Semi-Final Lectures

AN ILLUSTRATIVE THEORETICAL /NUMERICAL CASE-STUDY

TO HIGHLIGHT A PATH ENCOMPASSING A VARIETY OF TOPICS

- **MODELING FROM CONTINUOUS TO MINIMAL REDUCED ORDER**
- **ACCOUNTING FOR 'CLASSICAL' AND 'NON-CLASSICAL' NONLINEARITIES**
- **ANALYZING LOCAL BIFURCATION SCENARIOS AND WEAKLY/STRONGLY NONLINEAR RESPONSE**
- **EVALUATING DYNAMIC INTEGRITY**
- **APPLYING A LOCAL CONTROL AND ADDRESSING ITS GLOBAL EFFECTS**
- **EXPLOITING GLOBAL DYNAMICS TO SECURE/IMPROVE SAFETY**

A NONCONTACT ATOMIC FORCE MICROSCOPE (AFM):

NONLINEAR DYNAMICS AND CONTROL FOR SAFE RESPONSE

DAY	TIME	LECTURE
Monday 05/11	14.00 -14.45	Historical Framework - A Global Dynamics Perspective in the Nonlinear Analysis of Systems/Structures
	15.00 -15.45	Achieving Load Carrying Capacity: Theoretical and Practical Stability
	16.00 -16.45	Dynamical Integrity: Concepts and Tools_1
Wednesday 07/11	14.00 -14.45	Dynamical Integrity: Concepts and Tools_2
	15.00 -15.45	Global Dynamics of Engineering Systems
	16.00 -16.45	Dynamical integrity: Interpreting/Predicting Experimental Response
Monday 12/11	14.00 -14.45	Techniques for Control of Chaos
	15.00 -15.45	A Unified Framework for Controlling Global Dynamics
	16.00 -16.45	Response of Uncontrolled/Controlled Systems in Macro- and Micro-mechanics
Wednesday 14/11	14.00 -14.45	**A Noncontact AFM: (a) Nonlinear Dynamics and Feedback Control** (b) Global Effects of a Locally-tailored Control
	15.00 -15.45	Exploiting Global Dynamics to Control AFM Robustness
	16.00 -16.45	Dynamical Integrity as a Novel Paradigm for Safe/Aware Design

12.1a – A Noncontact AFM: Nonlinear Dynamics and Feedback Control

Giuseppe Rega

Department of Structural and Geotechnical Engineering
Sapienza University of Rome, Italy
Giuseppe.Rega@uniroma1.it

Coworkers: V. Settimi, O. Gottlieb

OUTLINE OF 12.1a

SYSTEM

1. **AFM PRINCIPLES**
2. **MODELING**

NONLINEAR DYNAMICS

3. **BIFURCATION/RESPONSE SCENARIOS**
4. **GLOBAL DYNAMICS AND INTEGRITY**

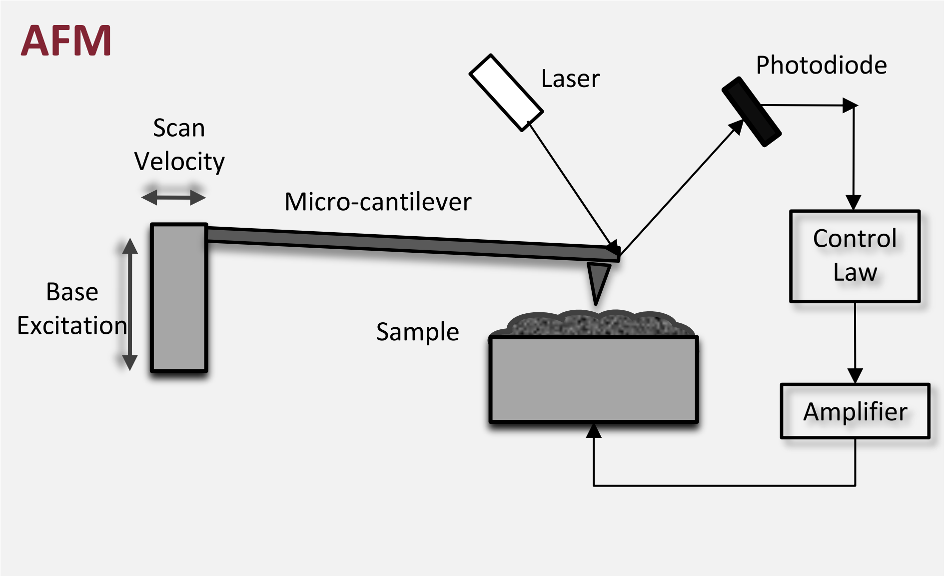
FEEDBACK CONTROL (a)

5. **EXTERNAL FEEDBACK CONTROL**
6. **WEAKLY NONLINEAR DYNAMICS**

ATOMIC FORCE MICROSCOPY (*Binnig et al 1986*)

- Devices to **TOPOLOGICALLY CHARACTERIZE SURFACES** up to **MICRO** and **NANO** RESOLUTION levels
- **TOPOGRAPHY**: sharp tip, fixed to the free end of a micro-cantilever vertically bending over the sample surface

→ MEASURE OF THE TIP DETECTION

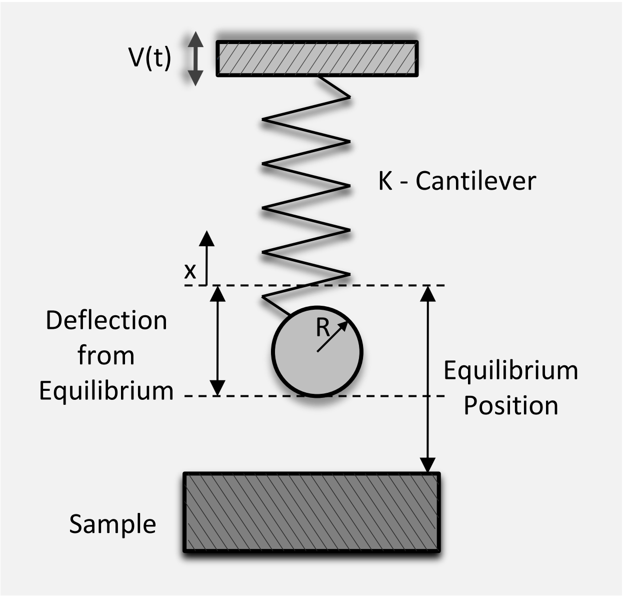


TIP-SAMPLE INTERACTION: modifies beam dynamics and allows to image surfaces and measure sample physical properties.

Long- and Short-range contributions:

- **VAN DER WAALS:** long-range interaction arisen from electromagnetic field fluctuations
- **ELECTROSTATIC FORCE:** when tip and sample have electrostatic potential difference
- **CHEMICAL FORCES:** empirical model potentials (*Morse p., Lenard-Jones p., Stillinger-Weber p., Tersoff p.*)
- **CAPILLARY FORCES:** caused by adhesion layers on tip and sample in ambient conditions
- **CONTACT FORCES:** various approximations*: Maugis, Muller at al., Hertz, JKR (Johnson-Kendall-Roberts), DMT (Derjaguin-Muller-Toporov)*

MATHEMATICAL MODELS

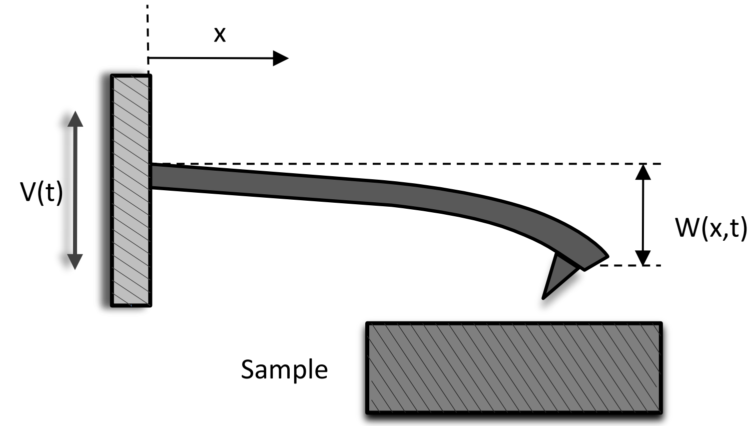


LUMPED-MASS SPRING DASHPOT SYSTEMS

Durig et al. (1992), Erlandsson and Olsson (1998), Ashhab et al. (1999), Couturier et al. (2002), Paulo and Garcia (2002), Yagasaki (2004), ……………………

CONTINUOUS MODELS

Rabe et al. (1996), Turner et al. (1997), Stark and Heckl (2000), Lee et al. (2002), Wolf and Gottlieb (2002), Turner (2004), Hornstein and Gottlieb (2008), ……………..

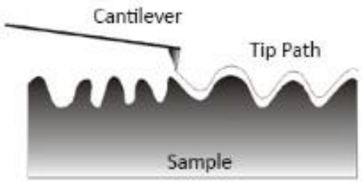


AFM OPERATING MODES

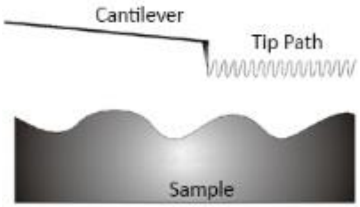
DYNAMIC AFM

- **CONTACT-AFM**: the tip is brought to a close proximity with the sample

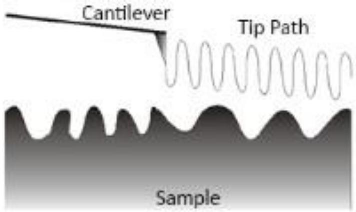
  REPULSIVE FORCES dominate the tip-sample interactions



- **NONCONTACT-AFM:** no contact between the atoms, tip-sample interaction governed by an **ATTRACTIVE** POTENTIAL INTERACTION

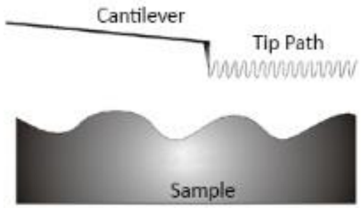


- **TAPPING-AFM**: the tip operates in the **ATTRACTIVE** and **REPULSIVE** FORCE region, and **TOUCHES** the surface only **FOR SHORT PERIODS**, in order to reduce damages to potentially fragile samples.



AIMS OF INVESTIGATION

- **NONCONTACT-AFM:** no contact between atoms, tip-sample interaction governed by an **ATTRACTIVE POTENTIAL INTERACTION.**



The tip has to maintain a design gap from the sample such to ensure that the beam elastic restoring force is stronger than the atomic attraction.

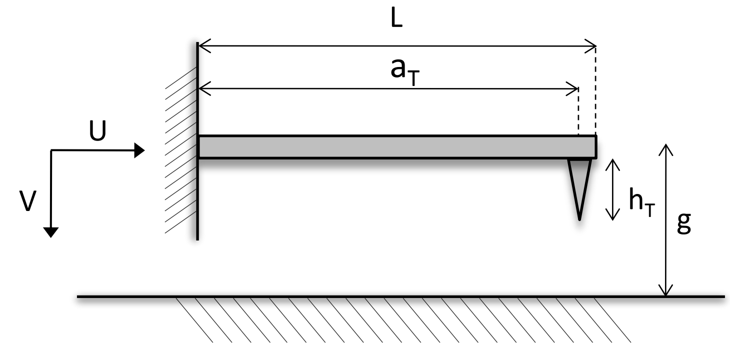
Otherwise → **INSTABILITY** of solution

“**JUMP TO CONTACT**", or **ESCAPE**

- **STUDY OF SYSTEM STABILITY** as a function of a varying excitation amplitude, or other (bifurcation) parameters: IMPORTANT ISSUE for NONCONTACT AFMs
- In view of involved dynamic scenarios, need to **KEEP SYSTEM RESPONSE TO A SUITABLE REFERENCE ONE** via an efficient **CONTROL TECHNIQUE**
- Verification of **EFFECTIVENESS** of control and its **INFLUENCE ON SYSTEM OVERALL STABILITY**

TECHNIQUES: Continuation, Asymptotic Solutions, Numerical Simulations, Dynamic Integrity Analysis

Hornstein and Gottlieb (2008)



- **MICROCANTILEVER** with a sharp tip close to its free end
- **PLANAR, INEXTENSIBLE, ELASTIC** *(Crespo da Silva (1979))*
- **INTERACTION FORCE**: Localized**, Van der Waals** like, derived from **Lennard-Jones potential** for a sphere-plane system

$$F_v^{\,A} = \frac{A_H R_T}{6\sigma_A^{\,2}}\left[-\left(\frac{\sigma_A}{g+\bar{v}-h_T}\right)^2 + \frac{1}{30}\left(\frac{\sigma_A}{g+\bar{v}-h_T}\right)^8\right]$$

- Extended Hamilton principle $\rightarrow$ **TWO COUPLED PARTIAL DIFFERENTIAL EQUATIONS (LONGITUDINAL** AND **TRANSVERSE)**:

$$\boxed{\begin{aligned} & m\bar{u}_{tt} - [EI\bar{v}_{rrr}\bar{v}_r - J_z\bar{v}_{ttr}\bar{v}_r + \lambda(1+\bar{u}_r)]_r = \bar{Q}_u \\ & m\bar{v}_{tt} - [-EI(\bar{v}_{rrr} + \bar{v}_r\bar{v}_{rr}^2) + J_z(\bar{v}_{ttr} + \bar{v}_{tr}^2\bar{v}_r) + \lambda\bar{v}_r]_r = \bar{Q}_v \end{aligned}}$$

$$\bar{Q}_u = \bar{g}_1\bar{u}_t - \bar{g}_2\bar{u}_{trr} - \bar{g}_3\bar{u}$$
$$\bar{Q}_v = \delta(r-a_T)F_v^{\,A} - d\bar{v}_t$$

+ BOUNDARY CONDITIONS:

$$\boxed{\begin{array}{lll} \bar{v}(0,t) = \bar{V}(t) & \bar{v}_{rrr}(L,t) = 0 & \bar{v}_r(0,t) = 0 \\ \bar{u}(0,t) = \bar{U}(t) & \bar{v}_{rr}(L,t) = 0 & \bar{u}_r(L,t) = 0 \end{array}}$$

CONTINUUM MODEL FORMULATION - 2 -

- Longitudinal *u(r,t)* as a function of transverse *v(r,t)* and *U(t)*
- Obtaining Lagrange multiplier
- Rescaling length/frequency and expanding multiplier to cubic order
- Obtaining modified transverse equation of motion (cubic term ignored)
- Transforming system to a moving reference frame : $v(s,\tau)=w(s,\tau)+V(\tau)$

SINGLE PARTIAL-DIFFERENTIAL EQUATION (VERTICAL)

$$\boxed{\begin{aligned}&w_{\tau\tau}+V_{\tau\tau}+w_{ssss}-\mu w_{\tau\tau ss}-Q_w=\\&\left[-w_s(w_{ss}w_s)_s+\mu w_s(w_{\tau s}w_s)_\tau+w_s\left(\left(1+\frac{1}{2}w_s^2\right)\left(U_{\tau\tau}-\int_1^s Q_u ds\right)-\frac{1}{2}\int_1^s\left(\int_0^s w_s^2 ds\right)_{\tau\tau}ds\right)\right]_s\end{aligned}}$$

$$Q_u=-g_1\left(U_\tau-\frac{1}{2}\frac{d}{dt}\int_0^s w_s^2 ds\right)+\frac{1}{2}g_2\left(\frac{d^3}{d\tau ds^2}\int_0^s w_s^2 ds\right)-g_3\left(U-\frac{1}{2}\int_0^s w_s^2 ds\right)$$

$$Q_w=\delta(s-\alpha)\bar{\Gamma}_1\left[\frac{1}{(\gamma+w+V)^2}+\frac{\bar{\Gamma}_2}{(\gamma+w+V)^8}\right]-\nu(w_\tau+V_t)$$

\+ HOMOGENEOUS BOUNDARY CONDITIONS:

$$\boxed{\begin{matrix} w(0,\tau)=0 & w_s(0,\tau)=0 \\ w_{ss}(1,\tau)=0 & w_{sss}(1,\tau)=0 \end{matrix}}$$

MODAL DYNAMICAL SYSTEM

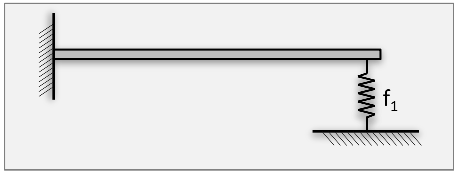
- **FIRST MODE approximation** (Galerkin method): $\boxed{w(s,\tau)=q_1(\tau)\cdot\Phi_1(s)}$

  Basis function: CLAMPED-SPRING LINEAR BEAM:

$$\Phi_n(s)=\cosh(z_n s)-\cos(z_n s)-K_n\left(\sinh(z_n s)-\sin(z_n s)\right)$$

$$K_n=\frac{\cos(z_n)+\cosh(z_n)}{\sin(z_n)+\sinh(z_n)}$$

$$\omega_n=z_n^{\,2}:z_n^{\,3}\left(1+\cosh(z_n s)\cos(z_n s)\right)-f_1\left(\sin(z_n s)\cosh(z_n s)-\cos(z_n s)\sinh(z_n s)\right)=0$$



- Noncontacting microscopy $\rightarrow$ REPULSION INTERACTION NEGLIGIBLE $(\bar{\Gamma}_2=0)$
- New state variable: $x(\tau)=q_1(\tau)\cdot\Phi_1(\alpha)/\gamma$
- Rescaling time: $t_N=\omega_1\tau$

[redacted]

$$-\left(\left(\ddot{V}_g+\nu_1\dot{V}_g\right)\right)\nu_2+\left(x\mu_1+\mu_2x^3\right)\left(\ddot{U}_g+\eta_1\dot{U}_g+\eta_2U_g\right)$$

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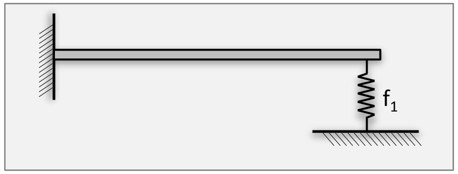

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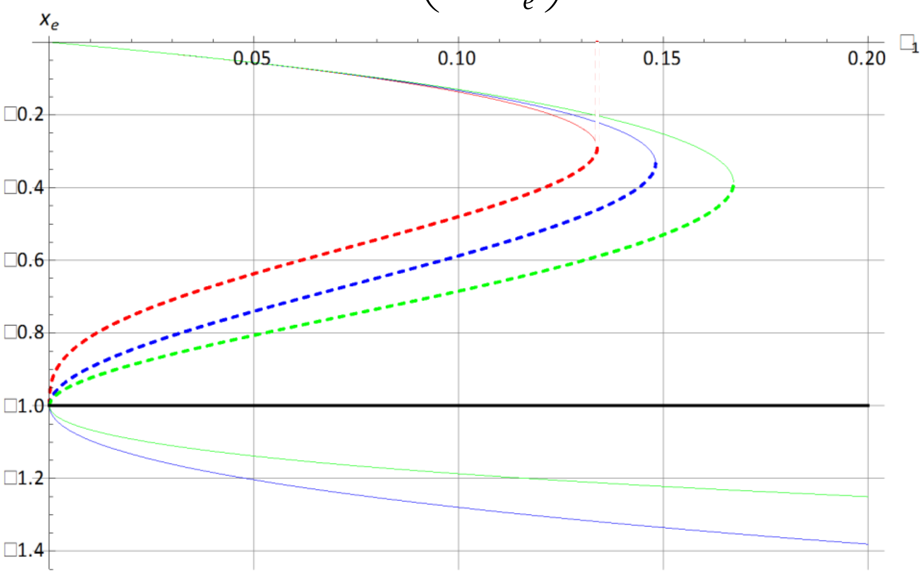
$$-\left(\boxed{\left(\ddot{V}_g+\nu_1\dot{V}_g\right)}\right)\nu_2+\boxed{\left(x\mu_1+\mu_2x^3\right)\left(\ddot{U}_g+\eta_1\dot{U}_g+\eta_2U_g\right)}$$

VERTICAL EXCITATION HORIZONTAL EXCITATION

UNPERTURBED SYSTEM

- **EQUILIBRIUM SOLUTIONS**

$$\alpha_1 x_e + \alpha_3 x_e^{\,3} + \Gamma_1 \left(\frac{1}{1+x_e} \right)^2 = 0$$



DEPENDENCE ON ATOMIC INTERACTION

$\alpha_1 = 1 \quad \alpha_3 = -1 \quad (red) \quad \alpha_3 = 0 \quad (blue) \quad \alpha_3 = 1 \quad (green)$

SADDLE-NODE BIFURCATION: 2 stable branches (solid), 1 unstable branch (dotted)

