Cellular Cavity Structure and its Building Technology for Shell Structure with Thin Sheet Materials

Geometrical Analysis and Structural Consideration in the Design and Building Processes

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Genehmigte Dissertation zur Erlangung des akademischen Grades Doktor-Ingenieur (Dr.-Ing.) im Fachbereich Architektur der Technischen Universität Darmstadt

Einreichung: 21.03.2017
Tag der muendlichen Pruefung: 23.05.2017

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Darmstadt 2017
Hochschulkennziffer D17

Verfassererklärung

Ich versichere hiermit, dass die vorliegende Dissertation:
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Acknowledgements

First of all, I would like to thank my supervisor, Prof. Dr.-Ing. Oliver Tessmann, for guiding my research and providing kindly supports in the toughest time. I am extremely grateful for his supervision throughout my research and for giving me opportunity to finish my Ph.D. study within the fruitful and inspiring environment of the Digital Design Unit in Technische Universität Darmstadt. I am particularly thankful to him for giving me sufficient time to guide my final writing of the dissertation and also for pushing me towards some fruitful paths. I want also to thank him for his generosity and all the resources he has provided me, including the opportunity to learn and use the robot arms, 3d printers and also the opportunity to teach in the seminars in his institute. His contagious enthusiasm in design and researches has given me an example to pursue in my future career. It is my great honor to have these years to work together with him and also him fantastic team at the research group.

Furthermore, I would like to express my gratitude to my second supervisor, Prof. Dr.-Ing. Jens Schneider. He has not only supported me with his valuable and inspiring suggestion and advices during my whole Ph.D. study, but also provided me many privileges to use the testing equipment in his institute as well as his personal contact to many relevant experts in the related fields. I am deeply thankful for his help to offer me the opportunity to have a cooperation design research with his master student Mr. Roland Werth in the design of my final pavilion. With the communications with the engineering students, I have learned a lot and also known the limitation and restriction of the thinking methods from both subjects.

I would also like to thank my former supervisor, Mr. Moritz Hauschild, for his two years' guiding of my previous research. Although the result is disappointing, he still has offered me the first opportunity to come to Darmstadt and start my personal research. Through the co-work with him and the academic debates, he helped me to firm my belief to pursue the real truth, instead of being sophisticated and compromising. Without the so many rethinking of the argument, I cannot enjoy my study so well and learn so many knowledge in various fields.

I would also like to Mrs. Juliane Huge, Mr. Peter Maier, Mr. Andreas Benz, Prof. Ariel Auslender and also Prof. Wolfgang Lorch from the Department of architecture. Only with their generous help I can finish my final project with the built pavilion. I would also like to mention Mr. Roland Werth, for his co-work in the structural analysis of the pavilion. At same time, I would also like to show my gratitude to every friend and student who has offered their help in the building process of the pavilion.

At last, I would like to show my most sincerely thanks to my wife. It is only with her steady support and believe that I can overcome so many obstacles and finish the impossible task in the Ph.D. study. Thanks very much for your endless support and being by my side!

Late in the night, 20.03.2017, in Darmstadt

Contents

Abstract

This research investigates a new structure concept and its building techniques for shell structures that can be built with thin sheet materials.

Shell structures have been a desired structure type for both architects and structural engineers due to its efficient load-bearing behavior and also the elegant curved geometry. In the past decades, innovational structure concepts such as grid-shells and spatial framework structures have continuously pushed the development of shell structures towards a comprehensive type of structures which provides a better structural behavior of building materials.

This dissertation argues for a material-driven design methodology of the new shell structure design through a well-rounded revisit of the development of shells in the past decades. And a new type of industrial materials has been selected for the innovational structure concept of the novel type of shells.

The research is based on a comprehensive analysis of the materials' application in the existing industrial and architectural design, as well as the theoretical analysis of the materials' behavior and the corresponding fabrication techniques and structural consideration in designs. A novel structure concept is then generated with also the inspiration of the natural cellular structures.

The main focus of the architectural analysis is the design procedure of the cellular cavity structure, including the form-finding process, the structural consideration of the stability problem of thin materials, the possible simple fabrication with the materials form a planar sheet form. At the same time, a collaborative design process is also discussed to enable a better cooperation between both architects and engineers.

A novel shell was finally built as a demonstrator of such a structure concept. With a dome which reaches a span of 5 meters and a height of 2.5 meters and was built only with 1mm paperboard as major materials, the load-bearing capacity and the construction details of the cellular cavity structure is tested and evaluated.

Finally, through the comparison with the recent parallel research and the popular experimental shell designs in the past several years, the cellular cavity structure is positioned into a series of relevant research fields. With the analysis of the limitations of this research, future works are also defined for the improvement and future development of this Ph.D. research.

Keywords:

Shell structure, thin sheet materials, cellular cavity, form-finding, design procedure, design variations, fabrication and assembly

1. Introduction

The first chapter of this dissertation introduces the main topic of this Ph.D. research, the establishment of the research questions and the internal logic.

The design of shell structures has been always considered as a special design field for both architectural design and structural engineering. However, due to its attractive curved forms and the efficient load-bearing behavior, shell structures are always welcomed and pursued by both the architects and engineers.

The research in the structural design of shell structures has established various tools to create basic prototypes and solve well-defined problems, so that a scientific analysis of the materials' property and the structure's behavior can be achieved. However, as this relies on the high degree of abstraction of the structural system and the precise definition of the problems, most of the tools and the research methods are only suitable in the structural analysis and the calculation for some complicated shells.

On the contrary, the architectural experiments in the shell designs are often standing on the point to pursue novelty in the geometry forms and innovational use of materials with new design tools in computer-aided design (CAD) methods. Such design tools in a structural design are usually individualized and based on the geometric logic.

This thesis seeks to find a possible way to establish a new structure concept for the shell structure, where the considerations from both realms can be simultaneously taken care of and a collaborative design procedure can be carried out between both architects and structural engineers. The goal is to make an integration of the powerful tools and theories form the structural design and the comprehensive criterion network with the consideration of the function, aesthetics, materialization and also the novelty.

1.1 Thesis statement

The design procedures and strategy of shell structures shows their great differences between architects and the structural engineers. This is mainly due to the different problems and objectives they have in the different design areas. Structural design concentrate itself mainly on the accuracy to estimate the structure's behavior and to provide the maximum safety and determinability while its architectural counterpart focuses more on the formal logic and the aesthetical expressions due to the individuality of the designers.

It is hard to find a bridge between the two realms unless a common emphasis can be found. To solve this problem, a first series of research questions are generated for the research.

---- What is shell theory and how do shell structures work?
---- What are the existing design theories and building methods of shell structures?
---- How have shell structures developed and how did architects do the design in the past?
---- How has the design method of shell structures been changed and what is the cause of the change?

With the establishment of the first research questions a rounded research of the development of the shell structure in the past decades has been made to find the connections of the architectural design and the structural considerations and the way how they are integrated in past shell designs. The pioneer works of the shell structure in the past have shown an answer to the research questions and was well expressed by the famous sentence from Eladio Dieste: “The resistant virtues of the structures that we seek depend on their form; it is through their form that they are stable, not because of the awkward accumulation of material”. (Allen & Zalewski, 2012)

The understanding of the most efficient way to transfer the loads in the structure by means of axial forces instead of bending has shown to be the very first and basic understanding the structural rule of the shell design. A morphological application of such a rule can also be later widely found in the shell experiments by pioneer shell designers such as Robert Maillart, Pier Luigi Nervi, Eduardo Torroja, Felix Candela and Heinz Isler, etc. In the pure structural analysis of the shell theory, such a characteristic is also mentioned as the most important feature of shell structures—the membrane behavior.

The method of manipulating the axial force flows in the structure system with the morphological designs of the shell forms can be considered the first feasible way to establish a collaborating design procedure for the shell designers. This has also lead to the rapid and fruitful development of the form-finding techniques of shell structures. As a powerful tool to give the architects a direct feedback with the structural consideration, such tools have been continually used by famous shell designers such as Antonio Gaudi, Heinz Isler, Frei Otto and so on. Nowadays, with the rapid development of the CAD design tools in various software in architectural design, many novel methods for controlling the form through a structurally-informed design have been applied in the new trend of shell designs.

However, when a well-rounded revisit of the development of the shell structure across the past century is made, it is still hard to choose a basic structure typology to start to use and develop the form finding techniques. It is easy to discover that the shell structures have been renovated for several time due to the application of new materials in the architectural designs. The efficiency is always the center topic of shell structures, as the structure itself shows the most efficient way to transfer the load in an elegant structure. It can be seen that many old types of shell are replaced one by one by the new forms which has been brought by the new materials. The most traditional masonry shells have been replaced by the modern concrete shells due to low cost in the processing methods and the availability for more free forms. The concrete shells have also once been replaced by the timber grid-shells and the spatial frameworks with the iron and steel component because of the benefits of the industrialization and the save of costs for the labor-consuming fabrication of moulds and scaffolds.

The cognition of the materials’ impact on the typology of the shell structures has also led to the second series of research questions in this research:

---- How have different materials been used in past shell designs and what is the suitable design method with the consideration of materials’ performance?
---- Are there some modern materials that have not been used in the shell structure designs?
---- How could a structure concept be established due to the materials’ behavior?
---- What should be the most important considerations in the morphological design process for

the generating of the structural form?

With this series of research questions a new kind of materials—the thin sheet material—is selected for the innovation structure concepts for shells. The fundamental mechanical behavior of such materials is researched and the existing industrial fabricating techniques are also summarized.

Based on all the above-mentioned internal logic, the new structure concept—the cellular cavity structure—is established with a thorough analysis of its geometry and the structural consideration of both the structure and the materials. The central question about the form finding techniques is then analyzed and introduced with case studies.

For the materiality of the structure, the fabrication techniques of the structure concept are also discussed with a final built experiment as a demonstrator of the feasibility of the structure concept.

Furthermore, the final purpose of this study is still to generate a suitable comprehensive design procedure to enable a collaborative design between architects and engineers. As there are always multiple criteria and a compromising as well as reciprocating process in the architectural design process, an iterative design process with the form-finding, shape optimization and also the structurally-informed revision of form is summarized as the final conclusion of this research.

1.2 Motivation and personal background

The author is educated as an architect with bachelor and master studies, and this dissertation is hence also made in an architectural-dominant perspective.

The initial motivation of the research on shell structures is from the master study of the author in China. Having participated in several digital design projects, the author has gotten his interest in the novel architectural form and the brand new digital design tools for the shell structures with double curvatures.

The motivation has also come from the reading and previous research in the digital design methods, where some form finding techniques and the geometrical optimization techniques have been studied as case studies in the algorithmic design procedure.

The aim of this research is mainly to establish a more efficient and suitable structural concept of shell structure with the thin sheet materials. So the main work will be concentrated on the geometrical analysis and the generation of the design system. At the same time, the structural points mentioned in this dissertation mainly serve for the analysis and the establishment of the structure concept in this study.

Consequently, this research defines its aim mainly to make its contribution to the architectural design experiments for the shell structure. No goals are made to make new innovations purely for the structural theory of shell structures. The necessary structural analysis and tests are done only to evaluate and testify the structural behavior of the cellular structure in this dissertation. A feedback and communication with experts from structural engineering and a collaborative research is considered as also an important criterion for the design of the final pavilion.

1.3 Key terminology

Some important terms and concepts which are frequently discussed in the dissertation will be briefly introduced in this section.

Cellular Cavity

The term cellular cavity is the specific summarization of the main characteristic of this innovative structural concept. Unlike the traditional shell structures, the design and building considerations are more from a discrete aspect of view. The shell space, in this concept, is tessellated into plenty of cellular structural elements, hence the structure works like a cellular structure. At the same time, the cellular elements are not in a solid form as the traditional masonry shell. Because the concept is designed specifically for the thin sheet material, the form of the cells are generated into a hollow form which is covered by membranes, the term "cavity" is used here in this dissertation.

Shell structure and membrane behavior

The shell structure mentioned in this dissertation refers to a series of structures which work similarly with an efficient load-bearing behavior. Due to the curvature of the structure itself, the load can be transferred mainly by the in-plane stresses in the shell surface instead of the bending moments. This behavior is also called membrane behavior, which is also introduced in detail in section 2.1.1.1.

Structural morphology

The word "morphology" comes originally from the same term in the analysis of biology. It focuses mainly on the study of the form and structure of organisms and their specific structural features. In the recent structural analysis and design of shell structures, this term has been brought back firstly in the IASS (International Association of Shell and spatial Structures) workgroup. Such term describes a design consideration in the shell structures by manipulating the variations of form according to the basic static rules and the materials' behavior.

Discrete and discretization

The term discrete and discretization here are used to define the characteristic of the structure which are not described and analyzed as a continuous structure with a traditional surface geometry. The discrete describes a way to design and analyze a structure with discontinuous system such as particle system and meshes. In the building process, it mainly assumes structures from the aggregation of cellular or modular elements without interconnections such as screw and gluing. In the final discussion of the fabrication technique, some connection is applied to deal with the possible deformation caused by the bending moment. This works as a supplement of the construction details, but not as a necessity of the structural requirements or the prerequisite of the design process.

Form finding

The term form finding is used here firstly based on the common definition of "form-finding" in shell structure designs, where natural systems are used as bases of processes and its deep principles of structural theory and growth are used as the methods to generate the basic rule to define the shape of shell structure.

There are mainly two types of form-finding techniques for shell structures--- the physical method and digital methods. Since ancient times, physical simulation methods of form-finding for shells are applied with many shell masters such as Antonio Gaudi(Nonell, 2000), Heinz Isler(Chilton & Isler, 2000) and Frei Otto (F. Otto, 1954). By observing the structures in nature and learning its principles, it has been researched that many structures in nature are following the efficient way of energy and structure (Thompson & Bonner, 2014). The digital methods are established later according to the understanding of the physical methods, and modern developments mathematics are used to fulfill the simulation in computer. Some form-finding techniques in digital ways are recently explained in several studies (Adriaenssens, Block, Veenendaal, & Williams, 2014)

For the cellular structures in this research, the form finding refers to several successive procedures to determine the final form of the different parts of the structure. Such procedure is from the architectural side most important to the final form of the structure. The related techniques are therefore also considered as a center part of the analysis.

Design procedure

The term design procedure is used for an integrated system of the geometric analysis of the innovational structure concept. It helps to describe the complicated and comprehensive considerations in the design process and their relationship to each other. The design procedure in the shell structure design should be a reciprocate and compromising process, where the designed form should be revised and improved continuously together with the consideration of other criteria and the structural feedback in the design and analysis process.
A design procedure is necessary for any artificial structures including all kinds of shell. Nowadays, the design procedure is becoming more and more complicated and many experts in related fields will also come into the decision-making process. This process, including the collaborative working method for architects and engineers, is also the main objective of this research.

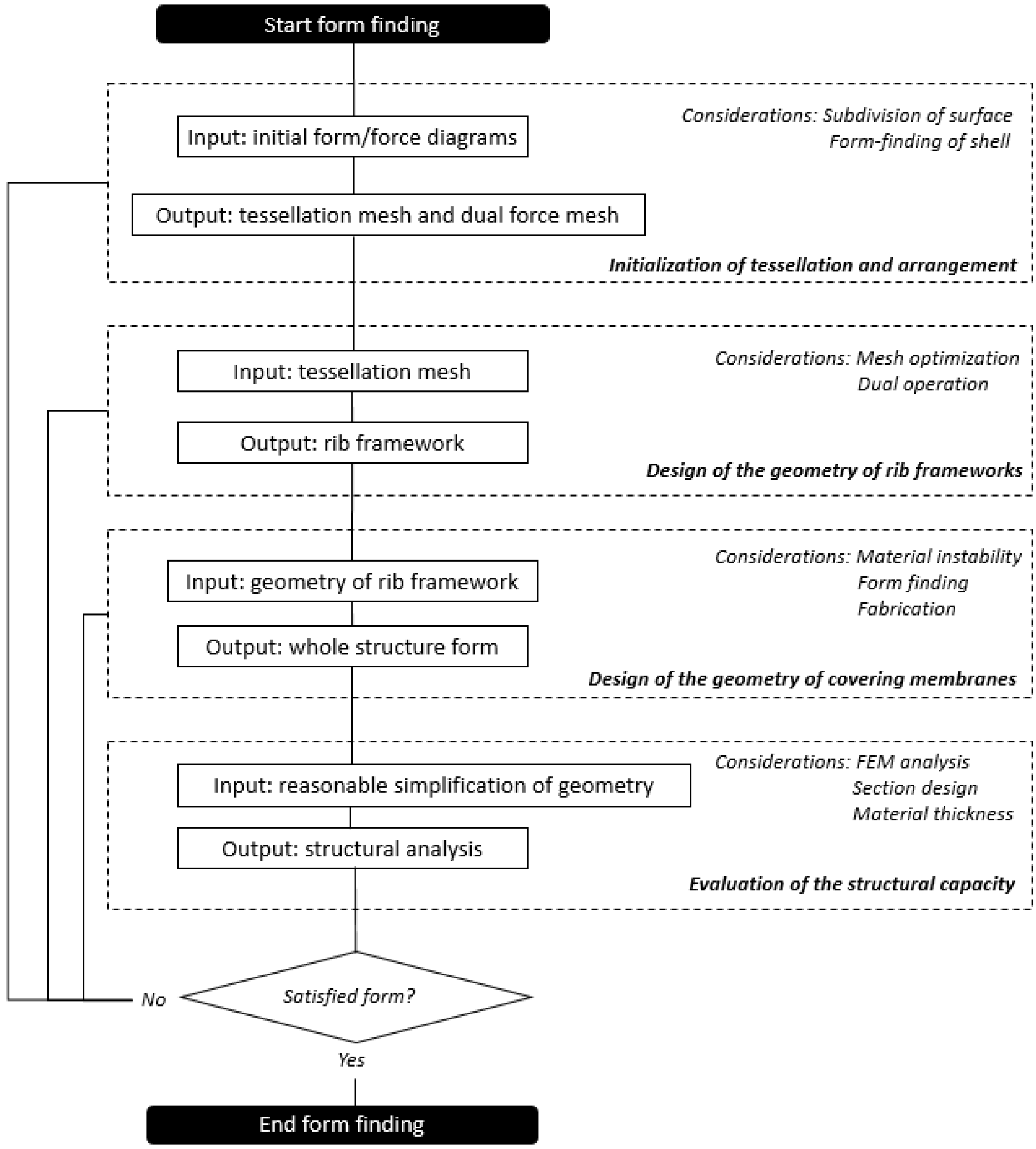


Figure 1.1 Simplified loop analysis of the design procedure

1.4 Thesis structure

Chapter 2: State of research

In the second chapter a well-rounded introduction of the related background studies about the shell structure theory and the design development of shell structures is given as a basis of the discussion of the research.
Apart from the ordinary description of the shell theory in the first part, a special point of view according to the structural morphology is made for the further discussion. As a conclusion, a design methodology based on the material-performance is introduced.

In the last part, a special development with cellular structures is introduced. This provides the background of understanding the basis of the cellular system design and also the previous studies of the nature-inspired morphogenesis.

Chapter 3: Fundamentals of thin sheet materials

In the third chapter an architectural and engineering analysis of the selected materials is presented. In the first part, the application of the new efficient thin sheet materials in the industrial design as well as the architectural design is reviewed and the characteristics in the form language is summarized. In the second part, the engineering analysis based on the related literature is reviewed. The similarity of such materials is described and the major problems that need to be considered in the structural design process, such as the bending and buckling problem, are emphasized.

For a reasonable design of the cellular structures with thin sheet materials, the architectonic approach for the design of such materials is suggested. Both the geometric consideration and the fabrication methods are introduced. For a complicated system composed of plenty of cells, the joining technique and the concepts of structure typology are also discussed as the fundamentals of the establishment of the structure concept.

Chapter 4: Cellular Cavity- the structure concept

In the fourth chapter, the whole development and analysis of the cellular cavity structure in this dissertation are presented.

The study of the design is based on a building project with the goals to use such materials to build a small full-scale pavilion. The foregoing structural experiments and the forming of the cellular cavity structure is firstly analyzed.

In the next sections in this chapter, the general analysis of the structure and the detailed form finding techniques are shown with simple diagrams. Related form finding techniques are introduced and also evaluated with different case studies. Based on the multiple possibilities to define the form and the consideration of the stability of the structure, three typical variations are also analyzed.

In the structural analysis for the stability problem, both simulations with the FEA software and pressing tests with full-scale physical models are done and compared. The difference in the design/ fabrication difficulty and the stability of the structure is summarized as the conclusion.

In the last part of this chapter, a comprehensive design procedure for such a structure concept is discussed. For a better collaborative design between architects and engineers, an iterative design process with the structurally-informed optimization has been argued as a conclusion.

Chapter 5: Building experiment and construction details

In the fifth chapter, a building experiment with a pavilion in a traditional dome form has been shown as a demonstrator of the structure concept. Both the research on the selected materials, the structural analysis, the construction details and the fabrication and assembly processes are illustrated in detail.

Both the material tests and the structural analysis with FEA software is made in the collaborative design process between the author and the master student Roland Werth from the department of civil engineering. The detailed analysis and illustrations are already presented in the Master Thesis of Werth (Werth, 2015b).

The fabrication process concentrates on the construction details of the cells made with thin paperboard with 1mm thickness. The geometric post-processing and the fabrication process are evaluated. At the same time, the final assembly procedure is also discussed.

Chapter 6: Conclusion and future works

In the last chapter, a conclusion of the contributions of this research is summarized. Furthermore, the recent research in the related fields or with the similar solutions is introduced and compared.

In the last part, the limitations of this research is also analyzed. A need of further research and the key points are mentioned for the future jobs of this Ph.D. dissertation.

2. State of research

This chapter reviews the development of shell structures and the recent research fields relevant to this dissertation. The discussion is defined by introducing the preliminaries of shell theory and design principles which are based on the engineering definition of shells. The developments in shell structures are then reviewed from a special aspect of the structural morphology, which concentrates itself on the architectural background philosophy of shell design and a material-based performative design methodology. Finally, the recent cellular structure research is presented to draw forth the main topic and key point of this dissertation.

2.1 Preliminaies of shell theory and shell designs

Shells, being the most ancient and effective structure forms in nature and continuously imitated by human beings, has always attracted both architects and structural engineers with its elegant curved form and outstanding load-bearing behavior. Unlike the other traditional structures which are based on the orthogonal or linear system, shells carry their external loads mainly by the internal forces due to their special form with spatial curvatures and this makes it possible to derive a long span of the structure with a very thin cross section. With this advantage shells are sometimes classified also as form resistant (Farshad, 2013) and surface-active structures (Engel, 1997).

Shells have been researched nowadays in various fields and aspects, from shell theory, geometry form finding to structure typology and building methods with different materials. This part will first introduce the basic research and concepts in general and then lead to the next introduction of the structural typology analysis and developments as background information. For the first step to expand the discussion, the most important principles and concepts are introduced as below.

2.1.1 Behaviors in shell structures

2.1.1.1 Membrane behavior

A shell structure carries the external load with a very special method, which is also called the membrane behavior. If the applied load is added onto the structure, internal stresses will be generated. In a thin shell, due to its curvature of geometry, and as it is discussed by Flügge (Flügge, 2013) as Figure 2.1.1, a membrane stress field will be caused and it enables the shell structure to carry the external even load by means of only in-plane normal and shear stresses and then transfer it to the supports. With its curved thin surface, the possible stress which is normal to the structure is hence much smaller than the internal forces and is therefore negligible.

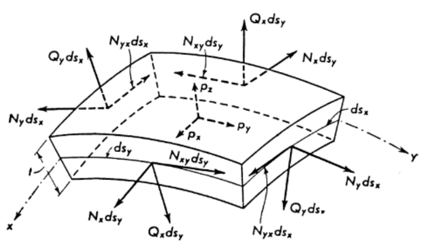


Figure 2.1. 1 Membrane Stress Field in shell element (Flügge, 2013)

This mechanism is very like the load bearing behavior of the membranes and is also very efficient—because the membranes have barely any flexure rigidity compared to the extensional rigidity, the loads can mainly be transferred by way of in-plane stresses. In shells, this behavior is not only determined by its small ratio of cross section to the span, but also by the curved geometry itself.

The membrane behavior is a more efficient way than bending in other structures such as beams, struts and plates, as most of the building materials have much more rigidity against stretching and contraction than bending. Hence that is also the reason why shell structures can be used to create a large span with only a very thin structure. In this sense, for the proper design of a shell structure, it must be allowed that a suitable and possible membrane state can be generated and a membrane stress field can be maintained. When the membrane state is insufficient in specific external load cases, bending behavior will appear and the whole structure will become more sensitive.

2.1.1.2 Bending behavior

The membrane behavior of a shell structure is a much-specified condition and relies much on the boundary conditions of load, support and geometry. In order to apply and maintain a pure membrane behavior in the shell, both the forces and deformation of the shell structure should be maintained as compatible, which means, the sudden change of the forces or a relative large deformation will not be acceptable, otherwise the membrane state will be disturbed.

The bending behavior is a compensation of the membrane behavior in shells, when a disturbance occurs, such as an incompatible support or a deformation constraint, a geometry discontinuity or an application of a sudden concentrated load. However, the bending compensation in the shell structure frequently happens only in a small region where the above disturbance appears, and leaves the other parts of the shell structure still in a membrane state. This special characteristic also helps the shell structure to work efficiently in general and can be applied so frequently in nature and also in human architectures.

The bending behavior as only a compensation relies a lot on the thin surface of the geometry, namely the thin shell. When a thick shell is also added into the discussion, the shell behavior can be more complicated that bending can be also the main load bearing behavior. As a result, the design of shells also should try to satisfy the property that the cross section of the shells should be kept to a small value. This implies a requirement of a double-curved geometry of shell surfaces, from the traditional mathematic surfaces (such as the HP-surface) to the complex curved geometry from the physical, computational form-finding or the topological optimization (Figure 2.1.2).

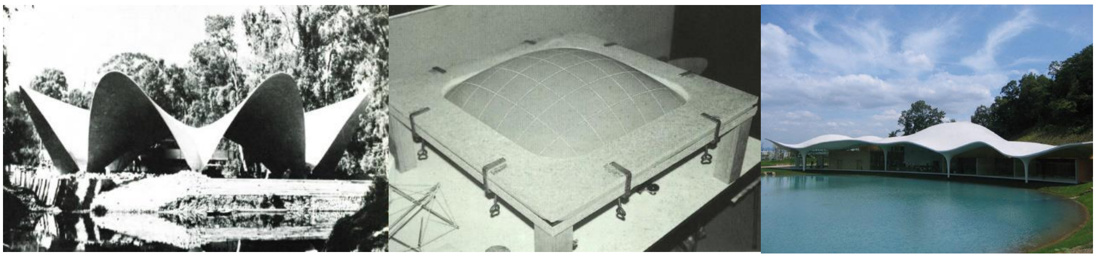


Figure 2.1.2 Double-curved geometry of shell structures

2.1.1.3 Structure failure

As it is described that the shell structure has often a small ratio of thickness to span and stays under a membrane stress state, it is reasonable to consider the most sensible structure failure as the instability problem, which is also called the buckling of the shell. The buckling problem comes from a mathematic bifurcation problem and in structural engineering, it also mentioned a sudden large deformation of the structural system under axial forces such as compression and tension. In shell structures, as the membrane behavior allows that the main forces in the shell are mainly axial forces, similar problem will also be easy to happen due to the thin cross-section of the structural system.

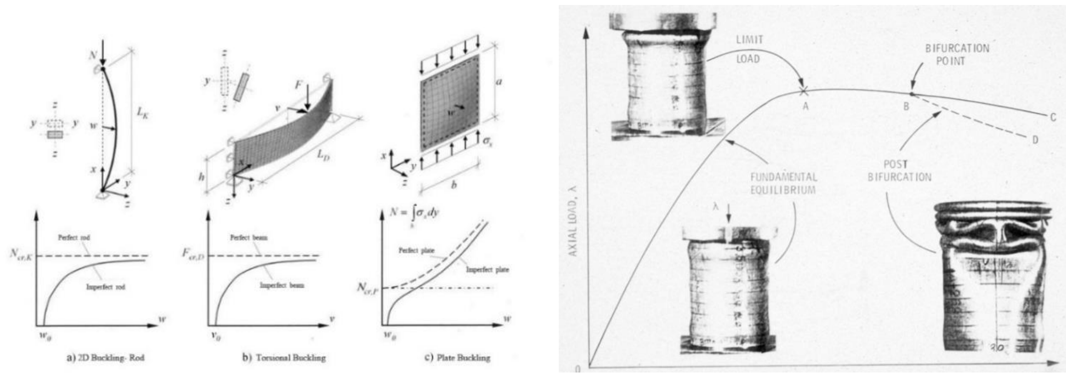


Figure 3.1.3 Buckling behavior of rods, plates and shells

The buckling behavior of the shells can occur due to many reasons, such as the applied load, material nonlinearity, thermal degradation or other imperfections of the shells. Compared to the material strength which is considered the most in the membrane states, the buckling strength of shells is much smaller, hence the shell is a typical imperfection-sensitive structure.

However, it is hard to forecast the buckling behavior of a shell accurately in advance, and therefore the stability analysis of a shell with considerations of imperfection is always important and a material test is also essential.

2.1.1.4 Design Consideration

Due to the special characteristic of shells, the design consideration is also unlike other structure designs. Because the membrane behavior has shown its special importance and can be used as the primary consideration in the initial design and analysis, the first aim of a proper design is also to offer a possible membrane stress field for the structure, and this refers to a suitable curved geometry of design.

At the same time, considering the sensitive behavior against the material property and the buckling behavior, more comprehensive structural analysis is often required for a final check of the whole structure. When the initial design is based on a logical consideration, this analysis should be a proof of the design or give some suggestions for the detail consideration or structural optimization at some local areas.

2.1.2 Geometry of shell structures

Just as the important role that membrane plays in the mechanism of shell structures, the curved geometry is vital to the design of shell structures from the architectural aspects. In this section, the geometry property of the curved surface will be introduced and analyzed.

2.1.2.1 Curvature and classification

Gaussian Curvature and Average Curvature:

The curvature of a curve or a surface can be described and calculated with mathematical methods in differential geometry. The Gaussian Curvature K is such a definition which equals the production of the principle curvatures k_1 and k_2 at a point on a surface:

$$K = k_1 k_2 = \frac{1}{r_1 r_2}$$

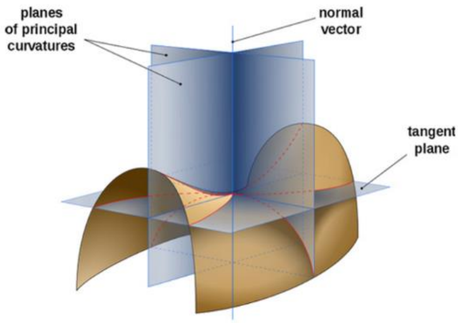


Figure 2.1. 2 Principle curvature of surfaces (Linton, 2012)

By every point on a curved surface in the R3 space, a Cartesian coordinate system can be established with a unit normal vector which is arbitrarily defined. (Figure 2.1.2) This will therefore create a unique direction that is tangent to the surface and enables a cutting plane perpendicular to the tangent plane as the normal plane. With the original surfaces and all the normal planes, intersection curves can be generated. The principle curvatures of surfaces on this point are the maximum and minimum values k1 and k2 of the curvatures. Therefore, they are also reciprocal of the radius of the osculating circle r1 and r2.

At the same time, the curvature can be also calculated as the average curvature:

$$K = \frac{k_1 + k_2}{2}$$

Classification of shell geometry:

Through the curvature analysis of the shell surface, a simple classification can be made into the positive, zero and negative- Gaussian curvature:

1, Synclastic Surface: Shell surfaces with positive Gaussian curvature are called the synclastic surface, which means, both the principle curvatures have the same sign. A typical synclastic shell will be traditional domes. The synclastic shell bears the load through the double curved form and mostly in ways of the meridian and circumferential stresses, and sometimes also the in-shape shear forces.

2, Monoclastic/ Developable: Curved shell surfaces which have only one non-zero principle curvature are called monoclastic surfaces and also developable surfaces. Because there is always one principle curvature that equals zero, it describes that the surfaces are always 'flat' in one direction. The developable surface has a special property that it can be built or reconstructed into a flat surface without any stretching, wrinkle or change of areas. The developable surface is easiest to build and is hence very important in shell structures. On the other hand, the monoclastic shell works always in method as a series of beams and arches, the membrane behavior is therefore reduced. Especially in some cases such as a long cylinder shell, the structure will be fragile in one direction, hence additional supports should be required.

3, Anticlastic Surface: An anticlastic surface is the surface with negative Gaussian curvatures. In this case, the behavior of the structure can be understood as a combination of arches under compression and cables under tension.

4, Minimal Surface: The minimal surface is a special type of surface in geometry whose average curvature always equals zero. The minimal surface is often considered the most efficient form of shell structures and saves energy and materials.

Figure 2.1. 3 classification of shell geometry (Peerdeman, 2008)

2.1.2.2 Continuous geometry of shell structure

The generation of the shell geometry can be divided into the geometric method and the non-geometric method. They refer to two different methods to create curved forms with classical geometry, modern geometry and also physical simulation. In classical shell designs and also classical geometry the curved surfaces remain continuous and can be generated in the analytical way where the generation of curves can be achieved or analyzed into a lower dimension by curves and with some shaping method. The shapes we can get with this method are often called the standard geometry and are dominant in the classical shell theory. In this group there are mainly rotational, translational and ruled surfaces which can be parameterized in the mathematical methods as following. (Pottmann, 2007)

Rotational Surfaces:

rotational surfaces (also called the surfaces of revolution) are the most popular spatial surfaces with the simple principle of generation. They can be built through a spatial rotation of a curve *c* along a spatial line as axis *a*. Through rotation, the structure of the rotational surfaces can be defined as a series of circumferential circles, which are derived from the rotation of every point on the original curve *c*, and also a series of meridian curves, which are the intersection of the section plane that includes the axis *a* and the final surfaces. (Figure 2.1.4 A) Therefore, the rotational surfaces can also be seen as a series of circles and curves which are orthogonal to each other.

The mathematical definition of rotational surfaces is easy to get as the rotational surface is the result of rotating a curve along an axis. If a coordinate system is built with the rotate axis as the Z-axis, and let the curve be laid on the XZ-Plane, then the curve can be defined as $m(v) = (x(v),0,z(v))$, then the final surface can also be defined as：

$x(u,v) = x(v)cos(u)$
$y(u,v) = x(v)sin(u)$

z(u,v) = z(v)

As the principle is easy, rotational surfaces have been most frequently used in ancient shell design. In most cathedrals and domes, the form has been chosen as a half-sphere and ellipsoids. At the same time, hyperboloids have also been widely used in thin-wall shells in modern times.

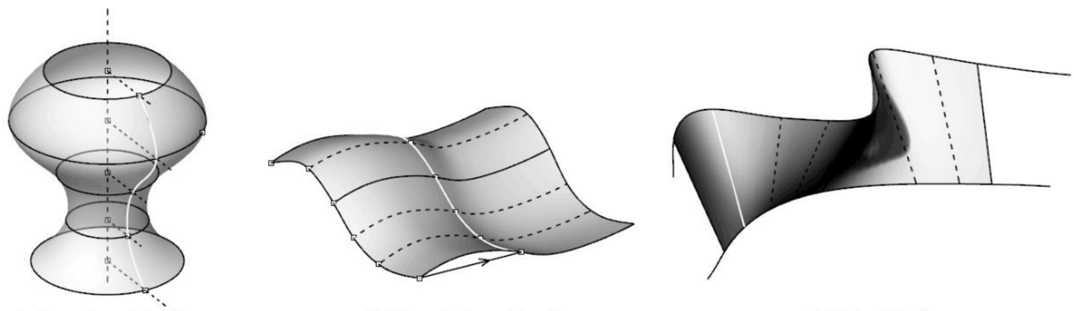


Figure 2.1. 4 Standard curved surfaces (redrawn from diagrams by H.Pottmann) (Pottmann, 2007)

Translational Surfaces:

The generation of the translational surface can be considered as the movement of two spatial curves. If the two curves *a* and *b* are intersecting at point *p*, it is possible to get the surface by swiping either curve along the other curve as the route. (Figure 2.1.4 B) In this way, the points on the translational surface can be calculated using the parameters on the both curves, which are called the *u, v* direction. In geometry, it is possible to describe the surfaces by adding two vectors *p* and *q* which are determined by the *u, v* parameters. If the curves are noted as *k* and *l*, the point *x* on the translational surface is then written as:

x(u,v) = k(u) + l(v)

The cylinder is the most simple and classical type of translational surfaces. It can be derived by translating a planar curve along a perpendicular straight line. In shell geometry, planar curves which are laid on two orthogonal planes are usually used in the form finding by architects.
At the same time, two types of special translational surfaces are the elliptic paraboloids and the hyperbolic paraboloids, which are also rotational ellipsoids and hyperboloids.

Ruled Surfaces:

Ruled surfaces are the surfaces which can be derived by moving and scaling a straight line, which can also be understood as swiping both ends of a line along their own curves. (Figure 2.1.4 C)

The general description of a ruled surface is by connecting a certain point on the double generating curves. The possible form of the ruled surface depending on two arbinary spatial curves can be infinite, because there are numerous ways to make a parametric definition of the curves.

As the ruled surface can be defined as series of straight lines, they are easy and efficient in the fabrication of most shells. In most of the classical and modern shell structures, the HP surfaces (hyperbololic paraboloids) are very popular and also the basic element of shape designs.

2.1.2.3 Discrete geometry of shell structure

Modern geometry of shell structures is developed with a new way of analyzing the geometry of curved surfaces. It is also catalyzed by the attempts and demands of projects of free-form structures. In this part, the typical category and application of methods and its derived geometry will be introduced. Then the details can be better analyzed with case studies in the next part.

Physical Simulation:

Using physical models to find the suitable forms for structures can be first dated back to Antonio Gaudi's work in his architecture works and has been also experimented with and explored in shell structures by later architects and engineers, such as Heinz Isler (Chilton & Isler, 2000) and Frei Otto (F. Otto, 1954). By observing the structures in nature and learning its principles, it has been researched that many structures in nature are following the efficient way of energy and structure (Thompson & Bonner, 2014).

Pneumatic forms are the ancient way of physical forms that interest architects. By inflating or adding water into membranes it is possible to find the final form which is influenced by the pressure. This technique has also been used by Isler in his shell experiments. (Figure 2.1.5 B) Another form which is often used is the minimal surface, which can be gotten by using bubble films with prefixed boundaries. The film will stay finally in a state where the area of membrane is minimal and hence an energy-efficient form.

The most popular physical method in shell designs is the hanging technique. By using a chain net or fabrics and adding weights onto every joint, it is possible to get a tension-only form with a curved surface. As the net cannot bear the bending moments, this bending-free geometry can also be used in shell designs only by making a reverse form. (Figure 2.1.5 A)

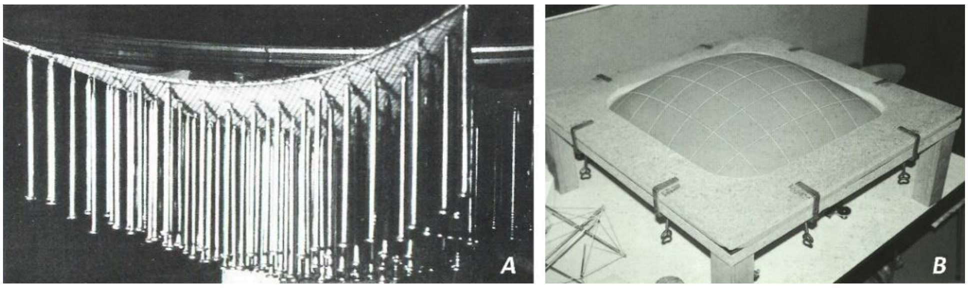


Figure 2.1. 5 Physical simulation(A: hanging and B: pneumatic) of shell forms (A: (F Otto, Hennicke, & Matsushita, 1974) B: (Chilton & Isler, 2000))

B-Spline Surfaces and NURBS Surfaces:

Along with the requirements or interests in curved forms in the industry and also the development of the Computer Aided Geometry Design (CAGD), new techniques of controlling the generating of surfaces in engineering projects have been invented. B-spline and NURBS (non-uniform rational B-spline) surfaces offer a new way to control free-form surfaces only from a series of splines which are defined by groups of control points. In this way, the algorithms of the control curves can be derived by limited points and the surfaces can be controlled or modified in a powerful way. With the recent rapid development of the CAD and CAM techniques in shell designs, this method is widely used in the description of the shell geometry.

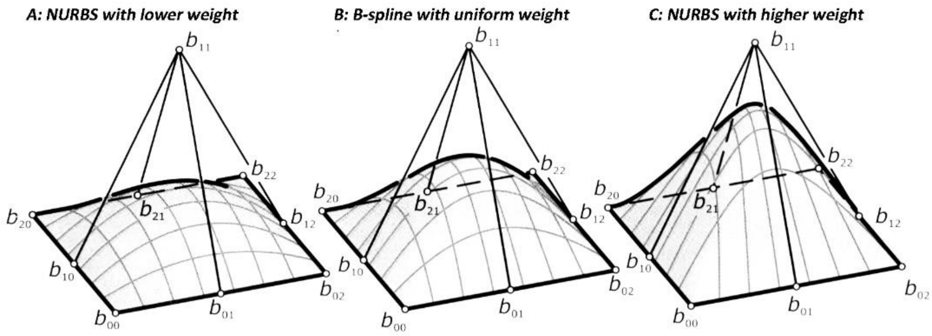


Figure 2.1. 6 B-spline and NURBS surfaces(redrawn from diagrams by H.Pottmann) (Pottmann, 2007)

Mesh Surfaces:

Although smooth surfaces can be parameterized and calculated in a computer, it will often take too much time to deal with so much information for a complex design. More importantly, except for several traditional surfaces, the complicated smooth geometry cannot be easily built with present technology. The mesh surface gives a solution to this problem by making a complete discrete way to represent a smooth geometry. By adding lines between the vertices on the surfaces, a grid system can be derived and the faces can also be defined by some closing boundary polygons.

The mesh surface is the most popular computational way to define a curved surface. At the same time, due to its highly discrete property, it has also been widely used in engineering simulations for static analysis. With such advantages, it is also an important method in the geometry analysis of recent shell research.

2.1.3 Design considerations of shell structures

In this part, the basic design procedures and problems will be discussed and popular methods will be introduced.

2.1.3.1 The Geometry and the form

As it is mentioned in above sections that the membrane behavior plays the vital role in the thin shell structure, the above-mentioned form-finding process also plays the most important role in the design of shell structures to activate this behavior. The aim of it is to find a suitable surface from which a proper membrane stress field can be generated by certain external forces and boundary conditions. An ideal form finding will determine the efficiency of a shell structure. As the basic definition is explained in Chapter 1, this section will illustrate the state-of-the-art techniques of such methods.

The physical form finding method is developed from the old hanging chain method according to Hooke's inverted catenary chain (Figure 2.1.7). Considering that most materials are efficient especially under compression or tension, this method has been invented to simulate a pure compression state for the masonry or brick arch.

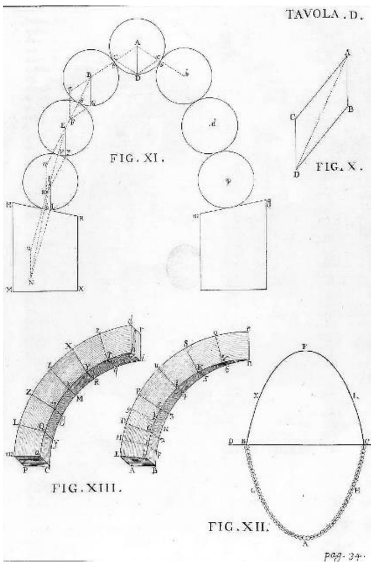


Figure 2.1.7 Diagram and simplification of the early form finding techniques (from the old drawing by Poleni) (Poleni & Poleni, 1748)

The hanging chain method can be easily developed into three dimensions by using a chain network or fabric with loads attached on each point. Under gravity a pure tensional form can be derived and by inversing the form vertically a pure compression form can be found. Former shell pioneers and masters, from Antoni Gaudi to Heinz Isler and Frei Otto, have all used and done deep research on the detail of this form finding technique. (Chilton & Isler, 2000; Nonell, 2000; F Otto et al., 1974)

However, the technique itself has also some limitations. For example, as also shown in Figure 2.1.7, the analyzed load cases can only be the application of some evenly or differently distributed load on the nodes which is connected by strings. A variation of the load factor or the distribution will totally lead to another form finding process and the combination of the horizontal forces such as the wind/earthquake loads which are common in any structure is often impossible. At the same time, as such analysis requires the discretization of the curved surfaces, the more complicated behavior in a continuous shell will also be much simplified in such form finding techniques.

With the same goals and method, architects and engineers are able to represent this method with more powerful tools with the help of computer techniques in modern times. With different mathematical methods, the following form finding methods are used widely nowadays.

Force Density Method (FDM): The analysis of the force density method is based on the simple rule of Hooke's law of elasticity with the goal of solving the static equilibrium of each node.

By considering the original network as some points with initial vertical loads, it is possible to get an equation system of the equilibrium of all the points. the state of the equilibrium of every point can be described with the following equation and is illustrated in Figure 2.1.8:

$$\frac{x_1-x_0}{l_a} \cdot F_a + \cdots + \frac{x_4-x_0}{l_d} \cdot F_d + P_z = 0 \text{ Where } F_t = \left[\frac{EA}{l_{0,t}}\left(l - l_{0,t}\right)\right]$$

With the analysis and the equation system, the aim of the simulation can be equaled to the solution of the equation system. However, it is possible to observe that such an equation system is non-linear because the distance $l_{0,t}$ should be calculated. The FDM method simplifies this problem by introducing the concept "force density" which means the stresses in every cable divided by the cable length. In this way the equation systems can be linearized and are hence easy to solve by giving each force density q and node loads p. The FDM method can be used for a rapid generation of the possible membrane force field, because only the direct solution of a linear matrix equation is required without any iteration. However, as the meaning of force density cannot be simply defined in geometry or mechanics, it is often used in the very first beginning of the shell designs. At the same time, as the input is only the topology of the system and the pre-determined load at points, the solution is independent of any material property and refers to the distribution of the stress field.

