9.2 – Achieving Load Carrying Capacity: Theoretical and Practical Stability

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Coworker: S. Lenci, M. Thompson

DAY	TIME	LECTURE
Monday 05/11	14.00 -14.45	Historical Framework - A Global Dynamics Perspective in the Nonlinear Analysis of Systems/Structures
	15.00 -15.45	**Achieving Load Carrying Capacity: Theoretical and Practical Stability**
	16.00 -16.45	Dynamical Integrity: Concepts and Tools_1
Wednesday 07/11	14.00 -14.45	Dynamical Integrity: Concepts and Tools_2
	15.00 -15.45	Global Dynamics of Engineering Systems
	16.00 -16.45	Dynamical integrity: Interpreting/Predicting Experimental Response
Monday 12/11	14.00 -14.45	Techniques for Control of Chaos
	15.00 -15.45	A Unified Framework for Controlling Global Dynamics
	16.00 -16.45	Response of Uncontrolled/Controlled Systems in Macro- and Micro-mechanics
Wednesday 14/11	14.00 -14.45	A Noncontact AFM: (a) Nonlinear Dynamics and Feedback Control (b) Global Effects of a Locally-tailored Control
	15.00 -15.45	Exploiting Global Dynamics to Control AFM Robustness
	16.00 -16.45	Dynamical Integrity as a Novel Paradigm for Safe/Aware Design

Outline

1. **Main stability concepts at a glance**
2. **Local versus global safety in statics and dynamics**
3. **Solution/attractor robustness in phase space; the relevant ‘safe’ basins** (through an archetypal model)
4. **Solution/attractor robustness and basin compactness in control parameter space** (through an archetypal model)
5. **Robustness/erosion profiles**
6. **Moving from theoretical to practical stability**

Achieving load carrying capacity

- **Load carrying capacity**: an old issue associated with the concept of **loss of stability**
- **Stability**: to be discussed by also considering the effects of (***static*** or ***dynamic***) **imperfections**, always present in nature/technology → A system must be able to sustain **changes** in both **initial conditions** and **control parameters**, without changing its desired outcome
- **Robustness**: a fundamental issue in analysis and design
- **Dynamic integrity**: a **global safety concept** essential to secure **practical stability** of systems
- Historical concepts and contributions at a glance ……..

Leonhard Euler (1707-1783)

- First fundamental contribution:
  Euler buckling load of a column

- Loss of load carrying capacity identified as the system instability occurring at the local bifurcation point of an equilibrium path when changing **a control parameter** (axial load) – talking, of course, in modern language

- A substantially **static** notion of stability

Aleksander Lyapunov (1857-1918)

- Rigorous formulation within a more **dynamically oriented** notion of stability
- Lyapunov (or classical) local stability roughly states that under

  infinitesimal changes in **initial conditions**

  the system must keep the reference response
- Major role in the solutions of a variety of engineering problems ensuing from modern technological developments

Warner Koiter (1914-1997)

Within the mechanical community, looking at the effects of changes of **control parameters**:

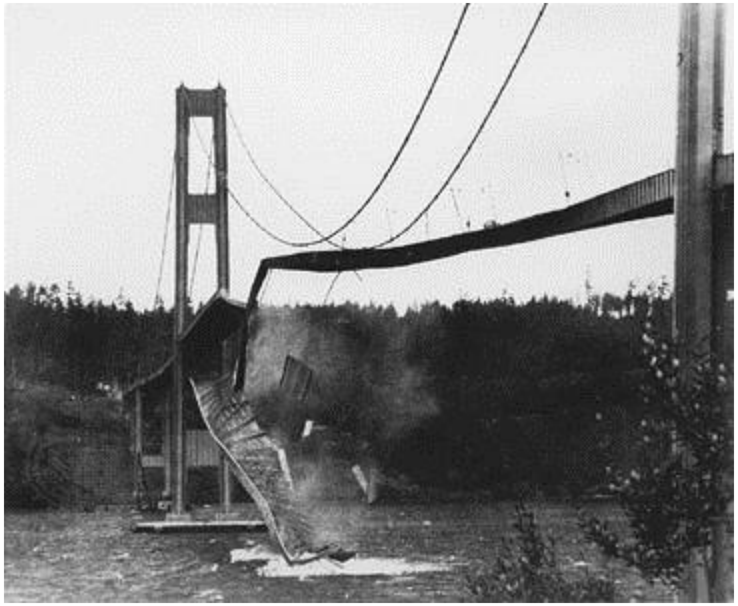
- Koiter realized that model imperfections are crucial in lowering the critical load
  - Due to imperfections, the branching point becomes a snap point, which (in the dangerous cases) occurs at a lower load threshold

- Dynamical character of stability was clear, but the reference framework was still **‘static’**

Structural stability

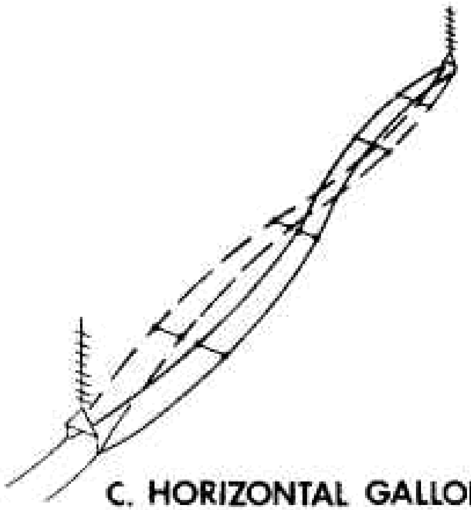
- Later on, **bifurcation theory** provided a mathematical background to this engineering intuition:
  - transcritical and pitchfork bifurcations (branching) are structurally unstable (i.e., unobservable in the real world, unless somehow forcing them) and become saddle-node bifurcations (snap) after system perturbations (imperfections in mechanical language)
- **Structural stability**: studying the effect of perturbations of the system with respect to **parameters** and not w.r.t. initial conditions, as in classical local stability

Dynamic stability



- When ‘flutter’ or ‘galloping’ of real systems came into play, **dynamics** definitely entered the concept of **loss of stability**

- In bifurcation theory language, the Hopf bifurcation was ‘discovered’ and experimentally observed, according to the fact that it is structurally stable



From theoretical to practical stability

Classical stability: **small changes** of **initial conditions** do not affect substantially the system response

Key point: how **small** have to be **perturbations**?