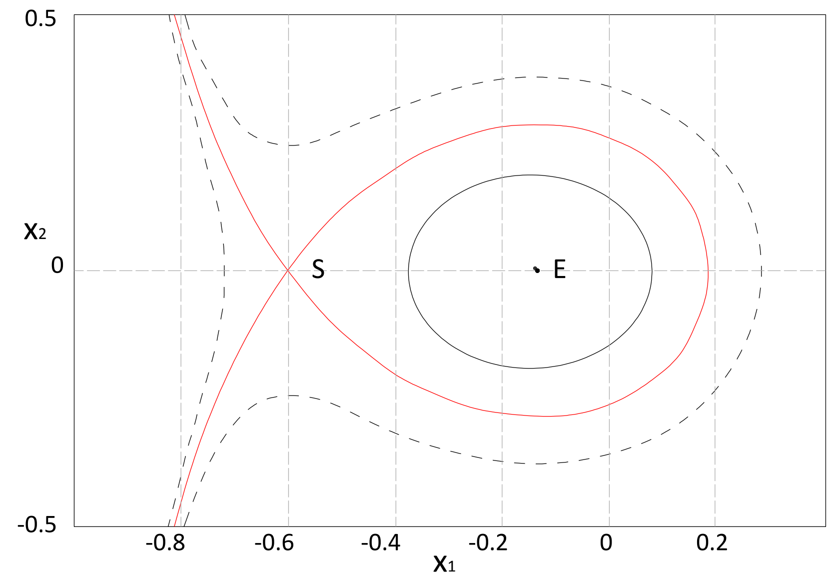
LOWER BRANCH: beyond the sample position $\rightarrow$ **NO PHYSICAL MEANING**

- **HAMILTONIAN SYSTEM**

$$\ddot{x} + \alpha_1 x + \alpha_3 x^3 = -\frac{\Gamma_1}{(1+x)^2}$$

$$\begin{cases} x_1 = x \\ x_2 = \dot{x} \end{cases} \qquad \begin{cases} \dot{x}_1 = \dfrac{\partial H}{\partial x_2} \\ \dot{x}_2 = -\dfrac{\partial H}{\partial x_1} \end{cases}$$

$$\boxed{H(x_1, x_2) = \frac{\alpha_1 x_1^{\,2}}{2} + \frac{\alpha_3 x_1^{\,4}}{4} + \frac{\Gamma_1}{1+x_1} + \frac{x_2^{\,2}}{2}}$$



E = Equilibrium solution **S** = Saddle

BOUNDED SOLUTIONS
HOMOCLINIC ORBIT
UNBOUNDED SOLUTIONS

BIFURCATION/RESPONSE SCENARIOS

- **MAPPING** all **BIFURCATIONS** prior to escape for several forcing frequencies

- **BIFURCATION DIAGRAMS** with continuation techniques for **VARYING FORCING AMPLITUDE**

- Numerical simulations : **HORIZONTAL PARAMETRIC** or **VERTICAL EXTERNAL** excitation

- Analyses around **FUNDAMENTAL (PRIMARY)** $\left(\omega_u\left(\omega_v\right)=\omega_1\right)$

  PRINCIPAL (SUBHARMONIC) $\left(\omega_u\left(\omega_v\right)=2\omega_1\right)$

  PARAMETRIC (EXTERNAL) resonances

- **ESCAPE THRESHOLD** as envelope of local bifurcation escape thresholds

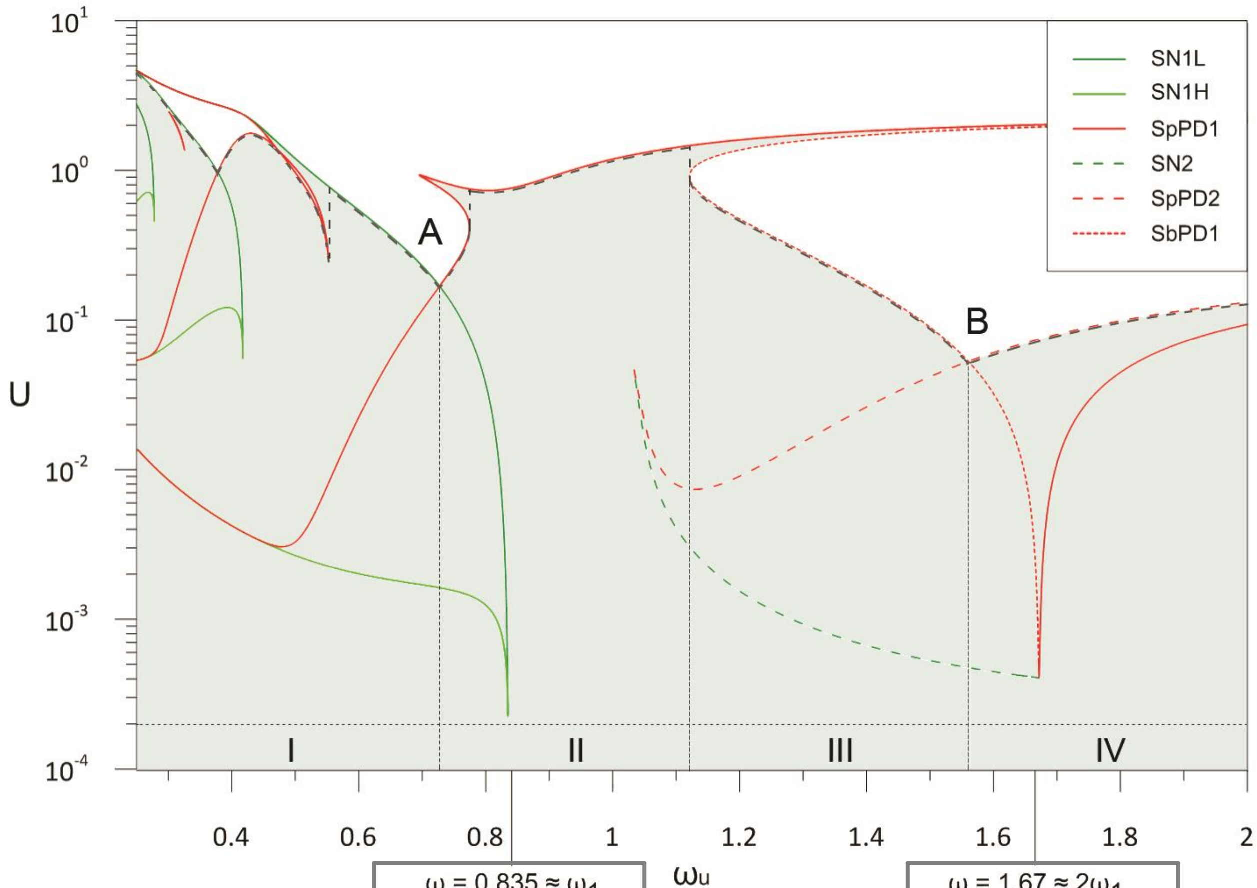
  → amplitude value, for several forcing frequencies, at which **basin annihilation** occurs

  → from **physical viewpoint**, the (**unacceptable**) amplitude value that would bring **TIP** oscillation **BEYOND** the **SAMPLE** location $(x=-1)$

PARAMETRIC EXCITATION - 1 -

$$\boxed{\ddot{x}+\alpha_1 x+\alpha_3 x^3=-\Gamma_1\left(1+x\right)^{-2}-\rho_1\dot{x}-x\mu_1 U\omega_u^2\sin(\omega_u t)}$$

ESCAPE THRESHOLD as **envelope** of **various** **LOCAL BIFURCATION ESCAPE THRESHOLDS**



$\omega = 0.835 \approx \omega_1$
fundamental resonance

$\omega = 1.67 \approx 2\omega_1$
principal resonance

$\mu_2 = \rho_2 = 0$

$\eta_1 = \eta_2 = 0$

$V_g = \alpha_2 = 0$

$\alpha_1 = 1$

$\Gamma_1 = 0.1$

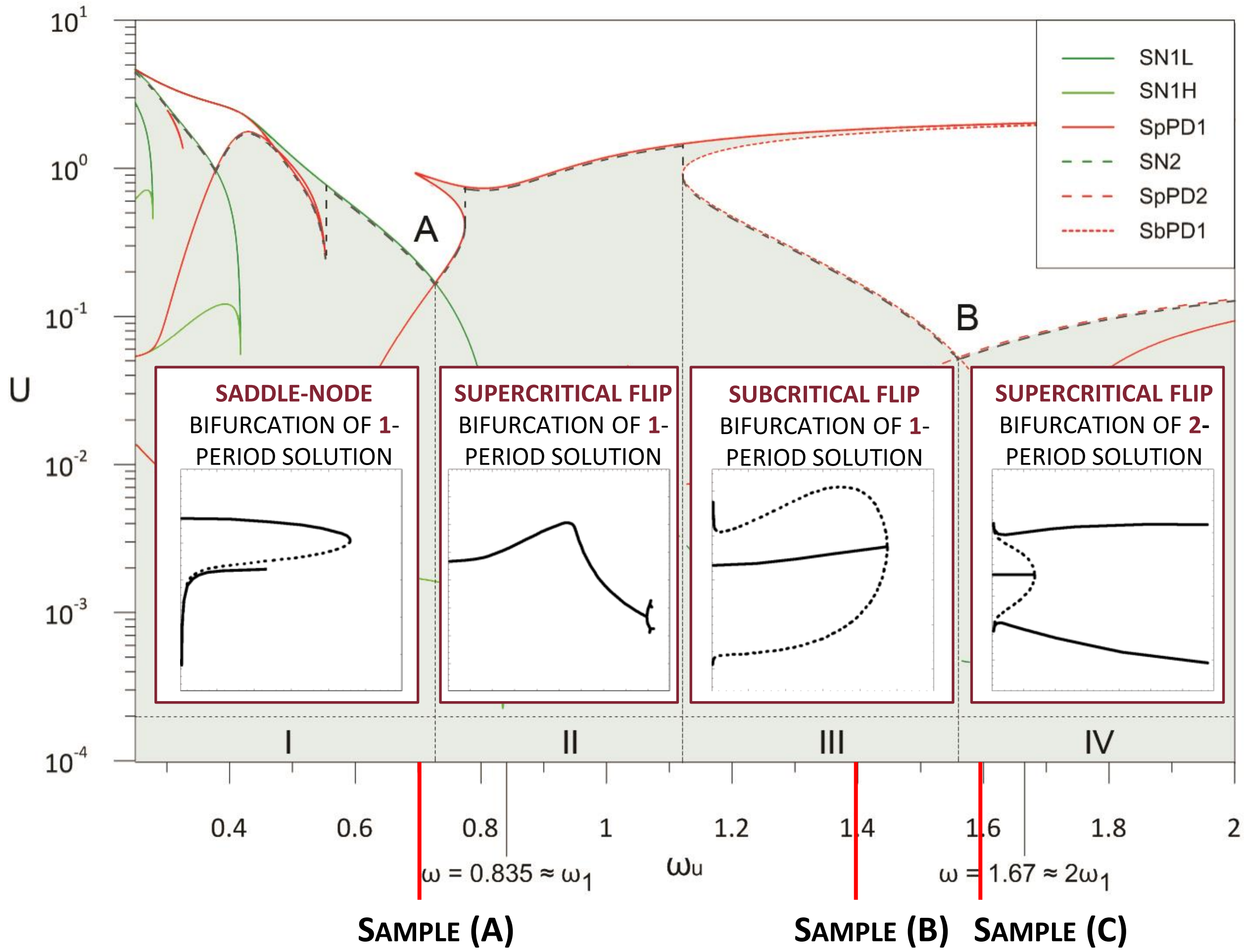
$\alpha_3 = 0.1$

$\mu_1 = 1.5708$

$\omega_1 = 0.8325$

PARAMETRIC EXCITATION - 2 -

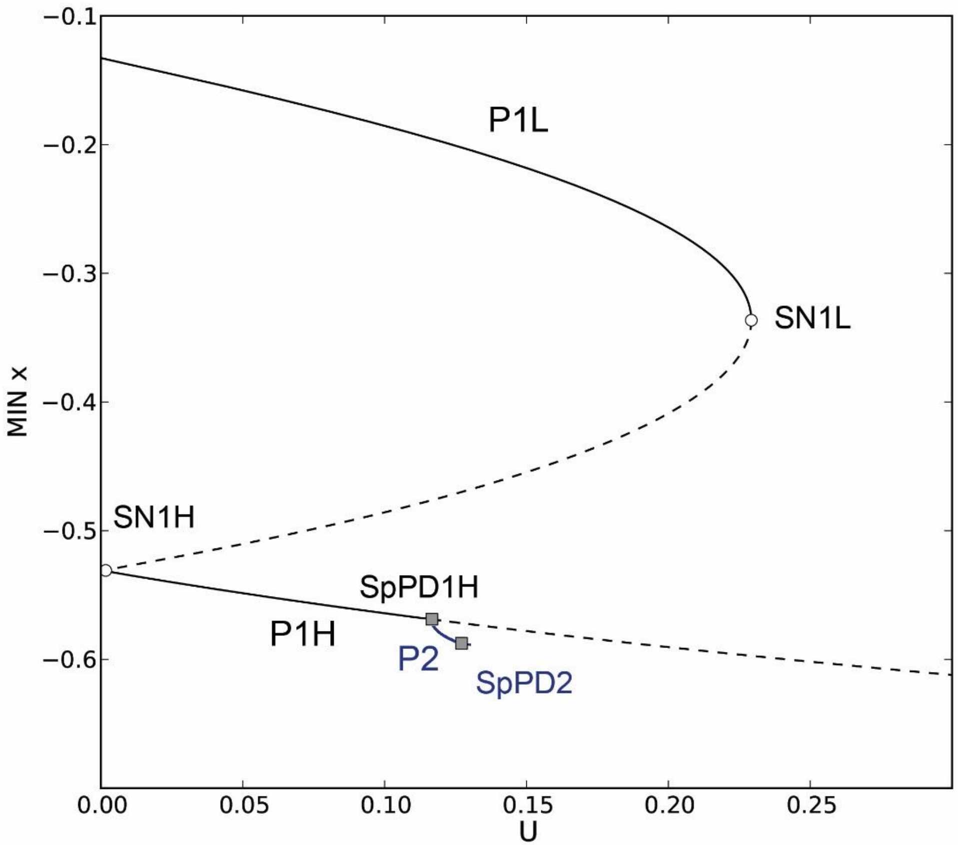
- **SUDDEN CHANGES** in **OVERALL THRESHOLD** → **DUE TO CHANGES** in **LOCAL STABILITY BOUNDARY**
- **ASCENDING** and **DESCENDING BRANCHES** → **ASSOCIATED** with **PARTICULAR BIFURCATION EVENTS** (localized sudden changes)



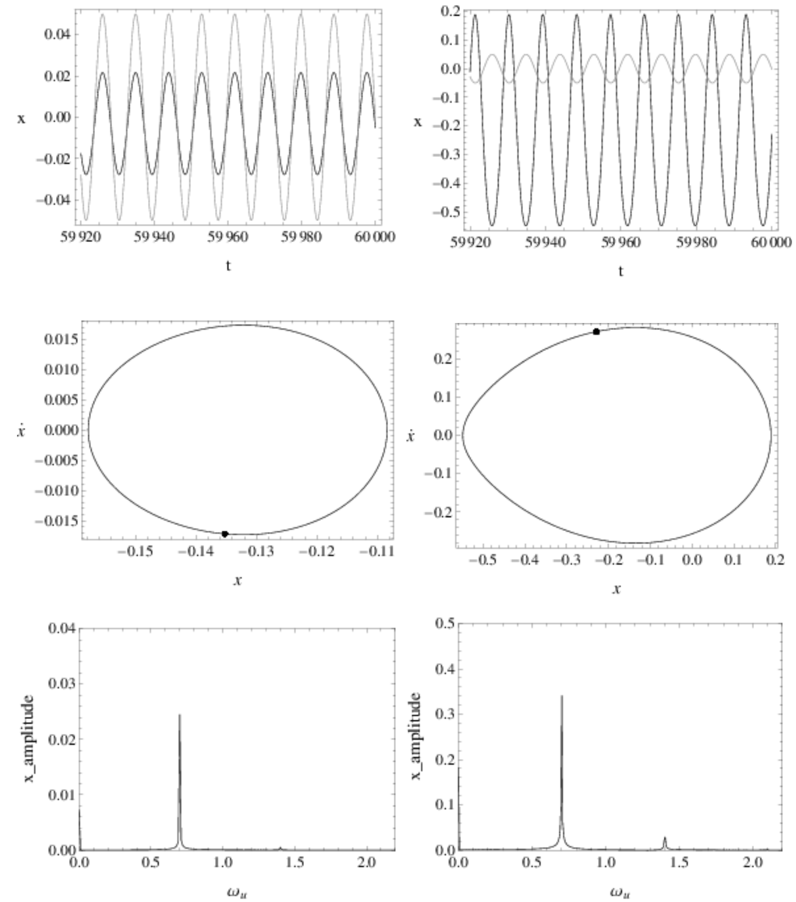
(A) SADDLE-NODE BIFURCATION - SUPERCRITICAL FLIP BIFURCATION OF 1-PERIOD

(SN1L- SN1H – SpPD1H)

ω_u=0.7 - P1L, P1H SOLUTIONS



U = 0.05 P1L P1H

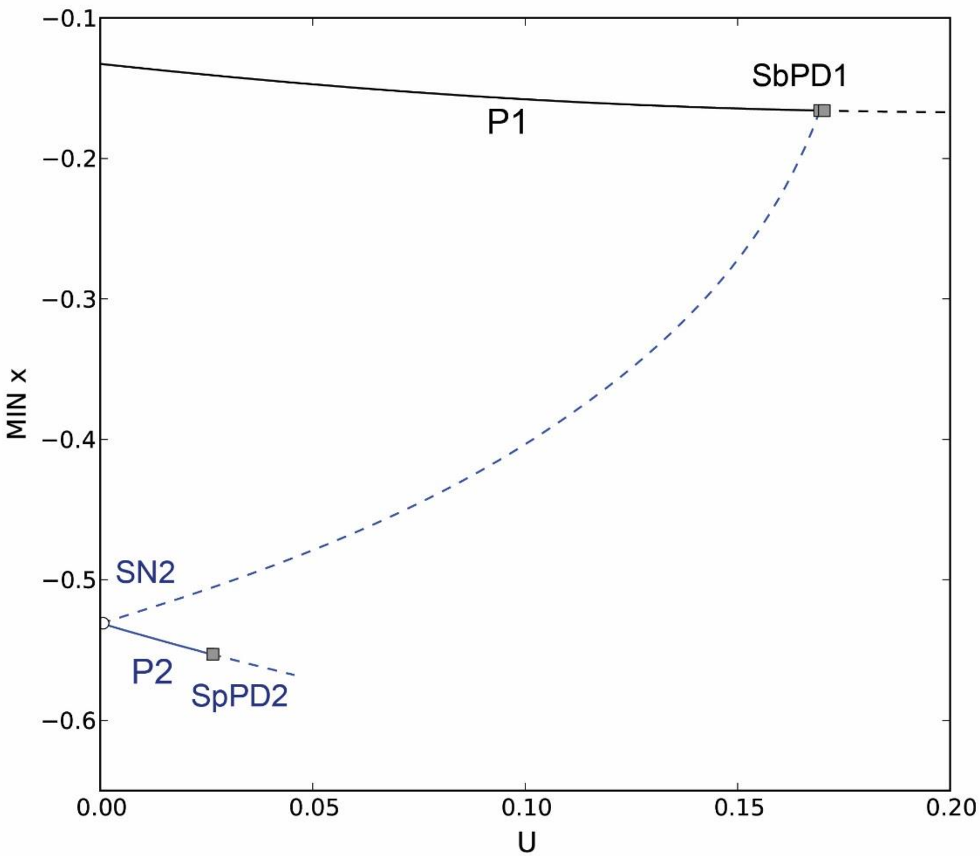


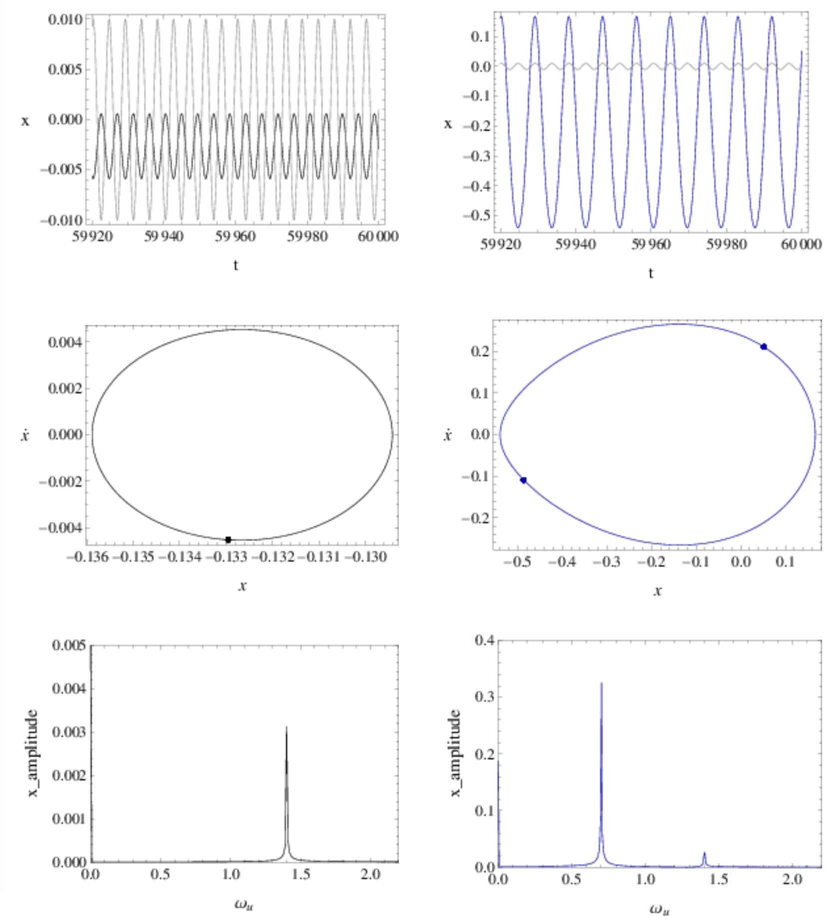
SAMPLE BOUNDED SOLUTIONS - 2 -

(B) SUBCRITICAL FLIP BIFURCATION OF 1-PERIOD SOLUTION (SbPD1)

