

DAY	TIME	LECTURE
Monday 05/11	14.00 -14.45	Historical Framework - A Global Dynamics Perspective in the Nonlinear Analysis of Systems/Structures
	15.00 -15.45	Achieving Load Carrying Capacity: Theoretical and Practical Stability
	16.00 -16.45	Dynamical Integrity: Concepts and Tools_1
Wednesday 07/11	14.00 -14.45	Dynamical Integrity: Concepts and Tools_2
	15.00 -15.45	Global Dynamics of Engineering Systems
	16.00 -16.45	Dynamical integrity: Interpreting/Predicting Experimental Response
Monday 12/11	14.00 -14.45	Techniques for Control of Chaos
	15.00 -15.45	A Unified Framework for Controlling Global Dynamics
	16.00 -16.45	Response of Uncontrolled/Controlled Systems in Macro- and Micro-mechanics
Wednesday 14/11	14.00 -14.45	<b>A Noncontact AFM: (a) Nonlinear Dynamics and Feedback Control (b) Global Effects of a Locally-tailored Control</b>
	15.00 -15.45	Exploiting Global Dynamics to Control AFM Robustness
	16.00 -16.45	Dynamical Integrity as a Novel Paradigm for Safe/Aware Design

# 12.1b – A Noncontact AFM: Global Effects of a Locally-tailored Control

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Sapienza University of Rome, Italy*

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**Coworker:** V. Settimi

## OUTLINE of 12.1b

1. **STRONGLY NONLINEAR DYNAMICS OF THE CONTROLLED SYSTEM**
2. **DYNAMICAL INTEGRITY OF THE CONTROLLED SYSTEM**

..... towards a **GLOBAL DYNAMICS-BASED CONTROL**

# NONCONTACT AFM WITH EXTERNAL FEEDBACK CONTROL

## RESPONSE OF THE CONTROLLED SYSTEM

$$\ddot{x}(1 + \alpha_2 x^2) + \alpha_1 x + \alpha_2 x \dot{x}^2 + \alpha_3 x^3 = -\Gamma_1 (1 + x + V_g + z - z_s)^{-2} - (\rho_1 + \rho_2 x^2) \dot{x} - (\ddot{V}_g + k_g (\dot{x}_{ref} - \dot{x}) + \nu_1 (\dot{V}_g + k_g (x_{ref} - x))) \nu_2 + (x \mu_1 + \mu_2 x^3) (\ddot{U}_g + \eta_1 \dot{U}_g + \eta_2 U_g)$$

$$\dot{z} = k_g (x_{ref} - x)$$

+

## REFERENCE RESPONSE

ATOMIC  
INTERACTION

$$\ddot{x}_{ref} (1 + \alpha_2 x_{ref}^2) + \alpha_1 x_{ref} + \alpha_2 x_{ref} \dot{x}_{ref}^2 + \alpha_3 x_{ref}^3 = -\Gamma_1 (1 + x_{ref} + V_g)^{-2} - \rho_1 \dot{x}_{ref} - \rho_2 \dot{x}_{ref} x_{ref}^2 - ((\ddot{V}_g + \nu_1 \dot{V}_g)) \nu_2 + (x_{ref} \mu_1 + \mu_2 x_{ref}^3) (\ddot{U}_g + \eta_1 \dot{U}_g + \eta_2 U_g)$$

- INCREASED D.O.F. : **RICHER BIFURCATIVE SCENARIO**
- New TORUS and TRANSCRITICAL bifurcations: **STABILITY BOUNDARY REDUCTION**

# STRONGLY NONLINEAR DYNAMICS

- System nonlinear response as function of MOST RELEVANT **DYNAMICAL PARAMETERS**:
  - **FORCING AMPLITUDE**  $U(V)$
  - **FORCING FREQUENCY**  $\omega_u(\omega_v)$
  - **ATOMIC INTERACTION**  $\Gamma_1$
  - **FEEDBACK CONTROL PARAMETER**  $k_g$
- BIFURCATION DIAGRAMS and RESPONSE CHARTS around **FUNDAMENTAL** and **PRINCIPAL** resonances
- **PARAMETRICALLY** and **EXTERNALLY** forced system
- **COMPARISON** with results obtained for **UNCONTROLLED** system



**INFLUENCE of EXTERNAL FEEDBACK CONTROL on DYNAMIC BEHAVIOR**

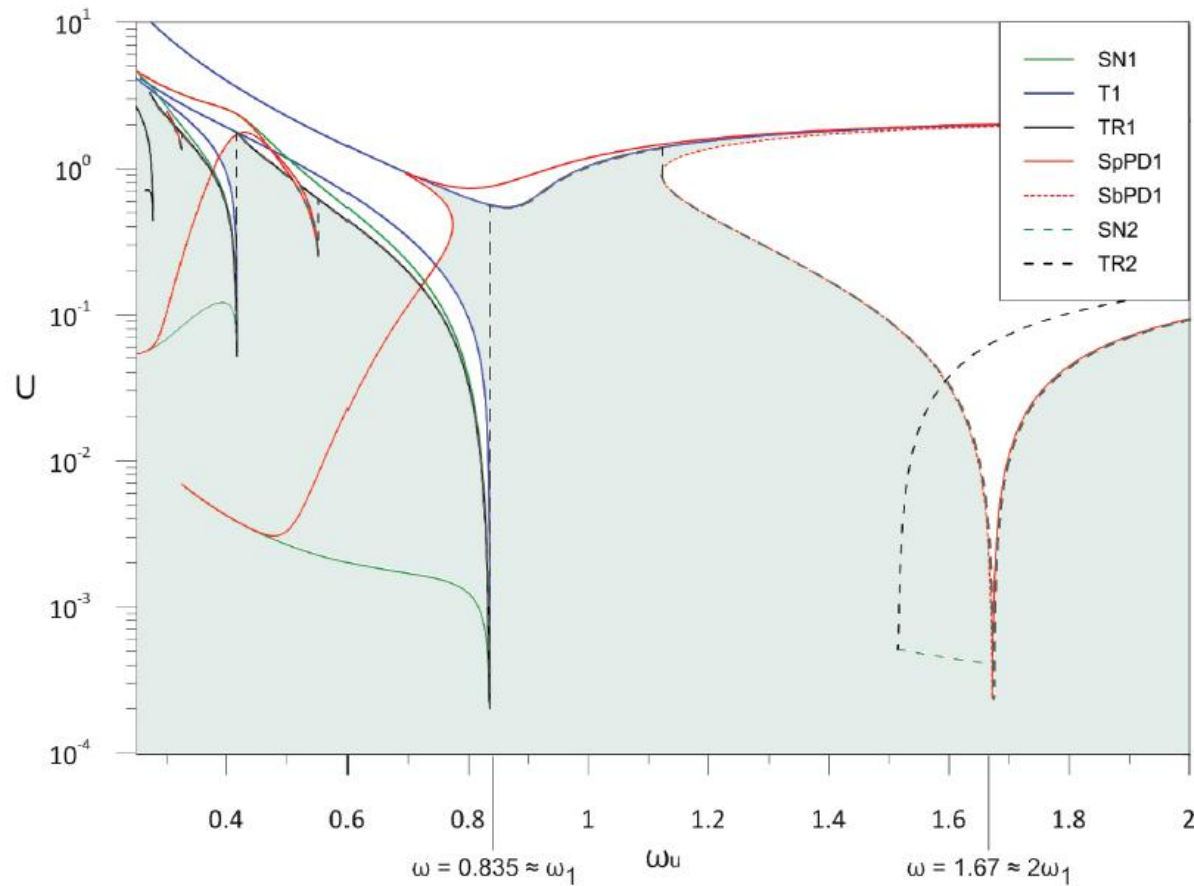
# PARAMETRIC EXCITATION - 1 -

$$\ddot{x} + \alpha_1 \dot{x} + \alpha_3 x^3 = -\Gamma_1 (1 + x + z - z_s)^{-2} - \rho_1 \dot{x} - x \mu_1 U \omega_u^2 \sin(\omega_u t)$$

$$\dot{z} = k_g (x_{ref} - x)$$

$\omega_u - U$

$\mu_2 = \rho_2 = 0$   
 $\eta_1 = \eta_2 = 0$   
 $V_g = \alpha_2 = 0$   
 $\alpha_1 = 1$   
 $\Gamma_1 = 0.1$   
 $\alpha_3 = 0.1$   
 $\mu_1 = 1.5708$   
 $\omega_1 = 0.8325$   
 $k_g = 0.001$   
 $z_s = 0.01$



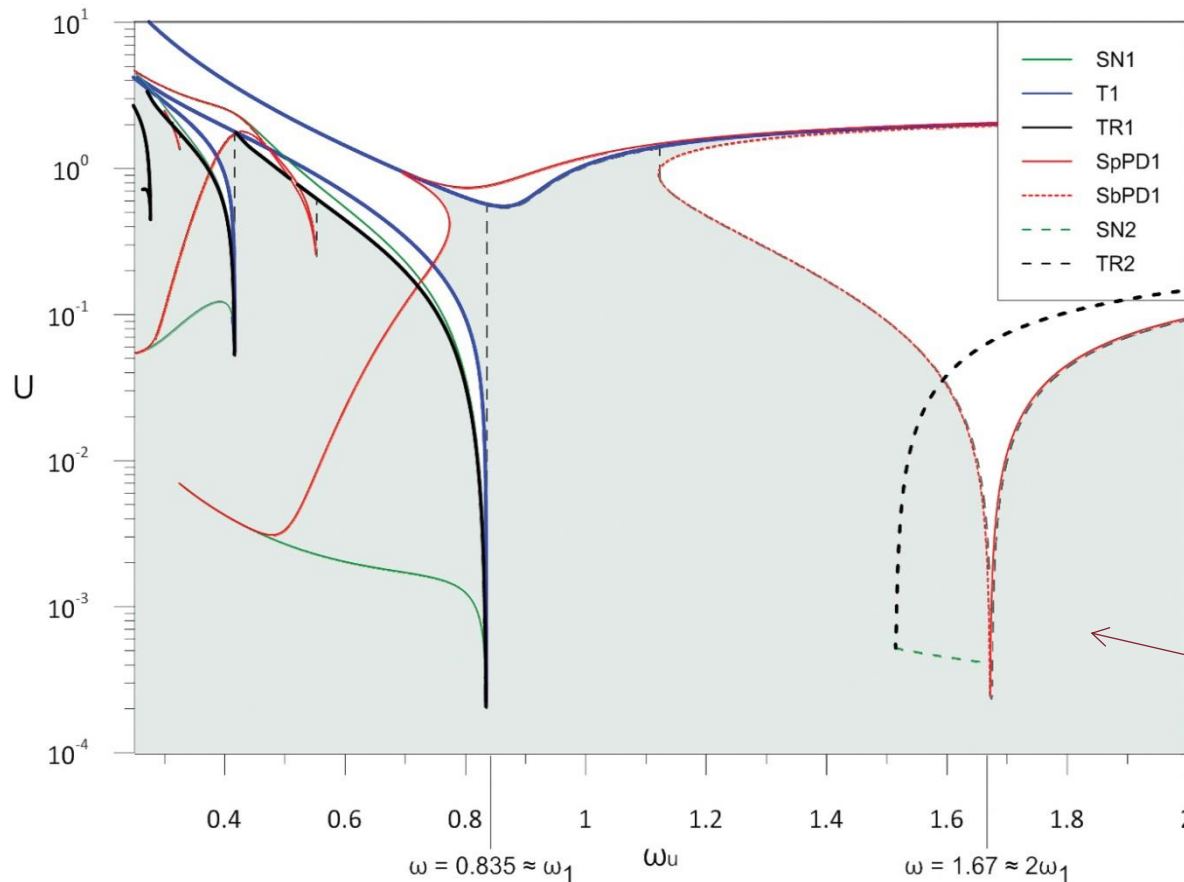
# PARAMETRIC EXCITATION - 1 -

$$\ddot{x} + \alpha_1 \dot{x} + \alpha_3 x^3 = -\Gamma_1 (1 + x + z - z_s)^{-2} - \rho_1 \dot{x} - x \mu_1 U \omega_u^2 \sin(\omega_u t)$$

$$\dot{z} = k_g (x_{ref} - x)$$

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 $\omega_1 = 0.8325$   
 $k_g = 0.001$   
 $z_s = 0.01$



NEW  
**TRANSCRITICAL  
AND TORUS  
THRESHOLDS**



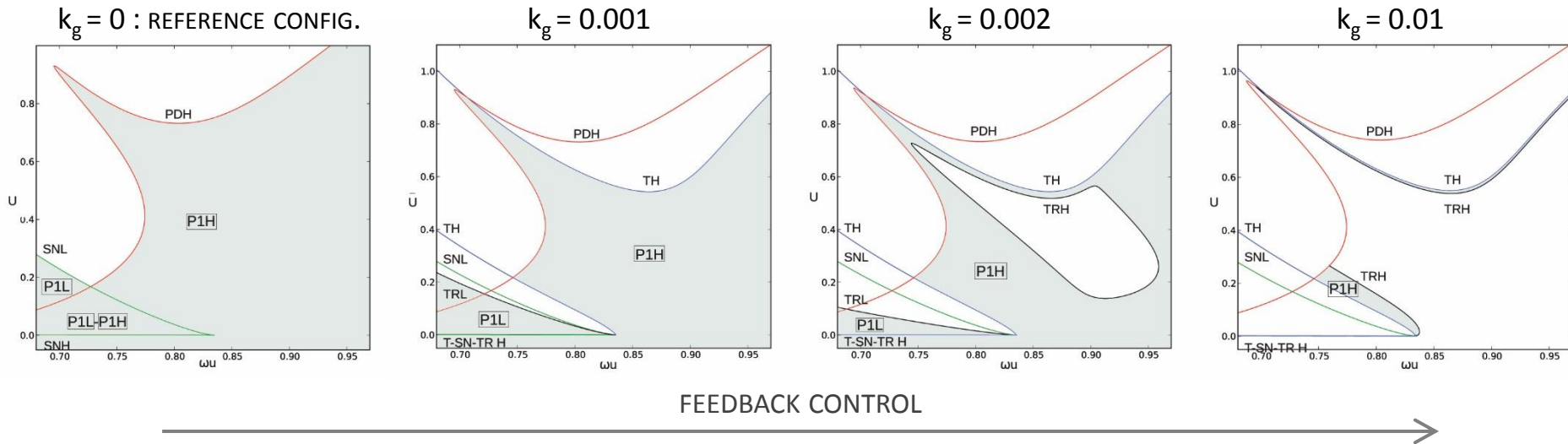
**DECREASE OF  
ESCAPE VALUE**

**STABLE REGION:**  
solutions for which  
**FEEDBACK CONTROL  
WORKS PROPERLY**

## FUNDAMENTAL RESONANCE

$$\omega_u - U$$

WITH VARYING FEEDBACK CONTROL



**TORUS** AND **TRANSCRITICAL** THRESHOLDS

- **UNSTABLE TONGUES** at **LOW** values of  $U$
- TRIANGLE region **REDUCED**
- No COEXISTENCE of P1L/P1H solutions



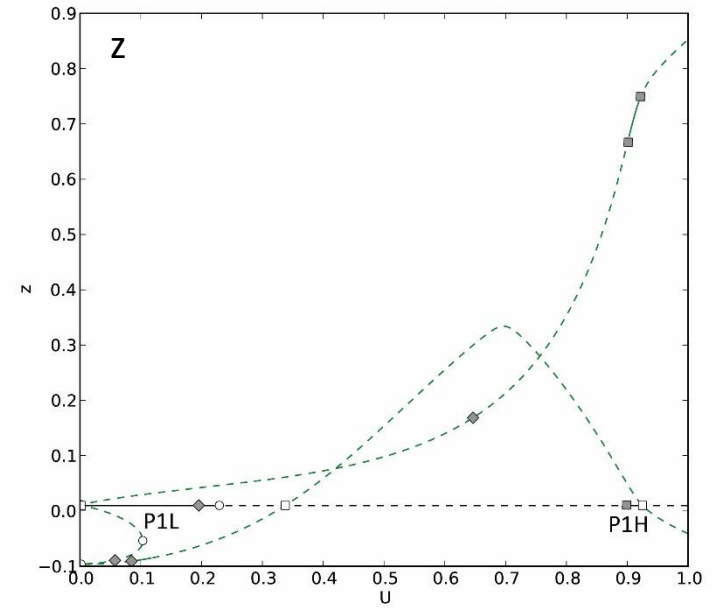
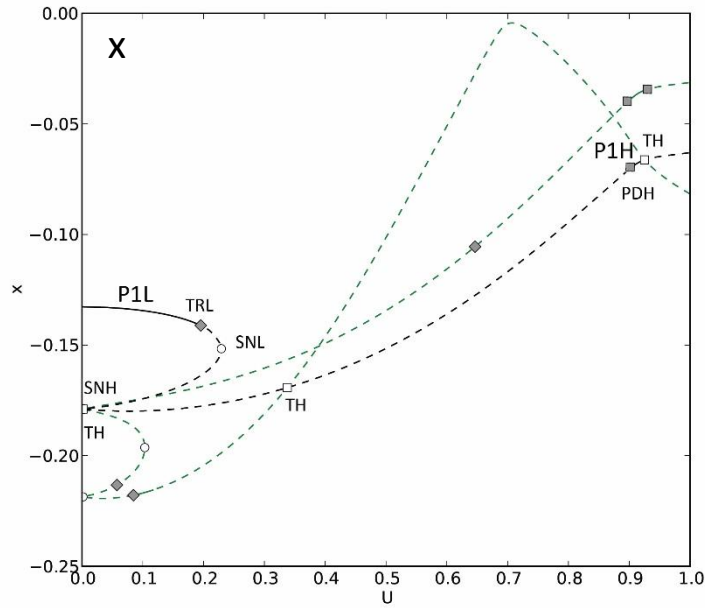
TOTAL **ESCAPE** occurs at **LOWER** VALUES OF  $U$



