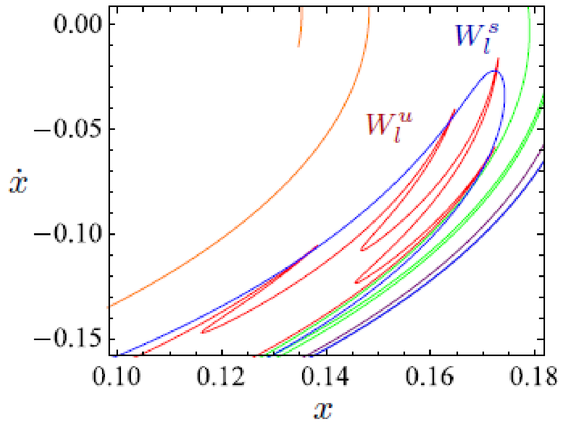
BEFORE
U=0.0063



TANGENCY
U=0.006375

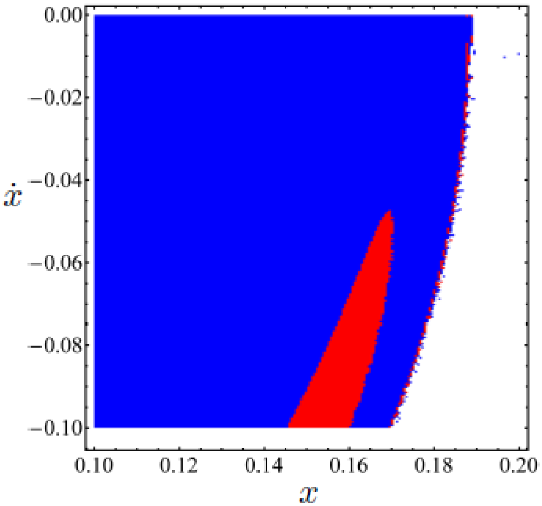


AFTER
U=0.0064



ω = 0.7

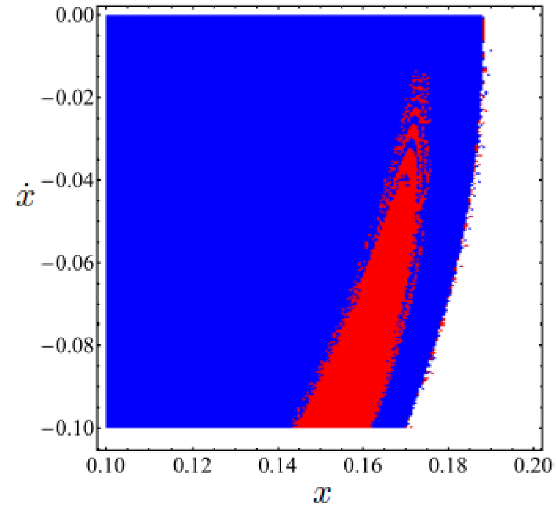
iii. U=0.006375 - HOMOCLINIC BIFURCATION OF THE IN-WELL SADDLE S_1



STARTING POINT FOR

IN-WELL BASINS BOUNDARY

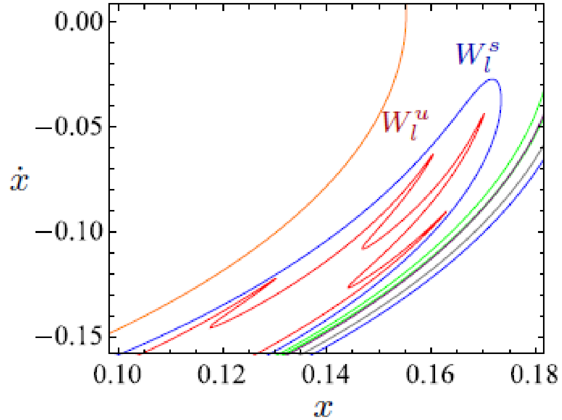
FRACTALIZATION !!