From a mathematical point of view the magnitude of perturbations is not important (e.g. 10^{-50} is ok)

But from a **practical** point of view it is important, since in our real world **imperfections** have a **finite** magnitude

Local (or classical, Lyapunov) stability is **not enough** for **practical** applications !!

Michael Thompson (1937-)

Practical stability of attractors
to be addressed in an
actually dynamical environment

Around the 90s:

- By considering a **global** approach, notion of **dynamic integrity** introduced, which is fundamental for properly pursuing the **safety of structures**

- **Basins of attraction** – and their variation with **a varying control parameter** - become fundamental tools

Solution robustness in phase space

Properly complementing the solely **local theoretical** character of the classical concept of stability with a **global practical** one

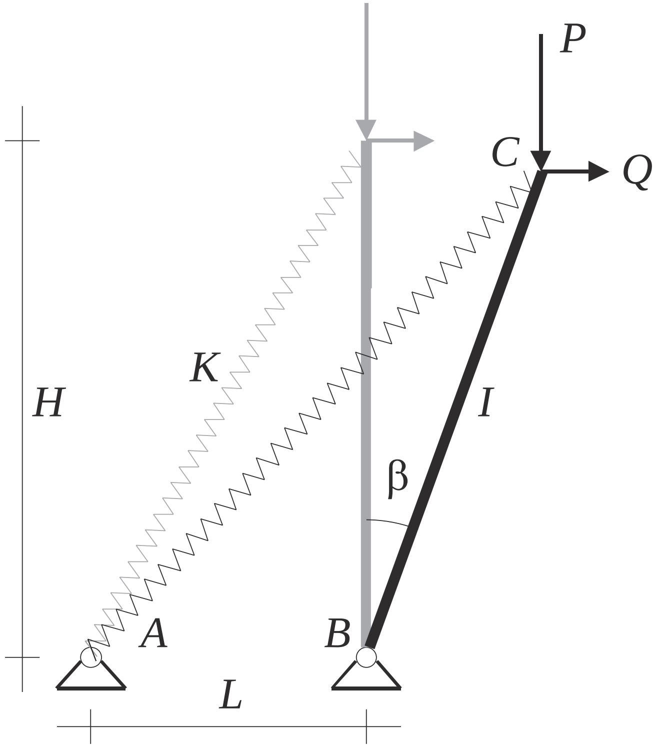
Already in the **static** case:

approaching a (local) bifurcation, the **basin** of reference solution shrinks to zero and becomes **unsafely small**, although the solution is still **stable** in the sense of Lyapunov

⇨ pursued response **non-robust** with respect to **finite** dynamic perturbations, though being its basin integer (no fractality)

An archetypal asymmetric model

- Single-dof mechanical model typically used to illustrate post-buckling behavior and imperfection sensitivity of structural systems liable to unstable buckling



- Q = "static" imperfection

- No damping and no dynamic excitation

$$\ddot{\beta} - p\sin(\beta) + \left[1 - \frac{1}{\sqrt{1+\alpha\sin(\beta)}} - q\right]\cos(\beta) = 0$$

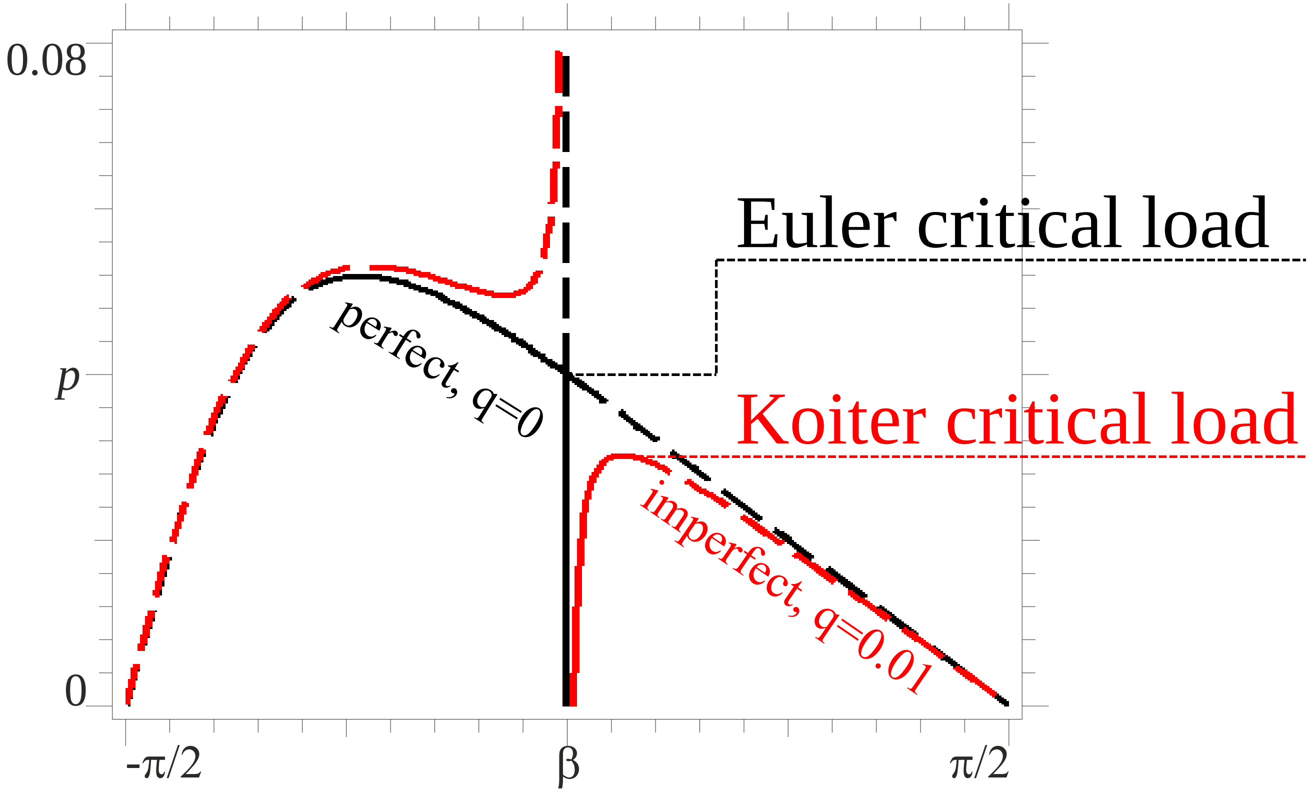
$$\alpha = \frac{2LH}{L^2+H^2} \in [0,1]$$