## FUNDAMENTAL RESONANCE

$\omega_u - U$

BIFURCATION DIAGRAM at  $\omega_u = 0.7$  and  $k_g = 0.001$

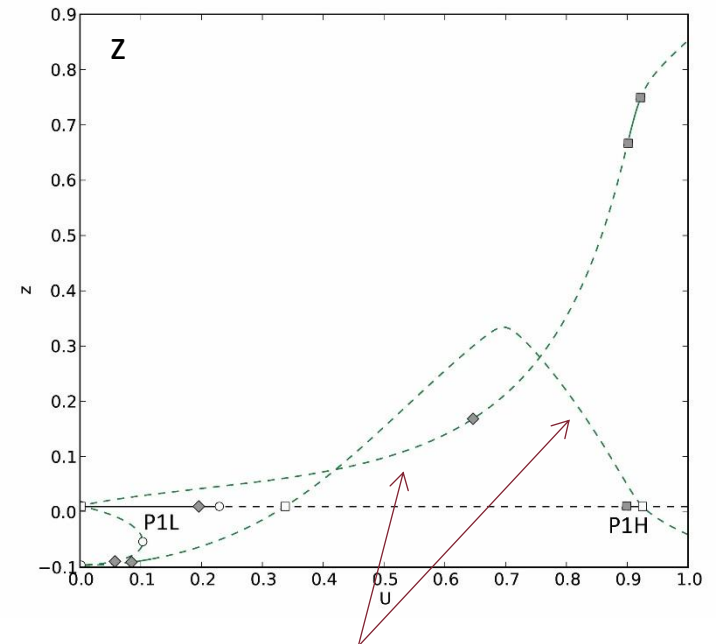
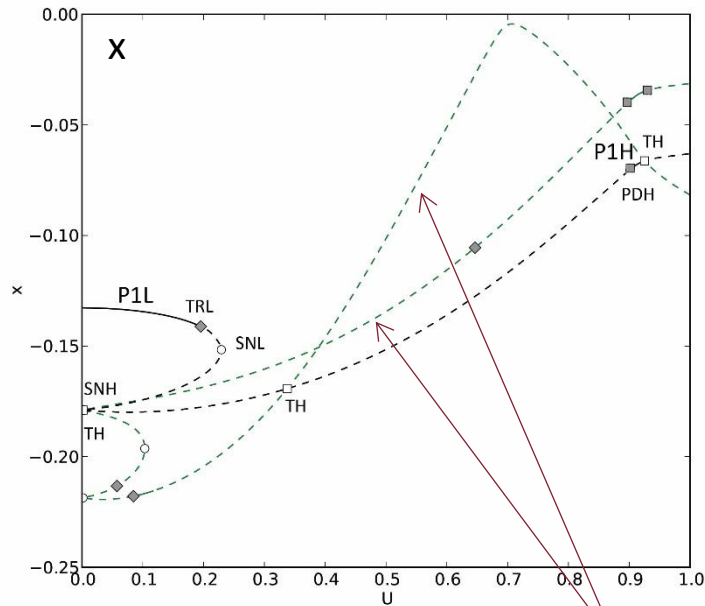


# PARAMETRIC EXCITATION - 3 -

## FUNDAMENTAL RESONANCE

$\omega_u - U$

BIFURCATION DIAGRAM at  $\omega_u = 0.7$  and  $k_g = 0.001$



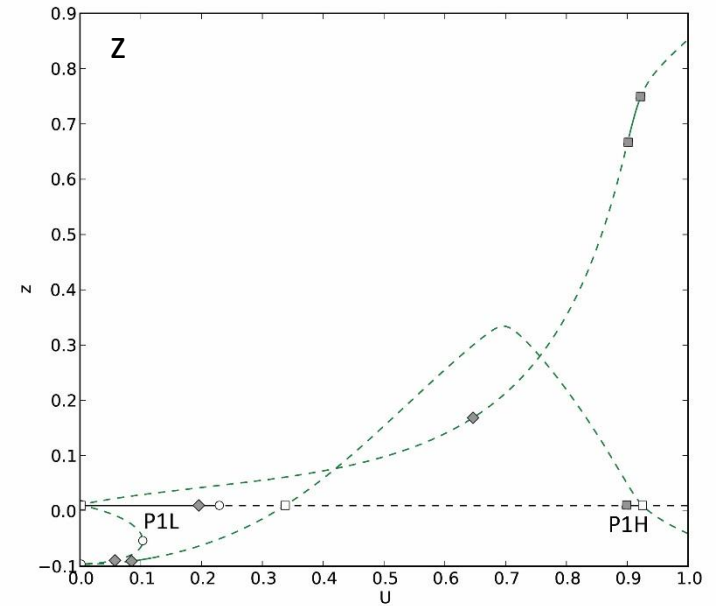
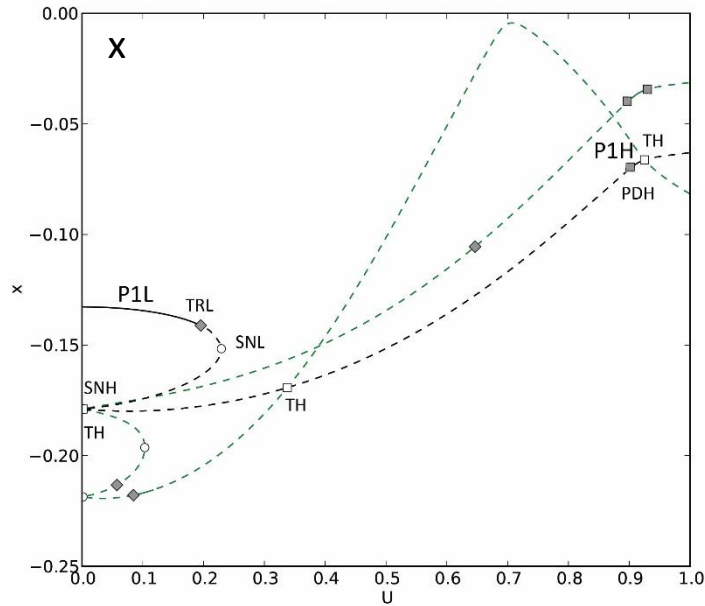
From **TRANSCRITICAL BIF.:** new **STABLE P1** solutions → **INEFFICIENCY OF CONTROL**

# PARAMETRIC EXCITATION - 3 -

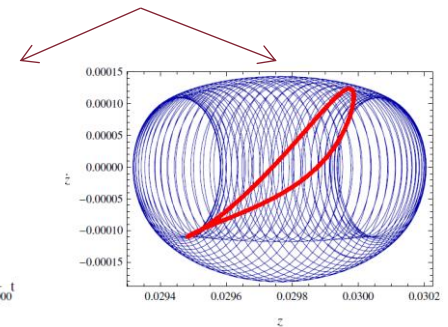
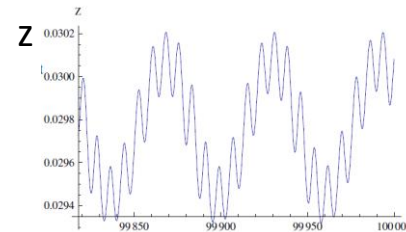
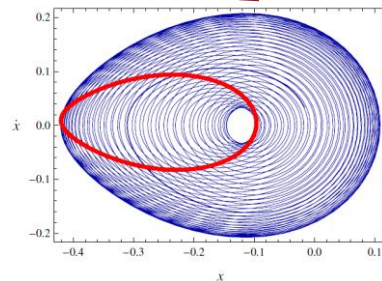
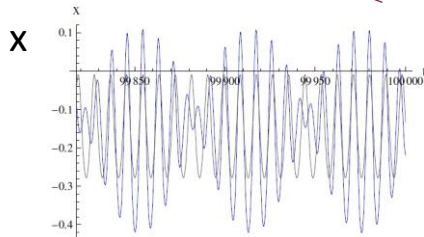
## FUNDAMENTAL RESONANCE

$$\omega_u - U$$

BIFURCATION DIAGRAM at  $\omega_u = 0.7$  and  $k_g = 0.001$

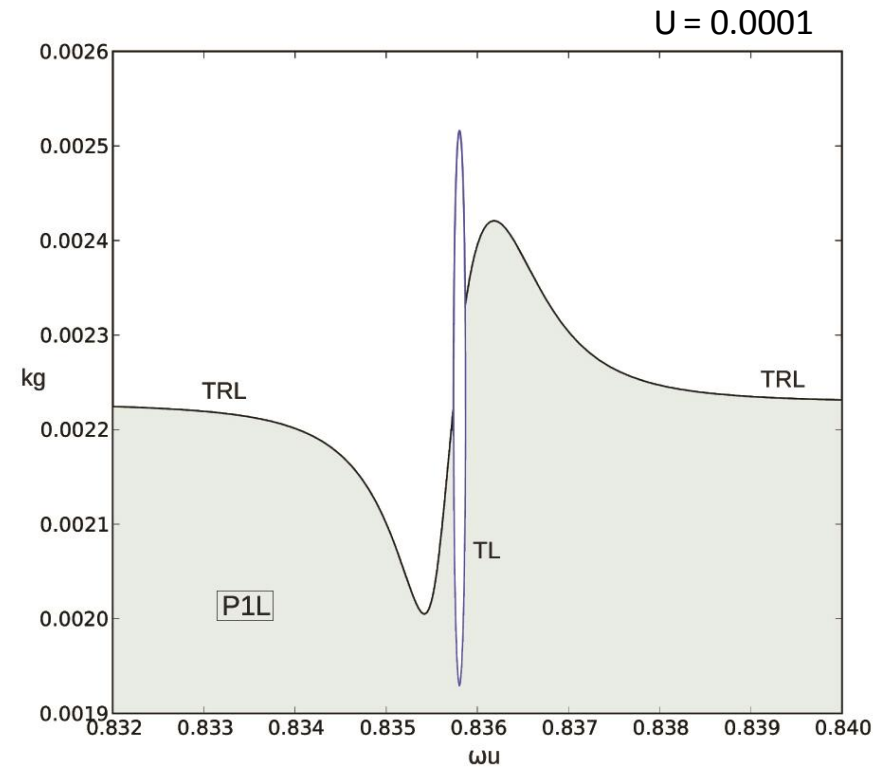


From **TORUS BIF.**: new **STABLE QUASIPERIODIC** solutions → **INEFFICIENCY OF CONTROL**



## FUNDAMENTAL RESONANCE

$$\omega_u - k_g$$

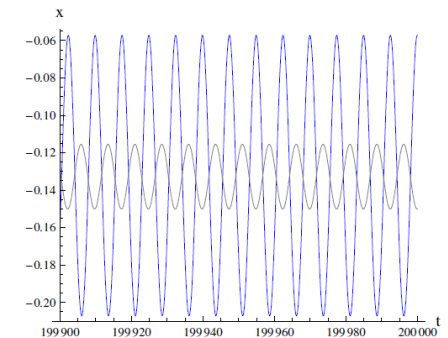
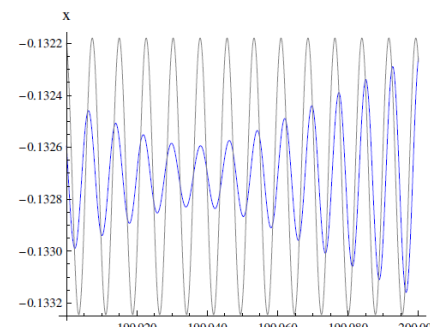
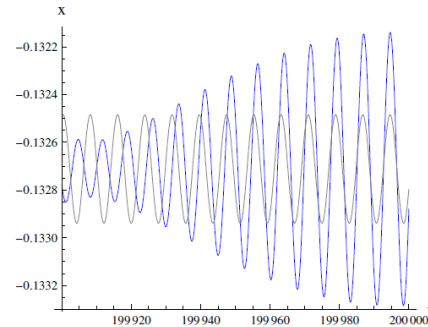
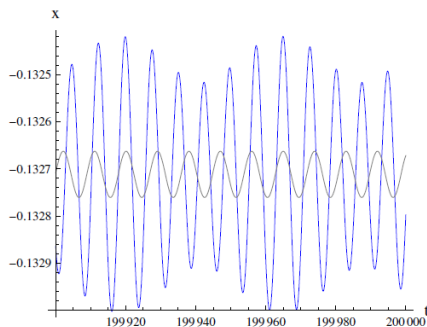
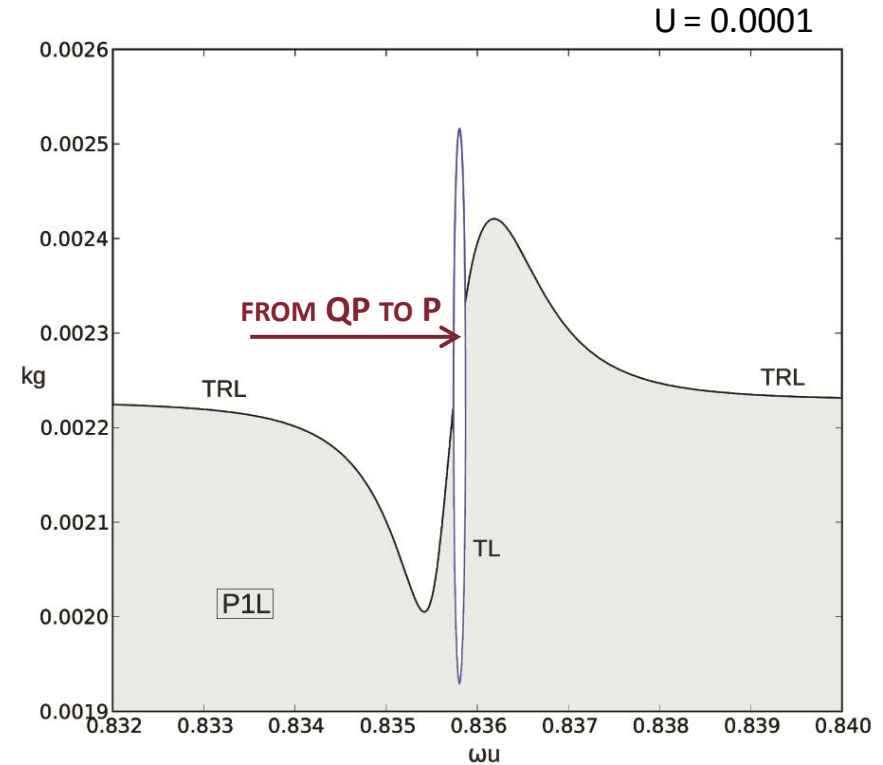


## FUNDAMENTAL RESONANCE

$$\omega_u - k_g$$

### PHASE LOCKING (from QP to P):

synchronization of response frequency  
to forcing one



## FUNDAMENTAL RESONANCE

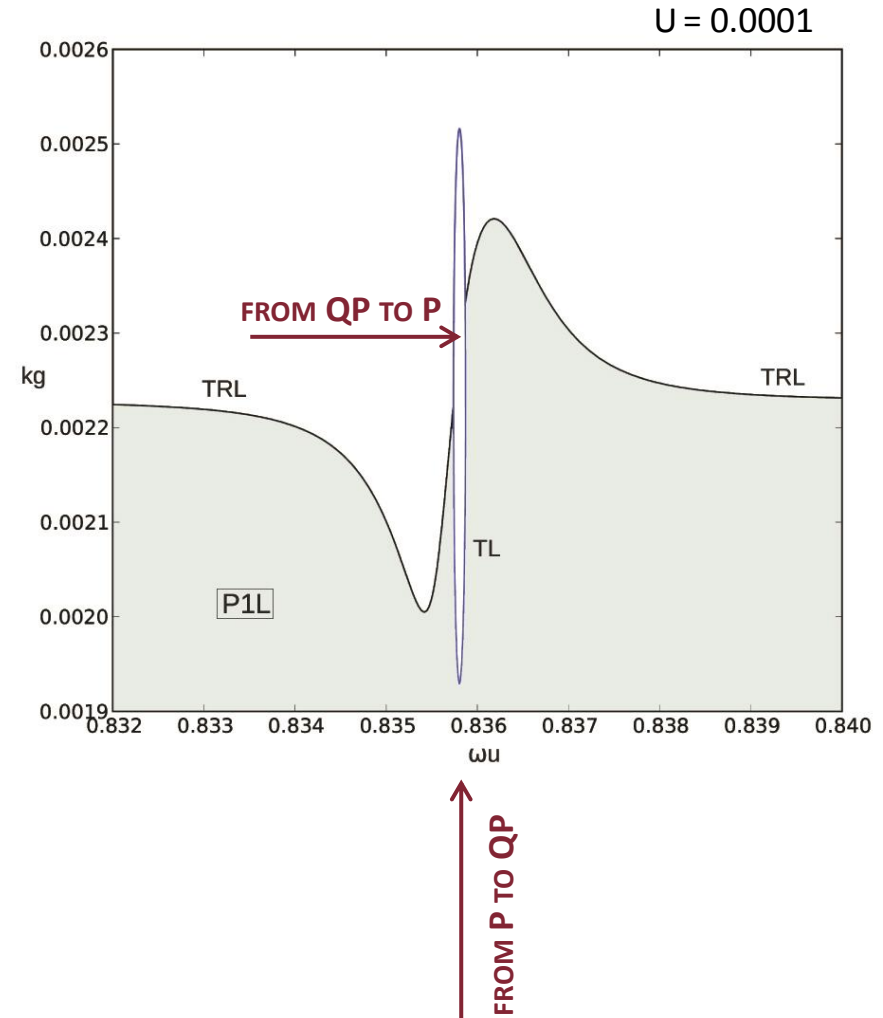
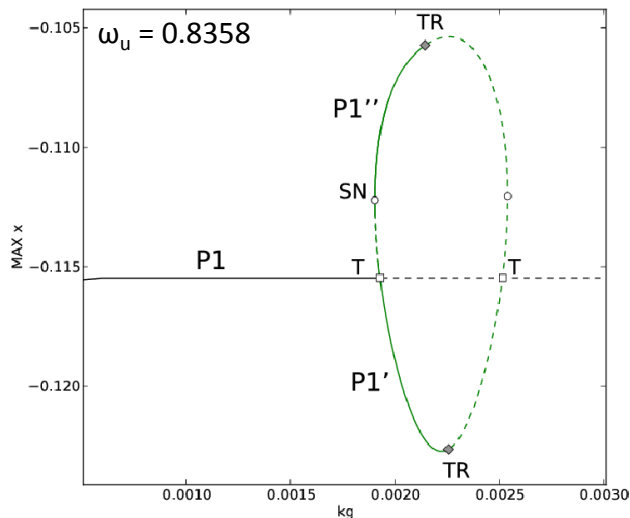
$$\omega_u - k_g$$

