11.1 –
Techniques for Control of Chaos

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Coworkers: S. Lenci, V. Settimi

DAY	TIME	LECTURE
Monday 05/11	14.00 -14.45	Historical Framework - A Global Dynamics Perspective in the Nonlinear Analysis of Systems/Structures
	15.00 -15.45	Achieving Load Carrying Capacity: Theoretical and Practical Stability
	16.00 -16.45	Dynamical Integrity: Concepts and Tools_1
Wednesday 07/11	14.00 -14.45	Dynamical Integrity: Concepts and Tools_2
	15.00 -15.45	Global Dynamics of Engineering Systems
	16.00 -16.45	Dynamical integrity: Interpreting/Predicting Experimental Response
Monday 12/11	14.00 -14.45	**Techniques for Control of Chaos**
	15.00 -15.45	A Unified Framework for Controlling Global Dynamics
	16.00 -16.45	Response of Uncontrolled/Controlled Systems in Macro- and Micro-mechanics
Wednesday 14/11	14.00 -14.45	A Noncontact AFM: (a) Nonlinear Dynamics and Feedback Control (b) Global Effects of a Locally-tailored Control
	15.00 -15.45	Exploiting Global Dynamics to Control AFM Robustness
	16.00 -16.45	Dynamical Integrity as a Novel Paradigm for Safe/Aware Design

Contents

- Background
- Classification of control methods
- Chaos control in mechanics
- The OGY method

Background (1)

Controlling nonlinear dynamics and chaos

- Huge number of publications (90s-2010)
- Monographs
- Journal Special Issues
- Dedicated Web sites (Fradkov, Chen)

"Control of chaos" today is commonly used for both

- **Suppression of chaos**
  chaos eliminated "tout court"
- **"True" control of chaos**
  exploiting chaos for its control

"…some techniques merely suppress or remove chaotic behavior…
…others actually exploit chaotic behavior…" Linder and Ditto [1995]

Background (2)

"Suppression of Chaos"

Simple elimination of chaotic effects

- Classical control techniques [Sifakis & Elliott, 2000]
- Empirical methods [Singer *et al*., 1991]
- Keen approaches [Pyragas, 1992]

Background (2)

“Suppression of Chaos”

Simple elimination of chaotic effects

- Classical control techniques [Sifakis & Elliott, 2000]
- Empirical methods [Singer *et al*., 1991]
- Keen approaches [Pyragas, 1992]

“Control of Chaos”

Pioneering work: **Ott, Grebogi & Yorke** [1990]

although other works appeared at about the same time

Seminal idea: ***exploiting*** the chaotic behaviour of systems to control their dynamics

from analysis to synthesis of nonlinear dynamics and chaos

Background (3)

- First of all. What is "control of chaos"?
- Eliminating chaos. But how?
- Two main approaches have been developed and used:

1) transforming a given, **single** chaotic attractor into a non-chaotic (equilibrium, periodic, quasi-periodic) solution – a **"local"** approach

2) regularizing the **overall** chaotic dynamics, by exploiting the chaotic properties of the systems – a **"global"** approach

Classification of control methods (1)

Possible classifications of the matter

- **Phenomenologically** based classification:

1. Acting on the **system parameters**

   (i) **Ott-Grebogi-Yorke** method **(OGY)** [Ott et al. 1990]

   (ii) **classical methods** of control theory **(CM)** [Isidori, 1995]

   (iii) **“control by system design” (CSD)** [Blazejczyk et al., 1993]

   (iv) **“parametric variation methods” (PVM)** [Chen and Dong, 1998]

Classification of control methods (2)

Possible classifications of the matter

- **Phenomenologically** based classification:

2. Acting on the **excitation**

(i) **classical methods** where **a modified input is applied (CM)** [Sifakis and Elliott, 2000], [Pyragas, 1992], [Isidori, 1995], [Pinto and Gonçalves, 2000]

(ii) **“control through operating conditions” (COC)** [Blazejczyk et al., 1993]

(iii) methods based on combining **parametrical/eternal excitations (PEE)** [Lima and Pettini, 1990]
or applying **weak periodic perturbations (WPP)** [Chacon, 2005]
or modifying the **excitation shape (SE)** [Shaw, 1990], [Lenci and Rega, 1998, 2004]

Classification of control methods (3)

Possible classifications of the matter

- **Performance** based classifications:

  (i) stabilizing an unstable zone of parameter space (CSD, COC, PVM)

  (ii) moving away from known chaotic zones (CSD, COC)

  (iii) stabilizing a given, erratic solution (CM, OGY)

  (iv) overall regularizing the system dynamics (PEE, WPP, SE)

Classification of control methods (4)

Possible classifications of the matter

- **Dynamical system** based classification. **Exploiting**:

  (i) the saddles embedded in the chaotic attractor (OGY)

  (ii) the ergodicity of the chaotic attractor [Shinbrot et al. 1990], [Boccaletti et al., 1997], [Bird and Aston, 1998]

  (iii) the sensitivity to initial conditions (S.I.C.) [Shinbrot et al., 1992]

  (iv) homo/heteroclinic bifurcations (PEE, WPP, SE)

Applications of control methods

Cross-disciplinarity of “Control of Chaos”

- Various application fields, including:

  (i) mathematics, physics

  (ii) chemistry, biology, medicine

  (iii) economics

  (iv) engineering

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Cross-disciplinarity of “Control of Chaos”

- Various application fields, including:

  (i) mathematics, physics

  (ii) chemistry, biology, medicine

  (iii) economics

  (iv) engineering, mechanics

  pendulums, beams and plates, systems with friction and/or impacts, spacecraft, vibroformers, microcantilevers, ship oscillations, tachometers, rate gyros, Duffing oscillators, robot-manipulator arms, earthquake civil engineering, milling processes, whirling motions under mechanical resonances, systems with clearance …

Chaos control in mechanics (1)

- Difficult categorization due to high variety

  (i) of the **systems (mechanical complexity)**

Discrete systems / Continuous systems

Archetypal nonlinear oscillators idealization | **Minimal discretized representations**

to be validated with experiments, refined theoretical and numerical models

Chaos control in mechanics (2)

- Difficult categorization due to high variety

  (i) of the **systems (mechanical complexity)**

  (ii) of the **involved dynamical processes (dynamical complexity)**

  In the last 30 years: several nonlinear dynamic phenomena (chaotic/regular) highlighted in mechanical systems

"Control of chaos"

- Chaotic nonlinear phenomena
  in strict sense
- Regular nonlinear phenomena
  nonlinear, wanted or unwanted, phenomena

Chaos control in mechanics (3)

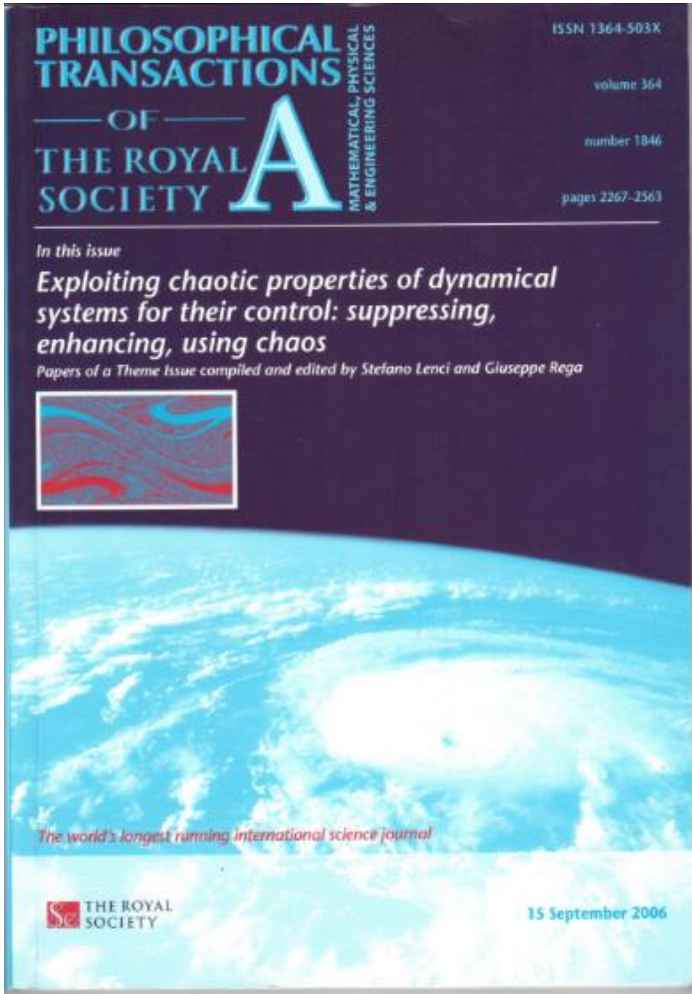
- Difficult categorization due to high variety

  (i) of the **systems (mechanical complexity)**

  (ii) of the **involved dynamical processes (dynamical complexity)**

  (iii) of the **specific control goals**

Rich and intriguing framework!



Chaos control in mechanics (3)

- Difficult categorization due to high variety

  (i) of the **systems (mechanical complexity)**

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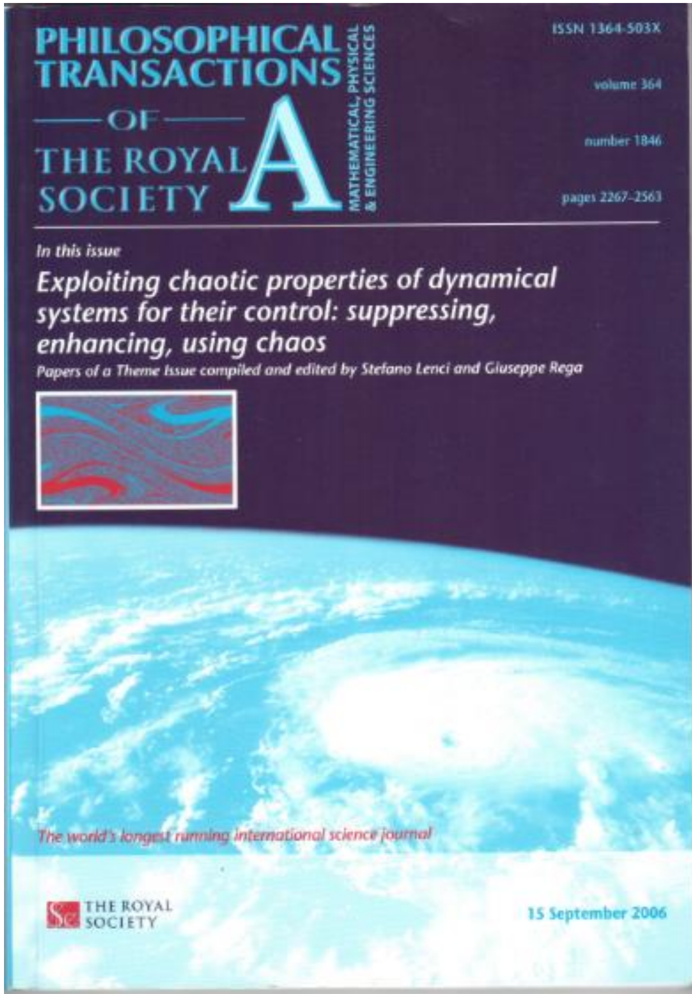
Rich and intriguing framework!