ω_u=1.4 - P1, P2 SOLUTIONS

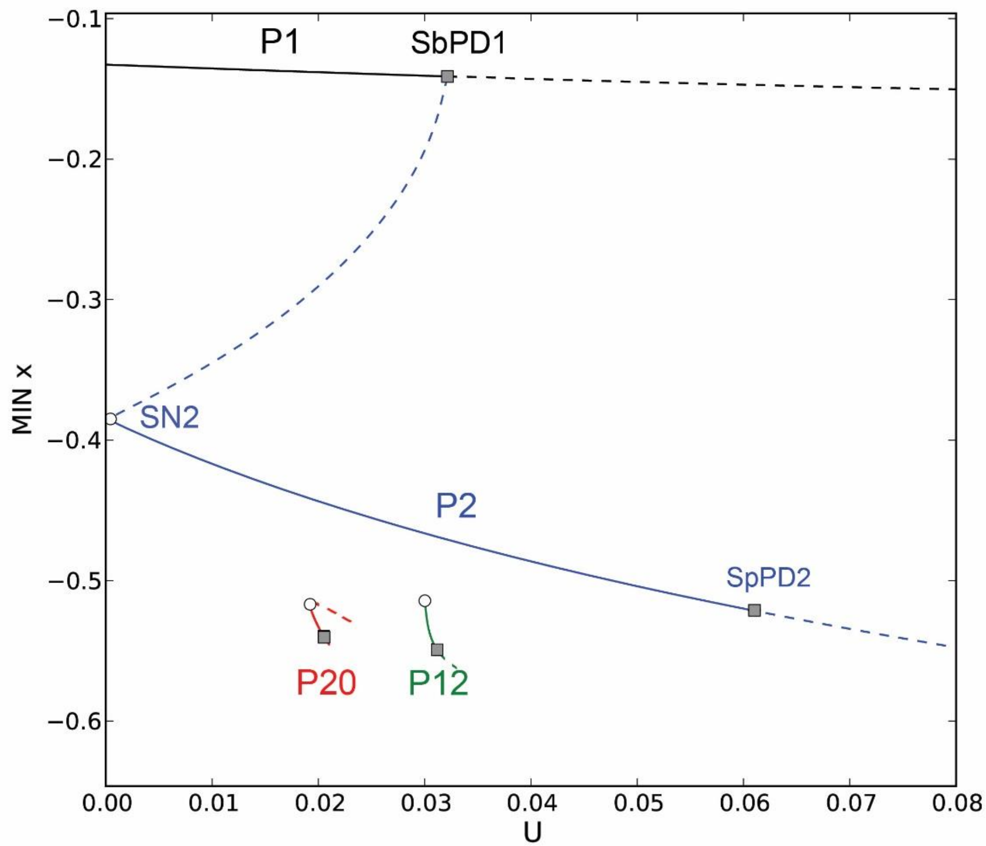
U = 0.01 **P1** **P2**



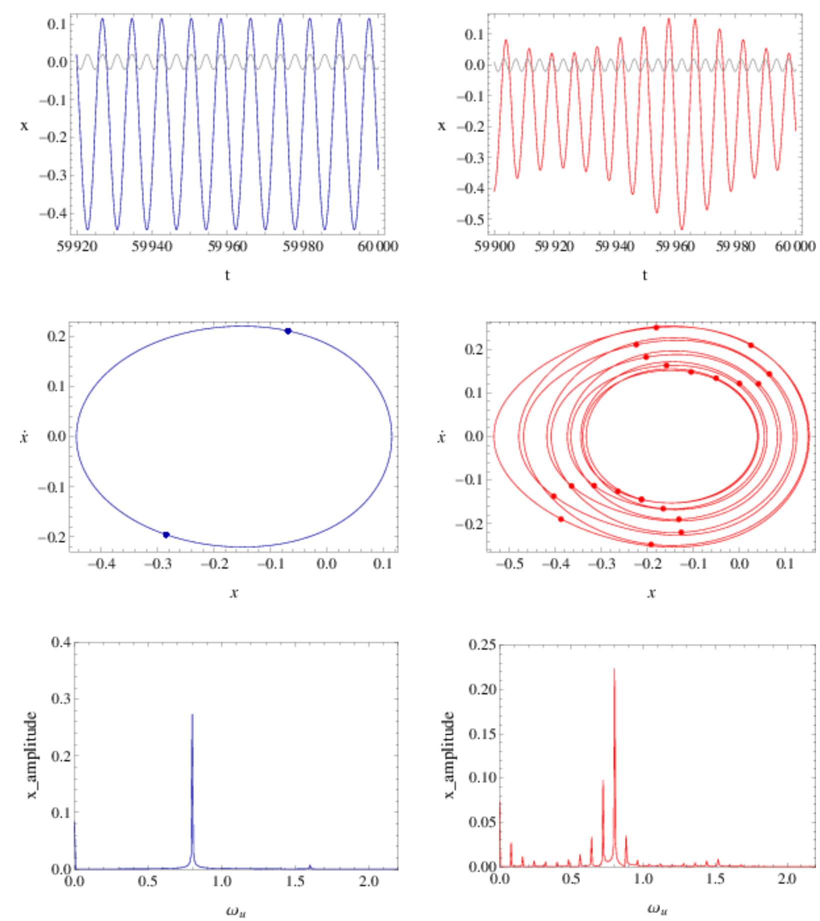


(C) SUPERCRITICAL FLIP BIFURCATION OF 2-PERIOD SOLUTION (SpPD2)

ω_u=1.6 – P2, P20 SOLUTIONS

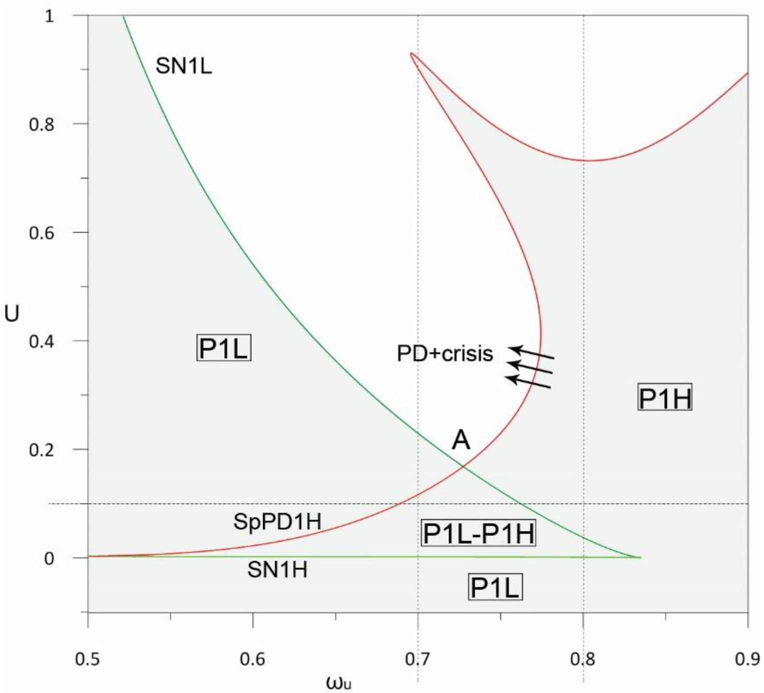


U = 0.02 P2 P20

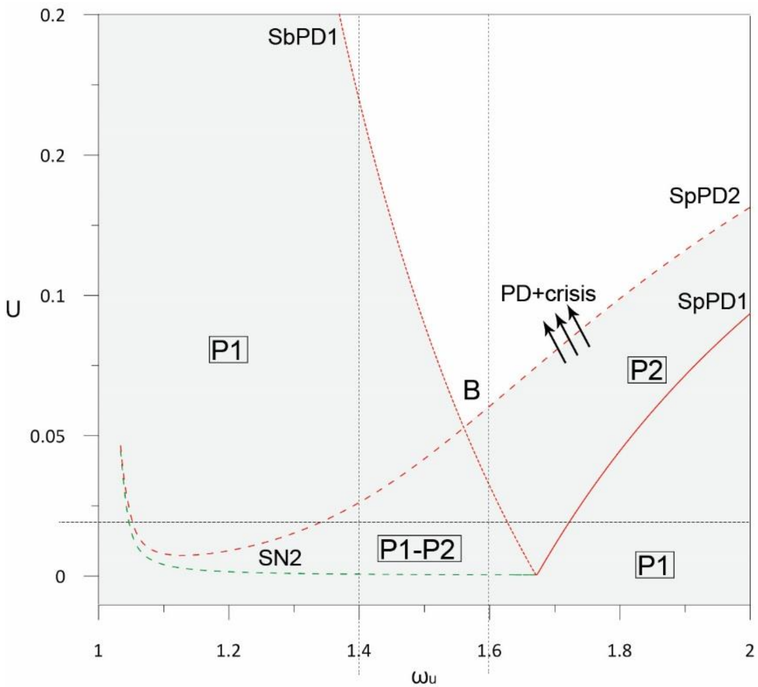


RESPONSE CHARTS - 1 -

- **FUNDAMENTAL** RESONANCE **(P1L/P1H)**



- **PRINCIPAL** RESONANCE **(P1/P2)**

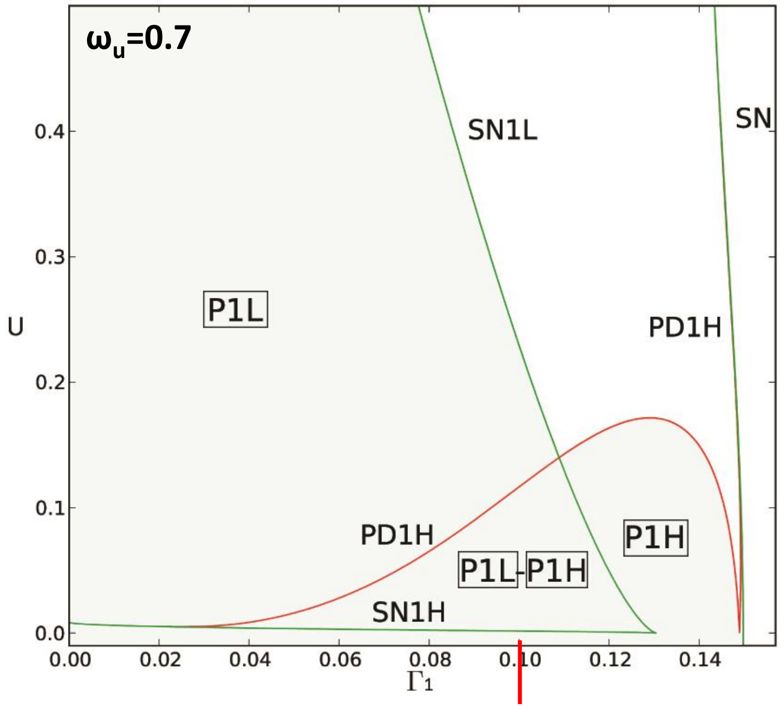


SAME **QUALITATIVE** BEHAVIOR OF **OTHER SOFTENING SYSTEMS** (Helmholtz oscillator, MEMs…)

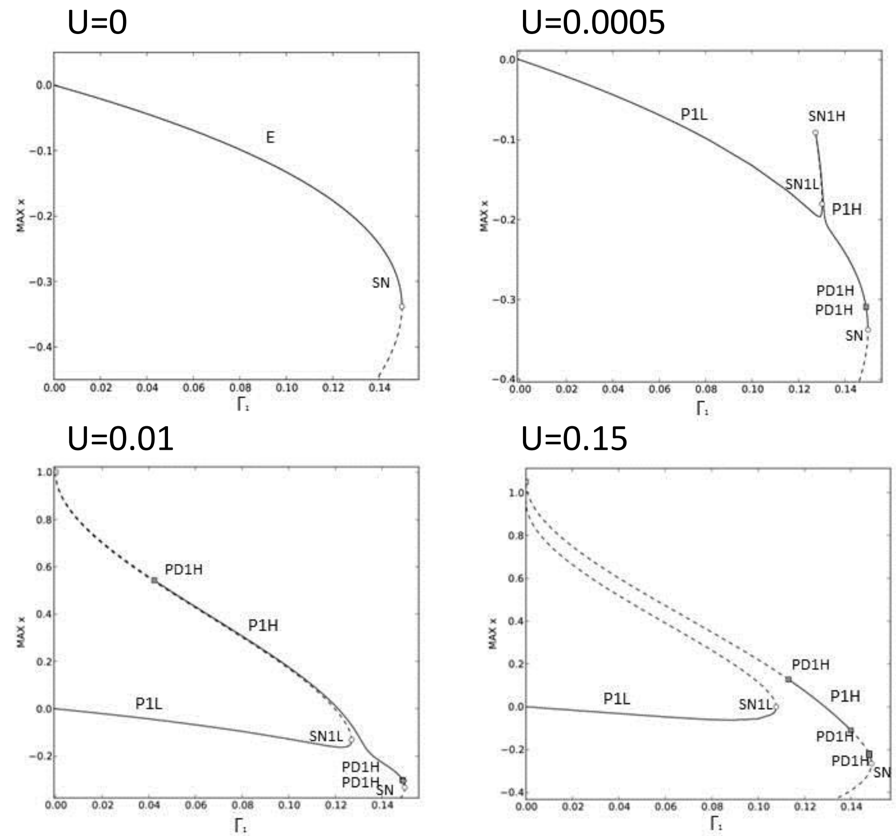
- V-shaped region of escape
- Triangle region with coexisting solutions
- Degenerated cusp bifurcation

INFLUENCE OF ATOMIC INTERACTION

RESPONSE CHART IN Γ_1-U PLANE



REFERENCE VALUE

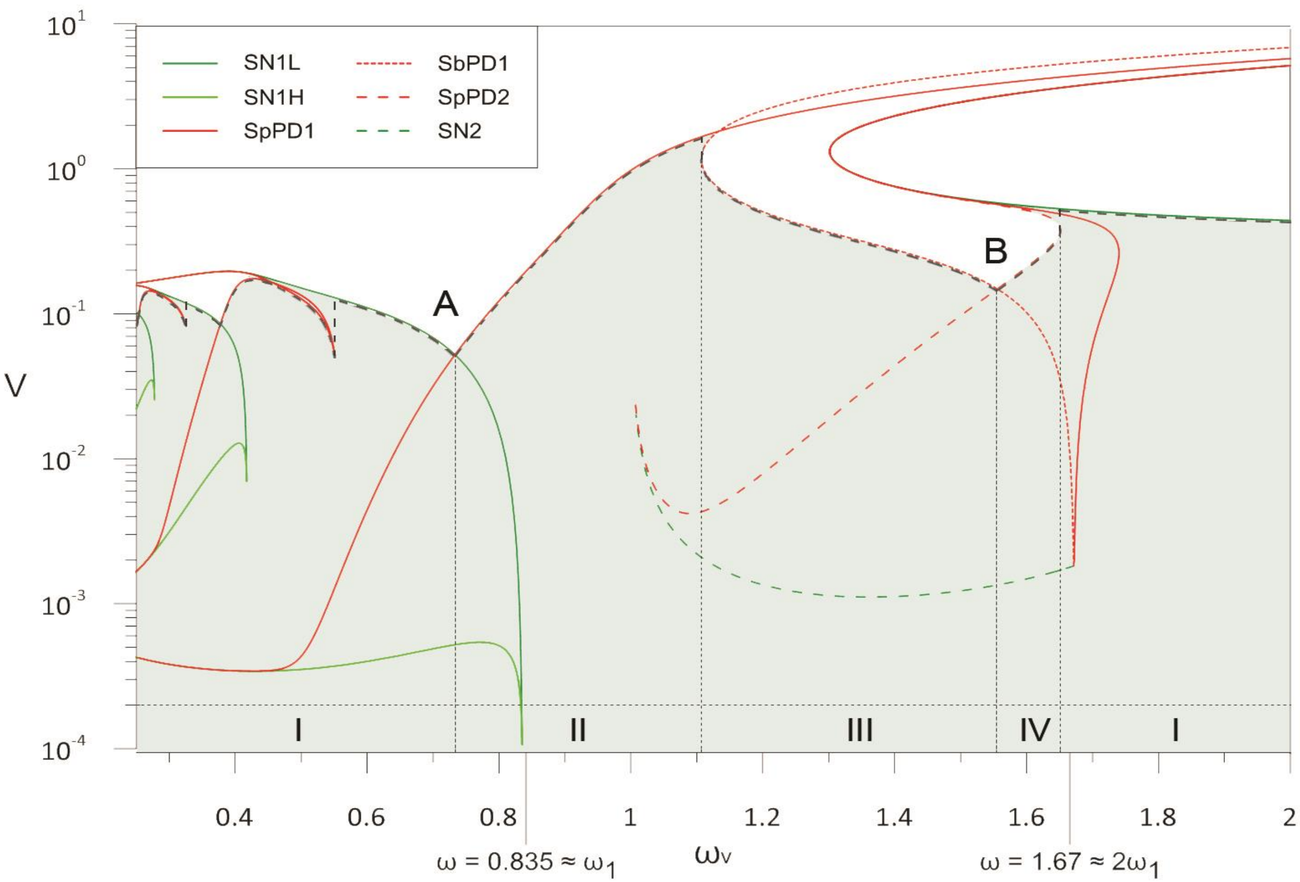


FOR DESIGN PURPOSES

- **Limiting values of forcing amplitude** to be used depending on sample properties
- **Calibration of tip-sample interaction** (tip/sample material) depending on AFM operation settings

EXTERNAL EXCITATION

$$\ddot{x}+\alpha_1 x+\alpha_3 x^3=-\Gamma_1\left(1+x+V\sin(\omega_v t)\right)^{-2}-\rho_1\dot{x}-\nu_2\left(\nu_1\omega_v V\cos(\omega_v t)-\omega_v^2 V\sin(\omega_v t)\right)$$



$\mu_2 = \rho_2 = 0$

$\eta_1 = \eta_2 = 0$

$U_g = \alpha_2 = 0$

$\alpha_1 = 1$

$\Gamma_1 = 0.1$

$\alpha_3 = 0.1$

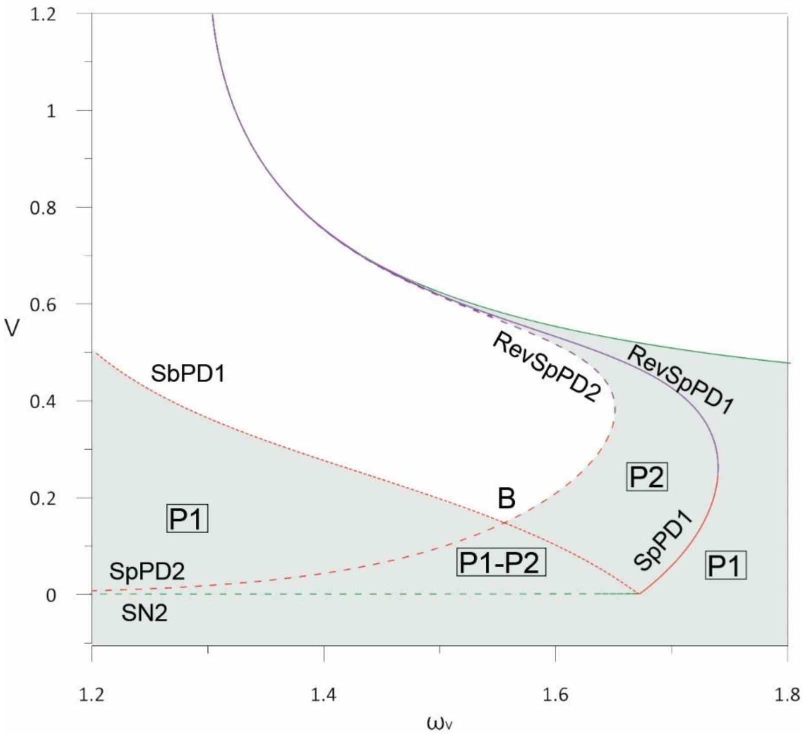
$\nu_1 = 0.01$

$\nu_2 = 0.01$

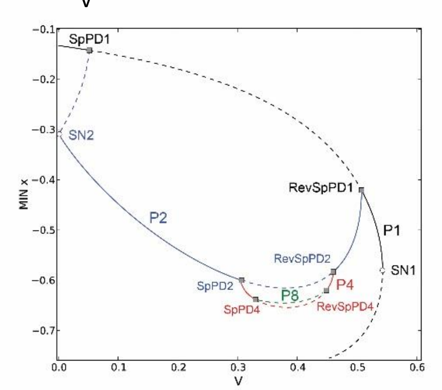
$\omega_1 = 0.8325$

- **Absolute minimum** shifted from **subharmonic** to **primary** resonance
- Same bifurcation scenarios at primary resonance, slightly different at subharmonic resonance