In the following

$\alpha = 0.8$

Equilibrium points and critical loads

$$p\sin(\beta) = \left[1 - \frac{1}{\sqrt{1+\alpha\sin(\beta)}} - q\right]\cos(\beta)$$



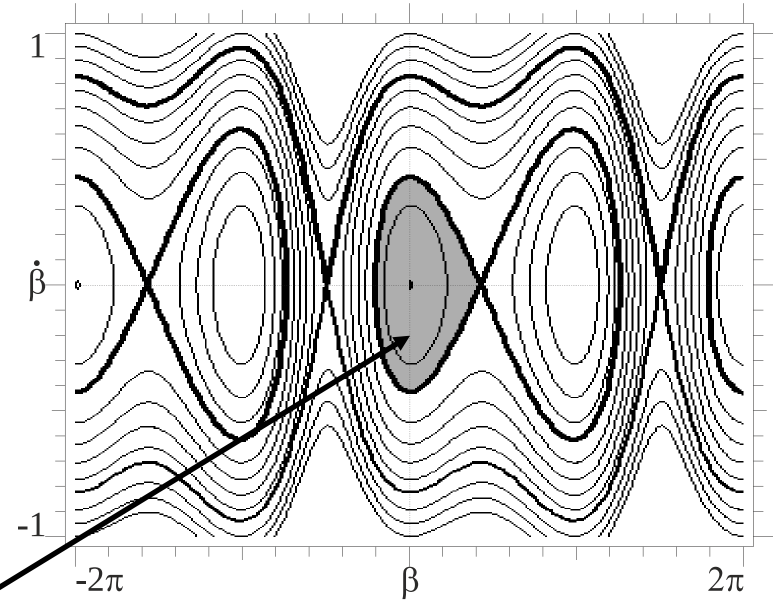
$$p_E = \frac{\alpha}{2}$$

$$p_K = \frac{\alpha}{2} - \alpha\frac{\sqrt{6}}{2}\sqrt{q} + O(q)$$

Global safety

- Phase portrait

p=0.05
q=0.01

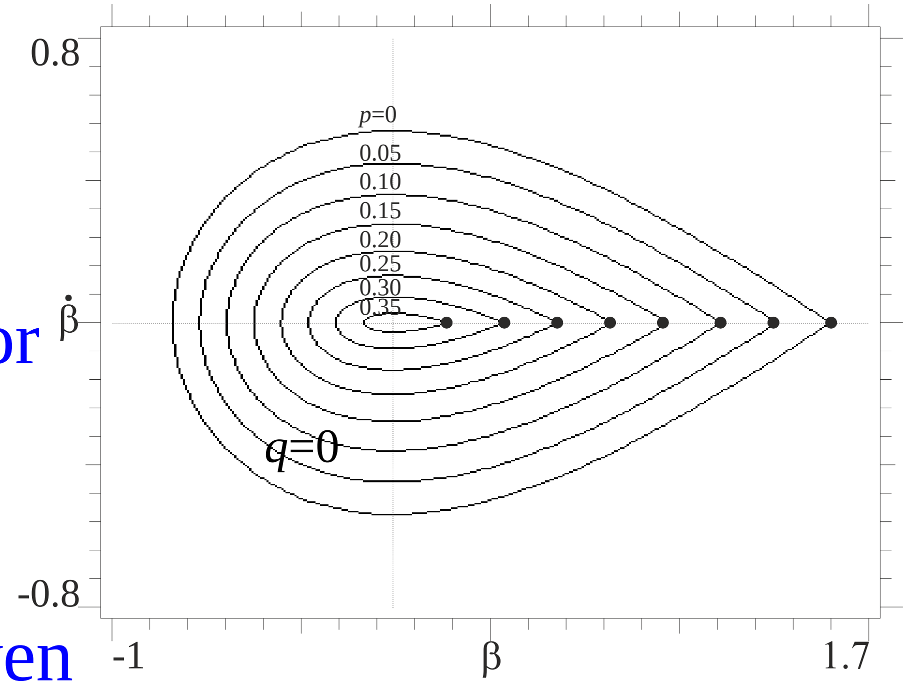


- ‘basin of attraction’ of equilibrium point
- The larger the area, the larger the ability of the system to support ***finite*** changes in i.c. $\rightarrow$

the larger the **safety** of the structure

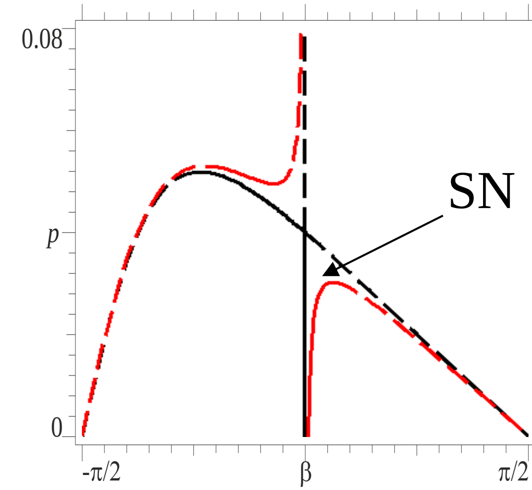
Basin and actual critical load reduction

- Area shrinks as approaching critical load, with or without 'static' imperfections, and 'rapidly' becomes too small for real world, where ***finite*** dynamic imperfections exist



- 'Basin of attraction' under (even transient) dynamic perturbations shrinks to the attractor

$\longrightarrow$ (Koiter) SN bifurcation **overestimates the actual critical load**