A qualitative look at

OGY Method



The OGY method

- Introduction
- Fundamental attributes of chaos
- Chaos control goals
  - Controlling Steadily Running Chaotic System
  - Targeting
- OGY Method in Mechanics
- From Local to Global Control

OGY Method

- First formulated in **E. Ott, C. Grebogi and J.A. Yorke, 1990, Controlling Chaos, *Phys. Rev. Lett. E*, 64, 1196**
  (University of Maryland Chaos Group)

- **Paradigmatic method: thousands of papers** referring to it, providing **explanations**, **clarifications**, **extensions**, **improvements** on theoretical and computational aspects, experiments… in both hard and soft science

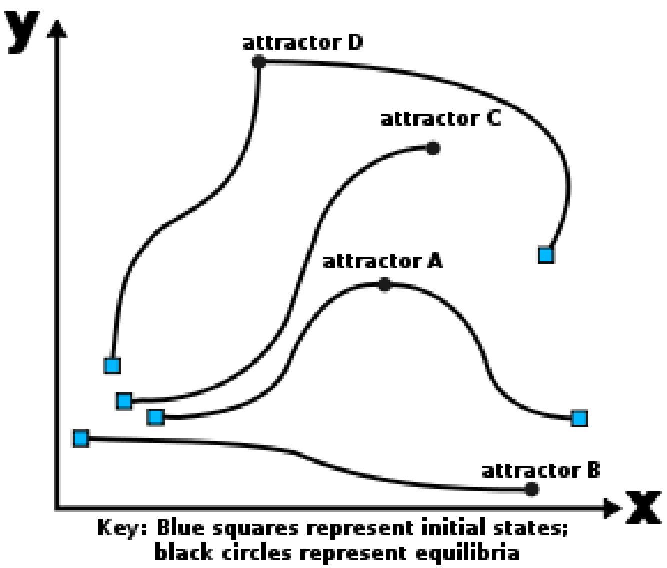
Fundamental attributes of chaos (1)

Based on two fundamental aspects of chaos:

(1) Exponential sensitivity to small perturbations

(largest Lyapunov exponent)

Difficulty in prediction of future states of the system



Fundamental attributes of chaos (1)

Based on two fundamental aspects of chaos:

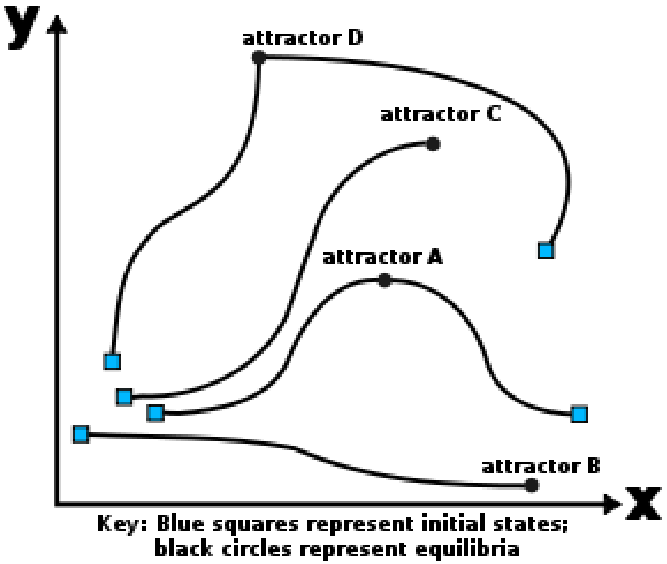
(1) Exponential sensitivity to small perturbations

(largest Lyapunov exponent)

Difficulty in prediction of future states of the system

For control purposes:

large changes in location of orbit points can be produced by only small changes in control variable



Fundamental attributes of chaos (2)

Based on two fundamental aspects of chaos:

(2) Complex orbit structure

(entropy measures)

Infinite set of unstable periodic orbits (UPOs) embedded within chaotic attractors

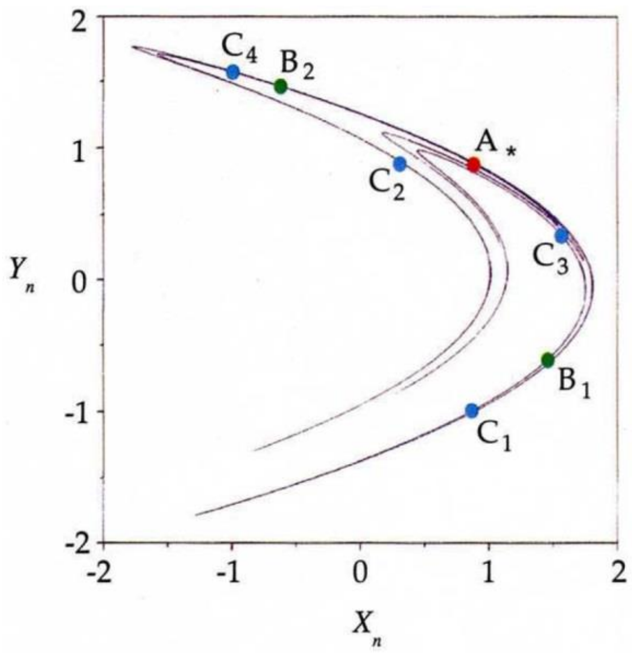


FIG. 1 Hénon attractor, with period-1 point, A_*; period-2 points, B_1 and B_2; and period-4 points, C_1, C_2, C_3 and C_4.