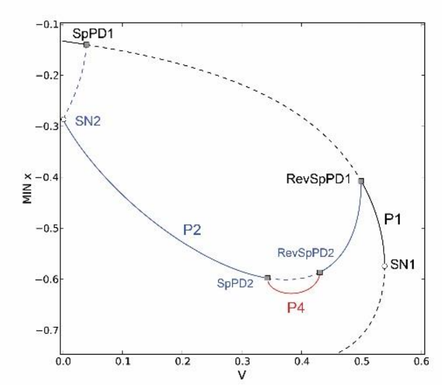
SUBHARMONIC RESONANCE



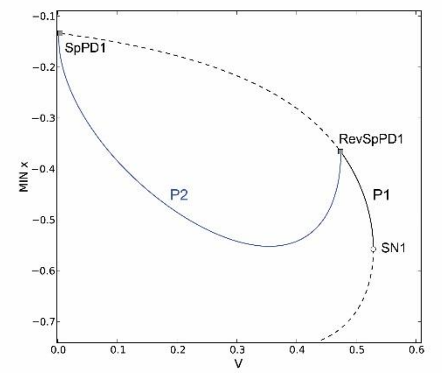
$\omega_v = 1.64$



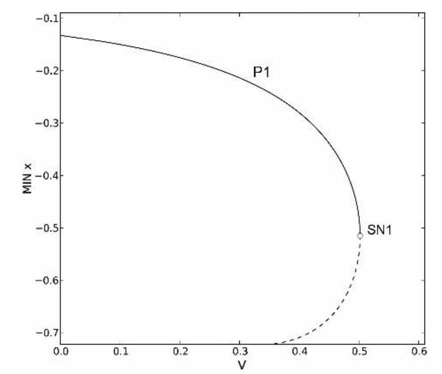
$\omega_v = 1.648$



$\omega_v = 1.672$



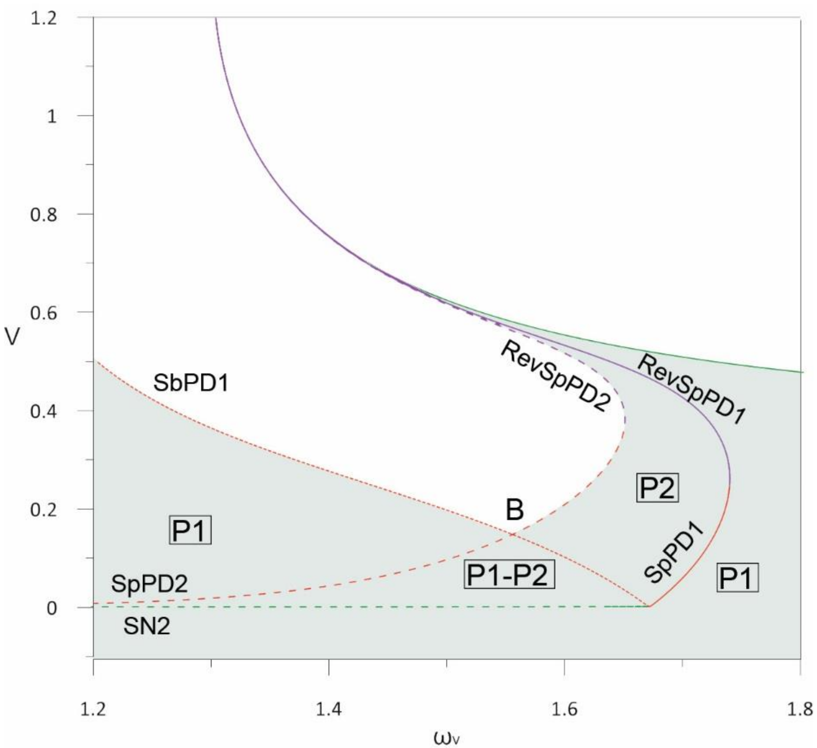
$\omega_v = 1.745$



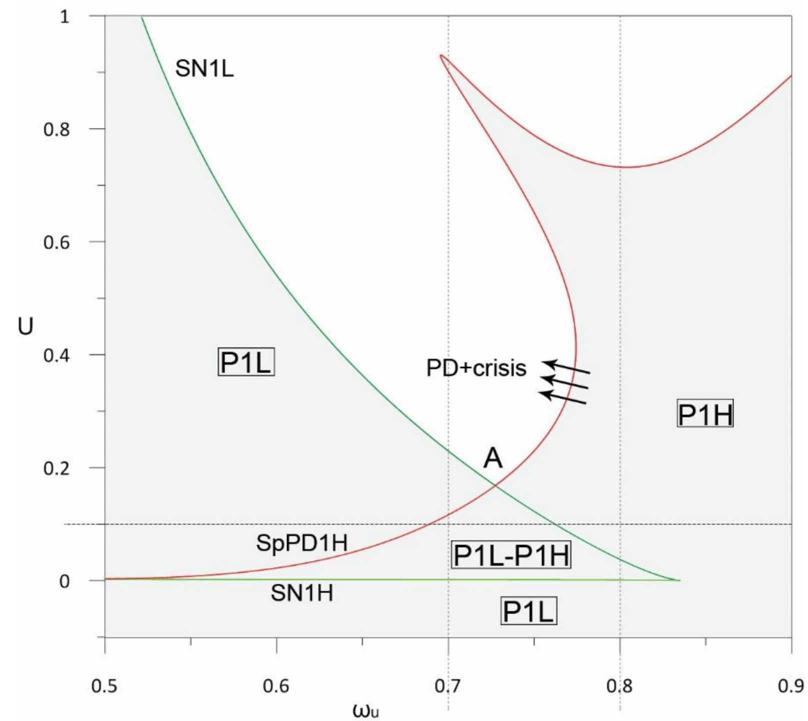
- **UNSTABLE BUBBLE** characterized by period doubling **TRANSITION** TO **IN-WELL CHAOS**
- P1 and P2 solutions regain stability through Reverse Supercritical Period Doubling bifurcations (RevSpPD)

EXTERNAL VS PARAMETRIC EXCITATION

SUBHARMONIC RESONANCE (External)



FUNDAMENTAL RESONANCE (Parametric)



- **PRIMARY (PRINCIPAL) RESONANCE**: **Dominant** for externally (parametrically) forced system → **typical** dynamical **behavior** of literature

- **SUBHARMONIC (FUNDAMENTAL) RESONANCE:** slightly **affected** by **ultrasuperharmonic** response → birth of a **period doubled superharmonic solution**

SAFETY of a nonlinear system **DEPENDS** not only on stability of its solution but also **on the UNCORRUPTED BASIN** surrounding each solution

⇩

- **ATTRACTORS** and **BASINS OF ATTRACTION** identification; **EVOLUTION** through local bifurcations and erosion profiles: subjects of **THEORETICAL** and **PRACTICAL IMPORTANCE**

- **INTEGRITY CONCEPTS:** → used to **THEORETICALLY CHECK robustness** of competing attractors and analyze **erosion** processes that bring to the **escape** from bounded regions

  → **PRACTICAL TOOLS** to validate experimental results and to furnish valuable information for **ENGINEERING DESIGN**

- **TOOLS**

  SAFE BASIN: here, union of all classical basins of attraction; transient dynamics ignored

  INTEGRITY MEASURES: **GIM, LIM, IIM, IF, AGIM** *(Soliman and Thompson, 1989; Rega and Lenci, 2005)*

  - **GLOBAL INTEGRITY MEASURE (GIM):** normalized **hyper-volume** (area in 2D) of the safe basin
  - **INTEGRITY FACTOR (IF):** normalized **radius** of the largest **hyper-sphere** (circle in 2D) entirely belonging to the safe basin

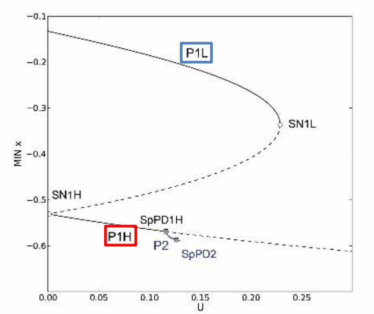
- **STEPS**

  1. **INVESTIGATION** OF **BASIN EVOLUTION** due to variation of system parameters (excitation amplitude)
  2. Integrity measure vs amplitude plot to provide the so-called **EROSION PROFILE**