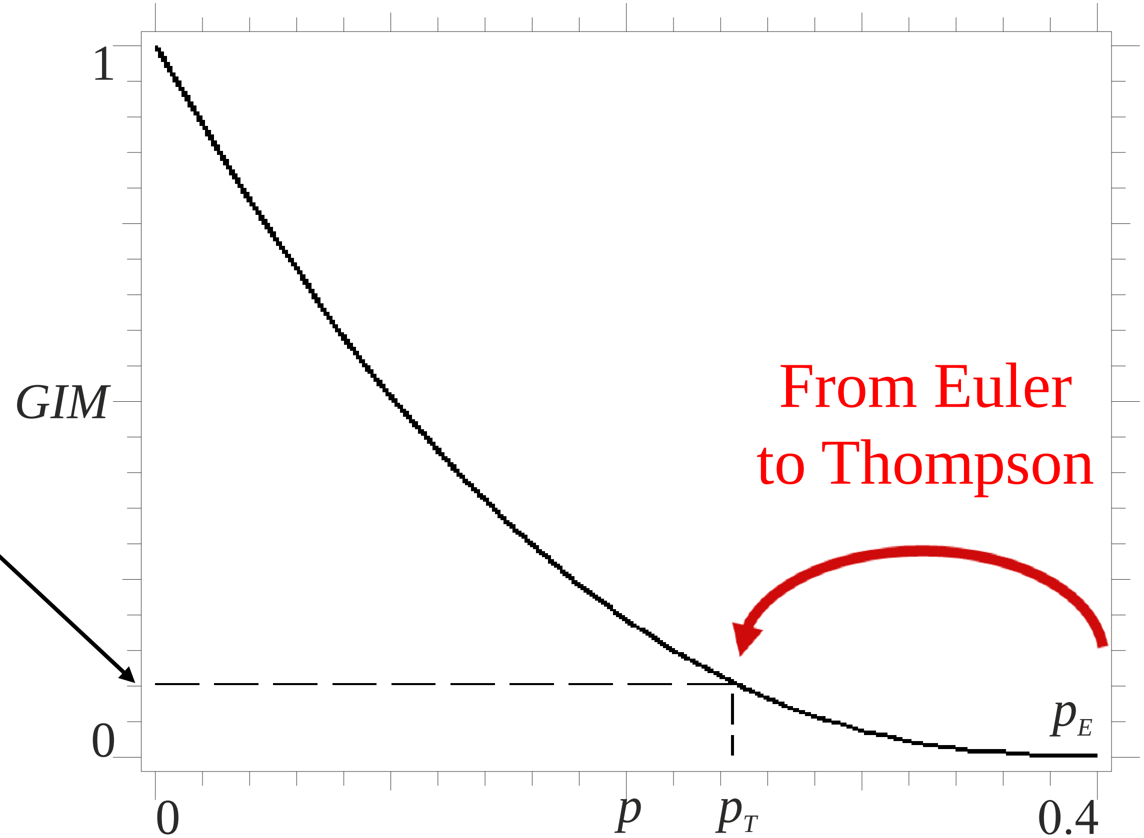
Area decrement without imperfections

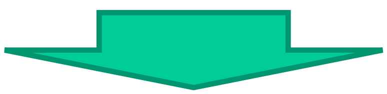
In parameter space

(q=0)

Let's accept a reduction to **10% (very low !) of initial area (GIM):**

then p_T is **59%** of p_E





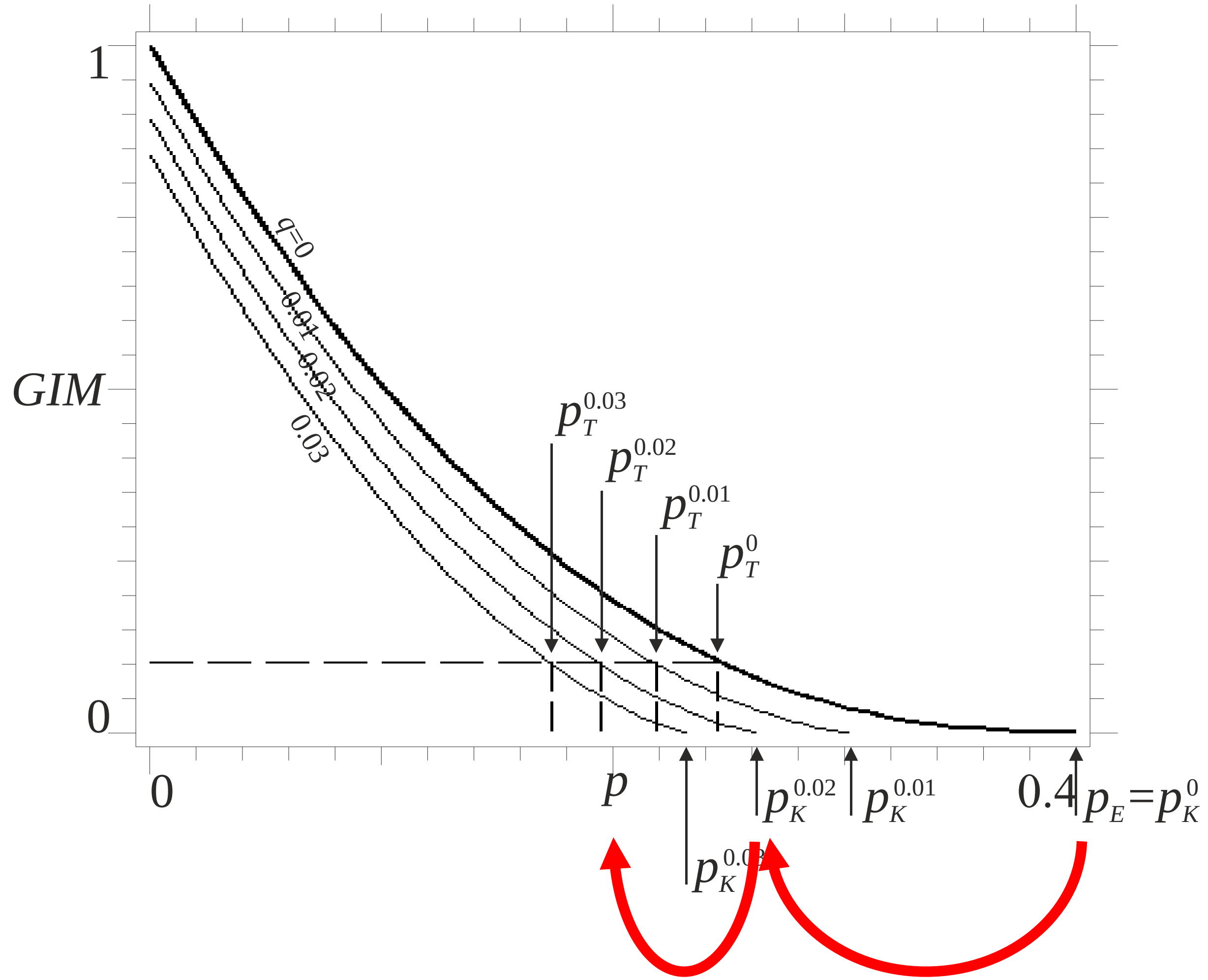
- In the neighbourhood of p_E the **safe** region is merely **residual** and unsafe ⟶ **practical p_T** ('Thompson') critical load **much less** than p_E (Euler)

Area decrement with ‘static’ imperfections

Same qualitative behaviour

($q \neq 0$)

practical p_T (‘Thompson’) critical load also **lower** than p_K (Koiter)



For q = 0.02 ⟶ From Koiter to Thompson

From Euler to Koiter

With dynamic excitation

- What happens when a dynamic excitation is applied, e.g., $q+q_1 \sin(\omega t)$?