[Shinbrot et al., 1993]

Fundamental attributes of chaos (2)

Based on two fundamental aspects of chaos:

(2) Complex orbit structure

(entropy measures)

Infinite set of unstable periodic orbits (UPOs) embedded within chaotic attractors

For control purposes:
Very diverse dynamical changes.
Great flexibility in dynamical behavior

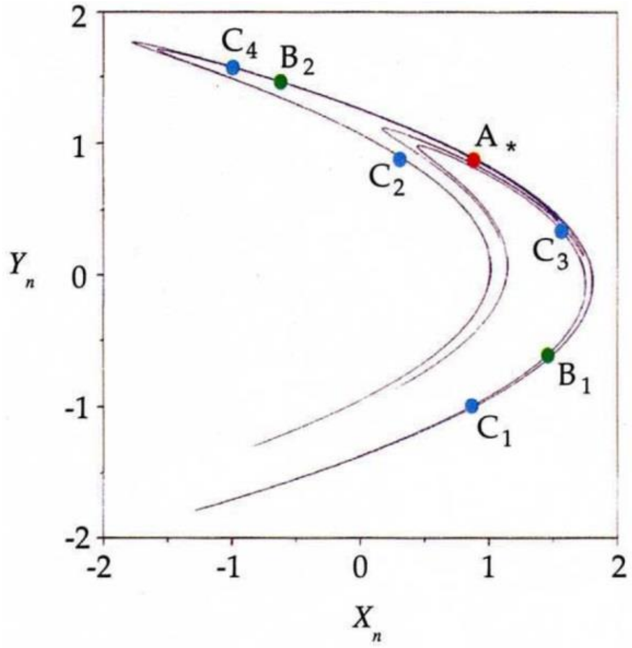


FIG. 1 Hénon attractor, with period-1 point, A_*; period-2 points, B_1 and B_2; and period-4 points, C_1, C_2, C_3 and C_4.

[Shinbrot et al., 1993]

Chaos control goals (1)

- Two ways of effectively using chaos:

  (i) Control : Stabilizing selected unstable orbits

  (ii) Targeting : Bringing an orbit to a desired location

- Thanks to chaos attributes **(1)** e **(2)** both goals can be achieved with small perturbations

  low-energy/low-force controllers

- Control = ***feedback*** *control*

  measurements of the state of the system are regularly taken, and, based on them, some controllable parameter (or set of parameters) is adjusted so as to achieve some goal

Chaos control goals (2)

(i) Controlling a Steadily Running Chaotic System

- Related to attribute **(2) *Complex Structure***
- **Goal**: improve the averaged **performance** of the system

Chaos control goals (2)

(i) Controlling a Steadily Running Chaotic System

- Related to attribute **(2) *Complex Structure***
- **Goal**: improve the averaged **performance** of the system

System Performance *P*

- Depends on its state $x(t)$ and its history
- $P = \langle f(x(t)) \rangle$: time average of some quantity f function of the state
- For each UPO i: $P_i = \langle f(x_i(t)) \rangle$
- For the chaotic orbit c: P_c = weighted average of P_i

$$\Downarrow$$

For some UPO: $P_i > P_c$

Control the system to such UPO = improve system performance

Chaos control goals (3)

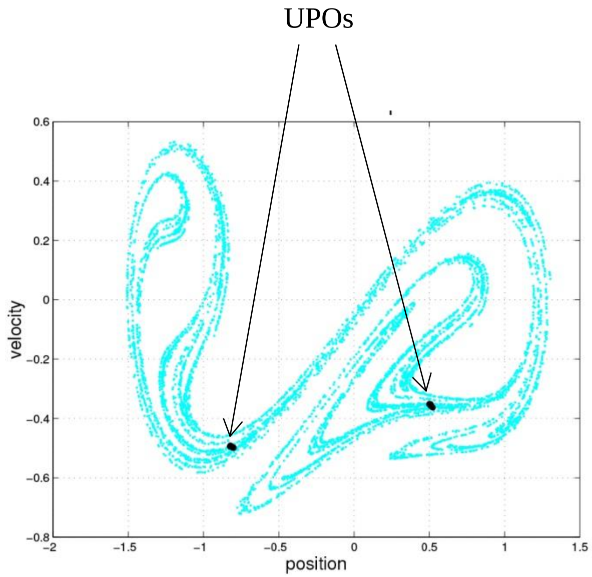
(i) Controlling a Steadily Running Chaotic System

- Two issues:

(a) Determination of UPOs

- If accurate **analytical model available**: standard numerical techniques (e.g., Newton's method)
- If **analytical model not available (experiments)**: state space embedding and attractors reconstruction techniques

[So et al., 1997], [Pierson and Moss, 1995]



[Witvoet, DCT 2005.36]

Chaos control goals (4)

(i) Controlling a Steadily Running Chaotic System

- Two issues:

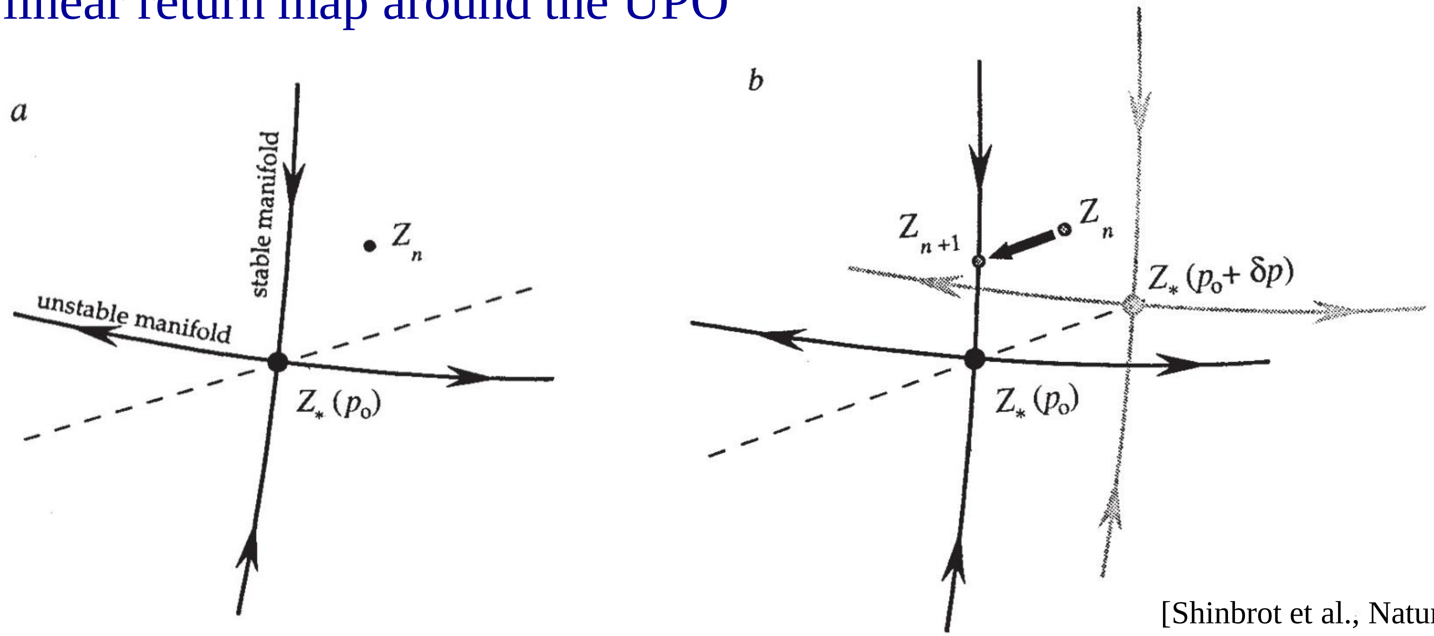
(b) UPO control Applied only to discrete data

Continuous time systems: discrete time (Poincaré map)

- Steps:

- Small kick to the chaotic orbit to place it very near to the UPO

Local linear return map around the UPO



[Shinbrot et al., Nature 1993]

Chaos control goals (4)

(i) Controlling a Steadily Running Chaotic System

- Two issues:

(b) UPO control Applied only to discrete data

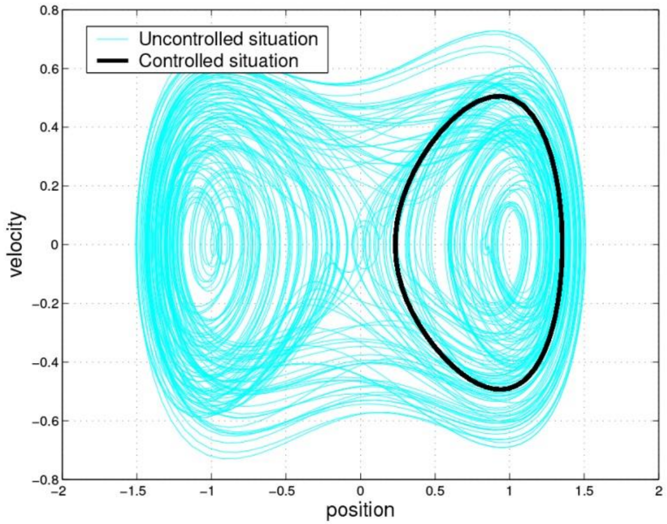
Continuous time systems: discrete time (Poincaré map)

- Steps:

- Small kick to the chaotic orbit to place it very near to the UPO

Local linear return map around the UPO

- Continually do small kicks to maintain the orbit close to the UPO



[Witvoet, DCT 2005.36]

Chaos control goals (5)

(i) Controlling a Steadily Running Chaotic System

- Two issues:

  (a) Determination of UPOs

  (b) UPO control

Algorithms:

- Original OGY technique [Ott et al., 1990]
- "Pole-placement" technique [Romeiras et al., 1992]
- Time-series measurements of a single scalar state variable [Dressler and Nitsche , 1992], [So and Ott ,1995]
- Control of very fast dynamics [Socolar et al., 1994]

Chaos control goals (5)

(i) Controlling a Steadily Running Chaotic System

- **Application: Duffing Oscillator**

$$\boxed{\ddot{x}(t) + c\dot{x}(t) - x(t) + x^3(t) = \boxed{p}\cos(\omega_0 t)}$$