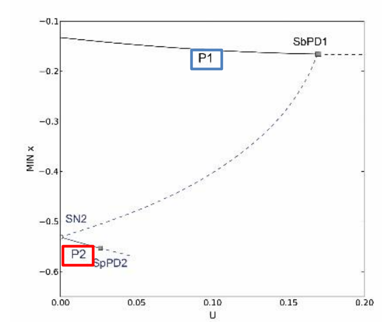
BASINS OF ATTRACTIONS AND EROSION - 1 -

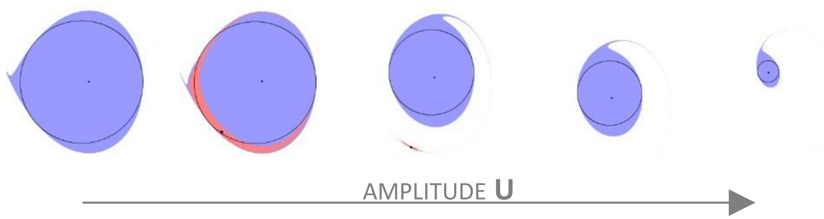
BEFORE RESONANCE

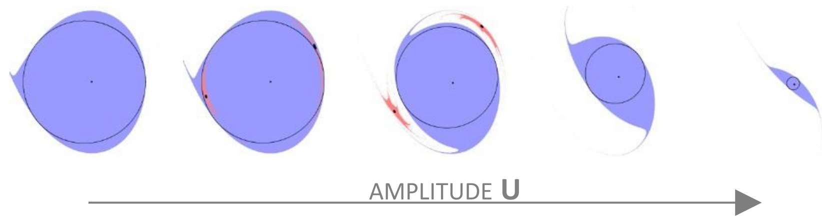
$\omega_U = 0.7$

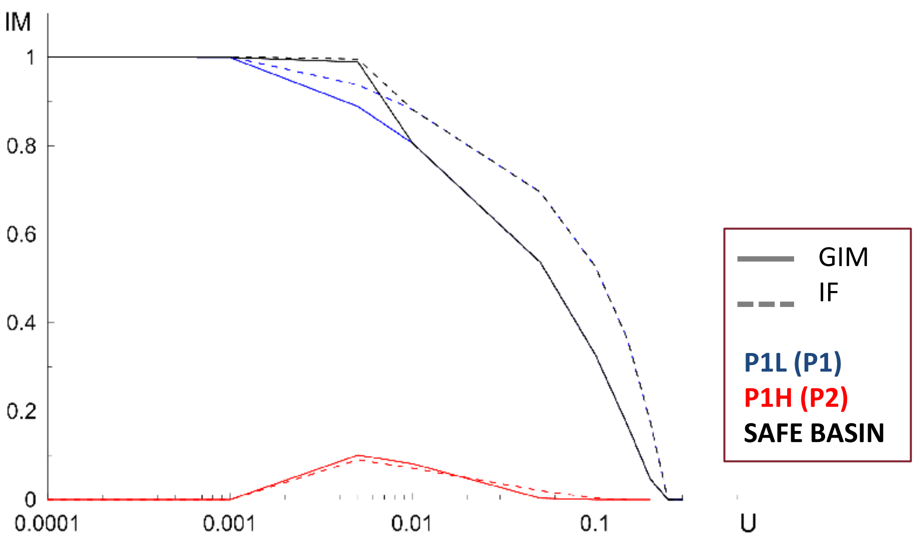


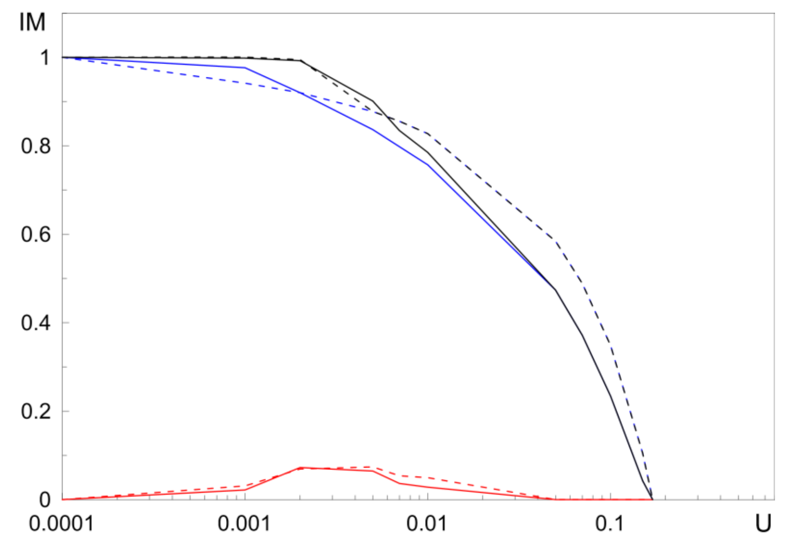
$\omega_U = 1.4$







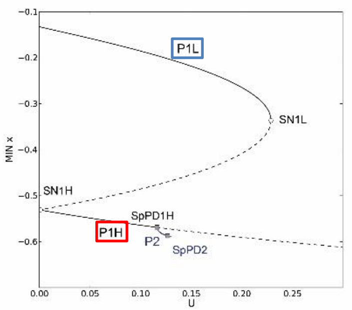




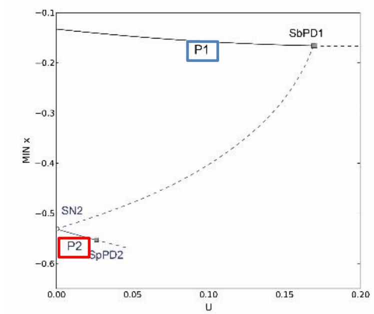
BASINS OF ATTRACTIONS AND EROSION - 1 -

BEFORE RESONANCE

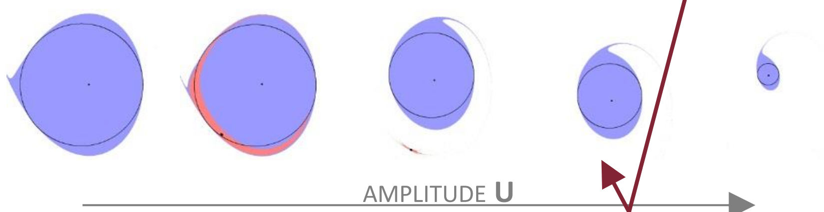
$\omega_U = 0.7$

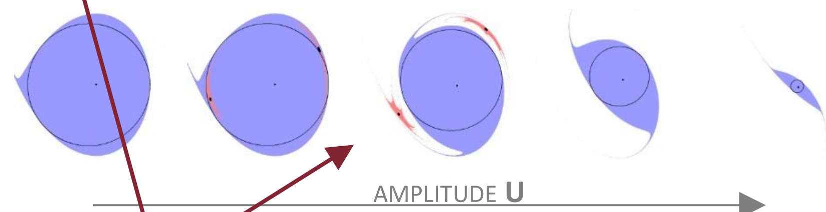


$\omega_U = 1.4$

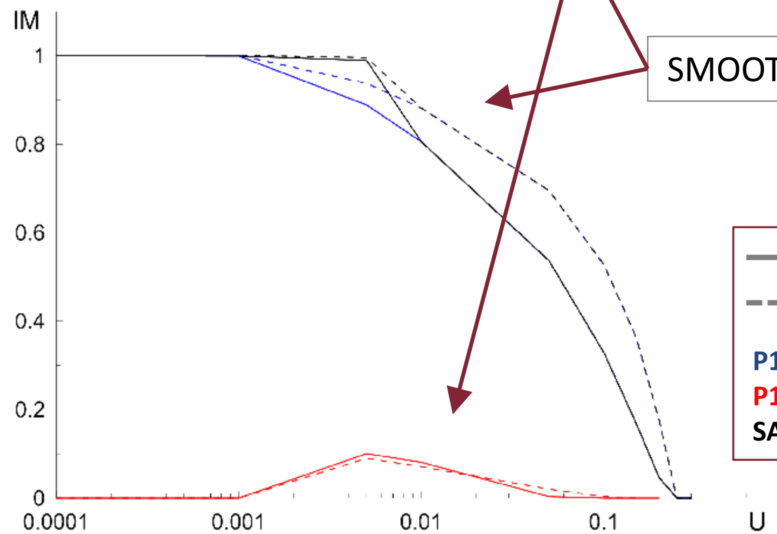


WEAK COMPETING SOLUTIONS



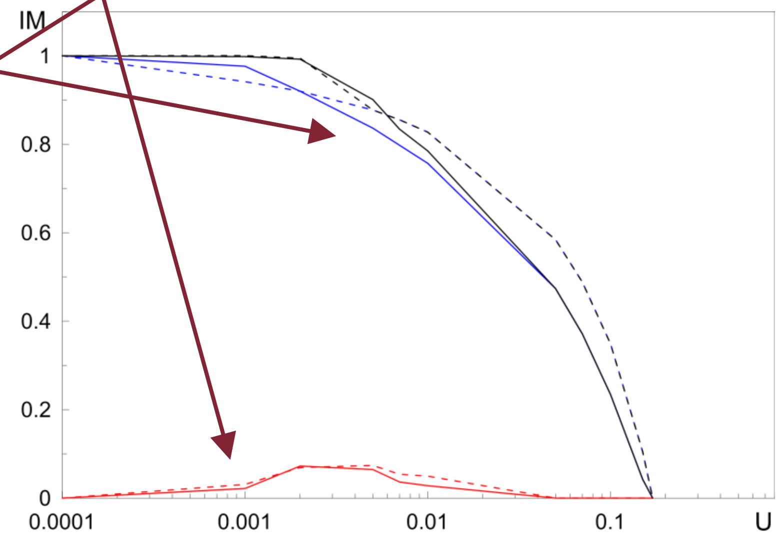


SMOOTH EROSION



GIM
IF

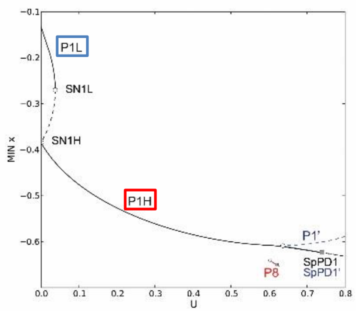
P1L (P1)
P1H (P2)
SAFE BASIN



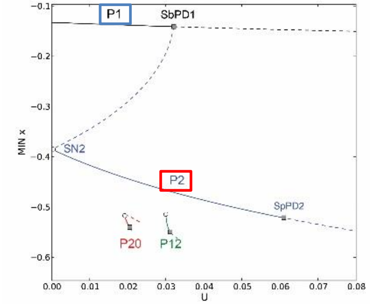
BASINS OF ATTRACTIONS AND EROSION - 2 -

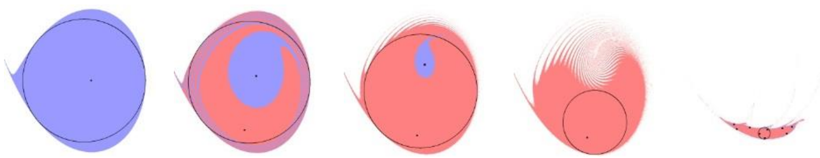
AROUND RESONANCE

$\omega_U = 0.8$

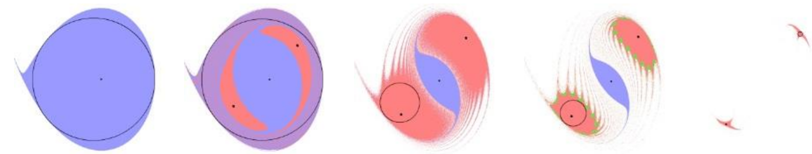


$\omega_U = 1.6$

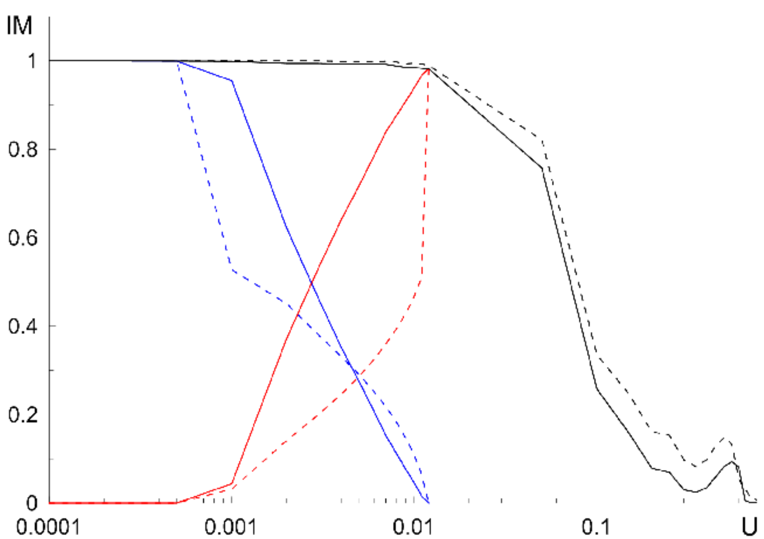




AMPLITUDE **U**

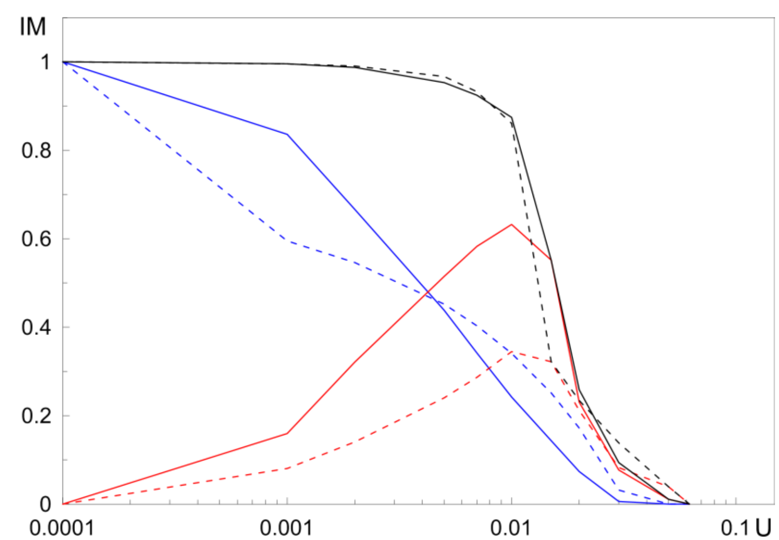


AMPLITUDE **U**



GIM
IF

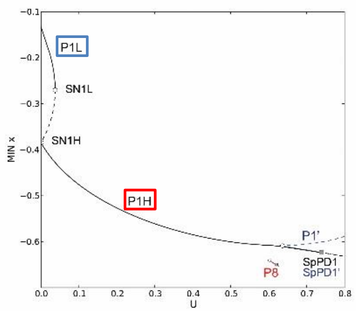
P1L (P1)
P1H (P2)
SAFE BASIN



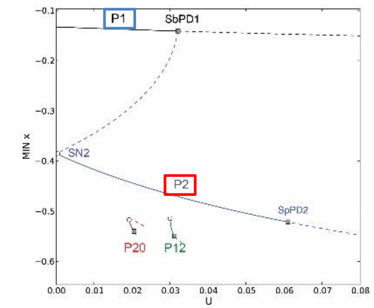
BASINS OF ATTRACTIONS AND EROSION - 2 -

AROUND RESONANCE

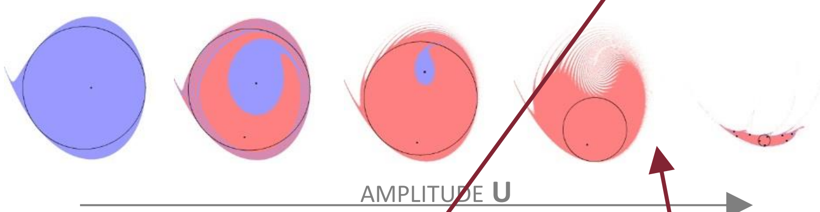
$\omega_U = 0.8$

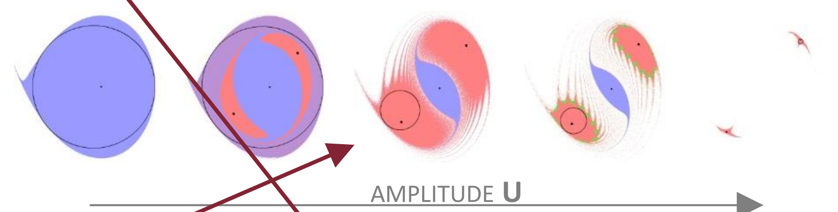


$\omega_U = 1.6$

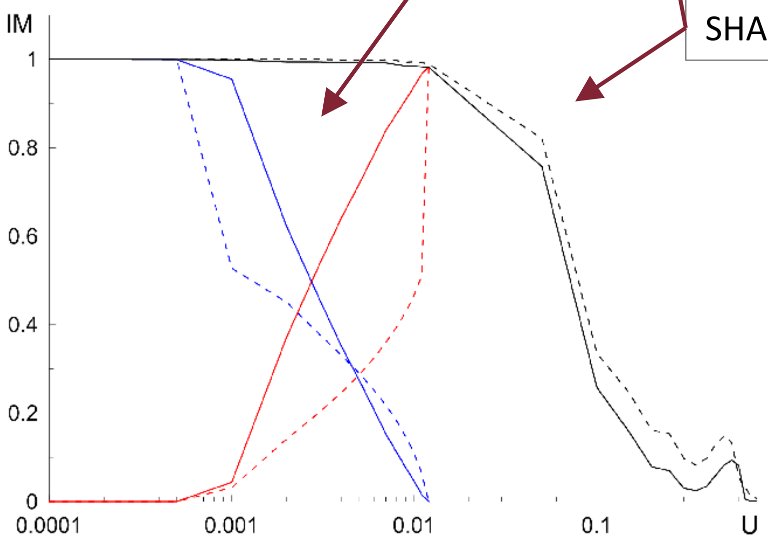


STRONG COMPETING SOLUTIONS



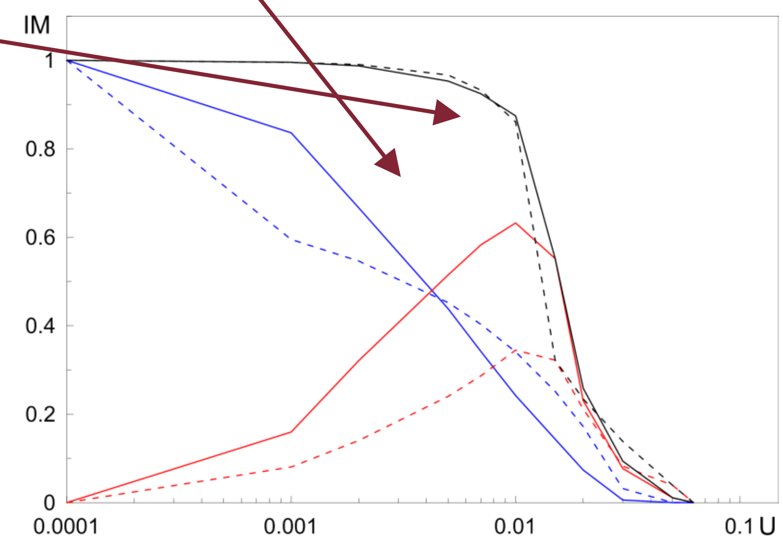


SHARP EROSION

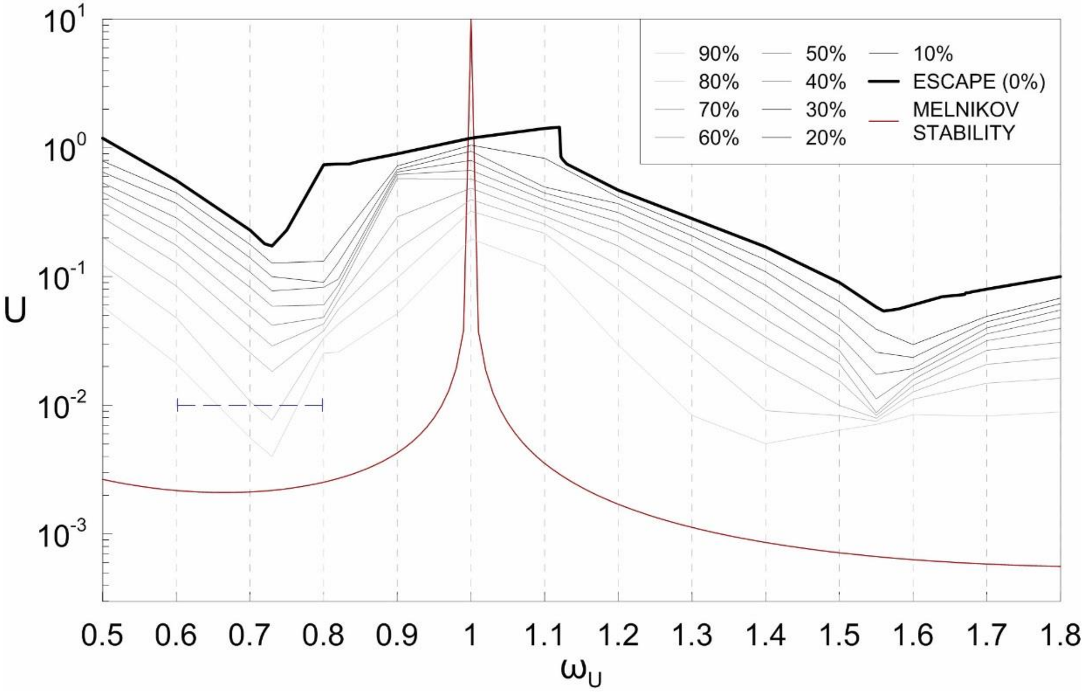


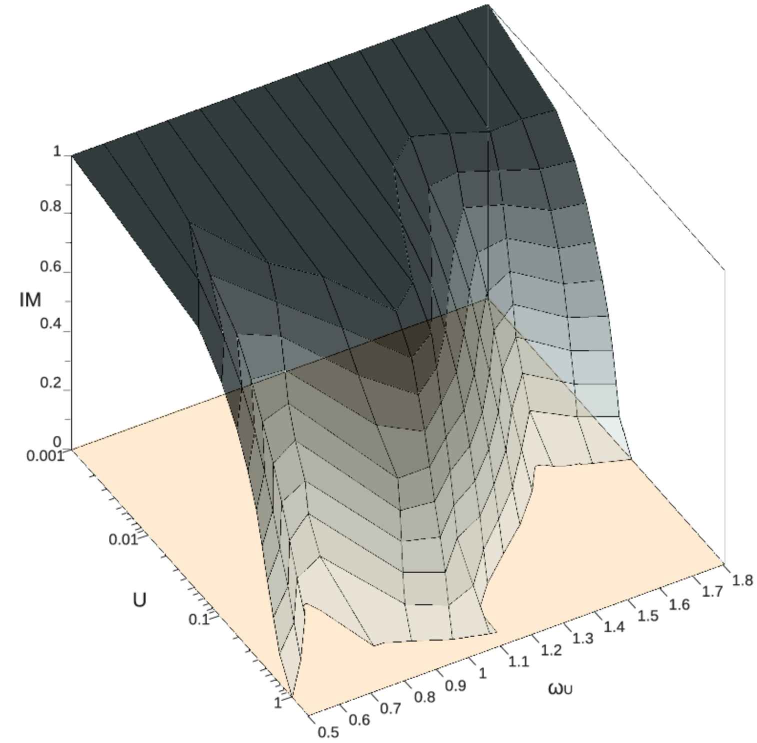
GIM
IF

P1L (P1)
P1H (P2)
SAFE BASIN



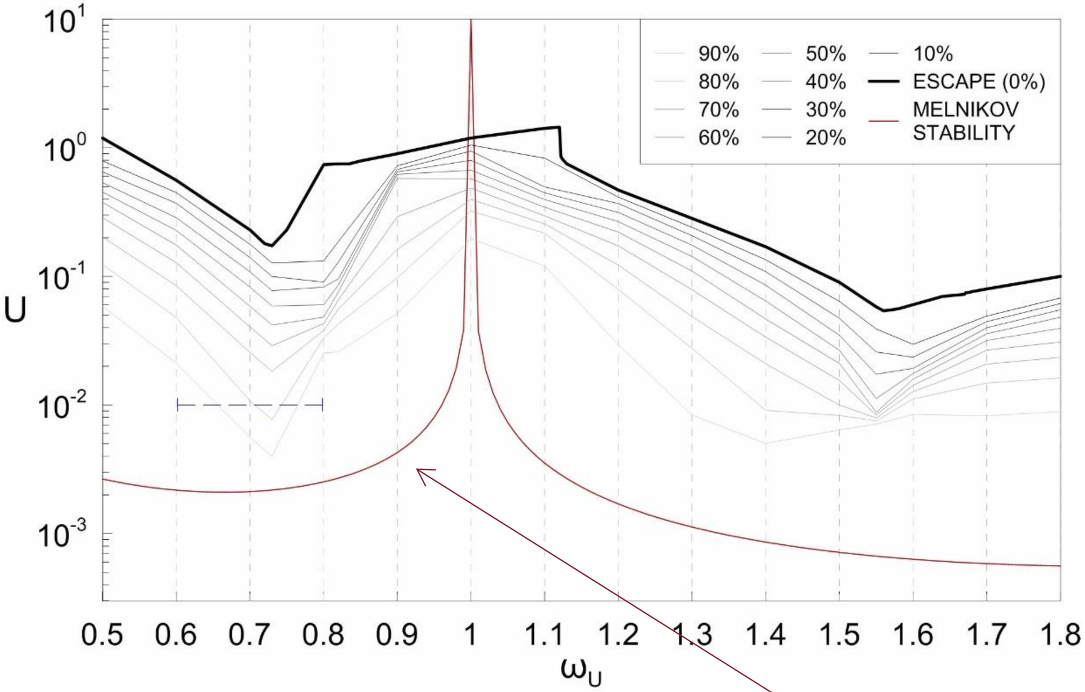
INTEGRITY CONTOUR PLOTS

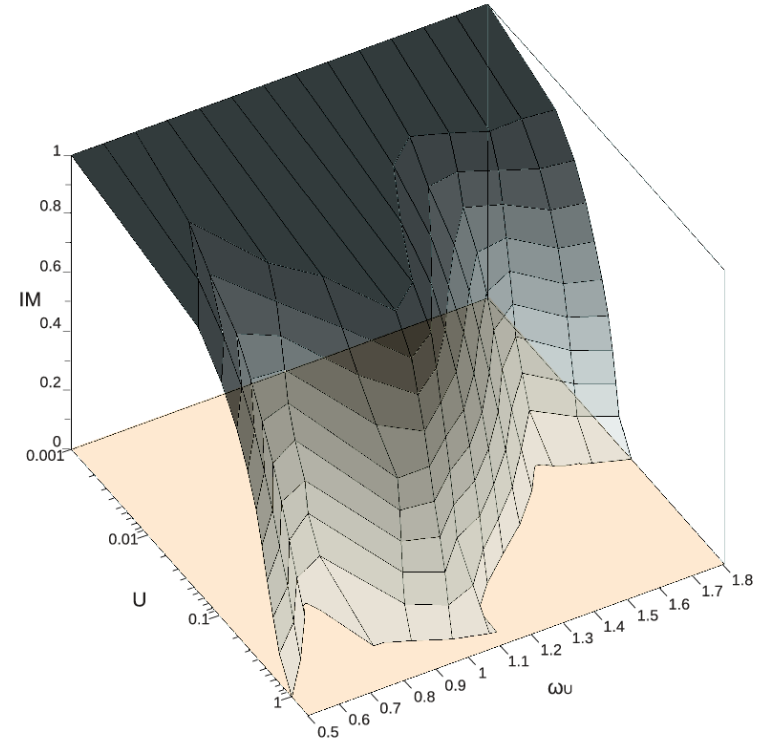




DYNAMIC INTEGRITY

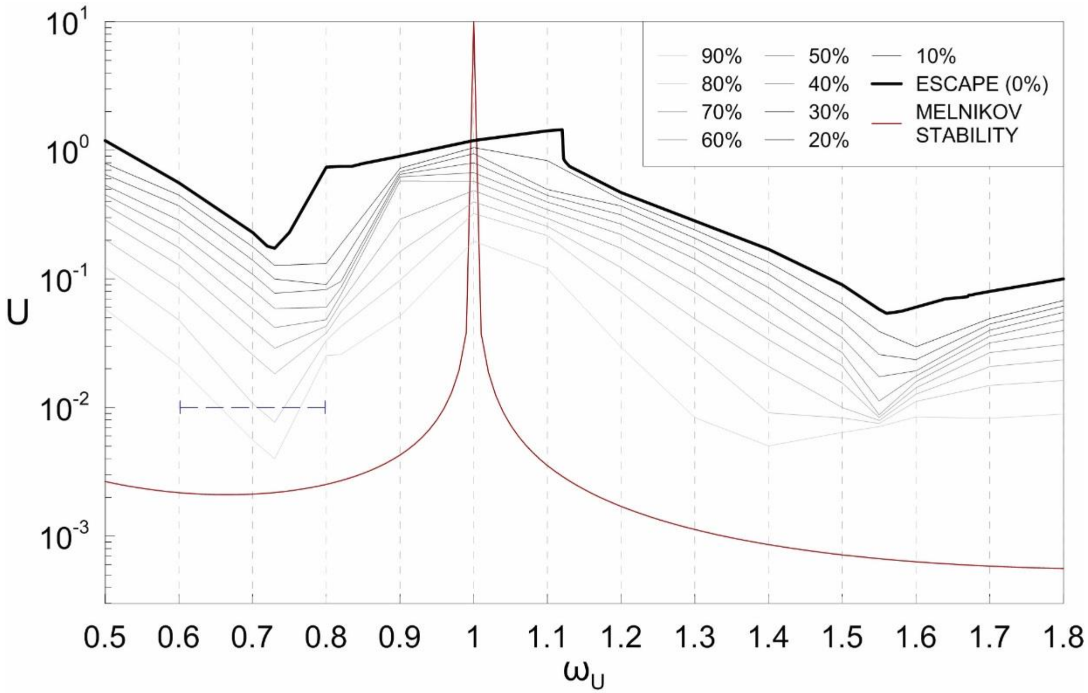
INTEGRITY CONTOUR PLOTS

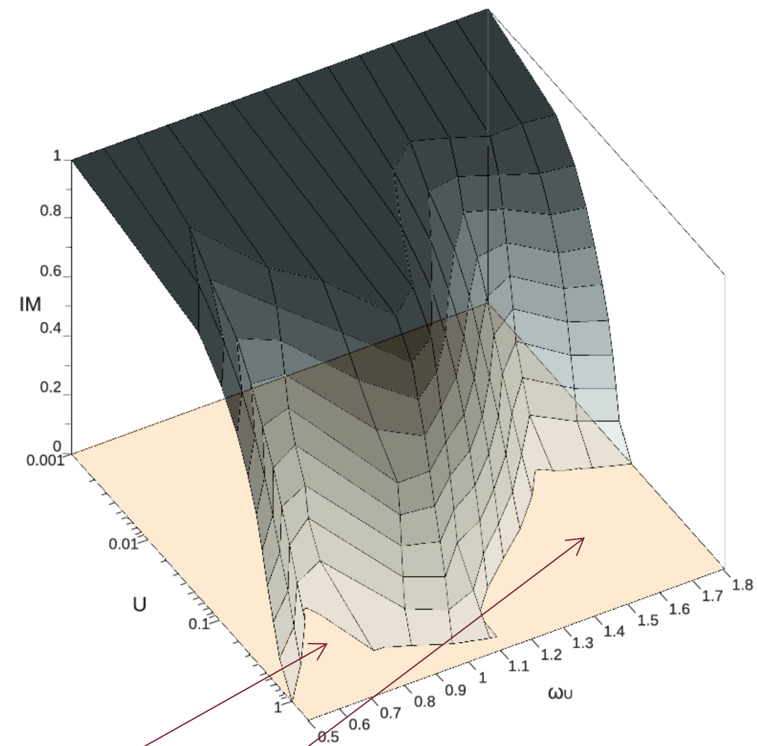




- **EROSION**: starts just **ABOVE THE HOMOCLINIC BIFURCATION** of hilltop saddle
  (GLOBAL EVENT)