### PHASE LOCKING (from QP to P):

synchronization of response frequency to forcing one

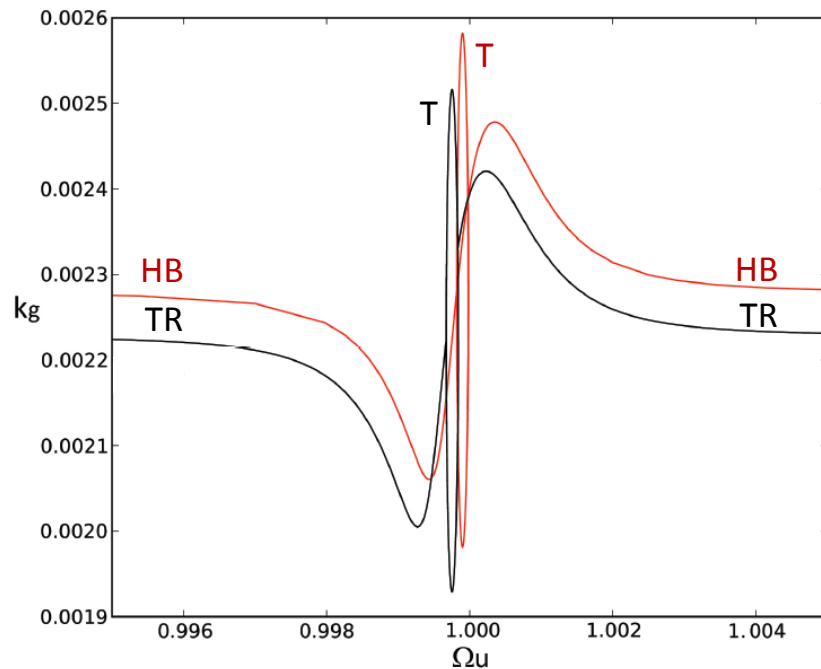
### RESONANCE FREQUENCY (from P to QP):

increase of response amplitude that feedback control barely dominates



GOOD AGREEMENT between **AMEs** and **ODEs**

- **BIFURCATION BEHAVIOR**
- **STABILITY THRESHOLD** OF BOUNDED REFERENCE SOLUTION

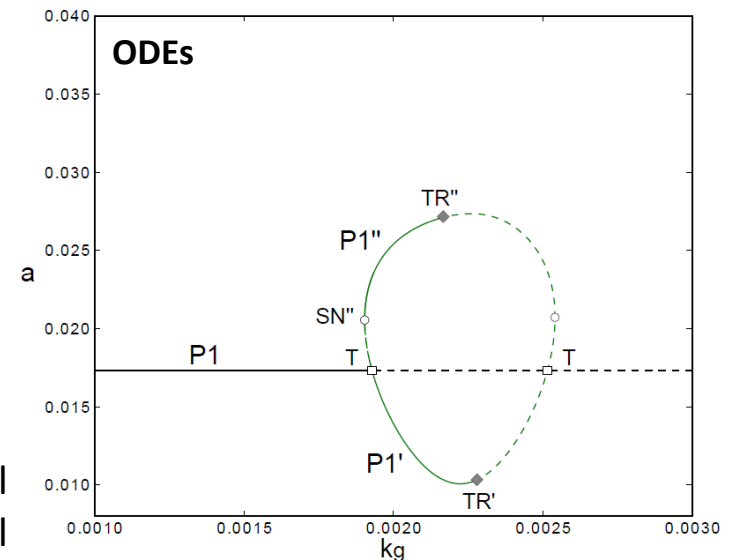
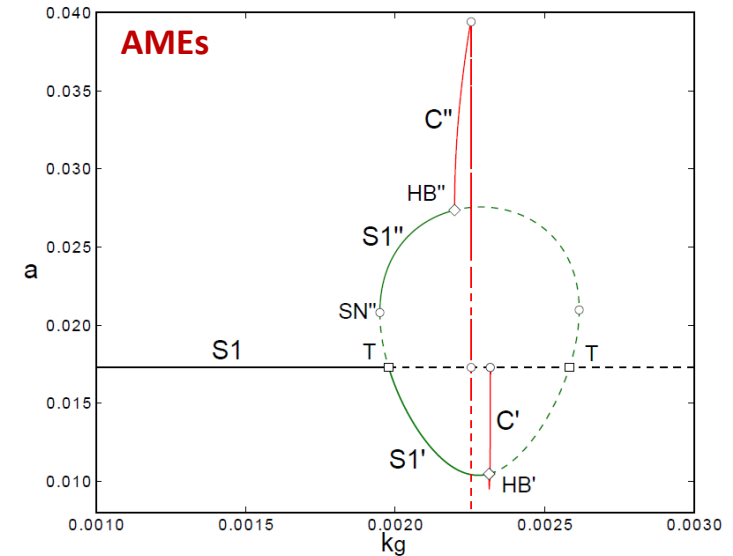
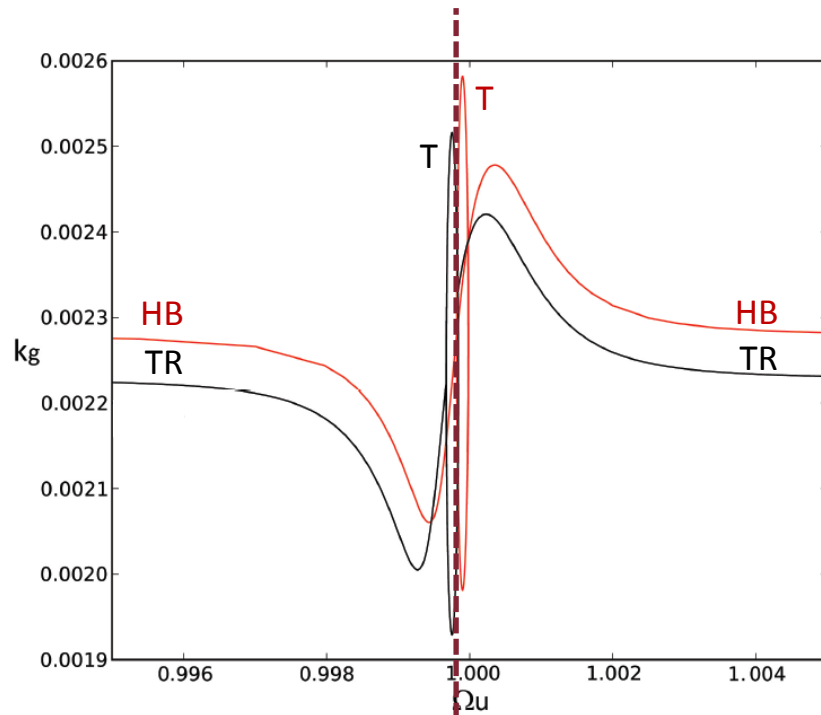


STABILITY THRESHOLD: **Hopf HB** (**Torus TR**) bifurcation

AROUND RESONANCE: **Transcritical T** bifurcation

## GOOD AGREEMENT between **AMEs** and **ODEs**

- **BIFURCATION BEHAVIOR**
- **STABILITY THRESHOLD** OF BOUNDED REFERENCE SOLUTION



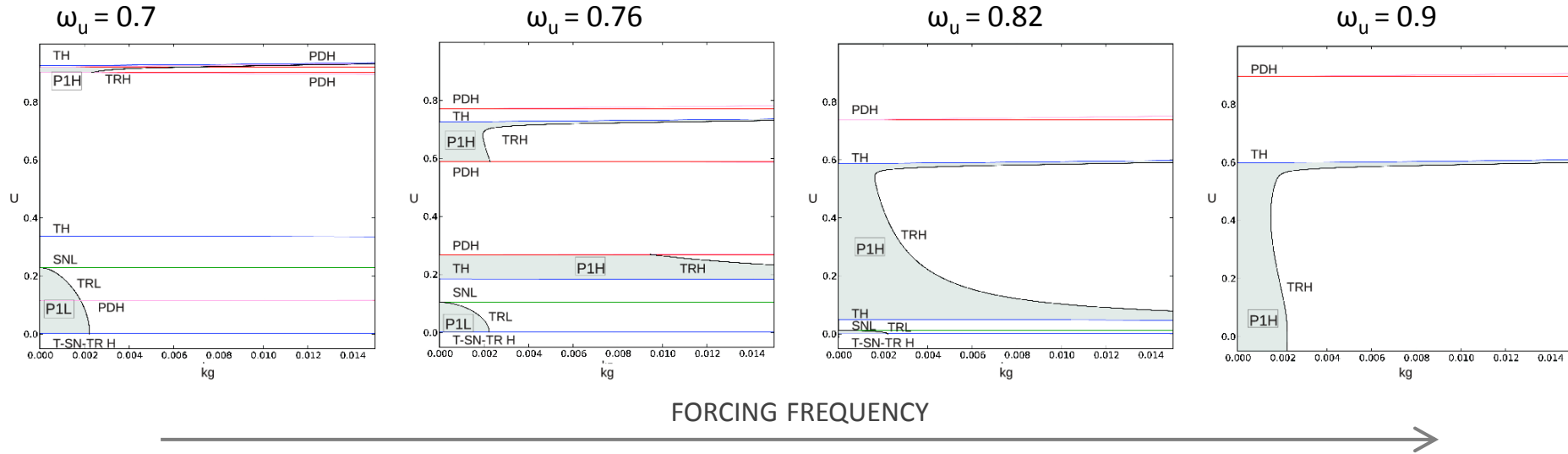
- |   |                      |
|---|----------------------|
| 1-period REFERENCE Solution <b>S1</b> (P1)              | → ok control         |
| 1-period NEW Solutions <b>S1', S1''</b> (P1', P1'')     | → failure of control |
| QUASIPERIODIC Solutions <b>C', C''</b> only <b>AMEs</b> | → failure of control |



## FUNDAMENTAL RESONANCE

$k_g - U$

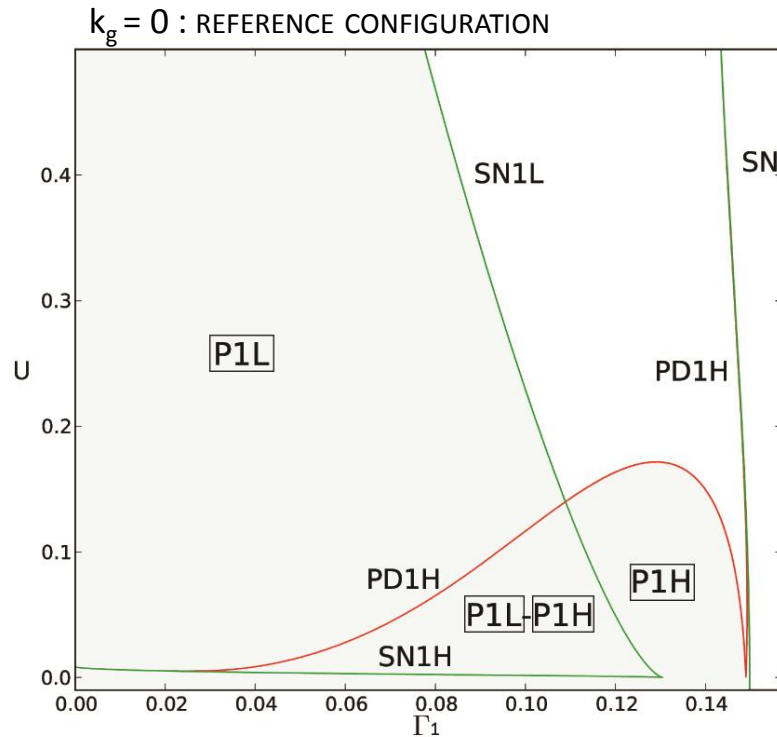
WITH VARYING FORCING FREQUENCY



PRESENCE OF **CONFINED STABLE REGIONS (P1H)**: - HIGH  $U$   
- HIGH  $k_G$

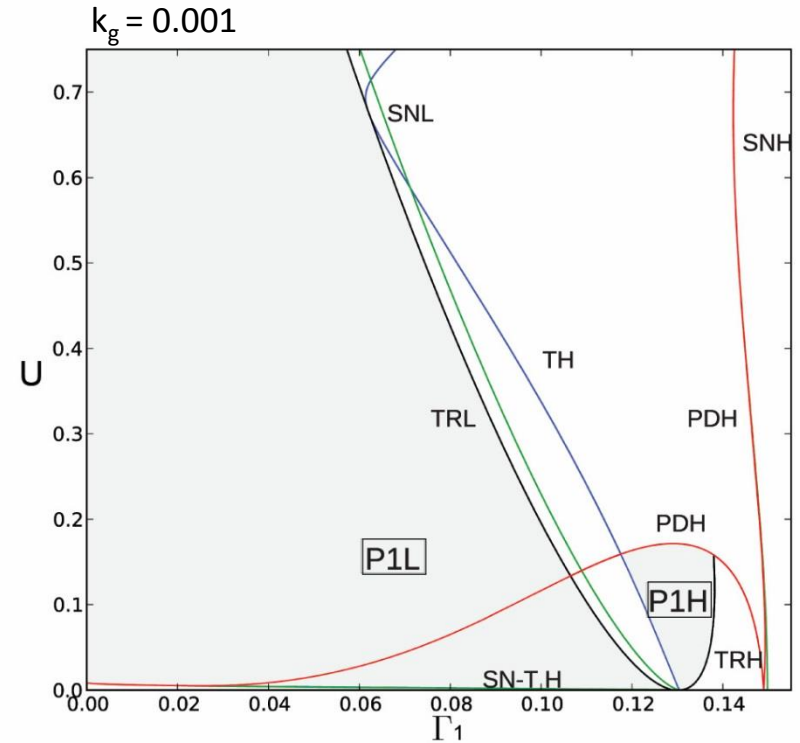
## FUNDAMENTAL RESONANCE

$\Gamma_1 - U$



UNCONTROLLED

WITH VARYING ATOMIC INTERACTION

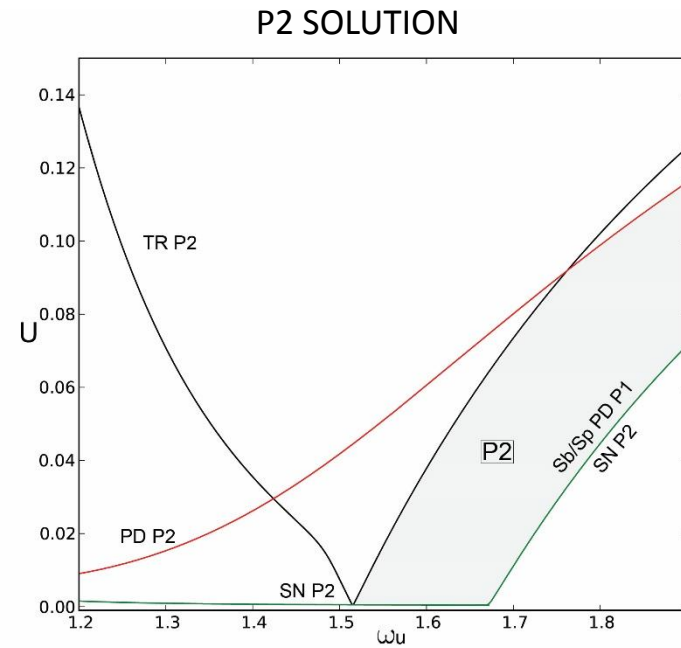
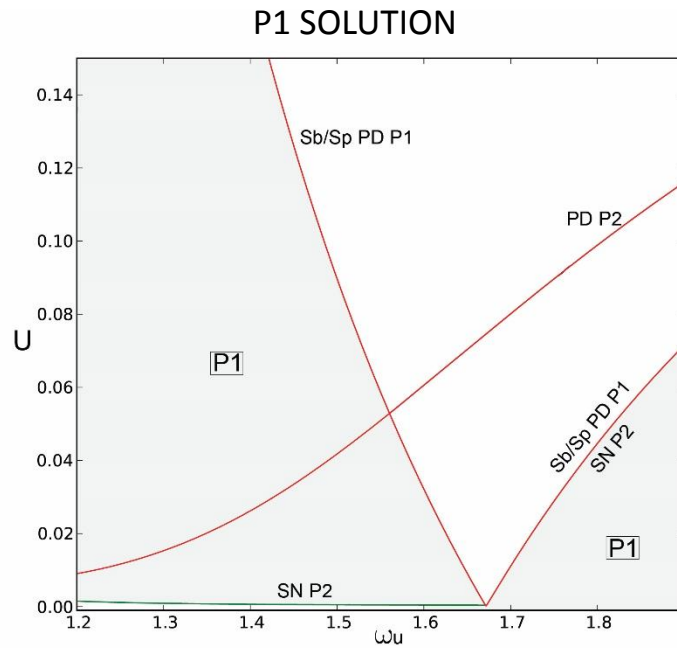