- Poincaré Map

$$[x_{n+1} \quad \dot{x}_{n+1}] = F\left([x_n \quad \dot{x}_n], p_0, c\right)$$

- Taylor expansion around x_F

$$[x_{n+m} \quad \dot{x}_{n+m}] - [x_F \quad \dot{x}_F] = \mathbf{A}\left([x_n \quad \dot{x}_n] - [x_F \quad \dot{x}_F]\right)$$

- Eigenvalues and eigenvectors of **A**

Control

$$\boxed{p = p_0 + C(x - x_F)f^u, \quad C = \frac{\lambda_u}{(\lambda_u - 1)}\frac{1}{(\mathbf{b}\ f^u)}}$$

x_F = fixed point of Poincaré map corresponding to periodic orbit

A = matrix of linearized coefficients

mT = period of UPO to be controlled

λ_u = unstable eigenvalue of **A**

b = $\delta x_F/\delta p$ relative shift of the fixed point as the **driving amplitude is perturbed** by δp

f^u = corresponding unstable eigenvector of **A**

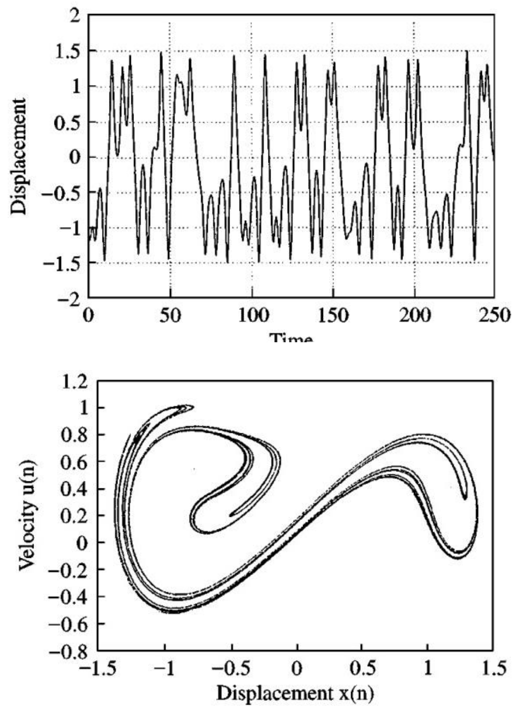
Chaos control goals (5)

(i) Controlling a Steadily Running Chaotic System

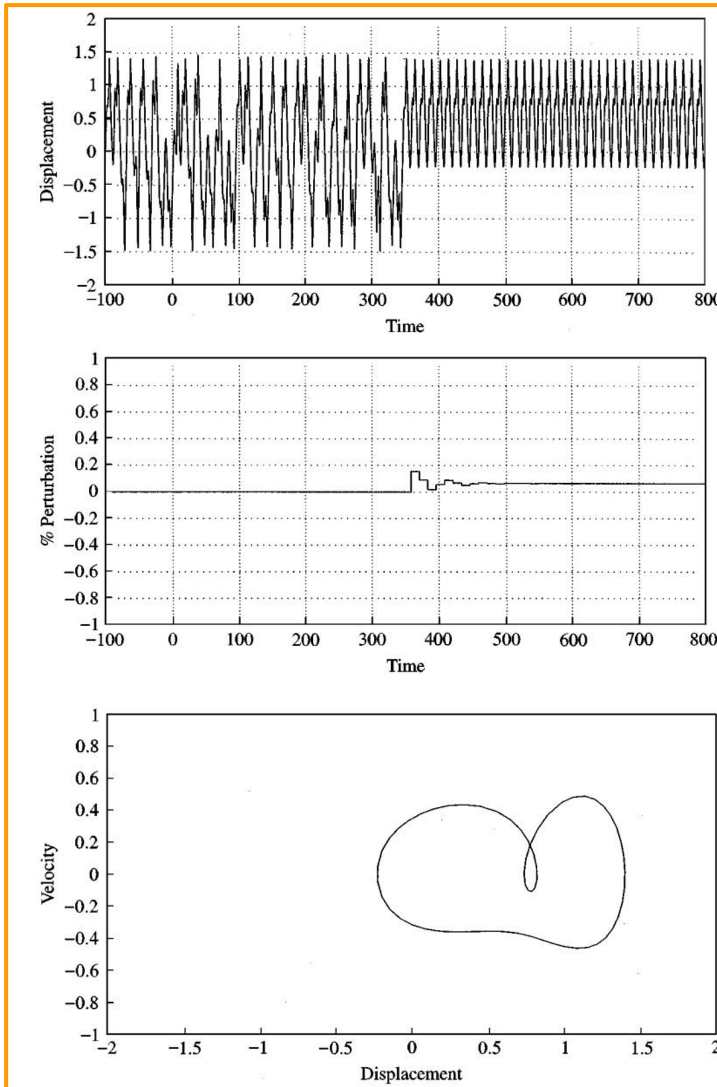
- **Application: Duffing Oscillator**

$$\ddot{x}(t) + c\dot{x}(t) - x(t) + x^3(t) = \boxed{p} \cos(\omega_0 t)$$

Uncontrolled



Control

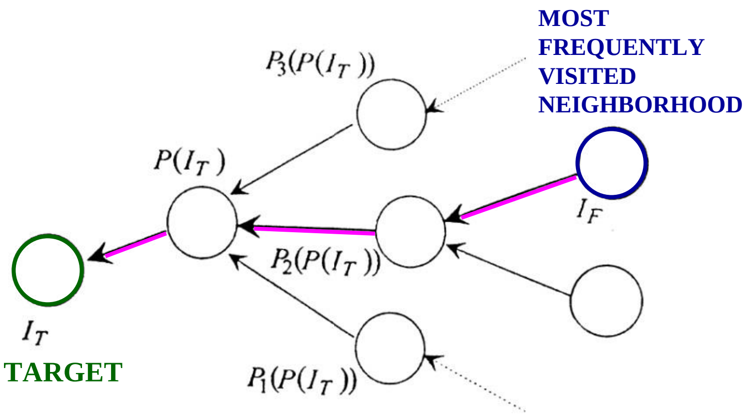


[Sifakis and Elliott, 2000]