INTEGRITY CONTOUR PLOTS





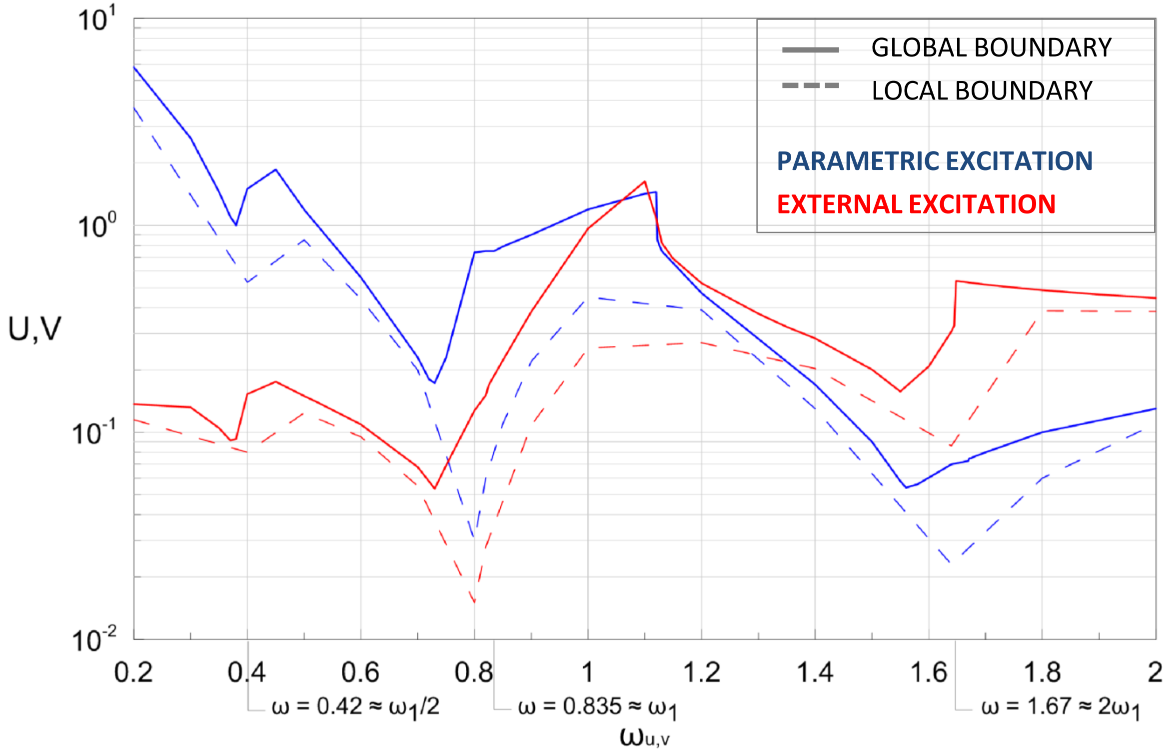
- **EROSION**: starts just **ABOVE THE HOMOCLINIC BIFURCATION** of hilltop saddle (GLOBAL EVENT)
- **DOVER CLIFF EROSION SURFACE**: slow decrease of safe basin volume, followed by sudden **fall down** to zero
- **Two depressions** near **fundamental** and **principal** resonance frequencies

THEORETICAL STABILITY BOUNDARIES - 1 -

ESCAPE: **NUMERICAL INTEGRATION** versus **BIFURCATION DIAGRAMS** threshold

"LOCAL" VS "GLOBAL" STABILITY BOUNDARY

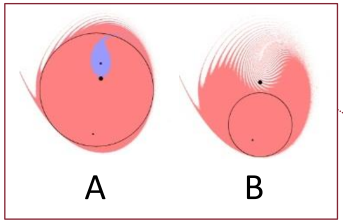
Integration from rest position (or other sets of i.c.)

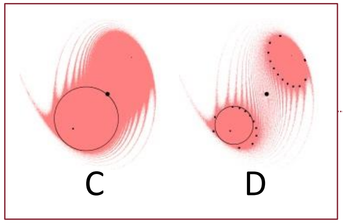


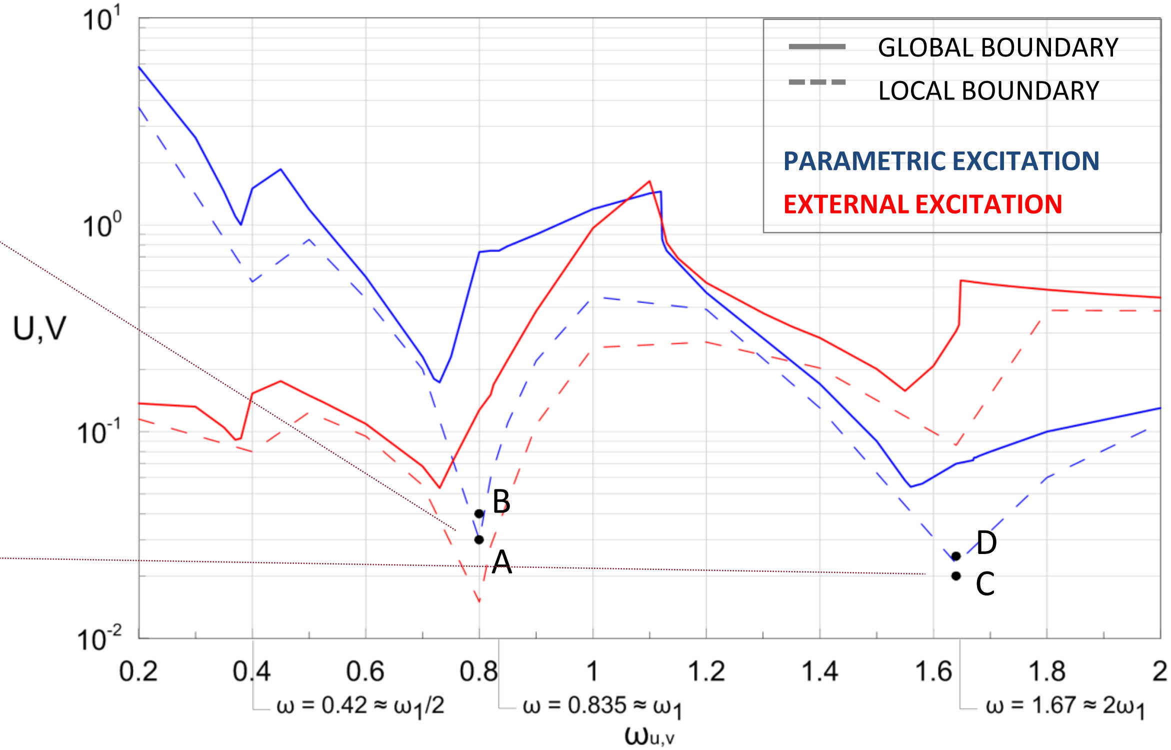
- **LOCAL ESCAPE: below GLOBAL** in whole frequency range
- **GLOBAL MINIMA**: **shifted left** due to **softening** nonlinear system

ESCAPE: NUMERICAL INTEGRATION versus **BIFURCATION DIAGRAMS** threshold

"LOCAL" VS "GLOBAL" STABILITY BOUNDARY







- **LOCAL** ESCAPE: **single trajectory** → seemingly **TOO CONSERVATIVE**
- **GLOBAL** ESCAPE: BASIN ANNIHILATION → **NO RESIDUAL SAFETY** → UPPER BOUND

.... but ....

THEORETICAL VS PRACTICAL STABILITY BOUNDARIES

INTEGRITY CONTOUR PLOTS

ACCEPTABLE

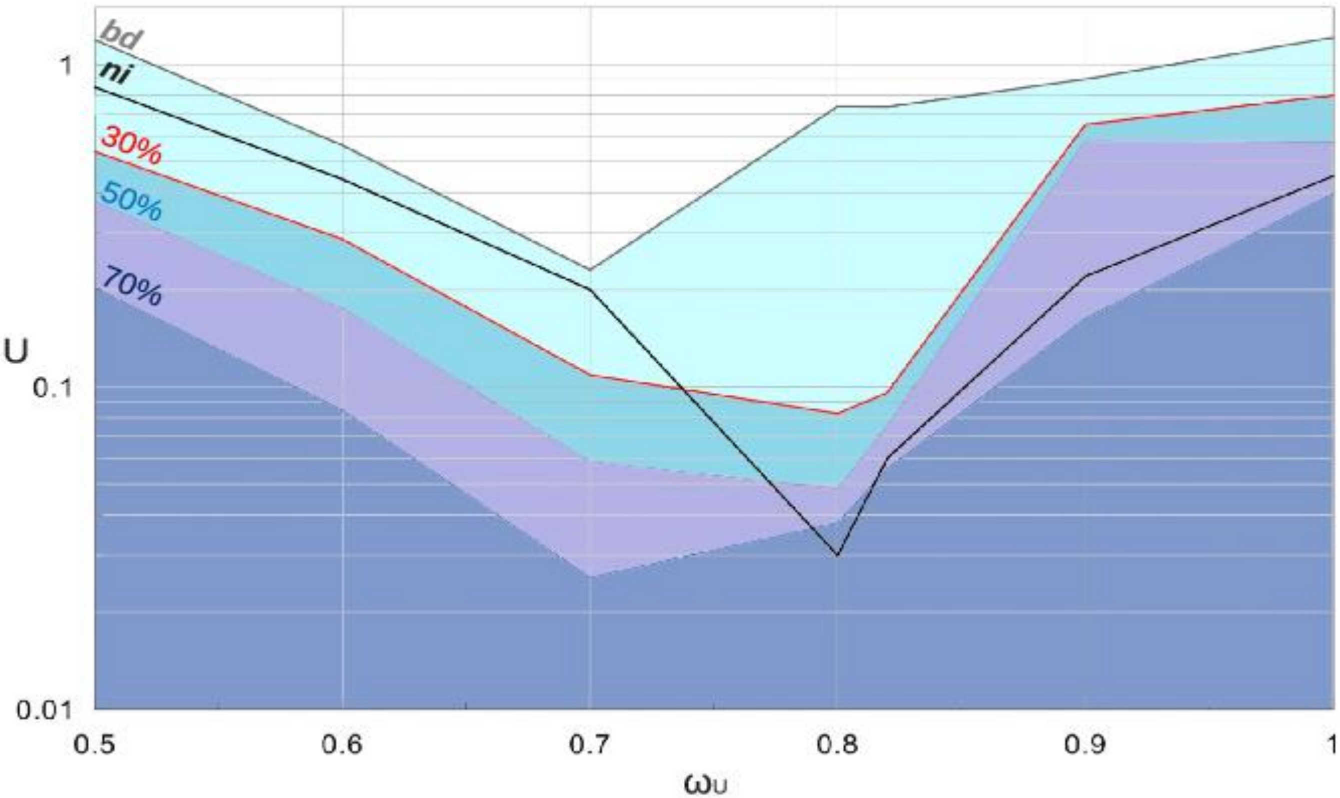
FREQUENCY-DEPENDENT

THRESHOLDS

ASSOCIATED WITH

a priori safe

design targets



LOCAL escape:

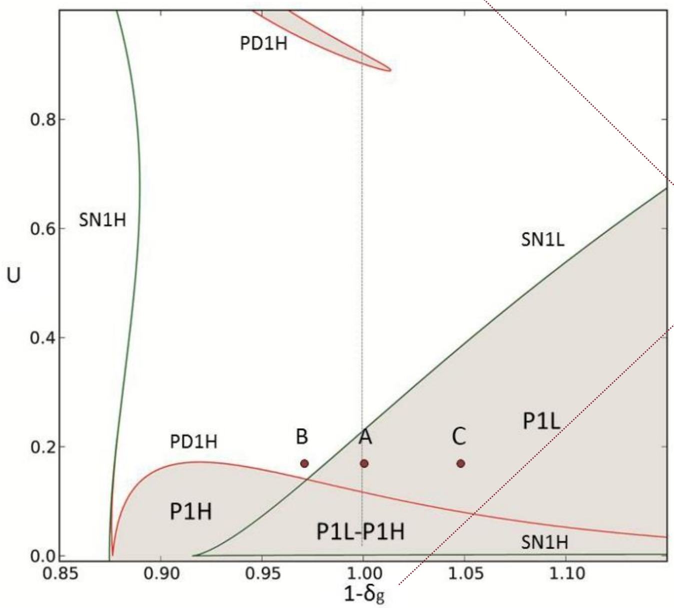
- strongly **VARIABLE RESIDUAL INTEGRITY**
- **overestimation** of system **PRACTICAL SAFETY** (0-30% of residual integrity left of resonance)

➡ need of a **GLOBAL** analysis **!!**

TOWARDS CONTROL: EFFECTS OF TIP-SAMPLE DISTANCE VARIATION - 1-

- REFERENCE SYSTEM : **confined UNSTABLE** REGIONS
- AFM DYNAMICS: strongly related to **TIP – SAMPLE DISTANCE**

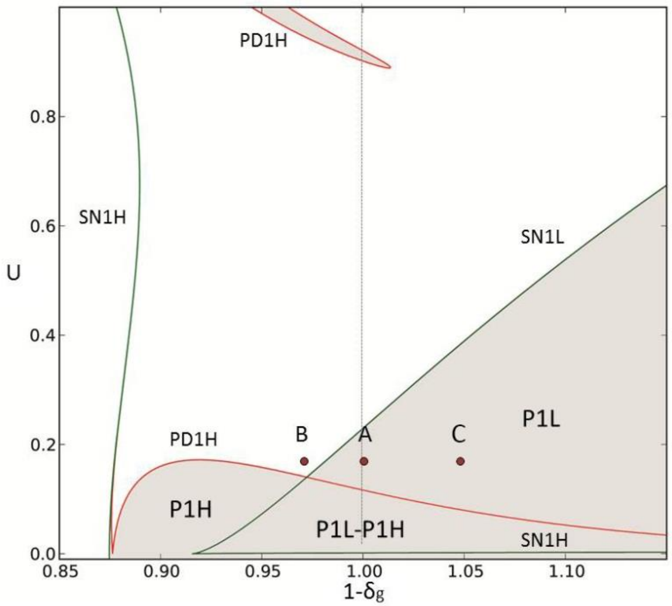
$$\ddot{x}+\alpha_1 x+\alpha_3 x^3=-\Gamma_1\left(1-\boxed{\delta_g}+x\right)^{-2}-\rho_1\dot{x}-x\mu_1 U\omega_u^2\sin(\omega_u t)$$



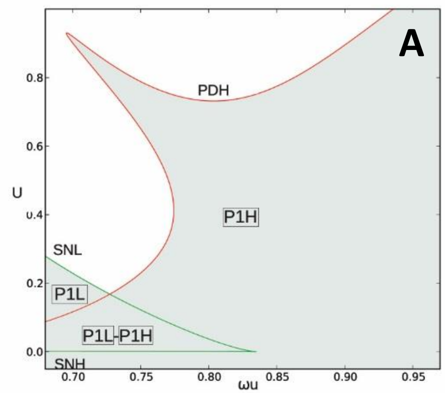
NONDIMENSIONAL REDUCTION RATIO OF TIP-SAMPLE GAP

- REFERENCE SYSTEM : **confined UNSTABLE** REGIONS
- AFM DYNAMICS: strongly related to **TIP – SAMPLE DISTANCE**

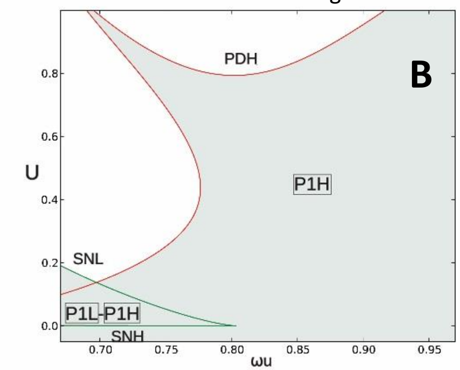
$$\ddot{x}+\alpha_1 x+\alpha_3 x^3=-\Gamma_1\left(1-\boxed{\delta_g}+x\right)^{-2}-\rho_1\dot{x}-x\mu_1 U\omega_u^2\sin(\omega_u t)$$



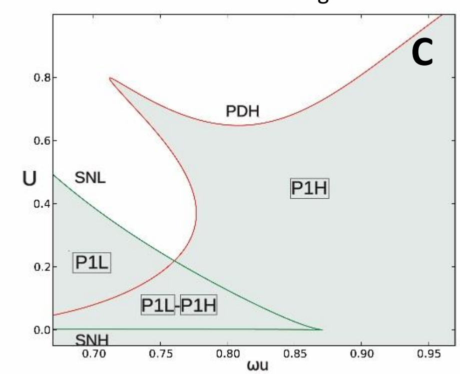
$\delta_g=0$



$\delta_g=0.03$



$\delta_g=-0.05$



Slight VARIATIONS of TIP-SAMPLE DISTANCE: → **critical CHANGES** in system DYNAMICS

→ **ERRORS** in sample topography

⇩

AFM: IMPORTANCE OF MOTION CONTROL TO AVOID UNSTABLE/CHAOTIC RESPONSE

CONTROL OF COMPLEX RESPONSE

CHAOS CONTROL

STARTING POINT: Ott, Grebogi, Yorke (1990) → OGY METHOD
Pyragas (1992) → EXTERNAL feedback control and DELAYED FEEDBACK CONTROL techniques

LAST TWO DECADES: **several control techniques**

- **STABILIZE** an **UNSTABLE ZONE** of parameter space
- **STABILIZE** a given, erratic **SOLUTION** (OGY method and its revisions/improvements)
- **OVERALL REGULARIZE** system **DYNAMICS**, irrespective of single solution behavior

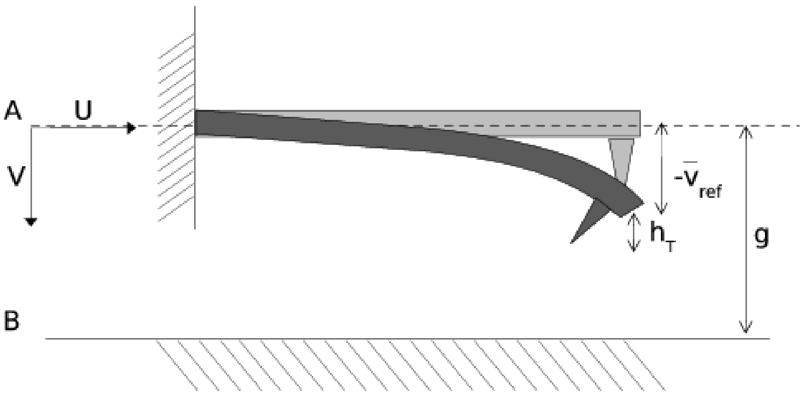
CONTROL IN AFM : Techniques based mostly on **FEEDBACK CONTROL**:

- state feedback control based on Melnikov function
- positive feedback system
- inversion based feedback/feedforward control
- nonlinear delayed feedback control
- **external feedback control** of **Tapping AFM** - *Yagasaki (2010)*
- Here: **external feedback control** of **noncontact AFM** *(with O. Gottlieb)*

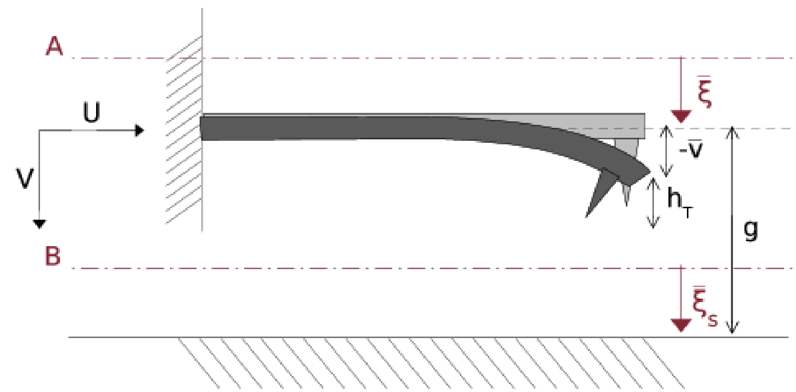
OBJECTIVE: **keep the cantilever vibration to a reference one** and simultaneously measure the sample surface (a **“local” control**)

PERIODIC REFERENCE RESPONSE of **uncontrolled system**

REFERENCE CONFIGURATION



ACTUAL CONFIGURATION



NEW VARIABLE $\xi(t)$: distance of the cantilever fixed side from the horizontal reference axis

NEW PARAMETER ξ_s : displacement of the sample surface from the selected reference position

- **1 NEW D.O.F.** to the general relations of the uncontrolled system:

$$m\bar{u}_{tt}-[EI\bar{v}_{rrr}\bar{v}_r-J_z\bar{v}_{ttr}\bar{v}_r+\lambda(1+\bar{u}_r)]_r=\bar{Q}_u$$
$$m\bar{v}_{tt}-[-EI(\bar{v}_{rrr}+\bar{v}_r\bar{v}_{rr}^2)+J_z(\bar{v}_{ttr}+\bar{v}_{tr}^2\bar{v}_r)+\lambda\bar{v}_r]_r=\bar{Q}_v$$
$$\bar{\xi}_t=\bar{k}\left(\bar{v}_{ref}-\bar{v}\right)$$

+

BOUNDARY CONDITIONS:

$$\bar{v}(0,t)=\bar{V}(t)+\bar{\xi}(t)=\overline{W}(t),\quad \bar{v}_{rrr}(L,t)=0,$$
$$\bar{v}_r(0,t)=0,\quad \bar{u}(0,t)=\bar{U}(t),$$
$$\bar{v}_{rr}(L,t)=0,\quad \bar{u}_r(L,t)=0.$$