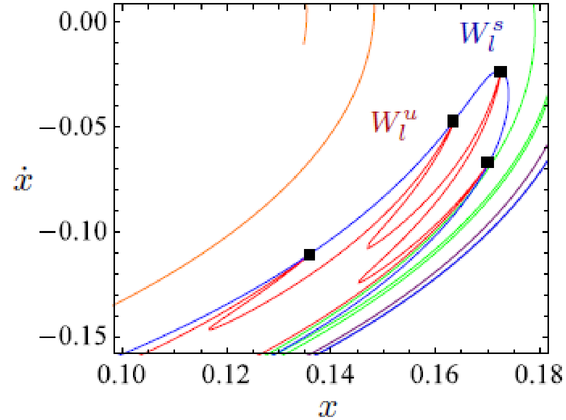


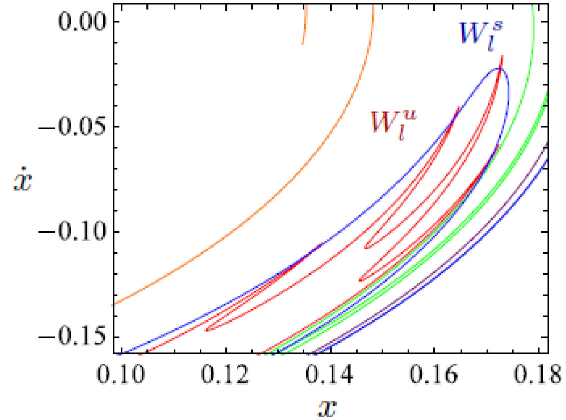
BEFORE
U=0.0063

TANGENCY
U=0.006375

AFTER
U=0.0064

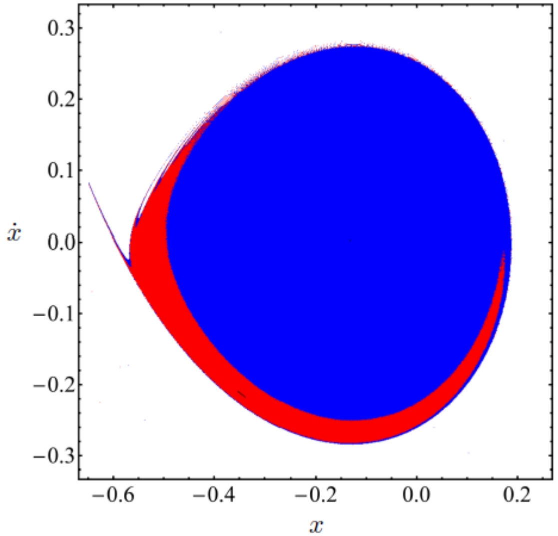




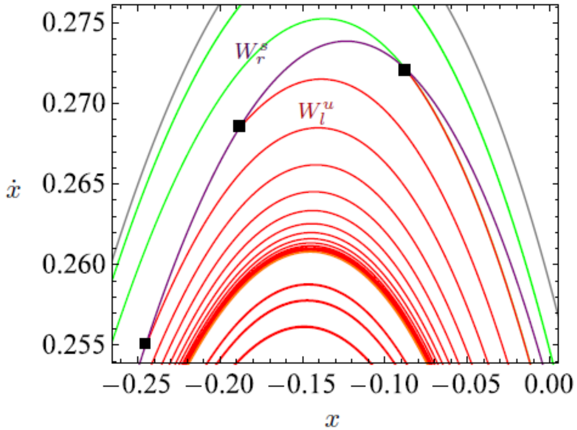


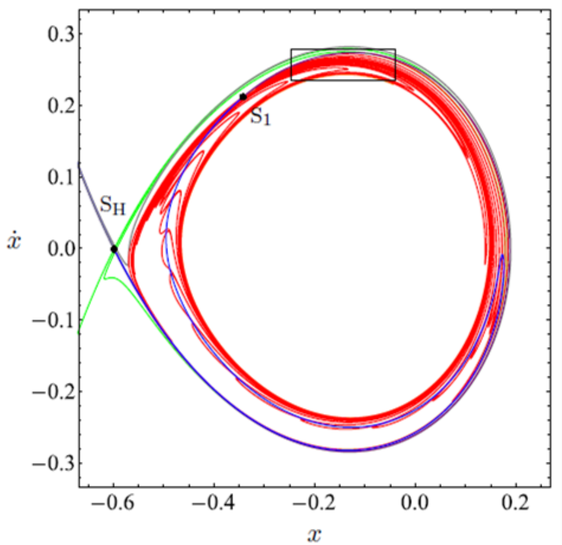
ω = 0.7

iv. U=0.006676 - HOMOCLINIC BIFURCATION OF THE IN-WELL SADDLE S_1

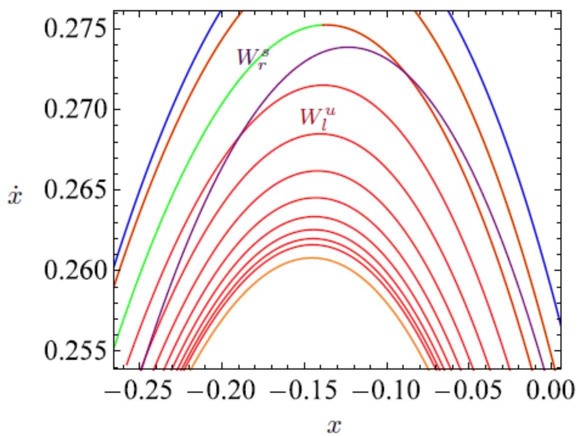


TANGENCY U=0.006676





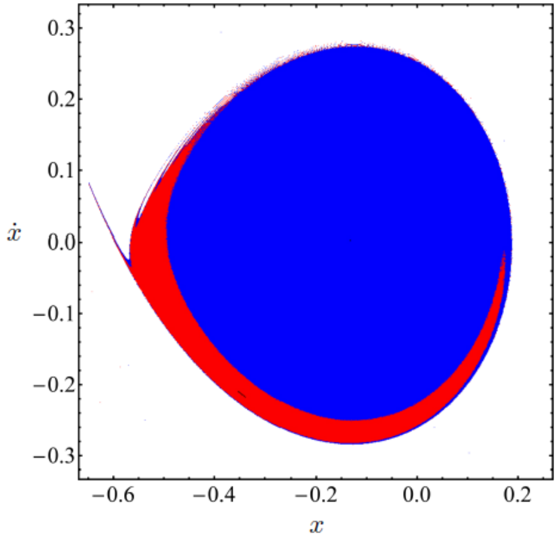
AFTER U=0.00668



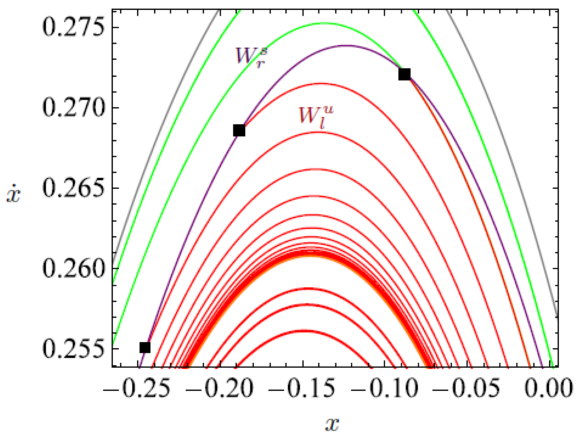
ω = 0.7

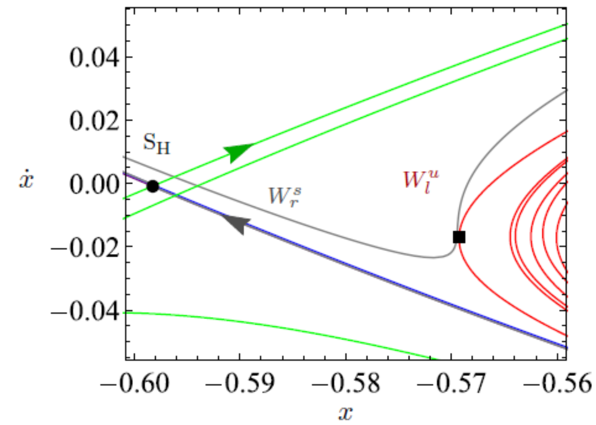
iv. U=0.006676 - HOMOCLINIC BIFURCATION OF THE IN-WELL SADDLE S_1

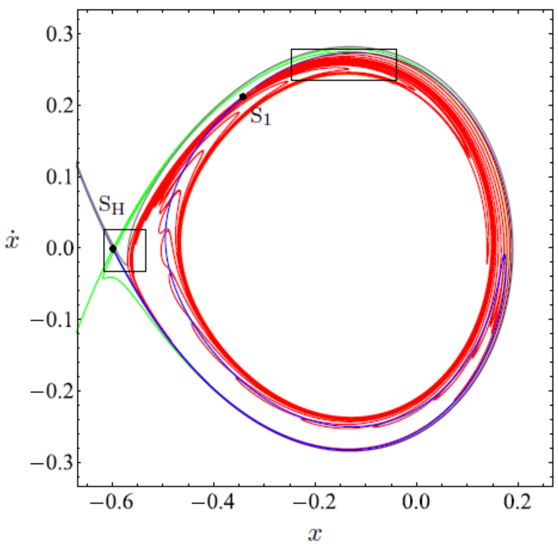
v. HETEROCLINIC BIFURCATION OF THE IN-WELL/HILLTOP SADDLEs S_1/ S_H