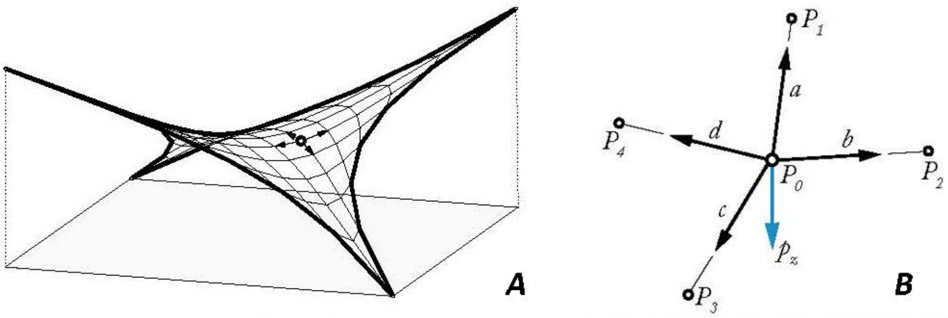


Figure 2.1. 8 Discretization of a shell surface (A) and the equilibrium state of a single node (B) (Adriaenssens et al., 2014)

Dynamic Relaxation Method (DR): Dynamic relaxation method is also the conventional computational form finding method like the FDM method. Instead of the static analysis in the FDM Method, the DR method relies on a dynamic analysis. This method was first invented and used by Alistair Day in solving the nonlinear equations.(Day, 1965) In summary, the DR method solves the problem with a dynamic iteration to check the residual forces of the system by every small movement step by step for small time increments. The calculation of the method relies mainly on Newton's Second law and with only the damping of the system in the x-direction as a degree of freedom. For a given time t, the equation can be written as follows:

$$F_x(\mathrm{t}) = \mathrm{M} \cdot a_x(\mathrm{t})$$

By considering the damping of the "springs" in the system $C_{ix}v_{ix}(t)$ with the stiffness K_{ix} and the displacement $\delta_{ix}(t)$, it can be rewritten as:

$$P_{ix} - K_{ix}\delta_{ix}(t) - C_{ix}v_{ix}(t) = M_i a_{ix}(t)$$

From the equation, the velocity and acceleration of the node are changing over time, and the residual forces are changing with the displacement of the nodes. Viscous damping helps the whole system of residual forces to converge quickly below a certain tolerance, and when the time step is correctly selected, the convergence can be quickly achieved. Some DR methods have been well researched and the computational algorithm has also been published and is available to be used in nowadays design software. (Vejrum, 2013) (Adriaenssens et al., 2014)

Thrust Network Analysis (TNA): The Thrust Network Analysis (TNA) method is a new form finding method which is an improvement of the FDM method, and also a better powerful tool for the architects and engineers to finish the interactive design. The TNA method was first introduced by Philippe Block in 2007 and developed in 2009 (P. P. C. V. Block, 2009).

The TNA method introduced a more direct way to find the equilibrium state with the help of the reciprocal relationship between the form and force diagram of the thrust line network. And by solving the horizontal equilibrium at the first stage, it enables a 2D graphic method to be applied in both the analyzing and designing process of the shell structure design. Instead of the FDM method, where the value of loads and the force density should be pre-decided, the design process in the TNA method can start with a graphic control of both the form and force diagram. As a product and an application of the TNA method, a design software plugin "Rhinovault" has been developed and is already widely used in shell designs (M Rippmann, 2012).

Particle-Spring system Method (Thompson & Bonner): A recent novel method of the shell form finding method is the Particle-Spring system Method (Thompson & Bonner). Like the DR method, it also relies on a dynamic analysis of the nodes of the whole system. The Particle-Spring model makes a similar and accurate system model like the traditional physical form finding method in architectural design. With a suitable abstract typological model of a network, the forces can be defined according to Hooke's law of elasticity and also Newton's second law.

The PS method is very like the DR method, and when only the viscous damping is considered, the PS and DR method are totally interchangeable. The PS method has another advantage to the DR method. It provides mainly a description of a physical system and its simulation but is not simply a way to solve the mathematical problems to a certain analysis. Unlike the DR method, the PS is a wide concept in which lots of solvers can be used and many other static problems can also be solved. A recent popular design software plugin Kangaroo for Grasshopper has used mainly the particle-spring system and a series of solvers to achieve many simulations with different goals. (Piker, 2013)

A simple comparison of the state of art form finding techniques is shown below:

Family	Name		Year	Element type
Stiffness matrix	Natural Shape Finding	NSF	1974/1999	bar + surface (1992)
Geometric stiffness	Force Density Method	FDM	1971/1974	bar + surface (1995)
	Surface Stress Density Method	SSDM	1998/2004	surface
	Thrust Network Analysis	TNA	2007/2007	bar
	Updated Reference Strategy	URS	1999/2001	bar + surface (1999)
Dynamic equilibrium	Dynamic Relaxation	DR	1977/1977	bar + surface (1977)
	Particle-Spring system	PS	2005/2005	bar

Figure 2.1. 9 Comparison of the state of art form finding technique for shells (Adriaenssens et al., 2014)

2.1.3.2 Structure optimization

The structure optimization method is the important process of shell designs, where the efficiency of the design can be maximized with special objectives and constraints. It always works based on an initial design and aims to make an improvement considering the additional sensitivities by taking the non-considering problem into account and using suitable mathematical tools.

The structure optimization in architecture and engineering is usually an integration of the establishment of specific disciplines, selecting an abstract design model, structural analysis, defining constraints and algorithms and mathematical programming. It emphasizes the optimization of the design model and the structural behavior.

In this comprehensive procedure, the establishment of the goals and constraints will always affect the result of the structural optimization analysis. At the same time, as there are lots of issues in the design and construction of shell structures, a multi-criterion optimization process is often welcomed where multiple objectives will be considered simultaneously to get a compromising best design of the shell structure.

Because the shell structure has shown its efficiency with its special structure behavior and design methods, it is often possible that the initial design of a shell structure also has a great risk to become an over-optimized structure. The design which is derived from a theoretical analysis always has a high sensitivity due to some small imperfection such as the changes of the local concentrate load or the buckling behavior caused by the geometry or material imperfections. In this sense, a better optimization taking such problems into consideration will cause an improvement of the initial design.

1, Size Optimization: The pure size optimization of the shell structure is mainly a better design of the thickness of the shell surface. As the theoretical thickness of a shell can be close to zero, a better optimization is to reduce the cross-section to as low as possible and to maintain the capacity of the shells under every possible load case.

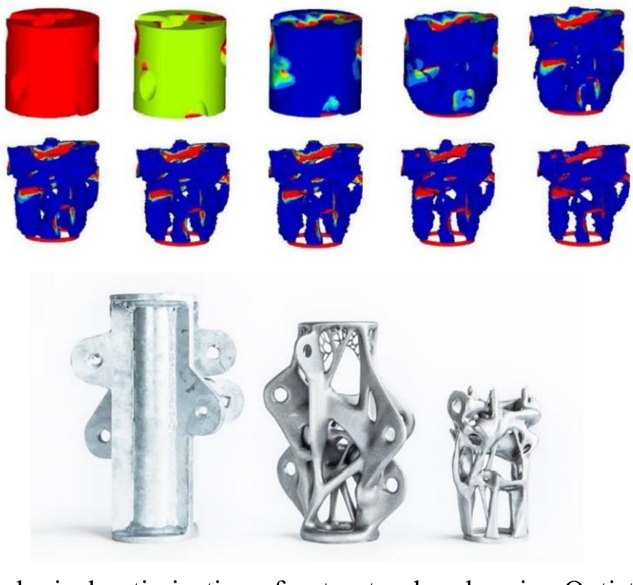


Figure 2.1. 10 Topological optimization of a structural node using Optistruct (top) and the optimized shape (bottom) (Galjaard, Hofman, Perry, & Ren, 2015)

2, Topology/shape Optimization: Similar to the discrete way to analyzing the form finding for shell structures, the shape and topology optimization also aims to find a better geometry for the shells. Actually, the form finding process is a kind of shape optimization, but in some other cases such as when the design is started from an arbitrary or traditional surface, such a shape optimization can be used to offer a better in-plane stress state. At the same time, the topology optimization can also be used in reducing the initial geometry to derive a more efficient form. In such techniques, iterative or genetic algorithms can be used to “cut down” the parts on the initial geometry, where no stress or low stress is occurring, to find the best solution at the end. (Figure 2.1.10)

3, Material Optimization: The initial design of shell structures is always done with a high abstraction of the objectives and constraints where the material properties are always ignored to some extent. However, in some detail design procedures of shell structures, such properties are often taken into account with parameters such as the anisotropies, reinforcement, pre-stress and buckling factors. The application of material optimization of shell structures are usually on the specific materials such as the composite materials (Stegmann & Lund, 2005).

The structural optimization of shell structures is often the specific part of designs which works for a more efficient and accurate analysis or design optimization. Sometimes, a multi- criterion optimization can also be applied to finalize a best design suggestion by taking as many considerations as possible. As this is not the key point of this essay, more details are omitted in this part.

2.1.3.3 Structural analysis

Same as that the form finding and the optimization methods help the shell designers to make a design of the shells, the structural analysis method plays the irreplaceable role for the engineers to verify the capacities of the shell structure. Instead of finding the possible form of the shell surface, the analysis procedure aims more to describe the mechanical behavior of the design, by understanding the possible stresses, strains and displacements which arise in the shell due to the applied load.

The structural analysis is the main target in structural engineering and varies greatly according to the possible abstraction of the structure model. In this part, the following techniques will be introduced.

1, 2D Force Diagram and the Graphic Statics: Among the numerous creative tools in the structure analysis, the graphic analysis is one of the most ancient and natural ways which has shown its application before advanced mathematics and the computer. The graphic method can be considered as the simplest and priori method where the simple principle of statics and algebra are translated into the geometrical representation.

The method with a polygon of forces is the basic principle of the graphical equilibrium, which is based on Newton’s first law that an object either remains at rest or continues to move at a constant velocity only if no forces are added or the several forces will balance each other.

The graphical analysis is first used in the 2D analysis of the arch equilibrium or the indeterminate cable system where the forces are abstracted into the line of thrust. For an arch that is constructed with blocks of masonry, the whole structure can be analyzed as a 2D chain system with applied loads laid on the center of gravity of the blocks. By every block which is in an equilibrium state, a force polygon can be constructed and analyzed geometrically. In graphic statics, which has been used widely by architects and engineers in the mid-20th century, the equilibrium state of a system can also be analyzed by means of a closed force diagram. (Figure 2.1.11) (Wolfe, 1921)

For shells, the same analysis can also be applied. As it is discussed with the membrane behavior, the ideal state of shells should be a field of membrane forces instead of the bending behavior. It is easy to presume that a linear force system should exist. In this sense, the 3D analysis of such 2D thrust lines should show a solution for the shell design or the analysis and can be developed with mathematical methods. Actually, the TNA method in the shell form finding is an extension of graphic statics and can also be used in the shell design process.

However, the graphical methods of the shell analysis also have their limitation, which is, the abstraction of the shell behavior should be based on a discrete force field and only linear behavior can be considered. Although the graphic representation of forces has shown the elegancy and simplicity of the physical mechanism, it slowly loses its application during the development of the modern theory of structure statics. But with the powerful computational capacity of the computer, this method has been mentioned more and more nowadays especially in the designs of shells. Today, it is well accepted that this method can be used as a helpful tool in the first design process of the shell structure.

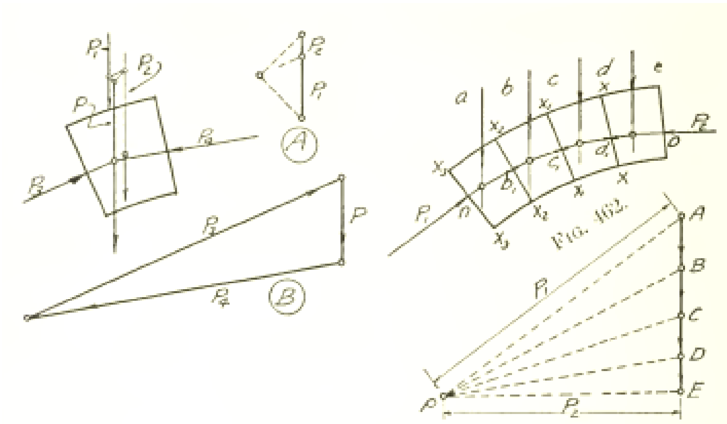


Figure 2.1. 11 2D structural analysis using graphic statics (Wolfe, 1921)

2, Classical shell theory and the analysis with traditional geometry: In the development of the pure analysis of a shell structure mechanism, the most theoretical way is the classical shell theory, which was first established by AEH Love in 1888 with Love's first approximation(Love, 1944). The shell theory is often considered as the deduction and extension of plate theory for the various surface-active structures. The theory shows its great contribution to the establishment of the membrane theory of shells with its reduction of the 3D behavior of shells into a 2D problem by suggesting the normal direction stresses are not significant to the shell behavior.

The shell theory prewar mainly used mathematical deduction and it is therefore limited in the traditional geometry for most shells. A thorough analysis has been done for domes, the cylinder shell and conical shells. As the analysis is based on the theory of elasticity, the large deformation of the materials has been taken into account and the result is hence more accurate and trustable for the analysis of the real shell behavior.

However, with the emphasis on the linear elastic theory, it has been shown in further research and experiments that the theory has far underestimated the buckling phenomenon of the thin shells. The post-war research has established lot of supplements with detailed research for the buckling sensitivities of thin shells and both of the geometry and material imperfections have been considered, and such research on the traditional forms has been improved.

The shell theory analysis has shown its better accuracy and well-around consideration so that it has become the basis of the modern scientific researches. At the same time, its limitation on the complicated mathematical calculation and several simple forms has also made it to be not the first choice in the general analysis method in engineering projects.

3, Physical Model Test: As it is shown before that the physical modeling form finding has been the most popular method for architects in the design process, this method is also one of the main methods to research the real behavior of shells.

By many of the pioneer architects of shells, such as Heinz Isler and Frei Otto, the small scale models are not only used to find the proper shell forms but also to simulate the possible actions of the real shell.(Isler, 1993; F. Otto, 1954) In the age of physical simulations of shell structures, many kinds of models have been invented to predict the different performances of the shell structure.

Unlike the form finding process where only the possible thrust line system should be found, the objectives of a physical model test of a shell could be complex. With different aims of the test, from verifying an initial design to watching the full-scale performance or the materials' behavior in the shell, the physical model should also be designed or improved for a more accurate simulation for the real circumferences. In most model tests, real materials and full-scale models should be used for the construction details. And by the testing procedure, long time performance and also the micro behavior are also needed to be observed.

The physical models are used in a long time to help the theoretical research such as the buckling test of shells, mainly due to the lack of suitable mathematical tools. With lot of experiments to test the behavior of the imperfections, the observed buckling behavior has also helped to enhance the theoretical researches in return.

4, Computational Methods: The attempts and the requirements of the free-form shell structure have urged the development of a new powerful analysis method of the shell behavior. Nowadays, a detailed analysis of the mechanism of an irregular shell structure is often done by engineers with the help of computational methods. As it is easier nowadays to solve problems such as a large equation system or complicated non-linear equations with computers, it is possible to get such simulations of free-form shells with the new techniques.

The most popular computational analysis method is the finite analysis method (FEM), where the whole system is divided into a finite set of small components, named as the finite elements, and the behavior is thus calculated by solving series of equation that is established the various boundary conditions. This subdivision of a complicated and hard describable structure leads to several advantages such as the easy and accurate representation of the complex system, the

reasonable result with consideration of the different material properties in the system and the attention to the local behavior and imperfection.
However, compared to its well use in the structural analysis, the FEM has seldom been widely used in the design of the shell structure. As the work flow of the FEM analysis requires too much professional knowledge in the structural engineering, most of the works can only be done by meshing a designed model into a suitable model for the calculation with all the definitions of the geometry, boundary condition and load cases. It provides the analysis only on the certain cases but cannot provide a general solution for a design problem. In this sense, such techniques can only be considered as a useful tool in the evaluation process of the design of shell structures.

The FDM analysis is provided nowadays by various softwares and has been used widely for the final evaluation of the validity of shell structures. (Chapelle & Bathe, 2010)

2.1.3.4 Special considerations in shell designs (opening, support, edge)

The principles of shell designs are mostly based on the structure behavior and the static mechanisms. However, for an architecture design, more details should also be considered to make a structuralized designing system for shells.

1, Thickness and cross section design: Structure thickness is an important property and also a paradoxical problem for shell structures. Being an efficient form of structure, the shell design requires a lot on a low-weight structure which can be built with less material. At the same time, the slenderness of the shell surfaces will enable a lower load and also a more prominent membrane behavior. Nevertheless, a shell structure with too little thickness can be too much sensitive for the bending and buckling behavior and hence will be dangerous with any kind of extra load.

A thickness or cross section design comes often at the first step of the shell designs, where the span, load cases and material property should always be considered. In a traditional way of the masonry shell or the concrete shell designs, the "thrust line" methods like the form finding of arches can be used. (Figure 2.1.12) By pre-defining the different dominating load cases and a proper subdivision of the shell, different thrust line fields can be generated and drawn geometrically. For such a structure with a solid element, according to the lower theorem, if all of the thrust line can be contained in the volume of the structure, a possible state of equilibrium can exist and be maintained. In this sense, an efficient shell design should at least satisfy the minimum shell thickness but can also be optimized with a non-uniform thickness design.

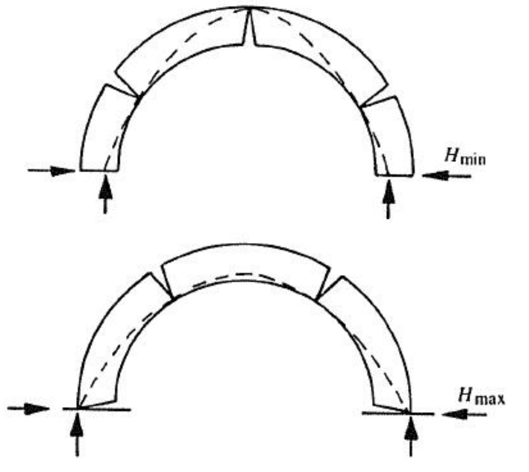


Figure 2.1. 12 Sections satisfying different load cases (Heyman, 1997)

For real built shells, the material property of the structure should also be taken into account for the cross section designs and this varies a lot from the material and depends also on the architectural requirements, such as the reinforcement of the concrete, covering and other technical requirements on the structures. It is also common to design an increasing structure thickness to avoid the local stress concentration and to confirm the successful load transfer to the supports.

2, Opening and strengthening: The theoretical shell structure should always be kept in a continuous surface or a complete topology to ensure the membrane behavior; however, in architecture design, the opening or holes on the surfaces are at most times inevitable for the functionality of the structure. As it is mentioned above, when the membrane behavior cannot satisfy the load in the local area, the bending behavior will come up as a compensation for the whole structure. In a shell with a primary membrane behavior, the bending behavior occurs in the local area and will rapidly vanish because of the dynamic mechanism in the shell. Globally the most part of the shell will still be kept in its original membrane behavior.

In the design of shells, this problem of the disturbance of the membrane behavior which is caused by the discontinuity of geometry should also be taken into account by the form finding and structural analysis process. A local change of curvature or a strengthening can both be the solutions to such problems. When the requirement of an opening is considered in the form finding process or the structure optimization, a better form with some local adjustment of geometry can be derived. With such changes a local change of the membrane stress field will be generated. This mechanism can also be understood with the forms of the edges of shell structures. In most the form finding experiments, the geometry cannot be kept continuous by either having edges caused by the point support or a local hole on the structure. As a result, it is easy to observe a change of curvature by such local areas and it will definitely lead to a new membrane force field which is suitable for the new geometry. (Figure 2.1.13 A)

As it is not simple to take so many special conditions into account in the form finding and the sudden change of geometry is hard to accomplish in the building process, it is common to add new local strengthening elements to solve the problem. With a continuous bending resistance element, such as a compression or tension ring, the local disturbance and resultant bending behavior can be resisted and the load can be helped to transfer successfully. (Figure 2.1.13 B)

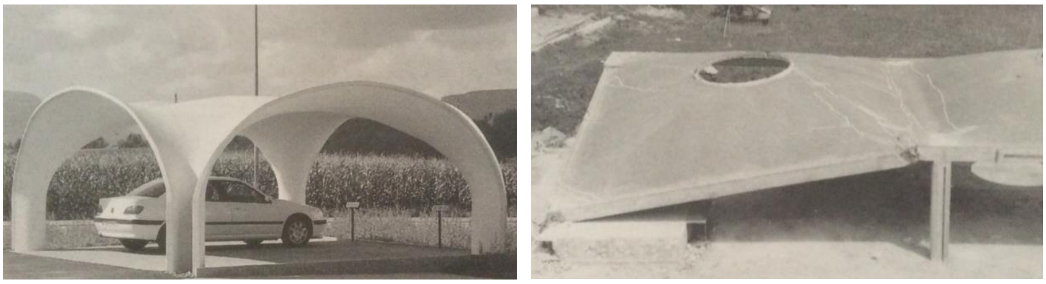


Figure 2.1. 13 Geometric and constructional solutions for the opening and edges in shell (Chilton & Isler, 2000) (A: change in curvature by physical form finding B: Compression ring)

3, Supports and edges: Edge members are the necessary parts of the shell structure to transfer the loads to the supports. In architecture design, it is also the key point where the whole structure should be articulated to the ground or other elements such as the walls. In the design of shells, both the positioning of the support and the structural construction details should be considered.

Position:

As the membrane stress field can be considered as the force flows in the shell structure, when the supporting condition is changed with the different designs of the supporting edge members, the membrane stress is therefore also changed. In this sense, the design of the position of the supports will determine the main mechanism of the shell. (Figure 2.1.14) In the classic designs of domes, most shells are fully supported around the bottom boundary with a continuous arrangement of beams and columns. Such symmetrical shells can easily be analyzed as a series of arches with a horizontal connection, with orthogonal meridian and circumferential stresses. However, along with the developments of new forms and aesthetic interests, to show the cross section and the free edge of shells has been popular due to this vitality and grace. (Candela & Faber, 1965; Webb, 2007)

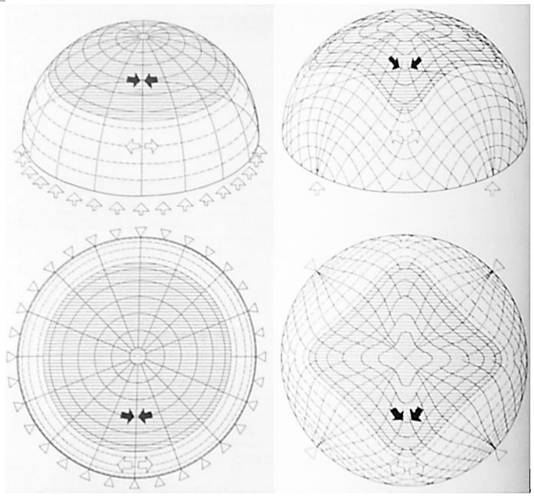


Figure 2.1. 14 Different stresses in shells with different supporting conditions (Left: fully supported shell, like a series of connected arches, will have a regular compressional and tensional stresses along the horizontal and the meridian directions. Right: a partly supported shell will have a disturbance of the internal stress.)

Design of a shell with free edges always leads to the local problem of the membrane stresses, which cannot get balanced to maintain the stability of the shell. In this sense, small changes of form are required to transfer the loads mainly to the supports. In return, a concentration of force will occur at the supporting point and will lead to a stronger edge disturbance that will affect a wider area of the shell. The most common way to solve this problem is also to use local strengthening at the free edges, such as a tension ring in the bottom of a rounded dome. At the same time, it is also useful to increase the thickness at the free edge area to offer a bigger bending stiffness or making a possible warped geometry to adjust the membrane stress.

Structural and construction details:

Designing a reasonable support construction is a delicate problem for the shell designers because a balanced design is always required according to the theoretical analysis of the shell behavior. A shell can be statically either determinate or indeterminate. A traditional and common way is to create the isostatic state of the shell structure, which requires theoretically the hinge connection between the shell body and the edge members of the structure. The reason is that only such hinge connections can always offer a possible tangent thrust to balance the membrane stress without causing the extra bending moments. For such a perfect isostatic condition, the edge members should also be simply supported, to allow a possible small deformation of the whole structure.

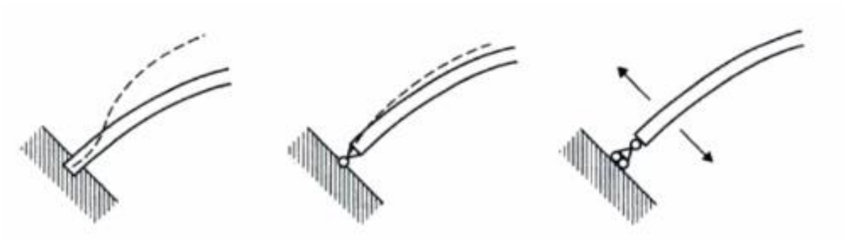


Figure 2.1. 15 Simple analysis of the different supporting conditions to the shell behavior (Left: fixed support will cause the local bending behavior; Middle: Pinned support will offer an ideal tangent force to the structure. However, the horizontal stress will also cause the deformation of the structure and that will be restricted by the support. The possible small bending moment will appear in this case; Right: A ideal roller hinged support can provide the possible small displacement in all directions.)

As it is shown in Figure 2.1.15, the typical supporting methods of shells can be simplified into the three conditions. Because it is required that only a tangent force should be generated and a small deformation should be allowed so that there is no bending moment in the shell, the theoretical solution of the supporting connection is only the roller hinged support (right one). However, such a construction detail is always too hard to build. In practice, some conservative methods are still usually used to simulate the ideal condition while at the same time offering a certain stiffness against the possible moments.

A tension ring method is the most popular practical method for the supporting of shell structures in the regular rotational geometry. In such designs the major problem is the design of the hinge connection between the shell and the ring. Normally there are several ways, such as offering an L-shaped groove to offer the horizontal and vertical force to the shell and accept the small deformation (Figure 2.1.16 A), or simply strengthening the bottom part of the shell and extend it into the ring to enable a bending stiffness (Figure 2.1.16 B)

In some more shell designs,various kinds of support details are developed. A hinge-like connection is always done with the half-hinge way. Such as in the concrete shell technique, the connection of the shell and the tension ring is usually made with a small angle in geometry that leads to a tangent direction of the shell geometry. At the same time, the hinge is always done by simply binding it with steel anchors which will offer a small rotation and displacement for its bending behavior. (Figure 2.1.16 C, D)

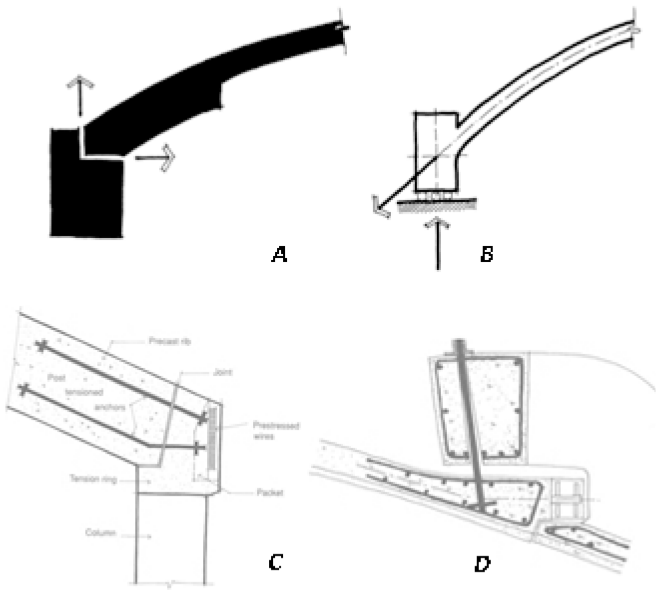


Figure 2.1. 16 Typical supporting details in concrete shell constructions (Melaragno, 1991)

2.2 A structural morphological review of the developments of shell structures

The traditional and basic principles of the shell designs are often simplified to show the fundamental requirement of the shell behavior. However, this is still involved too little in the architecture design methodology. In recent years, shells and spatial structures have come back into the topics of architectural and engineering research, not only showing a revisit on the traditional shell forms, but also accompanying the blooming emergence of various structure systems and forms inspired by the developments of the design tools, fabricating techniques and materials.

A popular and recent research topic of the shell structure design is the structural morphology, which has gotten its main meaning first from the biological research that focuses on the study of the form and structure of organisms and their specific structural features. In the research of shell structures, it has also shown its influence in the late 20th century mainly from the pioneer works of shell structures, where the new coupling association of forces and forms are emphasized by the shell designers (René Motro, 2011). With the new trend of design concepts of the spatial structures and the emergence of structure systems nowadays, the structural morphology is brought up again with the foundation of the working group "Structural Morphology Group" (SMG) in the International Association of Shell and Spatial Structures (IASS) in 1991. Focusing on the relationship between the structure concepts and the form, material and fabrication, lots of new structure systems and experiments have been done worldwide and opened a new era of shell development with various kinds of building experiments.

In this part, more practical cases of shell designs will be studied both chronologically and with a consideration of the structure morphology. The discussion emphasizes mainly the emergence and replacement of the structure concepts of shell-like structures with the understanding of the background development of materials application, building technologies and the structure theory. Finally, a new material and structural performance based design tendency will be clarified to lead the main research objectives of this dissertation.

2.2.1 Design the Force and Form: from arches to concrete shell

2.2.1.1 Ancient domes, intuitive Example of the shell typology

To analyze the importance of the structural morphology, the relationship between form and forces and the effects of materials, it is inevitable to take a revisit of the building history of shell structures from the ancient Roman civilization, when the first domes with large spans and curved roofs were built.

These domes, instead of being based on a structural consideration which is shown in the previous part, are built in a relative intuitive way. After the ancient applications of shell forms in the buildings of pseudo-vaults, tombs and stupas, the ancient builders started the curved roofs experiment mainly based on an aesthetic idea and the experience of using arches and vaults in the ancient constructions.

The Pantheon in Rome is famous today to be almost the first large scale dome still remains standing and it has also led the main building techniques of following concrete shells. In the

Pantheon, it shows the first typical structure morphology with the basic design rules of the shells: the whole structure is built by pouring the concrete using interior scaffolding as formwork; with the half-sphere form the structure was built by building concrete hoops step by step until the 5.9m oculus. It can be seen that the thickness of the shell varies from the height, the bottom rings are thickened by adding more mass on the base to act like a buttress for the stress concentration. For the oculus the typical solution of opening on shells is clearly displayed: by adding a compression ring which is made by 3 layers of hoops, 2 bricks thick, the compression concentration is therefore resisted. Although the successful design, which has been derived from the ancient Roman building experience, has showed its great structural rightness and lead the structure stand till today, a lot of cracks have also appeared in the inner side of the structure. With all the cracks in the meridian direction and the bottom tension part of dome, it has shown that limited static knowledge was known in those days and the structure design was mainly based on geometry and experience.

2.2.1.2 Masonry arches to the finding of form and forces

The exemplar of St. Peter's Dome in Rome (1505) presents another widely-used application of shell morphology with its double layered shell framed by 16 large arched ribs. Compared to the old casting techniques, this is not innovative because the building typology of domes between the drums and lanterns can be found earlier in Brunelleschi's dome in Florence and the rib structure based on the arches are the most common and typical building methods which are based on the use of concrete masonries in the Roman culture.

The St. Peter's however is so famous in the history of domes due to its cracks in 1742 and the following stability analysis which lead to the first application of the catenary theory from Poleni (Figure 2.2.1), and it is also the first use of the safe theorem of limit analysis on the masonry structure. Such an application has shown the powerful function of the graphical understanding of the coupling between form and force, and hence started the opening of the form-finding methods of shell structure later. (Schmidt, 2005)

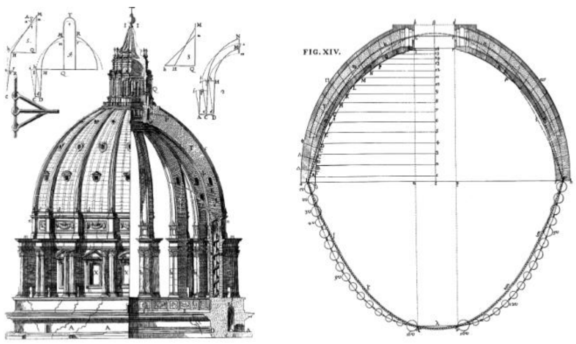


Figure 2.2. 1 St. Peter's Dome and the static analysis by Poleni (Schmidt, 2005)

Although the previous milestone has created the early structural typology of shells with many more arch domes from the ancient Roman time to the Renaissance and the Baroque era, the modern shell development with a change of the structural morphology didn't appear until the engineering use of iron and steels. The new structure form of the iron domes can be found as pioneer work in the renewal of the dome "Halle au blé" in Paris (1811). Although the iron dome got its structure system mainly due to the non-existent precursor of the binding-wood domes, which has been first developed by Philibert de l'Orme in the mid-16th century (Graefe & Andrews, 1989), the first iron dome structure form still shows a similar translation of the old arch and ring systems like the masonry dome, but offers a better future of a light-weight, high efficient structure typology. (Figure 2.2.2)

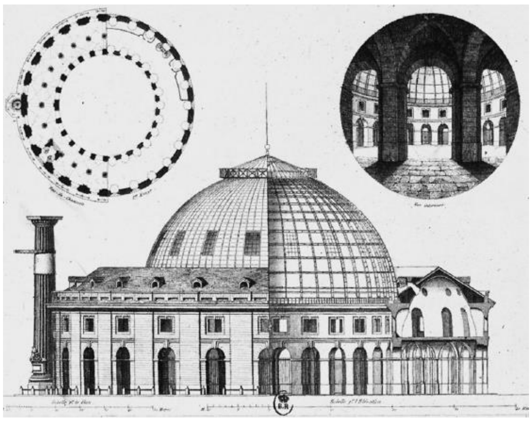


Figure 2.2. 2 Wood and steel succedaneum of arch typology (Schmidt, 2005)

2.2.1.3 Iron and steel to the establishment of the Zeiss-Dywidag-Method

The truss system with iron and steel has opened a new age for domes and the establishment of the spatial framework structures. However, it is not until the first coming of the triangular arrangement of the iron truss system that the modern "shells" can be developed from the domes. Based on the previous experiments such as the Zimmermann's dome and the theoretical research of the spatial framework from August Föppl, the beginning of the modern reinforced concrete shell is also considered with the development of the Zeiss-Dywidag-Method (Figure 2.2.3 B) with the building of the Zeiss planetarium shell (1925). Improving the old reinforced concrete system, it offered a better way to build thin shells with the tension capacity of steels for the concrete shells. Besides that, the on-site building process which was introduced also replaced the old techniques with huge scaffoldings or formworks. The triangular-based geometry also lead the future developments of the spatial framework structures, the free-form concrete structures and the numerical analysis and designs of the shell structure, such as the following concept of the "geodesic dome" by Buckminster Fuller.

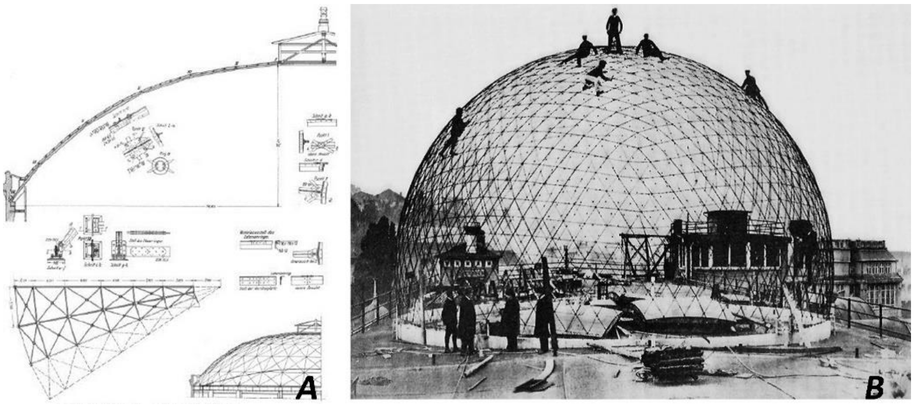


Figure 2.2. 3 Schwedler Dome and the prototype of the Zeiss-Dywidag-System (Schmidt, 2005)

The first bang of the inventions of the reinforced concrete shells has opened a new era for shell history, with summaries and practices and also the following technicalisation of shell theory, which had evolved in the tradition of the mathematical elastic theory. (Kurrer, 2012) With the great amount of applications of the Zeiss-Dywidag-Method, the blooming of concrete shell structures has been widely spread to Europe and the USA. In this blossomy era of shell designs, lot of improvements and experiments of structural morphology have been invented by lots of master works of many pioneers all over the world.

2.2.1.4 Graphic statics and the new design methods of shells

Unlike the pre-stage of modern concrete shells, where the limited analytical geometry reflected the limitation of the early shell theory and designs, the new works in shells showed not only a desire of designers to create new forms, but also a new method of understanding the relationships between form and forces with new design tools. Graphic statics is the most impressive tool in this background. The practical application of graphic statics in shell designs came after a long preparation of the theoretical discussion about the geometrical analysis of the equilibrium of forces, from the establishment of the theory from Karl Culmann's book "die Graphische Statik" to Wilhelm Ritter's comprehensive graphical analysis of several structures in his book "Anwendungen der Graphischen Statik".

The milestone of the graphical analysis of shell structures is Rafael Guastavino's research of the Guastavino tile (Figure 2.2.4 A), by which the graphical understanding of the force flow had inspired a new way to design more efficient structure forms that has been used by various modern shell pioneers such as Antoni Gaudi and also led the development of the form finding techniques.

Pier Luigi Nervi's works on shell structures are considered to be one of the typical morphological innovations of concrete shells. Like in other structures that are built by Robert Maillart, Gustav Eiffel and Eladio Dieste in the same age, Nervi has used graphic statics as the fundament for the design and calculation of his shell structures and his applications of ribs to stiffen the shell and lead the flow of forces (Figure 2.2.4 B) had enabled him to build thin shells with a low cost and less time-consuming way with prefabricated components. The pre-casting and erection of the lattice system was simple and fast and enabled a slender thin shell; it is also proved by Nervi that the separation into structural components didn't disturb the global quality of the shell behavior. (Nervi & Desideri, 1979) Such rib-shells have also been used in many later designs and became an important typology of the concrete shells.

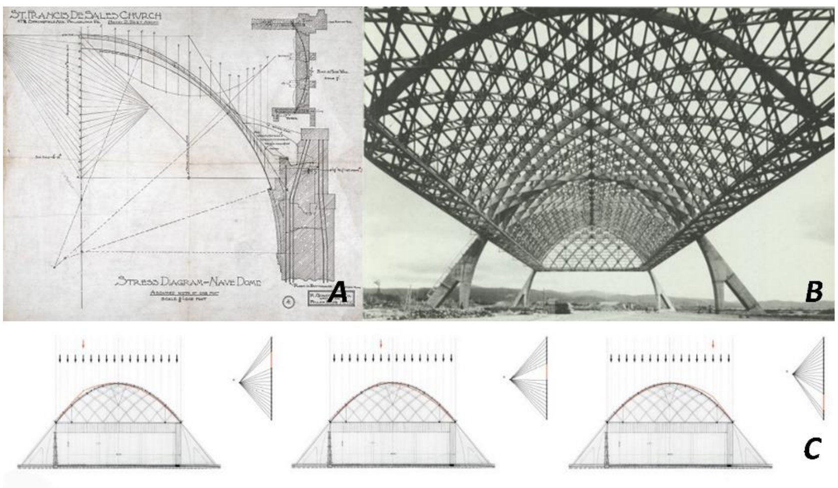


Figure 2.2. 4 Graphic analysis of Guastavino tile (A); The rib-shell built by Nervi (B) and its static

analysis (C)(Nervi & Desideri, 1979)

Being the founder of the IASS, Eduardo Torroja had led the modern concrete shell development in Spain and had also used graphic statics as the tool to do his design, especially on double-curved shells with complicated forms. In both of his works, the first large span concrete shell in Spain—the Algeciras Market hall (1933) and the Zarzuela Hippodrome (1935)—he used the concept of isostatic lines to generate two possible shell forms with double curvatures. This way to visualize the flow of forces referred to an original graphical form finding method where the forces guided the design of the form. (Figure 2.2.5)

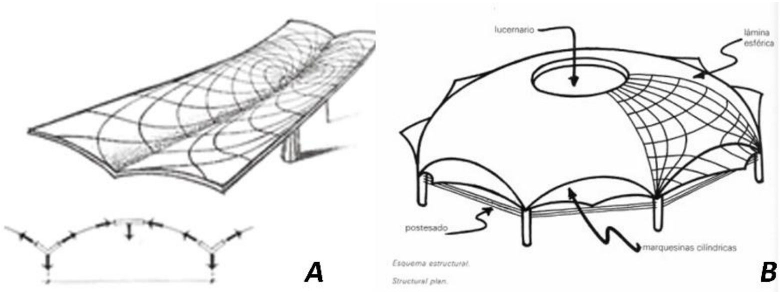


Figure 2.2. 5 Graphic visualization of the flow of forces in shells (Fernández Ordóñez & Navarro Vera, 1999)

2.2.1.5 Geometrical elegancy and the nature-inspired form finding technique

In structural morphology, the generation of the design can be both carried out from the form and the forces. In the later shell master Felix Candela's works, the geometry method has shown its dominant position in the shell designs and has also enlightened architects' enthusiasm in the forthcoming spread of shells all over the world. Candela has gotten his mathematical and statics education in a school founded by Torroja and learned the graphic way there. Although he had his dream to start a deep research in the theoretical analysis of shells following Dyckerhoff-Widmann, the Spanish Civil War "saved" him and lead him in another way to spread the HP shell with his numerous elegantly built shells in Mexico. (CASSINELLO & TORROJA, 2010)

Candela had a special preference in the HP form of shell structures, and this has given him a special way of doing the designs with a dominant geometrical method. In his academic works, the structural behavior of the HP thin shell has been well analyzed with the shell theory.(Candela & Faber, 1965) His design has also followed Torroja's method of the full scale model tests. With his success and numerous shells with appealing geometries, a fervor has also risen of shell in the later architecture designs. The simple formwork consisting of a system of straight planks supported by another system of straight lines also enabled the economical building technique which is favored by engineers for a long time.

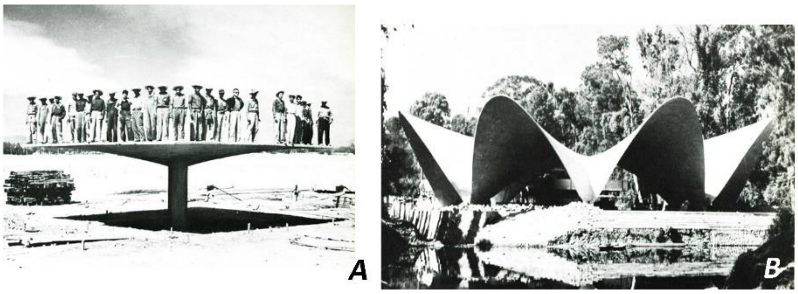


Figure 2.2. 6 Felix Candela's shells with analytical geometry (Candela & Faber, 1965)

Another shell designer with profound influence is the Swiss architect Heinz Isler, who is also considered as the founder of modern form finding and optimization techniques of shells with the nature-inspired geometry. In his numerous concrete shell designs between 1950s and 1970s, Isler invented several structure concepts of shells such as the "Bubble shells" (Figure 2.2.7 B), the expansion shells, free-form shells, inverted membrane shells and also many shells with analytical geometries. The numerous new imaginations and experiments on shells had an enormous impact on the contemporary development of shell designs and had opened the new trend of research of the structural morphology and the understanding of "form follows forces". In his thesis in IASS 1993, Isler summarized a classification of the morphology of shells keeping the analytical and the experimental models with his physical form finding methods (Isler, 1993), with the class "other methods" he also implied and predicted the future importance of the structural morphology in the development of shell designs. (Figure 2.2.7 A)

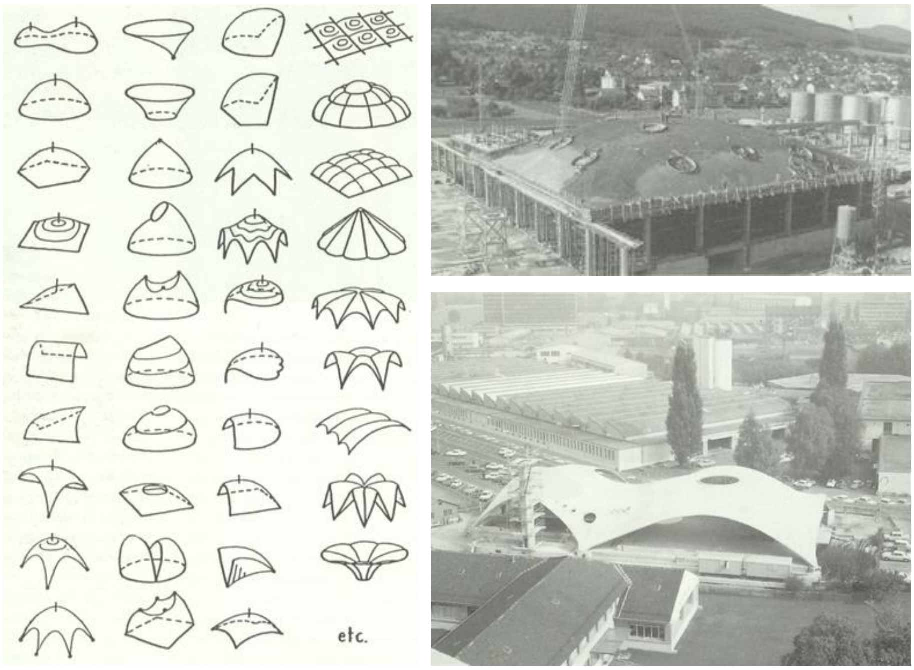


Figure 2.2. 7 Heinz Isler's form finding techniques of shell designs and their applications (Chilton & Isler, 2000)

The geometry-dominant design procedure has lead to the appearance of more concrete shells with free forms and also a tendency that architects came into the design process of the shells. This has also caused a multi-criterion situation of the shell design and a need of a complex well-designed design procedure. In the late 1950s, such a phenomenon occurred in the building of the Sydney Opera House by Jørn Utzon from 1957 and the Philips Pavilion by Le Corbusier in

1958. The different understanding of the rule of forms have caused an intense collision of ideas from the architects and engineers. For example, Utzon's persistence on the free-form of the shell to imitate sails (Figure 2.2.8 A) cannot provide an efficient and reasonable building plan of the shell for both the prefabrication and static analysis. This hence caused a more than one year of revision of the final shape and a great waste of budget. (Figure 2.2.8 B) Although in both projects, the form of the shells and the curvature were neither considered for the structural behavior and caused the hard work to revise the previous design, the innovational result of the cooperation between architects and engineers also showed its success with the attractive building and the reconsideration about the design methodology of the shell structure.