MODIFIED INTERACTION FORCE: $F_v^A=\dfrac{A_H R_T}{6\sigma_a{}^2}\left[-\left(\dfrac{\sigma_a}{g+\bar{v}-h_T-\bar{\xi}_s}\right)^2+\dfrac{1}{30}\left(\dfrac{\sigma_a}{g+\bar{v}-h_T-\bar{\xi}_s}\right)^8\right]$

- NONDIMENSIONAL HOMOGENEOUS BOUNDARY SYSTEM

$$w_{\tau\tau}+W_{\tau\tau}+w_{ssss}-\mu w_{\tau\tau ss}-Q_w=$$
$$\left[-w_s(w_{ss}w_s)_s+\mu w_s(w_{\tau s}w_s)_\tau+w_s\left(\left(1+\frac{1}{2}w_s{}^2\right)\int_1^s U_{\tau\tau}ds-\frac{1}{2}\int_1^s\left(\int_0^s w_s{}^2ds\right)_{\tau\tau}ds\right)-\left(1+\frac{1}{2}w_s{}^2\right)\int_1^s Q_u ds\right]_s$$
$$\xi_\tau=k\left(w_{ref}-w\right)$$

WHERE $Q_w=\delta(s-\alpha)\bar{\Gamma}_1\left[\dfrac{1}{(\gamma+w+W-\xi_s)^2}+\dfrac{\bar{\Gamma}_2}{(\gamma+w+W-\xi_s)^8}\right]-\nu\left(w_\tau+B_\tau\right)$

- **1 NEW D.O.F.** to the general relations of the uncontrolled system:

$$m\bar{u}_{tt} - [EI\bar{v}_{rrr}\bar{v}_r - J_z\bar{v}_{ttr}\bar{v}_r + \lambda(1+\bar{u}_r)]_r = \bar{Q}_u$$

$$m\bar{v}_{tt} - [-EI(\bar{v}_{rrr} + \bar{v}_r\bar{v}_{rr}^2) + J_z(\bar{v}_{ttr} + \bar{v}_{tr}^2\bar{v}_r) + \lambda\bar{v}_r]_r = \bar{Q}_v$$

$$\bar{\xi}_t = \bar{k}(\bar{v}_{ref} - \bar{v})$$

FEEDBACK CONSTANT

REFERENCE DISPLACEMENT

+

BOUNDARY CONDITIONS:

$$\bar{v}(0,t) = \bar{V}(t) + \bar{\xi}(t) = \bar{W}(t), \quad \bar{v}_{rrr}(L,t) = 0,$$

$$\bar{v}_r(0,t) = 0, \quad \bar{u}(0,t) = \bar{U}(t),$$

$$\bar{v}_{rr}(L,t) = 0, \quad \bar{u}_r(L,t) = 0.$$

MODIFIED INTERACTION FORCE:

$$F_v^A = \frac{A_H R_T}{6\sigma_a^2}\left[-\left(\frac{\sigma_a}{g + \bar{v} - h_T - \bar{\xi}_s}\right)^2 + \frac{1}{30}\left(\frac{\sigma_a}{g + \bar{v} - h_T - \bar{\xi}_s}\right)^8\right]$$

- NONDIMENSIONAL HOMOGENEOUS BOUNDARY SYSTEM

$$w_{\tau\tau} + W_{\tau\tau} + w_{ssss} - \mu w_{\tau\tau ss} - Q_w =$$

$$\left[-w_s(w_{ss}w_s)_s + \mu w_s(w_{\tau s}w_s)_\tau + w_s\left(\left(1+\frac{1}{2}w_s^2\right)\int_1^s U_{\tau\tau}ds - \frac{1}{2}\int_1^s\left(\int_0^s w_s^2 ds\right)_{\tau\tau} ds\right) - \left(1+\frac{1}{2}w_s^2\right)\int_1^s Q_u ds\right]_s$$

$$\xi_\tau = k\left(w_{ref} - w\right)$$

WHERE

$$Q_w = \delta(s-\alpha)\bar{\Gamma}_1\left[\frac{1}{(\gamma + w + W - \xi_s)^2} + \frac{\bar{\Gamma}_2}{(\gamma + w + W - \xi_s)^8}\right] - \nu\left(w_\tau + B_\tau\right)$$

- MODAL DYNAMICAL SYSTEM

$$w(s,\tau) = q_1(\tau) \cdot \Phi_1(s)$$

$$w_{ref}(s,\tau) = q_{ref\,1}(\tau) \cdot \Phi_1(s)$$

- GALERKIN PROCEDURE + NONDIMENSIONAL FORM

ATOMIC INTERACTION

$$\ddot{x}\left(1+\alpha_2 x^2\right) + \alpha_1 x + \alpha_2 x\dot{x}^2 + \alpha_3 x^3 = -\Gamma_1\left(1 + x + V_g + z - z_s\right)^{-2} - \left(\rho_1 + \rho_2 x^2\right)\dot{x}$$
$$-\left(\ddot{V}_g + k_g\left(\dot{x}_{ref} - \dot{x}\right) + \nu_1\left(\dot{V}_g + k_g\left(x_{ref} - x\right)\right)\right)\nu_2 + \left(x\mu_1 + \mu_2 x^3\right)\left(\ddot{U}_g + \eta_1 \dot{U}_g + \eta_2 U_g\right)$$

$$\dot{z} = k_g\left(x_{ref} - x\right)$$

z = new control variable

EXTERNAL EXCITATION

PERIODIC REFERENCE RESPONSE $x_{ref} = \bar{x}_{ref} + \tilde{x}_{ref}(t)$

MEAN COMPONENT

TIME-DEPENDENT OSCILLATING COMPONENT

- **EQUILIBRIUM SOLUTIONS**

$$\tilde{x}_{ref}(t)=0 \rightarrow x_{ref}=\overline{x}_{ref} \qquad \dot{x}=\ddot{x}=\dot{z}=0 \quad \Rightarrow \quad x=\overline{x}_{ref}, \quad z=z_s \quad \Rightarrow \quad \alpha_1\overline{x}_{ref}+\alpha_3\overline{x}_{ref}^{\;3}+\Gamma_1\left(\frac{1}{1+\overline{x}_{ref}}\right)^2=0$$

expected value of control variable

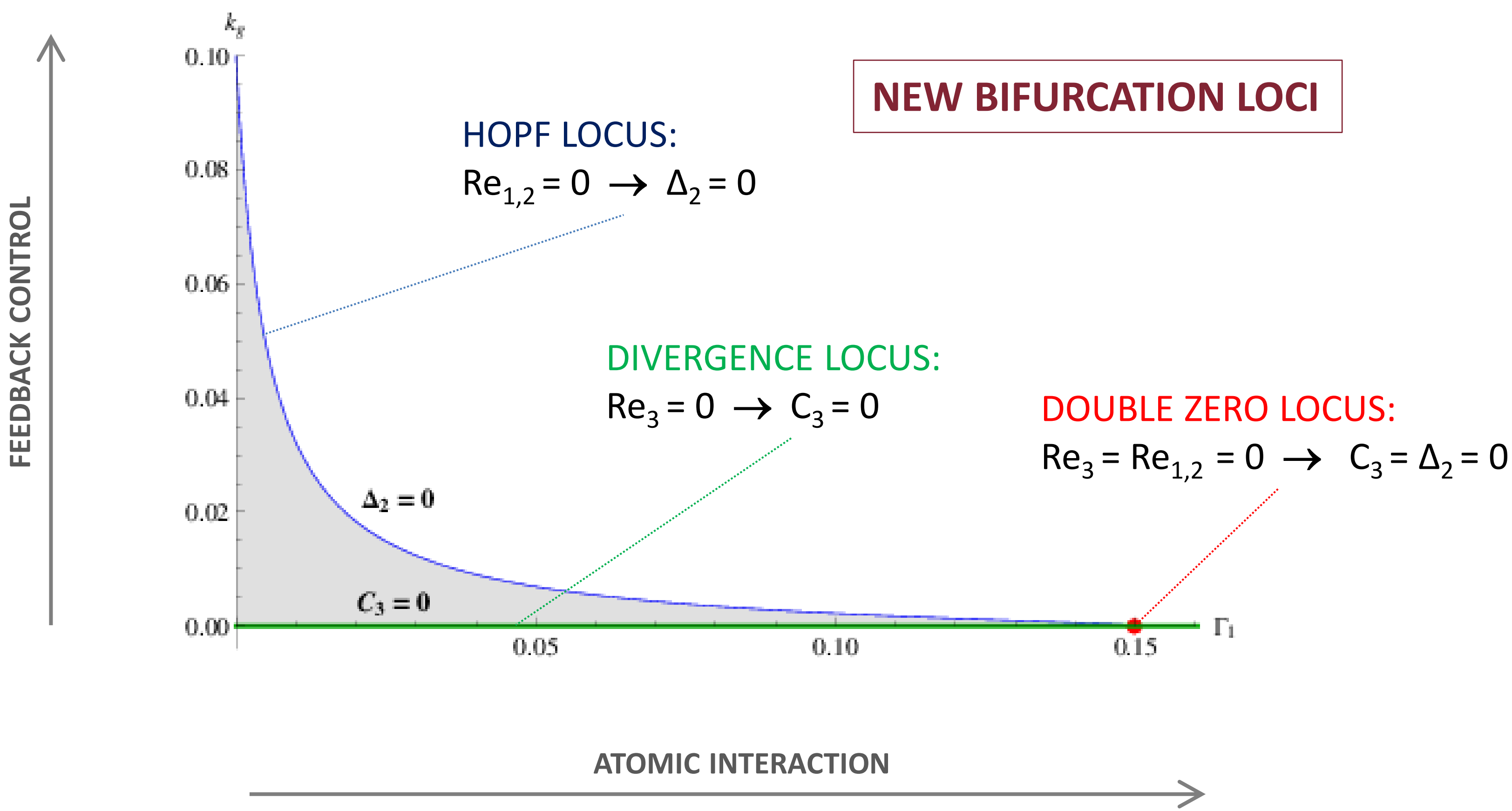
system equilibria not influenced by feedback control parameter k_g

- **STABILITY OF EQUILIBRIUM SOLUTIONS**

JACOBIAN MATRIX
$$J=\begin{bmatrix} 0 & 1 & 0 \\ a_{21} & a_{22} & a_{23} \\ k_g & 0 & 0 \end{bmatrix} \qquad a_{21}=f\left(\Gamma_1,k_g\right), \quad a_{22}=f\left(k_g\right), \quad a_{23}=f\left(\Gamma_1\right)$$

CHARACTERISTIC POLYNOMIAL
$$p_J\left(\lambda\right)=\lambda^3+C_1\lambda^2+C_2\lambda+C_3=0 \;\rightarrow\; \lambda_{1,2}=\mathrm{Re}_{1,2}\pm i\,\mathrm{Im}_{1,2}, \quad \lambda_3=\mathrm{Re}_3$$

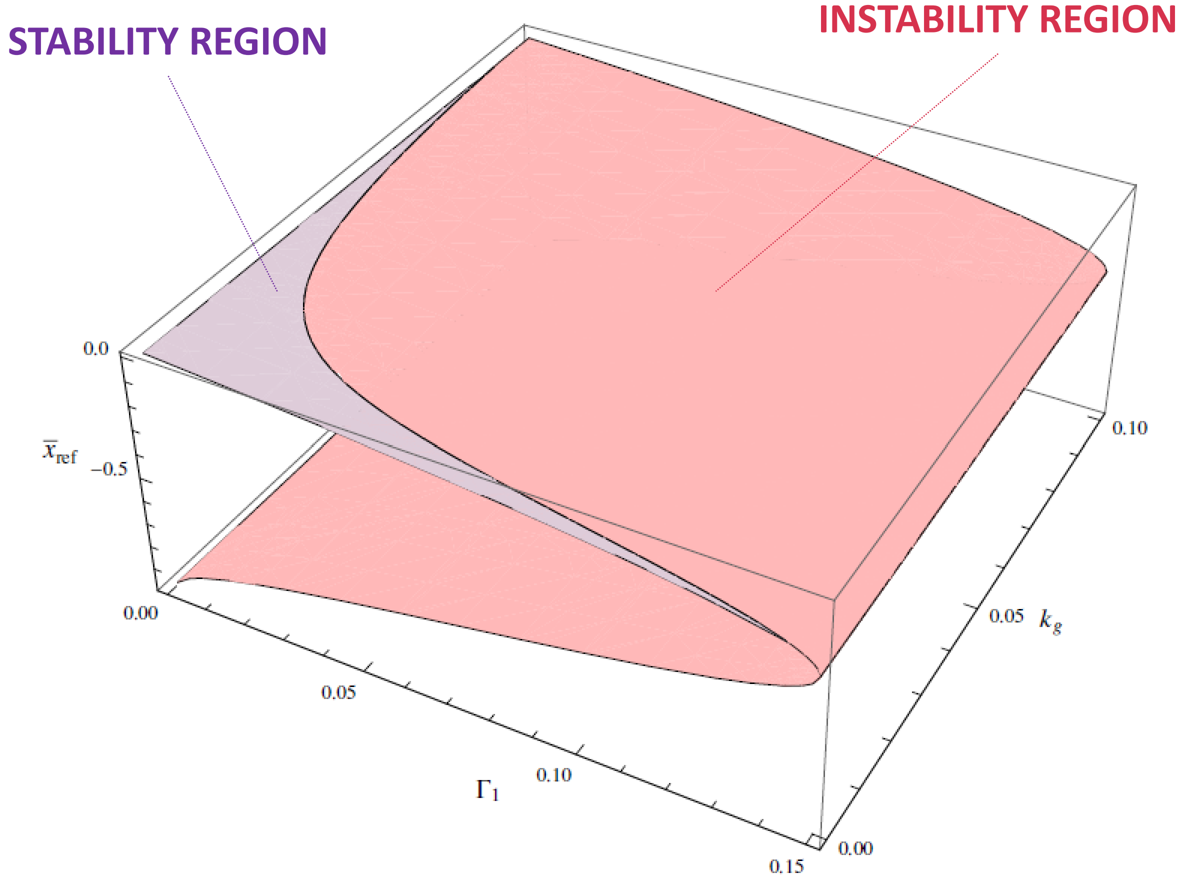
ASYMPTOTICAL STABILITY :
$$\begin{cases} C_i>0, \quad i=1,2,3 \\ \Delta_2=C_1C_2-C_3>0 \end{cases}$$



$$\mu_2 = \rho_2 = \eta_1 = \eta_2 = V_g = \alpha_2 = 0 \qquad \alpha_1 = 1 \quad \alpha_3 = 0.1 \qquad \omega_1 = 0.8325$$

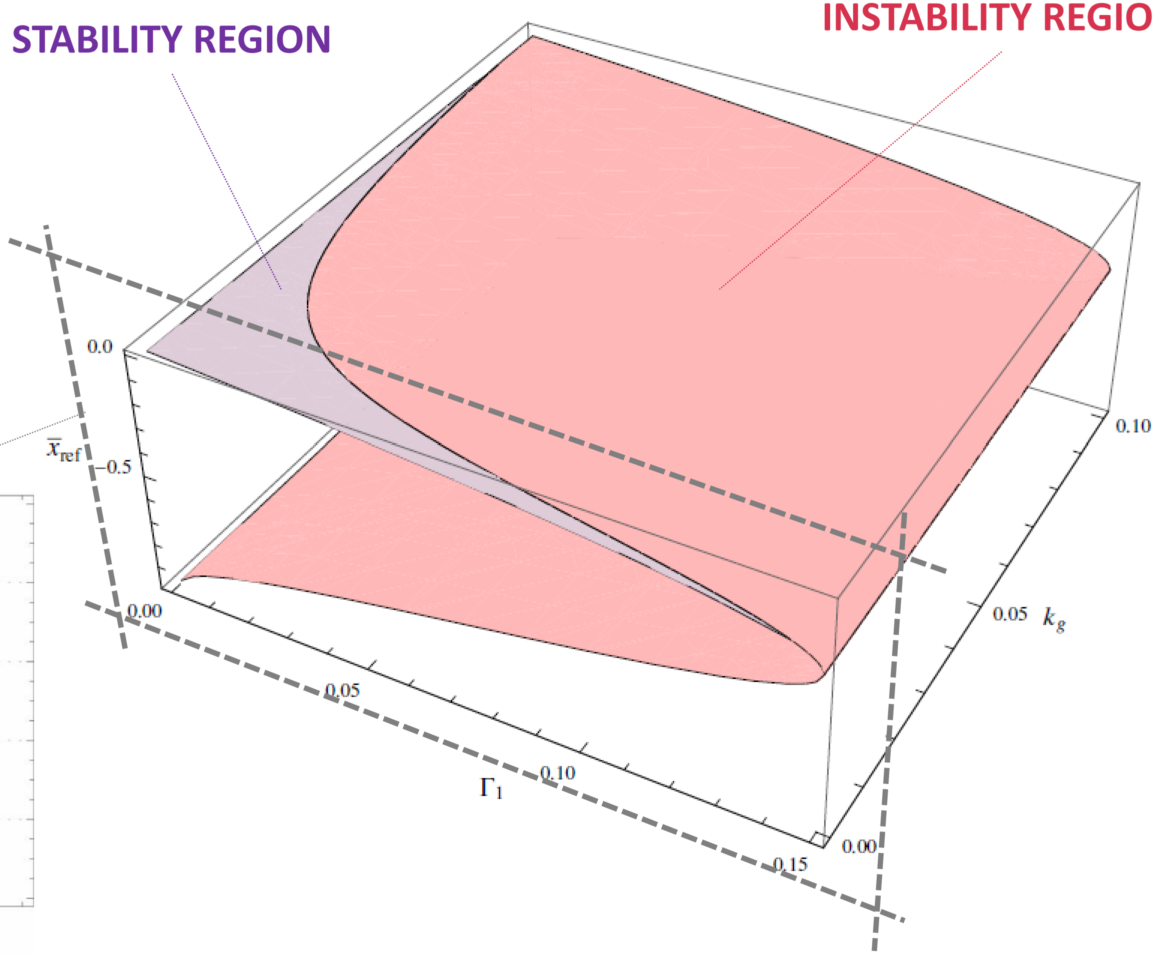
$$\nu_1 = 0.01 \quad \nu_2 = 0.01 \quad \mu_1 = 1.5708 \quad z_s = 0.01 \qquad U_g = 0.0001$$