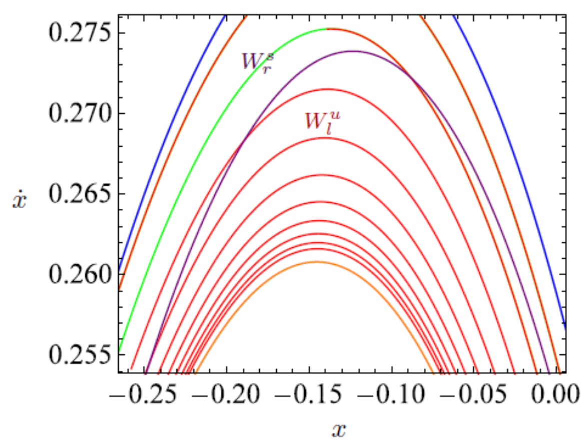
TANGENCY U=0.006676

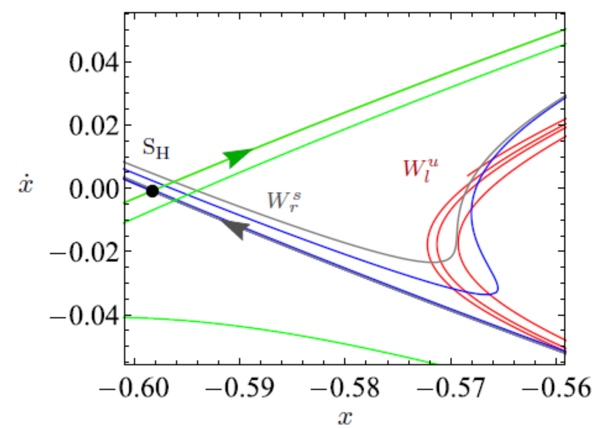






AFTER U=0.00668

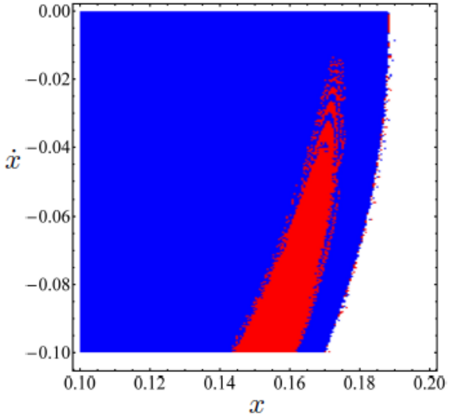




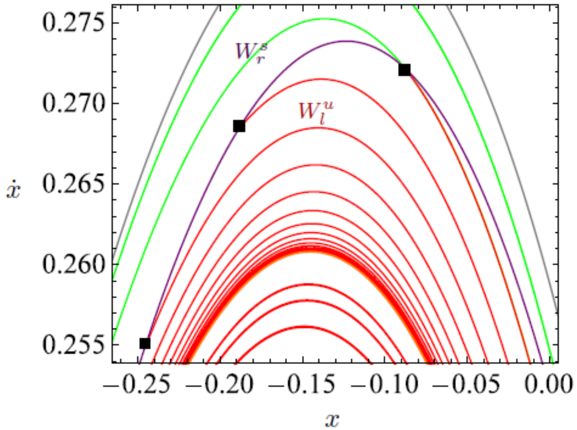
$\omega = 0.7$

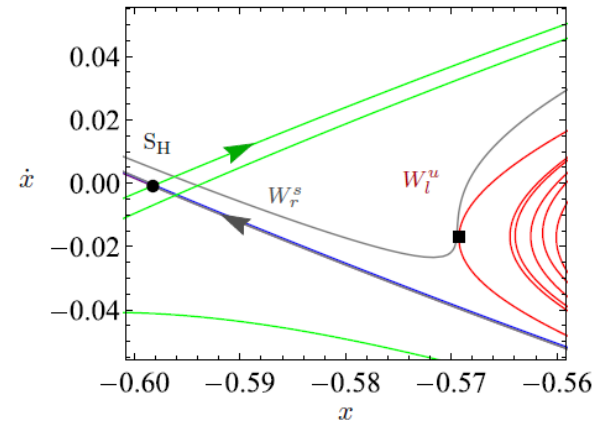
iv. U=0.006676 - HOMOCLINIC BIFURCATION OF THE IN-WELL SADDLE S_1