CONTROLLED

## PRINCIPAL RESONANCE

$$\omega_u - U$$

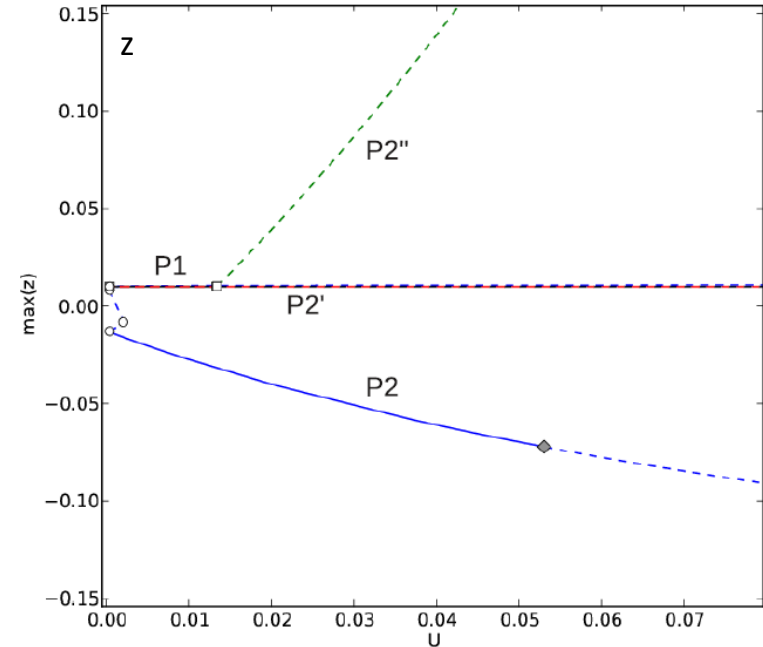
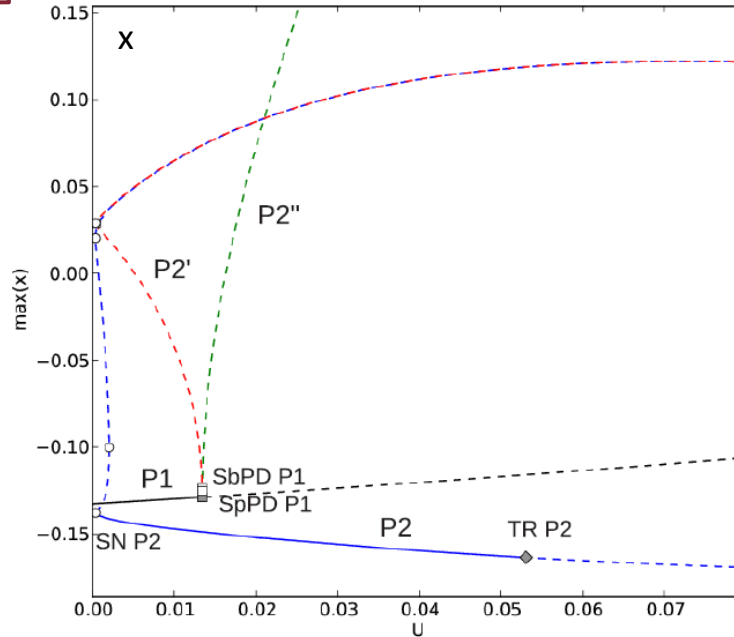
## P1+P2 SOLUTIONS



## PRINCIPAL RESONANCE

$\omega_u - U$

BIFURCATION DIAGRAM at  $\omega_u = 1.64$  and  $k_g = 0.001$

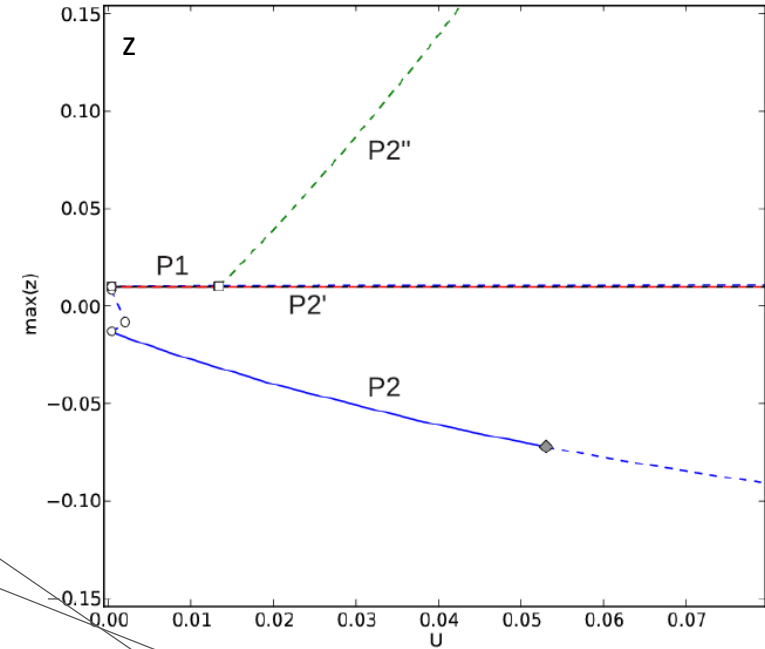
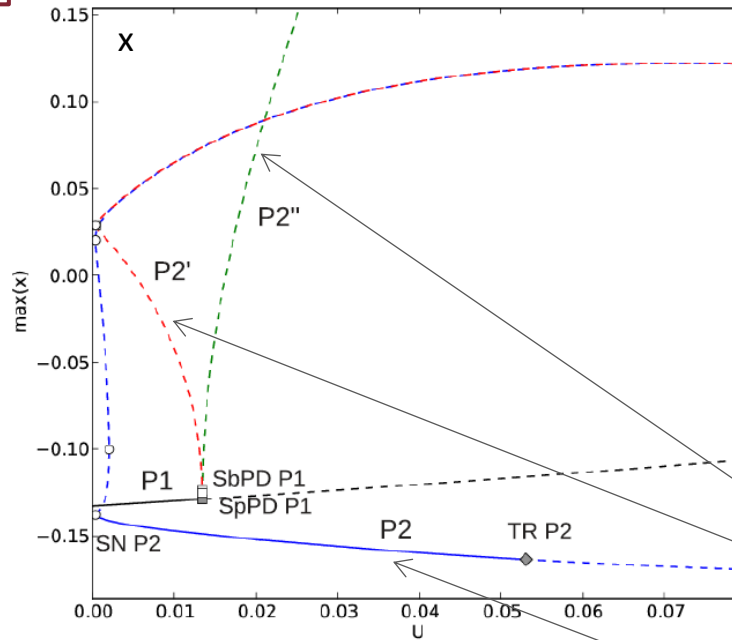


**Three P2 SOLUTIONS:** from Subcritical Period Doubling (SbPD P1) **P2'**  
 from Supercritical Period Doubling (SpPD P1) **P2''**  
 disconnected **P2**

## PRINCIPAL RESONANCE

$\omega_u - U$

BIFURCATION DIAGRAM at  $\omega_u = 1.64$  and  $k_g = 0.001$

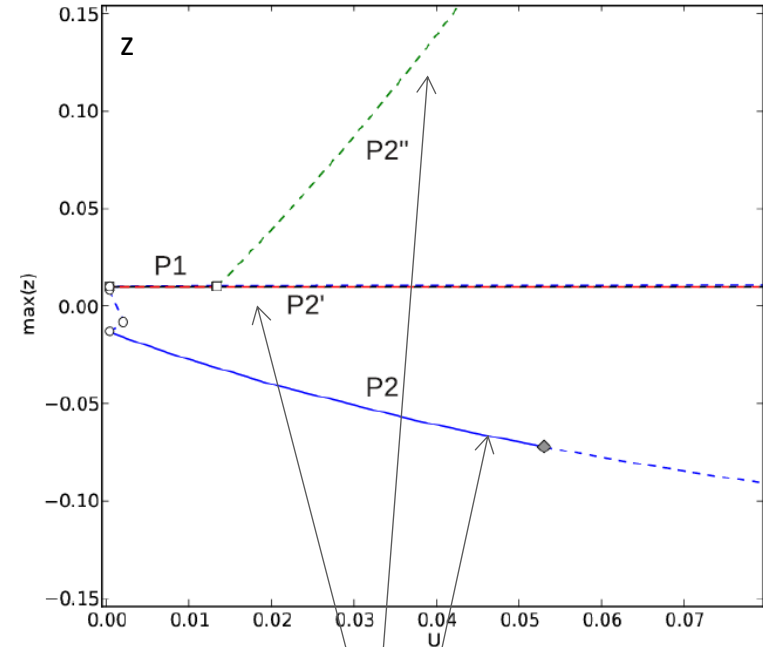
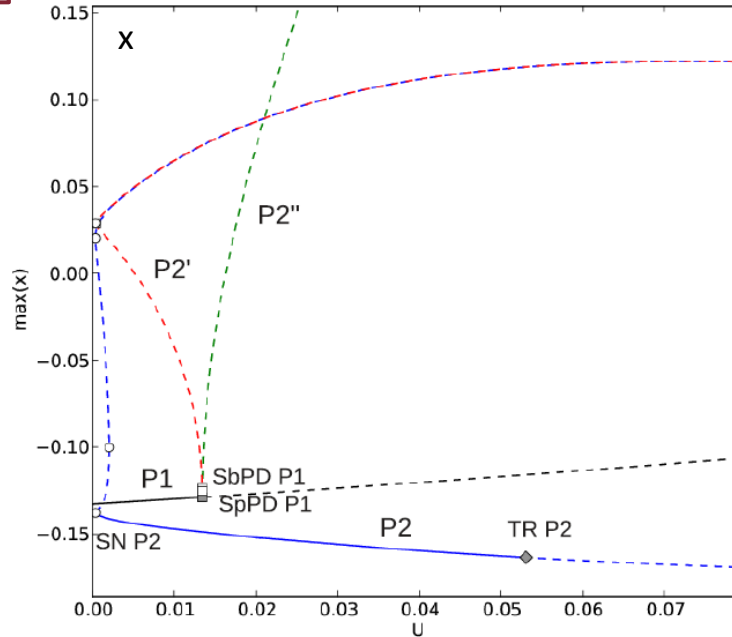


**Three P2 SOLUTIONS:** from Subcritical Period Doubling (SbPD P1) **P2'** UNSTABLE  
 from Supercritical Period Doubling (SpPD P1) **P2''** UNSTABLE  
 disconnected **P2** STABLE

## PRINCIPAL RESONANCE

$\omega_u - U$

BIFURCATION DIAGRAM at  $\omega_u = 1.64$  and  $k_g = 0.001$



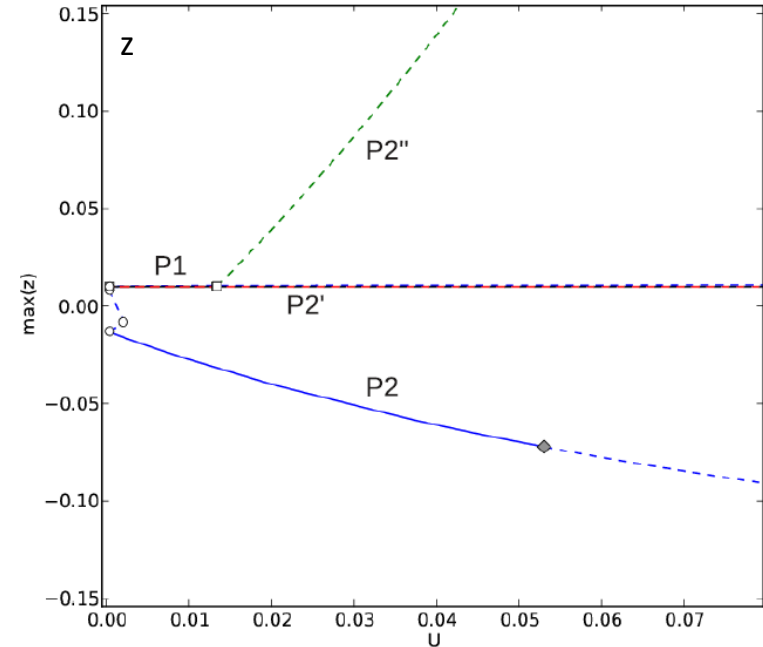
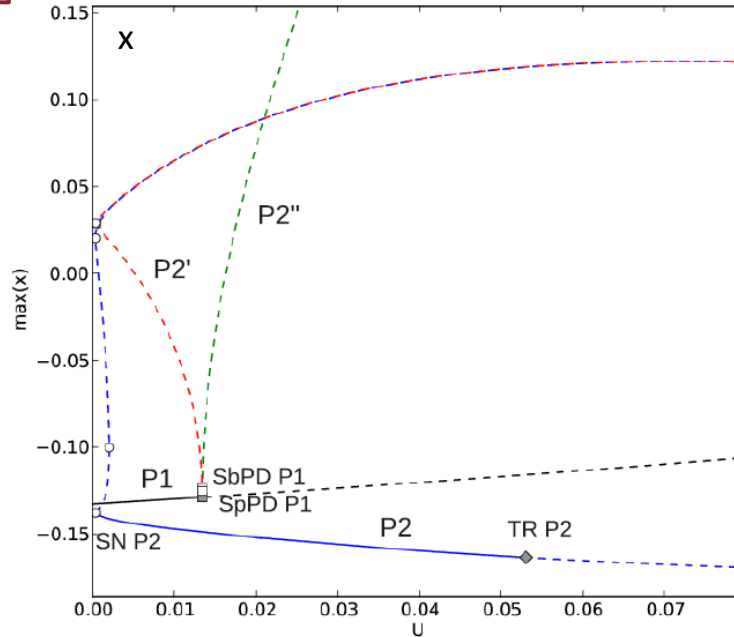
Three P2 SOLUTIONS: from Subcritical Period Doubling (SbPD P1) from Supercritical Period Doubling (SpPD P1) disconnected	P2'	UNSTABLE	OK CONTROL
	P2''	UNSTABLE	NO CONTROL
	P2	STABLE	NO CONTROL

**Control works only on P2', the only 2-period solution of uncontrolled system, here unstable**

## PRINCIPAL RESONANCE

$\omega_u - U$

BIFURCATION DIAGRAM at  $\omega_u = 1.64$  and  $k_g = 0.001$



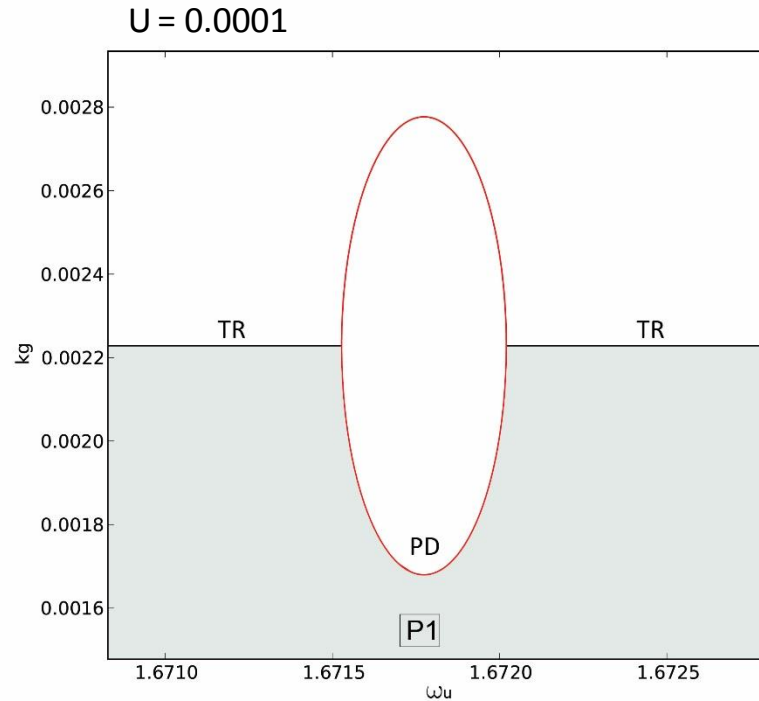
**Three P2 SOLUTIONS:** from Subcritical Period Doubling (SbPD P1) **P2'** UNSTABLE OK CONTROL  
 from Supercritical Period Doubling (SpPD P1) **P2''** UNSTABLE NO CONTROL  
 disconnected **P2** STABLE NO CONTROL



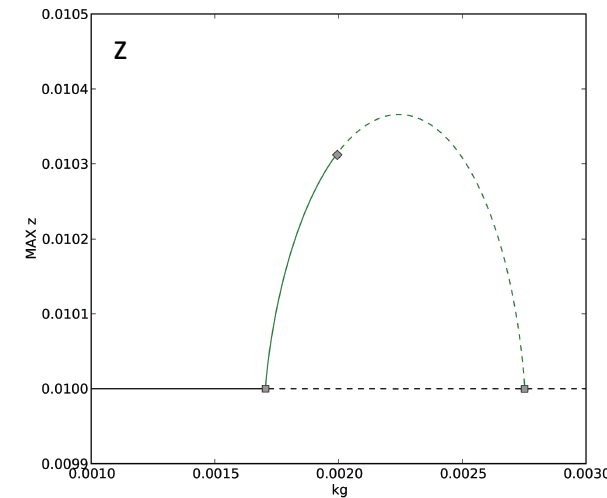
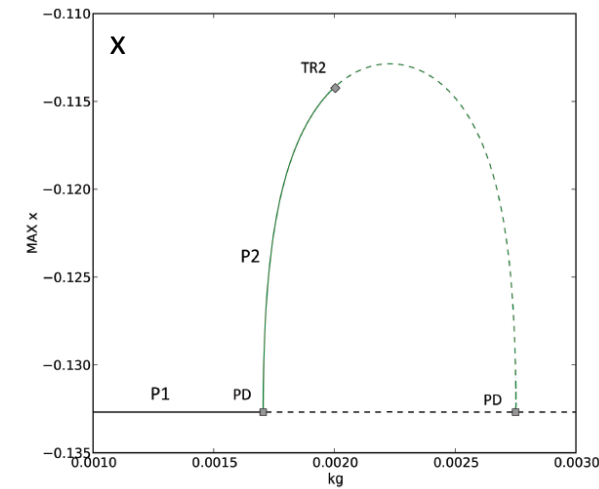
**ESCAPE THRESHOLD:** RELATED **ONLY** TO **P1** SOLUTION

## PRINCIPAL RESONANCE

$$\omega_u - k_g$$



$$\omega_u = 1.6717$$



## RESONANCE LOOP : PERIOD DOUBLING

- period-doubled solution  $\rightarrow$  coherent with principal resonance
- loop size increased  $\rightarrow$  PRINCIPAL resonance : MAIN region for PARAMETRIC system (with respect to fundamental resonance)