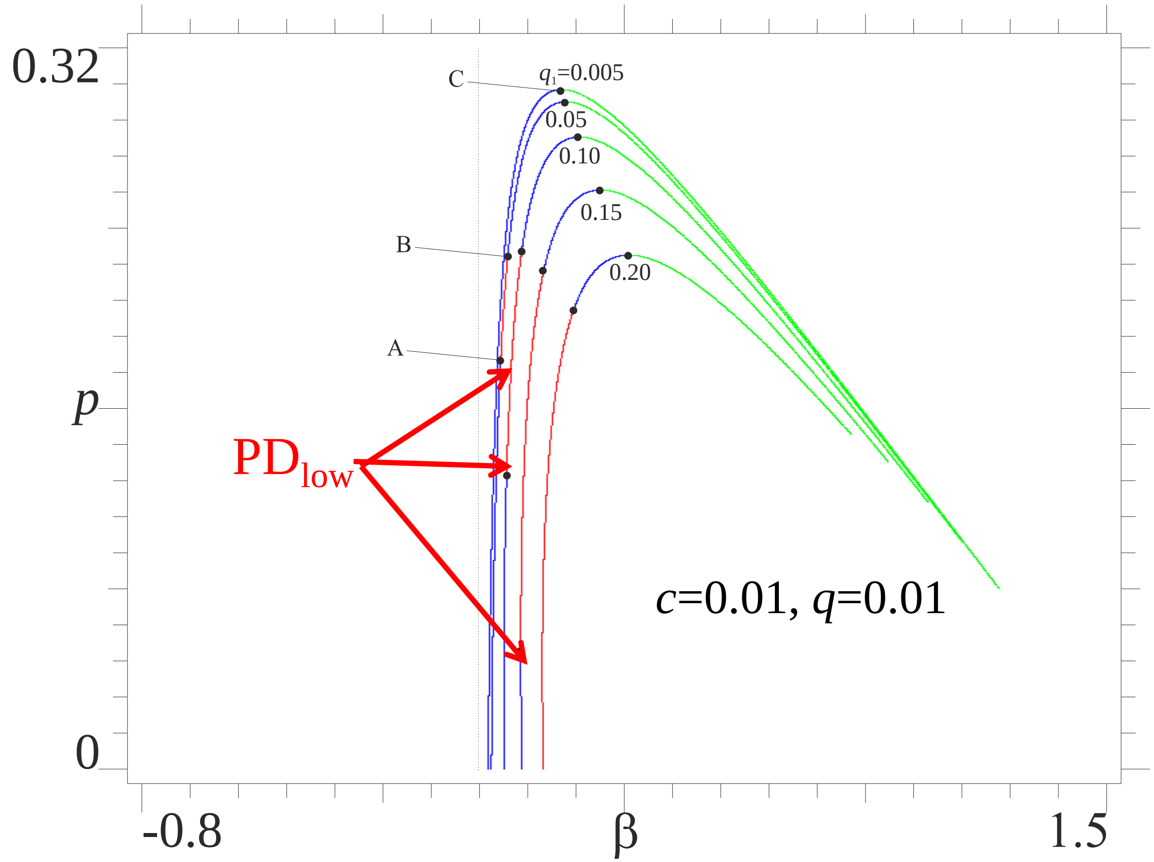
$$\ddot{\beta} + c\dot{\beta} - p\sin(\beta) + \left[1 - \frac{1}{\sqrt{1+\alpha\sin(\beta)}} - (q + q_1\sin(\omega t))\right]\cos(\beta) = 0$$

- The phase space augments of one dimension, but this is not a problem, and can be overcome, e.g., by considering Poincaré sections

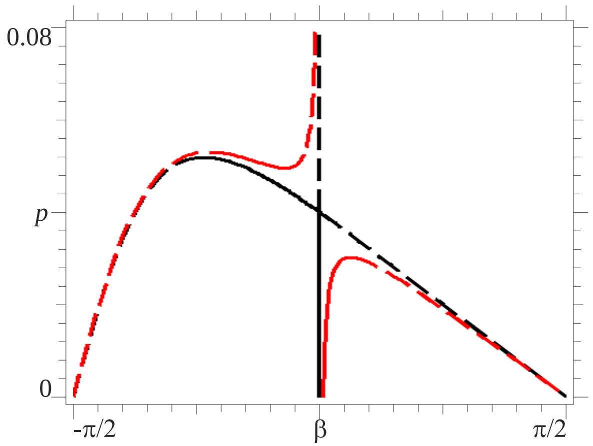
- Also damping is added for realistic engineering analysis

Periodic solutions

($q_1 \neq 0$)



Reference case without dynamical excitation (q_1=0)