Chaos control goals (6)

(ii) Targeting

- Related to attribute **(1)** ***Exponential Sensitivity***
- **Goal**: quickly bring an orbit to a selected location in state space



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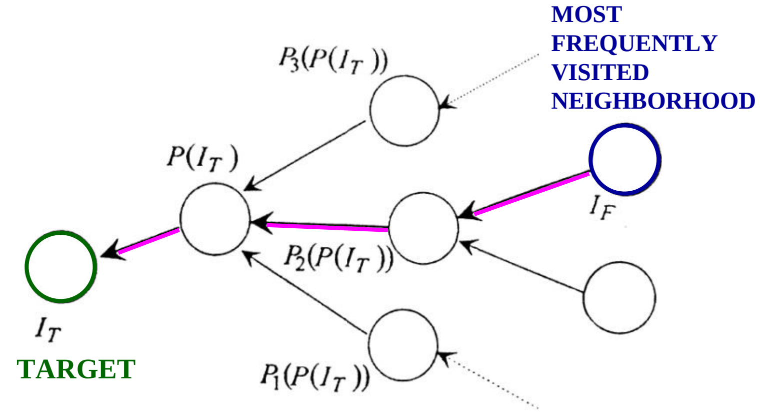


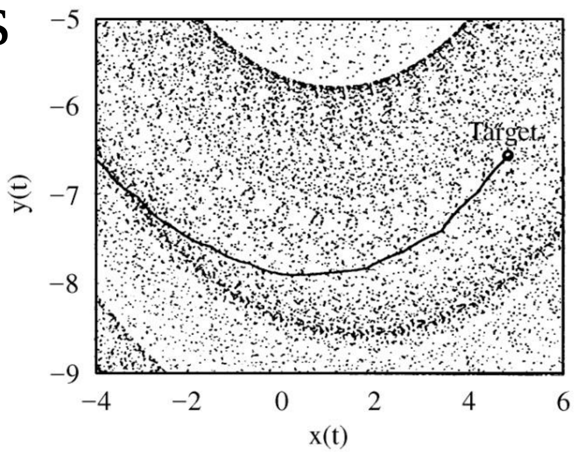
- Perturbations carefully chosen: small controls
- Several techinques

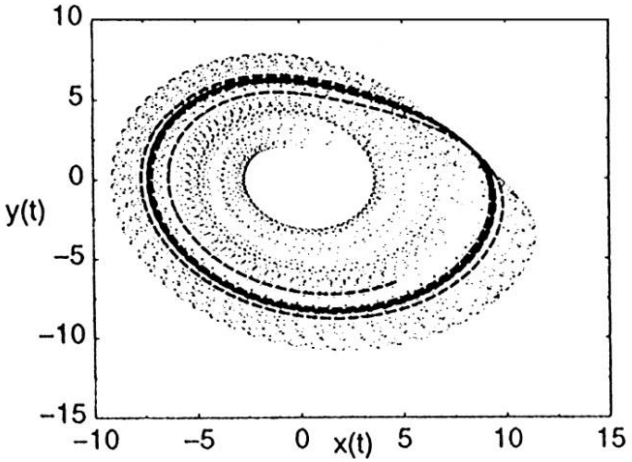
[Shinbrot et al., 1990]

[Kostelich et al., 1993]

[Bollt and Meiss, 1995]

[Schroer and Ott, 1997]





[Boccaletti et al., Phys. Rev. E, 1997]

OGY Method in Mechanics (1)

- **For OGY control, two attributes are needed:**

(i) Presence of one or more **unstable fixed points** (saddle-node type) within the strange attractor, to be possibly stabilized

(ii) **Accessible system parameter** for changing location of the unstable fixed point in phase plane

OGY Method in Mechanics (2)

- **Limitations:**

  - Systems with large **noise**

  - **Sensitive** systems (e.g., quickly changes in dynamics)

  - Systems with **long chaotic transient**

- **To overcome them:**

  Several improvements of OGY method, e.g.:

  - **SOGY method** [Shinbrot et al., 1990]

  - **Control of high periodic/high unstable UPO**

    [Hubinger et al., 1994], [Ritz et al., 1997]

  - **Time delay coordinate method**

    [Dressler and Nitsche, 1992], [So and Ott, 1995], [DeKorte et al., 1995]

OGY Method in Mechanics (3)

- **Applications:**

(i) **Pendulum System** [Hübinger et al., 1994], [DeKorte et al., 1995], [Starrett and Tagg, 1995], [Baker, 1995], [Bishop at al., 1996], [Christini et al., 1996], [Yagasaki and Uozumi, 1997], [Yagasaki and Yamashita, 1999], [Wang and Jing, 2004], [Pereira-Pinto et al., 2004, 2005], [Alasty and Salarieh, 2007]