# EXTERNAL EXCITATION - 1 -

$$\ddot{x} + \alpha_1 x + \alpha_3 x^3 = -\Gamma_1 (1 + x + V \sin(\omega_v t) + z - z_s)^{-2} - \rho_1 \dot{x}$$

$$-\left(-\omega_v^2 V \sin(\omega_v t) + k_g (\dot{x}_{ref} - \dot{x}) + \nu_1 (\omega_v V \cos(\omega_v t) + k_g (x_{ref} - x))\right) \nu_2$$

$$\dot{z} = k_g (x_{ref} - x)$$

$\omega_v - V$

$$\mu_2 = \rho_2 = 0$$

$$\eta_1 = \eta_2 = 0$$

$$U_g = \alpha_2 = 0$$

$$\alpha_1 = 1$$

$$\Gamma_1 = 0.1$$

$$\alpha_3 = 0.1$$

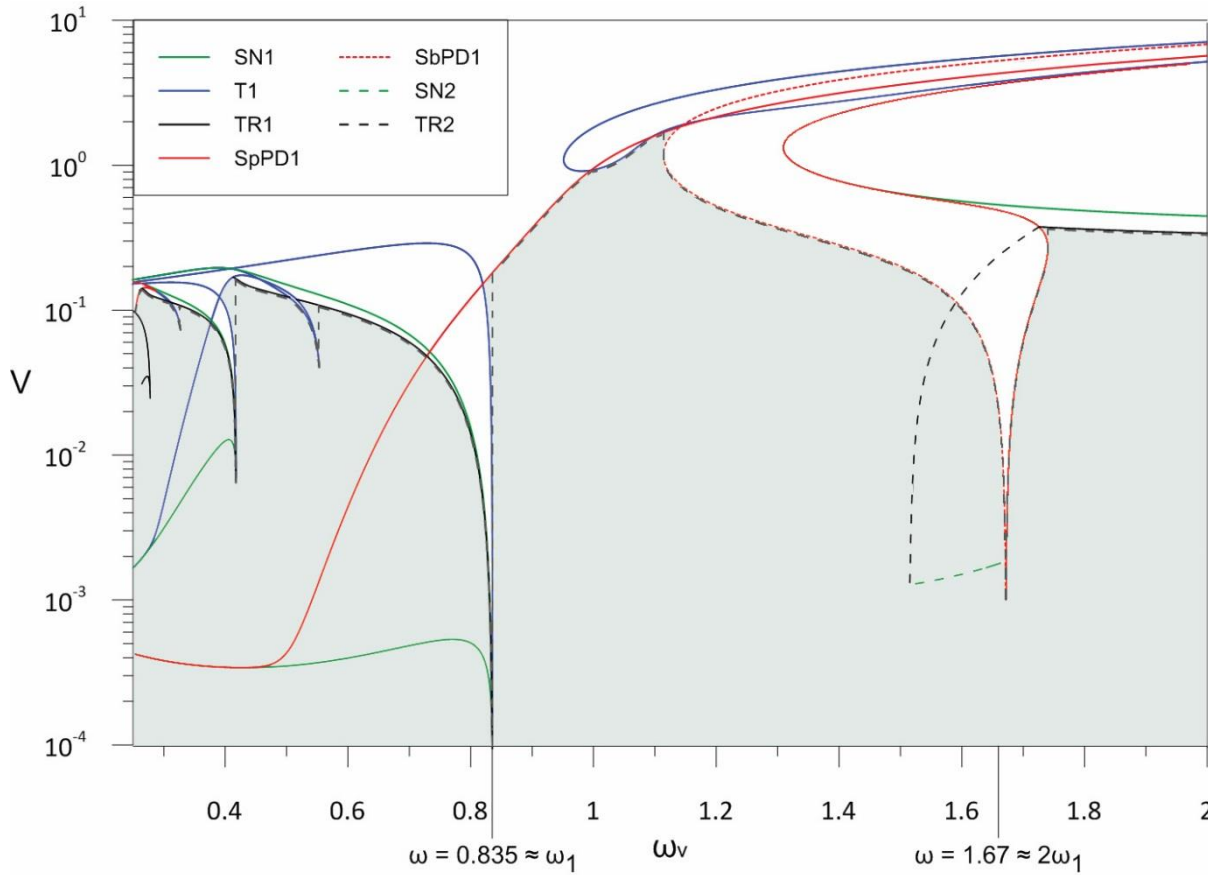
$$\nu_1 = 0.01$$

$$\nu_2 = 0.01$$

$$\omega_1 = 0.8325$$

$$k_g = 0.001$$

$$z_s = 0.01$$



## EXTERNAL EXCITATION - 1 -

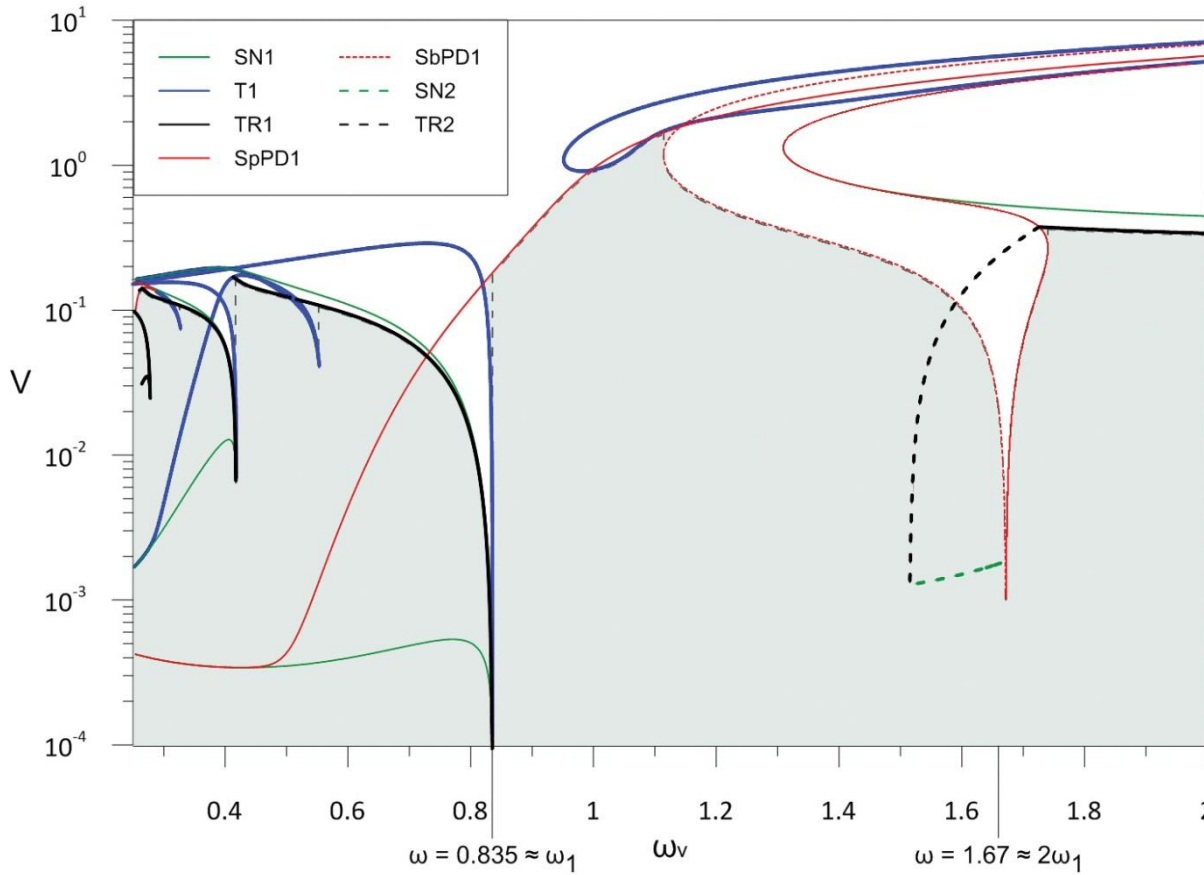
$$\ddot{x} + \alpha_1 x + \alpha_3 x^3 = -\Gamma_1 (1 + x + V \sin(\omega_v t) + z - z_s)^{-2} - \rho_1 \dot{x}$$

$$-\left(-\omega_v^2 V \sin(\omega_v t) + k_g (\dot{x}_{ref} - \dot{x}) + \nu_1 (\omega_v V \cos(\omega_v t) + k_g (x_{ref} - x))\right) \nu_2$$

$$\dot{z} = k_g (x_{ref} - x)$$

$\omega_v - V$

$\mu_2 = \rho_2 = 0$   
 $\eta_1 = \eta_2 = 0$   
 $U_g = \alpha_2 = 0$   
 $\alpha_1 = 1$   
 $\Gamma_1 = 0.1$   
 $\alpha_3 = 0.1$   
 $\nu_1 = 0.01$   
 $\nu_2 = 0.01$   
 $\omega_1 = 0.8325$   
 $k_g = 0.001$   
 $z_s = 0.01$



**SAME BEHAVIOR OF  
PARAMETRIC CASE**

**NEW  
TRANSCRITICAL  
AND TORUS  
THRESHOLDS**

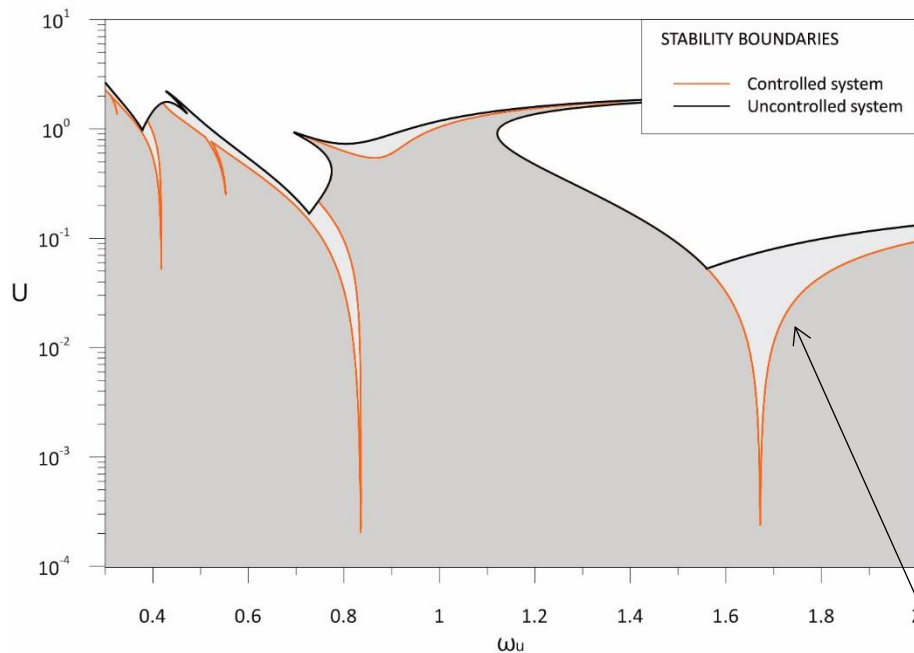


**DECREASE OF  
ESCAPE VALUE**

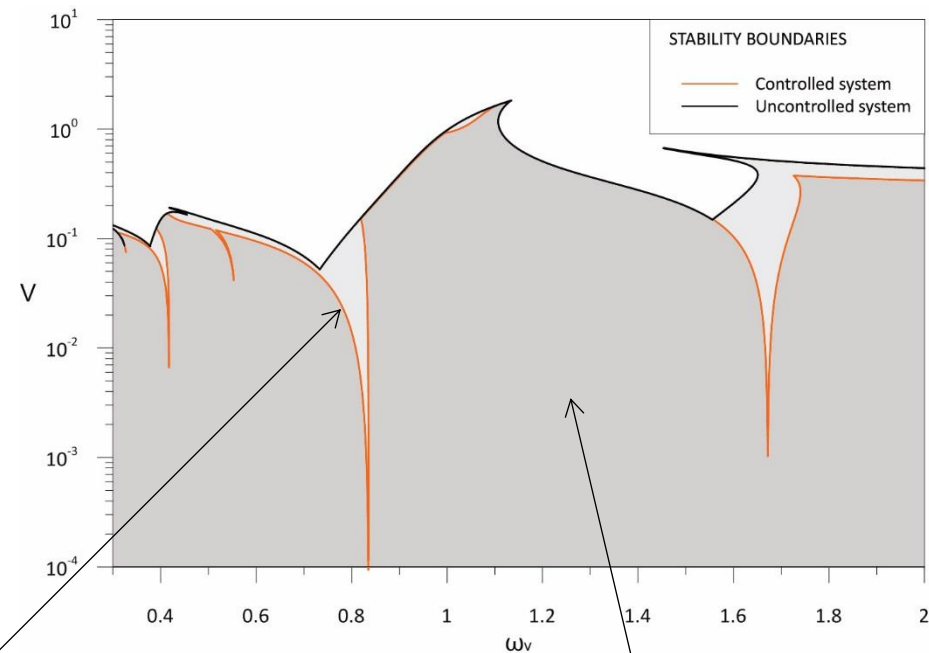
**STABLE REGION:**  
solutions for which  
FEEDBACK CONTROL  
WORKS PROPERLY

# STABILITY REGIONS WITH/WITHOUT CONTROL

## PARAMETRIC EXCITATION



## EXTERNAL EXCITATION



MAIN RESONANCE FREQUENCIES: deep **INSTABILITY TONGUES**

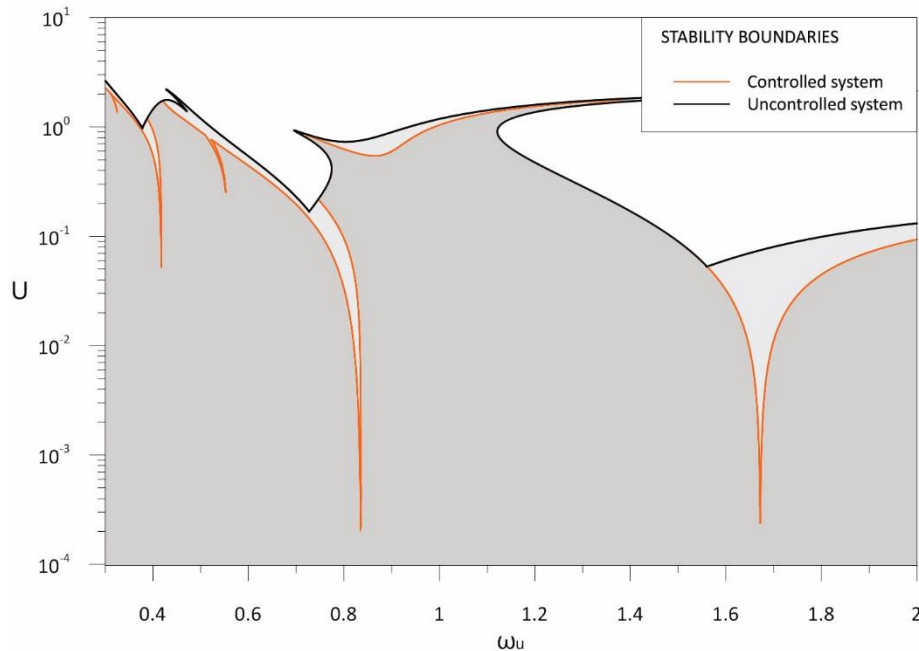
→ reductions of  $\approx 99,9\%$

**stable region:**  
solutions for which **feedback control works properly**

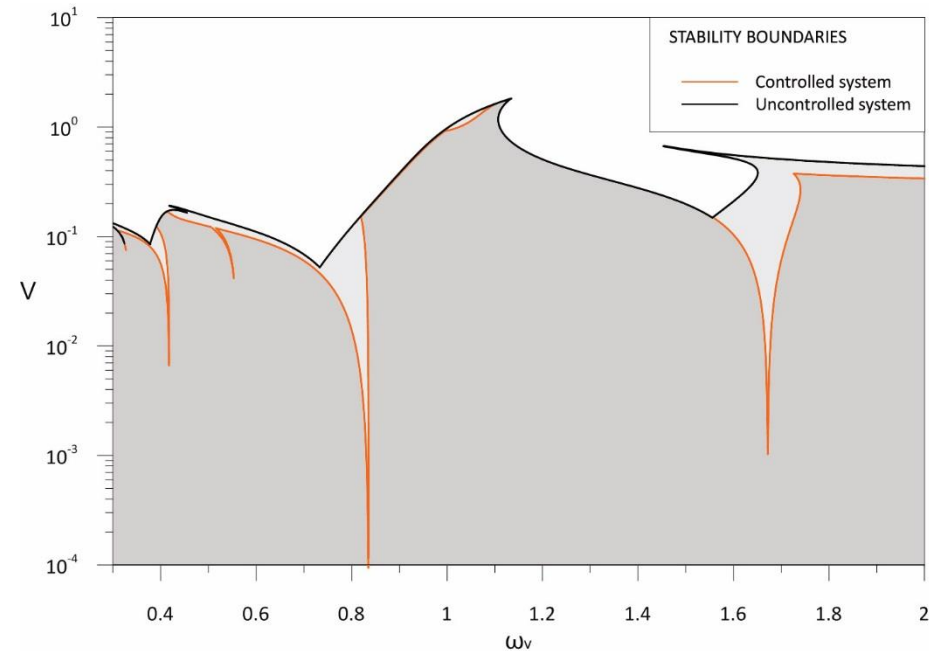
due to a substantial **increase of response amplitude** that **LOCAL feedback control barely dominates** ➡ **need of a GLOBAL control !?**

# STABILITY REGIONS WITH/WITHOUT CONTROL

## PARAMETRIC EXCITATION



## EXTERNAL EXCITATION



**STABILITY of CONTROLLED SYSTEM: ONLY solutions on which the CONTROL works PROPERLY**

AROUND  $2\omega_1$ : UNCONTROLLED : governed by **P2** response

CONTROLLED : ESCAPE related to **P1** solution  
**P2** solution **NOT** acceptable



**ADDITIONAL REDUCTION OF ESCAPE VALUE**

# DYNAMICAL INTEGRITY OF THE CONTROLLED SYSTEM

- AIM:**
- complete the evaluation of the **GLOBAL PERFORMANCE** of a **LOCAL** external feedback **CONTROL** technique
  - properly identify the **DESIGN PARAMETERS RANGES** able to guarantee the **SECURE OPERATION** of the AFM

- NUMERICAL ANALYSES:**
- BASINS EROSION PROCESS as a function of the most relevant dynamical parameters around the resonance frequency
  - EROSION PROFILES by means of two integrity measures (IF, GIM)
  - THRESHOLDS of RESIDUAL INTEGRITY in system parameters space

# CONTROLLED SYSTEM UNDER PARAMETRIC EXCITATION (HARMONIC)

Orders of magnitude of coefficients in commercial AFMs  $\rightarrow$  feedback controls  $(\eta_1, \eta_2, \rho_2)$  and the nonlinear term related to  $\alpha_2$  can be neglected

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\alpha_1 x - \alpha_3 x^3 - \frac{\Gamma_1}{(1+x+z-z_s)^2} - \rho_1 y - x \mu_1 \omega_u^2 U \sin(\omega_u t) \\ \dot{z} = k_g (x_{ref} - x) \end{cases}$$

EXTERNAL FEEDBACK CONTROL

$$\begin{cases} \dot{x}_{ref} = y_{ref} \\ \dot{y}_{ref} = -\alpha_1 x_{ref} - \alpha_3 x_{ref}^3 - \frac{\Gamma_1}{(1+x_{ref})^2} - \rho_1 y_{ref} - x_{ref} \mu_1 \omega_u^2 U \sin(\omega_u t) \end{cases}$$