v. HETEROCLINIC BIFURCATION OF THE IN-WELL/HILLTOP SADDLEs S_1/ S_H

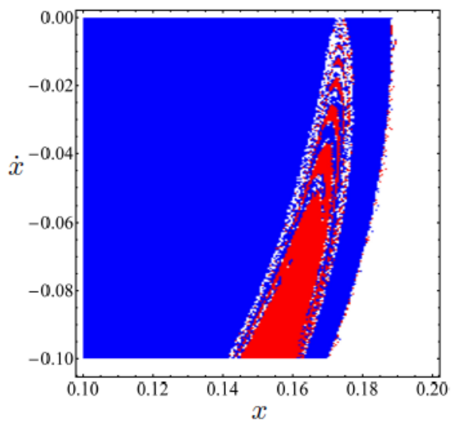


TANGENCY U=0.006676

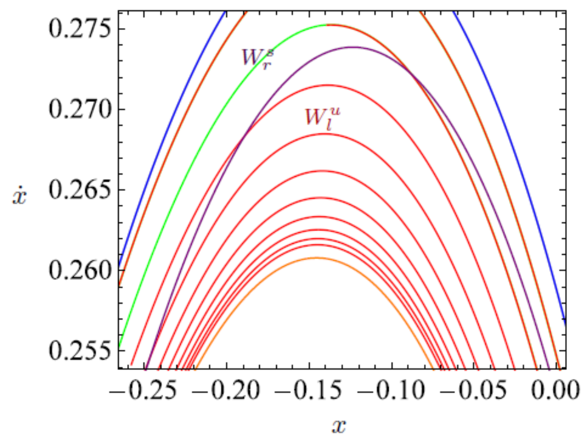


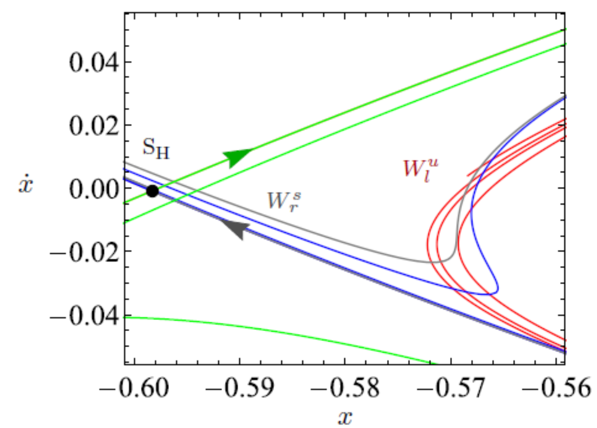


STARTING POINT FOR ACTUAL **IN-WELL** BASINS **SEPARATION** VIA **TONGUES FROM ESCAPE !!**



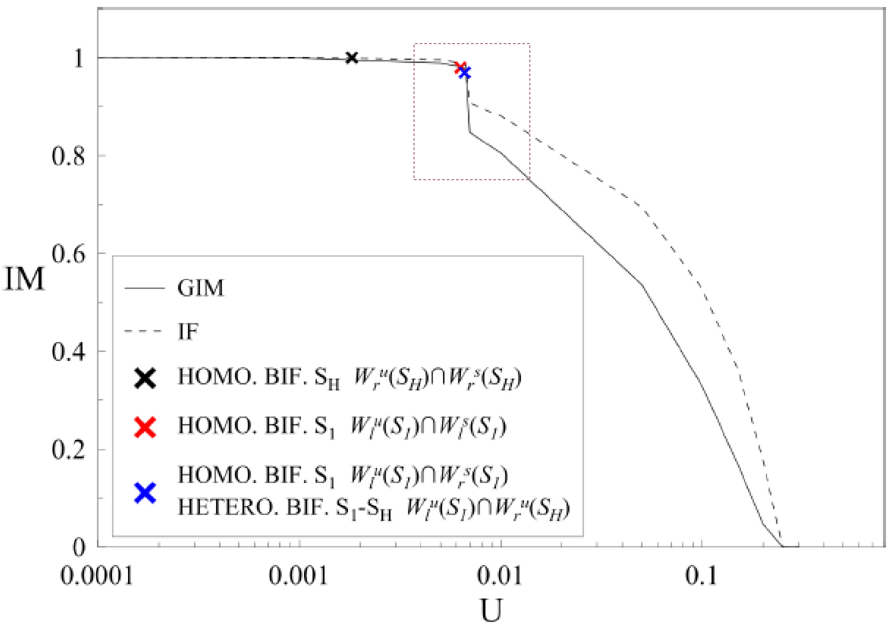
AFTER U=0.00668

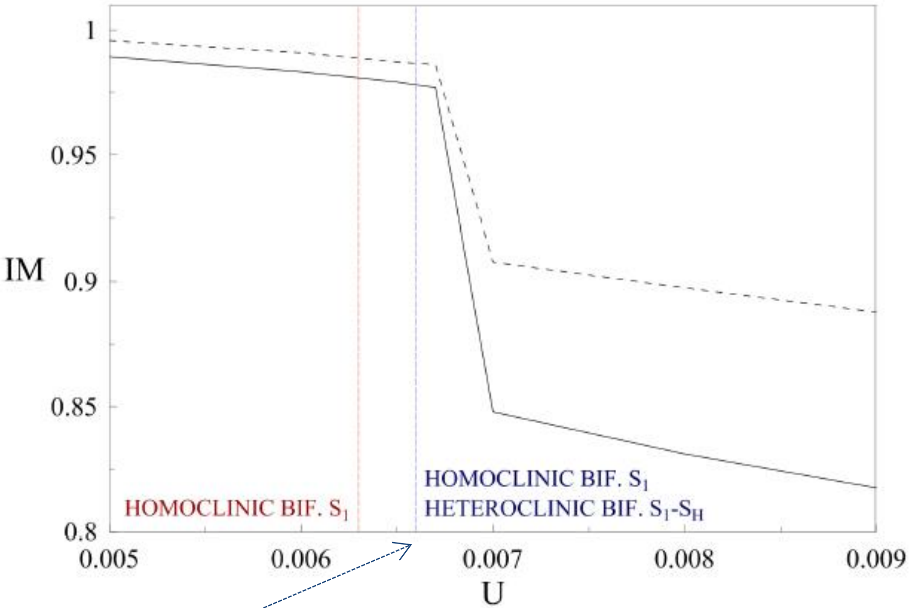




ω = 0.7

BIFURCATIONS VS EROSION PROFILES





HOMOCLINIC BIFURCATION S_1 / HETEROCLINIC BIFURCATION S_1-S_H:

actually **TRIGGERING** the **SHARP REDUCTION** OF **SAFE BASIN INTEGRITY !**

NUMERICAL CONTROL PROCEDURE

AIM

Delay **HOMO/HETEROCLINIC** BIFURCATIONS involving **ANY SADDLE** of the system

STEPS

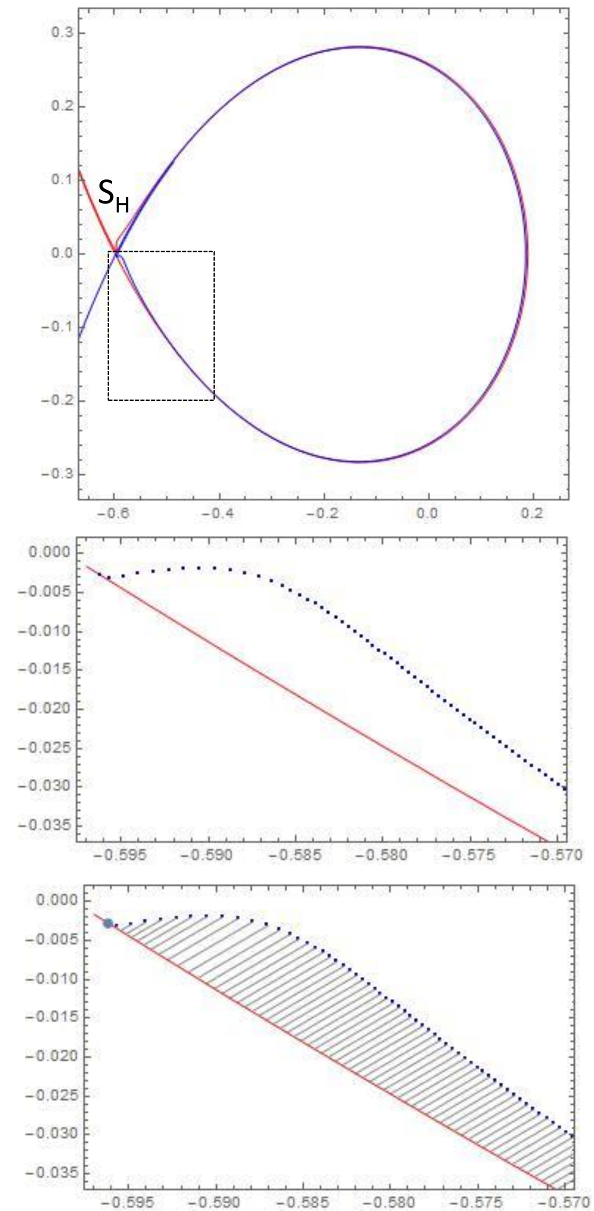
- **IDENTIFY** a PROPER **REGION** in state plane (including all possible global bifurcations)
- NUMERICALLY **DETECT** stable and unstable **MANIFOLDS**