In the design process, the design of shells got their successes mainly on the intensive collaboration procedure. (Tessmann, 2008) And in such a collaboration, the shell theory and the calculation didn't help as the main solution to the problems. However, the tool that has worked was the role which the structural understanding of form and forces played as the feedback to the architects to revise the forms and the existing tools of the structural morphology for the materials such as the ribbed system in the Sydney Opera House and the combination of HP shells with the physical form finding techniques in the Philips Pavilion. (Figure 2.2.8 C, D)

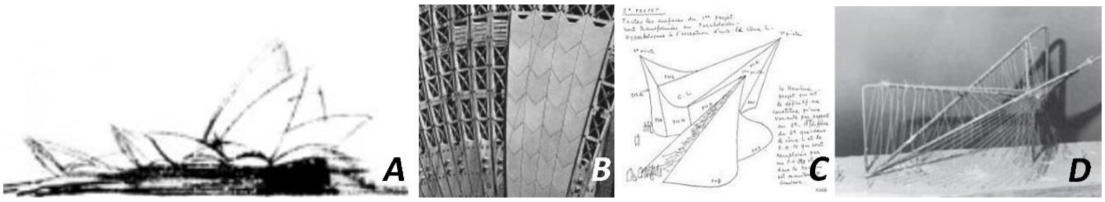


Figure 2.2. 8 Original design of the Sydney Opera and the Philips Pavilion and their revise of shapes based on the materials' behavior and static principles (Capanna, 2015; Kerr, 1993)

2.2.2 Discrete linear system: spatial framework with wood and steel

After the 1970s, the development of concrete shells fell into a sudden disappearance due to the relatively huge cost of the formwork and human power for such complicated geometry. At the same time, the emergence of using the spatial frameworks in the shell-like structures has guided a new trend for the pure "shell" structure to the "shell and spatial" structure, (Ramm, 2011) where the advantages of new materials and the design methods of new forms with computers had presented its great impact on the history of shells.

2.2.2.1 Spatial Framework structure with steel and the discrete analysis with computers

The popularization of the spatial framework structure showed the benefits of new materials with a proper structural morphological design. Dated back to the steel dome system such as the Schwedler Dome and Zeiss-Dywidag-Dome in Jena, the success of trussed framework based its structural logic mainly on the property and behavior of the new steel productions. On the theoretical side, from Föppl's establishment of the spatial framework theory to Mengeringhausen's summarization and systematization (Föppl, 1892; Mengeringhausen, 1975), the main topic of the new structure theory has also been around the structural morphology: the geometry laws, the structural laws and the construction laws.

The geometry constitution is in the central consideration of the designs, where the discretization

of the curved surfaces into a discontinuous linear system should be achieved with as few types of elements as possible. Buckminster Fuller's geodesic dome (Figure 2.29 A) and the following studies of the space structure (Loeb, 2012) and the law of spatial frameworks (Mengeringhausen, 1975) had inspired the first period of shells with spatial framework structures.
The use of computers in structural engineering has started the wide spread of spatial framework structures in the free-form shell structure design. With the development of modern differential geometry and the subdivision methods of the surfaces and the FEM theory, the shell designer is enabled to control the free-form structure and understand the complex structure behavior inside. Hence a new era came with the large projects of stadiums, museums and large span spaces with curved forms which are built worldwide. (Figure 2.29 B, C) In designs, the various concepts were coming from bionics, geometry and the recent definition of the structural morphology was therefore defined.

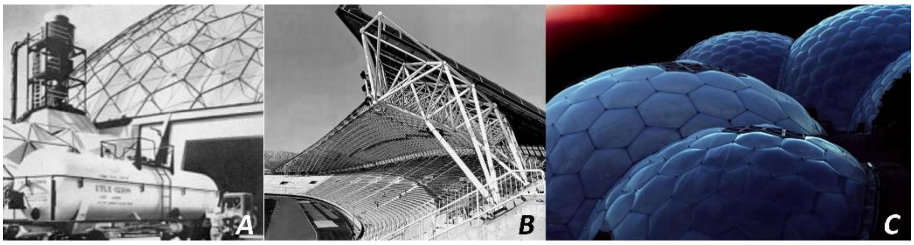


Figure 2.2. 9 Discretization of the spatial framework structures and their applications (Bollinger & Architektur-Dokumentation, 2011; Fuller, Krausse, & Lichtenstein, 1999; Knippers, 2010)

The application of the spatial framework made an interesting change of the original design goal, where only the pure compression condition is welcomed. With the new structure system, the structure can be either in tension or compression. With the development of such a linear system, the computational form finding technology and the static analysis method developed also rapidly with the theory from mathematics and computational graphic. At the same time, such structural typology also got its advantage from the high quality steel products which can be efficiently supplied in any linear forms. With both appropriate ways to use the new efficient material and structural rules, such structures are still widely used nowadays.

2.2.2.2 Grid-shell, physical form finding and timber behavior

The grid-shell is a morphological innovation of the traditional shell which can be understood in a structuralism way. It can be considered that the rest parts of the shell are removed and only the possible thrust lines remain. As in continuous shells where there are infinite possible membrane fields, in the grid-shell the membrane behavior is also defined and restricted by the design pattern.

The grid-shell has come with a good understanding and usage of the physical hanging form finding technique and with the innovational use of timber material. After Frei Otto's research on the hanging roof (F. Otto, 1954), a summarization of such a technique and the structure concept of grid-shells are then established with experiments. (F Otto et al., 1974) After his several small scale grid-shells with timber and bamboos (Figure 2.2.10 A), the first large scale grid-shell, the Mannheim Multihalle, was built in 1975 and is considered to be the first application of such a structure concept. (Figure 2.2.10 B)

The appearance of the grid-shell concept is also an optimization with timber as material to build shells. Although the concept of designing the membrane force field can be applied to various materials theoretically, only the timbers are flexible enough for such building techniques so that they can deform and bend during the erection phase to reach the ideal shape. It is also the distinct characteristic of timber that the building element has a limitation of length so that a lattice pattern is therefore needed. The pattern of a grid-shell is often in quadrilateral forms so that the deformation can appear. In this sense, it is also similar to the physical form finding methods with a chain net or fabric. After the erection, the lattice can be stiffened by adding extra rods to change the pattern into a triangular grid so that a local stiffness can be guaranteed.

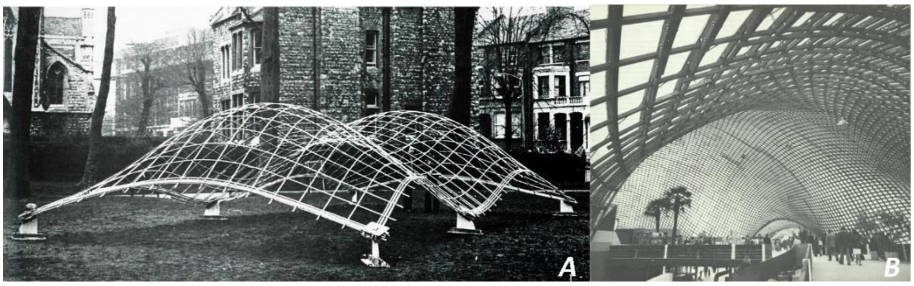


Figure 2.2. 10 Frei Otto's experiments for grid-shells and the Multihalle in Mannheim (F Otto et al., 1974)

The grid-shell is welcomed by architects because the design process is mainly begun with the architectural choice of forms and it also shows its efficiency with a similar membrane behavior. After the Mannheim Multihalle, the grid-shell has also been used in several designs of the landmarks such as the Japan Pavilion in 2000 (cooperation design between architect Shigeuban and the engineers from Buro Happold) and the Weald and Downland Museum (cooperation design between Edward Cullinan Architects and the engineers from Buro Happold) in England. The grid-shell is also widely researched nowadays with its computational form finding and optimization, and also with its building technique with new formworks. Due to the use of timber, it is also welcomed in contemporary building with a good sustainability.

2.2.2.3 Tension structures—Cable, Membrane and Tensegrity

Contrary to the traditional compression-only shells, tension structures considered the situation where only tension existed. It has the similar goal to find the efficient use and stresses for certain materials that are stronger in tension, such as the membranes, rods and cables. As materials such as membranes are too sensitive with compression, the development of such structures was first started with the structural consideration and also the form-finding methods. It has already been realized by the shell pioneers that in the old form finding methods, the hanging model has a reverse effect in that it enabled designers to find both the compression and tension-only from. Afterward, other methods such as the soap film method were also found by Frei Otto to be able to generate the minimal surfaces. In his design of the Germany Pavilion in EXPO 67 in Montreal and the German Olympic Stadium in Munich, such physical methods had been well used. In later designs, the form finding has been developed and improved a lot with more analyzing methods based on the computational way and more optimization algorithms. (Figure 2.2.11)

Figure 2.2. 11 German Olympic Stadium in Munich (A, B) and the modern membrane structure with computational designs (C) (Saechtling, 1973)

The tension structure with membranes and cables showed an example to build a new type of self-supporting structures, and based on the same idea, an innovational structure has been one of the most recent research topics of the spatial structures. Presented first by Emmerich, Fuller and Snelson, the tensegrity structures are shown in form of patents as the modular self-supporting structure composed by a linear system with continuous tension elements and discontinuous compression rods (R. Motro, 2003) (Figure 2.2.12 A). The tensegrity structure opened the new future of the structural morphology with its simple structural behavior and geometry logic. During the past 30 years plenty of experimental applications have been raised to expand its form diversity and potentials in architecture. Although the pure tensegrity structure has not been completely used in large scale designs, such a morphological concept has given birth to many innovative cable-struts systems, such as the Cable-Dome in the Olympic Stadium in Atlanda and the Suspend-Dome in many of the recent Japanese and Chinese Stadiums. In the research fields, the design and calculation of new forms of tensegrity remain popular in structural engineering; ideas such as the free-form structures and foldable structures with tensegrity have appeared in its application. (Figure 2.2.12 B)

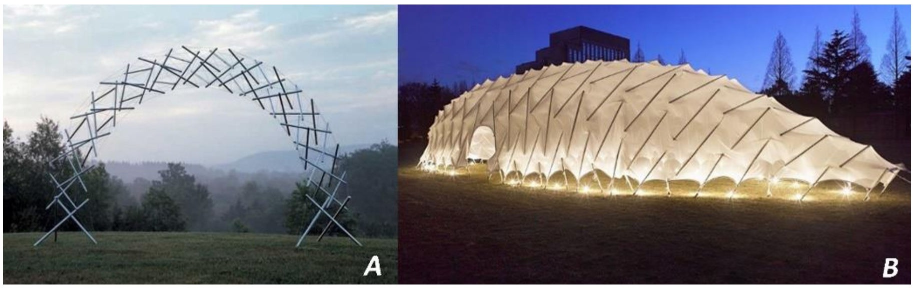


Figure 2.2. 12 Tensegrity and its application in architectural experiments (Associates, 2011; Heartney & Snelson, 2009)

2.2.3 Emerging experimental Structural Morphology

2.2.3.1 Plate structures- Origami, Panelization:

Due to the outstanding membrane behavior in double-curved shells and the easy abstraction and calculation of the linear shell system, the planar materials are usually used in shell structures as skins and covering rather than a structural component. However, with more interest on the application in wood panels, steel panels and glass in the shell design, the plate shell structure became one of the topics of the structural morphology in recent research. The common key point in the research and experiments can be divided into the geometry discretization, the static analysis of plate actions and the material behavior. In the 1960s, Fuller has already applied paperboard and aluminum in several designs of domes to show the geometrical possibility to use panel material in his geodesic dome concept. (Figure 2.2 13 D)

Figure 2.2.13 Applications of planar sheets in architectural design (Ayan, 2009; Fuller et al., 1999)

On the structural side, Wester proposed the duality between the spatial plate structure and the linear lattice system and suggested the corresponding design and analysis rules for such structures. (Wester, 1984) Even in recent research, such a structural rule has been reused in an experiment of the TU Stuttgart to build their one-layered shell with thin wood panels. (La Magna, Waimer, & Knippers, 2012) With the development of computational techniques and the mesh method to describe curved surfaces in designs, the panelization method has been popular because of the easy fabrication and the increased choice of materials. Several algorithms have been proposed for mesh optimization and the generation of a planar panel based free-form building technique, and they have enabled more experimental buildings with panel materials. In the field of new structural concept designs, such a plane based morphology has also inspired the recent topic of the origami structure which based itself on the application of the folded plate structure. In its developments both the computational geometry and the robotic prefabricating methods are used to develop new possibilities for research.

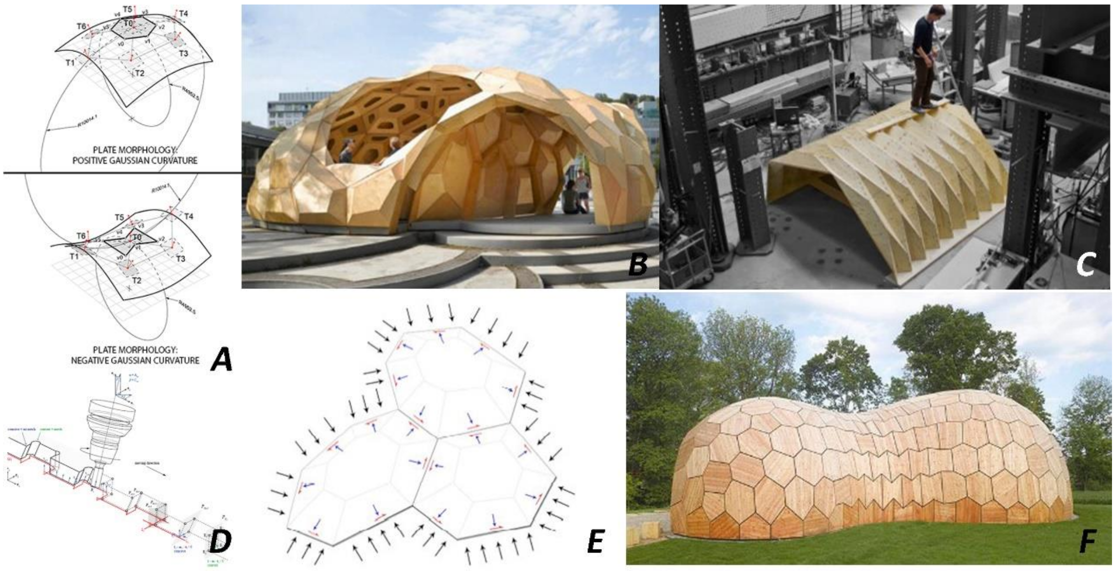


Figure 2.1.14 Recent developments of planarization and joining techniques for plate structures (Krieg et al., 2015; La Magna et al., 2012; Robeller, 2015; Schwinn & Menges, 2015)

On the side of material application, the plane based concept has also inspired the innovative use of glass plates in the thin free-form structures. Since Wester with his ideas of the morphological design with glass, the facetted glass shell structure and its structural behavior have been tested and researched several times. Since 1998, the frameless glass shells came back in the research of new possibilities of shell structures and several tests has been done in the Netherlands and Germany. (Aanhaanen, 2008) (Figure 2.1.15 A) In Bagger's Ph.D. research of plate glass shells, the details of the form and force and design methods are well discussed and a guideline of the design of such shells is suggested. (Bagger, 2010) (Figure 2.1.15 B C)

Figure 2.1.15 Application of planar glass in shell structure experiments (Aanhaanen, 2008; Bagger, 2010)

2.2.3.2 Curve and bending—Active-bending structure:

With a better understanding of the shell behavior and the design methods of shells, the curved structures are also coming back into the central part of shell research. By determining and exploring the fundamental relationship between geometric curvature and mechanical properties of shell and membrane structures, the curved forms are used today more and more in the initial design of the shell structure. With this aim, both research of the curved structure behavior and plenty of designs of "Bending-Active" structures are coming out to play an important role.

An "bending-active" class of structure was categorized and summarized in 2011 as an approach to generate a similar structural behavior like arches and shells through the elastic deformation mechanism of material, i.e. bending. (Lienhard & Knippers, 2014) Unlike the traditional categorization of structures which is based on the structural mechanism and the load-bearing behavior, the bending-active structure offers a new strategy of design that is inherent from a physical deformation and its derived structural benefit. This new morphological topic doesn't refer only to a trend of various emergences of new structure concepts with the design method or the dominance of physical understanding of the relationship of form and force, but also shows a fruitful design potential in an age when the structural analysis methods (e.g. FEM methods) are involved as a collaborative procedure in the architectural and structural design innovations. (Bollinger, Grohmann, & Tessmann, 2010)

In the new experiments of bending-active structures, structures based on curved elements are sought and the potential of elastic materials such as plywood, bamboo and paper are developed. The bending-active structures are hence a popular research field worldwide since 2010. (Figure 2.2.16)

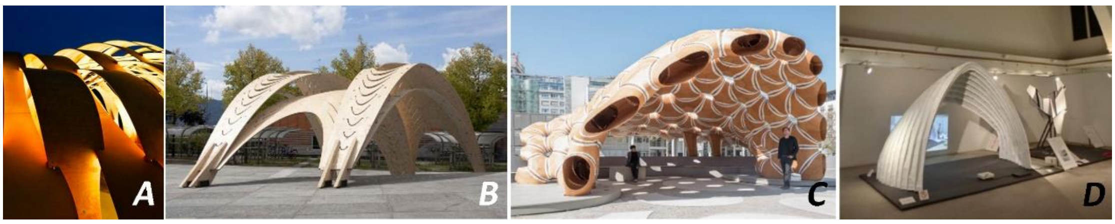


Figure 2.2.16 Recent experimental designs of bending-active structures

2.2.3.3 Transformation of old structure with new materials:

Apart from the direct generation of structure systems with the understanding of structural behaviors or design considerations, in many of the recent shell designs there are also many experiments with some new materials, which are used to replace some parts of the old structure system. With the fast development of the science of building materials and fabricating techniques such as laser cutting, CNC technique and additive manufacturing, many new ecological and efficient materials are developed and used in building experiments.

However, this fever has brought not only some new applications and efficient designs but also a method of transforming the existing structural concepts. In designs from Menges, Clifford, Yu and Ban in many countries, the old structures of masonry shells and the spatial framework system have been renovated with the application of new materials and recent fabricating techniques. (Ban & Bell, 2001; Brownell, Swackhamer, Satterfield, & Weinstock, 2015; Gramazio, Kohler, & Langenberg, 2014) These novel attempts have opened a new way of shell designs to recall and improve the traditional structure system by showing a new appearance and energetic surface. (Figure 2.2.17) Nevertheless, when it is checked from the side of structural efficiency and the materials' behavior, a simple replacement of the old system with new materials often returns a meaningless and costly design. A suitable system of structures today which follows the structure morphology is hence required for the development of shells today and even in the future.

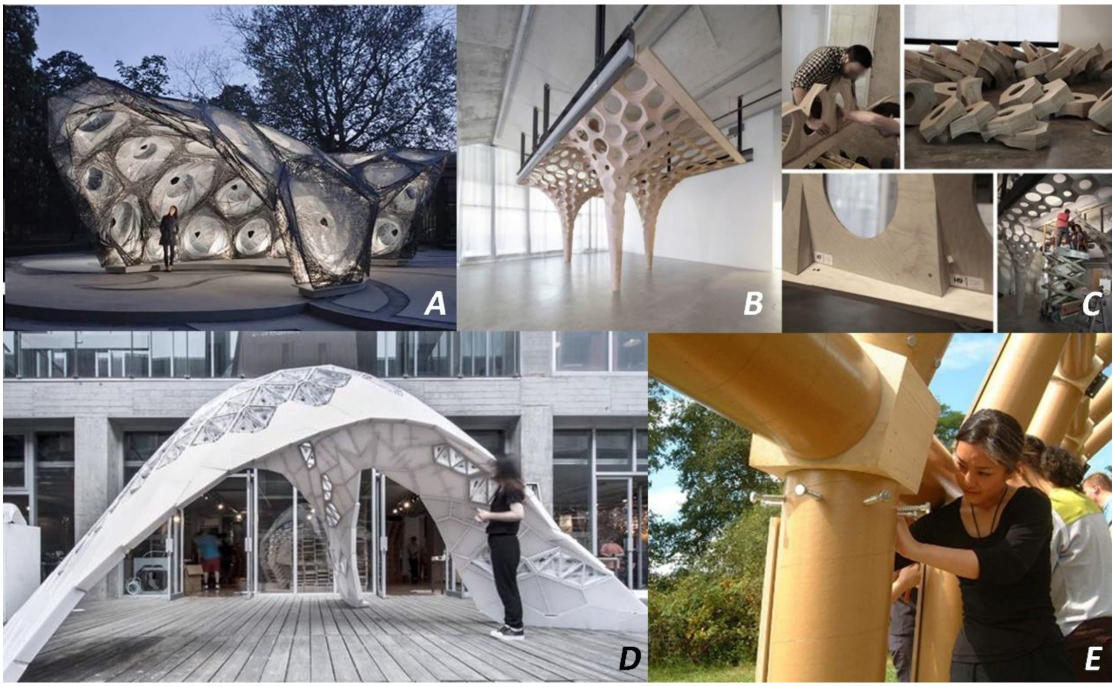


Figure 2.2.17 Recent experimental structures with novel uses of materials

2.2.4 Conclusion: a material-driven tradition of performative design

As it is shown in the previous review of the development of shell structures in the aspect of structural morphology, it is a comprehensive and multi-criterion process of shell structures in the design process. For a proper shell structure concept, not only should the structural behavior be taken care of and properly designed, the geometry and form, the form-force duality, the building and fabricating technique and the materials' behavior also have their importance. With the structure analysis based on a typological method, the materials' application will often raise a great innovation of the shell structure typology. In this sense, such a tradition is called a material-driven tradition of the shell structure development.(Karana, Barati, Rognoli, & Zeeuw Van Der Laan, 2015) At the same time, the form of shells doesn't follow any single element such as function, forces, nature or geometry, but a combination of these objectives. In conclusion, the development of shell structures is also a history of the structure morphology of material, and also a material-driven tradition of performative designs.

2.3 Morphogenesis: Nature-inspired shell designs and cellular structures

With material based design philosophy the morphogenesis can be seen as the most recent design methodology of the structural morphology of shell structures. It refers to a new thinking of the relationship of form and structure, that a structure design is not a decision-making process which is based on hierarchical commands of the structural mechanics, but rather a natural way which comes out from the materials' logic, a way how the material finds its form under nature's rules. (DeLanda, 2004; Menges, 2012)

It is the most original and long-lasting design method of architecture and structures of human-beings where people copy the structure forms in nature by understanding its reason of growth and generation. Since Darwin's theory of evolution in the mid-19th century, the relationship between organic forms and nature's rules has been reviewed by humans in a scientific way. It can also be seen that art of nature forms have also affected and inspired people's understanding and creations in artworks since an early age. (Haeckel, 2013) The learning about the background principles of nature were highlighted by d'Arcy Wentworth Thompson's work in the early 20th century (Thompson & Bonner, 2014), where he used theories in mathematics, geometry, physics and his theory of transformation to explain the generation of forms in nature, such as bodies, cells, tissues and so on. This tradition has also laid the foundation of research on biology, bionics and especially the morphogenesis method in structure design nowadays.

2.3.1 Cellular structures in shells: Pioneer projects

2.3.1.1 SFB (Sonderforschungsbereich) 64 and 230: cells in natural shells and cell mechanics

Cells are considered to be the central units and elements of natural structures and represent a simple way of how the numerous complicated forms in our daily life are built. Between the 1970s and the 2000s, a research interest of the light-weight structure is raised in Germany with the establishment of several special research areas (Sonderforschungsbereich: SFB). Especially in the SFB 64 (on large span surface-structures by University Stuttgart) and its following program SFB 230 (on natural constructions by University Stuttgart and Tübingen), topics about shell structures and the cell forms are first researched with an architectural and engineering aspect. In both research areas, the morphogenesis of cells in natural shell structures has been researched and the mechanics of cells are also discussed.

As an architect, Frei Otto was most important to this topic with his numerous research during the time. In his book about shells in nature and the technics for the SFB 64, the natural micro structure of diatom valves and honeycombs were researched in detail and the theoretical morphogenesis of the cell forms are established. (Bach & Burkhardt, 1984) (Figure 2.3.1)

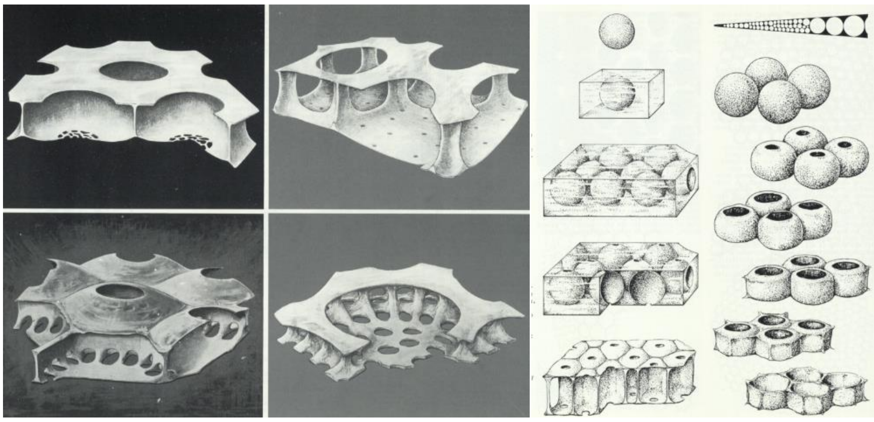


Figure 2.3.1 Research on the cellular structure in SFB 64 (Brinkmann & Flächentragwerke, 1990)

The method of research by Frei Otto was mainly based on the observation of the formation of the micro cell system, and it emphasized the global morphology and related that to the load bearing behavior in the growth process (Figure 2.3.2). In this way, the early stage of the natural cellular structure research based itself also on this simplification of cells as a series of frames, and the studies focused mostly on the typological relationship and possible form finding technique with physical simulations with network materials. In the following special colloquium of the SFB 230 about "cell mechanics" in 1986, it was also suggested by Schnept that the general morphogenesis of cells in plants was the genesis of the cell walls. And Frei Otto had also proposed in his article his interests with the energetic form which was generated with the natural force principles, and had presented his method to find the possible linear grid-system with cellular elements with a physical simulation with pneumatic methods.

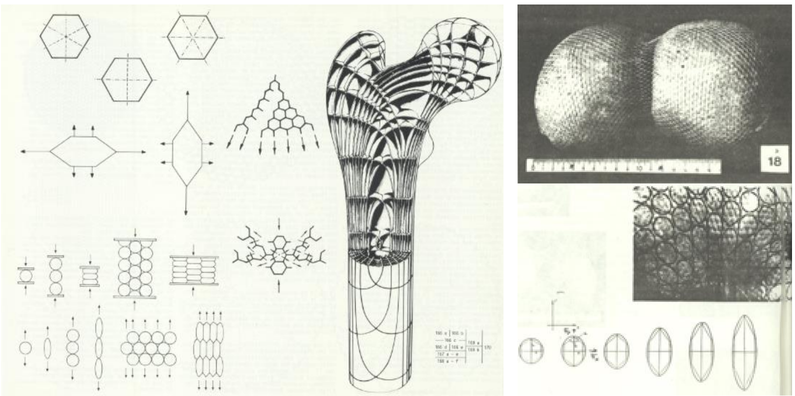


Figure 2.3.2 Diagrams of cell forms and the growth process (Teichmann & Konstruktionen, 1996)

2.3.1.2 Bionic designs of cells and modular building systems

The result of the research from the SFB 64 and 230 was based on a bionic ideological trend, which explained most of the emerging structures at that time and also affected the later architects and engineers a lot. The first uses of the cellular idea with a network system in shell structures can be considered the rib systems such as Nervi's innovation in his reinforced concrete shell. With the organic research and the understanding the flow of force, more energetic rib shells were later built. Another innovation of the building method during this time is the modular system in all kinds of structures. Such examples can be seen in Manleitner's

“Waldbühne” of the Olympic Stadium in Berlin with the individual hexagonal components (Figure 2.3.3 A) and also Torroja’s attempts in building nature-inspired shells with separated triangular elements.(Figure 2.3.3 B)

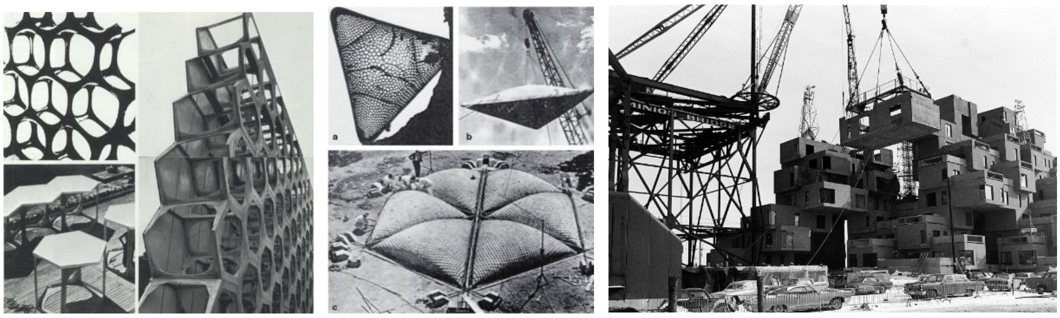


Figure 2.3.3 Cellular and modular building methods (Nachtigall & Pohl, 2013; Teichmann & Konstruktionen, 1996)

However, the most far-reaching applications of such modular methods happened in the development of dwelling buildings; it is famous that the modular building in Expo Montreal 67 had brought the revolution and new tradition to modern architecture. (Figure 2.3.3 C)

2.3.2 Material Morphogenesis and recent cellular structure developments

The cellular structure for shell structures didn’t develop so fast in the 20th century mainly due to the restriction of material. With the most common shell building materials such as concrete and steels, a modular based structure is not so welcomed because the building technique preferred a large scale curved surface which should be built on the formwork and the linear system. Although the natural ideas have inspired the pioneer work such as the “Eden-Project” and that the “Plate-lattice Duality” has lead the plate glass domes, the large numbers of new experiments with cellular concepts in shell structures come recently in the 21th century with the new trend of material morphogenesis and computational technology.

The emergence of cellular structures of shells got its appearance with the building experiments and geometrical research of the structures which use plate materials. With a large demand of free-form spatial framework structures, a structural optimization of the spatial structure geometry with planar covering elements are studied first in the geometry fields to fulfill the projects’ requirement. For the geometrical consultation on several projects, parametric strategies for freeform glass structures using triangular or quadrilateral planar facets are proposed. (Glymph, Shelden, Ceccato, Mussel, & Schober, 2004; Schober, 2003) As a result, the strategy and the optimization method for the form finding of such meshes are generated. These research for plate materials started the following study of another structure type: the structures which can be built with a series of ribs that connects two layers of meshes and hence form plenty of cell walls. Such studies were quickly made and the definition of “offset-mesh” is then established. (Pottmann, Liu, Wallner, Bobenko, & Wang, 2007) With the same aim and as form tests in the architectural studies of computational fabrication, such forms were also built in building experiments since 2000, in which mostly simple triangulation and its reciprocal hexagonal and voronoi forms are tested. (Figure 2.3.4)

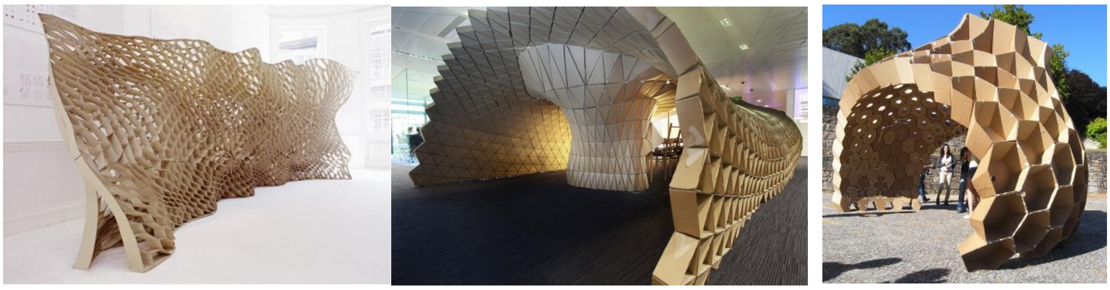


Figure 2.3.4 Cellular frames in triangular, hexagonal and Voronoi forms

With the success in building such structures, the most recent research of such structures have focused on the potential of new patterns of such cell-walled structures. In the geometrical research, more variations of the existing structures are found with concepts such as reciprocal structures. (Song et al., 2013) And in architectural building experiments, such geometries are also being tested and the construction details for the digital fabrication are also discussed. (Scheurer, Schindler, & Braach) (Figure 2.3.5)

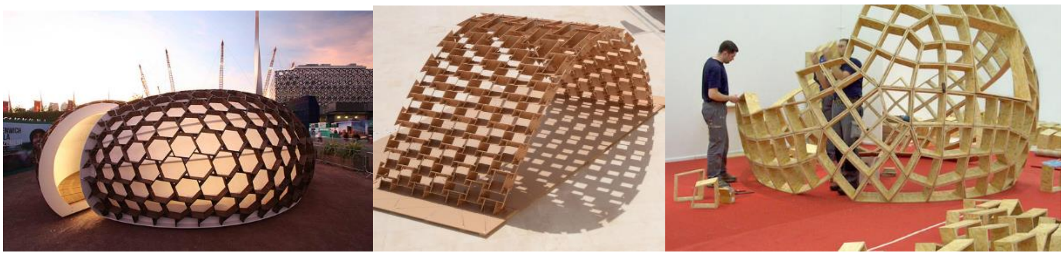


Figure 2.3.5 Variation of cellular frames in various forms

Besides the form design and the geometrical research, the materialization and the fabricating techniques are also an important research topic nowadays. It is also a hot area in architectural research to find the possibility of new kinds of materials and to test the application of new digital fabrication of the materials in the complex design and construction process. Such techniques and materials like recycled cans, paperboard and CNC laser cutting, 3D Printing techniques are being tested in a lot of small scale structure research. (Figure 2.3.6) It has been argued that the application of new materials is changing the design of free-form shell structures with their advantages such as low cost, environmentalfriendliness and easier prefabrication. At the same time, the digital and parametrical design and fabricating process today have also shown the urgency to establish a proper design strategy of such structures for architecture.

Figure 2.3.6 Experiments of cellular frames with new techniques

Since 2010, the shell behavior and the static principle have been considered and added into the new developments of the digital design and fabrications as a determination rule. The traditional form finding techniques and design strategies have been reused and improved with new developments with computational techniques. (Figure 2.3.7) With the multiple structural design software and plugins such as RhinoVault, Grasshopper and Kangaroo, the generation of such structure forms can be achieved by a modular design process with different algorithms. However, such algorithms worked like a "black box" in the design process so that the real behavior and generating process still need to be researched.

Figure 2.3.7 Recent developments of cellular structures in architecture

The cellular shell structure concept is nowadays the cooperating and gap area between architectural design, geometry research and structural engineering. With the advantages in the materials' property and application, such a structure concept has also shown its importance in the architectural research for the shell structure designs. As a recent topic in the structural morphology for plate materials, new issues of the geometrical form finding, the materials' behavior and fabrication, the structural mechanisms and the designing procedure are integrated to the argument of an urgent research field in order to realize it in the building practice.

3. Fundamentals of thin sheet materials

Based on the previous discussion of the developments of material-driven shell designs, this research aims to focus on the recent development and applications of new building materials—the thin sheet material—and to develop a new corresponding structure concept. Being some newly widely-applied materials, the thin sheet material implies a series of industrial materials which are in sheet forms and usually very thin (usually less than 10mm[1]) compared to the other traditional materials in architecture. In this chapter, such materials, their characteristics and the existing applications will be explored and analyzed mainly from two points of views, the constructive and architectonic sides. As a conclusion, the design considerations of such an innovational structure concept will be summarized as the basis of the next chapter.

3.1 Thin sheet materials and their application in industry and architecture

3.1.1 Innovational construction awareness of materials and application of thin sheet materials

The 20th century has witnessed a blooming development of building materials and techniques. Until the mid-19th century, only a few materials have been applied in construction and architecture and the selection was highly limited to the availability of natural materials such as stone, wood, sand, straw, masonry, concrete and a few metals. With the large application of steel and the modern architecture movement, a great number of architectural experiments and research started on the new possibilities of structure and building techniques for new efficient materials. As a result, the architecture materials' category has been largely expanded with the new material types and the emerging member products.

In the past decades, this explosion has come to a high level and has started to impact more on architecture and constructions. As it is mentioned in the discussion of the changes of manufacturing methodology (Kieran & Timberlake, 2004), although the explosive increases of material products has been exponential in the past 150 years, the large application of new materials in the architecture designs has only been well developed since the last quarter of the past century and still remains at a relative low level. (Figure 3.1.1)

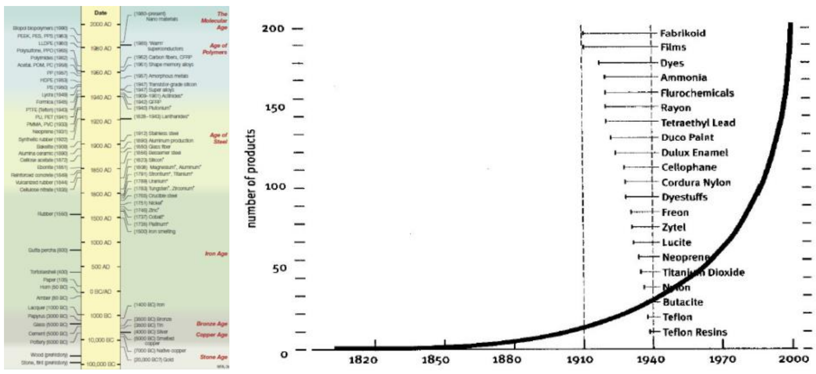


Figure 3.1.1 Explosion of new materials (Ashby & Cebon, 1993; Kieran & Timberlake, 2004)

[1] For different raw materials, there is some diversity of the definition, usually due to the industrial supply and the manufacturing technique. For example, the thin metal sheet is defined as thin as 0.5-3mm, for thin glass to be specifically defined as 1.65mm, for wood sheets 0.5-12mm and for paperboard to be 0.5-7mm

The renovation and development of architecture has also a close relationship with the industrialization, the new construction techniques as well as the new characteristics which the new materials have brought. Apart from the aesthetical infatuation, the newly applied building materials have in general often better structural efficiency—higher ratio of strength to density, and yield significantly greater strength than the traditional materials. At the same time, the modern industrial manufacturing has also enabled a large and fast prefabrication and mass customization of building components from new materials such as plastics, steel and glass. In recent years, along with the development of digital design methods and the robotic application in architecture design, several novel on-site fabrication techniques such as 3D-printing, robotic stone and wood cutting, CNC techniques, have also affected and changed the traditional building techniques and structure types of several materials.

The innovational construction awareness and experiments will finally be reflected in the form and building system of the development of architecture itself and will then catalyze the generation of a new tradition. As it is described in Giedion's book (S. Giedion, 1967), many innovational methods of assembly, such as the balloon frames which benefited from the large use of metal nails in wood structures, had totally changed the traditional building system of wood structures and even the American residential architecture.

The thin sheet material is one type of the new popular materials in architecture and refers to a certain type of industrial materials, which are often semi-finished products in thin sheet forms. Although it is known that most of daily life products and even cars and aircrafts are often made from such materials like plastic panels and thin metal sheets, such materials have just started to be popular in architecture design. In this chapter, their characteristics in the application of architecture will be analyzed and used as a foundation for the further discussion.

3.1.2 Manufacture, advantages and industrial application of thin sheet materials

3.1.2.1 Prefabrication and Manufacturing

As a type of material collection, the thin sheet material can be made by many raw materials such as wood, steel, plastic, glass and paper. The basic performance of such materials varies due to the characteristics of the raw material so that the density, strength and the E-modulus are at different levels. However, with the same form of thin sheets, such materials also share some similarities even from the manufacturing process. All thin sheet materials are made from a long industrial manufacturing process from the raw material to the final sheet. In this part, several typical types—plastic, steel, glass, wood/paper—will be introduced and discussed.

1. Raw material preparation

The raw materials for the thin sheet materials are originally found in nature, such as the natural ore for steel (Figure 3.1.2 B), Tripoli for glass, timber and plant fiber for wood (Figure 3.1.2 C) or paper sheets, and there are also some artificial source materials like the polymer plastic raw material (Figure 3.1.2 A). Such raw materials cannot be directly used in many industries or architecture and need to be further treated.

In the raw material preparation, complicated physical and chemical processes will be applied to prepare the source material for the final semi-products of the thin sheets. (Figure 3.1.2 D, E) For example, in the paper making process, the raw materials (wood chips) will be treated through the mechanical/ chemical pulping process which separates the fibers, the refining process which increases the flexibility and conformability of fibers to achieve good bonding, and the final finishing process to add some other components, such as mineral fillers and chemicals, to obtain a water-based furnish ready for papermaking. (Niskanen, 2012)

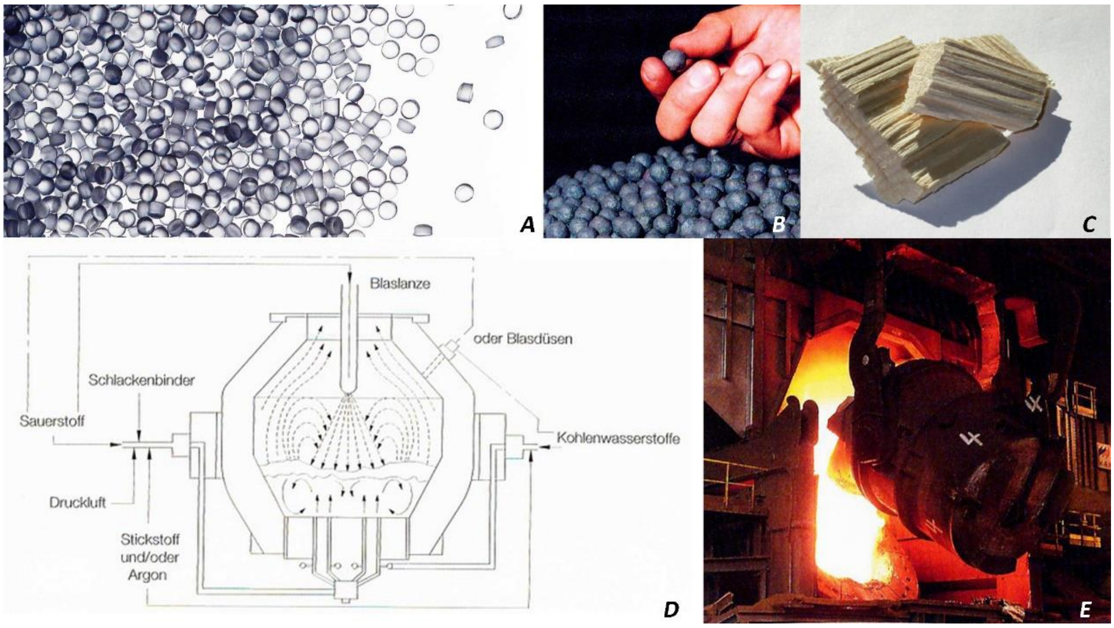


Figure 3.1.2 Raw Materials and the preparation (Bollinger & Architektur-Dokumentation, 2011; Knippers, 2010; Niskanen, 2012)

2. Forming Process and Semi-Product Preparation

The raw material preparation is followed by the forming and pressing process to make the final semi-product of the sheet material. In the forming process, large machines and production lines (Figure 3.1.3) will be used to make the raw material into the sheet form, and also make some other preparation to improve the performance of the product. For example, in the forming process of plastic sheets, different processing steps and also different additives are used between the polymer reactions and the end process of the plastic sheet manufacturing, in order to make the different performances of the material (thermo-plastic, elastic-plastic, duro-plastic). (Knippers, 2010) And for different end-products, the processing technique varies a lot too.

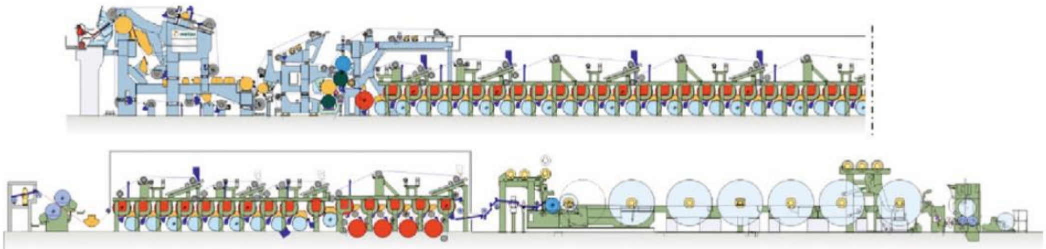


Figure 3.1.3 Production line of paper manufacturing (Niskanen, 2012)

3.1.2.2 Advantages and industrial applications of thin sheet materials

The large increase of building materials has not only expanded the architectural form but also has brought an increase of other parameters which determine the building material choice and hence promotes the new trend of multi-functionalism. Such multi-functionalism can also be seen from the construction industry and the change of most products and the designers' attitude to the material selection. Because of this, it is necessary to analyze the advantages and industrial applications of thin sheet materials for its use in architectural design.

1. Advantages of thin sheet materials

Strength and lightweight

The load capacity is usually the most important characteristic for a structure and also for the construction material. At the same time, a lightweight structure is also being pursued by every designer.

In general, conventional materials have a relatively low ratio of strength to density. A great deal of materials are required to yield relatively little strength. (Kieran & Timberlake, 2004) As can be seen in Figure 3.1.4, through the development of engineering materials, light materials with more strength are invented and recent popular materials such as plastics, CFRP and alloy metals all share a higher value of the ratio of strength to density.

At the same time, the new thin sheet materials have also a relatively thin cross-section and this enables a light weight of the structure unit. In contrast, traditional masonry and concrete which are in the form of blocks and massive solids become too clumsy and counterproductive.