EQUILIBRIUM ANALYSIS AND STABILITY - 3 -

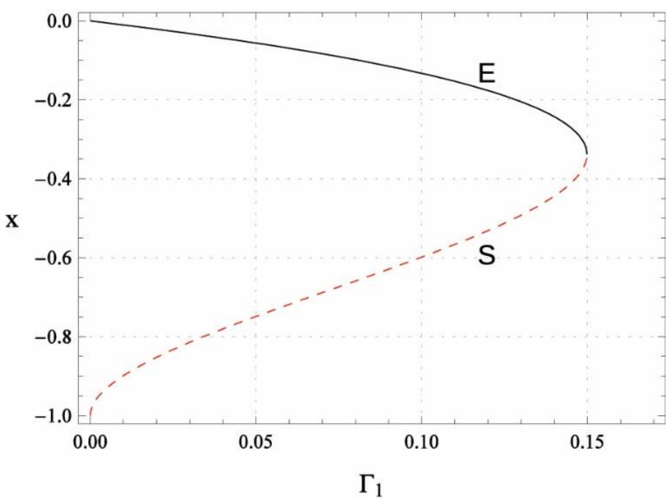


STABILITY REGION

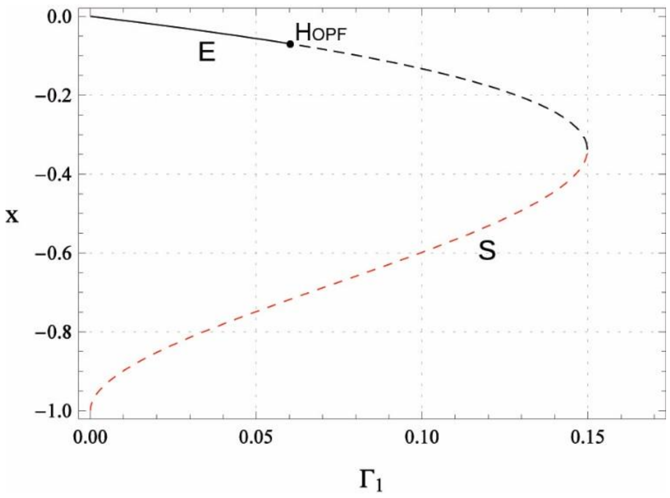
INSTABILITY REGION

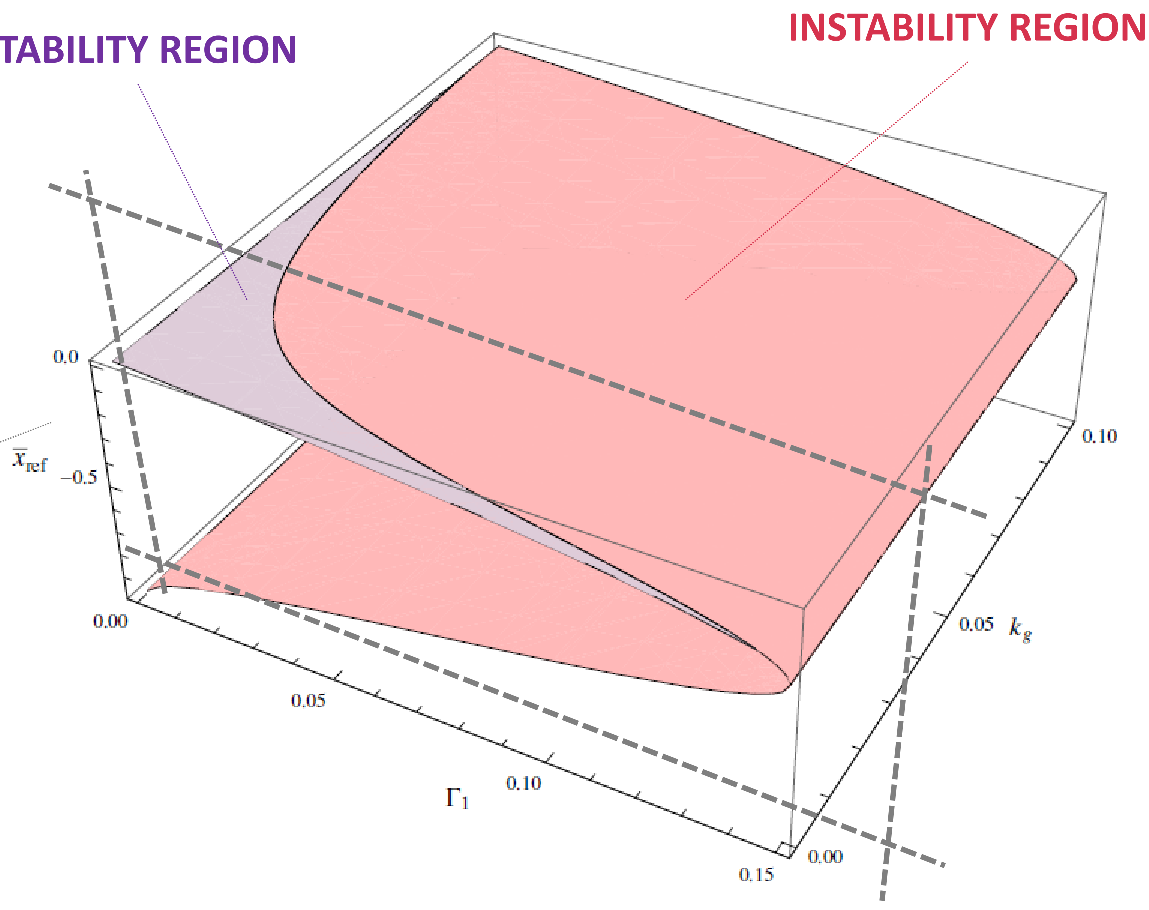


k_g = 0:
UNCONTROLLED SYSTEM



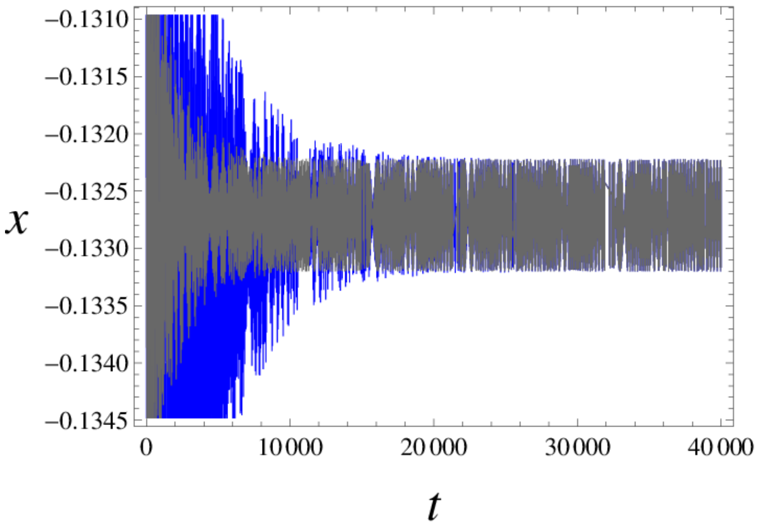
$k_g = 0.005$:
CONTROLLED SYSTEM

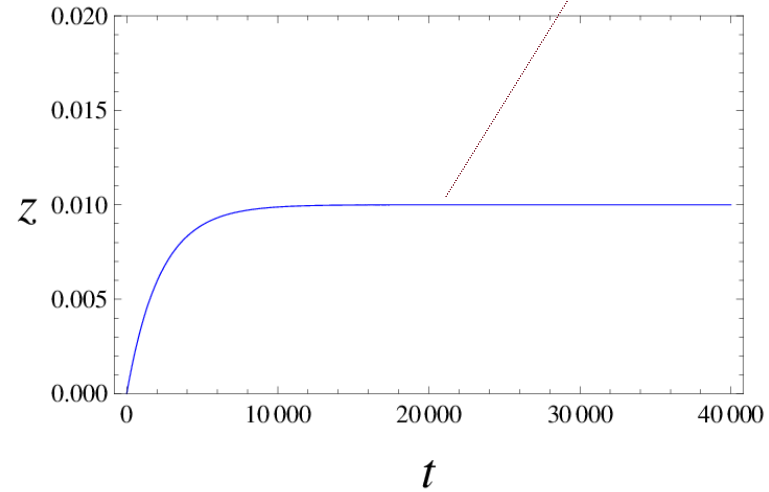


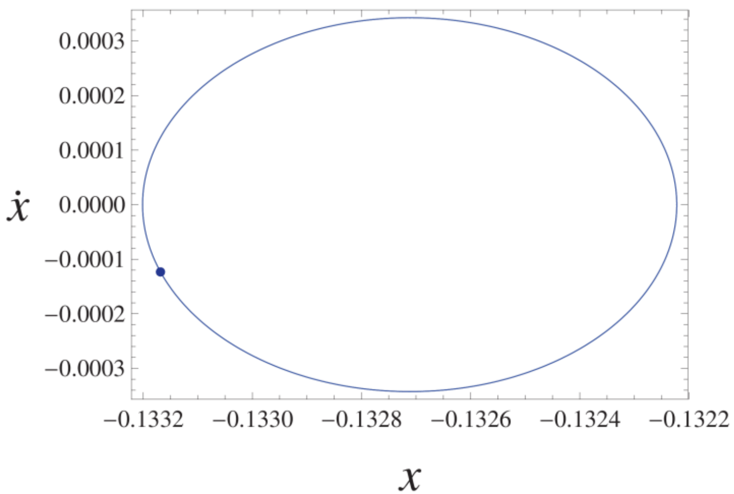


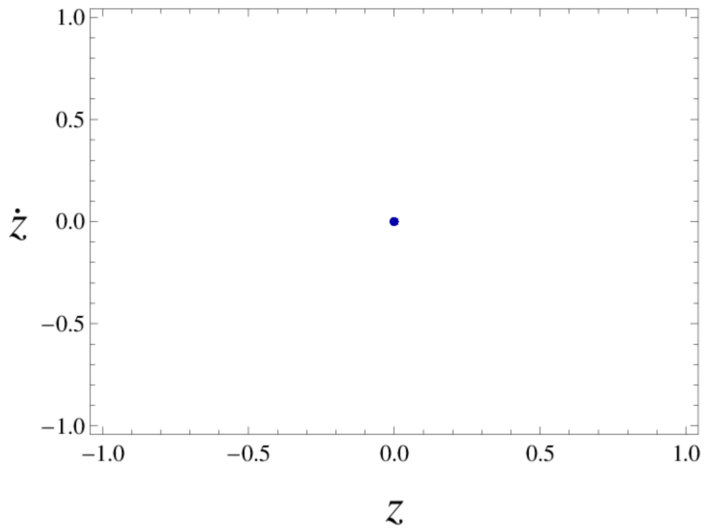
NUMERICAL CHECK OF THE CONTROL TECHNIQUE

The system response is kept to the reference one, i.e. $\boldsymbol{z}$ is equal to the expected value $\boldsymbol{z_s}$









Behavior of system around reference position (x_{ref}, z_s)

$$\begin{cases} y = x - x_{ref} = x - \bar{x}_{ref} - \tilde{x}_{ref} \\ p = z - z_s \end{cases}$$

Control works when *p*=0

$$\ddot{y}\left(1+\alpha_2\left(y+\bar{x}_{ref}+\tilde{x}_{ref}\right)^2\right)+\left(\alpha_1+\alpha_2\dot{y}^2+\alpha_3\left(y+\bar{x}_{ref}+\tilde{x}_{ref}\right)^2\right)\left(y+\bar{x}_{ref}+\tilde{x}_{ref}\right)=$$

$$-\Gamma_1\left(1+\left(y+\bar{x}_{ref}+\tilde{x}_{ref}\right)+V_g+p\right)^{-2}-\left(\rho_1+\rho_2\left(y+\bar{x}_{ref}+\tilde{x}_{ref}\right)^2\right)\left(\dot{y}+\dot{\tilde{x}}_{ref}\right)+\left(\ddot{V}_g-k_g\dot{y}+\nu_1\left(\dot{V}_g-k_g y\right)\right)\nu_2$$

$$+\left(\left(y+\bar{x}_{ref}+\tilde{x}_{ref}\right)\mu_1+\mu_2\left(y+\bar{x}_{ref}+\tilde{x}_{ref}\right)^3\right)\left(\ddot{U}_g+\eta_1\dot{U}_g+\eta_2 U_g\right)$$

$$\dot{p}=-k_g y$$

METHOD OF MULTIPLE TIME SCALES: HIGH-ORDER SOLUTION

- INDEPENDENT **TIME SCALES** UP TO THE FOURTH ORDER $\quad T_0 = t,\quad T_1 = \varepsilon t,\quad T_2 = \varepsilon^2 t,\quad T_3 = \varepsilon^3 t$
- **SCALING** OF VARIABLES

$$y\left(T_0,T_1,T_2,T_3\right)=\varepsilon y_1\left(T_0,T_1,T_2,T_3\right)+\varepsilon^2 y_2\left(T_0,T_1,T_2,T_3\right)+\varepsilon^3 y_3\left(T_0,T_1,T_2,T_3\right)+\varepsilon^4 y_4\left(T_0,T_1,T_2,T_3\right)$$

$$p\left(T_0,T_1,T_2,T_3\right)=\varepsilon p_1\left(T_0,T_1,T_2,T_3\right)+\varepsilon^2 p_2\left(T_0,T_1,T_2,T_3\right)+\varepsilon^3 p_3\left(T_0,T_1,T_2,T_3\right)+\varepsilon^4 p_4\left(T_0,T_1,T_2,T_3\right)$$

- **ORDERING** $\quad V=\varepsilon^3 V \quad U=\varepsilon^3 U \quad \rho_1=\varepsilon^2\rho_1 \quad \rho_2=\varepsilon^2\rho_2 \quad \nu_1=\varepsilon^2\nu_1 \quad \nu_2=\varepsilon^2\nu_2 \quad k_g=\varepsilon^2 k_g$
- **DETUNING** PARAMETERS $\quad \varepsilon^2\sigma_u=\omega_u-\omega_1=\omega_1\left(\Omega_u-1\right),\quad \varepsilon^2\sigma_v=\omega_v-\omega_1=\omega_1\left(\Omega_v-1\right),\quad \Omega_i=\omega_i/\omega_1$

REFERENCE SOLUTION $\tilde{x}_{ref}$: modulated response of uncontrolled system

solution of perturbed uncontrolled system

considered as a further variable:

$$\tilde{x}_{ref}\left(T_0,T_1,T_2,T_3\right)=\varepsilon\tilde{x}_{ref\,1}\left(T_0,T_1,T_2,T_3\right)+\varepsilon^2\tilde{x}_{ref\,2}\left(T_0,T_1,T_2,T_3\right)+\varepsilon^3\tilde{x}_{ref\,3}\left(T_0,T_1,T_2,T_3\right)+\varepsilon^4\tilde{x}_{ref\,4}\left(T_0,T_1,T_2,T_3\right)$$

- Solving **multiple scale** equations at each order and eliminating secular terms

AMPLITUDE MODULATION EQUATIONS (AMEs)

$$\begin{aligned}\dot{A}=&\beta_6 AB+\beta_9 A+\beta_5 A_{un}B+\beta_8 B\cos\left(\sigma_u t+\phi_u\right)-\beta_{10}B\sin\left(\sigma_u t+\phi_u\right)\\&+\beta_{11}B\cos\left(\sigma_v t\right)+i\Big(\bar{A}_{un}\left(A^2\left(\beta_1 B+\beta_4\right)+2AA_{un}\left(\beta_1 B+\beta_4\right)+\beta_1 A_{un}B\right)\\&+A^2\bar{A}\left(\beta_1 B+\beta_4\right)+2\beta_4 A\bar{A}A_{un}+AB\left(2\beta_1\bar{A}A_{un}+B\left(\beta_2 B+\beta_3\right)+\beta_7\right)\\&+A_{un}\left(B\left(\beta_1\bar{A}A_{un}+B\left(\beta_2 B+\beta_3\right)+\beta_7\right)+\beta_4\bar{A}A_{un}\right)\\&+B\left(\beta_8\sin\left(\sigma_u t+\phi_u\right)+\beta_{10}\cos\left(\sigma_u t+\phi_u\right)+\beta_{11}\sin\left(\sigma_v t\right)\right)\Big)\end{aligned}$$

$$\dot{B}=\frac{2C_{214}k_g}{\omega_1^2}\left(A\bar{A}_{un}+A\bar{A}+\bar{A}A_{un}\right)+\frac{C_{212}k_g}{\omega_1^2}B^2+\frac{C_{11}k_g}{\omega_1^2}B$$

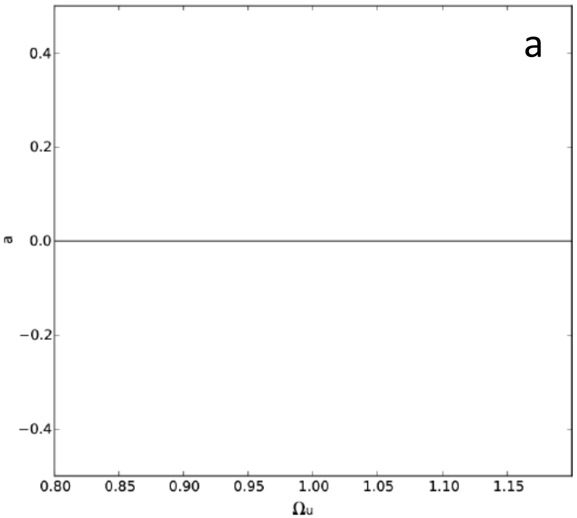
+

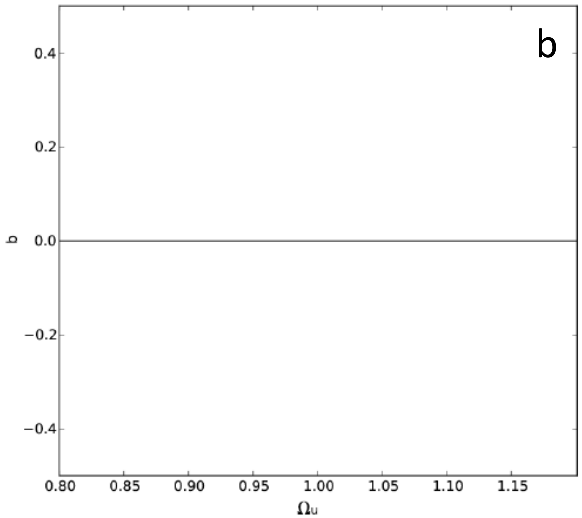
AMEs UNCONTROLLED SYSTEM

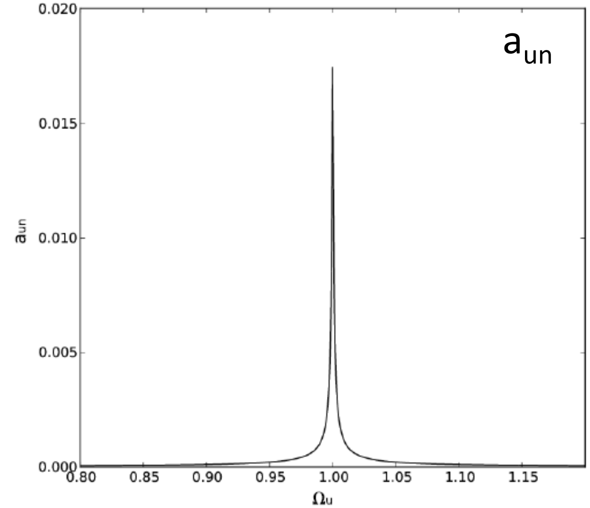
$$\begin{aligned}\dot{A}_{un}=&-\frac{C_{35}}{2}A_{un}+\frac{C_{sv}}{4\omega_1}\cos\left(\sigma_v t\right)\\&+\frac{C_{su}}{4\omega_1}\cos\left(\sigma_u t+\phi_u\right)-\frac{C_{cu}}{4\omega_1}\sin\left(\sigma_u t+\phi_u\right)\\&+i\left(\frac{C_{301}}{2\omega_1}A_{un}^2\bar{A}_{un}+\frac{C_{sv}}{4\omega_1}\cos\left(\sigma_v t\right)\right.\\&\left.+\frac{C_{su}}{4\omega_1}\sin\left(\sigma_u t+\phi_u\right)+\frac{C_{cu}}{4\omega_1}\cos\left(\sigma_u t+\phi_u\right)\right)\end{aligned}$$

VALIDITY OF ASYMPTOTIC SOLUTION

AMEs EQUILIBRIUM POSITION $(a,b)=(0,0)$ $\rightarrow$ **REFERENCE** STATE $(x,z)=(x_{ref},z_s)$







STEADY STATE RESPONSES

$$y=f_{y0}+f_{yc1}\cos(\omega_1 t)+f_{ys1}\sin(\omega_1 t)$$
$$+f_{yc2}\cos(2\omega_1 t)+f_{ys2}\sin(2\omega_1 t)$$
$$+f_{yc3}\cos(3\omega_1 t)+f_{ys3}\sin(3\omega_1 t)$$
$$p=f_{p0}+f_{pc1}\cos(\omega_1 t)+f_{ps1}\sin(\omega_1 t)$$

a_{un} **corresponds**
to the asymptotic reference response

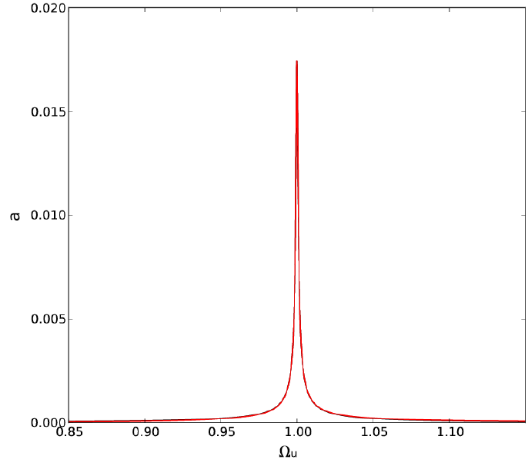
$\mu_2=\rho_2=\eta_1=\eta_2=V_g=\alpha_2=0 \quad \alpha_1=1 \quad \Gamma_1=0.1 \quad \alpha_3=0.1 \quad \omega_1=0.8325$

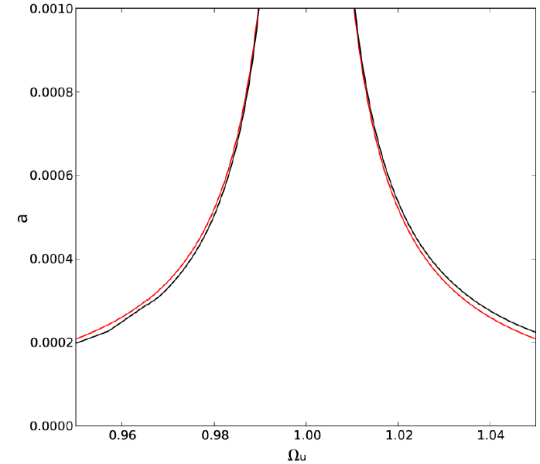
$\nu_1=0.01 \quad \nu_2=0.01 \quad \mu_1=1.5708 \quad z_s=0.01 \quad k_g=0.001 \quad U_g=0.0001$

COMPARISON WITH **NUMERICAL INTEGRATION** OF **SYSTEM EQUATION**

Amplitude = $\left(x_{max} - x_{min}\right)/2$

PARAMETRIC EXCITATION $U_g = 0.0001$





VERY GOOD

AGREEMENT BETWEEN

AMES AND **ODES**

EXTERNAL EXCITATION $V_g = 0.0001$