- **COMPUTATION** of MANIFOLDS **DISTANCE**:
  - One manifold → DISCRETE

    Other manifold → CONTINUOUS (interpolating function)

  - Projection of DIRECTION of hilltop saddle UNSTABLE EIGENVECTOR on each point of discrete manifold
  - MEASURE of DISCRETE-CONTINUOUS SEGMENT via Arclength Method

  DISTANCE equal to **ZERO** → **GLOBAL BIFURCATION**



VALIDATION

HOMOCLINIC BIFURCATION OF HILLTOP SADDLE S_H

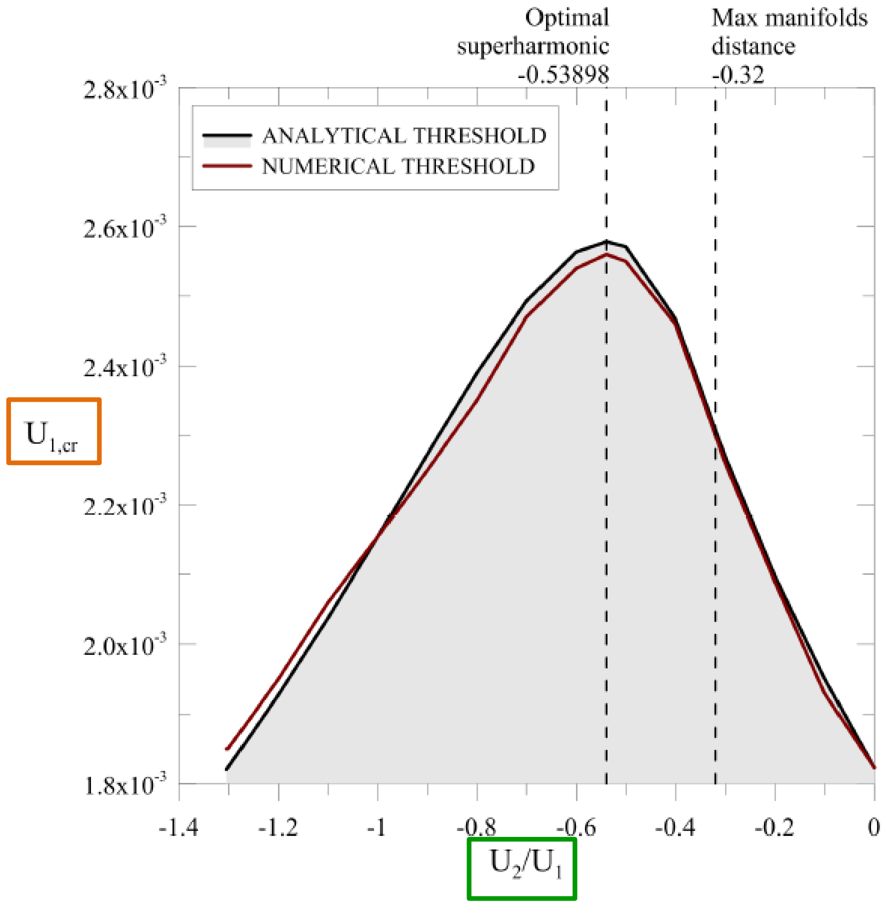
$$\ddot{x}+\alpha_1 x+\alpha_3 x^3+\frac{\Gamma_1}{(1+x)^2}=$$
$$-\rho_1\dot{x}-\mu_1\omega^2 x\boxed{U_1}\left(\sin(\omega t)+\boxed{\frac{U_2}{U_1}}\sin(2\omega t)\right)$$

COMPARISON between

analytical **MELNIKOV** method

and **NUMERICAL** method

(1 superharmonic, ω=0.7)



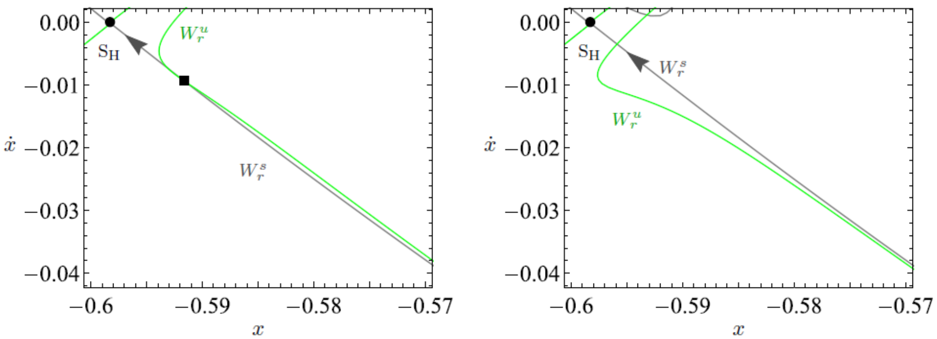
- Good **ACCORDANCE** between results
- The **numerical method** is able to **DETECT** the value of **OPTIMAL SUPERHARMONIC** to be added for shifting the global bifurcation to the highest value of forcing amplitude

VALIDATION

HOMOCLINIC BIFURCATION OF HILLTOP SADDLE S_H

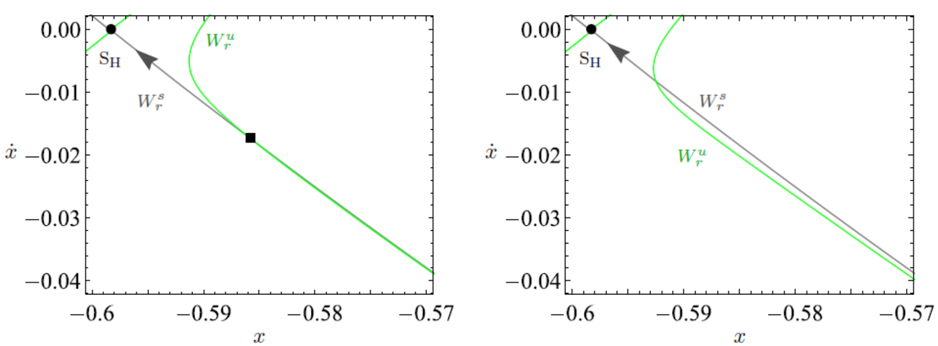
Analytical vs numerical max manifolds distance