REFERENCE RESPONSE

5 state variables  $\rightarrow$  **5-DIMENSIONAL BASINS OF ATTRACTION**

- COMPUTATIONALLY DEMANDING task
- RESULTS INTERPRETATION considerably DIFFICULT



**PLANAR SECTIONS** in  $(x=x_{ref}, y=y_{ref})$  plane with fixed  $z$

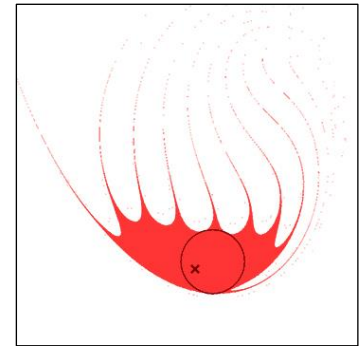
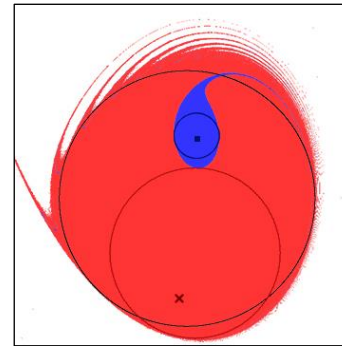
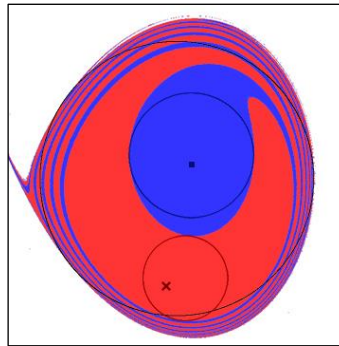
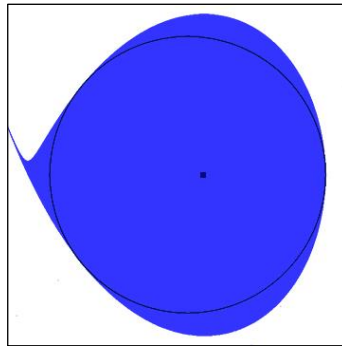
# INFLUENCE OF THE EXCITATION PARAMETERS - 1 -

## BASIN EROSION FOR INCREASING FORCING AMPLITUDE ( $\omega_u=0.8$ )

FORCING AMPLITUDE



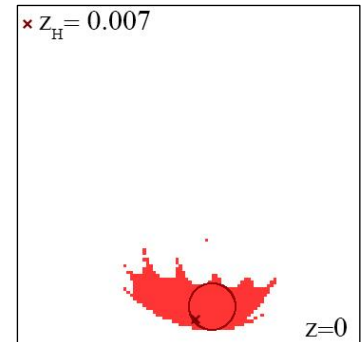
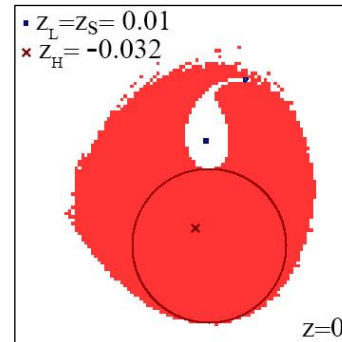
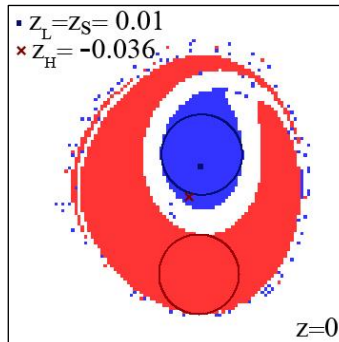
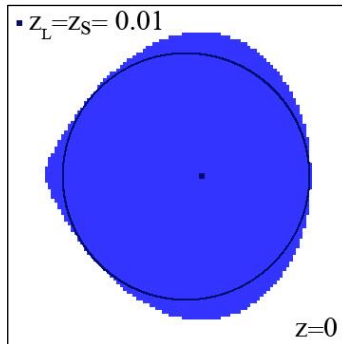
REFERENCE  
(UNCONTROLLED)  
SYSTEM



CONTROLLED  
SYSTEM

$$z(0) = 0$$

$$z_s = 0.01$$

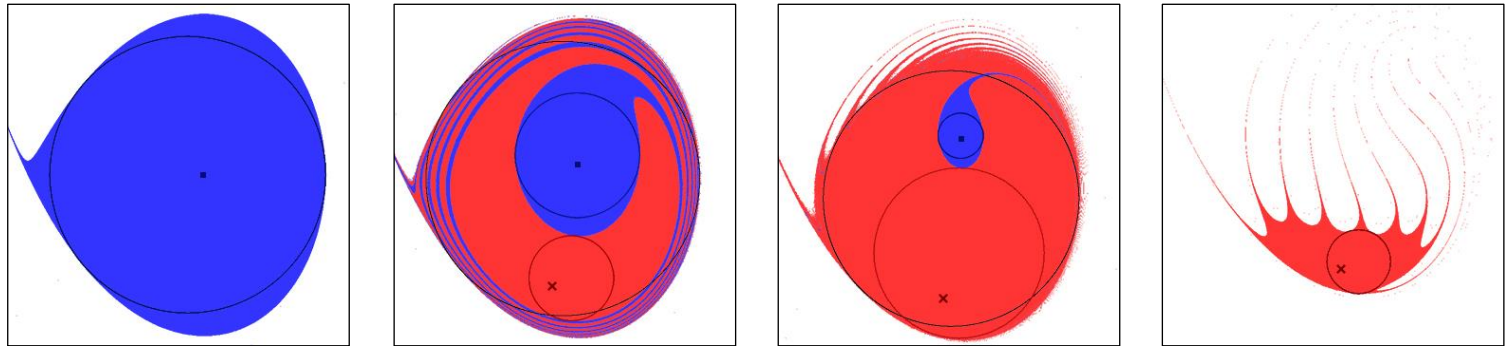


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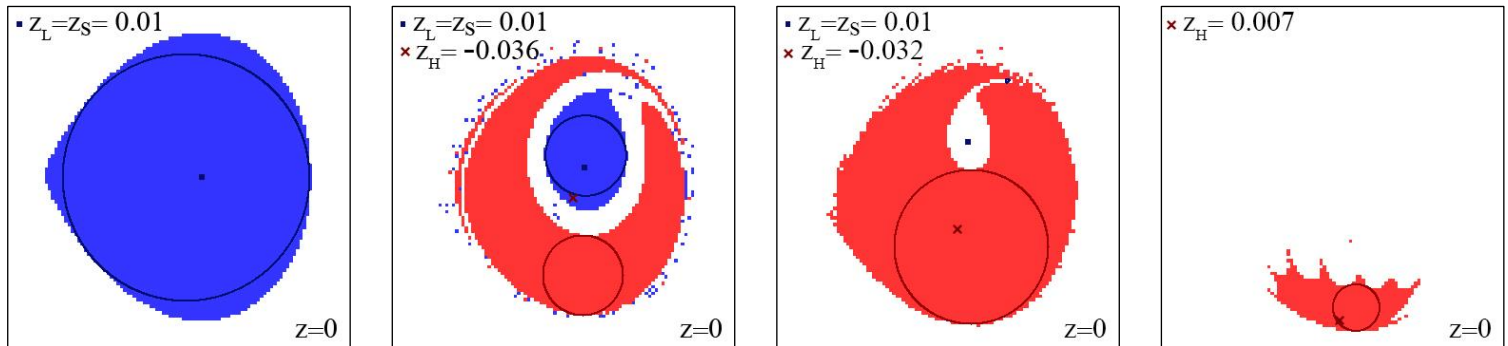
FORCING AMPLITUDE →

REFERENCE  
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CONTROLLED  
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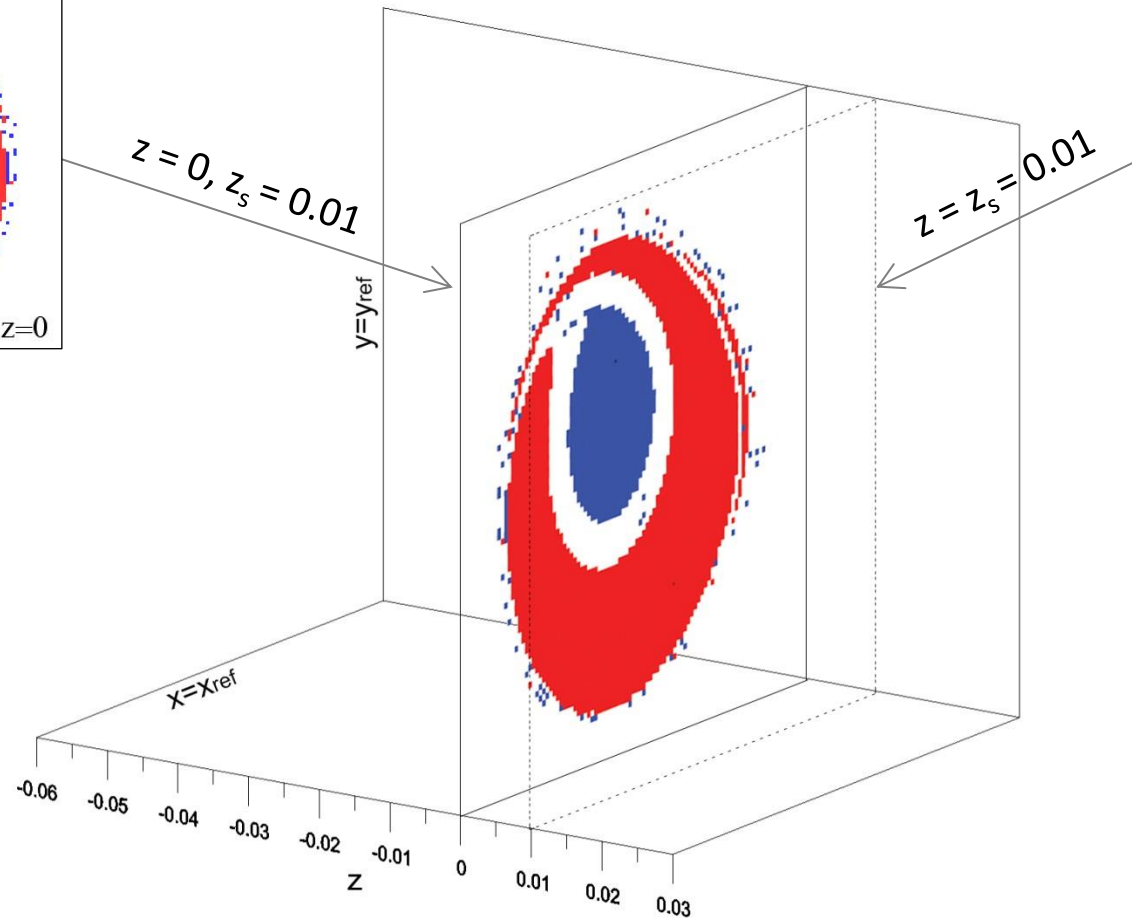
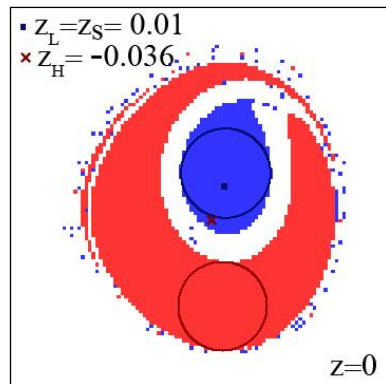


- **CONTROL: TONGUES** of the **UNBOUNDED** solution basin (white) inside the potential well
- BASIN SEPARATION for LOW amplitude
- **NONRESONANT** basin: strongly **REDUCED** → **TOPOLOGICAL SCENARIO MODIFIED**

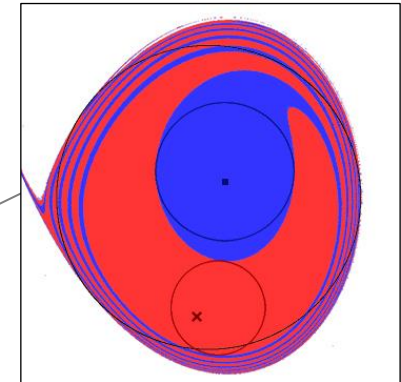


## INFLUENCE OF THE EXCITATION PARAMETERS - 2 -

### BASIN EROSION FOR INCREASING FORCING AMPLITUDE ( $\omega_u=0.8$ )

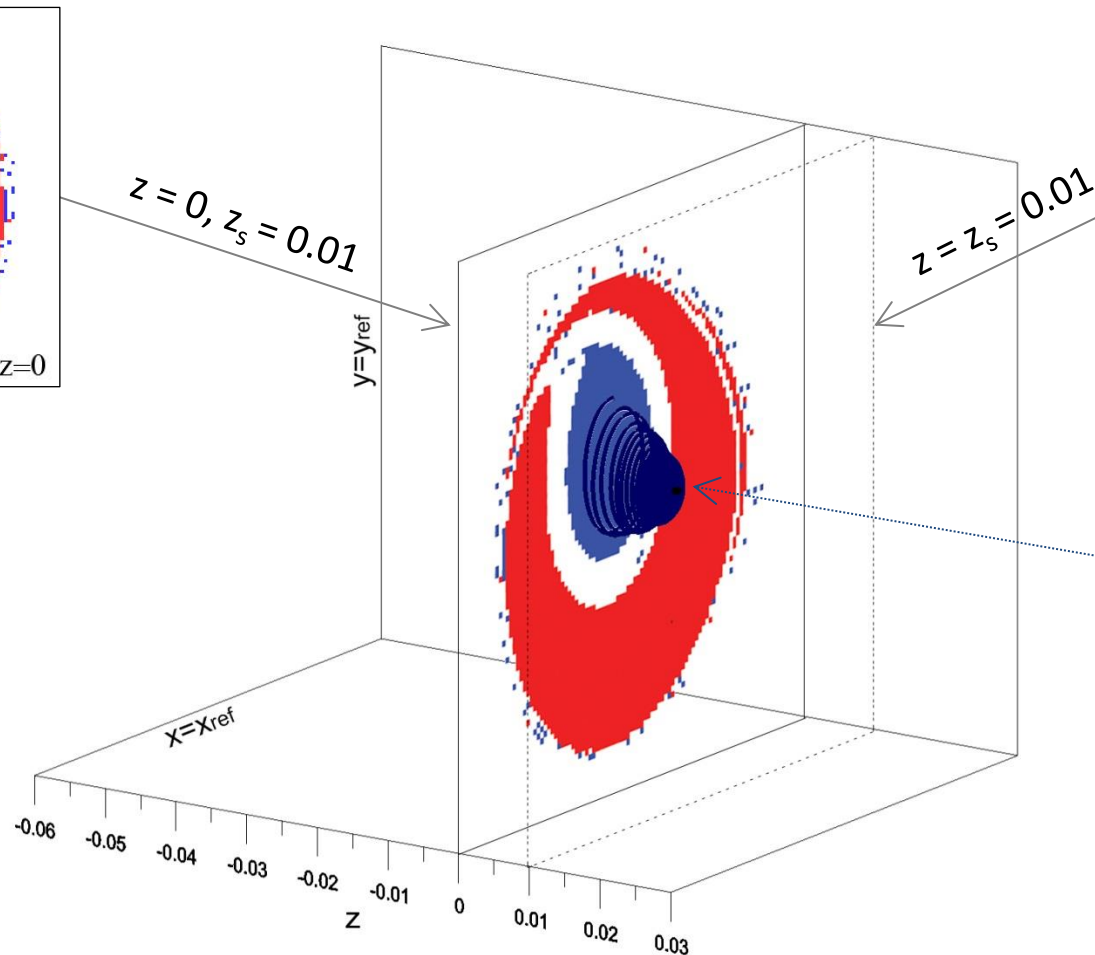
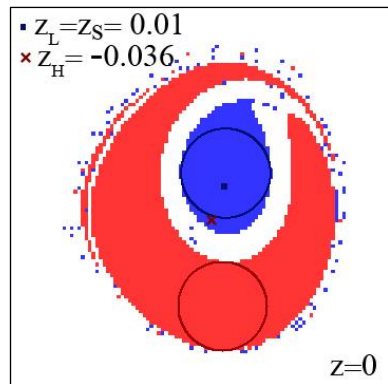