(ii) **Smooth Archetypal Oscillators** [Ditto et al., 1990], [Moon, 1992], [Moon et al., 1996], [Hunt, 1991], [Ding at al., 1994], [Dressler et al., 1995], [Ding et al., 1996], [Sifakis and Elliott, 2000]

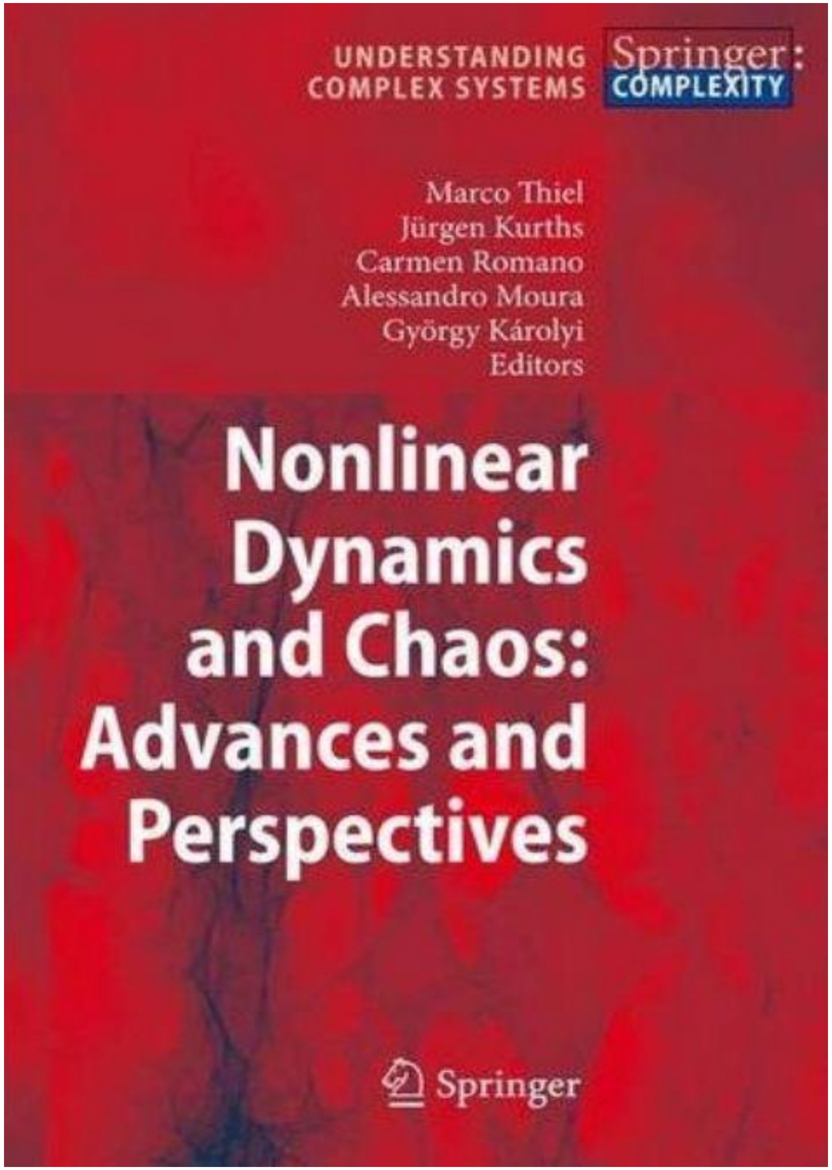
(iii) **Vibro-Impact and Friction Systems** [Nordmark, 1991], [Kalagnanam, 1994], [Chatterjee et al., 1995], [Bishop et al., 1998], [Lenci and Rega, 2000], [Galvanetto, 2001], [Gutiérrez and Arrowsmith, 2004], [Moon et al., 2003], [de Souza and Caldas, 2004], [de Sounza et al., 2005]

(iv) **Coupled Mechanical Systems** [Feudel et al., 1998], [Alvarez et al., 1999], [Agiza, 2002], [González-Hernández et al., 2001, 2004]

(v) **Targeting in Astrodynamics** [Macklay et al., 1984], [Lai et al., 1993], [Lai et al., 1993], [Macau, 1998,2000,2003]

(vi) **Atomic Force Microscopy** [Ashhab et al., 1999], [Basso et al., 2000], [Jamitzky et al., 2006], [Arjmand et al., 2008], [Misra et al., 2008]

OGY Method in Mechanics (4)



2010

Controlling Chaos: The OGY Method, Its Use in Mechanics, and an Alternative Unified Framework for Control of Non-regular Dynamics

G. Rega, S. Lenci, and J.M.T. Thompson

From Local to Global Control (1)

- **OGY method:** **"local" approach**

  Exploiting chaotic properties to **stabilize a selected unstable saddle** embedded in chaotic attractor

- **In alternative:** **"global" approach**

  Exploiting chaotic properties to **eliminate (shift) a homoclininc/heteroclinic intersection** triggering unwanted phenomena

From Local to Global Control (2)

- **Different methods for "global" approach:**

  - Modifying system parameters

  - Adding controlling parametric excitation to the external one, o viceversa [Lima and Pettini, 1990]

  - Modifying the excitation by adding controlling terms (i.e. controlling sub/superharmonics) [Chacon, 2005], [Lenci and Rega, 1998-…….]

From Local to Global Control (2)

- **Different methods for "global" approach:**

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  - Modifying the excitation by adding controlling terms (i.e. controlling sub/superharmonics) [Chacon, 2005], [Lenci and Rega, 1998-…]

To be addressed in the next lecture !