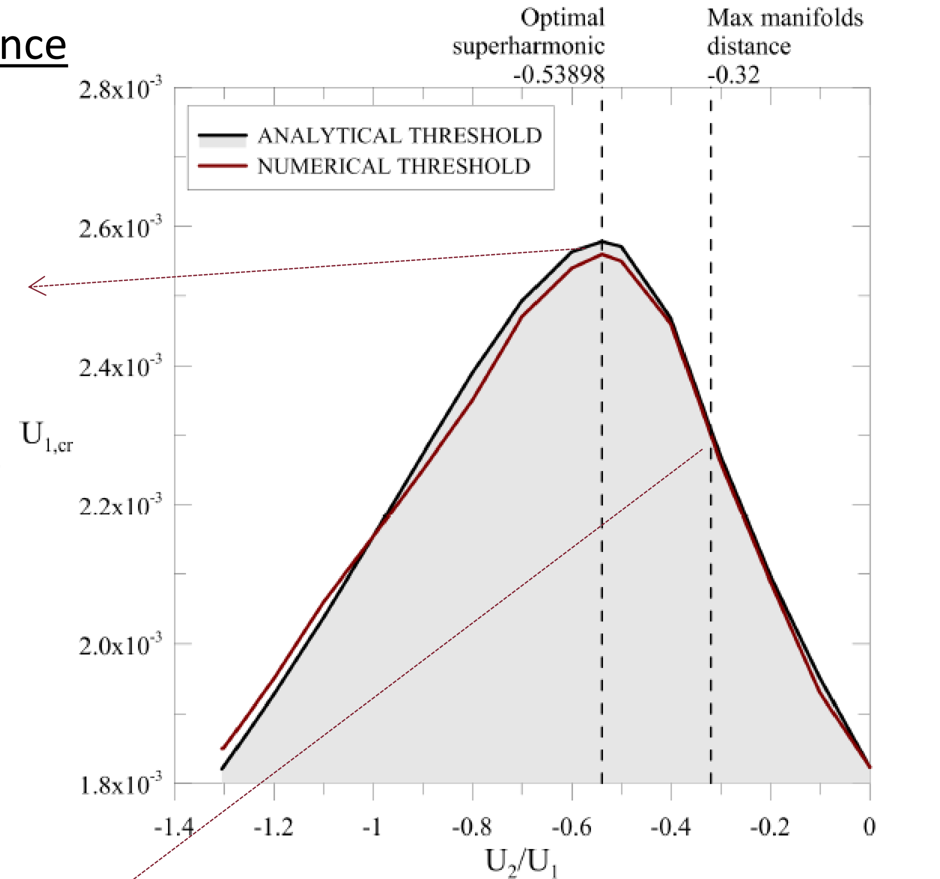
ANALYTICAL



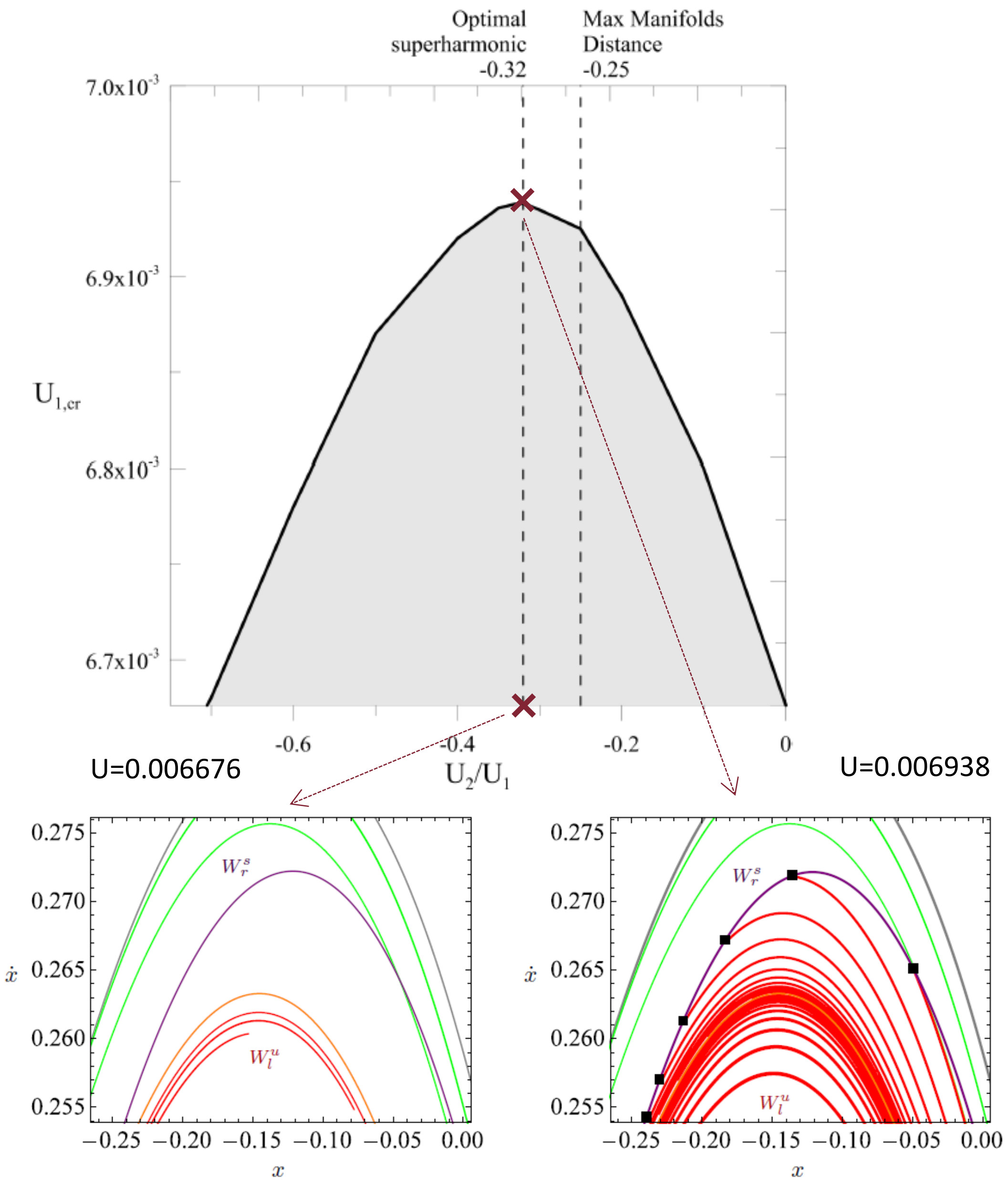
- Strongly nonlinear behavior of manifolds
- Different way of approaching each other
- Shift of the bifurcation point