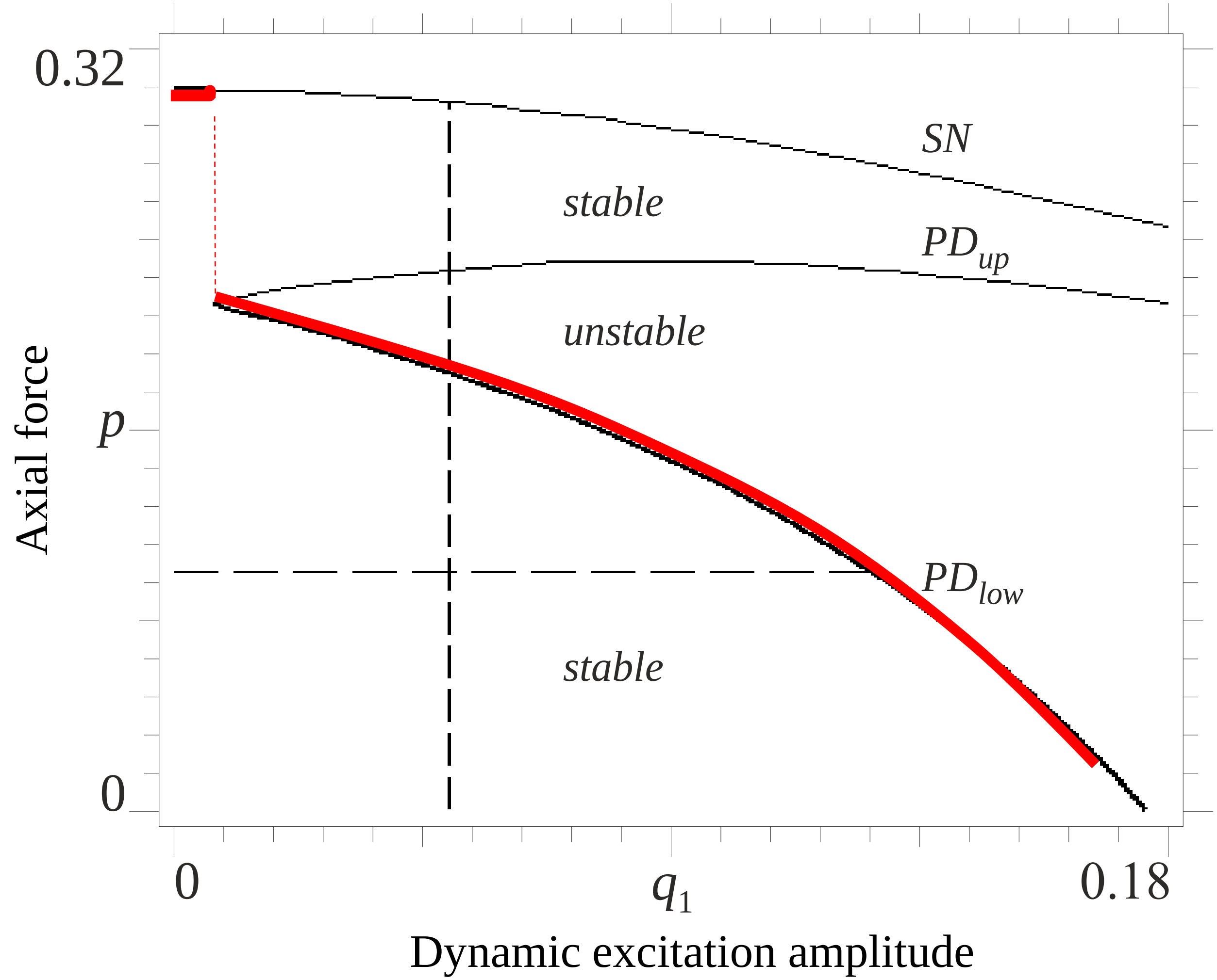
- The saddle-node (SN) decreases by increasing q_1
- A period doubling (PD) reduces the stability threshold (above PD$_{low}$ the solution may jump out of well)

Stability threshold with dynamic excitation

$q+q_1 \sin(\omega t)$

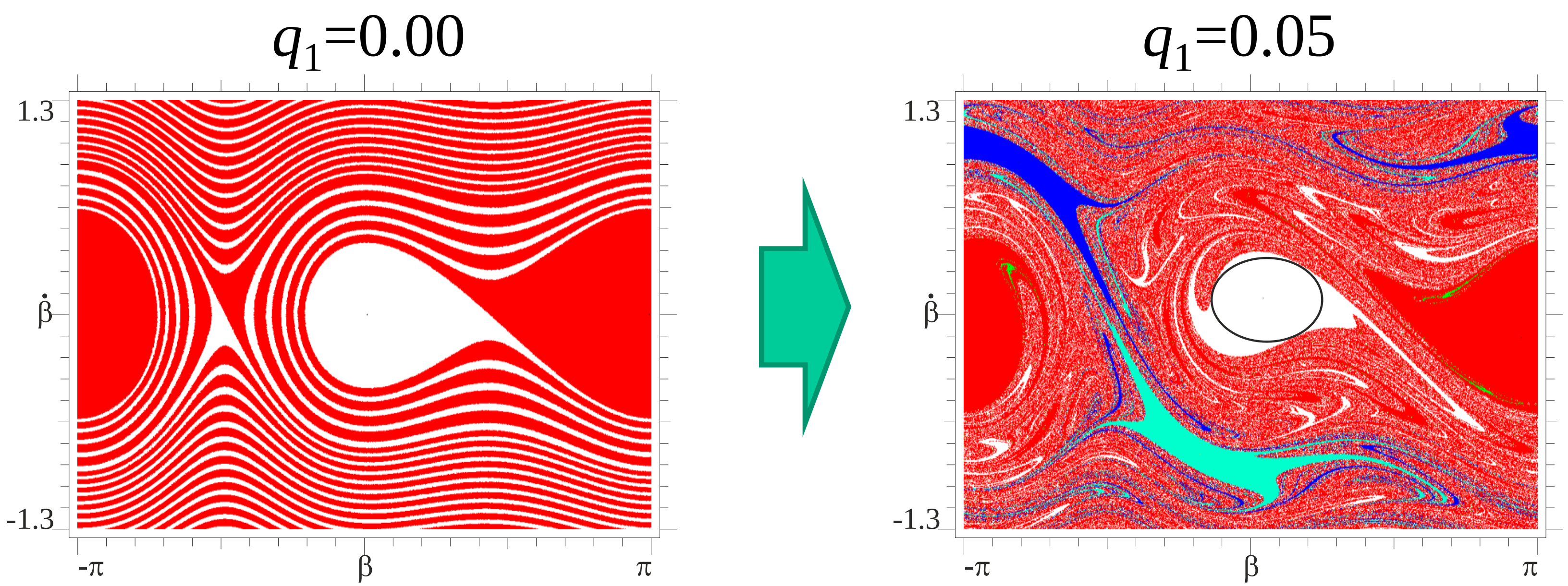
- **periodic,**
- **quasiperiodic,**
- **chaotic**

attractors



- **Interaction** between **static** (p) and **dynamic** (q_1) loads causes meaningful **loss of load carrying capacity** (w.r.t. Koiter one)