Phase-plane of the uncontrolled system

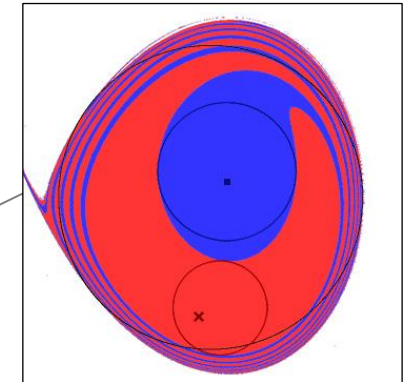


## INFLUENCE OF THE EXCITATION PARAMETERS - 2 -

### BASIN EROSION FOR INCREASING FORCING AMPLITUDE ( $\omega_u=0.8$ )



Phase-plane of the uncontrolled system



**NONRESONANT  
SOLUTION:**

$$z = z_s$$

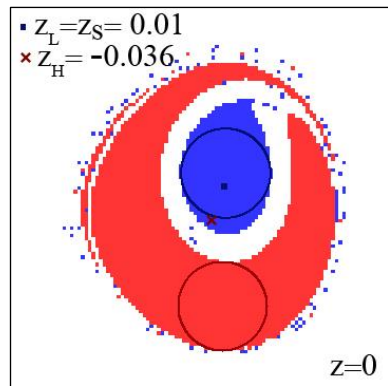
EFFICIENCY of  
CONTROL



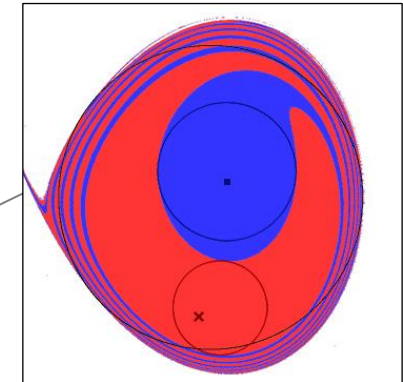
**SAFE BASIN**

## INFLUENCE OF THE EXCITATION PARAMETERS - 2 -

### BASIN EROSION FOR INCREASING FORCING AMPLITUDE ( $\omega_u=0.8$ )



Phase-plane of the uncontrolled system



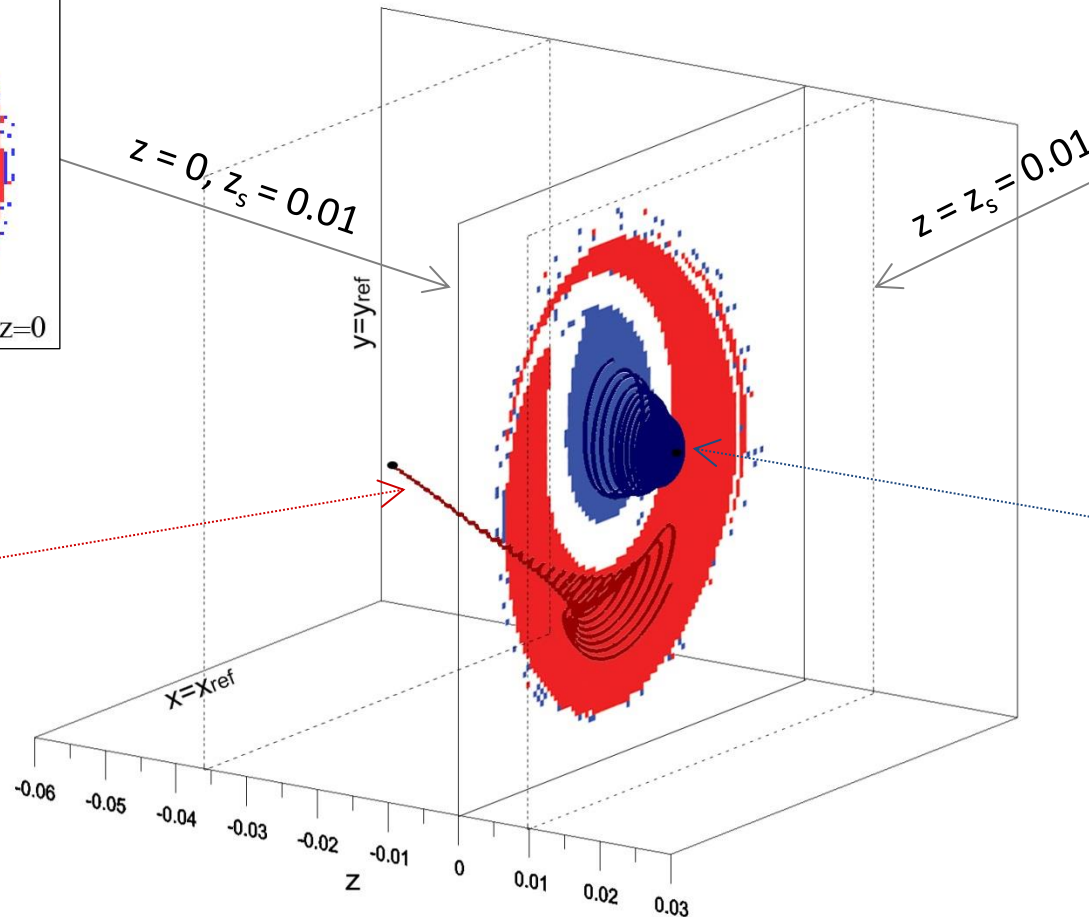
**RESONANT  
SOLUTION:**

$$z \neq z_s$$

INEFFICIENCY of  
CONTROL



**UNACCEPTABLE**



**NONRESONANT  
SOLUTION:**

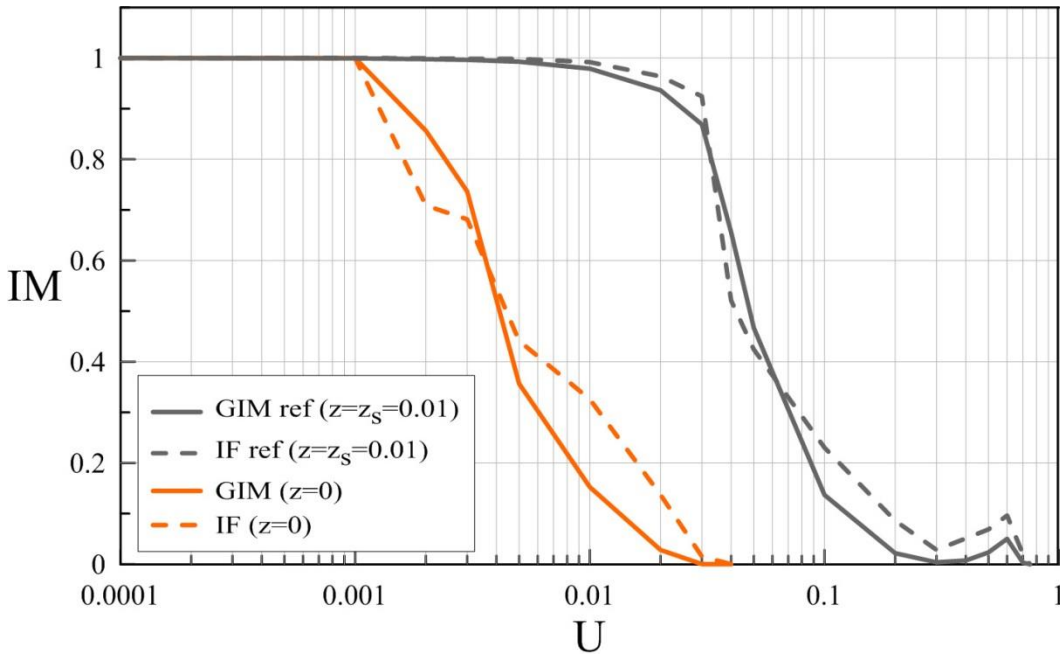
$$z = z_s$$

EFFICIENCY of  
CONTROL



**SAFE BASIN**

## EROSION PROFILES FOR INCREASING FORCING AMPLITUDE ( $\omega_u=0.8$ )



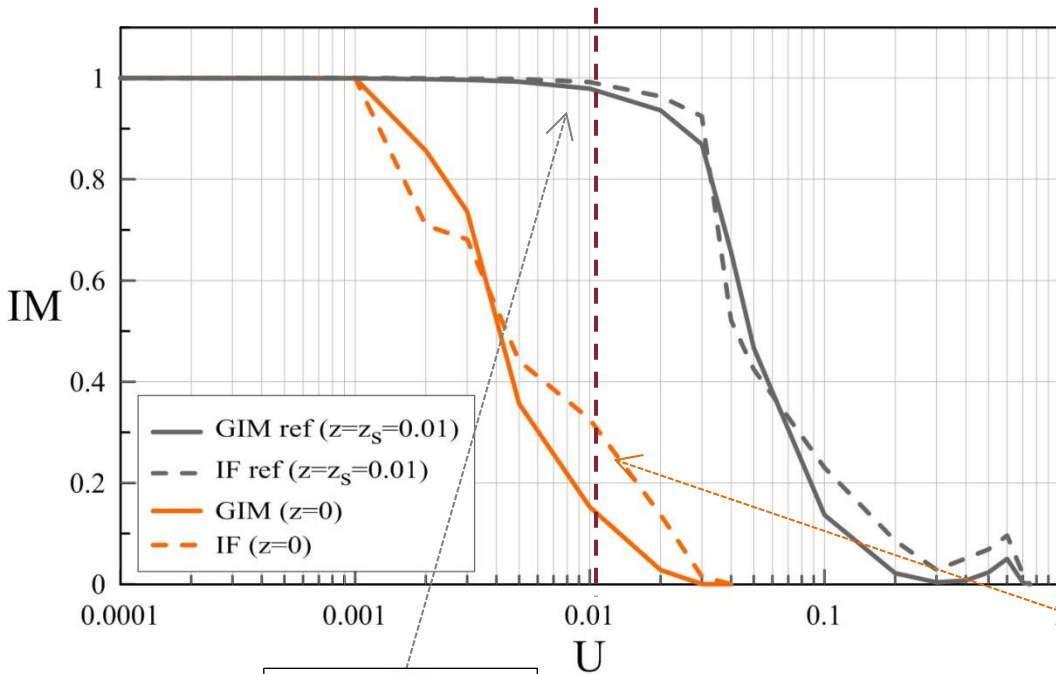
## INTEGRITY MEASURES

Global Integrity Measure (GIM):  
normalized AREA of the safe basin

Integrity Factor (IF):  
normalized RADIUS of the largest CIRCLE entirely BELONGING to the safe basin (sole compact part)

# INFLUENCE OF THE EXCITATION PARAMETERS - 3 -

## EROSION PROFILES FOR INCREASING FORCING AMPLITUDE ( $\omega_u=0.8$ )

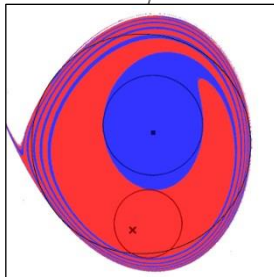


### INTEGRITY MEASURES

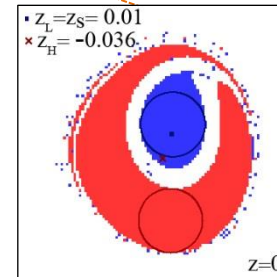
Global Integrity Measure (GIM):  
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### SAFE BASIN



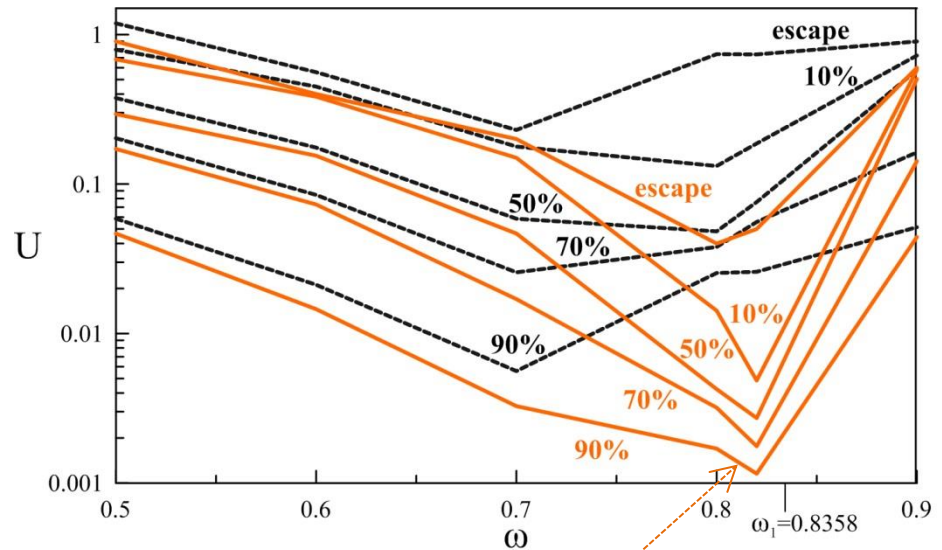
UNCONTROLLED system:  
**RESONANT** +  
**NONRESONANT** basins



CONTROLLED system:  
**NONRESONANT** basin

**CONTROLLED SYSTEM** : much **MORE DANGEROUS** THAN the UNCONTROLLED one

## RESIDUAL ISO-INTEGRITY CURVES



AROUND RESONANCE FREQUENCY: severe **WORSENING** of **PRACTICAL STABILITY**

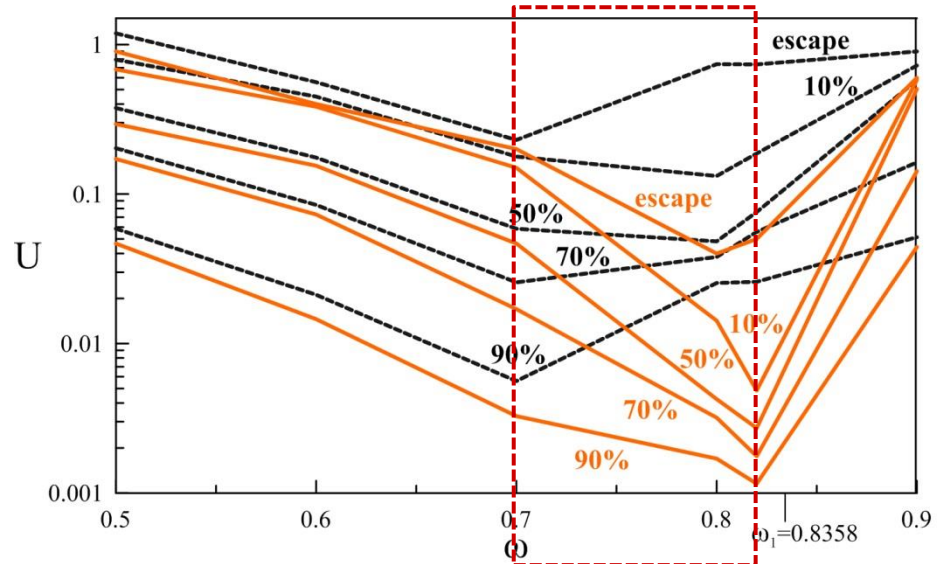
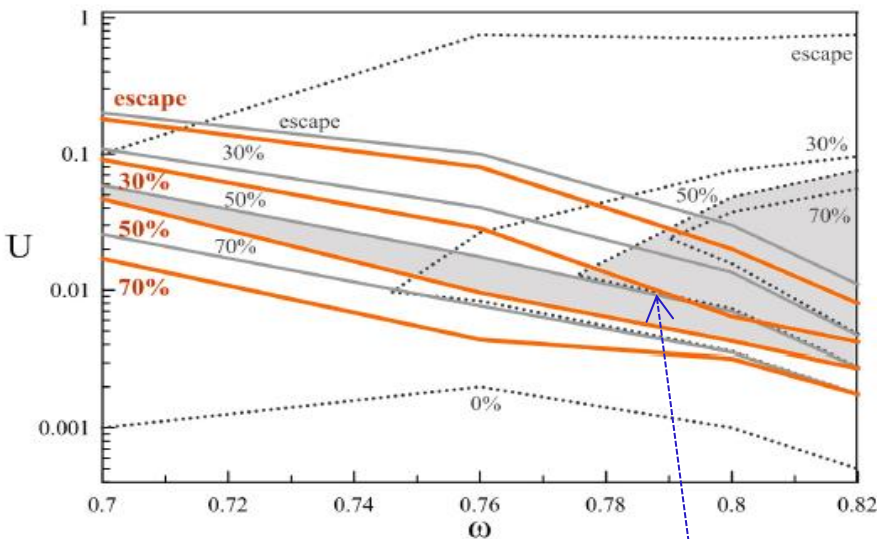
(residual integrity from 90% to 10% in  $\Delta U = 3.6 \cdot 10^{-3}$ )

**SHIFT** of **LOWEST PEAK** from nonlinear (**uncontrolled**) to linear (**controlled**) resonance frequency



# INFLUENCE OF THE EXCITATION PARAMETERS - 4 -

## RESIDUAL ISO-INTEGRITY CURVES



**AROUND RESONANCE FREQUENCY:** severe **WORSENING** of **PRACTICAL STABILITY**  
(residual integrity from 90% to 10% in  $\Delta U = 3.6 \cdot 10^{-3}$ )

**SHIFT** of **LOWEST PEAK** from nonlinear (**uncontrolled**) to linear (**controlled**) resonance frequency

RESONANT solution P1H: **no longer acceptable** for the system

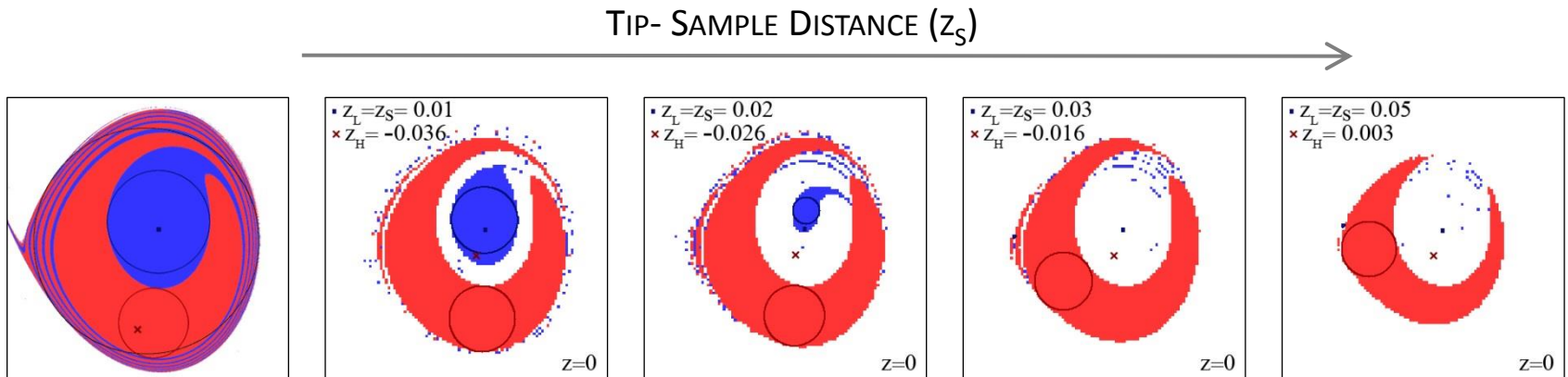
NONRESONANT solution P1L: **solely governing** the response robustness



**Meaningful loss of controlled (orange curves) stability domain with respect to uncontrolled (grey curves) one for given (e.g. 50%) iso-integrity**