NUMERICAL

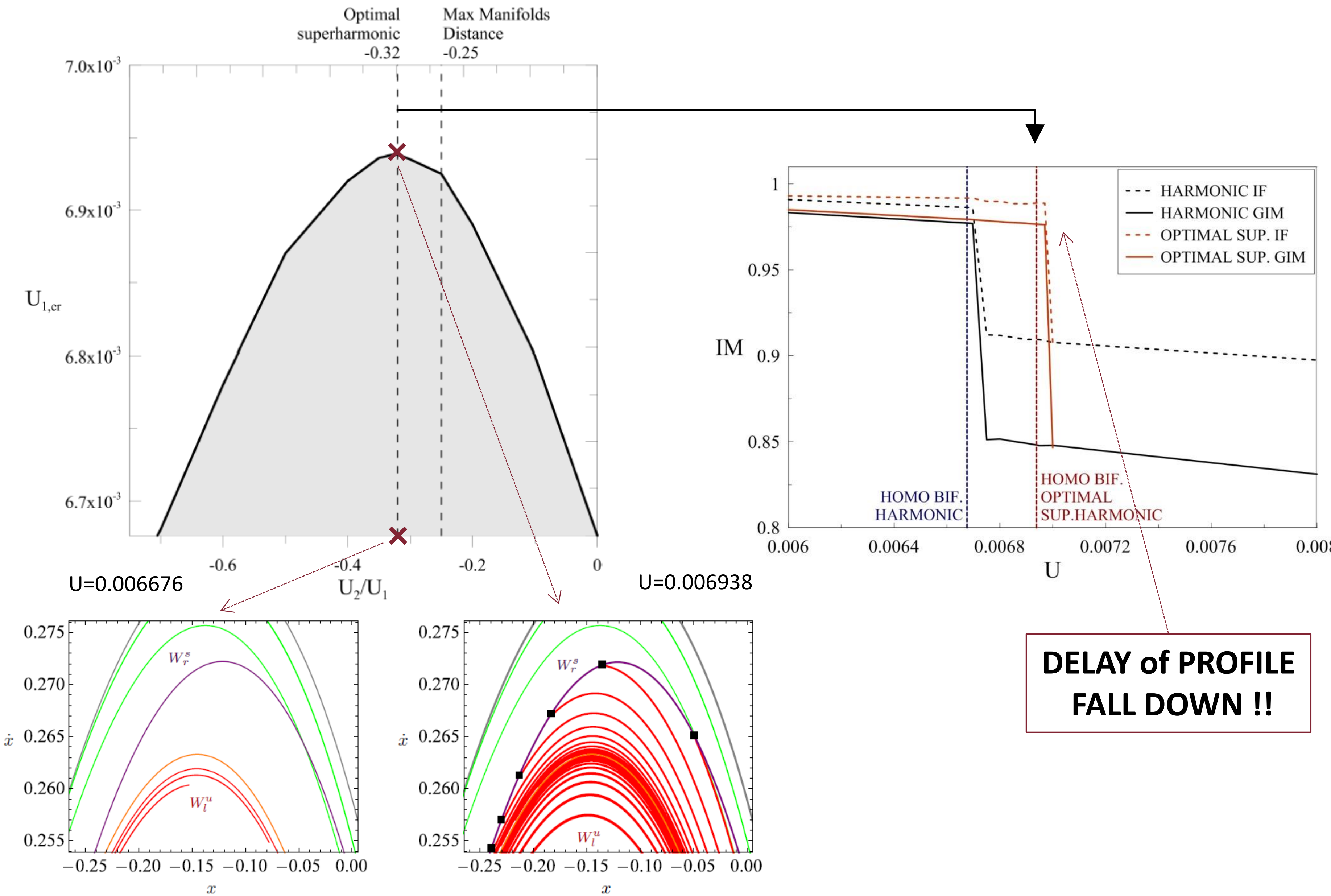




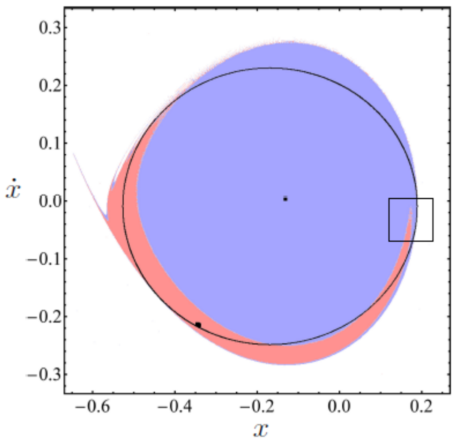
HOMOCLINIC BIFURCATION OF IN-WELL SADDLE S_1 - 1 -

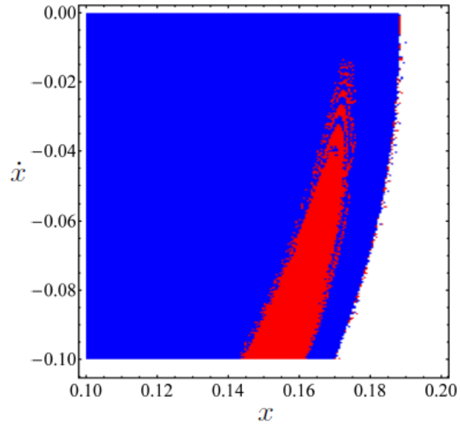


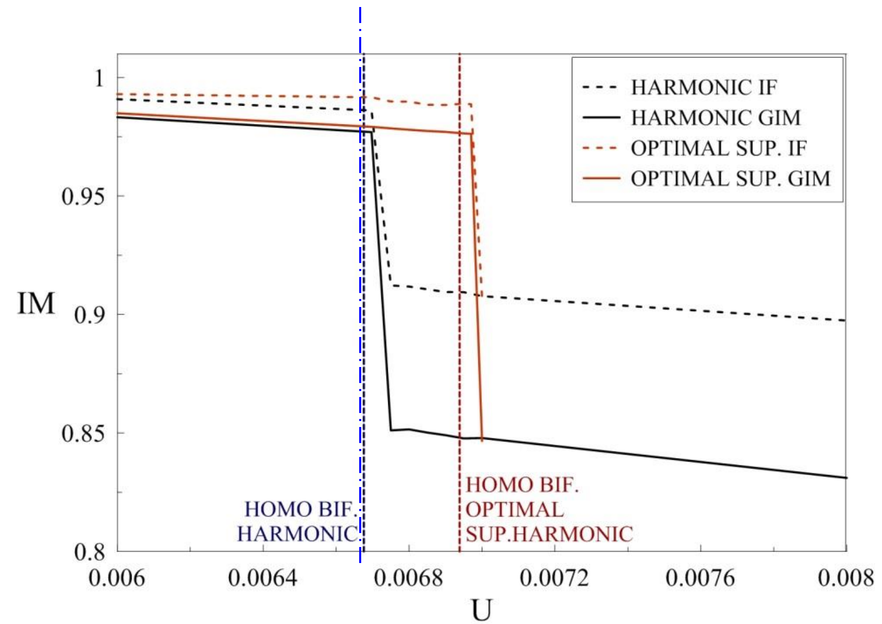
HOMOCLINIC BIFURCATION OF IN-WELL SADDLE S_1 - 1 -