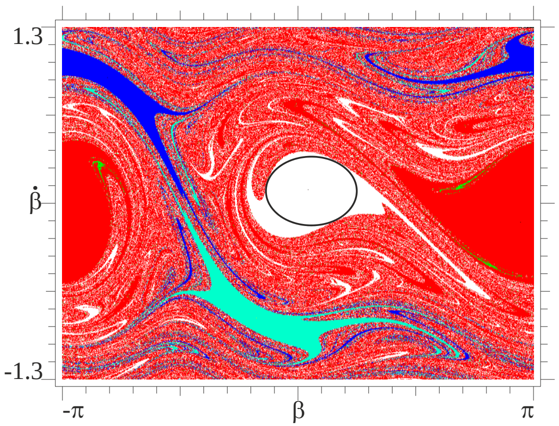
Fractalization



- Existence/competition of **more attractors**
- Basin of attraction **no longer safe** against small but **finite** incidental changes of i.c.
- Basin is **eroded** and **loses** its **compactness/integrity**
- **Load carrying capacity** depends on **practical stability** under **imperfections/perturbations**

Major effects of dynamic excitations

- Attractors are no longer equilibrium points, but **periodic, quasi-periodic, … chaotic orbits**
- The topology of the basins of attraction changes significantly; **fractality** commonly appears
- **Dynamic integrity:**
  ⟶ a **major role**
  in determining the
  load carrying capacity



Practical stability under **imperfections/perturbations**

W.r. to **dynamic** imperfections: ***initial conditions*** in ***phase space***

solution/attractor robustness and basin properties

Static solution: **robust** if **large** safe basin

Dynamic attractor: **- robust** if **large** and **compact** (i.e. **integer**) basin

- non-robust if **large** but **fractal** basin

W.r. to **system** imperfections: ***parameters*** in ***control space***

how **solution/attractor robustness and basin compactness** in phase space **evolve with a varying control parameter**

Static solution: **robustness** profile of safe basin

Dynamic attractor: **- robustness** profiles of (integer) competing basins

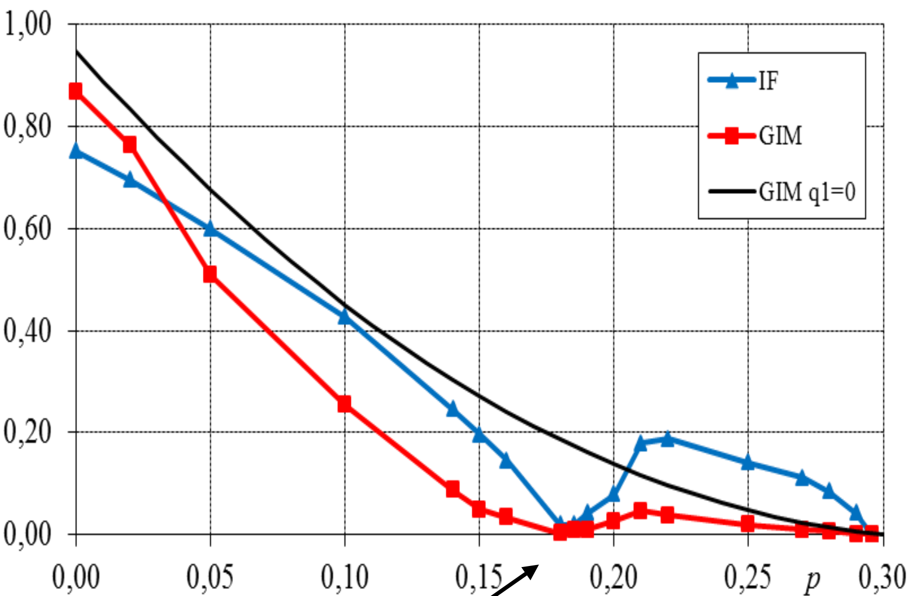
- erosion profile with **integrity reduction**

Robustness profiles: size reduction/increase of integer basin vs competing one

Erosion profiles: reduction of basin integrity, to be explained also in terms of global bifurcation phenomena (homo/heteroclinic tangencies, crises, etc.)

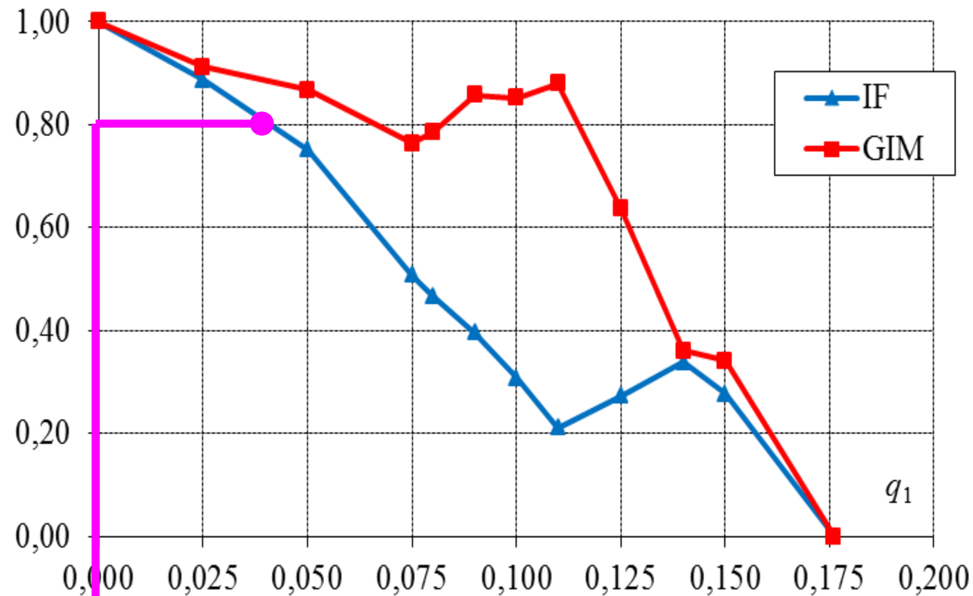
Robustness profile	Erosion profile

Increasing axial load, fixed dynamic excitation



- **Practical** (Thompson) stability threshold about **1/3** of **theoretical** (Koiter) critical load

Increasing dynamic excitation, fixed axial load



- **Practical** (Thompson) load carrying capacity **much lower** than Koiter one, e.g.:

residual ***IF*=80%** (practically uneroded basins) → Thompson threshold = **22 %** Koiter threshold