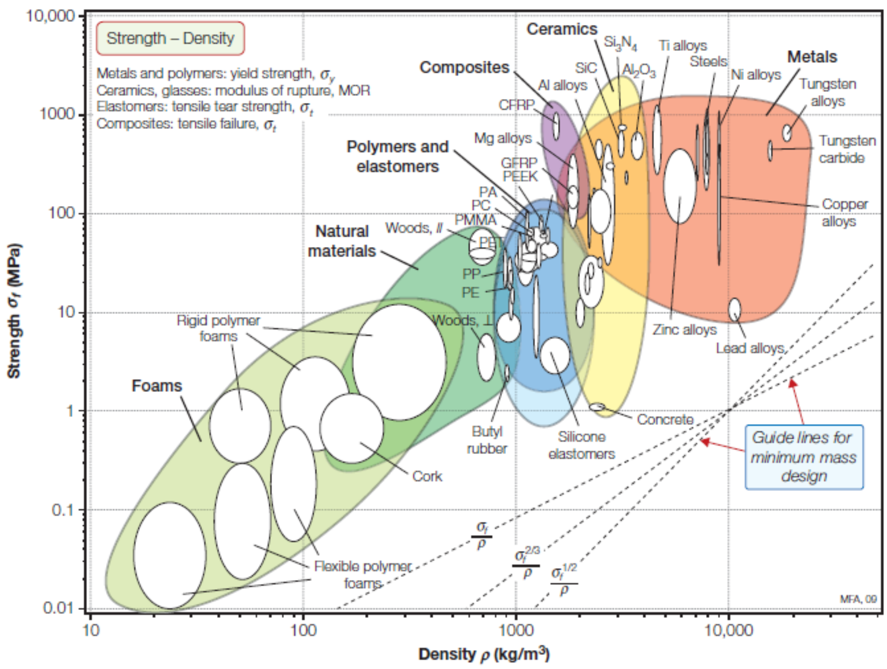


Figure 3.1.4 Strength-Density diagram of engineering materials (Ashby & Cebon, 1993)

Cost-efficiency and mass customization

The cost-efficiency of construction materials involves not only the cost of the materials themselves but also more about the manufacturing costs through the whole fabricating process of the final products. Although it is true that many of the traditional materials offer a very good structural strength per cost (Figure 3.5), the development of material selection has changed itself a lot with the consideration of mass production and mass customization.

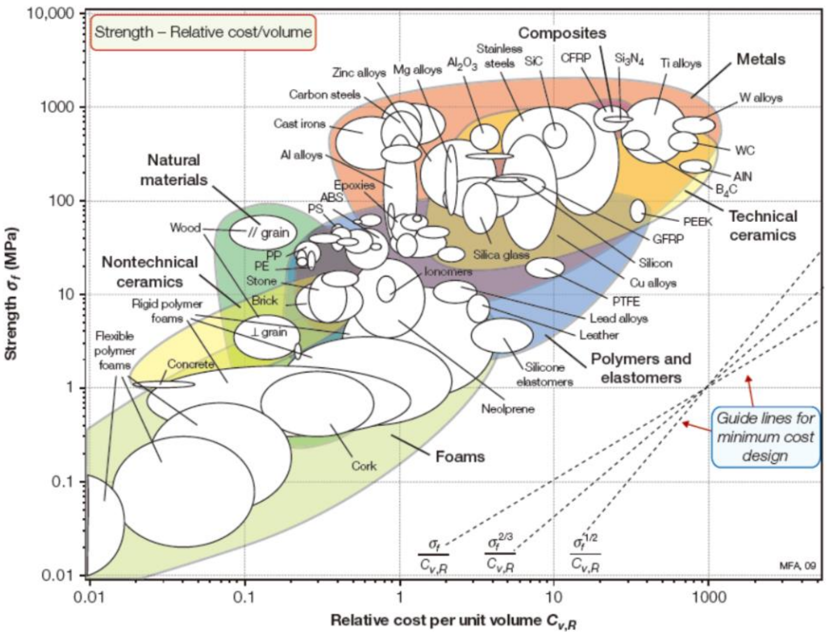


Figure 3.1.5 Strength-Cost diagram of engineering materials (Ashby & Cebon, 1993)

The idea of mass production introduced by Henry Ford with the success of the ModelT of the Ford Motor Company has given an innovational awareness of the important prior developments in the production of interchangeable parts, the idea of continuous flow and the rise of an efficiency movement. (Hounshell, 1985) According to Giedion (Siegfried Giedion, 1955), such a transition comes not at the start but at the end of the mechanistic phase which has also arisen to be the first great development of the construction materials. The new application of machines and the high demand after war had also required for low cost products, which inevitably improved the mass production and standardization techniques.

Nowadays, the new innovations with computer and digital designs have also changed culture and art and have also arisen as a rapid replacement of mass customization to mass productions. Apart from the aesthetic requirement of customers, such a phenomenon can also find its cause with the limitation of materials, the fabricating process and the humanlabor costs. Taking the concrete building as an example, for a good cost-efficiency of a concrete structure, it is required to get rid of complicated moulds or form-work which cost a lot of humanlabor; hence this leads to a restricted form to satisfy the large quantity of repetition of prefabricated components. In this sense, the advantage of mass customization of the thin sheet material as a well-developed industrial material shows us the potential of low cost, light weight and being less complicated for manufacturing.

Plasticity and formability

Plasticity and formability are also the most important reasons that make the thin sheet materials widely used in the construction industry nowadays. Free forms which are derived from the plasticity have become the popular preference of product designs and also architectural designs in the last decades. The thin sheet materials, which can be easily shaped in a desired form, have greatly satisfied the designers' lust.

Compared to the traditional massive materials such as the concrete and masonry, due to the cost issues, construction inefficiencies, weight and also the detailing disadvantages, new sheet materials and cladding materials are also more and more popular in modern structure designs as a surface material or façade material. For the cladding materials, the construction efficiency is not much improved because of the complicated manufacturing process. It always requires special moulds and complex hand work to cast and shape the material. (Figure 3.1.6 A, B) In contrast, the thin sheet materials can nowadays be shaped and formed easily by hand or with a lot of new no-mould processes (Figure 3.1.6 C, D) and hence have become the most efficient new materials in both industrial and architectural construction.

Figure 3.1.6 Formability and forming techniques of thin sheet materials (Castaneda, Lauret, Lirola, & Ovando, 2015)

2. Industrial applications of thin sheet materials

The industrial application of thin sheet materials are far earlier and wider than its uses in architectural construction. Due to the development of the vehicle and aircraft industry, it is since the 1930s that such materials as metal sheets have started to be used for vehicle bodyworks and aircraft wing and body-skins. (Petsch & Petsch-Bahr, 1982) (Figure 3.1.7 A-C) Nowadays also the composite materials made by polymers and reinforced by fibrous and particulate materials are gaining their popularity in such applications because they are stiffer, stronger, more ductile and light-weight. (Happian-Smith, 2001)

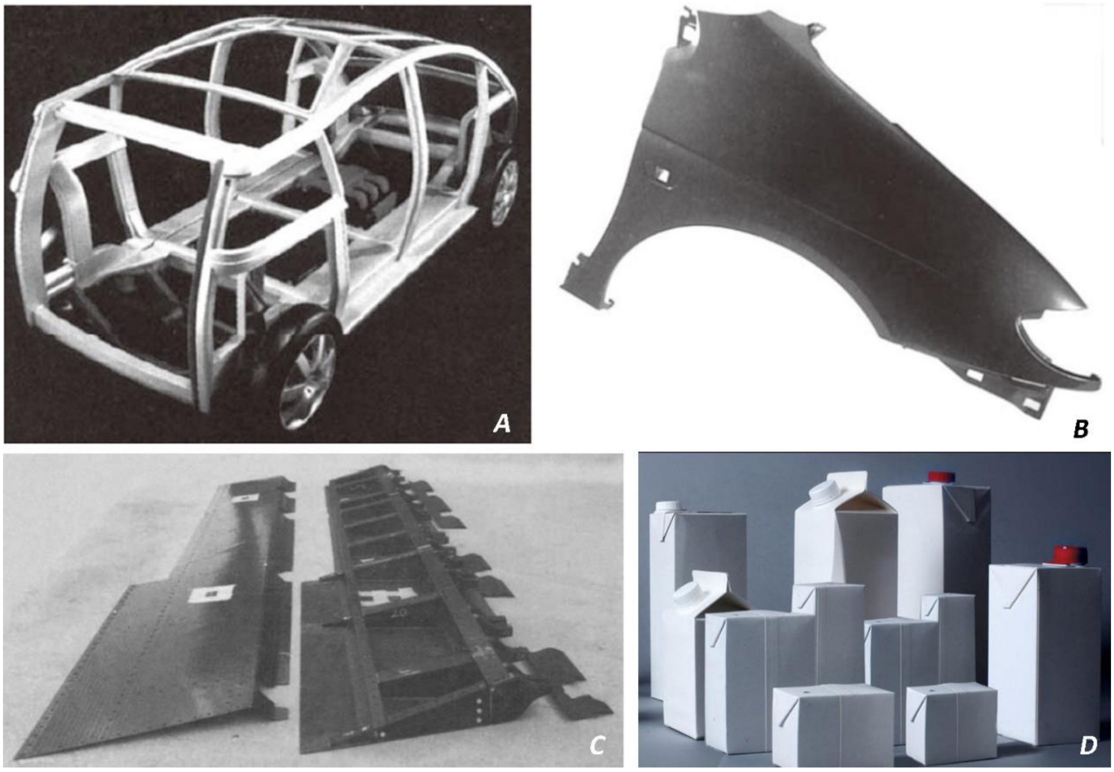


Figure 3.1.7 Industrial applications of thin sheet materials (Baker, 2004; Happian-Smith, 2001)

Apart from the uses in the large manufacturing industry, thin sheet materials are also now widely used in our daily life, such as food packaging (Figure 3.1.7 D), model making and also the covering of smartphone displays. With the rapid development and the high efficiency of the materials, they are being used nowadays in more and more fields including architectural experimentation.

3.1.3 Experiments and applications of thin sheet materials in architecture and lightweight structure design

The experimental architectural application of thin sheet materials has started in the 1950s, just after the war and along with massive rebuilding movements. Such materials have also arisen from the inspiration of architects to create more efficient construction techniques with novel form languages. In general, there are four types of thin sheet materials which are popular in architecture designs: plastic and polymer composites, thin metal sheets, thin glass panels and timber/ paperboards.

1. Plastic and polymer composites

Building with glass-fiber reinforced plastics (GFRP) has started with the large manufacturing of glass fibers in 1932 and the discovery of unsaturated polyester in the USA in 1942. (Genzel & Voigt, 2005) Since the 1950s, experimental buildings with plastic sheets have been built in the USA and Europe.

The well-known and wide spread use of plastic structures should mainly be owed to Buckminster Fuller’s many experiments with his geodesic dome theory and his patent for the radome, which served as the construction to protect a wide-range of military or weather radars. The special function shows a particular requirement for such structures which must be metal-free and cannot be disturbed by electromagnetic waves. In the time, plastic and polymer composites just satisfied this requirement and enabled Fuller to build domes with a diameter of 16.8m with a laminated composite from glass fiber mats and epoxy resin. (Figure 3.1.8) During the time there were more than 200000 such structures which were built all around the world, and it just stimulated the rapid development of plastic architectures. (Genzel & Voigt, 2005)

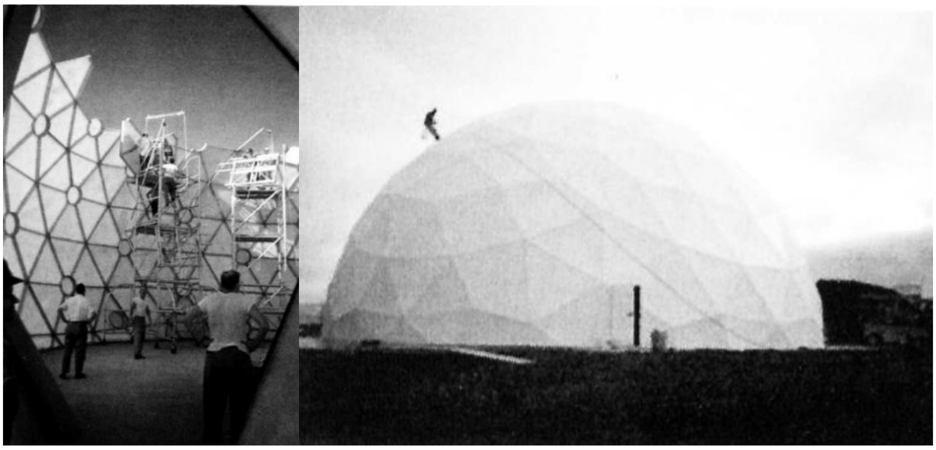


Figure 3.1.8 Radome structure by Buckminster Fuller (Genzel & Voigt, 2005)

Because the manufacturing of plastic composites at the time was still relying on the laminating techniques and manual operation with certain moulds, more free-form pre-fabricated sheets are produced and used in architecture with new forms. In 1954, the Monsanto Chemical Company had started the design of a plastic house research with the idea from the Massachusetts Institute of Technology, and then had built the "House of Future" in Disney California. (Figure 3.1.9 A, B) With the patent of Monohex from Fuller, (B.F. Richard, 1965) the Fly Eye's dome had been designed by a modular structure concept and worked as an "autonomous dwelling machine" in 1965 for many sheet materials such as metals and wood, and then first realized in 1975 with GFRP. (Figure 3.1.9 C) Such projects had shown a great future for the plastic structures and inspired more architects to create their innovations with such materials.

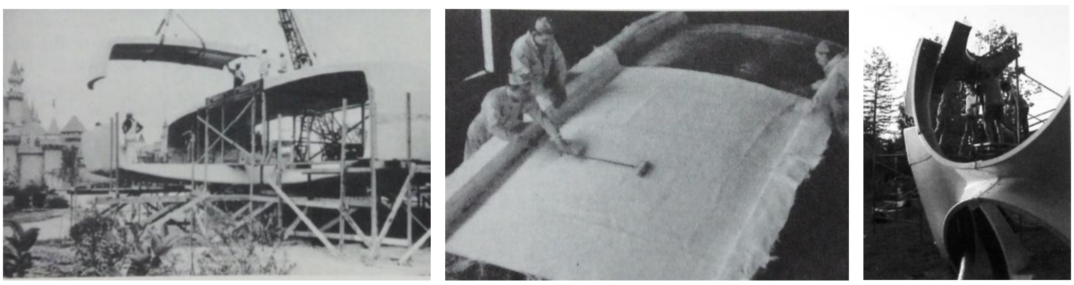


Figure 3.1.9 Project "House of future"(A,B) and "Fly-eyes's dome"(C) with GFRP (Genzel & Voigt, 2005)

With the desire to build the structure much lighter, architects had also been trying to apply thinner materials in the building experiments. However, it is the inherent characteristic of plastic materials that the lower stability with thinner cross-sections will cause a great deformation. To solve such problems, apart from the former use of curve shaped sheets which were laminated with moulds, the planar industrial sheets had started to be used with folding forms. In 1966, the Renzo Piano Architects and the research institute had built a barrel vault based on a uniformed

FRP panel in rhombus form. Every panel was 14kg and measured 2.72*1.20m. (Figure 3.1.10 A,B) In 1968, the research group "Tragende Kunststoffbaukörper" had focused on the folding structures for plastic sheet panels which are suitable for the industrial prefabrication and had built a prototype of such a structure also with the uniform GFRP panels. (Figure 3.1.10 C)

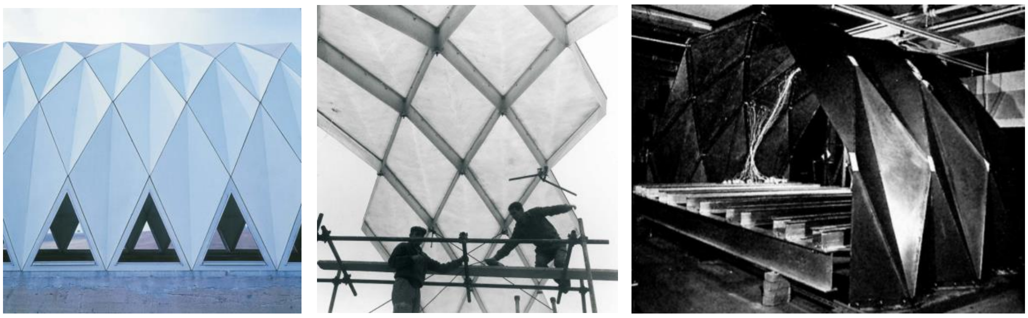


Figure 3.1.10 Applications of planar sheets in the architectural experiments (Genzel & Voigt, 2005)

The use of the planar industrial thin sheet panels had led to the emergence of some new structure forms which were rooted from the folded structure and combined with the spatial grid-framework system. In the late 1960s, such structures were designed and well researched in the University of Surrey (Saechtling, 1973) and were also realized with a lot of variations in real building projects by architects such as the Renzo Piano Architects. (Figure 3.1.11)

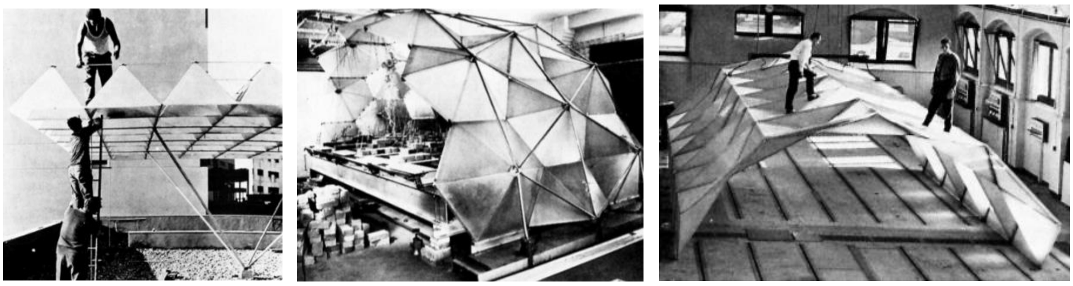


Figure 3.1.11 Spatial grid-frameworks with planar thin sheets (Genzel & Voigt, 2005)

The plastic thin sheets are nowadays also used widely in architecture designs, mostly as the façade element and covering skins for many structures. In many present projects and research, such materials are being used in combination with spatial frameworks and also as self-supporting structures. (Figure 3.1.12)

Figure 3.1.12 Modern application of plastic thin sheets in spatial framework structures or as self-supporting structures. (Knippers, 2010)

2. Metal sheets

Among all the thin sheet materials, the metal sheets are the most common materials in architectural construction due to their high strength. Metal components had been brought into architectural designs since the industrial revolution and were well developed in the past century. It is almost in the 1920s that the forming techniques of metal sheets were well mastered and the thin sheets were also from that time widely used in all aspects of building structures, such as furniture, structures, façades and roof details.

For the metal sheets' application, Jean Prouvé could be one of the first masters of metal sheet forming and its early developments in architecture. After the previous experience of forging techniques and his artworks with iron and metal, Prouvé started to advocate the new form possibilities of metal sheets with new fabricating techniques such as stamping, welding, edge bending and roll bending, which can be finished by special machines. (Hachul, 2006) During the time, Prouvé devoted himself to designs of such metal sheets of 1-4mm in various scales. At first, experiments had been done in all kinds of small constructions such as tables and chairs. (Figure 3.1.13 A) The series of products and the connection details he created gave him the enough experience to make some small building projects like walls, doors and elevator cabins (Figure 3.1.13 B) finally in 1929.

The building project Maison du Peuple in Clichy in 1939 was one of the first building projects that was built entirely from metal sheets of less than 4mm width. With the edge bending technique Prouvé was able to establish a robust structure system to create a large space, opening roof, detailed canopy and large façade element from metal sheets with a maximum size of 1.2 by 4 meters. (Figure 3.1.13 C)

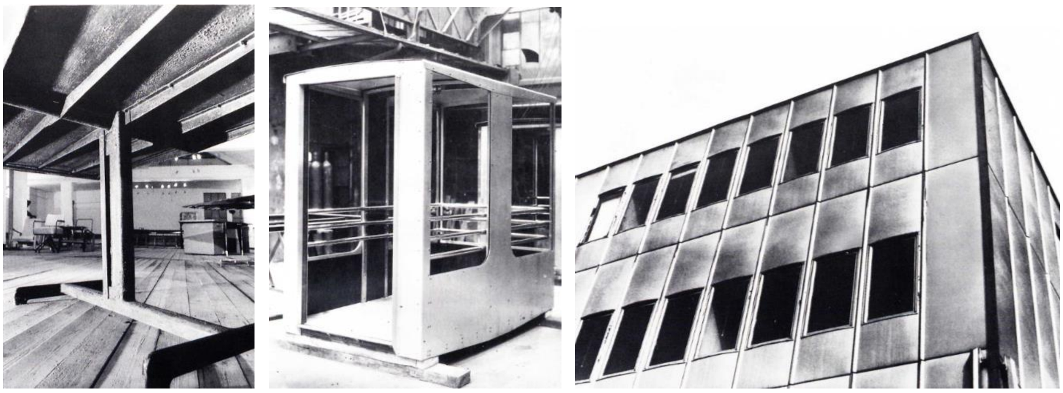


Figure 3.1.13 Project Maison du Peuple in Clichy by Jean Prouvé (Sulzer & Prouvé, 1991)

Almost at the same time, structures of metal sheets were also researched by Buckminster Fuller in the USA. Fascinated by the performance of aluminum such as its light weight and natural qualities, long term durability, recyclability and lack of maintenance or painting, Fuller started to design his first metal housing project—the Dymaxion House—since 1927. Although the first several designs of the Dymaxion House were not satisfiying to Fuller and finally not realized, the experimentation of a subordinated design—the Dymaxion Washroom—had a surprising success and impacted Fuller's future designs significantly in plan, and clearly reflected his many intentions in this project. (Figure 3.1.14 A) In this project, Fuller has designed a fully packaged toilet which was composed of 4 pieces of metal parts. For every part, layered copper metal sheets were used and stamped previously with moulds. The whole structure was only 210kg and also cost only 300 US dollars. The design has shown the impact of the industrialized pre-fabrication technique on Fuller’s design method and also effected the future designs of Fuller for the lightweight structure with thin sheets.

Between 1944 and 1946, after several new optimized designs of the Dymaxion House and the prototype of the Dymaxion Deployment Unit in 1940, Fuller had designed and built a pioneer project—the Wichita House. (Figure 3.1.14 B) Affected by his experiences with formed thin metal sheets, Fuller tried to optimize the design with more advanced techniques from the aircraft industry and specialized structure concepts. With a central mast which integrated tensile members within the top of the roof and the crisscrossing tube elements as a system of strut framework, it also became a prototype and a test for Fuller’s geodesic dome structures. (Neder, 2008) (Figure 3.1.14 C-E) The material in the Wichita House was mainly aluminum, and it was shaped and prefabricated in the aircraft industry in a factory.

The first experiments of Fuller’s showed his preference of formed sheet materials and his interests in mass-production. (Fuller et al., 1999) After his patent of the geodesic dome structure, such interests have become a typical structure logic of his to use modular elements to build surface-active hybrid structures. In his many later works, it can be seen that this tactic has been tested and optimized in construction details for several times. In his Kaiser Dome project in 1957, Fuller designed a double-layered structure, where the creased aluminum sheet stabilized the rhombus structure as a local shell membrane. (Figure 3.1.14 F) And in the Union Tank Car Company Dome in 1958, 3mm planar steel plates were used to build a larger dome with a diameter of 130m. The structure system and its geometry form are always the key aspect of the research by Fuller, and this can also be seen his many patents as well as the construction details. (Buckminster, 1954; Richard, 1959; Buckminster Fuller Richard, 1965)

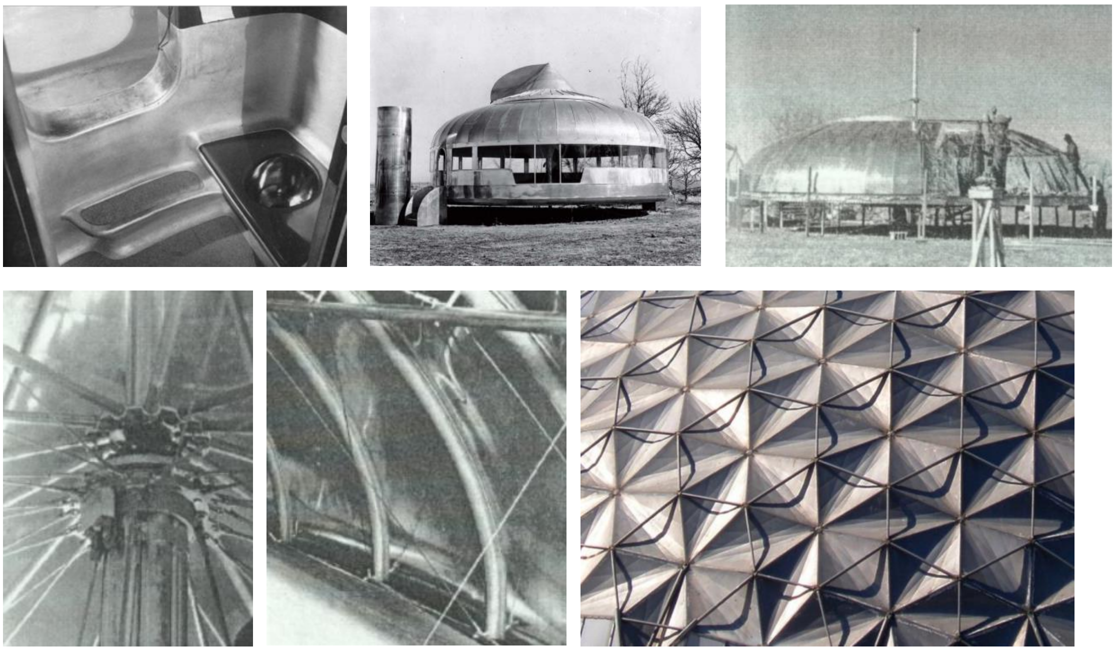


Figure 3.1.14 Applications of thin metal sheets in the experimental designs by Buckminster Fuller (Fuller et al., 1999)

The aircraft industry and the new construction and forming techniques have not only affected the building technique in the USA, but also changed the structural design in Europe. Like Fuller, Hugo Junkers brought the thin metal sheet into the use of architecture after he had invented the full-metal aircraft and opened his own company. Accompanying Bauhaus and his colleges such as Okkokar Paulssen and Siegfried Ebeling, he opened his own architecture office and research group for the "metal house" and started to develop a small metal cabin in 1924, which was also the start of research by Junkers for lightweight structures. (Walter Scheiffele, 2003) The Lamellenhalle was the first large scale structure by Junkers in 1926, which was based on the Zollinger structure concept for the spatial framework system and used thin metal sheets 2-6mm thick and 1-3m long. (Figure 3.1.15 A,B) As a large scale experimental structure, the Lamellenhalle has shown a great example for the advantages of thin metal sheets with industrial prefabrication techniques. Although the C-profile was only a substitute for an existing structure form of wood and metal rods, the new technique has shown people the new possibility of mass-production, high accuracy, low transport fees and a simple assembling process.

At the same time as the Lamellenhalle, innovational structures with thin metal sheets were also being researched by Junkers' team in the "metal house" group in Bauhaus. Inspired by the simultaneous invention of the Dymaxion House by Fuller, Junkers also focused on the new possibilities of metal houses. (Hachul, 2006) Based on his experience in aircraft design and the forming technique of thin metal sheets, Junkers invented a mostly thin-walled building system, where walls, floors and elements were built with metal sheets and used a special structure form to improve their stability. The target of Junkers was to spare the structure weight and the use of materials as much as possible. Instead of using massive solid materials or simply adding to the cross-section of the material with laminating, Junkers created several sandwich structures and also hollow structures. The most creative characteristic of his inventions was the concave membrane sheets which were bent with the cold-bending technique and were facing each other. (Figure 3.1.15 C,D,G) By doing so, the surface would gain at first a tendency to bend inwards and with local stiffness by either using filling materials (Figure 3.1.15 F) or perpendicular supports (Figure 3.1.15 E), the structure itself would be well stabilized. (Hugo, 1933)

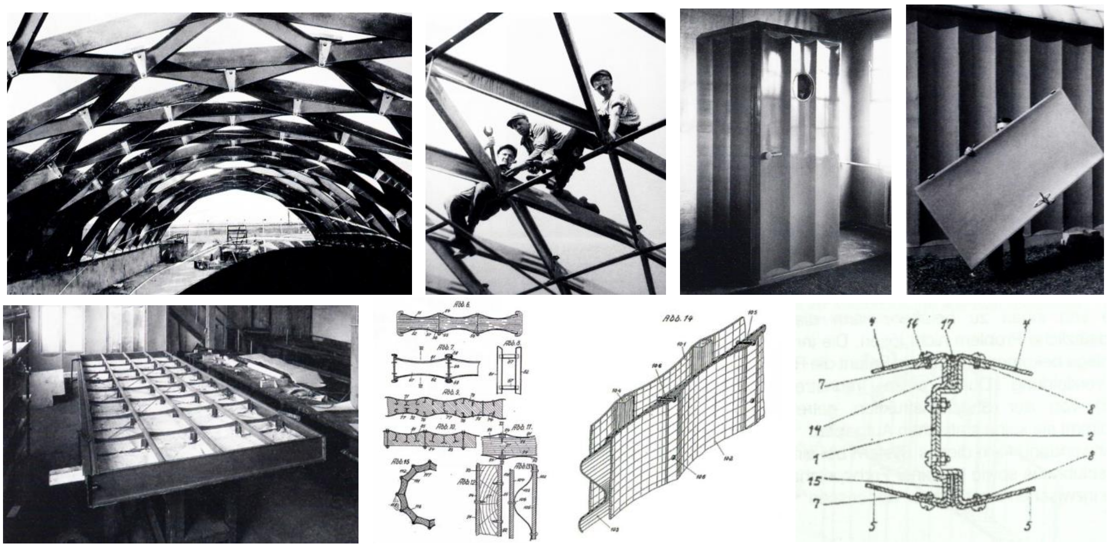


Figure 3.15 Structures and patents of constructions with thin sheet metals by Hugo Junkers (W. Scheiffele, 2016)

The pioneer works between the 1920s and the 1950s have established an ingenious tradition of using thin metal sheets as a structural material, and they are also followed by too many architects to use such materials in the building projects. With the development of the steel industry and the popular use of spatial framework structures in architecture, thin metal sheets are more and more widely used as cladding and surface materials instead of structural materials. (Figure 3.1.16 A,B) In recent years, a renaissance of research and experimental constructions with metal sheets have arrived due to the rapid development of digital design techniques and the new tradition of building with new materials for lightweight structures. Most of the modern forming techniques, such as laser-cutting, CNC cutting, and incremental sheet forming techniques are used together with the new structure concepts and design logics. (Figure 3.1.16 C-G)

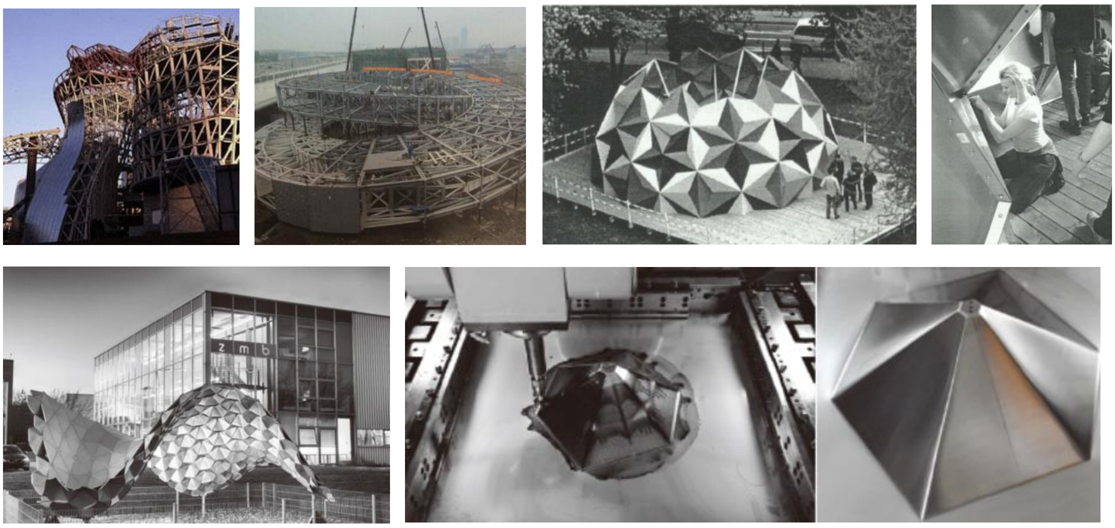


Figure 3.1.16 Modern architectural applications with thin metal sheets and the development of forming techniques (Bailly et al., 2014; Bollinger & Architektur-Dokumentation, 2011; Hachul, 2006)

3. Flat glass sheets

Although it has been 2000 years since flat glass was firstly used in architecture, the continuous technique development and the industrial manufacturing and refining process have made it one of the most modern building materials which is present modern architecture today. (Wurm, 2007) Nowadays, glass has gotten a new significance in architecture due to the pursue of architects for enhanced transparency. With the development of the steel structure and spatial frameworks, glass was integrated mainly in many large projects with skeleton structures which minimized the structural function and materiality of glass. (Figure 3.1.17)

Figure 3.1.17 Integration of flat glass sheets in the spatial framework structures (Wurm, 2007)

In the glass industry there are many semi-products such as the glass block, glass profiles and flat glass. The material discussed here is the flat glass, which is often used as a building material. Over 70 percent of float glass today is used in new buildings or the renovation of old glass skins.

(Klindt & Klein, 1977) The discussion in this part is not only limited to thin glass, which sometimes has a special definition with a thickness of less than 1.65mm due to the special uses as photos, lantern slides and coating materials in LCD/LED displays. (Commission, 1985) Because it is discussed here in the possibility and experimental uses of planar sheet materials which all have similar properties such as a relative thinness, the industrial manufacturing process and also the sheet forms, the sheet glass (mostly manufactured up to 10mm) is used for a specific definition in this part.

The use of sheet glass as structural elements has just started in recent decades, and was first designed to replace traditional structural elements in the orthogonal structure systems, such as the column, beams and fins in the frame constructions. It is since the 1950s that the glass fins started to be used to stabilize shop windows and since the 1980s that glass beams were increasingly used in roof structures. (Wurm, 2007) Later in 1994, the first glass beam that subjected to bending was used in a horizontal glass roof in Paris. (Schittich, Staib, Balkow, Schuler, & Sobek, 2007)

Since the 1990s, research about the structural uses of sheet glass has been started worldwide. In 1995, an experimental glass pavilion was built for research in RWTH to transform the traditional frame structure with glass elements. (Figure 3.1.18 A and B, column 3*12mm, beam 2*6mm roof and wall 12mm) In most of this research, it is found that the glass structure could provide a sufficient capacity for structures with reasonable designs to be placed in the compression area of the structure and with adequate stabilizations. In 2002, a "glass cube" pavilion was built by ILEK (Institute for Structural Systems and Structural Design) from Stuttgart with a large span roof of 5.5m. For the roof panels which were made from VSG of 2*10mm TVG, 6 supporting GFRP ribs were added to make the stabilization. (Figure 3.1.18 C)

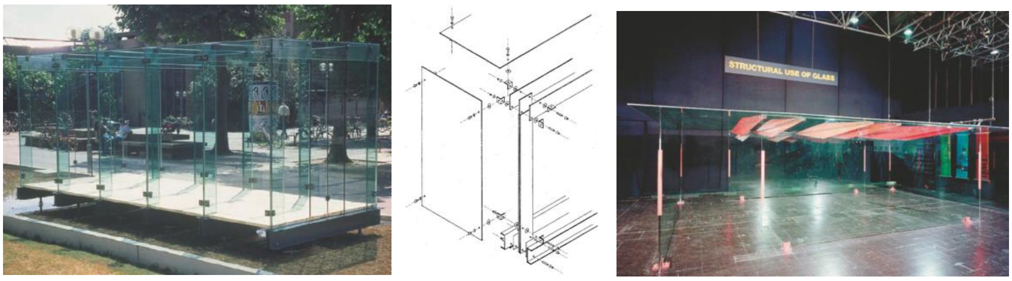


Figure 3.1.18 Experiments of flat glass sheets in self-supporting structures (Wurm, 2007)

With plenty of experiments and experiences for understanding the materiality of flat glass, structural forms were also developed to fit the specific performance of the material. In the later experimental research, developments were mainly done in a typological way to establish a corresponding structural system for the efficient use of flat glass.

In such experiments, the first principle which was generated was an abandonment of the pure structural uses of flat glass in a hierarchical structure system. Being a brittle material which is very sensitive and can fracture suddenly under a critical load, it is not suitable to be used in the hierarchical structure where a sudden failure of the primary elements will cause the failure of the whole structure. At the same time, due to the bearing behavior of glass in that it prefers compression to bending moments, compression only structures such as arches, barrel vaults, grid-shells and facetted shells have started to be developed in the experimental flat glass structures.

Although it was proved with the "Stuttgart glass shell" in 2003 that the pre-formed glass could be able to construct shell structures with only 10mm thickness and reach a ratio of thickness to span of 1:850, the double-curved glass is still too costly for the present fabrication technique. Structural and morphological innovations which make use of the planar characteristic of flat glass panels are developed nowadays, such as the glass rib structure, which was renovated from the Lamelle shell concept (Figure 3.1.19 A) and the facetted shell, which was based on the concept of lattice-plate duality. (Figure 3.1.19 B,C) (Bagger, 2010)

Figure 3.19 Development of the recent applications of thin glass sheets in the experiment building projects (Wurm, 2007)

Meanwhile, cellular structures which are made from flat panels have also become a trend for the flat glass structures. The modular building concept fits well with the industrial products of sheet glass and the elements can be easily cut from standard float glass into different sizes. The polyhedral shape of the cells can be made with planar surfaces and the connection can also be achieved with adhesives and solders. Since the work of Fuller (the discovery and application of geodesic domes), the cellular forms started to be used in architectural experiments. In 1980, Renzo Piano invented a semicircular pavilion for the IBM travelling exhibition where a tunnel vault was composed of 34 self-supporting segments, each of which contained a row of 12 polycarbonate pyramids. The segment used a three-hinged arch structure that was composed of a series of three-chord beams, which were supported by glulam rods and stiffened by the polycarbonate pyramids. (Figure 3.1.19 D) In this way, a complex cellular structure which was under mainly axial loads was made by a mixture of materials and started the innovations with transparent materials. In 2000, a similar "tetra arch" with a span of 8m was built as a prototype and presented at the exhibition Glasstec 2000. The structure also used a cellular form which composed of tetrahedrons and half-octahedrons to conform a fixed-ended three-chord trussed arch. (Figure 3.1.19 E) The shape was controlled with the section design to ensure the structure was mainly subjected to compressions and to enable the whole structure to be built out of 2 x 6 mm heat-strengthened glass and fixed by anodized aluminum joints. The cellular structure is today still an experimental structure form which is being tested by architects and engineers and many variations of form details such as the polyhedron packing and the tetra grid (Figure 3.1.19 F) are being developed.

4. Timber and Paperboard (Natural fiber-based materials)

Thin timber sheets and paperboards are extensively used in daily industry from food packaging to interior elements such as furniture. It has made such fiber-based materials and their byproducts welcomed mainly due to advantages such as their relatively low cost, high degree of recyclability and environment friendliness. The structural use and the analysis of such products have also been studied and it has been proved that paper and cardboard have been shown to have analogous strength, stability and stiffness in comparison with other common building materials. (Sedlak, 1975)

Meanwhile, due to limitations such as their vulnerability to humidity, fire, chemicals, UV rays, and temperature changes, timber sheets and paperboard are relatively less used in large scale architectural design due to their weakness compared to metal and plastic sheets. However, for experiments of light-weight structures, such materials were often applied in the prototypes in relatively small scales in the last decades.

Since the first emergence of the cardboard building, the 1944 House, paperboard became popular for temporary experimental buildings and accommodations for low-income people and refugees. (Critchlow, 1970) After the first uses of plastic panels and the application of geodesic structures, in 1960 Fuller developed the Geospace Dome for Monsanto Chemical Co. as temporary buildings, which were made of 12.7mm layered carton panels. (Figure 3.1.20) The panels were mainly composed of 2 layer paperboard, one for water-proofing and the other for stabilization, whose in-between space was sprayed with PUR foam. The dome showed its great efficiency with a covered space of 32.5m2 and a cost of only 345 US dollars. In 1966, Hirshen and van der Ryn built a new experimental structure, the Plydome, for migrant agricultural workers in California, where a pre-folded shell was built from a 10mm kraft paper panel with a polyurethane core. The structure first applied the origami technique in architectural and structural design and enabled a very efficient building with a total cost within 1000 US dollars.

Figure 3.1.20 Early applications of cardboard in temporary architectures (Critchlow, 1970)

In both of the Geospace Dome and the Plydome the applications of the basic structure concept of folded plates can be observed, which encouraged the later developments of the spatial structure built with planar materials, such as Fuller’s various geodesic plate structures and the innovation of composite polyhedrons and the building of the dome “Stéréometrique” in 1970 by D.G. Emmerich. (Emmerich, 1990; Georges, 1967) However, until the 1990s, the development of the structural use of thin wood sheets and paperboard was slow mainly because the materials were being researched by the specific industry to overcome the disadvantages and limitations. With the development of the material industry it became possible to see many building experiments in the 1980s and 1990s. During these years, a dome artist, Steve Millers, built several domes with the Self-strutted geodesic plydome patent by Fuller (Richard, 1959) and showed the new possibilities of thin plywood as a building material. (Figure 3.1.21) The different projects in various scales all applied the same principle of structure and got full use of the bending behavior of the thin plywood sheets.

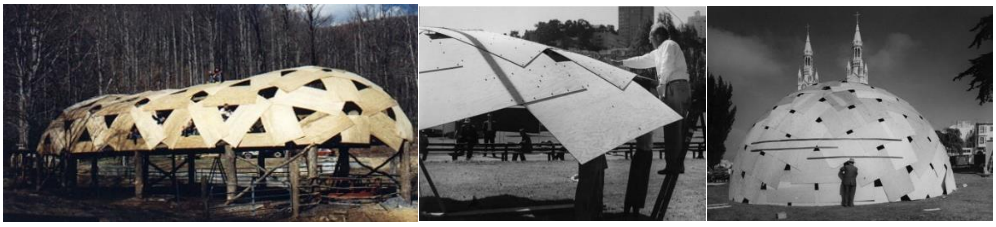


Figure 3.1.21 Application of bent plywood in free-form architectural experiments

Pioneered by the previous architects and artists, the recent 10 years have witnessed an emergence of such experimental projects with prototypes with the thin sheets of wood panels and paperboards. And with the new development of the digital design research and the industrial fabrication technique, theoretical research about the form finding and the modern joining and assembly techniques were tested in different projects. In architectural projects, the bending form which is brought by the using of thin sheets are often used to present a modern plasticity and formability to create impressive space with a lightweight structure. Such projects can be seen in the Winnipeg skating shelter in Canada in 2011 (Figure 3.1.22 A), where the 2 layered 10mm plywood was used and bent into a shelter form to accommodate a few people. It can also be seen in the temporary fire shelter in Copenhagen in 2013(Figure 3.1.22 B,C), where a 4.7m high structure was built from 2-9mm plywood pre-fabricated with the CNC-technique. Such a bending form of the materials has also inspired a recent upcoming category of structure typology—the bending-active structures. In the Ph.D. dissertation of Julian Lienhard (Lienhard & Knippers, 2014), the structural characteristics and projects were analyzed in aspects of form-finding techniques, structural analysis and the prefabrication techniques. Such a structural concept was also tested in the ICD/ITKE research pavilion in 2010 (Lienhard & Knippers, 2013), which covered the space with a diameter of more than twelve meters and was constructed using only 6.5 millimeter thin birch plywood sheets. The bending-active structure is one of the popular research topics in lightweight structure research, mainly because the sufficient use of bending behavior enhanced the load-bearing capacity of materials and enabled the thin materials to be used in large scale structures. Such examples can also be seen in the on-going research of the timber-fabric structures from the IBOIS (Figure 3.1.22 D, E) (the laboratory for timber construction of the EPFL) and also the research pavilion from EmTech (AA) and ETH Zurich. (Figure 3.1.22 F, G)

Figure 3.1.22 Modern applications of timber sheets and paperboard in architectural experiments

Similar to the recent applications of plywood, paperboard is nowadays also widely used in the experimental projects for lightweight structure designs. Although the problem of water-proofing and moisture-proofing for the paper products is not yet well solved, such materials have already been used in indoor structures and temporary structure experiments. At the TU Darmstadt, structural cardboard has been used in experimental structure courses and new special structure concepts were suggested, analyzed and tested in full scales. (Figure 3.1.22 H) At Tongji University, a worldwide academic structure competition was held every year to promote the structure innovations with new materials, and cardboard has been chosen for several years. (Figure 3.1.22 I, J) As the one-layered thin paperboard is too weak and soft for its mechanic performance, in most cases multi-layered corrugated cardboard or honeycomb cardboard are used to derive a better stabilization. The possible using of thinner paperboard is still to be developed. In the experimental prototype of this dissertation, 1mm press-paperboard is used to build a large scale dome structure.

3.2 Materials' behavior and physical characteristics

The development of a structure system relies on the reasonable understanding and use of the materials' characteristics. And after reviewing the existing thin sheet materials' applications in both industrial and architectural designs, it is the aim of this part to work on the basic description of the materials' mechanical and physical characteristics. The target is to make an overview of the materials and further data collection and comparison. After that the special bending behavior and buckling problem will be characterized and the geometrical stabilization and forming techniques based on such features will be introduced. Because the aim is to use this understanding to develop a novel structure system, the discussion aims to derive a guideline and empirical deduction from both the pioneer works and the engineering theory. Comprehensive actual material behavior as well as the calculation and tests will only serve as the tool for deduction of the understanding or the later design's basis, but will not be further developed.

3.2.1 General characteristics of materials

As it is discussed in the previous sections, the thin sheet materials are mainly classified into 4 categories: the metal sheet, polymer and plastic composites, glass and the natural fiber-based materials. There is a wide range of material properties such as density, strength and stiffness. (see Table 3.1) Metal is stiff and strong, glass is brittle, paper is soft and polymer is lightweight and flexible. However, from the case studies above it can be seen that all the materials are adequate for structural uses and the structural form and typology also share some similarities.