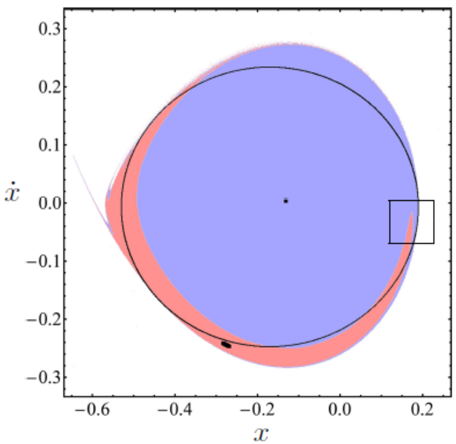
UNCONTROLLED (HARMONIC)

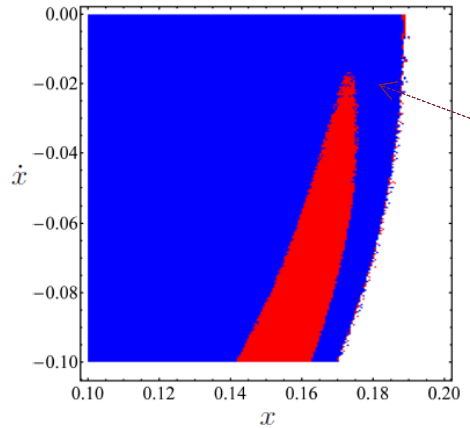






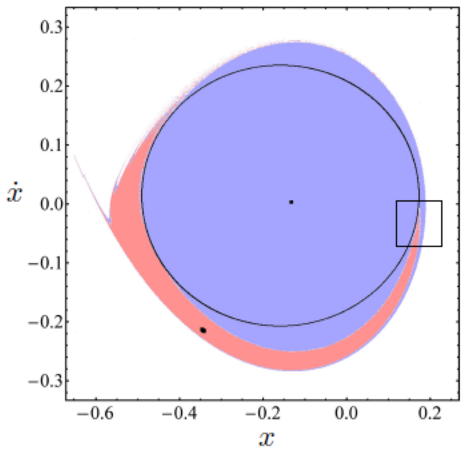
CONTROLLED (OPTIMAL SUP.)

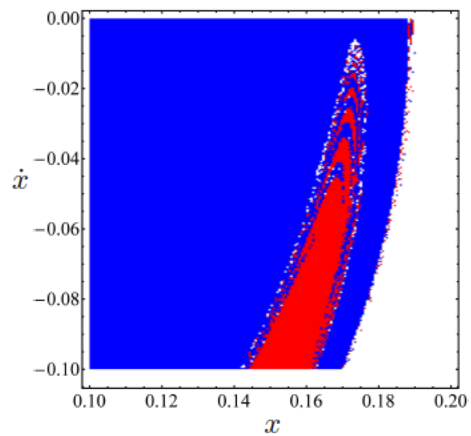


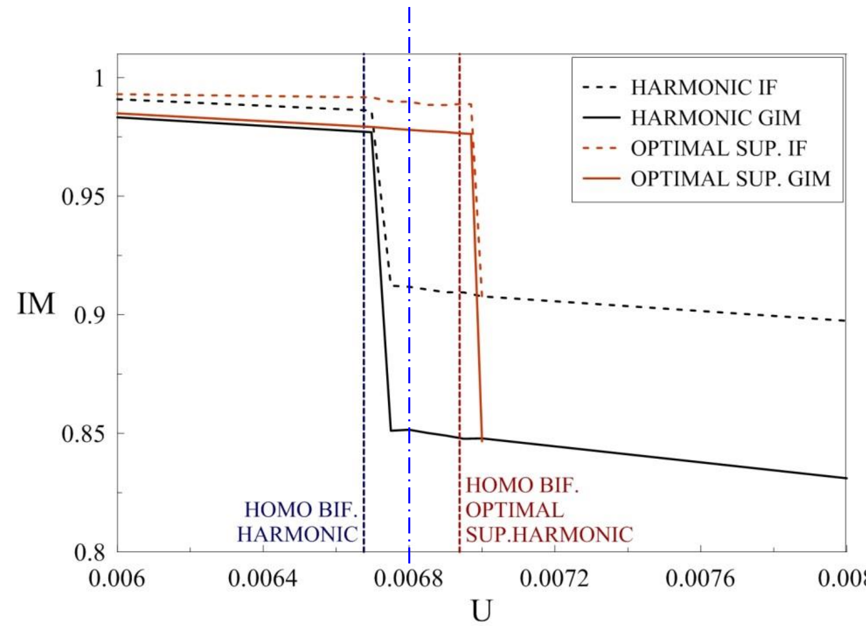


REDUCTION OF BASINS EROSION

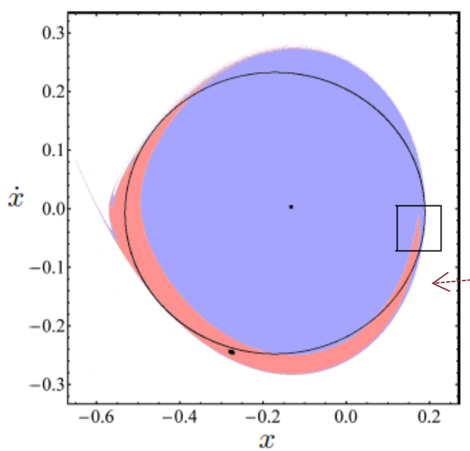
UNCONTROLLED (HARMONIC)

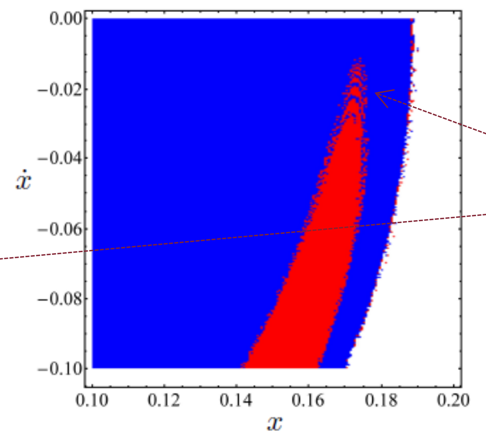






CONTROLLED (OPTIMAL SUP.)





DELAY OF BASINS SEPARATION

SUMMARY AND COMMENTS

- Transition from **LOCAL** to **GLOBAL SAFETY** in **engineering design**: major implications also as regards **FEASIBILITY/EFFECTIVENESS** of **CONTROL**
- **GLOBAL control procedure EXPLOITING** some associated **GLOBAL BIFURCATION** event to **favorably affect** system **stability** in terms of **EROSION DELAY**

⇓

MAIN PROBLEM: DETECTION of GLOBAL BIFURCATIONS/SADDLES involved in erosion triggering

→ **HILLTOP SADDLE**: **analytical asymptotic MELNIKOV METHOD** to compute distance between perturbed stable and unstable manifolds

→ **OTHER INTERNAL SADDLES**: need for a **FULLY NUMERICAL METHOD**

Cross-validation and **differences**