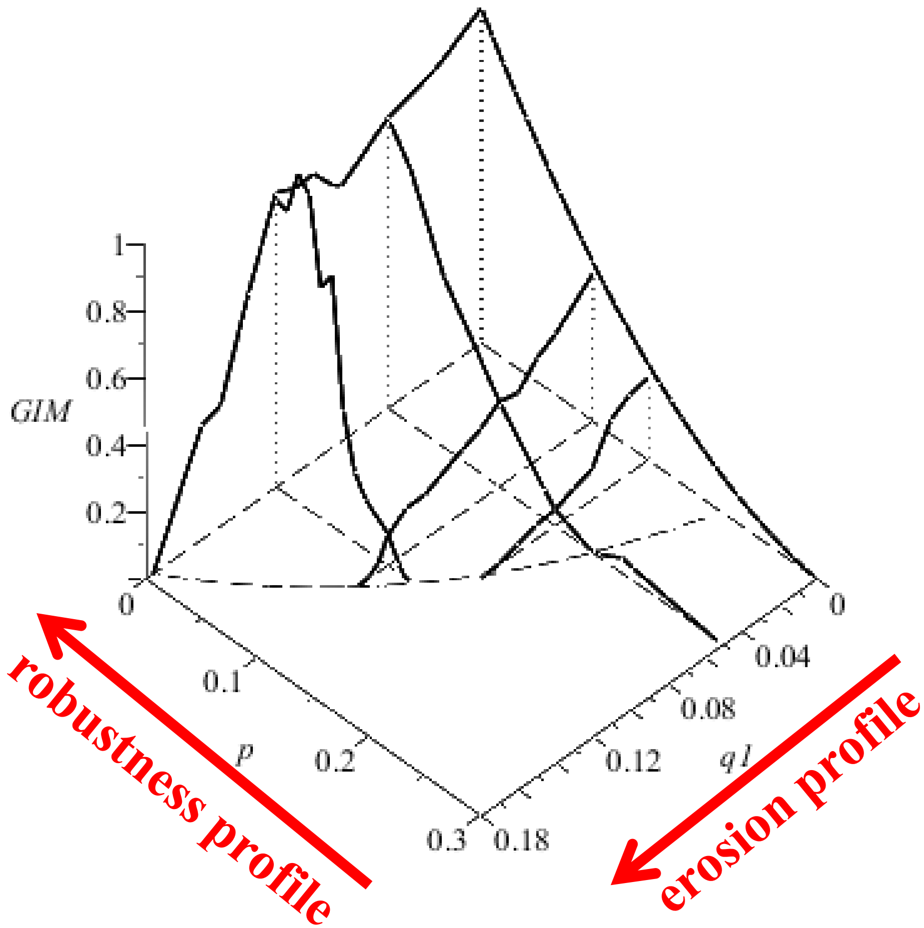
Hints for design

- Koiter load can be determined upon fixing the value of the expected static imperfection q
- Thompson load can be determined upon fixing the acceptable **minimal integrity**

  (which corresponds to fixing the maximum allowed change in i.c. that can be safely supported by the system; in other words, this corresponds to fixing the "**safety factor**")

- Both Koiter and Thompson theories are thus 'applicable' with the knowledge of q and GIM

A summary interaction picture

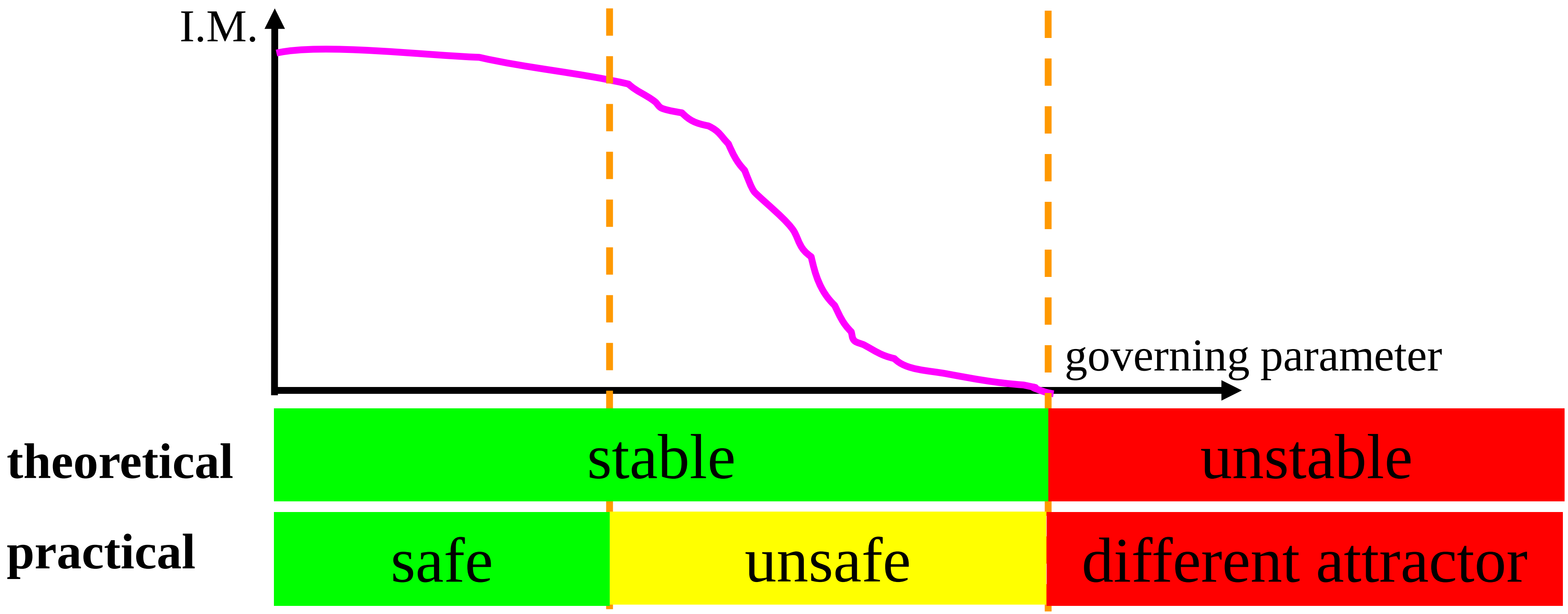


- **dynamic** excitation **reduces** Koiter practical critical load
- **static** axial load **reduces** Thompson escape dynamic excitation

- **Interaction** of **static axial load** and **dynamic excitation**
- **Dangerously residual** robustness/compactness occurs **well before disappearance** of solution/attractor

Theoretical vs practical stability

end of **robustness/erosion** profile corresponds to attractor disappearance, i.e. to **loss of stability**



threshold computed (approximated) by a local (global) bifurcation analysis

dynamic integrity** is **necessary

threshold easily computed by local stability analysis