## INFLUENCE OF TIP-SAMPLE DISTANCE

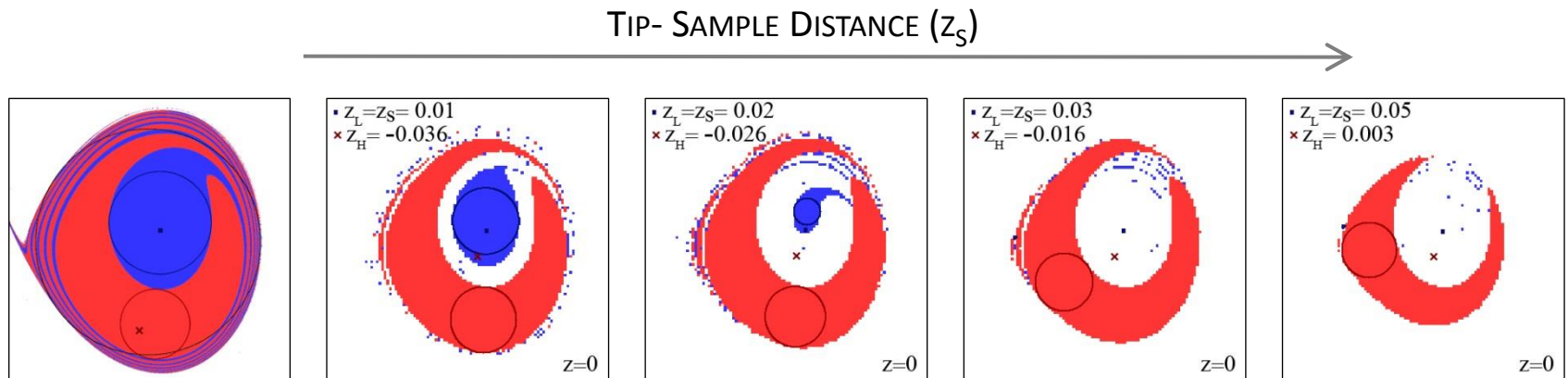
- DEPENDENCE on the ROUGHNESS of the SAMPLE to be scanned → high VARIABILITY during the AFM scanning OPERATION
- its EFFECT on the global behavior: particularly IMPORTANT to assess the system ACTUAL SAFETY in operating conditions





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TIP-SAMPLE DISTANCE INCREASE → **ENLARGEMENT** of the **UNBOUNDED** solution basin (white)  
→ **REDUCTION** of the **NONRESONANT** (controllable) basin (blue) up to its disappearance

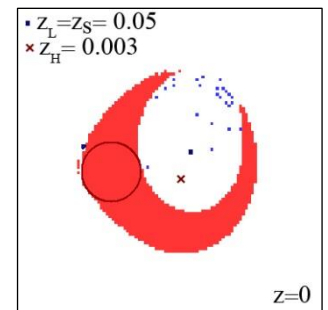
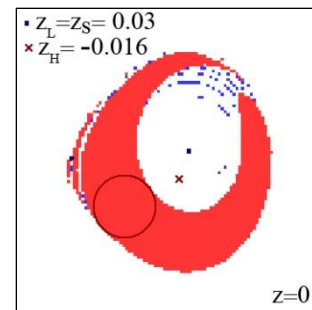
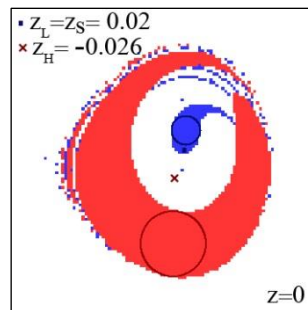
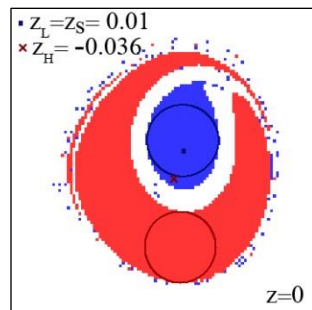
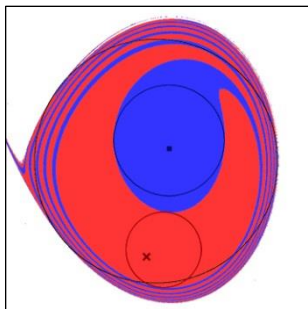
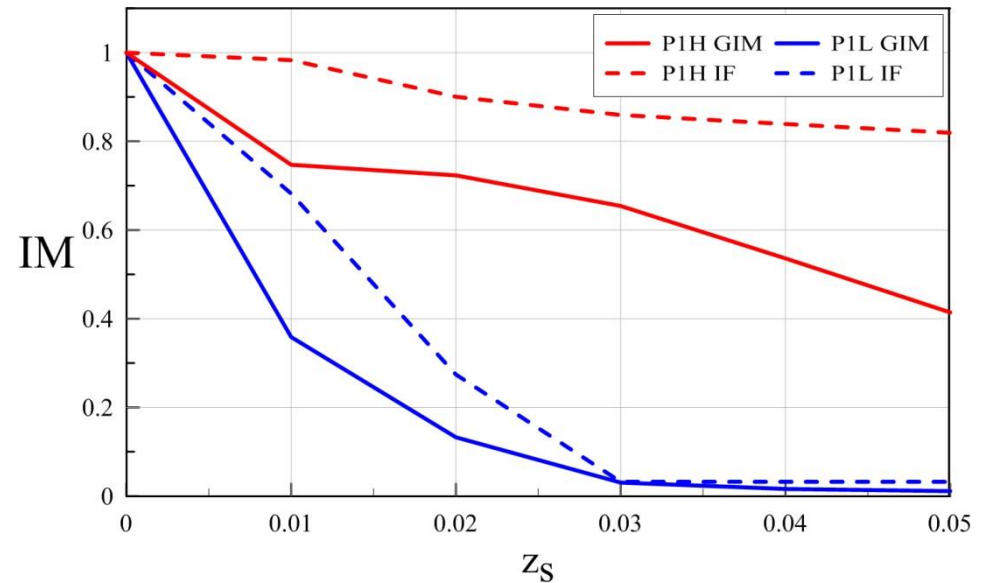
# INFLUENCE OF INTRINSIC PARAMETERS - 1 -

## INFLUENCE OF TIP-SAMPLE DISTANCE

- **NONRESONANT** profile: much **LOWER** than **RESONANT** profile
- **BASIN EROSION**: from the outer edge preserving the compact part of the basins

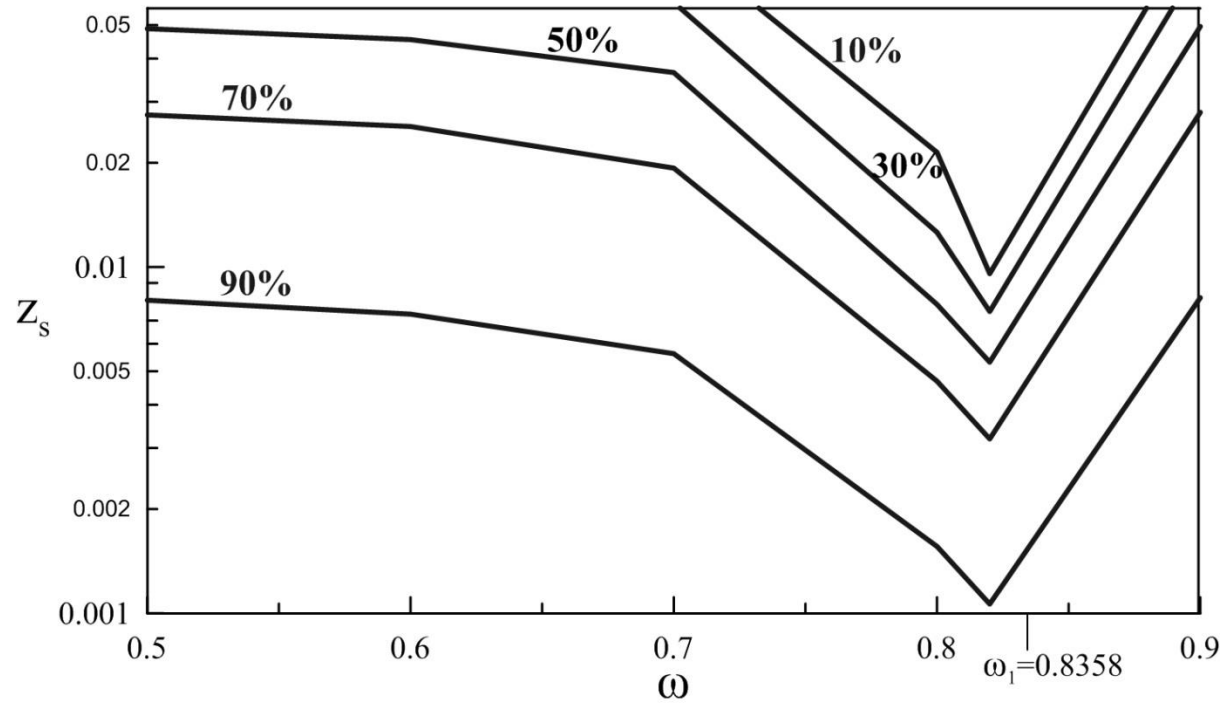


**GIM** more **CONSERVATIVE** than IF



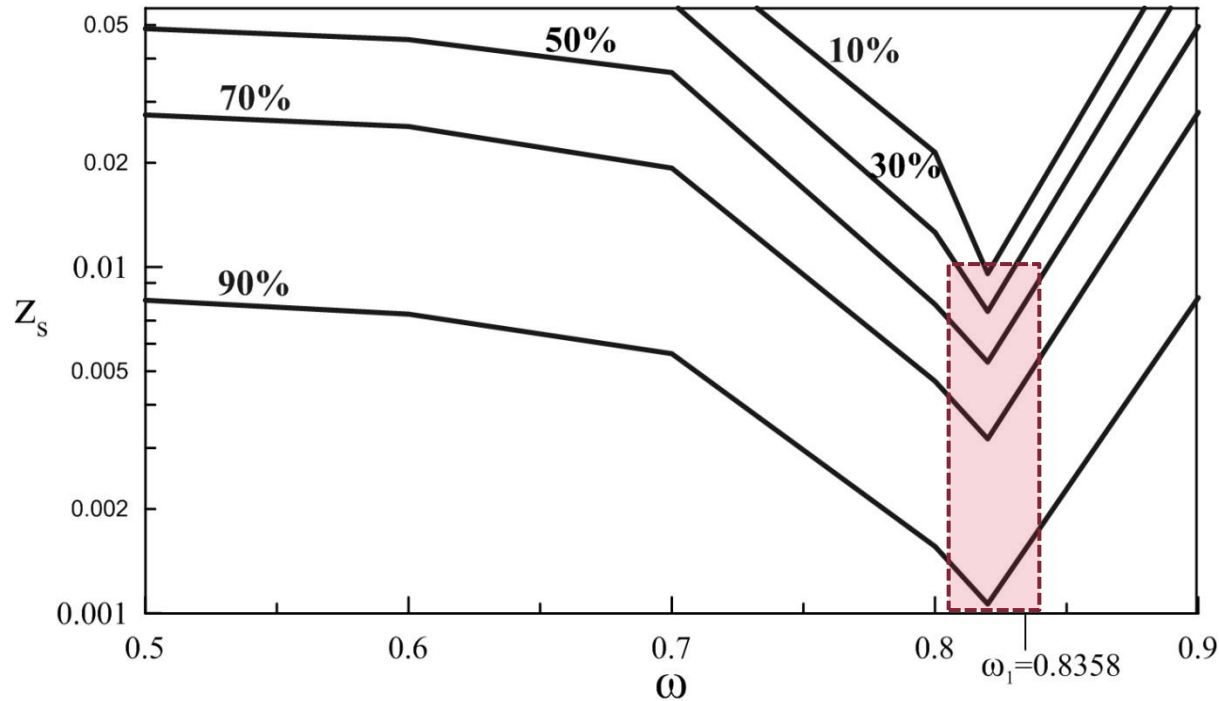
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### RESIDUAL ISO-INTEGRITY CURVES



**RESONANCE REGION: CRITICAL** also with respect to VARIATION of the TIP-SAMPLE DISTANCE

### RESIDUAL ISO-INTEGRITY CURVES

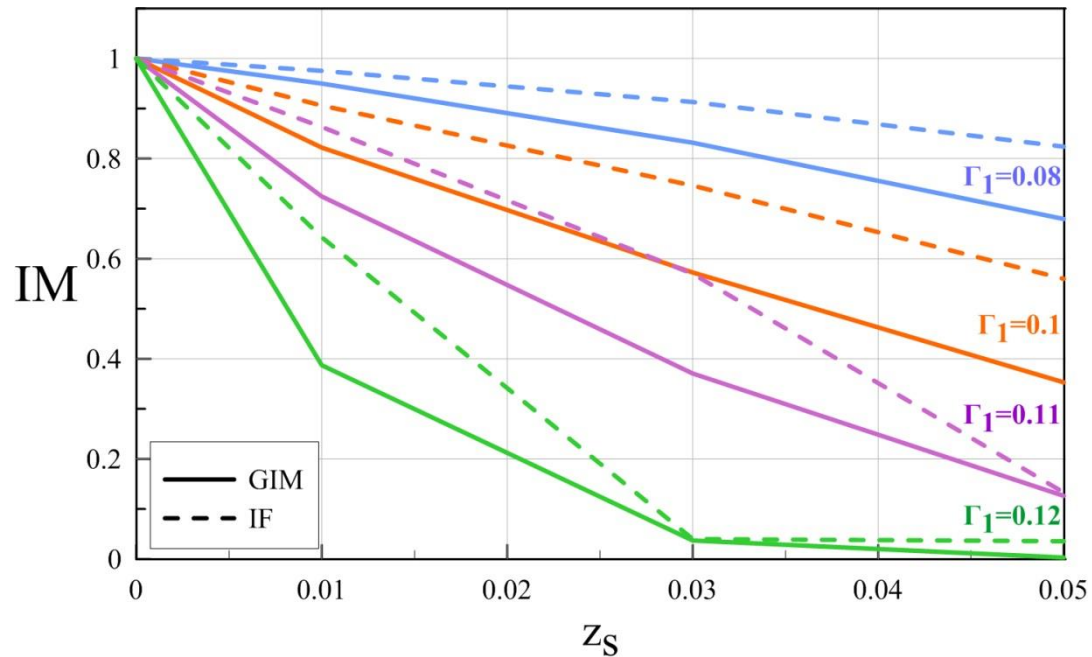


**RESONANCE REGION: CRITICAL** also with respect to VARIATION of the TIP-SAMPLE DISTANCE

$z_s$  from **0 (uncontrolled)** to **0.01** → **DYNAMICAL INTEGRITY** from **100%** to **10%**

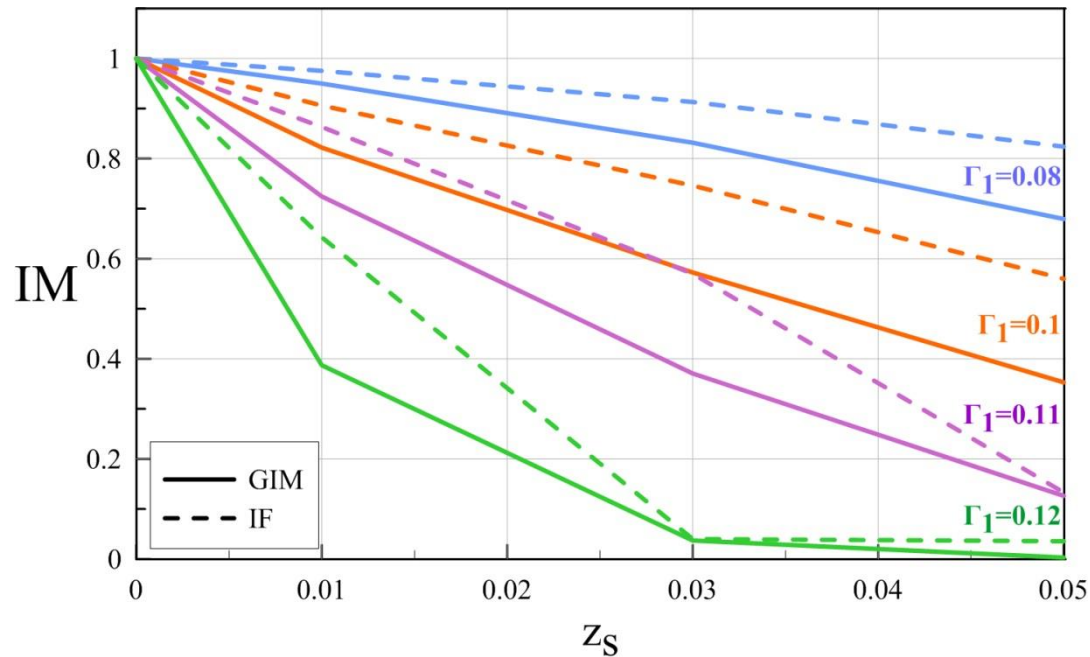
### INFLUENCE OF NONLINEAR INTERACTION ( $\Gamma_1$ )

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### PRACTICAL CONSEQUENCES:

- **ROUGH** sample **SURFACE** and/or a **STRONG** atomic tip-sample **INTERACTION** represent **DANGEROUS** situations for the application of the external feedback control to an AFM
- useful **HINTS** to **CALIBRATE** the tip-sample interaction (e.g., tip material choice) depending of the sample characteristics

### STRONGLY NONLINEAR DYNAMICS OF AFM WITH EXTERNAL FEEDBACK CONTROL

- STRONGLY NONLINEAR DYNAMICS analysis for **PARAMETRICAL** and **EXTERNAL** excitation, around **PRIMARY** and **SUBHARMONIC** resonances

#### RESULTS:

- INCREASED D.O.F. : **RICHER BIFURCATIVE SCENARIO**
- New TORUS and TRANSCRITICAL bifurcations: **STABILITY BOUNDARY REDUCTION**
- **ESCAPE THRESHOLD: DEPENDENT** on the **ACTUAL** existence of **SOLUTIONS** which are the **GOAL** of the **CONTROL** procedure
- **CONTROL WORKS PROPERLY** for **SPECIFIC DESIGN PURPOSES**, but **STRONGLY REDUCES ESCAPE THRESHOLD** when operating at resonances



### GLOBAL DYNAMICS OF NONCONTACT AFM WITH EXTERNAL FEEDBACK CONTROL

- **DYNAMICAL INTEGRITY** as a function of the most relevant system parameters
- CROSS SECTIONS of 5D basins of attraction
- **BASIN EROSION** and **INTEGRITY CHARTS** providing thresholds of constant residual integrity
- **COMPARISON** with the results already obtained for the **UNCONTROLLED** system to highlight changes and criticalities in the system global response due to the control

### RESULTS:

- Generalized **DETRIMENTAL EFFECT** of the **CONTROL** on the system **ROBUSTNESS**
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  - COEXISTENCE of resonant and nonresonant solutions **DISAPPEARS**
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### NEXT LECTURE

### GLOBAL DYNAMICS-BASED CONTROL of AFM