In structural design and material selection, it is always a complicated decision making process with multiple criteria to seek the best match between design requirements and the properties of the materials that might be used. (Ashby & Cebon, 1993) A proper combination of the properties of materials can result in a better choice of materials than considering only a singular factor of the mechanical behaviors. (Lienhard & Knippers, 2014) It is also the reason why metal sheets are popular with curved bent forms even though they have a relatively large Young's modulus, the concomitant high yield strength enabling the deformation and the load carrying behaviors.

Objectives and constraints are the main important factors that determine the material choice and the geometrical structure morphology of architectural designs. In Ashby's book of material selection, such a general principle is described with the method to evaluate the performance of the structural elements. (Ashby & Cebon, 1993) For a building or a structure, the basic units are considered to be the structural elements, they carry the load, define the space, transfer the heat and store the energy. The different structural elements together compose the structure system. In a structural design, the decision-making process also equals finding their best performance, which is often determined by several factors of design: the functional requirements, the geometry and the materials' properties themselves, and sometimes the local considerations of the section design and construction details. The performance of these elements can be simply written as the following form:

$$\mathrm{P} = \left[\begin{pmatrix} Functional \\ requirements, F \end{pmatrix}, \begin{pmatrix} Geometric \\ parameter, G \end{pmatrix}, \begin{pmatrix} Material \\ properties, M \end{pmatrix}\right] \text{ or } \mathrm{P} = \mathrm{f}(F, G, M),$$

An optimum design is therefore a better selection of both materials and structural forms which maximize or minimize *P* by an analysis according to the different combinations of the functions, objectives and constraints. In reverse, for a given material property and the target of design, a proper design of the geometry of the structure can also be analyzed to optimize the performance of the structural elements.

For thin sheet materials, as the initial forms are often the flat, planar forms of slabs, the calculation methods of panels are usually applied, and different from the ties and rods, the main objectives are often the stiffness of the structure rather than the strength. Therefore, the density, yield strength and flexural strength, Young's modulus are usually the most important mechanical parameters of such materials. A table of the corresponding parameters is presented below:

	Density(Mg/m^3)	Yield Strength (MPa)	Young's Modulus (GPa)
Metal			
Cast irons	7.05-7.25	215-790	165-180
Carbon steels	7.8-7.9	250-1155	200-216
Stainless steels	7.6-8.1	170-1000	189-210
Al Alloy	2.5-2.9	30-500	68-82
Cu Alloy	8.93-8.94	30-500	112-148
Mg Alloy	1.74-1.95	70-400	42-47
Ti Alloy	4.4-4.8	250-1245	90-120
Zn Alloy	4.95-7	80-450	68-95
Glass			
Borosilicate Glass	2.2-2.3	264-384	61-64
Slica Glass	2.17-2.22	1100-1600	68-74
Soda-lime Glass	2.44-2.49		68-72
Polymer and Polymer Composite			
PC	1.14-1.21	59-70	2-2.44
PVC	1.3-1.58	35.4-52.1	2.14-4.14
PE	0.039-0.96	17.9-29	0.621-0.896
PET	1.29-1.4	56.5-62.3	2.76-4.14
PP	0.89-0.91	20.7-37.2	0.896-1.55
ABS	1.01-1.21	18.5-51	1.1-2.9
PMMA	1.16-1.22	53.8-72.4	2.24-3.8
CFRP	1.5-1.6	550-1050	69-150
GFRP	1.75-1.97	110-192	15-28
Natural Fiber			
Wood			
OSB Plate	0.55	11.9	4.3
Hardboard	0.85	26	4.8
Medium board	0.65	9	3.1

MDF Plate	0.65	18	3.1
Bamboo	0.6-0.8	35-44	15-20
Paperboard(wood paper)	0.8	21-30	2.53-3.5

Table 3.1: basic mechanical parameters of thin sheet materials (the values of the table are taken from (Ashby & Cebon, 1993), Euro-code 1993-1996 and 1999, and also the other norms such as the DIN 622-2,3,5, DIN 17721 and DIN 12369. For glass the compressive strength is used as the yield strength, as it is more relevant in the structural designs.)

3.2.1.1 Isotropic and anisotropic

Not all the sheet materials are isotropic or anisotropic, and it depends on the characteristics of the raw materials and the manufacturing process.

For most of the natural fiber-based sheet materials, such as wood panels and paperboards, the mechanical performance shows a great difference between the directions parallel and perpendicular to the arrangement of the grains and fiber. Apart from the natural constitution of fiber, the anisotropy also comes from the forming process, such as the inhomogeneous shrinkage caused by the rolled-method in the drying-process of the thin wood sheets and paperboard manufacturing.

For the other materials which have a network of molecular structures, even though typical molecular networks have some basic forms (like the hexagon grids and tetrahedron forms in glass molecules), their arrangement in the materials is often irregular and this can make the overall performance of such materials also isotropic. The glass sheets are one of the typical materials in this type.

Metal and polymer plastics are also the materials which are composed of regular molecular networks with irregular arrangements, hence the materials also show an overall isotropic property before the forming process. However, during some of the forming process of the sheet manufacturing, the rolled-method which is used for the metal sheet and the shrinkage after the extrusion of the thermal plastic also makes the global arrangement ordered. Similarly, a so-called quasi-isotropic property appears in composite materials such as laminated plywood and fiber-reinforced plastics. Laminated with different materials or the same anisotropic materials in different directions, the final products will show an isotropic in-plane property but not with an isotropic out-plane (bending) property.

The anisotropic nature of materials often results in a different strength and stiffness in the manufacturing direction and the cross direction, and this also leads to an orthogonal physical performance of the material. Such a performance can be observed with parameters such as:

--- The strength of materials in the manufacturing direction is often higher than in the cross direction. This difference is in most cases linear with a factor. Analogical differences also happen in stiffness with the E-Modulus in both directions.

--- For the manufacturing direction the E-Modulus in both the compressive and tensile-zone are very similar and can be considered the same in most cases.

For the architectural design procedure, it is often necessary to consider the anisotropic property of some materials in advance. A reasonable arrangement of the materials' direction in the structure system will possibly enhance the performance of the structure. However, although it is possible to clearly recognize the anisotropy of the material through the shape of products, it is usually too hard to guarantee the material's best direction, especially when a complicated prefabrication process (such as complex nesting by cutting the form), complicated load cases or geometry for the structure is needed.

The anisotropic property of the material is usually more important to be noticed in the structural analysis process. The different yield strength and Young's Modulus in different directions will cause a complicated stress and strain state and also the complex failure condition of the materials. In this case, for the anisotropic materials, the special failure criterion is usually applied in the structural analysis. For isotropic materials, criteria such as the Von-Mises and the Tresca can be applied, and for the anisotropic materials, various criteria such as Tsai-Wu, Tsai-Hill, and maximum stress can be used in different cases.

Although the failure criterion method can give an exacter result of the structural analysis, it can still only be used in limited simplified cases. Due to the above difficulties to estimate the material's direction or the complicated application of materials in three dimensional structures, the compromising assumption of a quasi-isotropy is also used, where for the orthotropic materials the arithmetic mean value of the materials' property is used in the structure analysis. In the cooperation research of this dissertation, such a method is applied in the detailed structural analysis of the demonstrative pavilion with paperboard. For details and discussions see (Werth, 2015b).

3.2.1.2 Strength and stiffness

Strength and stiffness are the most important properties of materials in the structural design. As it is listed in the table of material parameters above, different parameters of properties are chosen from several design codes of corresponding materials. The relationship and the combination of material parameters determine the material choice and performance of the structure. For each different objective of the design tasks, different matches of material properties are required. A practical and effective method in material selection has been introduced by Ashby to use the material property chart to define "the guideline of design" and "the search region" to find the adequate selection of materials with different design requirements. (Ashby & Cebon, 1993) In the example of "design requirement of a light, stiff panel", the fatal relationship of "$E^{1/3}/\rho$" is deducted and the better choice of materials can be visualized with the property chart "$\mathrm{E}-\rho$". (Ashby & Cebon, 1993)

Similarly, in the material's characteristic selection in the "bending-active structures" introduced by Lienhard (Lienhard & Knippers, 2014), the sufficient strength of materials is often a prerequisite of the structure with bendable materials. In comparison, the stiffness problems are usually the most important problems because in most cases the 3-dimensional forming of planar sheets (such as folding and bending) and the local insufficiency of stiffness (buckling behavior) cause the structural failure at first. For such materials' behavior it has been discussed by Lienhard that the factor of material property—the $\sigma_{M,R_k}/E$- takes command and divides the materials into different categories. (Lienhard & Knippers, 2014) A detailed discussion about the elastic and bending behavior will be discussed later in the next section.

3.2.1.3 Dampness and hygroscopicity

Natural fiber-based materials such as timber and paper are hygroscopic materials which respond to the air moisture content and water contact; at the same time, the structural properties of such materials are affected by the moisture content, and deformations such as expansion and shrinkage will occur. Water molecules are kept and held by the molecular forces in the fiber cells, they become attached by the hydrogen bonds to the OH-groups of the cell wall components, especially the cellulose. (Ridout, 2000) In natural timber, such access to the groups may be blocked by the extractives within the timber, and it has been demonstrated that the removal of these will increase the hygroscopicity. (Skaar, 2012)

Natural raw materials have a large amount of moisture content. In the manufacturing process of plywood and paperboard the raw materials are treated in several procedures, where for example the drying process removes the moistures in the materials. In some preparation processes in the paper production, the extractives such as lignin are removed from the natural fibers. These treatments of the sheets result a relatively large hygroscopicity of such materials and a great capacity to absorb the moisture content in the air.

The hygroscopicity makes the thin sheets from such materials highly dependent on the climate conditions and sensitive to the sudden change of the air moisture content (and slightly to the temperature). When the hygroscopicity increases, the water comes into the cell wall of the materials and makes the winding of the fibers loose, and hence decreases the structural strength and stiffness of the material. (Figure 3.2.1A shows the decrease of strength and stiffness in wood and Figure 3.2.1B shows the decrease of E-Modulus and tensile strength in paperboard). At the same time, due to the water absorption, some deformations which are critical to the structure will also occur.

The hygroscopicity also affects the viscoelastic behavior of the materials. The instability of relative humidity will increase the growing trend of long time deformation. Especially under the long-time stable and quasi-stable compression loads, it has been tested that both the creep progress and creep value of such materials are greater and faster in the humid summer rather than the dry winter. (Pohl, 2009) Paper products are a little different to the timber sheet when it faces a sudden change in humidity. Contrary to the reaction of wood panels over several hours or days, it reacts very fast with the changes of relative humidity almost without delay. It is always necessary to consider the water-proofing process to isolate the structure to water and moistures. For materials such as timber and paperboard, some coating materials is already in use or being developed.

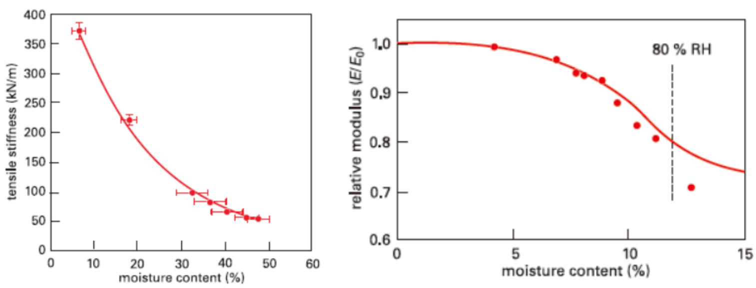


Figure 3.2.1 Decrease of strength and modulus of paper products with higher humidity (Niskanen, 2012)

3.2.1.4 Fatigue, Creep and relaxation

The special characteristic of creep and relaxation shows the properties of materials with a time-dependent nature. And this time-dependence of the mechanical response extends across large time scales from microseconds to decades. At very short times, where relaxations have not started, most of the materials will appear brittle and stiff. But at long time-scales, they will appear more ductile and compliant.

Such behavior appears remarkable on most of the sheet materials except the brittle glass and also shows the special viscoelastic (plastic) behavior of such materials. In general, the creep compliance and relaxation modulus can be functions of the stress and strain in different directions, temperature, and moisture content. (Niskanen, 2012) In this research, this time-dependent behavior is considered as an effector of the materials' behavior, especially the parameter for the structural calculation. As a result, it enables a non-linear material behavior of the time-dependent delay of strains to the long-time applied stresses. For wood, plastics, paper and steel under a high stress or temperature, it can be observed that an obvious non-linear behavior occurs in the stress-strain diagram. (Figure 3.2.2) As a method for the analysis of many materials, the "master-curve" method can be used to describe such a behavior in a linear expression or a simple non-linear expression with a master-curve. (Niskanen, 2012) For a brittle material such as glass, although it is demonstrated that the creep behavior exists especially under bending, the deformations is still so small as to be in the nanometer scales, so the behavior of such a material can be understood as linear-viscoelastic. (Vannoni, Sordini, & Molesini, 2011)

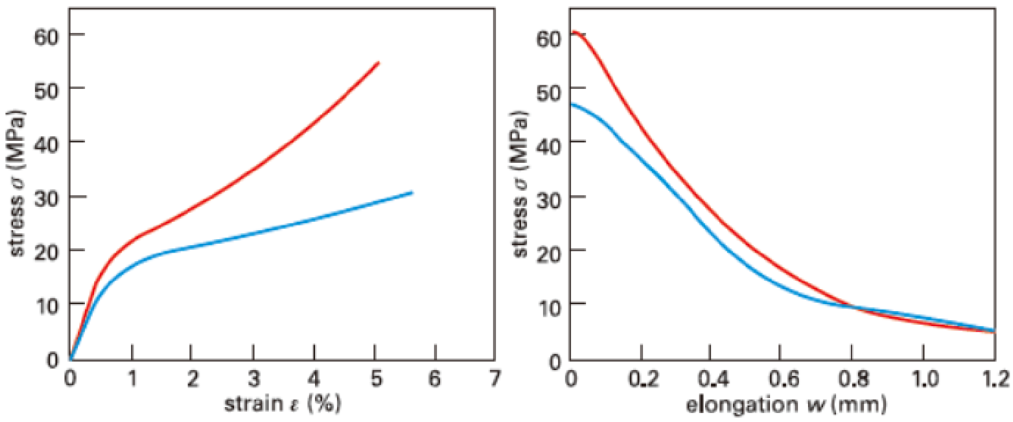


Figure 3.2.2 Ordinary tensile stress–strain curves (left) and post-peak cohesive stress (right) of a 70g/m^2 Kraftpaper in the Manufacturing Direction (Niskanen, 2012)

The fatigue of materials is related to the cyclic long-term loading, and the fatigue problems sometimes need to be considered in the elastic kinematic structures. For this research such problems are not deeply considered and only the references and design codes will be considered as a reduction of stress limit according to a safety factor in design.

According to the Ph.D. research by Lienhard (Lienhard & Knippers, 2014), taking GFRP as an example, both the referenced research projects showed that a given maximum stress under approximately 60 % of the permissible stresses would not lead to the fatigue problems.

3.2.1.5 Design codes

The current Eurocodes are adequate for some of the materials which are considered in this research, including steels, aluminum and timber panels. For the other structural materials such as thin glass sheets, paperboard, plastic panels and GFRPs, the establishing of new Eurocodes are still being developed and some of the German guidelines are collected and used in this research.

The following design codes and standards are available nowadays to determine the design parameters of the material property of the thin sheet materials.

A, Design codes and standards for the design of structures:

Eurocode EN 1990	Basis of structural design
Eurocode EN 1993-1-1	Design of steel structures
Eurocode EN 1995-1-1	Design of timber structures
Eurocode EN 1999-1-1	Design of aluminum structures
DIN 18008-1	Glass in building- Design and construction rules
BÜV 2010	Tragende Kunststoffbauteile im Bauwesen

b, Design codes and standards for the material properties and semi-products

Steel	DIN EN 10027	Designation systems for steels
	DIN EN 10025	Hot rolled products of structural steels
Timber	DIN EN 300	Oriented Strand Boards (OSB) - Definitions, classification and specifications
	DIN EN 312	Particleboards - Specifications
	DIN EN 622	Fibreboards - Specifications
	DIN EN 636	Plywood - Specifications
Glass	DIN EN 572	Glass in building
Plastic and composites	DIN 18820	Glass fibre reinforced unsaturated polyester
	DIN EN ISO 1163	Plastics -- Unplasticized poly(vinylchloride) (PVC-U) moUlding and extrusion materials
	ISO 17855	Plastics -- Polyethylene (PE) moulding and extrusion materials
	ISO 19069	Plastics -- Polypropylene (PP) moulding and extrusion materials
	DIN EN ISO 8257	Plastics -- Poly(methyl methacrylate) (PMMA) moulding and extrusion materials

For the design value of the permitted stress limit, the safety factors are mainly used to consider the influence of the effectors such as the environment and the load duration and the product's uncertain property. In the Eurocode DIN EN 1990 (Institution, 2002), the load and resistance factor design (LRFD) method is used and the design value of materials is expressed as:

$$X_d = \eta\, X_k / \gamma_m$$

Where: X_k -- the characteristic value of the material η -- mean value of the conversion factor (volume, scales, load duration, moisture, temperature, etc.)

γ_m -- partial factor for the material

For different materials, the design resistance for the ultimate limit states takes different major factors into consideration, and in most cases, the modification factor k_{mod} is used to take the effect of the duration of load and moisture content into account. However, in some cases like steel and thermal pre-stressed glass, the k_{mod} is not considered or is just replaced simply with $k_c = 1$. See the table below:

Steel	Glass (thermal pre-stressed)	Glass (not thermal pre-stressed)	Timber	Plastic
DIN EN 1993-1-1	DIN 18801-1	DIN 18801-1-1	DIN 1995-1-1	BÜV 2010
$R_d = \frac{R_k}{\gamma_M} = \frac{1}{\gamma_M}(\eta_1 X_{k,1}; \eta_i X_{k,i}; a_d)$	$R_d = \frac{k_c f_k}{\gamma_M}$	$R_d = \frac{k_{mod} k_c f_k}{\gamma_M}$	$X_d = k_{mod} \frac{X_k}{\gamma_M}$	$R_d^u = \frac{R_k}{\gamma_M A_{mod}}$ $f_d = \frac{f_{k0,05}}{\gamma_M A_{mod}^f}$

Table 3.2: Definition of the design resistance and the safety factors in different design codes

3.2.2 Elastic behavior, bending and local stabilization

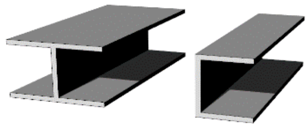
One of the major characteristics of the thin sheet materials is the planar sheet form, which makes the material relatively flexible and easy to deform. Due to this flexibility, the stability problem and the stiffening method need to be particularly considered in structural design. In this sense, the particular characteristics of such materials are the bending and buckling behavior. In this part, such behaviors and the corresponding stabilizing method will be discussed.

3.2.2.1 Nonlinearity and stiffness

The major mechanical behaviors of thin sheet materials rely on the deformation of the materials and hence the structure systems will also show a non-linear behavior.

The stiffness of structures defines the basic difference between the linear analysis and non-linear analysis of the structures system. In structural analysis, the linearity of the structure system implies, if the external forces of the structure system are multiplied by a factor of n, the displacement and internal stress are also multiplied by a factor of n. In many cases, especially considering the stability of structures or the large deformations, the system behaves differently with a so-called nonlinear behavior.

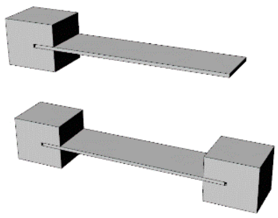
The concept of stiffness can be used to simplify and unify the analyzing method for these behaviors, and it can be affected by three main factors: the geometry, material, and boundary conditions. Hence three basic types of structural nonlinearity can be differentiated in the following aspects:



a, Geometry and shape: a steel beam with an I shape has a different stiffness compared to a C shape. At the same time, during deformation, there is also a non-linear relationship between external forces and the deformation.

The geometrical nonlinearity is often necessary to be considered in structures with thin sheet materials, as a relatively large deformation is usually very common in such structures.

b, Material's property: with the same shape, the steel beam often has a greater stiffness as the iron beam. For the same material, during the increase of stress, the strain will also show a nonlinear behavior with the plastic deformation. For a simple analysis of normal structures, the material's nonlinearity is often not considered and needs to be considered with the special material model.



c, Boundary conditions: the same beam is likely to bend more with only one edge fixed than with both edges fixed, hence it has less stiffness. For the nonlinearity of structure systems, it can also be caused by a changing of boundary conditions possible due to the large deformation and contact of components of structure systems. This kind of nonlinearity is not so common for it to be considered in structural design and analysis.

The concept of stiffness (stiffness matrix K) also describe this behavior and make it possible to be analyzed. In static finite element analysis (FEA), the relationship between the force vector *F* and the displacement vector *d* is expressed by the equation of an equilibrium:

$$[F] = [K]\,[d]$$

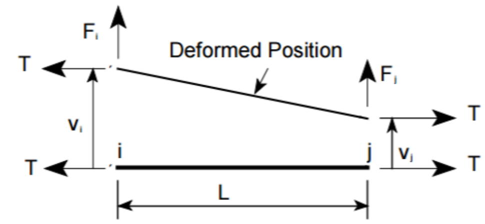
In the analysis of the linear structure system, the equation of the equilibrium is often formulated considering an undeformed system or a system under a relatively small deformation, the stiffness matrix is hence to be considered as constant. When the external forces are multiplied with a factor of n, the displacement is clearly also to be multiplied by a factor of n. But in the non-linear analysis of a structure, which often has a large deformation that cannot be ignored, the stiffness matrix is not constant either and relies on the actual instant deformation. So the n times forces will equal a displacement multiplied by a factor of m. In the nonlinear FEM analysis, the calculation will be done by an iterative process, where the deformations of every step will be considered as an effector of the system's stiffness matrix.

It needs to be mentioned that all the material properties, geometrical deformation and the boundary conditions all affect the stiffness matrix. However, as it is shown above, it is usually considered in most cases and also in this research that the plastic deformation and change of the supporting method are not welcomed in designs, and hence not to be considered. The main aspect of the structure nonlinearity refers mainly to the geometric nonlinearity and the geometric stiffness.

3.2.2.2 Geometric stiffness, stress stiffening and pre-stress

As mentioned, the geometric stiffness is the property of structures to deform under external forces. It relies on the geometry and also the initial load on the structure, and is especially prominently affected by the normal forces on the structure.

According to a related analysis (Wilson, 1998), it is possible to describe the effect of geometric stiffness on structure analysis with a simple example of a horizontal cable, which is under an initial tensile load of *T*. For the lateral displacement v_i and v_j, external forces F_i and F_j must be added to maintain the equilibrium.



For this structure system, with the assumption that the deformation is small so that the tension doesn't change, considering the moment about point *j* and the vertical equilibrium,

it can be expressed as:

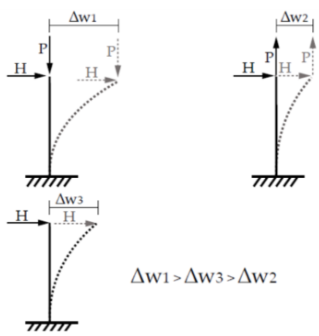
$$F_i = \frac{T}{L}\left(v_i - v_j\right) \;\; ; \;\; F_j = -F_i$$

Combining both equations together, a matrix form of lateral forces can be gotten as:

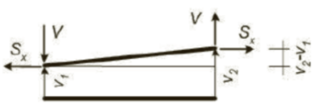
$$\begin{bmatrix} F_i \\ F_j \end{bmatrix} = \frac{T}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} v_i \\ v_j \end{bmatrix} \text{ or symbolically } F_g = K_g v$$

In this form, the characteristic of the geometric stiffness matrix can be observed, that it isn't dependent on the material's property but only on the structure's geometry and initial load *T* and *L*. So this is also the structure's property, also called the "initial stress" stiffness.

In the nonlinear analysis of structures, the geometric stiffness also describes the nonlinear behavior of the structures, especially the effect of normal forces to the stiffness of the system. In general, a tensile normal force causes an increase of stiffness for a structure component while a compressive one will cause a decrease.



A simple example can be shown with a linear rod which is under the normal external loads. When the rod is under a large tensile force, the stiffness will be largely enhanced and vice versa when under a compressive load. When an external force is added perpendicular to the rod, a displacement in different scales will occur due to the preset normal forces. This is also called the "P-Delta" effect in the structural analysis, especially in the second order analysis that considers the nonlinearity of structures.



A typical analysis of the effect of the geometric stiffness can be shown with a simple 2D rod considering the deformation. (Werkle, 1995) For a small displacement, with the similar consideration of the moment about point *a*, the equation can be gotten as:

$$V \cdot L = S_x \cdot (v_2 - v_1) \text{ or modified as } V = \frac{S_x}{L}(v_2 - v_1)$$

and plus the axial elongation based on the strength of rod the equation can be rewritten as:

$$\left(\frac{EA}{L}\begin{bmatrix}1 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\\ -1 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}+\frac{S_x}{L}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & -1\\ 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 1\end{bmatrix}\right)\cdot\begin{bmatrix}u_1\\ v_1\\ u_2\\ v_2\end{bmatrix}=\begin{bmatrix}F_{x1}\\ F_{y1}\\ F_{x2}\\ F_{y2}\end{bmatrix}$$

Or simplified as $(K_e + K_g)u_e = F$

In this analysis, it can be observed that the stiffness matrix of a structure system consists of both the elastic stiffness matrix and the geometric stiffness matrix, where for elastic deformation of materials the K_e remains constant. For geometric stiffness, it can be seen that when supported, a P-Delta effect will occur in the perpendicular direction of the rod, and this stiffness is determined by the factor the normal force S_x divided by the length *L*. For comprehensive cases, more complicated considerations must be done such as in the cubic beam, where the 3 dimensional displacement will be considered as: (Przemieniecki, 1985)

$$\begin{bmatrix}F_1\\ F_2\\ F_3\\ F_4\\ F_5\\ F_6\end{bmatrix}=\left(\frac{\boldsymbol{EI}}{\boldsymbol{l^3}}\begin{bmatrix}\frac{\boldsymbol{Al^2}}{I} & & & & & \\ 0 & 12 & & & & \\ 0 & 6l & 4l^2 & & & \\ -\frac{\boldsymbol{Al^2}}{I} & 0 & 0 & \frac{\boldsymbol{Al^2}}{I} & & \\ 0 & -12 & -6l & 0 & 12 & \\ 0 & 6l & 2l^2 & 0 & -6l & 4l^2\end{bmatrix}+\frac{F}{l}\begin{bmatrix}0 & & & & & \\ 0 & \frac{6}{5} & & & & \\ 0 & \frac{l}{10} & \frac{2l^2}{15} & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & -\frac{6}{5} & -\frac{l}{10} & 0 & \frac{6}{5} & \\ 0 & \frac{l}{10} & -\frac{l^2}{3} & 0 & -\frac{l}{10} & \frac{2l^2}{15}\end{bmatrix}\right)\cdot\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6\end{bmatrix}$$

In more complicated cases such as in surface structures like plates, more complicated forms of geometric stiffness will be applied and the in-plane forces including the normal and sheer force and the splitting values of the membrane forces need to be considered. (Werkle, 1995) Such membrane forces will enhance the complexity of the equation and lead to a very complicated formulation.

However, it can summarized from the analysis of the geometric stiffness that the normal in-plane stress and strains within the structure system will enhance or reduce the stiffness of the structure and also cause or prevent the risk of an out-plane deformation. Such a phenomenon also explains the behavior mentioned above, that the stiffness of a rod under a large tensile force will be largely enhanced. In structural engineering, it enables a stress stiffening effect and the pre-stress method for the stabilization of the structure design.

The stress stiffening usually means the tension stiffening in structure design, where a preloaded tension stress will enhance the structure's geometric stiffness. As it is dependent on the initial stress and the geometry of the structure, such stiffening is also mentioned as geometric stiffening, initial stress stiffening and so on. Especially for the structures which are slender and weak in bending stiffness, such a stiffening method is usually considered and used in structural design. The pre-stressing will also result the residual forces in the structure, which will affect the stiffness, stability and also the eigenvalue of the structure.

In the nonlinear analysis, the stability problem is often important for the thin materials. The imperfection approach is often used in such an analysis, where a structure is considered with some initial geometric imperfection. For an approximation, the first eigenform is often used to describe the imperfect shape. It is determined by means of the eigenvalue analysis. The dynamic eigenvalues are related to the stiffness of the structure and hence related to the pre-stressing. The relationship will also affect the failure of the structure. It was found that the first eigenvalue of a column under compression becomes zero when instability is reached. This is also known as the Euler critical buckling load that is related to the buckling behavior of the structure. So, as a conclusion, the initial stresses and geometry of a structure system are important for the structure's stiffness and also determine the stability of the structure system.

3.2.2.3 Bending of thin sheets

One of the typical and important deformations is the bending of planar thin sheets. It enables designers to create curved shapes for the structure. In this part, two major questions will be discussed. On the one hand, the mechanical behavior of bending will be researched to understand the relationship between the stress and strain. On the other hand, the minimal radius problem will also be analyzed to know how to control a possible bent form of the thin-sheets.

1. Bending Theory—Beams

The bending theory in structural mechanics is developed and refined depending on the simplification of structure forms and behaviors. Being the most simplified and common forms of structure, the bending of beams in one dimension is often first considered and used as a preliminary for other complicated cases.

The most classic and important model that discusses the bending of beams is the Euler–Bernoulli beam model (Figure 3.2.3 left), which describes the relationship between the beam's deflection ω and the applied load q with the equation:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2\omega}{dx^2}\right) = q$$

where the bending moment of the beam $M = -EI\frac{d^2\omega}{dx^2}$

In the Euler–Bernoulli beam model, the basic assumption of the structure is based on:

1, the beam is modelled as a one-dimensional object

2, the cross section of the beam remains planar after bending and is always perpendicular to the neutral axis of the beam.

After the Euler–Bernoulli beam model, Timoshenko gives another assumption in his beam model where there is a shear deformation and the planar cross section of the beam after bending rotates about the neutral axis (Figure 3.2.3 right) and got the more complicated equation:

$$\frac{d^2}{dx^2}\left(EI\frac{d\varphi}{dx}\right) = q \quad and \quad \frac{d\omega}{dx} = \varphi - \frac{1}{\kappa AG}\frac{d}{dx}\left(EI\frac{d\varphi}{dx}\right)$$

where φ is the angle of the neutral axis' rotation, A is the cross section area, G is the shear modulus and κ is the Timoshenko shear coefficient (κ = 5/6 for the rectangular cross section)

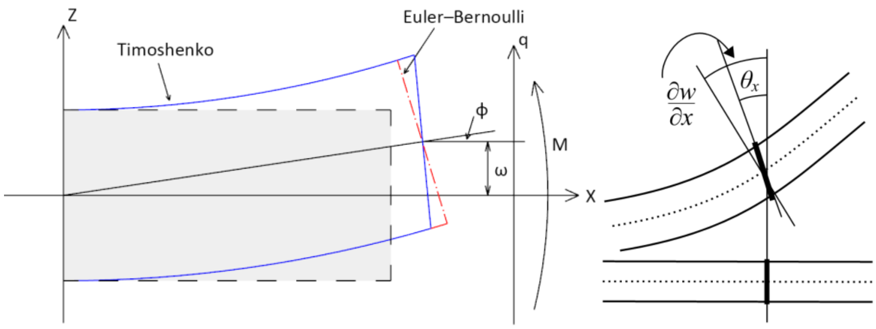


Figure 3.2.3 Diagrams of Euler–Bernoulli and Timoshenko beam models

It can be observed in the equations between the two models that a small deviation of the angle of the neutral axis' rotation occurs after bending. It takes the first-order shear deformation into consideration and is based on the deviation from normality. The difference is greater when considering a thick beam or a short beam where the length of the beam is not much more than the thickness. However, when considering the thin beams or long beams, both equations can be considered the same since the last part of Timoshenko's equation for the rotation angle can be neglected.

2. Bending Theory—Plates and shells

Compared to the beam's bending theory, the thin sheets have another property in that they usually have a planar form where there is not only the length but also the width of the geometry. In this case, the one-dimensional model for both the beam's theories become insufficient. In this case, the theory of plates and shells should be taken into consideration.

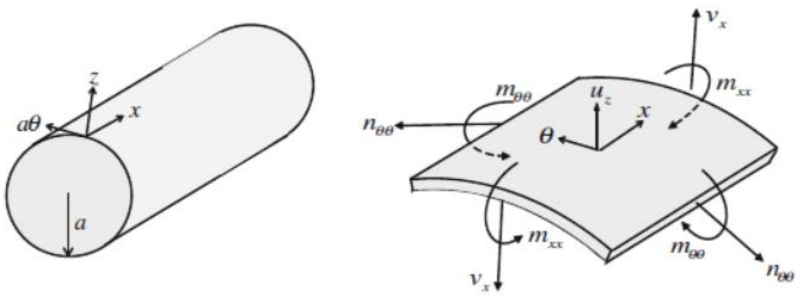


Figure 3.2.4 In-plane stresses by curved shell (Blaauwendraad & Hoefakker)

The plate and shell theory uses two dimensional and three dimensional models for the structure, where the stresses and strains in the structure consist of several components of the membrane stress field. Like in the flat plate, normal stresses σ_x and σ_y, and a shear force σ_{xy} occur; in the curved shell these are named n_{xx}, n_{yy} and n_{xy} respectively. And considering the possible hinged-support or clamped edge which prevents the normal displacement of the shell, the bending moment can also occur in the shell and can also be named m_{xx}, m_{yy} and m_{xy}.(Figure 3.2.4)

As it can be considered that the bending of a planar sheet will result in a curved deformation in one direction, the deformed shape will turn into a cylindrical or conical form with one principle curvature. The constitutive equations of stress and strain of the cylinder shell can be used here to describe the relationship in the thin sheets.

According to an analysis (Blaauwendraad & Hoefakker), as such single-curved shells are mainly curved in one direction θ, the y direction of such shells is rewritten as θ, hence the stresses and moments which exist in the cylinder shell are the n_{xx}, $n_{\theta\theta}$, $n_{x\theta}$, m_{xx}, $m_{\theta\theta}$, $m_{x\theta}$. And according to the Morley bending theory for circular cylindrical shells, the equations will be:

$$\begin{bmatrix} n_{xx} \\ n_{\theta\theta} \\ n_{x\theta} \\ m_{xx} \\ m_{\theta\theta} \\ m_{x\theta} \end{bmatrix} = \begin{bmatrix} D_m & vD_m & 0 & 0 & 0 & 0 \\ vD_m & D_m & 0 & 0 & 0 & 0 \\ 0 & 0 & D_m\frac{1-v}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_b & vD_b & 0 \\ 0 & 0 & 0 & vD_b & D_b & 0 \\ 0 & 0 & 0 & 0 & 0 & D_b\frac{1-v}{2} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{x\theta} \\ \kappa_{xx} \\ \kappa_{\theta\theta} \\ \rho_{x\theta} \end{bmatrix}$$

where the membrane rigidity D_m and flexural rigidity D_b can be defined as:

$$D_m = \frac{Et}{1-v^2} \ and \ D_b = \frac{Et^3}{12(1-v^2)}$$

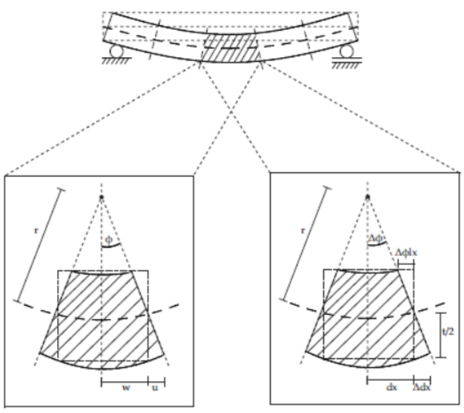
It can be observed that when the deformation of a plate gives a rise to a curvature $\kappa_{\theta\theta}$, a corresponding bending moment $m_{\theta\theta}$ will also arise. The full analysis of the stresses and strains in shells will need a particular definition of a load case and the boundary condition, and will need to be solved in a relevant differential equation system. As this will not be included in this research, a more detailed analysis will not be introduced.

3. Calculation of the minimum bending radius

For the most important consideration in the architectural design or the geometry design of the bent form, it must be known to what extent the materials can be bent and how to control the curvature in the structure. In this sense, the bending theory could be used here to determine the minimum bending radius, which equals the reciprocal value of the maximum principle curvature.

Because the form is simplified only as one dimensional in the beam theory, it is also only suitable to the thin sheets where the length is much larger than the width, also referred to the long sheet. In this case, according to the theory, the minimum radius occurs when the maximum stress in the structure arrives at the permitted stress of the material σ_{Rd}.

In this case, based on the geometry relations and the analysis in bending-active structures (Lienhard & Knippers, 2014) the following equations can be derived:



$$\frac{t/2}{r} = \frac{\Delta d_x}{d_x} = \varepsilon = \frac{\sigma}{E} = \frac{Mt}{EI}$$

So the maximum permitted stress:

$$\sigma_{Rd} = \frac{Et}{2r}$$

Hence the minimum radius of bending:

$$r_{min} = \frac{Et}{2\sigma_{Rd}}$$

For the other case, when the width cannot be ignored compared to the length of the thin sheet, the bending moment $m_{\theta\theta}$ in the θ direction of the cylinder will be generated according the curvature $\kappa_{\theta\theta}$:

$$m = \frac{Et^3}{12(1-v^2)} \cdot \kappa = \frac{Et^3}{12(1-v^2)} \cdot \frac{1}{r}$$

And the maximum stress at the edge of the cross section:

$$\sigma_{Rd} = \frac{12m}{t^3} \cdot \frac{t}{2} = \frac{Et}{2r(1-v^2)} \text{ and } r_{min} = \frac{Et}{2\sigma_{Rd}(1-v^2)}$$

The calculation of the minimum radius of bending relies on the isotropy of the material. When it comes to the anisotropic materials, as the formulation of stress and strains becomes too comprehensive, it is not deeply researched due to the restriction of this research's content. Meanwhile, the analysis above is based on an assumption of the elastic bending procedure for a linear material property, and the stress stiffening effect is also neglected for the simplification of the analysis.

Through a comparison, for different materials under bending, the proportion *t*/*r* often remains constant, and it also corresponds to some materials' research of bendability. For many materials, the relative bend radius *t*/*r* is tested to be constant, such as for metal sheets (Semiatin, MArquard, & Lampman, 2006) and glass sheets. (Figure 3.30-a) Other conditions such as temperature will also affect the bendability and the minimum bend radius. Thermally or chemically tempered glass will show a different relative bend radius due to the pre-stress states with different preprocessing methods. (Figure 3.30-b)

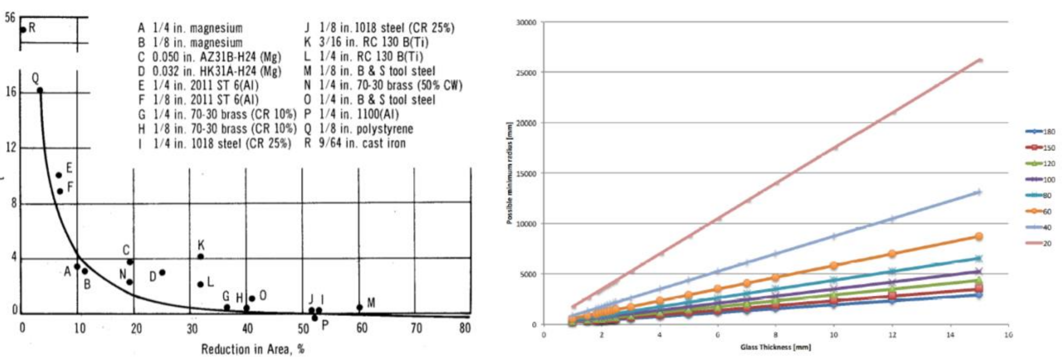


Figure 3.2.5 Relationship between minimal bending radius and material thickness (Semiatin et al., 2006)

3.2.2.4 Buckling behavior

1. Buckling behavior of structure elements

The buckling behavior is also an instability problem of the structure system, where the structure collapses before the yield strength of materials arrives. It can often be observed in many structures, such as the long beam, thin plate and also cylinder cans with thin walls. The buckling behavior can be obvious and better explained with examples when a structure is under a compressive axial load. From engineering's point of view, the buckling phenomenon happens when the load in the structure is over a buckling critical load and causes the sudden failure of the structure.

Mathematically, the buckling of a structure is a problem of bifurcation. When the load in the structure increases, the deformation of the system will show an uncertainty, that there exists more than one solution to maintain the equilibrium. This phenomenon is also comparable to the three possible equilibrium states of the structure, where an equilibrium is considered stable if the movement requires energy and always returns back after small disturbances (Figure 3.2.6-a). If the system moves away from the equilibrium after small disturbances and the energy is released or remains the same, then the equilibrium is unstable. (Figure 3.2.6-b,c)

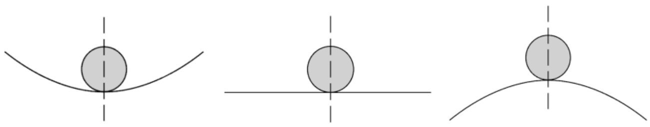


Figure 3.2.6 Three equilibriums states

Such an equilibrium state can also be transferred to the buckling behavior. The ideal bifurcation problem with all cases of possibilities at the same time only exists in the ideal system and the calculation process. In reality, all the possible causes of the imperfection of the system (such as the material's deviation or the stress eccentricity) will lead to a possible buckling behavior that cannot be exactly predicted and calculated.

The broader buckling behavior happens in various types of structures mainly under the compressive load. As shown in Figure 3.2.7 A, an ideal column or rod which is under an eccentric axial compression will tend to bend into a curved shape as the typical buckling form. If the eccentricity of the axial force also has a slope, the stability of the beam or a long plate will be under the combined effect from the normal force and also the bending moment. In such cases a torsional-buckling will happen. For the plate which is hinged supported at the four edges, the normal forces along the neutral surface will result in a bulking form in the center of the plate and the spatial deformation of the plate will occur with double curvature. Such a similar phenomenon can also be found in the case of walls (Figure 3.2.7 B), for walls under the vertical load, when the side of the walls are fixed and stabled on different edges, because the deformation of the arch-like curve will be prevented, the buckling forms will be similar to the buckling of 2D rods, torsional long plates and plates with fixed edges.

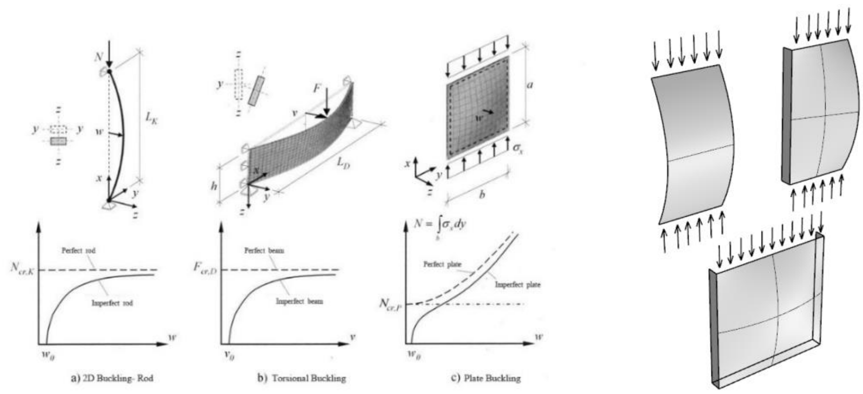


Figure 3.2.7 Buckling of rod and plates

As shown in the discussion above, it can be described from the engineering's point that the buckling reflects the uncertainty of the structural stability and will be enhanced by the imperfection of the structure. As shown in figure 3.2.8 A, in the buckling process of a cylinder shell which is under the axisymmetric load, several critical points will appear. Before the buckling, axisymmetric deformation of the structure continues until a critical load is reached at point A. If the load is not relieved by the structure's stiffness, a dynamic "snap-through" will occur in a way of nonlinear buckling to point B. At point B, as the equilibrium state of the structure comes to a bifurcation point, both possible deformations can happen in the next step, either still in an axisymmetric way as in the path BC, or non-asymmetrically along the path BD. Apart from the theoretical possibilities of the buckling, such structures will also be very "imperfection-sensitive", as shown in Figure 3.2.8 b, if the structure is considered with an initial imperfection (with a small deviation to the ideal shape), the critical buckling load will be smaller than the load for the perfect structure. This can be demonstrated, in that the critical buckling load from the tests will be much smaller than the theoretical value especially for the thin plates and shells.

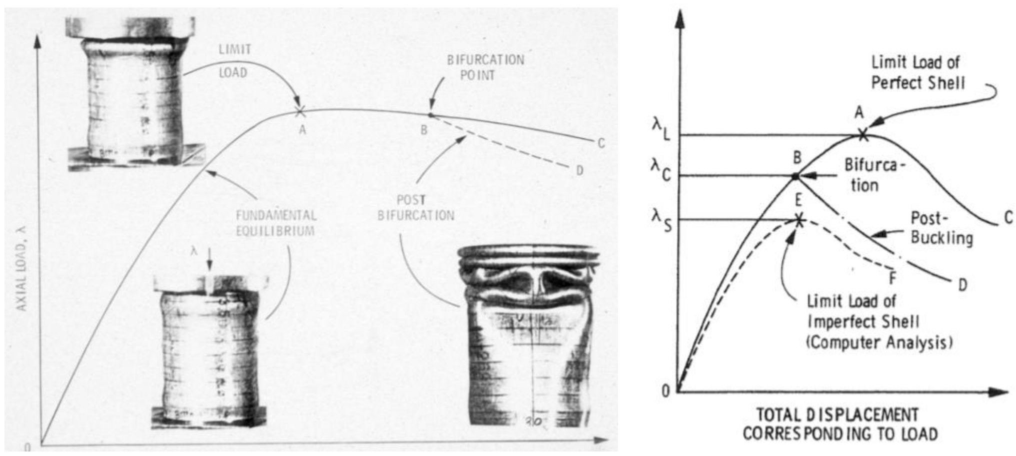


Figure 3.2.8 Buckling stages of plates and shells

2. Linear buckling analysis

To analyze the different processes of the buckling behavior, different methods have been developed and applied. The easiest way to approach a simplification is to consider the buckling process as an eigenvalues problem with the equations of stiffness. As it is described above, because the relationship between the external forces and the displacement of the system relies on the stiffness matrix *K*, the buckling criterion of point *A* will also mean that the stiffness matrix *K* is singular and the displacements are indeterminate.

In the real structure with the nonlinear behavior, the stiffness matrix *K* can be considered the sum of two parts: the normal matrix of a linear problem K_L and the nonlinear part K_{NL} which depends on the load *P*. The equation can be gotten as:

$$K = K_L + K_{NL}(P)$$

In linear buckling analysis, K_{NL} is assumed to be proportional to the load P_0, so that:

$$K = K_L + \lambda K_{NL}(P_0)$$

As described, to find the stiffness matrix as singular means that its determinant is zero, and:

$$\left(K_L + \lambda K_{NL}(P_0)\right) \cdot u = 0$$

In this case, the minimal eigenvalue factor λ is the critical load factor and the corresponding eigenmode u will be the buckling shape of the structure.

For an arbitrary structure, it can be concluded that the critical buckling load will be determined together by the material, geometry and boundary condition. And for a calculation, the equilibrium equations should be first written out. For the possible forms of thin sheets, the equilibrium equations are often complicated partial differential equations for the two dimensional surfaces; it is usually too hard to get a direct answer except in some simple cases such as the planar plates with hinged supported edges. A calculation of a simple case can be shown as below:

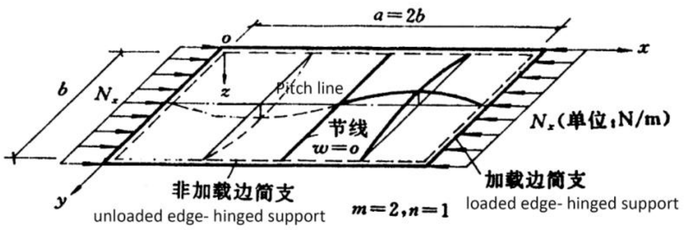


Figure 3.2.9 Diagram for buckling analysis

For the analysis of a plate with hinged supports at all sides, the equilibrium equation can be derived from the differential equations of the elastic surfaces with small deformations:

$$D_b\left(\frac{\partial^4\omega}{\partial x^4} + 2\frac{\partial^4\omega}{\partial x^2\partial y^2} + \frac{\partial^4\omega}{\partial y^4}\right) = N_x \cdot \frac{\partial^2\omega}{\partial x^2} + 2N_{xy}\frac{\partial^2\omega}{\partial_x\partial_y} + N_y \cdot \frac{\partial^2\omega}{\partial y^2}$$

where the flexural rigidity of plate $D_b = \frac{Et^3}{12(1-v^2)}$

As shown in Figure , where the force $N_y = N_{xy} = 0$, so the equation comes to:

$$D_b\left(\frac{\partial^4\omega}{\partial x^4} + 2\frac{\partial^4\omega}{\partial x^2\partial y^2} + \frac{\partial^4\omega}{\partial y^4}\right) - N_x \cdot \frac{\partial^2\omega}{\partial x^2} = 0$$

with boundary condition:

when $x = 0$ or $x = a$, $\omega = 0$ and $\frac{\partial^2 \omega}{\partial x^2} = 0$ and

when $y = 0$ or $y = b$, $\omega = 0$ and $\frac{\partial^2 y}{\partial y^2} = 0$

and the displacement of the deformed surface can be described as:

$$\omega = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

so the equation will be changed into:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\left[\frac{m^4\pi^4}{a^4} + 2\frac{m^2n^2\pi^4}{a^2b^2} + \frac{n^4\pi^4}{b^4} - \frac{N_x}{D_b} \times \frac{m^2\pi^2}{a^2}\right] \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} = 0$$

According to the equation, the buckling condition of the plate will be:

$$\frac{m^4\pi^4}{a^4} + 2\frac{m^2n^2\pi^4}{a^2b^2} + \frac{n^4\pi^4}{b^4} - \frac{N_x}{D_b} \times \frac{m^2\pi^2}{a^2} = 0$$

$$\text{or } N_x = \frac{\pi^2 a^2 D_b}{m^2}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2$$

for such load cases, the critical buckling load will be the minimal load when the plate is a little bent and take $n = 1$,

$$N_{x,cr} = \frac{\pi^2 a^2 D_b}{m^2}\left(\frac{m^2}{a^2} + \frac{1}{b^2}\right)^2 = k\frac{\pi^2 D_b}{b^2}$$

And here k will be the critical buckling factor

$k = \left(\frac{m}{\beta} + \frac{\beta}{m}\right)^2$ and $\beta = \frac{a}{b}$

So the critical load is

$$\sigma_{x,cr} = \frac{N_{x,cr}}{t} = \frac{k}{12(1 - v^2)}\frac{\pi^2 E}{(b/t)^2}$$

Then it can be seen that the critical load of a planar plate under average compression will be determined by the buckling parameter *K* and also the ratio of width to thickness.

By finding k_{min} in the buckling parameter curve (Figure 3.35), the minimal buckling load can hence be calculated.

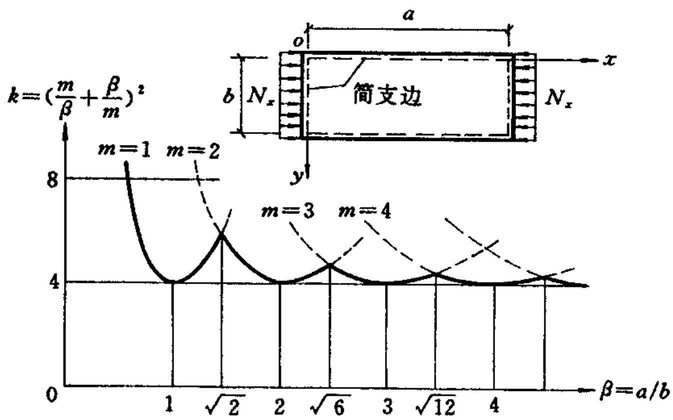


Figure 3.2.10 Critical buckling factor

3. Post buckling and nonlinear analysis

The linear buckling analysis shows only a minimal critical buckling load, at which the ideal structure will come to a buckling state and a pronounced large deformation will occur. But in any real states, the nonlinear behavior of the structure cannot be ignored, hence there is always a deviation of the real buckling behavior to the linear buckling analysis.

The post buckling behavior refers to the structure's nonlinear behavior when the critical load has been reached. Taking the plate under axial load as an example, when the critical load has been reached, the structure will deform suddenly to a spatial form with double curvature. The deformed structure also offers a stiffness and hence the rise of the applied load is still possible. As the large deformation and the corresponding nonlinear geometry behavior takes control in the post-buckling of structures. The nonlinear analysis in the third-order should be applied instead of the linear analysis in the second-order. And the iterative method should be used to take a full observation of the deformation's process of the structure system.

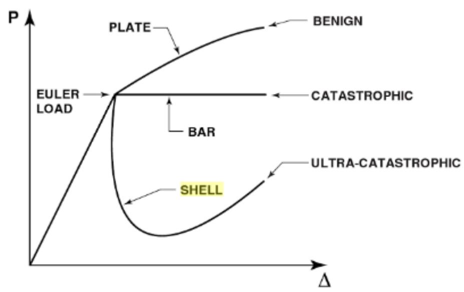


Figure 3.2. 11 Post-bucklings of plate, bar and shell (Brush & Almroth, 1975)

It need to be mentioned that the post-buckling behavior is different in various types of structure and leads to a different consideration and will affect the determination of the safety factors of design. For the ideal buckling mode of a rod, plate or shell, the deformation at the critical load point will cause different behaviors later. As the buckling of a rod is Figure 3.2.11 (Brush & Almroth, 1975) considered to be relatively catastrophic (Figure 3.2.11), the design safety factor of the applied load should be defined with a correspondingly higher value. For the planar plate, as the post-buckling behavior will also enhance the structure's stiffness, therefore this buckling can be understood as benign and the post-buckling process can also be used as a load carrying mechanism. (Figure 3.2.11) For shells, which have curvatures in the initial geometry, the buckling behavior also depends on the type of loading on the structure. When the shell is under an axial compressive load, the buckling load will be only 10%-20% of the linear critical load, but when the load is a compressive load as lateral pressure, the buckling load will be near the linear analysis result also with a very large deformation. (Brush & Almroth, 1975)

Because the buckling of some flexible structures such as thin plates and shells will also depend on the imperfections from manufacturing or the initial load, such a structure is also called "imperfection-sensitive". For such structures, the buckling load in real cases will possibly be much lower than the ideal critical load. So, for the design of such structures, a reasonable way to consider the possible imperfection is also a necessity for the design and analysis.

4. Design consideration for buckling behavior

Due to the causes and sensitivity to buckling of thin sheet materials, it is necessary to consider the stability of structures in the design process and sometimes the local stabilization of structures should be applied. As the buckling of structures is related to the material's property (flexural rigidity), the shape of structure (geometric stiffness) and also the type of loading on the structure, all of the factors should be considered in the design process.

A simple method to decrease the possibility of buckling on a structure is to increase the material thickness. As it can be found in the analysis of the plate buckling under axial load, that the critical buckling load will be directly proportional to the square of the thickness *t*. However, due to the manufacturing of such types of materials, a maximum material thickness will be restricted to a certain value.

There are not so many design standards for all of the materials mentioned in this research. However, the stability of steels has been well analyzed and the design consideration has been mentioned in some design standards. For the buckling problem and the related problems, discussions and safety factors are given chapters in following standards, where different safety factors such as the reduction factors of k in relation to the slenderness λ_p and the partial safety factor η and γ_M have been mentioned:

· DIN ISO 18800-3 Steel structures- Part 3: stability- buckling of plates
· Eurocode 1993-1-5 Chapter 4: Plate buckling effect due to direct stresses at the ULS
· DASt Richtlinie 012: Buckling safety verification for plates (Beulsicherheitsnachweise für Platten)
· DASt Richtlinie 017: Buckling safety verification for shells (Beulsicherheitsnachweise für Schalen)

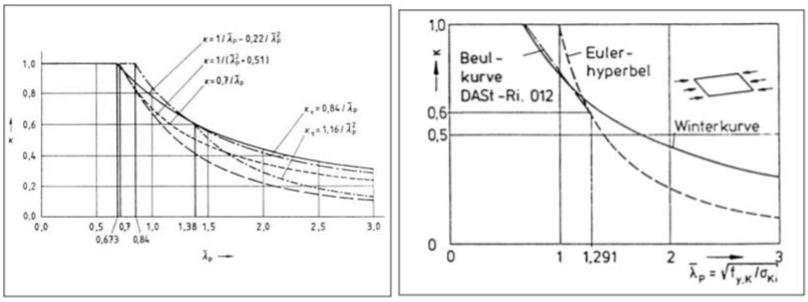


Figure 3.2.12 Definition of safety factors for buckling

For special materials or a complicated structure design, it usually needs a special material buckling test or a specific stability analysis by using techniques such as the nonlinear FEM analysis. But for a first design progress, it is usually necessary for the designers or the architects to know the weakness of the structure and the stiffening techniques should be used to increase the local stability of structure. In this research and in the next section, such stiffening techniques in structure design will be mainly introduced.

3.2.2.5 Stabilization of thin sheet materials

A simple planar thin sheet has often a bad load-bearing capacity due to its weakness under bending or the buckling behavior. Although by increasing the thickness with techniques such as laminating will enhance this capacity a little, it is usually best to consider a better structure form with better geometry and the resulting improvement of the structure mechanics. Such a design methodology can be seen in the analysis of the development of the shell typology and also the different morphologies for materials in chapter 2, and also can be simply illustrated by Figure 3.2.13. By folding the structure (to increase the stiffness) or by improving the support condition with reinforcements, the structure can hence get an increase of limit stresses.

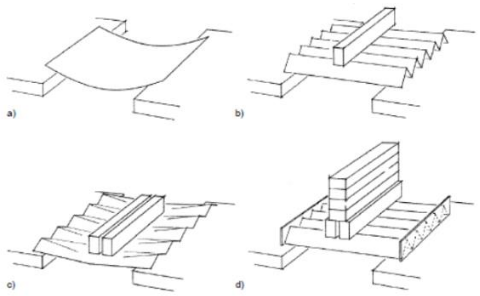


Figure 3.2.13 Stabilization of plates (Siegel, 1960)

From the architectural point of view, the following techniques can be applied to win a stiffening effect for the thin sheet materials in general (although some of them cannot be directly applied to all of the materials here mentioned, the alternative for design can be used with other techniques in this section):

1. Folding

The folding technique is one the easiest and most common techniques to increase the stiffness of a planar surface, and serves as a global stiffening strategy. By disassembling the spatial structure into planar segments that meet at adjacent edges, the simple planar surfaces have turned into a spatial system of load bearing slabs. With a derived complicated spatial form, the possible large bending moments in the center of a thin plate will change into a series of in-plane compression and tensions. With a divided relative small area of every slab, the stability is hence also enhanced.

By folding of a simple plate with several creases (Figure 3.2.14 left), structure height will be generated and hence the strength of the structure is activated. The weight of the structure will be lead to the edge at the bottom side through the in-plane forces rather than the bending behavior. The upper side of the crease will be in this sense under the maximum compressive load and the lower side will be under the maximum tension. (Figure 3.2.14 right). However, because this effect is also an in-plane axial load, the buckling behavior of the slab elements will also need to be taken care of. The larger the size of every element is, the bigger risk of buckling will exist.

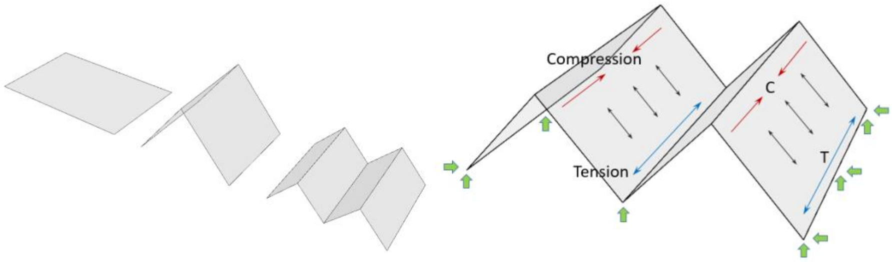


Figure 3.2.14 Folding and the internal stresses

Such a folding technique was already applied for a long time in the handcraft techniques to form interesting spatial structure forms (Figure 3.2.15 A) and it has been well known as the "Origami" through the wide spread of handmade artworks from China and Japan. By applying suitable connection techniques and controlling the dimension of the plate element, it has also been experimented with in the architectural designs. As buckling behavior can be prevented by the thickness of plates, it is usually the most important part to solve the problems form the connection of the discrete elements. (Figure 3.2.1 B,C)

Figure 3.2.15 Folded Origami structure and joinging detials (Buri, 2010)

2. Bending

The bending stiffening can be at first recognized as a variation of the folding technique, where the small number of fold creases are expanded into infinite creases, and hence this brings also a global increase of stiffness. (Figure 3.2.16 A) Unlike the folding technique, bending will cause a continuous principal curvature along the bending direction and will activate the more effective membrane behavior for the structure. This enables the understanding of the bent plate into a series of linear arches which are bound together by the axial strips. The external forces will be transferred through the continuous arches to the supports, at the meantime, as they are bounded by the strips in-between, the lateral displacement of the arches are prevented also by the in-plane forces. (Figure 3.2.16 B) From a global view, the behavior of the simply bent form and the folded form under the same load will show a certain similarity. The top side will be under the compression and the bottom side under tension. (Figure 3.2.16 C) But due to the continuous curved surfaces, the membrane behavior prevents the discrete structural form and also the forces in the structure (which will cause the break in the folded system above). And the risk of buckling under a uniformly distributed load will also be decreased as the structure is at nowhere planar but always with continuous curved surfaces.

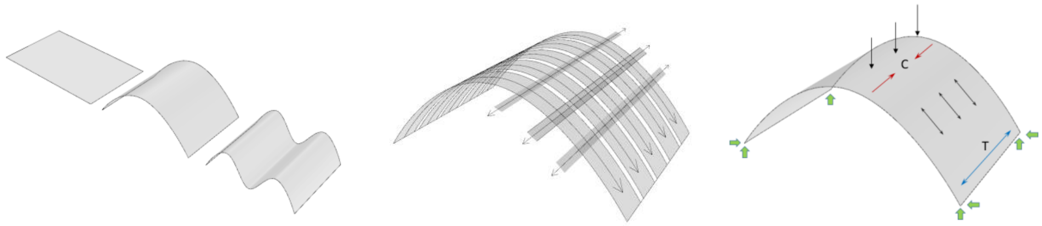


Figure 3.2.16 Bending and internal stresses

According to recent research (Pini et al., 2016), the two-dimensional bending can extraordinarily stiffen the thin sheets. It uses not only the total change of the structure load-bearing mechanism, but also benefits from introducing the in-plane stress by activating the curvature. Simple tests have been done to show the stiffening effect, when the thin sheet remains flat, it is easy to bend the material into a curved shape, but when the initial curvature is activated, to bend the structure in the other direction will be hard. (Figure 3.2.17) It is also explained from the energy aspect and demonstrated with vibration frequency tests that even a small amount of initial two dimensional curvature will stiffen the sheet to a large extent.

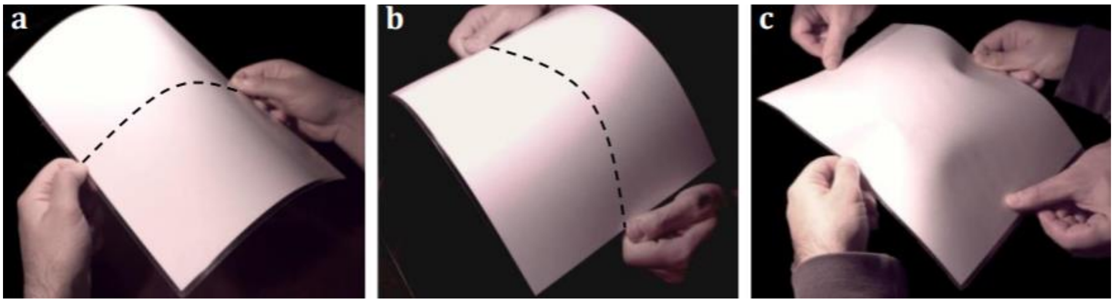


Figure 3.2.17 Stiffness of paper agianst 2D bending and 3d bending (Pini et al., 2016)

Although the shell behavior provides to the bent material a more efficient load-bearing behavior, it still depends on the design details of the load case and boundary condition. As shown in Figure 3.41-c, when loaded by a vertical load, compression and tension areas will be generated in the surface and with a vertical hinged support, an outward thrust will also appear at the edge. Such shortcomings of the design will also cause the bending moments in the structure and it needs to be specially considered in the design process. As also mentioned above, because the shell is an imperfection-sensitive and buckling-sensitive structure, the stability also needs to be testified and analyzed.

3. Edge folding, curling and local ruffling

Apart from the global stiffening methods, local stiffening also using the folding and bending techniques can be used to stabilize the structure, especially against bending and buckling. As shown in Figure 3.2.18, if an axial load or a point load is added on the structure, the buckling behavior or the bending behavior can easily happen with the thin and flexible sheets. By folding the edge to a certain angle, the flange will work as a supporting beam to stabilize the structure and prevent the possible deformation. In this sense, compression and tension zones will appear in the flange plane and the whole structure will hence have an enhanced bending stiffness.

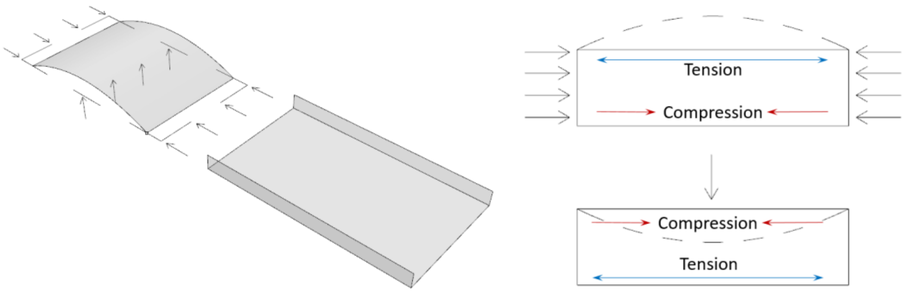


Figure 3.2.18 Edge folding and the internal stresses

With the folding edges, the structure from the thin sheets will work like a profile structure. But although the flange will help to prevent the deformation, its planar form will also result in a lateral buckling behavior of such constructions when a large load is applied. (Figure 3.2.19) For this problem, special geometry can be used as the shape of the flange to increase the local stiffness with the edge curling method. Both by creating a curved surface or adding additional folding will enhance the structure's stiffness against buckling as they will affect the buckling parameter of the structure. (Figure 3.2.19)

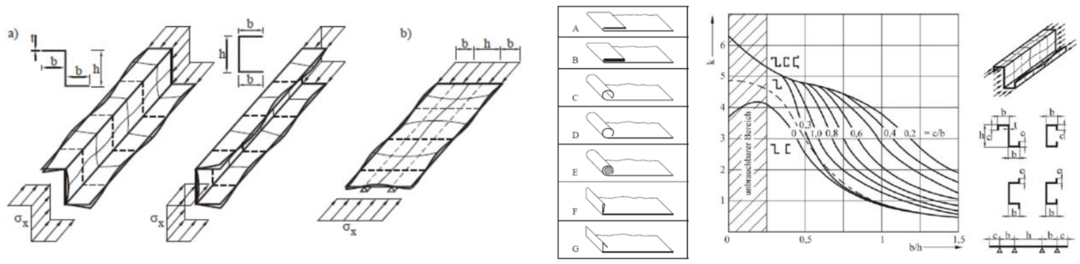


Figure 3.2.19 Different edge folding and their effects on buckling (Klein, 2013)

The folding of the structure can be used not only by the edge stiffness, but also can be applied in other parts of the structure. It is commonly used to change the planar surface to a spatial shape with the sinking techniques. As it has been analyzed, the critical buckling load of the plate can be simply calculated as:

$$\sigma_{x,cr} = \frac{N_{x,cr}}{t} = \frac{k}{(1-v^2)}\frac{\pi^2 EI}{b^2 t}$$

Where it is not only dependent on the width and the thickness of the plate (b and t), but also on the moment of inertia of the plane area I. After sinking and the deriving shaping effect I has also been changed, hence an increase of the stiffness can be achieved. (Figure 3.2.20 A, B) Similar methods are also used to stiffen the rim holes on the thin sheets such as steel plates. When large holes need to be designed on the structure or the load is too high, the local stiffness and the stress concentration need to be considered. By punching the rims into an uprising curved surface (Figure 3.2.20 C, D), the stiffness can be increased with similar behavior.

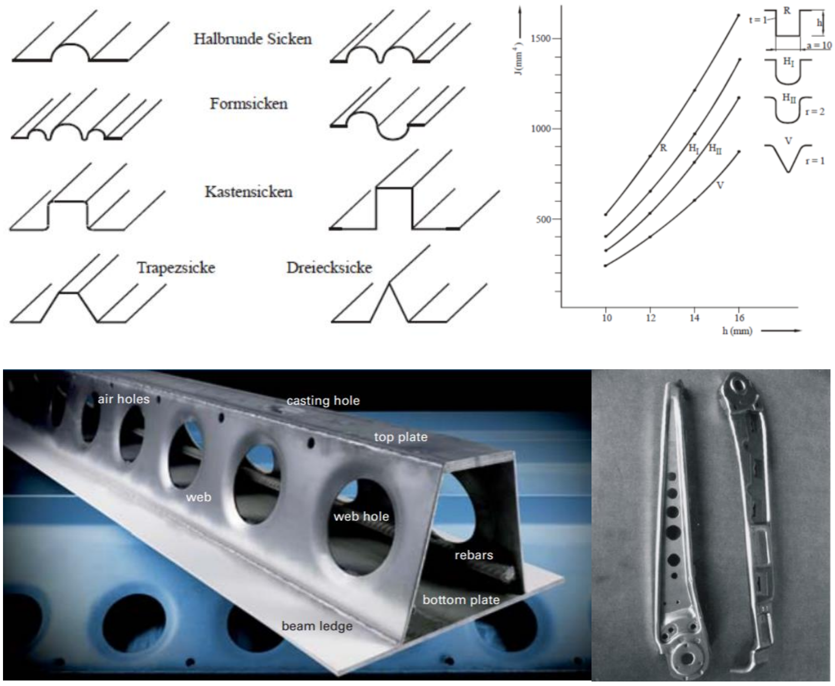


Figure 3.2.20 Stiffening methods with local folding and bending (Klein, 2013)

4. Rib stiffening and sandwich structure

Apart from the active forming methods above to stiffen the sheets, another passive method—rib stiffening—is one of the most popular and widely used techniques. Similar to the folding edges and the sinking shaping, the function of the ribs will enhance the bending and buckling stiffness by changing the geometry of the structure.

The ribs can be formed in many shapes and should be connected to the sheet materials. Techniques such as welding (Figure 3.2.21 A,B), screwing (Figure 3.2.21 A: c-e) and extruding (Figure 3.2.21 B: f-i) are usually applied in industries.

The rib stiffening can be considered similar to the sinking stiffening. But the arrangement and setting of the ribs should be at best in the tension area of the structure. So, it is very common that the ribs are put on the bottom side of the structural elements.

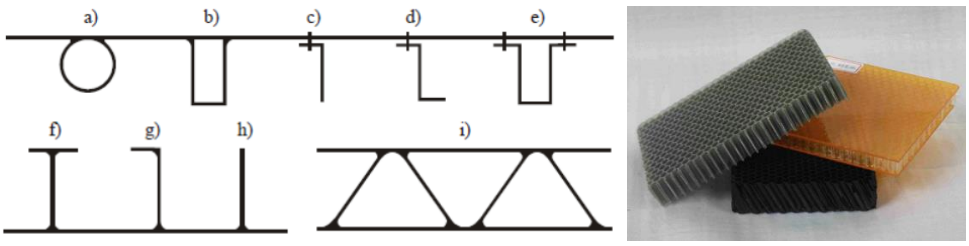


Figure 3.2.21 Rib structure and sandwich structures as stiffening (Klein, 2013)

For some cases, the ribs can be designed and constructed into a rib system, where a non-directional stiffening can be achieved. With such a concept, a new material type of the composite material has been developed as sandwich materials. (Figure 3.2.21) Such a typology can be seen in many semi-products of the steel, plastic and paper panels. With the normal thin sheet as the skin material and the ribs to offer a stiffening to bending or buckling, a lightweight material can be made without simply adding to the thickness of the structure. This method is also suitable for some anisotropic materials such as paper products. As the stiffening layer will help a lot in the load-bearing behavior, a quasi-isotropy property can hence be generated.

3.3 Architectonic approach and system typology

After reviewing and analyzing the special physical and engineering properties of the thin sheet materials, a basis of the design principles can be summarized to find the design emphasis of such structures. In this part, it will be discussed mostly from an architectonic way, where a system thinking of structure concepts is emphasized. At the same time, the availability of techniques, the strength and stiffness of structures and also the cost or time efficiency in the manufacturing process will be comprehensively balanced.

3.3.1 Geometrical Consideration and forming Techniques

3.3.1.1 Geometric consideration and cutting techniques

1. Developability and formability

Due to the materiality, the geometry of any structure should be based on the availability of the forming technique for the materials. However, there is an essential difference between the design process to determine the shape or form and the real constructing process. A software dependent and the NURBS based shape definition and representation will often result in an impossibility for the imagined arbitrary structural forms to be realized. For most of the designers of the free-form structures, an optimizing procedure or the background knowledge which is based on the constructability works as the foundation of designs so that such rationalizing can be widely seen in the design process by many of the architects and studios, e.g. the Frank Gehry partners. (Shelden, 2002)

Like one can never form a planar paper sheet into a sphere shape without stretching or deformation, the developability is often the first consideration of the shape optimization for sheet materials due to the economic and pre-fabrication efficiency. As introduced in chapter 2, the developable surfaces refer to a series of surfaces which are characterized by the property that they can be mapped isometrically into a plane and hence the planar isometric image S^d is the planar unfolding of S (also the development of S). (Pottmann, 2007) As it can be imagined with a simple planar paper sheet, it can be bent in any single direction, but cannot be bent together in both directions. This means that a developable surface will have always at any point a principal curvature $K = 0$, also considered to be constructed by a series of straight lines. Such surfaces can only be one of the following types: planes, cylinders, cones and tangent surfaces of spatial curves. (Figure 3.3.1)

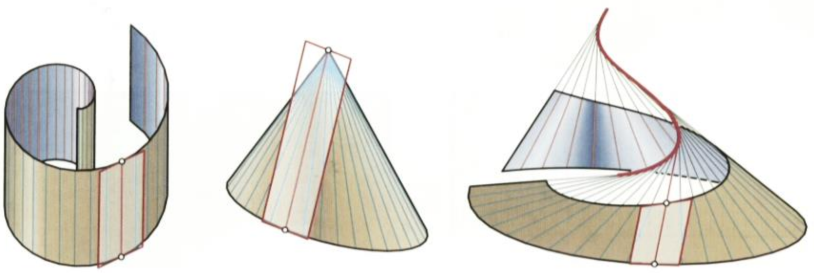


Figure 3.3.1 Diagram of developable surfaces (Pottmann, 2007)

Except for the cylinder and cones as the classic geometry, the construction of a free-form developable surface usually takes the use of the discretion methods. By considering a surface as the combination of some planar or twisted surfaces that are defined by several verteces, one can get a complicated developable surface by controlling the planarity of every single face in the mesh system. In this way, several geometrical methodologies, such as the planar quadrilateral (PQ) mesh, combinations of tangent surfaces of a continuous spatial curve can be used to generate or optimize the design geometry. (Figure 3.48) At the same time, special techniques have already been implemented in the design software, such as the loft functions in Rhinoceros, or the D. Loft function collection in the plugin EvoluteTools.

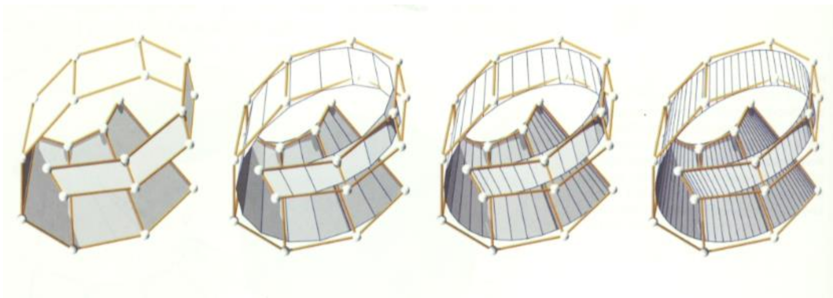


Figure 3.3.2 Planar-quad mesh (Pottmann, 2007)

From the economic point, using the developable surfaces is the most optimizing way in the architectural projects with the thin sheet materials. It can be achieved by simple cold bending or forming techniques. And in some special cases, it needs also some special double curved surfaces such as the requirements in the car body skins and the packaging industry. These forms in architecture should be designed with the special forming techniques such as pressing, stretching and incremental forming. These techniques will be introduced later and as the deformation of material has been caused in such techniques, the changes of materials' properties should be specially taken care of.

2. Cutting technique and nesting

The thin sheet materials in industry are also due to the economic reason supplied in standard sizes. Hence in most cases it needs to be cut into the specialized forms. The traditional cutting techniques usually require the shear forces to split the material into pieces, and due to the complexity of the machines used and the final products, it includes normal shear cutting and the fine cutting techniques. And with the development of the digital and the modern industry, more new techniques are used nowadays in the thin sheets' processing.

· Traditional shear cutting techniques

The traditional shear cutting techniques are based on the shear forces caused by the fast punching of two cutting tools. The shear force will cause a fracture of the cross-section of the material plate and the material is hence cut into pieces. (Figure 3.3.3 A) For a more efficient cutting process, the cutters are often made with an angle, and for friction to be avoided, a small rotatable angle is often also added. (Figure 3.3.3 B) When cutting the materials, a local pressing should also be added to prevent the bending of the material, hence to guarantee the cutting edge. (Figure 3.3.3 C) For the normal thin sheets, a hand-lever scissor is for most cases enough and for some cutting machines, a curved edge cutting is even possible. For the fine cutting techniques, large modern cutting machines should be used and the high quality products can be achieved with a smooth cutting edge. (Figure 3.3.3 D-F) This is mostly used in precise steel cutting.

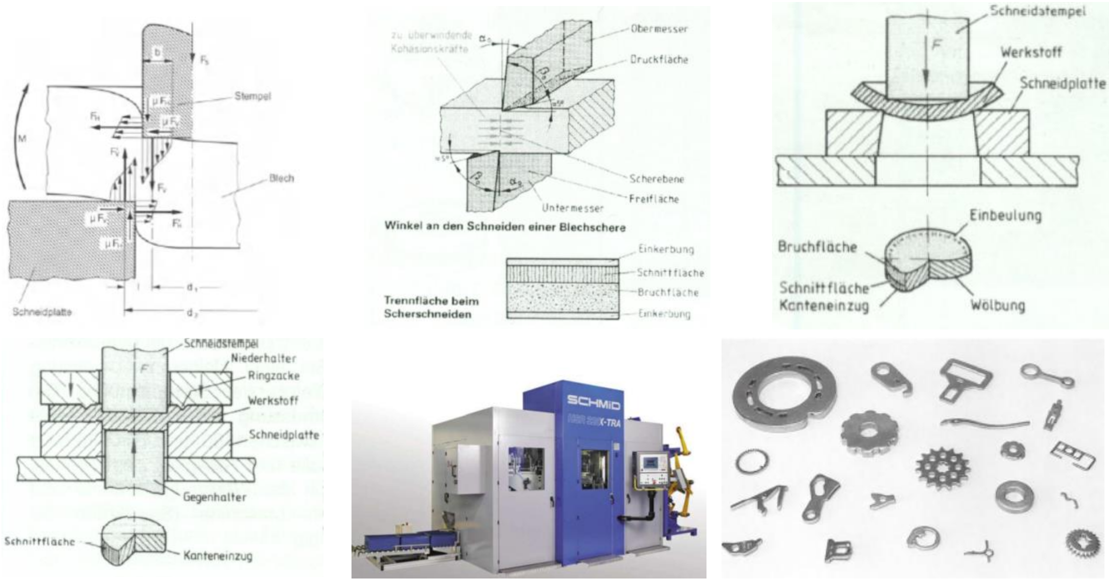


Figure 3.3.3 Traditional shear cutting techniques (König & Klocke, 1990)

· Laser cutting techniques

The laser cutting techniques are a modern digital cutting technique, where the cutting is finished with a high-energy laser ray by either fusing, flame burn or sublimation. (Figure 3.3.4 A) The laser ray shoots at a small area of the materials (for thin sheet materials it can be less than 0.3mm (König & Klocke, 1990)) without any direct contact, and with the help of special gas it can heat the material so extremely that it melts or even vaporizes. Argon and nitrogen are used in fusion cutting, where such inert gases don't react with the molten material by the kerf and the fluid part of the material will be blown away by the gas sprays. In flame cutting, oxygen is often used to amplify the energy of the laser beam by the local reaction. With such high and concentrated energy, it enables a fast cutting speed and a possible large thickness or a small radius of contour and hence complicated patterns. However, the burning of material at the kerf will also cause oxidation which should be avoided by some materials such as timber, paper and plastics.

The sublimation cutting is nowadays mostly widely used as it provides a high-quality edge and without the oxidation problems. It uses a large power laser to vaporize the material with as little melting as possible. In the kerf, the material vapor creates a high pressure that expels the molten material from the top and bottom sides. The inert gases are sprayed to provide a shield to ensure the kerf to be oxide free. For this reason, sublimation cutting such as CO2 cutting is suitable for most of the thin sheet materials and is with less tolerance. With the 5-Axis and 6-Axis equipment it is also possible to realize the 3D cutting for complicated shapes. (Figure 3.3.4 B) As here more energy is used, its speed will also be slower than the flame cutting.

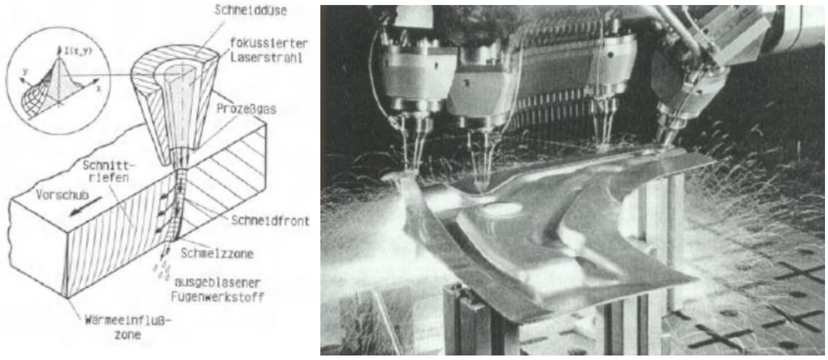


Figure 3.3.4 Laser cutting technique (Hachul, 2006)

· Water jet and plasma cutting techniques

Apart from the laser cutting, other high power cutting techniques such as water jet cutting and plasma cutting are also used in the industry especially for metal or glass fabrications. For the water jetting, a very high-pressured water jet (about 4000 Pa) or a mixture of water and an abrasive substance is used to cut hard materials. The high-speed water jet offers at the same time a very precise cutting area (0.08-0.5mm) and the ability to cut relatively thicker materials.

The plasma technique is used particularly for electronic conductive materials such as the metal sheet. It creates a completed electric circuit to enable a super high temperature at the kerf. However, both the two cutting techniques are not much better than the laser cutting technique when it comes to the thin sheet materials and from the cost side they are much more expensive. So the laser technique is more widely seen in both industry and the model making process.

· Anisotropic material and the nesting consideration

As it is discussed above, because some of the materials have an anisotropic property, the mechanic behavior defers in a different direction of manufacturing. As it needs also to be cut, the following considerations should be taken in the nesting process:

---- for the strip forms which are mainly under axial load or bending, the nesting should be oriented along the manufacturing direction for the maximum strength. And it is vice versa for the elements which are mainly under shear forces.

---- as in the edge area of the standard sheets there will be a decrease of material strength, a safety zone should be considered in the nesting process. For the elements which will be under higher loads, they should be arranged at best in the center of the sheet.

---- for the structural elements which are in a surface structure, the load direction should be diagonal to the manufacturing direction. This is also used in the built prototype and pavilions in chapter 5, because the connection curve of the membrane surfaces was perpendicular to the manufacturing direction of the paperboards.

3.3.1.2 Forming technique

The forming process relies usually on the plastic deformation of the materials under external forces, where the yield strength is exceeded and the yield strength R_e for the elastic deformation and the breaking strength R_m is not reached. (Figure 3.3.5) And sometimes, it is possible to be reached within the elastic deformation process and the spring back behavior should be prevented by extra fixings.

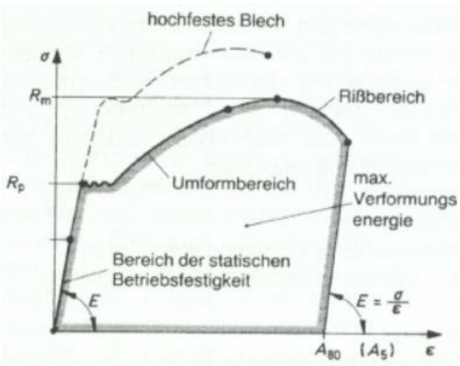


Figure 3.3.5 Yield strengths in sheet forming (Hachul, 2006)

The formability of materials depends on the stress and strain curve of the material test. For materials with lower yield strength, a wider area of plastic deformation and large capacity of strains, more forming techniques and a large extent can be achieved. On the contrary, for some special materials such as glass, it is hard to realize a plastic deformation as the material is brittle and easy to break.

The forming also depends on different temperatures. In warm forming, as a high temperature will increase the capacity of strains and decrease the strength for most sheet materials, a larger deformation can be made. But in the cold forming, a change of the micro-structure of material will appear in the plastic deformation and it will cause an increase of strength and decrease of strain—so called strain hardening. Such a behavior will in return lead the material to break easily in a large deformation.

The overall forming technique for the thin sheet material is summarized here according to the DIN 8582 as compressive forming, compressive-tensile forming, tensile forming and bending forming. Except for the bending forming, most of the other forming techniques are mainly used in the local change of shape so that the overall material quality doesn’t change. For bending, as the plastic deformation will change the materials’ behavior and it is not available for some of the materials, it is only considered here the elastic bending as a global forming technique.

1, Compressive forming

The compressive forming technique is rooted in the traditional technique of the forging and hammering in the old manual working. The machine-supported compressive forming is mainly suitable for the rotational symmetric form, where the original material is placed between the halter and a pressing mould and a spindle help to rotate the form while a compressing roll will press the sheet into the desired form. (Figure 3.3.6) This technique is divided to the conventional pressing with constant thickness of walls and the projective pressing with a change of the thickness. In the conventional pressing, the strength of the material remains almost the same before and after the forming, as the material thickness remains the same. However, in the projective pressing, with the loss of thickness the material’s strength also decreases. The compressive forming can also be achieved by hand for any free form with soft metals and plastic sheets or paperboard, where a prefabricated mould is needed. Due to the disadvantages of the strength lost and the heavy human power, it is only suitable in the prototype making or a design model.

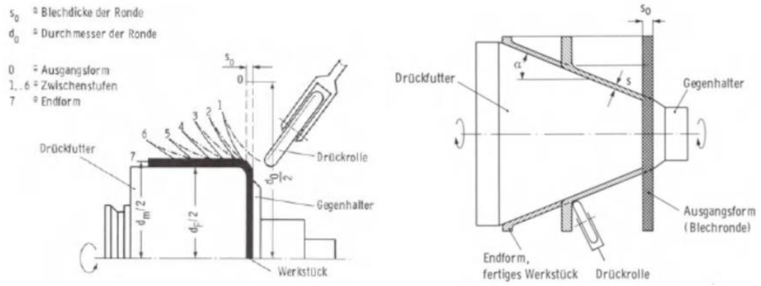


Figure 3.3.6 Compressive techniques(König & Klocke, 1990)

2, Compressive-tensile forming

Deep drawing and collar drawing

The deep drawing technique refers to the forming process that is widely used in sheet metal forming, in which a sheet metal blank is radially drawn into a forming mould by the mechanical action of a punch. (Figure 3.3.7 A) With this technique, it enables the making of some complicated concave forms from the planar sheets, and in normal deep drawing the thickness of the wall remains constant.

In the forming process, the external forces of the punch will cause a radial drawing force (tensile) and also a tangential compressional force (hoop force) in the flange area. To avoid the cracks caused by the compressional force, a holder will be used to fix the material to deform along the radius of the mould. The mould determines the form of the deformation and controls the thickness of the wall and the gap between the mould and the punch will be of great importance to control the thickness and also the quality of the drawing process. It is also possible to control the thickness of the wall by using ironing forming in the edge part. (Figure 3.3.7 B)

For the drawing process with multiple forms and tasks, elastic tools such as rubber or a membrane can be used as the punching tool. (Figure 3.3.7 C) Although it requires more drawing load to generate enough drawing forces in the sheets, such a technique shows some advantages such as a low cost and a possible sequential processing. With the similar techniques of deep drawing, it is also possible to make a local stabilization of the rim boundary, where the technique is also called collar drawing. (Figure 3.3.7 D) Apart from the soft metals, the deep drawing technique has also been used in the applications of plastic forming and paperboard forming. (Vishtal & Retulainen, 2012)

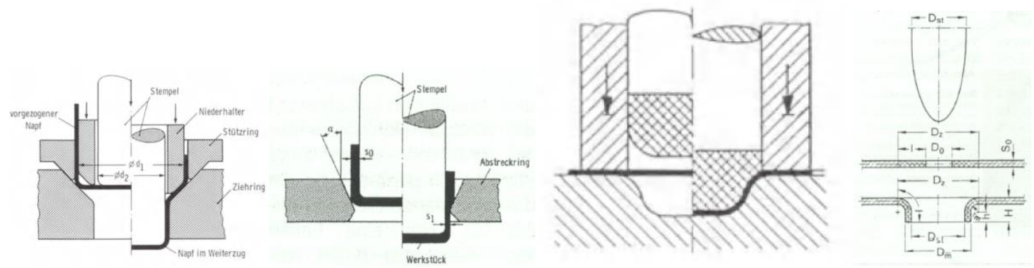


Figure 3.3.7 Compressive-tensile forming techniques (König & Klocke, 1990)

3, Tensile forming (Stretch forming)

The tensile forming process is like the deep drawing process, but only results in the dominant tensile forces. Compared to the deep drawing, the height of the deformation is usually smaller than the diameter of the sheet material, hence the hoop compressive force does not arise. With a hydraulic forming stamp or some CNC forming mould pressing from the bottom side, a curved deformation will occur on the planar surface. (Figure 3.3.8 A) At the same time, both the shrinkage of the materials' thickness and enlargement of the area of the surface will happen, and this technique works well especially for the large scale sheet material's forming. (Figure 3.3.8 B)

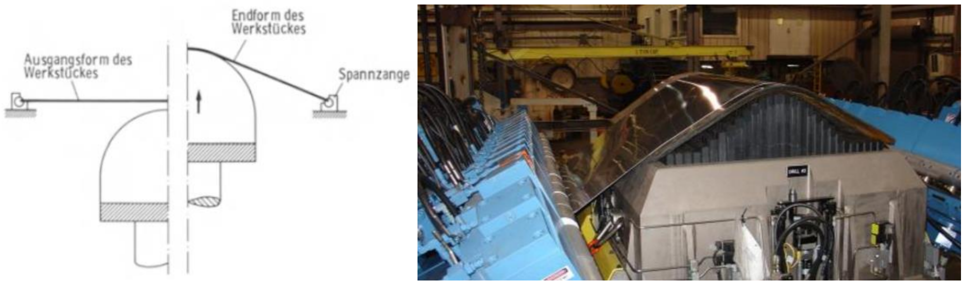


Figure 3.3.8 Tensile Forming techniques (König & Klocke, 1990)

4, Bending and folding

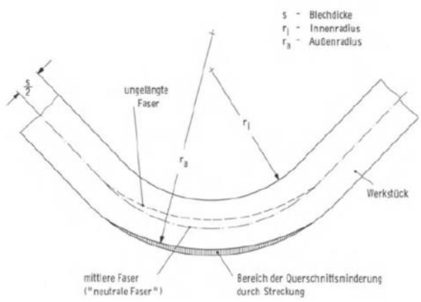


Figure 3.3.9 Displace of neutral surface in bending (König & Klocke, 1990)

Unlike the discussion in the bending theory, where many idealized simplifications were applied for the analysis and calculation, in real bending processes both elastic and plastic deformations can happen. By bending, it is clear that the outside of the material is under tension and the inner side is under compression. In an elastic deformation, the yield strength of material cannot be reached at both sides.

At the same time, the neutral surface will also remain at the center of the cross section. However, in the plastic bending, as the plastic deformation can arise, the materials' elongation is different on both sides and the displacement of the neutral surface will also be inwards at a relatively small radius. (Figure 3.3.9)

In the bending process, the force of the spring back will be introduced with the increase of the curvature. This force is dependent on the materials' property, such as the breaking strength, flexural modulus, thickness and the bending radius, and it will directly affect the minimal bending radius of the material. The analysis of the bending theory above has shown a theoretical maximum radius for an elastic bending of isotropic materials. In the engineering bending with plastic deformations, the minimal bending radius is also proportional to the materials' thickness and can be defined as the thickness *t* multiplied by the c-factor. (König & Klocke, 1990)

When a local bending is achieved with a very small radius, it is almost equal to the local folding technique. In this way, the plastic bending is also used in the folding of sheet panels such as metal sheets, paperboards and plastic sheets. In the bending forming technique, many methods and tools can be used, such as the pure free bending with bending moments, the press brake bending with moulds, swing folding and the rolling bending with both small and large radii. (Figure 3.3.10) The elastic bending is applicable for nearly all of the thin sheets mentioned here and can be achieved manually. The small radius bending and folding should rely on some special tools to guarantee the geometric quality.

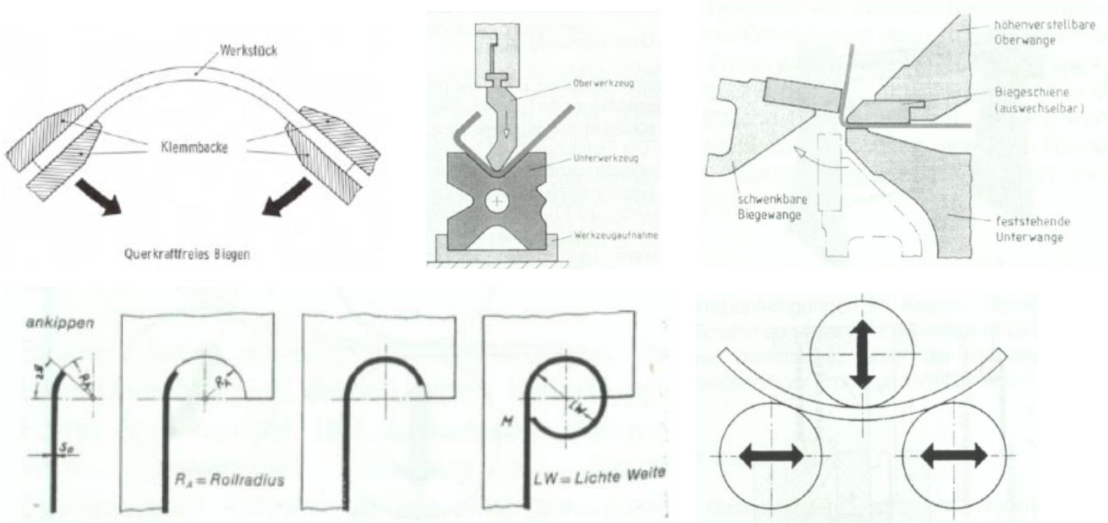
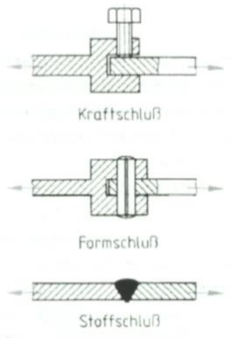


Figure 3.3.10 Different bending forming techniques (König & Klocke, 1990)

3.3.2 Joining Techniques for structural elements



The joining of thin sheets enables the assembly and disassembly of complicated structure systems for the formed sheet materials. According to the different objectives and constitution of the joining, it can be divided into three groups: stress conclusive, form-conclusive and material-conclusive. (Figure 3.3.11) In the existing standard of manufacturing methods, the mechanical connections which is stress conclusive are mainly defined in the DIN 8580. At the same time, some other techniques without certain structure forms such as the adhesive joining, are defined specifically in DIN 8595.

Figure 3.3.11 Three basic joining methods (Hachul, 2006)

3.3.2.1 Stress conclusive joining -- Mechanical joining

The mechanical joining provides in most cases the stable connection to transfer stress between the structural elements. With such connections, the stresses from one element will concentrate at the connecting materials and then be transferred to the adjacent element by means of tension, shear force and even bending moments. The most commonly used mechanic joining for sheet materials are the dismountable screwing and the non-removable nailing and riveting techniques.

The screwing technique is stress-conductive when the bolt and nut are used together, especially when the tensile forces between the elements are very high. In this case, the bolt and nuts should be highly pressed, and the high frictions between the elements will prevent any possible displacement. (Figure 3.3.12 A) Without such fixing the screwing will be similar to the nailing method and can only provide shear forces. (Figure 3.3.12 B) In both cases, the screw thread should be longer than the total thickness of the material and for materials such as wood and paperboard, as a creep deformation will happen at the screwing point caused by stress concentration, large spacers should be used to enable a dispersion.

Compared to the screwing technique which enables a dismounting of the joint, the riveting technique gives a permanent fixed joint. In general, the rivet fixes itself by the deformation of its head and provides a point fixing, which is also similar to the nailing method. (Figure 3.3.12 C)

Because the riveting technique doesn't need coupled materials like the screwing with nuts, it is suitable to be used in connecting a closed structure, such as the cellular cavity structure in this research.

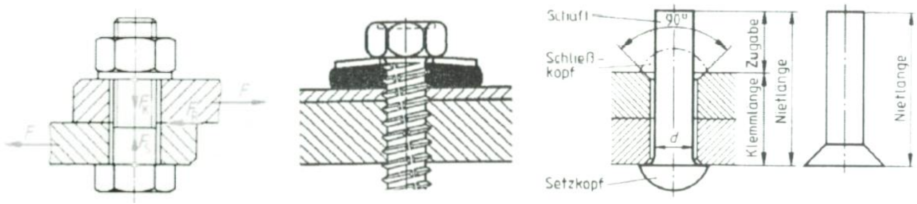


Figure 3.3.12 Mechanical Joining techniques (Hachul, 2006)

3.3.2.2 Form conclusive joining -- Geometrical joining

The geometrical joining techniques prevent the displacement of the elements through the form of the connection (usually the interposing of the elements).

With the thin cross sections of the sheet material, the geometrical prevention can only easily be realized in two dimensions. Such joints are often seen as the zipped connection. (Figure 3.3.13 B, C) In three dimensional connections of the structural elements, more complicated joining methods should be applied. In experimental constructions or the packaging industry, inserting methods are used in some small products and for the connections which don't require too much strength. In large scale architectural constructions, such techniques have not been used for the spatial joining of relevant materials.

Figure 3.3.13 Geometric Joining techniques

3.3.2.3 Material conclusive joining -- Adhesive joining

The adhesive joining technique is more suitable for almost all the thin sheet materials. And as a material conclusive technique, the joining function is mainly caused by the bonding of an intermediate layer to connect substrates of different materials. Due to such a principle, the adhesive joining makes itself an attractive alternative to mechanical joining and the advantages can be summarized as following:

---- with different adhesives, the joint can be either soluble or insoluble

---- the adhesive joining can be effective between different materials

---- as no holes should be drilled before the adhesive joining provides a cleaner finishing and the distribution of the stress load can be evenly made over a broad area

In the design of the adhesive joining the type of adhesive should be considered due to the material's property and the design requirements. As in the building experiment in this research, the type of adhesive for the paperboard is determined according to the requirement of the wood structure, hence according to the standard DIN 68141 and DIN 302 which is mentioned in DIN 1993-1-1.

The structural adhesive must provide enough bonding strength and also a long-lasting bonding capacity. It also needs to provide a bit of elasticity to adapt for the small deformation of the material due to the environment. For most of the architectural uses a good stability under a relative high temperature and relative humidity is also required.

The adhesive joining usually has a greater shear strength than peeling strength, so the structure should be also designed with the shear forces. When a large peeling force is caused by the possible local bending, the bonding strength should be testified and in some cases the mechanical joining should be used together. For a better bonding strength, a smooth and planar bonding area should be required and the cleaning surface needs to be prepared. Because the bonding strength is distributed in all of the bonding area, a sufficient area or enough width of bonding is also necessary.

4, Hybrid joining and other joining techniques

For some special materials such as the metal sheets, some particular joining techniques such as welding are also widely used in industry. With the development of the manufacturing methods, new digital methods such as robots are used more and more in the corresponding fields.

With the complicated structures where different material forms and stresses are being considered, a comprehensive hybrid joining method is often applied according to the specific requirements. As it is the aim of this discussion to guide the design method of the construction details for the structure concept in this research, the hybrid joining refers here to an optimization of the mechanical, geometrical and adhesive joining techniques.

3.3.3 Structure typology and multi-hierarchical building system

With both the discussion from the applications of thin sheet materials, to the engineering properties and its special structural behavior, then to the architectural aspects of forming and connecting different elements, the aim of this research with this comprehensive materials review is to get a necessary basis and foundation for a novel structural system or a structure concept.

According to the concept from Engel (Engel, 1997) (Figure 3.3.14), from the aspect of structure typology, thin sheet materials can be used in the following spatial structures:

· ***Section active***-- Ribbed frameworks with dense grids of sheets (requires the inserting connection for a honeycomb like structure form)

· ***Vector active***—Truss systems or spatial frameworks (requires the local bending and folding technique to form the planar surface into profiles or tubes for a truss element with high bending-stiffness)

· ***Surface active***—Plate or shell structures from the spatial joining of formed thin sheets (requires the global bending and folding technique and also the connection techniques to transfer the load; form-finding must be applied to align the surfaces to activate the membrane behavior, stability problems for thin sheets should be taken care of)

· ***Form active***— Planar sheets to be formed into suitable structural elements, and the arrangement of the elements should be designed with the form-active mechanism, such as the cellular structure system

· ***Hybrid system***— Structure elements from different forming techniques compose a coupled system with different internal load transmission behaviors

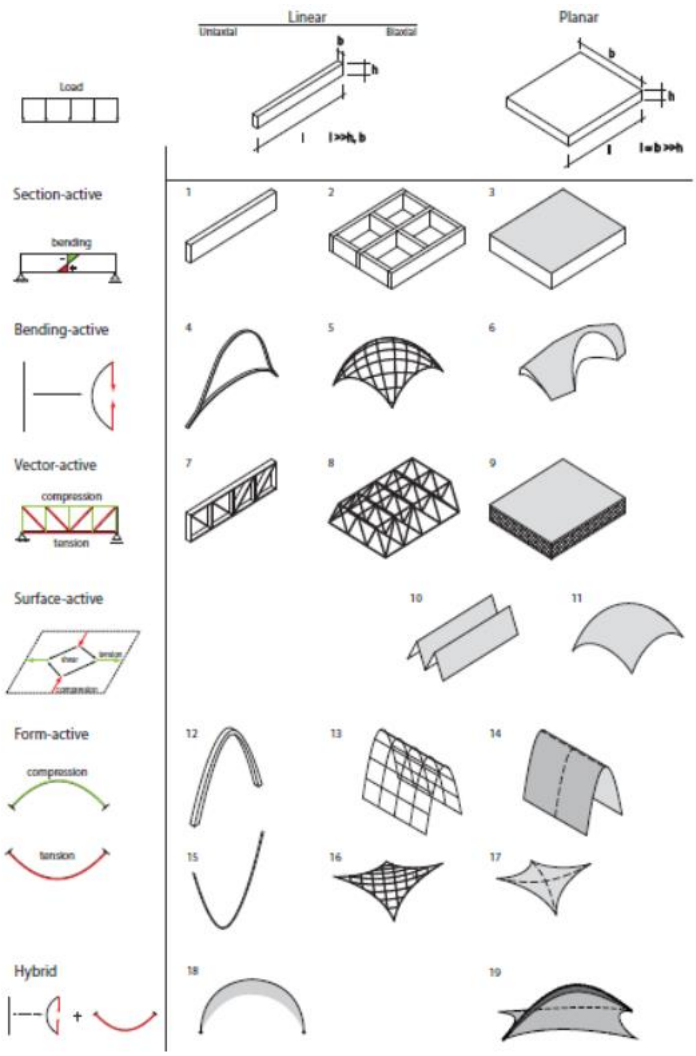


Figure 3.3.14 Diagram of definition and categorization of structural typology (Lienhard & Knippers, 2014)

Because of the obvious structural problems such as the stability problem of thin sheet materials, a pure application of a single structure typology for such materials is often difficult, especially when a small thickness or a large scale of structural elements would be used.

In both of the experimental designs and the practical projects, more and more hybrid structure systems are being carried out, where a multi-hierarchical structure is composed by coupling different systems together. With a different form and the inherent load bearing behavior, the reciprocal systems enable a balance and double-benefit together.

Similar to the typical hybrid structures such as the coupling of section- and form-active structures or the vector- and surfaces-active structures, in this research it is also an objective to establish an innovational coupling of such systems.

With such objectives, a modular cellular structure, which is based on the characteristics of thin sheet materials, is analyzed and experimented with in this research. With hollow cells composed of the ribs and membranes, a coupling of the ribbed shell and membrane shell is developed, and the geometrical and structural analysis will be discussed in the next chapter.

4 Cellular cavity—the structure concept

In this chapter the comprehensive discussions and analysis of the experimental structure concept of this research is presented. In the first part, some foregoing experiments of using the thin sheet materials with the existing shell structure typologies are shown and analyzed. The generation of the structure concept is then described with both the discussions of the problems and objectives. In the main sections of this chapter, the structural concept is then analyzed from both the geometrical side and the structural side. Finally, as a result of this analysis, the design process and important methods are summarized.

4.1 Foregoing experiments and the generation of the structural concept

4.1.1 Basic objectives and geometrical requirements of design

The most important target of the structural experiments and research in this dissertation is to evaluate the possibility of using thin sheet materials in the existing structure forms for shell structures and to establish an innovational structure concept that is specifically efficient for such materials. Or in other words, to develop new structure forms for shells based on multiple criteria which are composed of all the efficiency requirements from the architectural, structural and fabricating aspects of materials. According to the analysis of the structural morphology in chapter 2, a key point in this research should be based first on the analysis of the boundary of the materials and the restrictions of the semi-products for construction. And this refers to that all the disadvantages and limitations of such materials (such as the limited thickness, material size and the stability problems) and the available forming and fabricating techniques should be first taken into consideration.

This principle can be simplified into a trinity based on the definition of the structure forms by Siegel (Figure 4.1.1) (Siegel, 1960). The design of the structure system should depend on the above-mentioned multiple criteria, and the geometrical form is hence the subordinate and basic requirement of the designs.

In this sense, the analysis for the generation of the structure concept in this dissertation also starts from the establishment of the basic boundary of the criterion and the design requirements.

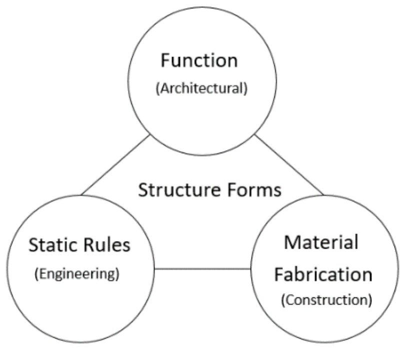


Figure 4.1. 1 Trinity of the factors in structure design (Siegel, 1960)

For the thin sheet materials that are discussed in this research, such a relationship or the basic boundaries can be classified into mainly three groups:

1, Architectural boundary: The architectural design of the shell structure aims to determine the main goals of the design and the priority of factors. As this is often complicated in the real designs, it works mainly as the guideline to organize and compromise between all factors such as the functional, aesthetic, cost, structural and technological requirements.

2, Structural boundary: For the shell structure in general, a suitable load bearing mechanic with the global membrane behavior is the basis of the design of such shells. But for the thin sheet materials also discussed here, the local stability problem should also be cared for. Due to the special characteristics of materials such as planar surfaces and small cross sections, the dimension problems and local stiffening against bending and buckling must also be taken into consideration.

3, Material boundary: The material boundary of this research is not about the precise property of each kind of thin sheet material, but about some common characteristics such as the material thickness (0.5-10mm), the transportable form (uniform in meters) and the formability of such materials. In the consideration, the forming techniques and its efficiency should also be considered.

Considering all the aspects above, the efficiency for shells with thin sheet materials is complicated as many aspects will lead to some paradoxical requirements. For example, the double curvature which is preferred in the structural considerations for shell geometry is not easy or economic to prefabricate in the mass-production with forming techniques nowadays. And the continuous geometry which is familiar in the concrete shells is also hard to achieve due to the specific material properties. The modern structural typology such as the spatial framework structures is not very suitable for such materials as the prefabrication of the semi-products is also inefficient from the economic side (such building experiments have been seen in the building examples of the paper tubes by architects such as Shigeru Ban). In this sense, a suitable change of the existing structure concept and the improvements of the structure system Are necessary and meaningful for the development of the new lightweight structures.

All the discussions together will help establish the basic criteria for the design of the structure system for such materials. And from the architectural side, to develop a new structure concept and system, all these criteria will directly affect the consideration of the structural forms, or in other words, the geometrical requirements of the new structure. As a conclusion for the geometrical analysis of the structure concept in this chapter, the following geometrical requirements can be summarized as the basic starting point of the generation of the structure concept in this research:

A, Diversity of the geometry:
The primary goal of the structure concept is to make it available for various free-form geometries so as to satisfy the various design requirements from the early design process for new shell structures. As the double-curved geometry and the well-designed membrane force field is preferred from the static side, a global double curvature is also demanded.

B, Discretion of continuous curved shells:
The discretion of the shell is due to the specific industrial form of the thin sheet materials. With the sheet form in standard sizes, every part of the shell should be formed from these semi-products. Unlike the traditional shells, this requires a consideration of the connection system and also the dimensioning of the building elements.

C, Developable surface or economically customizable free-form:
The manual and mechanical forming techniques nowadays also require an easily achievable

geometry of the structural elements. According to the above chapter, it is required for manually forming the developable surfaces and for the mechanically forming a small amount of types for bent or folded geometries.

4.1.2 Design experiments: structural concept with simple hierarchy

In order to improve the existing structure typology or develop an innovational structure concept, the first step of the research and experiments is to test the possibilities of using new materials in the existing structural typology with a design of a full-scale shell. Due to the budget and the availability of materials, paperboard was selected for the experiment, and the possibilities for other thin sheet materials were also considered. The original geometry of the shell is a half-sphere form which covers an area of almost 3m*3m. In the first several tests, structures with a simple hierarchy are tested with the inspirations from grid-shells, rib-shells, etc. In cooperation with Roland Werth in a pre-study, the structures are both evaluated from the architectural side and the structural side(Werth, 2015b).

4.1.2.1 Weaving shell/ Grid-shell and its variation

The first experiment is made with the inspiration of the handicraft of weaving baskets, which are mostly made of bamboo strips and compose a self-supporting lightweight structure. The idea is to use the bent strips to compose a lattice or grid, which can transfer the membrane forces and offer the local elements a better stiffness against buckling with its curvature.
With the half-sphere form a simple concept for a pavilion is generated as in Fig. 4.2. Structural strips made from the thin paperboard together conform the shell system and are connected to each other by the joining points. The structural behavior is analog to existing grid-shells built with wood strips, such as the Multihalle in Mannheim.

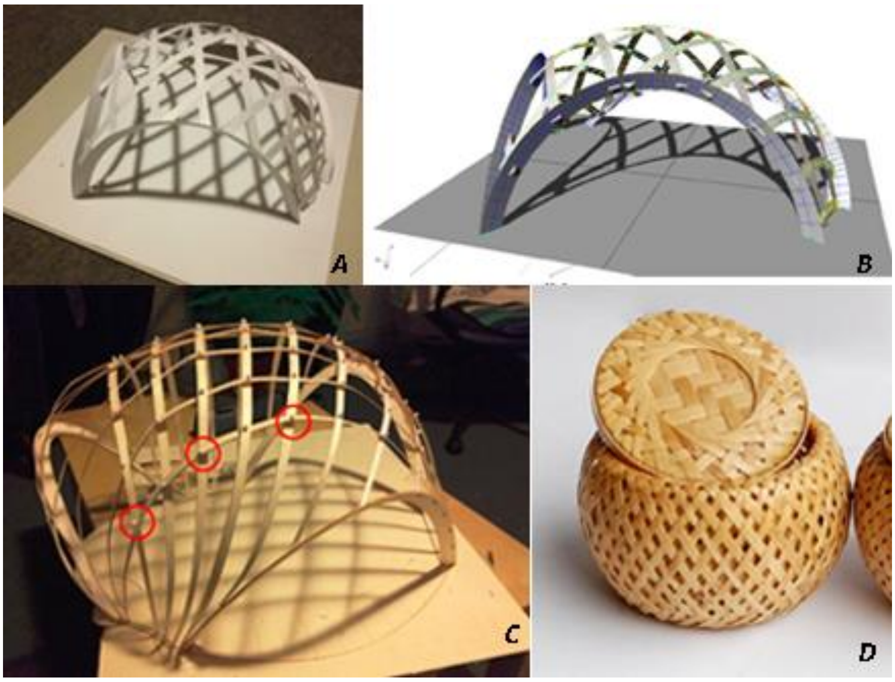


Figure 4.1. 2 Design experiments of weaving structure and form tests.

In the geometric analysis of the structure form, difficulties are found especially when the arbitrary free-form is the target of the design. As the grids of the structure should be organized along the principal curves in different directions, the double curvature in most of the free-forms will cause a deviation of the two strips that are across one point, because the bending of the planar strips can only offer a curvature in one direction. This is only suitable for structures which have a small and smooth change of curvature, then the flexibility of thin materials can offer a small bending capacity. But for the structures with some large curvature or the rapid change of curvature especially at the edge area, this will cause a large deviation which will give problems for the connecting techniques. (Figure 4.2 c)

In the handicraft artwork, such problems are mostly solved with very large bending with quite thin materials or by dividing the structures into some parts that are connected by other structure elements. (Figure 4.2 d) Or it can also be solved by decreasing the density of the grids to make large gaps between the lattices and to make the strips more like linear elements. However, both methods will cause a stability problem for the structure and will make the structural elements too weak and soft under some concentrated load or even under its self-weight. For the existing structures, the analysis of the lath size has been done by an existing study (Harris, Dickson, Kelly, & Roynon, 2004), which shows the sufficient size of cross sections should be in centimeters. (Figure 4.1.3) And this also shows that for the full scale structures this structure typology is not quite suitable of the selected materials here.

Gridshell	Span	No. of layers	Lathe Size	Material	Bracing
Mannheim	60m x 60m	4	50 x 50mm at 0.5 m.	Hemlock	Twin 6mm cables every 6th node
Japan Pavilion	72m x 35m	2	120mm dia at 1.0m.	Cardboard tube	Glulam ladders
Earth Centre	6m x 6m	2	32 x 15 at ??	Oak	Twin 6mm cables alternate nodes
Downland Gridshell	48 m x 15 m	4	50 x 35mm at 1.0 m.	Oak	Timber cladding rails, alternate nodes
Savill Garden Gridshell	90 m x 25 m	4	80 x 50 at 1.0 metre	Larch	mm plywood roof

Figure 4.1. 3 Summary of structural elements in several grid-shells (Harris et al., 2004)

In the analysis, a variation of the structure has been made to change the grid form into an interpolated grid system, where the strips are oriented almost perpendicular to each other. This helps to improve both the building process and the structural behavior of the system: as the intersection of the strips are in the form of an insertion, it is possible to use structural elements to fix the connection (screws, angle irons, etc.); and as the bent strips are flexible and sensitive to the local buckling especially when a low density of grid is applied, the change of another strip to the perpendicular direction will enhance their load-bearing behavior and resist the deformation to some extent.

With such models, two variations were analyzed with the FEM software (SOFiSTiK), where the geometry remains the same but only changes to the directions of each group of strips of 90 degrees were made. In both cases, the static analysis under the self-weight and the stability problems are carried out.

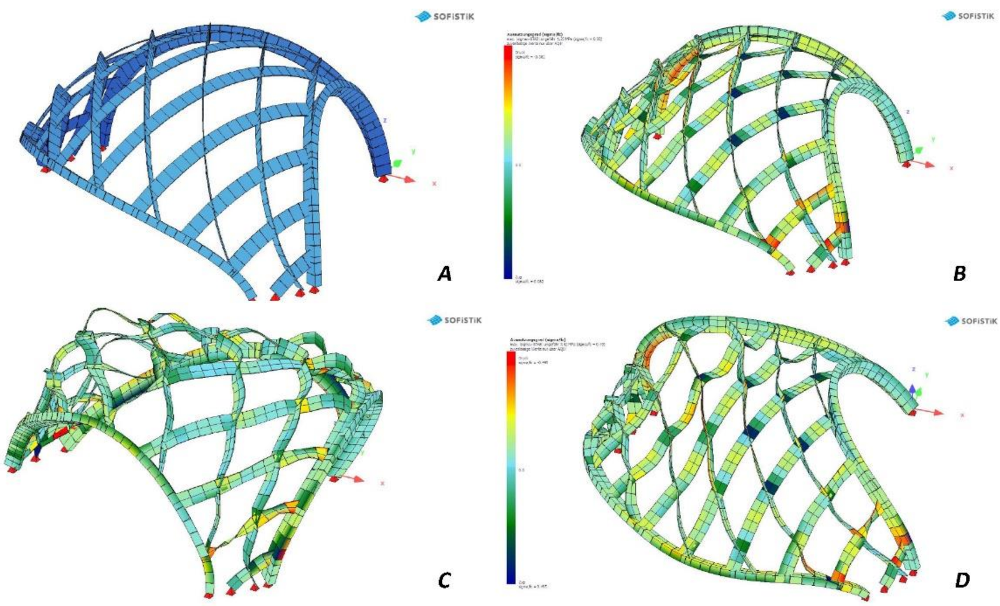


Figure 4.1. 4 FEM Analysis of Variation 1 (A, structural system; B, load case self-weight; C, 1st eigenform D, break figure by η_Ki=6.00)

According to the master thesis of Roland Werth (Werth, 2015b), by both of the static analysis under self-weight in different cases, similar results are received: the load-bearing capacity of the whole system first relies on the boundary ribs of the structure, which are deformed to a large degree due to the lateral thrust. The strips whose surfaces are parallel to the shell geometry are fragile to the vertical load and easily to deform by bending. On the contrary, the perpendicular ones help to straighten them in-between and hence resist the global deformation. (Figure 4.1.4 B and Figure 4.1.5 B) This has conformed to the assumptions.

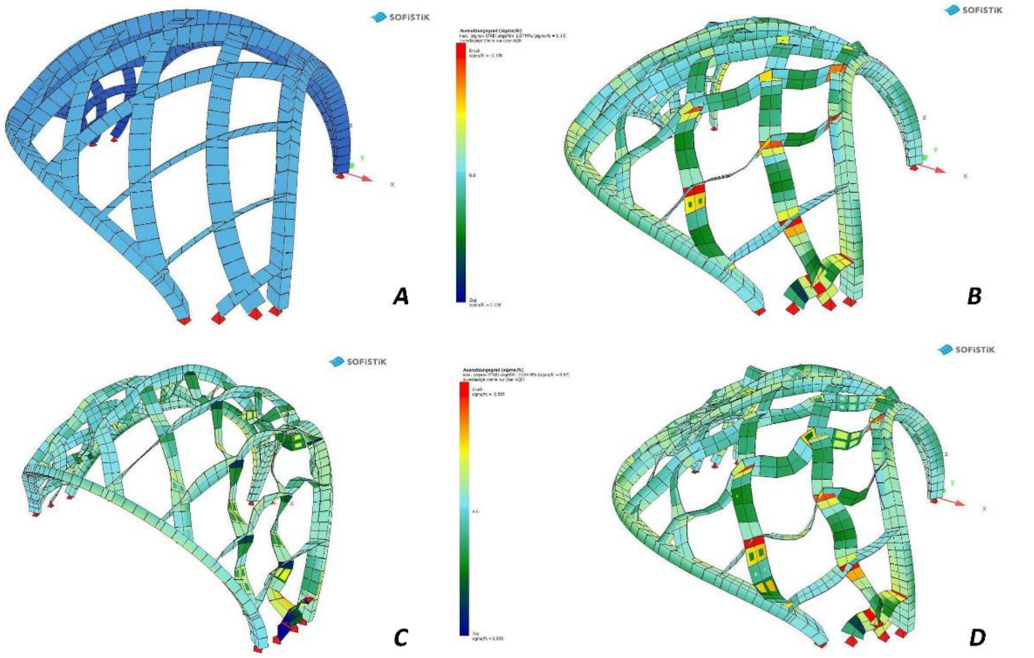


Figure 4.1. 5 FEM Analysis of Variation 2 (A, structural system; B, load case self-weight; C, 1st eigenform; D, break figure by η_Ki=6.39)

To check the stability problems of both systems, an FEM analysis has also been done with the perfect form as well as the 1st to 3rd eigenform as imperfections under self-weight and also considering the initial inclination of the structure in the x and y axis as a load iteration. Considering the difficulty to differentiate the manufacturing direction of material, the simplification of an isotropic material parameter is used. In the analysis of variation 1, all the cases show similar results and the limit load factor comes at about $\eta_{Ki} = 6.00$. For variation 2, part of the structural elements shows an instable state already at the factor $\eta_{Ki} = 3.00$, but the whole structure fails at a higher factor of about $\eta_{Ki} = 6.39\ to\ 6.46$.[2] (Figure 4.1.4 D and 4.1.5 D)

Only from the structural side, the comparison shows a better behavior by variation 2, as it offers a previous caution of the structural failure and a higher limit load. The reason can be that the shorter length of the strips perpendicular to the shell surfaces provides a better stiffness for the structure. However, if the safety factor $k_{mod} = 0.10$ is taken into consideration, both structures will fail at a load factor $\eta < 1$.

The analysis shows that the hypothetical size of the structural strips with section 150mm*4mm is not sufficient for the structure even under its self-weight, for which a minimum thickness 8mm is required. And with this result, it can be judged that such a form is also hard to be applied on a full-scale structure with thin sheet materials.

[2] For the evaluation of the testing structures, only the summary of results are used here to generate the discussion and conclusion. More details discussed in the master thesis of Roland Werth.(Werth, 2015b)

4.1.2.2 Shells with cellular frames of planar ribs

With the understanding that the perpendicular ribs will enhance the structural stiffness, the pure cellular rib shells are then tested with different patterns. (Figure 4.1.6) Compared to the geometrical restriction of the weaving forms, the rib framework shows a better capacity for its application into various arbitrary free-forms. In the half-sphere model, as there is the center point of the geometry and the form is both axisymmetric and centrosymmetric, it is obvious that the planar ribs can be modeled by projecting a mesh on the outer surface. Even in the free-form or the hanging form, the planarity of the ribs can also be derived by projecting in the z-axis or by other mesh optimizing methods. (The methods will be discussed in later parts.)

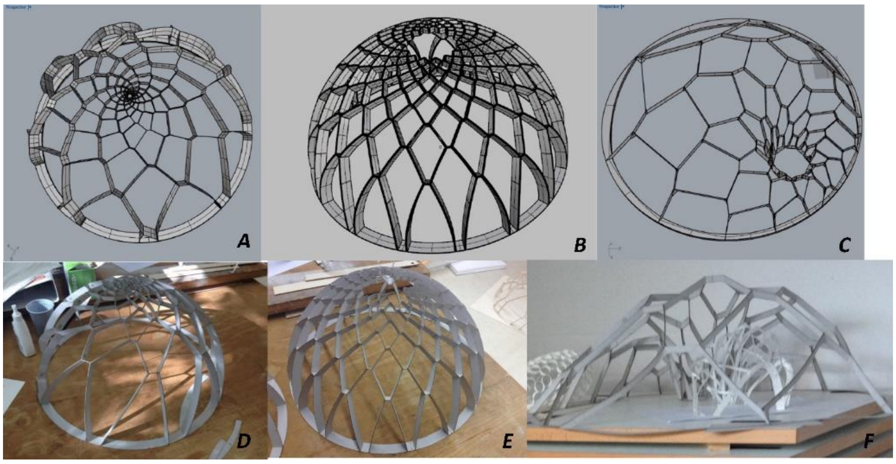


Figure 4.1. 6 Experiments with cellular ribs framework systems

Both the model in 1:8 and the static analysis of the structure show the low stability of the structure system with very thin materials. The test model with the non-uniform sizes of cellular frames causes the different length of the ribs in the whole structure, where the longer ones show the greatest possibility to deform. With the asymmetric pattern of the frames, the deformed shape of the structure also shows an irregular shape, and the large bending of the ribs at the foot areas can be obviously observed. (Figure 4.1.7 A) While in the one with regular patterns, the regular deformation of the system can be observed and the necessity to enhance the stiffness of the long ribs and also the connecting angles between ribs can be analyzed. (Figure 4.1.7 B) At the same time, such problems are also found in the real models in Figure 4.1.6 F, where the stability problems already happen with the paperboard ribs with a length of 200mm and thickness of 0.5mm.

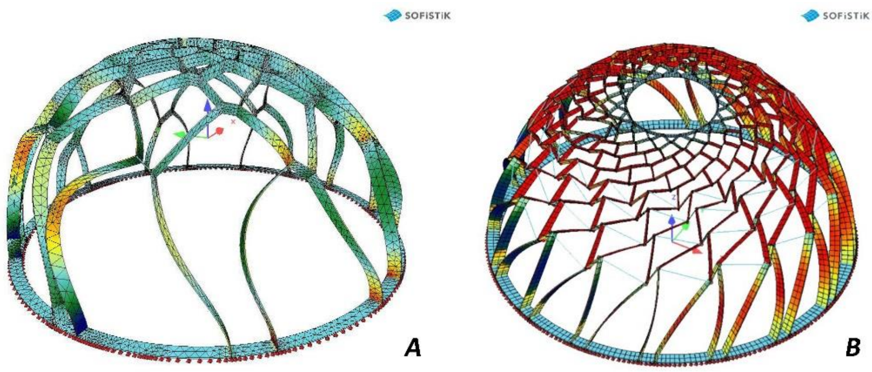


Figure 4.1. 7 1st eigenform of both cases of experiments

4.1.2.3 Curved packing rib shells

Realizing that the stability problems happen in the planar ribs which exceed a certain length, another variation of the structure form is tested. By bending the ribs of every cellular frame into a closed loop surface, the local stiffness can be enhanced by the curved geometry itself. With the basic geometry of the half-sphere, a variation is directly modified based on the geodesic dome from Buckminster Fuller. By constructing an interpolation circle in every triangle, a system can be gotten with circles connected to each other. After projecting to the center point, all ribs are generated with conical surfaces. (Figure 4.1.8 A, B)

The general geometric rule of the packing system would be similar as the circle packing theory. However, the pure circle packing is not necessary here from both the architectural and the structural aspects. This helps to simplify the problems and make it possible not to go too deep into solving the mathematical problems.

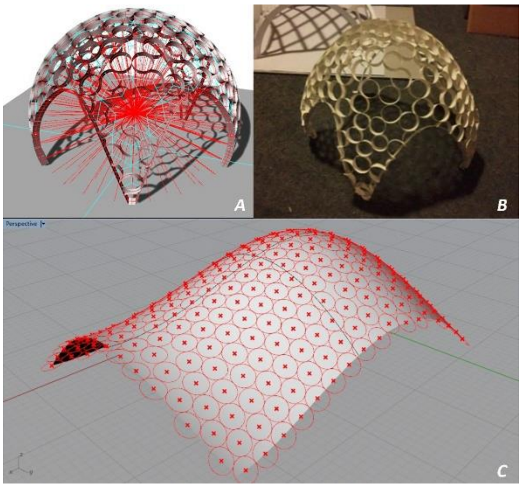


Figure 4.1. 8 Experiments and diagrams of packing rib shell

As the circle packing method also relies on the same method of the discretization of the geometry (Stephenson, 2005), the same techniques can be used in the form finding of such structures: after a reasonable subdivision of the geometry into meshes, interpolation circles/ curves can be generated in each unit of the meshes. When the meshes are regular and well-distributed, a balanced form of the packing will come out, which means no great deviations will happen at the connecting points between circles. (Figure 4.1.8 C) In architectural construction, as small tolerances are usually acceptable, this can be understood as the perfect point connection between structural elements. The developablity of the rib curves can be solved either by offseting the meshes and creating developable loft surfaces between the two curves on both meshes or simply extruding the original curve in the z direction. For experiments with arbitrary surfaces, geometrical tests are also made. (Figure 4.1.9)

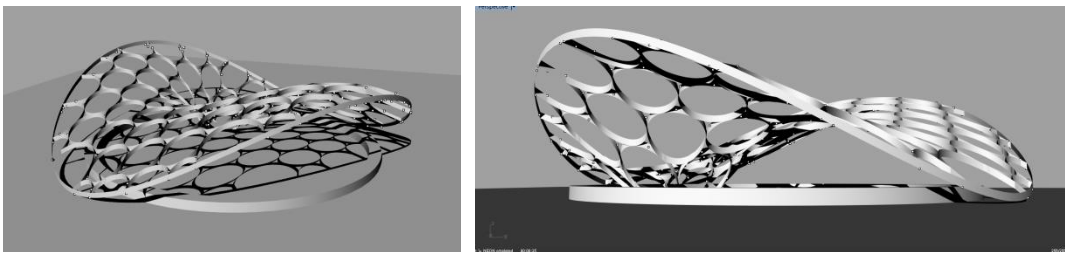


Figure 4.1. 9 Geometrical test with HP surface

Although the stiffness of the elements can be enhanced by the curved geometry, problems are observed in the prefabricating process: as the manufacturing of the structural elements relies on the bending of the planar strips, it is hard to control the perfect curved geometry with very thin sheets. Industrial customization should be required for the cylinder or conical elements and a minimal thickness is also necessary. To provide both the sufficient stability of the structure and make a feasible building process, laminating techniques can be used with customized section shapes of sheets. Such experiments have already been tested with cardboards in 2010 with pavilions designed by ETH Zurich. (Michele Leidi, 2010)

Figure 4.1. 10 Laminating technique for the curved packing structures with thin sheets

4.1.3 Design experiments: hybrid structures and cellular structures in small scales

After several tests with the existing structure typologies and their variations, the possibilities and difficulties of using thin sheet materials in such structures are argued. It can be discussed that the thin cross-sections of the structural elements will always first lead to a stability problem globally or locally. To solve such problems, extra stiffening methods should be added on the structure, such as using a profiled shape with folded edges on the long strips in the ribbed shell experiments. These methods raise a consideration to use a hybrid structure with a multi-hierarchy.

4.1.3.1 Double-layered shells with in-between ribs

The lateral bending and buckling problems of the rib systems lead to a consideration about how to stiffen the slender strips with the structural forms. By adding materials into the gaps between the frameworks the concept of a double-layered shell is suggested. The method of the discretization of the shell volume is inspired by the work of Khabazi (Khabazi, 2011), where the structural ribs have been changed into solid forms that can be defined as chambers. In the experiment such techniques are also used (Figure 4.11 a), and the generation of the surface geometry is due to the requirements of the materials improved with other algorithms. As shown in Figure 4.11 b, the surfaces are divided into a central triangle and also the side developable loft surfaces from the segments of the controlling curve. And by unrolling the developable surfaces, all the geometry of the singular structural element can be cut with one piece of material (Figure 4.12), hence such a form is a simply applicable structure concept for the prefabrication.

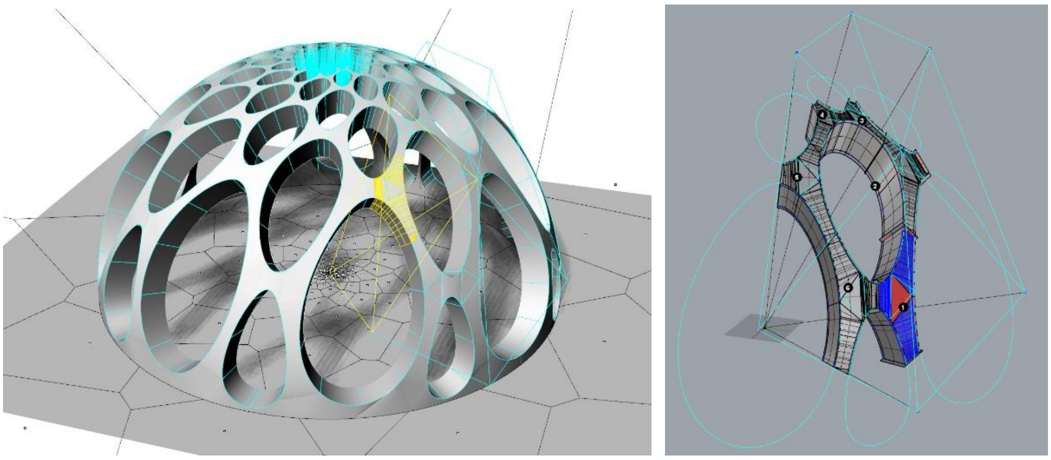


Figure 4.1. 11 Geometric rules for the experiment for the double-layered shell concept

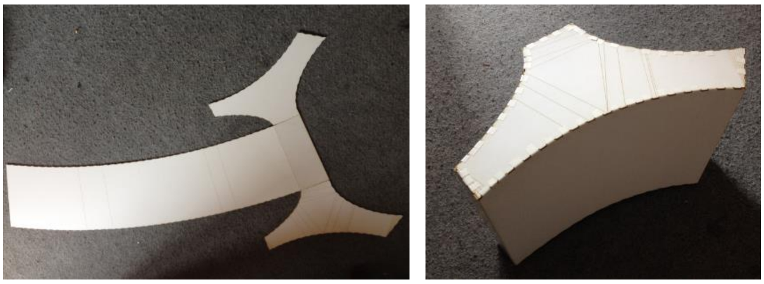


Figure 4.1. 12 Prefabrication test of the structural element

The static analysis of the structure is not carried out as a better solution with the cellular cavity structure comes and shows to be better as the hybrid system, no matter from the structural or the fabrication aspect of view. But both the geometrical capacity and the improvement of the stability can be tested and expected with both the models and the theory of the double-layered shell. (Flügge, 2013) As the inner and outer surfaces of the elements together form two continuous curved surfaces and the side walls together make the in-between supports of the two surfaces, it is similar to the classic homogenous double-layered shell. In this case, both the shell surfaces and the in-between ribs conform to a coupling system, and the structure hence gets a better stiffness. The structure generally works like a shell with the global membrane behavior and at the local side, the thin walls are better stabilized with a smaller division from the large scales. For the thin sheet materials, this can be considered as an effective improvement of the structure and also leads to the generation of the cellular cavity structures in this dissertation.

4.1.3.2 Cellular cavity structures in small scales

The cellular cavity structure leads to the final concept of the innovational structure in this research. With the experience of the double-layered shell to conform to a hybrid system, such a concept can also be applied with the tested rib shells. With the ribs to define the composition of the shell volume and with also the upper and bottom surfaces to stabilize the ribs against lateral deformation, the coupling system can work together to achieve a similar behavior as the traditional continuous double-layered shell.

As the definition of the ribs will rely on the discretization of the surfaces, such a structural concept can be applied to most of the arbitrary shapes or the shapes with the form-finding techniques. The coupling of both systems will emphasize the effects of the ribs and the covering membranes in the hybrid system. In comparison, such an effect of the covering membranes is different as in the similar spatial framework structures where only the linear trusses are considered as the structural elements and the covering panels are only used to take the loads. The division of the large scale structural volumes into small scales by cellular and cavity forms will largely decrease the weight of the structure and also make full use of the thin sheet materials.

In a design study of the geometry application of such a concept, a structure with a hyperboloid form has been divided into many small cells according to a simple and random triangulation of the surfaces. (Figure 4.13 a) The prefabrication of a 1:10 model has been tested to evaluate the difficulty to make the structural element. (Figure 4.13 b) The deeper discussion of the geometry and structural behavior of such structures will be carried out in the later sections of this chapter.

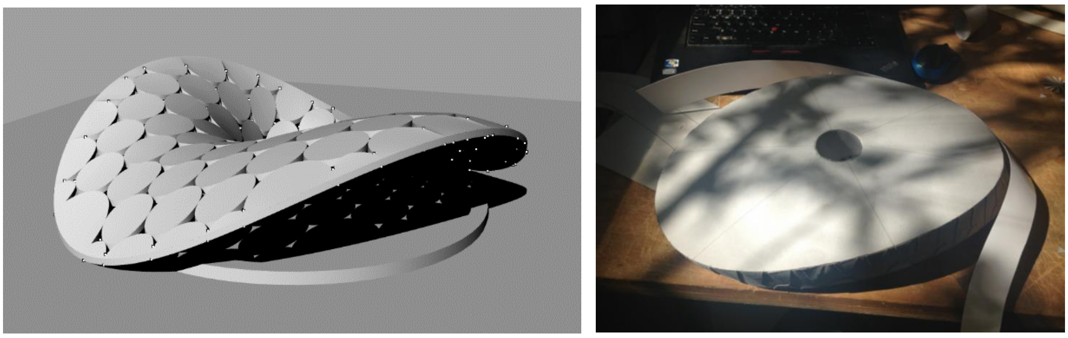


Figure 4.1. 13   Experiment of the original concept of the cellular cavity structure

4.2 Spatial tessellation and concept of cellular cavity forms

4.2.1 Spatial tessellation and the packing patterns

According to the analysis and generation of the structural system researched here, the cellular structure is first based on a discretization of a continuous shell space and the arrangement of the cell units. To provide the property that every cell can offer a bonding to its adjacent cell, which also means that a global membrane force field between cells can be created to make the system work like shells, a closed packing system is required here. In this part of the analysis, such principles as the spatial tessellation theories from the global side will be first introduced and discussed.

4.2.1.1 Diversity from modularization

The architectural requirements for any structures include the one for diversity, as there are always special requirements from the aesthetic or the functional aspects. However, this is usually paradoxical with the limited types of resources such as materials and the types of semi-products. The industrialization and the development of mass customization has resulted in a trend of standardization in structural design nowadays, where the efficiency requirements catalyze the development of the new structure typology with a minimum inventory of component types and maximum diversity of the form.

In this research, such a structure concept is a basic goal due to the above-mentioned criteria. In a fundamental way, standardization is also a principle of modularization, and such diversiform structures can be accomplished by modular systems which are more efficient in their use of the limited types of building materials. (Pearce, 1990) Therefore, the discussion of the cellular structures should first be the study of the rules for the orderly subdivision and the geometry basis of the cellular array and space.

4.2.1.2 2D Plane Tessellation—the principle

The tiling problem is the typical name of the packing problems—the plane filling arrangement of planar figures or its generalization to higher dimensions. When only some typical figures are considered, the tiling of regular a polygon (2D), polyhedron (3D), or polytope (nD) is called a tessellation. The plane tessellation is by definition a set of the infinite number of polygons which fit the whole plane area without gaps or overlaps, so that every side of each polygon belongs also to another. (Coxeter, 1973) For the shell space discussed here, the same goal is also desired for a continuous membrane force field between cells. And for the establishment of a general rule, the regular tessellation and its variations should be mentioned.

1, Regular Tessellation:

A regular tessellation of the plane is a typical form and the basic form of the 2D tessellation, which is also a pattern of uniform regular polygons filling the whole plane, where all the vertices are laid regularly on the plane.

There are only three types of the regular tessellation geometry due to the even subdivision of 360 degrees into parts. Such tessellations are only the tessellations to triangles (with 60°angles), rectangles (90°) and hexagons (120°). No more polygons with a face angle of more than 120°

will be able to form a regular tessellation.

The diagrams for such tessellations are presented in Figure 4.14 a-c. (Grünbaum & Shephard, 2013)

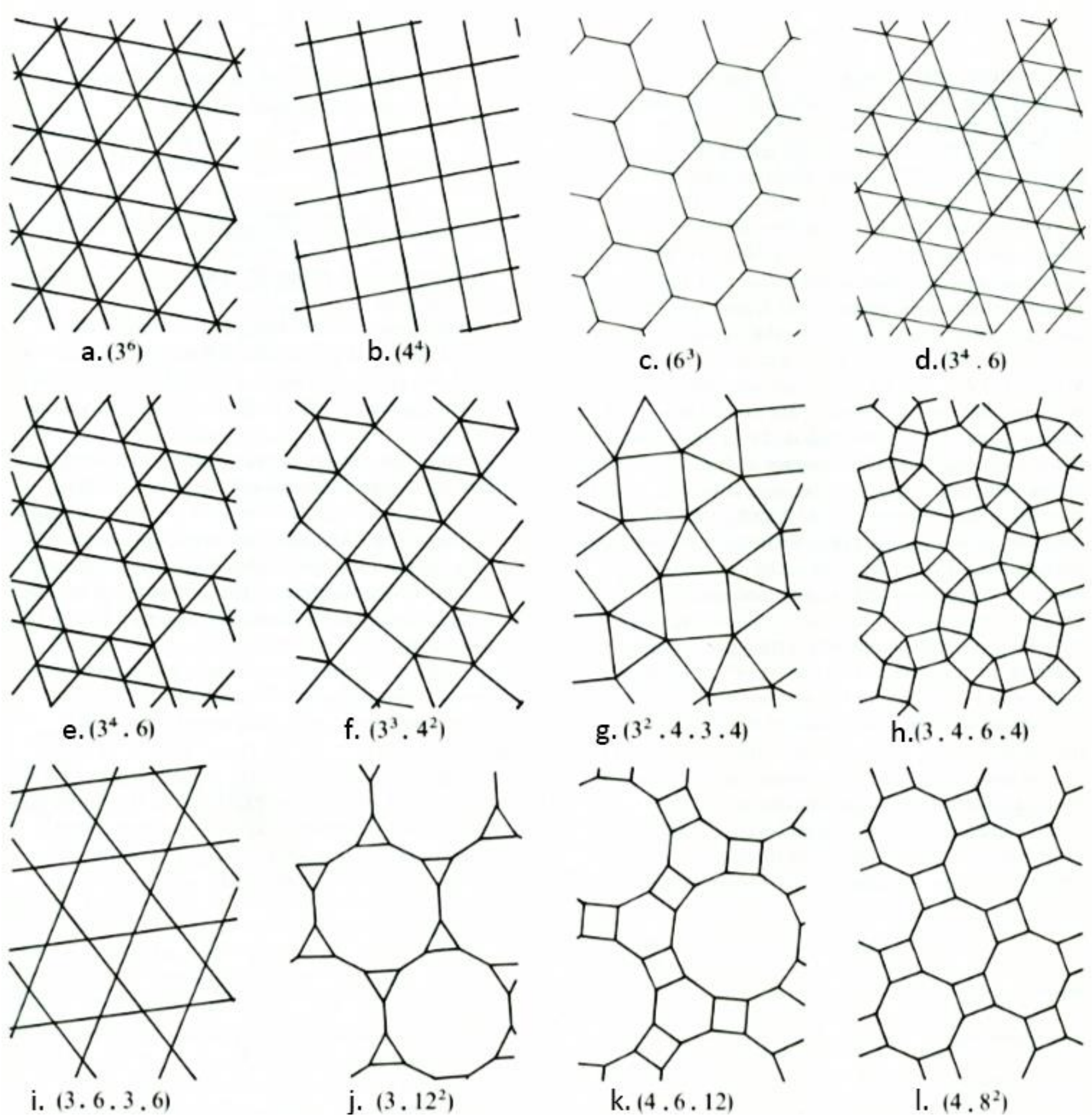


Figure 4.2. 1 Diagram of Regular Tessellation

2, Semi-regular Tessellation:

Regular tessellations of the plane by two or more convex regular polygons so that all the vertices are surrounded by the same type and order of polygons are called semi-regular tessellations, or sometimes Archimedean tessellations. This class requires all the polygons to be regular and the order of the adjacent polygons remain the same for all the vertices.
In the plane, as 360°can be matched by the sum of different kinds of face angles, there are eight such tessellations, illustrated by Figure 4.14 d-l. (Grünbaum & Shephard, 2013)

3, Demi-regular Tessellation (2-Uniform Tessellation):

Tessellations with regular polygons are called to be k-uniform if there are precisely k different kinds of vertices in the tessellation. It can be observed that there is only 1 type of vertex in all the regular and semi-regular tessellations. When some of the conditions of the limitation can be relaxed for the cases, a new range for the tessellations can be found.

The demi-regular tessellation, also called polymorphal tessellation, is used to describe such a kind of tessellation. In mathematics, no specific definition has been given to the demi-regular tessellations but the 2-iniform tessellation can be considered such tessellations in this case. According to the mathematician Otto Krotenheerdt, (Grünbaum & Shephard, 2013) 20 such tessellations can be enumerated with the illustrations in Figure 4.16. (Weisstein)

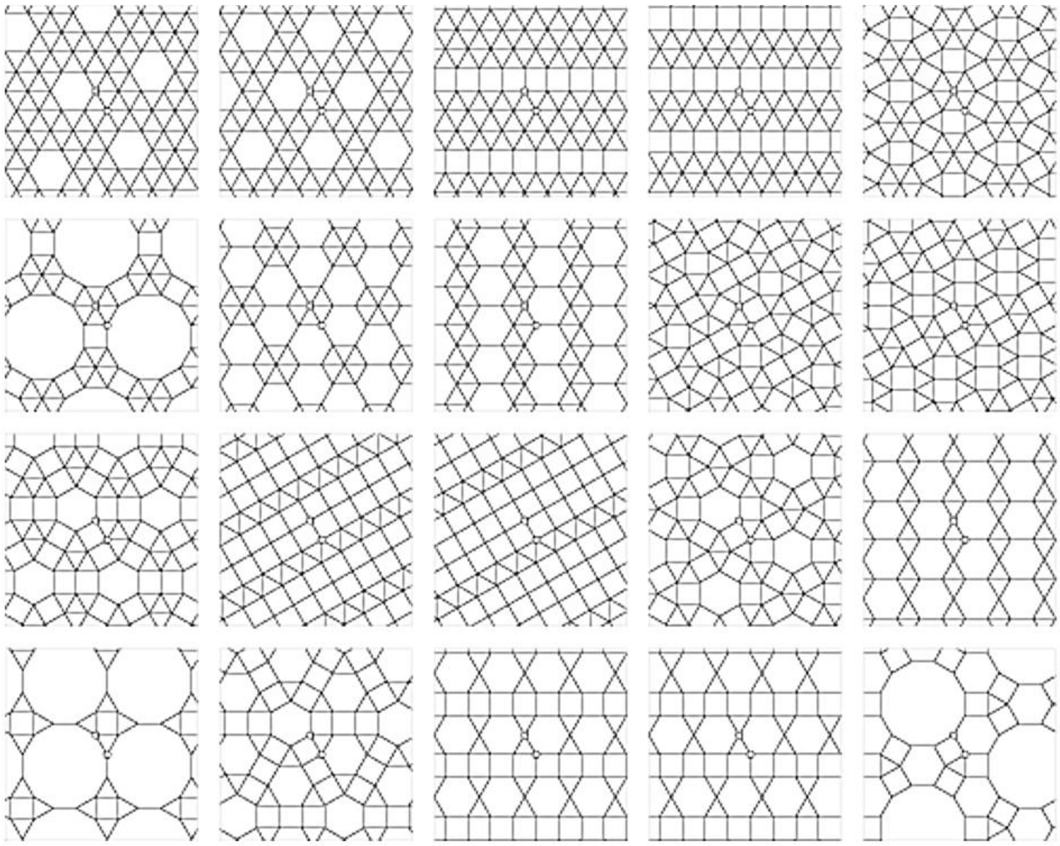


Figure 4.2. 2 Diagram of 2-Uniform Tessellation

4, 3 and higher-Uniform Tessellation:

When the number k goes higher than 2, the figure of the tessellation becomes more complicated. The diagrams will not be given out in this dissertation.

According to the research by Otto Krotenheerdt, the following table will show the possibilities for each case of such tessellations. (Grünbaum & Shephard, 2013)

Value of k	Types of tessellation	Value of k	Types of tessellation
1	11(Archimedean)	5	15
2	20	6	10
3	39	7	7
4	33	8	No tessellation

Table 4.1: 3 and higher-Uniform tessellations

5, Symmetry and transitivity of tessellations:

Besides the method to create regular tessellations as above, the property of the symmetry and transitivity of tessellations leads to the methods of constructing irregular tessellations from the mutation of the regular tessellations. Four isometric transitions (Figure 4.18a a, rotation; b, translation; c, reflection and d, glide-reflection) can be applied to the regular tessellations. Starting from every basic form of the regular tessellation (triangle, rectangle and hexagon), new forms of tessellations can be derived by using a symmetrical transformation. At the same time, with the operations to transform the edges of the tessellation, more forms can be generated with different transitivity classes. (Figure 4.18 b-d)

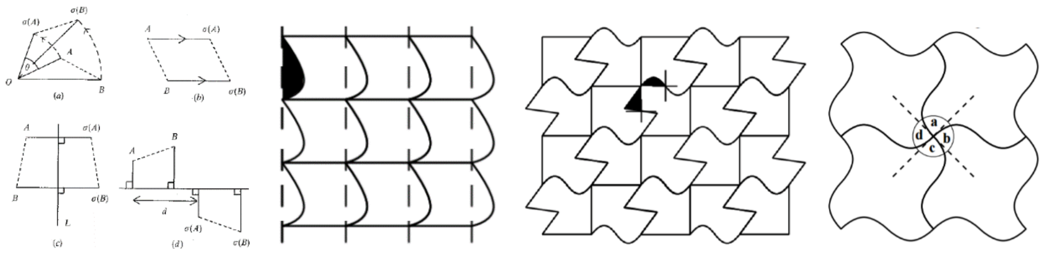


Figure 4.2. 3 Symmetry of tessellations and the construction of irregular tessellations

Such geometrical techniques are a useful tool for both architects and artists to create novel art forms or structure forms. M.C.Escher (1898-1972) is well known as a famous graphic artist with his pioneer work and research into using the geometric transformation to create impressive art works of tessellations. His famous works have also great impacts in the area of architecture design.

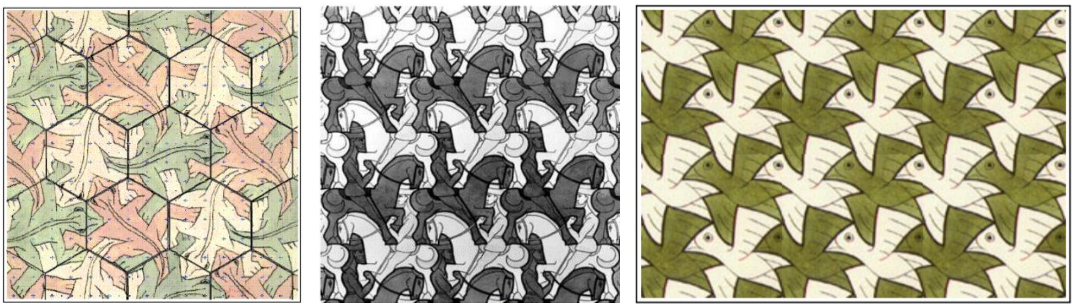


Figure 4.2. 4 M.C. Escher's symmetric tessellation drawing

6, Duality and valence of tessellations

Duality is one of the important concepts in the graph theory, where the dual graph of a plane graph *G* is a graph that has a vertex for each face of *G*. (Figure 4.20) The duality of the tessellation is often used as a tool to analyze the connectivity of the adjacent cells and also the distribution of the cells. As it can be drawn and observed, the tessellation of the congruent triangles and the tessellations with equilateral hexagons are also a dual diagram to each other. The duality is well used in many areas nowadays such as geography, computer vision and also computational geometry. One of the best applications of the duality of tessellations is the reciprocal property between the Delaunay triangulation and the Voronoi Tessellation. Such properties are also used in the following analysis and will be discussed more.

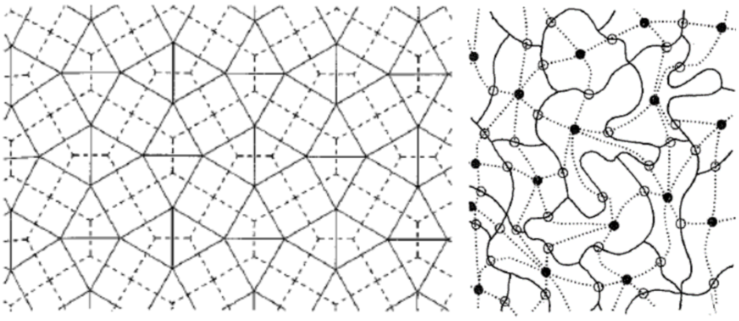


Figure 4.2. 5 Duality of plane tessellations

As it can be observed in the semi-regular and the 2 or more uniform tessellations, the space is often divided by many types of the basic units of shapes, such as the combination of triangles, rectangles and polygons. And in most of the cases, the arrangement of the basic units shows a diversity that different numbers or orders of polygons can be found at different vertices. The valence of tessellations is also a way to analyze the order of a tessellation on each vertex and also the order of its dual tessellation. It refers to the degree of freedom of the mesh elements. For the open mesh with a closed boundary, it is regular when all the non-boundary vertices have the same valence. In Figure 4.20 a, the original mesh is regular with 5-valence and the dual mesh is irregular with 3 and 4-valence vertices.

7, Triangulation and quadrangulation of manifolds:

After understanding the properties and principles of tiling a plane, it is necessary to know how to tessellate an arbitrarily formed 2-manifold with curvature.

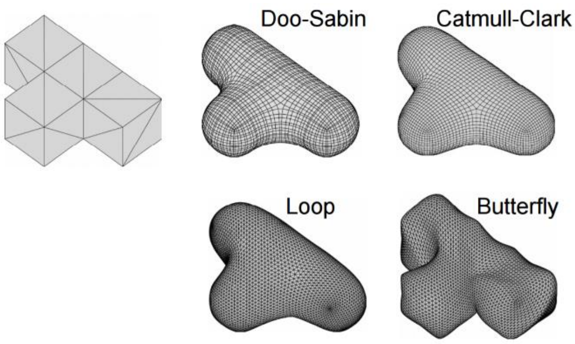


Figure 4.2. 6 Subdivision of curved surfaces

In computational graphics, the discretion of an arbitrary curved surface is finished with the subdivision of the surfaces. Like the regular tessellation with the triangles and rectangles, similar subdivision schemes can be applied to refine the structure of the meshes without changing their original topology. (Figure 4.21)

Another most important kind of triangulation in the structural morphology is the Delaunay triangulation and its dual Voronoi diagram. In mathematics, the Voronoi diagram is a partitioning of a plane into regions based on the distance to points in a specific subset of the plane. For every such seed point, a region can be enclosed that consists of all the points which are closer to this seed point than any other points, and also the nearest area can be found as the Voronoi cells. (Figure 4.22) The Voronoi diagram and its dual triangulation is nowadays widely used in many fields, especially for the analysis of the cellular structure. In biology or material engineering, such diagrams are both the important geometrical tool to understand the physical constraints that drive the organization of cell systems.

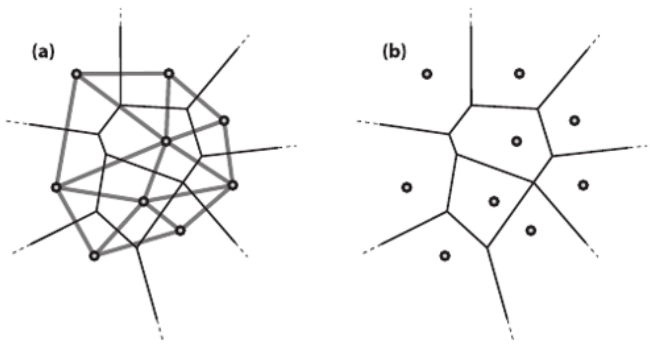


Figure 4.2. 7 Delaunay Triangulation and Voronoi Tessellation

4.2.2 Packing in nature and the morphological root of the cellular cavity structure

Other than the analysis or understandings from the theory in geometry, the cellular cavity structures in this research has also gotten its original inspiration from the natural forms of the closest packing in nature. From the natural structures of metal and crystals, to the micro structure of plant cells or biological tissues, such structural forms can be easily and frequently observed as the most efficient structure typology. In this part, such considerations and inspirations will also be mentioned.

4.2.2.1 Low energy closest packing and hexagonal tessellation:

Nature has its unique preference of structures with low energy costs. Already since the 1940s, such rules have been considered and researched with “cell aggregation” of bubbles and tissues. (Thompson & Bonner, 2014) The concept of the surface energy shows the way of the formation of the minimal surfaces between and proportional to the equilibrium state of the adjacent cells. For any isolated cell, it is well accepted and easily understandable that the ideal prototype of its form is the sphere. And when several similar cells are packed together, the same volumes and materials will result in a homogenous division of the 360° by the connecting point, hence it will also lead the form directly to a regular tessellation. If only one layer of cells is considered, it can be commonly observed that the hexagonal tessellation is the most popular of such forms in nature. (Figure 4.23 a) To explain this, the static rules will also be explained in the following parts.

Apart from the efficiency rules of growing and the structural stability, there are also reasons of material efficiency for the dominance of the hexagonal tessellation. As there are only 3 types of regular tessellation, it can also be simply testified that the hexagon tessellation is the one with fewest materials for the same area of space. The most common example for the natural structure which is made not by human beings is the honeycomb, where the hexagon form helps to store the greatest amount of honey with the least amount of beeswax. (Figure 4.23 b) (Tóth, 2014)

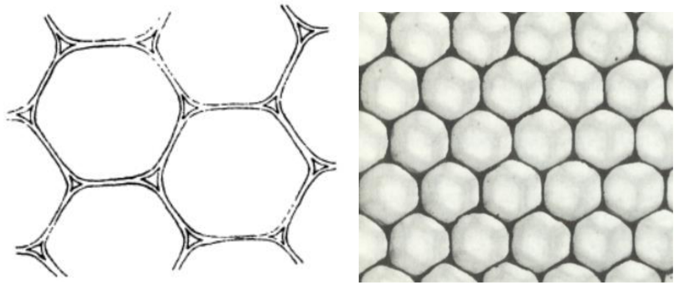


Figure 4.23 Hexagonal tessellations in nature

4.2.2.2 Stability and the dual of 3-valence tessellation:

The best way to explain the phenomenon that only the hexagonal tessellations can be most commonly seen in nature is from the stability analysis. As described also since the 1940s (Thompson & Bonner, 2014), the case of a cellular complex with four cells is discussed. (Figure 4.24 a) As presented in diagram A with the rectangular tessellation, the state of equilibrium of the cells is only instant and follows rapidly with the gliding of cells into diagram B. At the same time, if a detailed observation is made for the conjunction of the cells, it can also be found that a pure 4-valence type of tessellation doesn't exist. Either the "polar furrow" as shown in Figure 4.24 b-B or the "Polgrübchen" as the little separated space of the conjunction in Figure 4.24 b-C will be grown as a solution of the instability of the system.

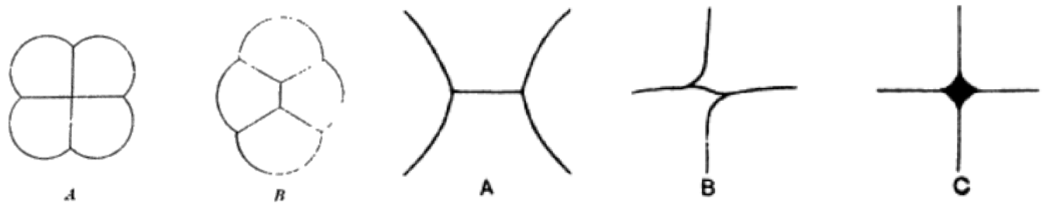


Figure 4.24 Stability difference in natural tessellation and the growth of the conjunctions

This can be explained by the duality of the tessellation. In structural engineering, the triangular system is considered as statically determinate and also stable for the truss system or the similar linear system. And when the cell volumes can be considered as a stable point which is connected by the connecting lines (through the contact of cells with adjacent walls), the equilibrium state of the cellular complex can also be generated with the dual diagram of the tessellation.
In this way, the 3-valence tessellation is more valuable to be considered in the analysis of natural tessellations and the hexagonal tessellation can also be considered as a perfect form of the homogenous 3-valence. In fact, if more observations are done with more natural cell forms, it can be seen that the 3-valence tessellation is actually the adequate way for a description, as not every cell will have the perfect congruent form so that in many cases pentagons and also polygons will be often observed. (Figure 4.25 a-e) At the same time, the transformation of the 3-valence tessellation with cell walls to the triangle-based linear system can also be found in other research. (Figure 4.25 f)

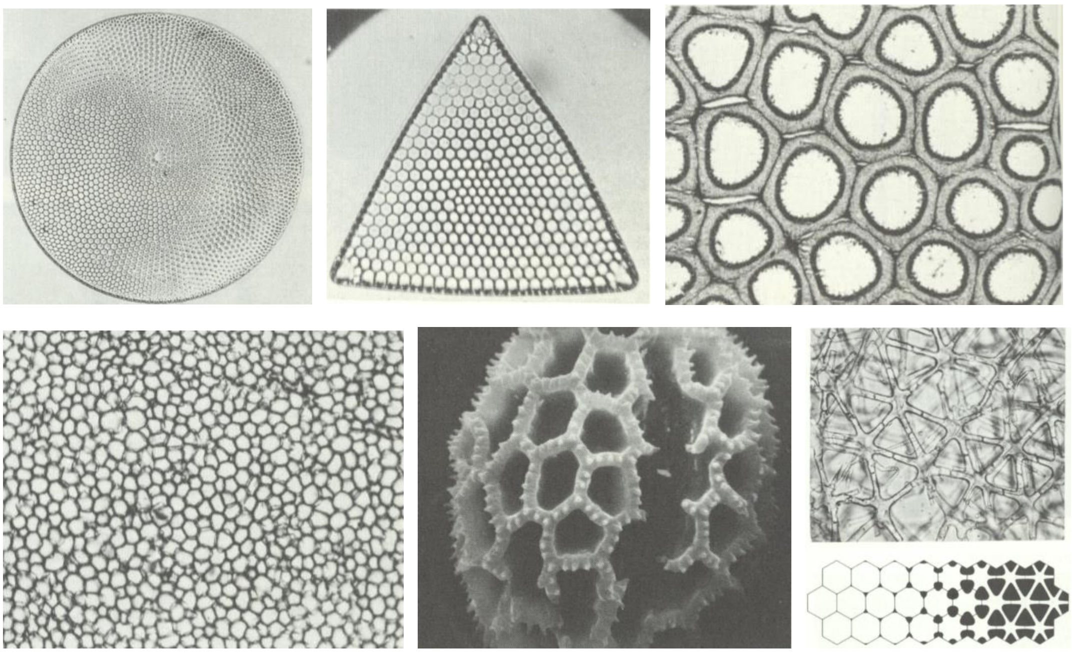


Figure 4.2. 8 3-Valence tessellation forms and their duality to triangulations in nature (a, a circular diatom (400X), b, a triangular diatom (300X), c, underside of a mushroom (30X), d, cork cells (35X), e, thrift pollen (400X) f, pith cells of the rush and the diagram showing the transformation of the hexagon tessellation (200X) copyright: a-d and f: (Jírovec, Fiala, & Bouček, 1959) e: Patrick Echilin and Cambridge Scientific Instruments)

4.2.2.3 Hierarchy and the concept of smaller cellular structures with relatively thinner materials：

When a large amount of cases are studied, it is realized that the cellular packing system is independent of the absolute dimension and size of the cells. And according to that, it is also possible to generate the idea of using such structure concepts in the multi-hierarchy structure system. In nature, the growth sequence causes the different hierarchies of structures such as the strong "veins" or ribs and the in-between cells which follow the rule of the packing geometry. In recent research, similar structures were also used in the design of lightweight plate-based wood structures. (La Magna et al., 2012)

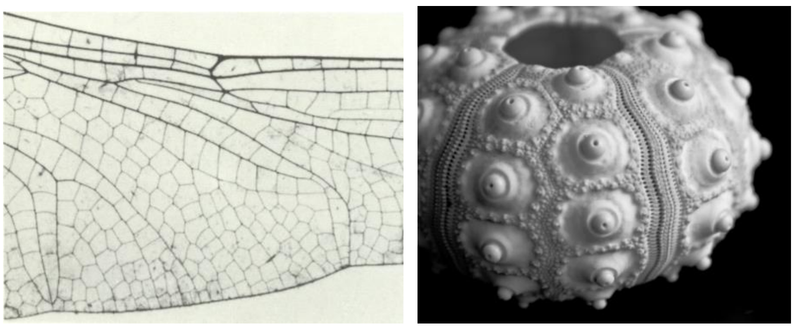


Figure 4.2. 9 Cellular tessellations in multi-hierarchical systems (a, Dragonfly wing b, Sand dollar shell)

4.3 Geometry of cellular cavity structures and the form finding methods

4.3.1 General analysis of the structure components and their properties

A full geometrical analysis of the form finding for the structure concept is challenging and also the main objective of the morphological research. And for the architectural design of shells, the design procedure should also be taken care of so that the innovational structure concept can not only provide a reasonable structure, but also provide a diversity in the structural forms to enable its capacity for different designs which can be started either from architectural or structural considerations. In the general analysis of the structure components, a definition of the elements for the structural concept will be first made. Based on such definitions, the diversity of structures and the principles of design should be discussed in detail.

4.3.1.1 Single layer close packing (thin shell):

As described above in the discussions about the natural tessellation, simple and regular tessellations are often considered as single-layered in the 2D plane in most of the cases. When it comes to the multiple layered packing systems, complicated forms such as the polyhedral tessellations will need to be discussed with the complex spatial tessellation. As it is the aim of this research to generate a suitable cellular building method for shell structures, such complicated structure behavior will not be considered. In this sense, the first definition of the cellular packing for the shells is the single-layered close packing, which means the thin shell volume should be packed with structure cells according to some pattern and sequence to fill in the space without extra gaps or overlapping.

4.3.1.2 Definition of the geometry components:

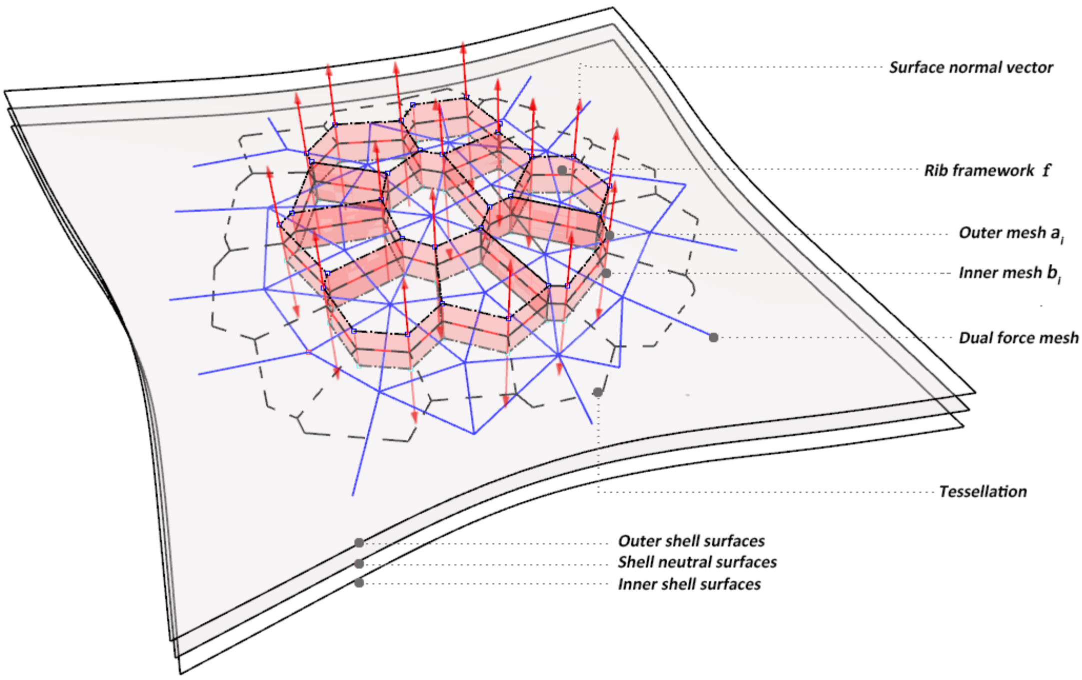


Figure 4.3. 1 Diagram of the cellular tessellation and its geometric components

As shown in Figure 4.27, the basic definition of the structure system can be presented with the diagram. For a shell with a designed neutral surface corresponding to a certain membrane force field, an offset of the neutral surface can be made to define the thickness of the shell space with both the upper and bottom surfaces. With a suitable and reasonable tessellation of the space, a single-layered packing system with structural cells can be generated. The close tessellation guarantees a single contact surface between every two adjacent cells. In this way, the first structural components which define the global arrangement of the cells will be the in-between ribs f_{ij}, which is defined by the corresponding vertices a_i, a_j, b_i and b_j from the upper and bottom meshes a and b. As every segment of the rib system is only defined by the four corner vertices, a simple quadrilateral geometry can also be considered as the prototype of forms for such structural components.

With the concept of a multi-hierarchical structure with a coupling system of ribs and covering membranes, the closed cavity system is the second generated structural form. With two series of covering surfaces S based on the controlling vertices on each mesh, complicated forms can be made to close the cell and also to stabilize the ribs against the lateral deformation. (Shown in Figure 4.28) In the traditional industrial and architectural projects, it is due to economic reasons that the planar surfaces are favored in such forms. But as in different tessellation geometries the number of the controlling vertices varies in each cell, it is sometimes not possible to provide all the covering membranes to be planar surfaces. In this research, more variations of the geometry are compared as prototypes for the design of the cellular cavity systems and their advantages and disadvantages in form finding, structural performance and also the fabrications are discussed. In structural experiments of lightweight structures nowadays, more forms derived from the combination of planar elements are also starting to be considered and tested. (La Magna et al., 2012)

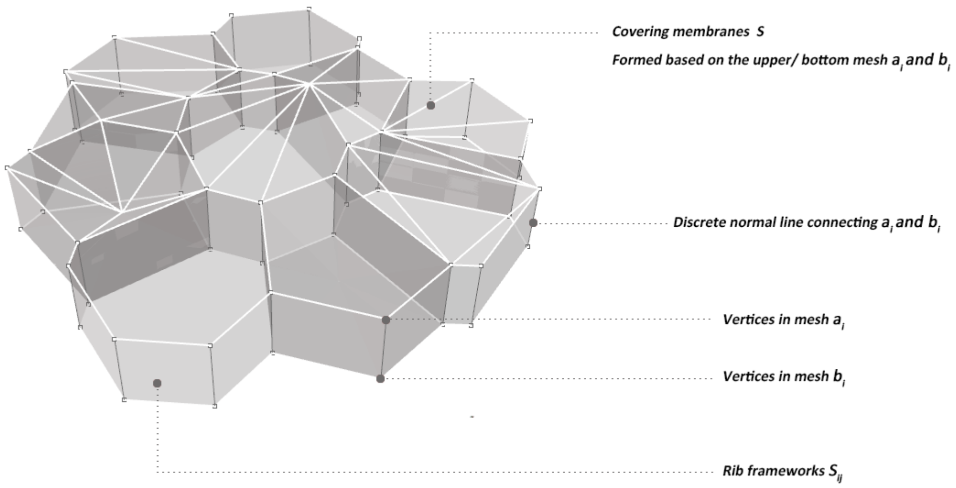


Figure 4.3. 2 Diagram of the full cellular cavity system

4.3.1.3 Procedure of the structural design and the considerations:

With the discussion and the definition of each structural component of the cellular cavity system, the requirements and the goals of the form finding process for each part of the structure is explained.

Before it comes to the detailed discussion of the form finding techniques, the work flow or the procedure of such designs shall be generalized with the following diagrams. (Figure 4.29) As the structure can be considered as the composition of two parts, the rib framework and the covering membranes, the form finding of the structure can also be summarized as a linear sequence with the solution of the two parts. And in each part of the form finding analysis, different technologies of the operations on meshes and the surfaces should be applied.

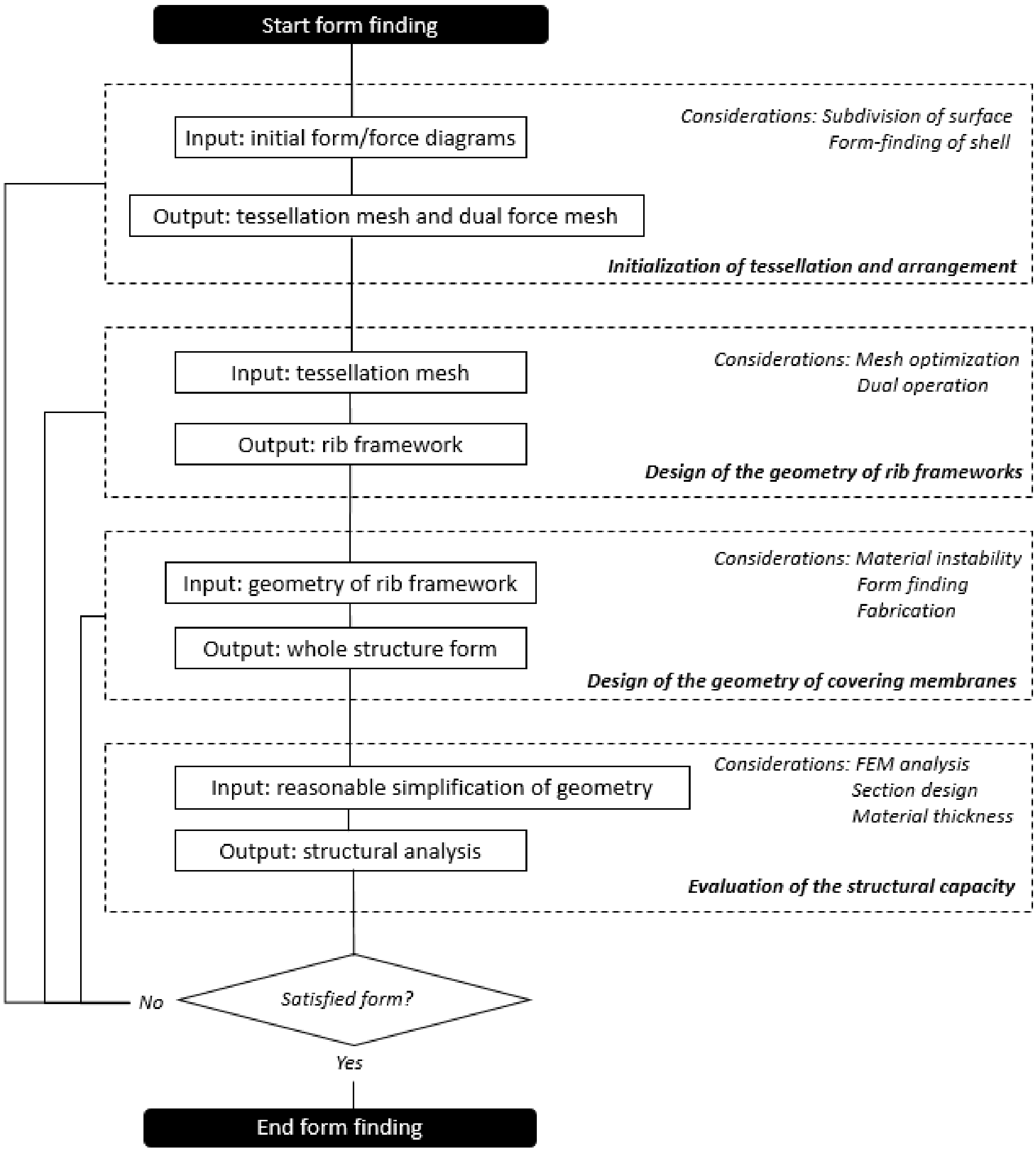


Figure 4.3. 3 Abstract of design procedure of the cellular cavity structure

1, For both architectural and engineering designs of shell structures, the complicated curved geometry is usually started with simple logics with the original diagrams as initialization. In this process, the structural form can be generated from either an arbitrary curved shape or a planar truss system for the hanging networks. The jobs and requirements of this process should be mainly the establishment of the logic of form finding and the discretion of the structure, which enable the subsequent form findings of the sub-system.

2, The generation of the basic tessellation form and also the planar rib frameworks can be seen as the first task of the form finding process, in which both the subdivision of the surface and the basic form of the membrane force field will be determined. For the economical building techniques with thin sheet materials, the meshes should be optimized to require additional properties, such as the planarity of the rib frameworks.

3, With a suitable arrangement of the cells, the geometry of the covering membranes should then be determined. With multiple variations the possible forms should be evaluated from many aspects such as the difficulty of the form finding and the fabricating process, and also its effects on solving the instability problems of the thin rib frameworks. With a comprehensive decision making process, the final form of the design can be determined.

4, In the preliminary design, the structural behavior of the whole system is only analyzed with a very abstract simplification, and the local problem of stability is still uncertain. For a final evaluation of the structure, a full structural analysis is still required and this will also possibly bring some changes for the previous processes.

The above-mentioned work flow for the form finding process of the structural concept works as a basic framework of the designing procedure. More details will be discussed and explained in the following parts according to the sequence of this framework.

4.3.2 Form finding of the tessellation and the supporting ribs

In this section, several methods of designs will be introduced to generate the geometry of the initial tessellation diagrams based on the possible design requirements of shell structures. Regular subdivision and tessellation forms will be discussed and the method of steering form and force with the duality of graphs will be explained. After getting an initial diagram of the global arrangement of cells with meshes, more operations and optimization methods will be shown to create the supporting rib frameworks with only planar surfaces.

4.3.2.1 Generating initial discrete diagrams

As shown in Figure 4.30, the starting process will be expanded as detailed operations in the form finding progress. In this procedure, both directions can be started from, either an architectural oriented form optimization from an arbitrary form or designed curve geometry which satisfies some special functional requirements, or a force-based diagram which often starts from the generation of a 2D network and its 3D transformation. In both work flows, similar techniques such as the subdivision of surfaces, and the form finding techniques can be applied.

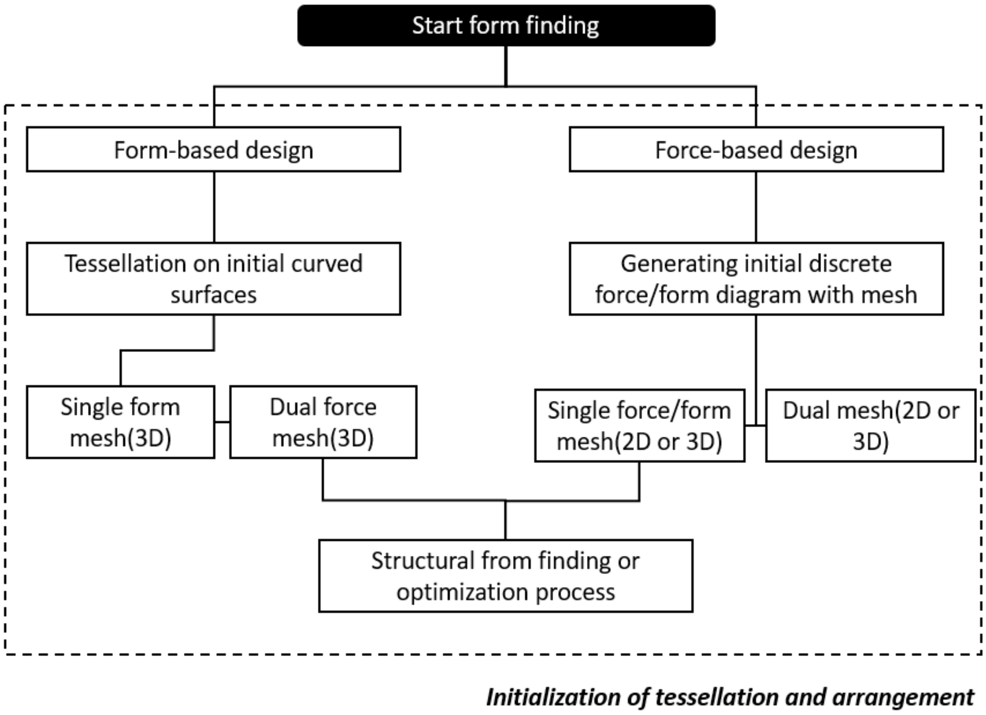


Figure 4.3. 4 Initialization of tessellation and the force diagrams in 3D

1, Defining the boundary conditions and the initial tessellation:

As the calculation of the form finding process is essentially the solution of the equation system which describes the equilibrium state of a system consisting of both the mass points and the connecting springs, it is an important process that the designers set up the basic boundary conditions of the system. In this process, a careful design should be made to define the distribution of the discrete mass points and also their connectivity, especially the distinct points which have special support conditions, such as the supporting points of the whole structure, the special opening edges and so on. At the same time, a reasonable subdivision of the system, the tessellation pattern, should be determined. According to the previous discussion about the methods of tessellation, the suitable types here will be the triangle and rectangle based regular tessellations and also their dual 3-valence and 4-valence tessellations.

2, Typical tessellation as form diagram and its dual force diagram:

In the tessellation process, not only the form diagrams which illustrate the arrangement of the cells but also the dual force diagrams which show the network of the internal forces should be taken care of. As shown in Figure 4.31, diagram A shows that the regular triangular tessellation has the dual diagram of a regular hexagonal tessellation, and diagram B shows that an irregular triangular tessellation will have a relatively complicated 3-valence tessellation, where the form of the cells is not always a hexagon. And in diagram C, it can be seen that the regular quadrilateral tessellation will have a similar dual quadrilateral tessellation. But if a triangulation is made from a quadrangulation such as in diagram D, a more complicated 3-valence tessellation will be generated. This duality of tessellation will also enable it to simplify the internal forces between cells in the tessellation system. If a cell of a tessellation is considered as a point in the polygon center and a connecting spring exists when two cells are adjacent, the force diagram can be derived by the dual operation of the form diagram. This operation and its application in the traditional analysis of the masonry shells is well explained in the Ph.D. dissertation by Block. (P. P. C. V. Block, 2009) In this research, similar techniques will be used to analyze the form of the cellular tessellation and the simplified global internal force networks.

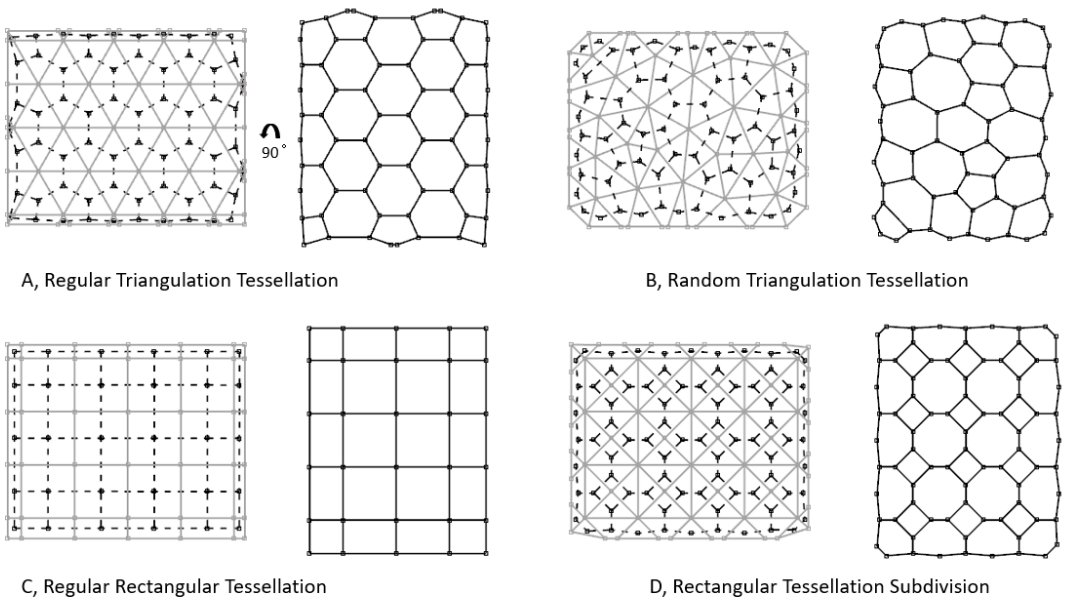


Figure 4.3. 5 Typical tessellations and their dual diagram

3, Subdivision from quadrangulation:

Any reasonable tessellation of the planar surface or the curved manifold should start from the basic triangulation or quadrangulation of the initial geometry, and if the relationship of the form and force diagram can be reversed, a random 3-valence diagram can also be gotten as the form diagram of the cellular structure from generating the random triangulation of the force diagram.

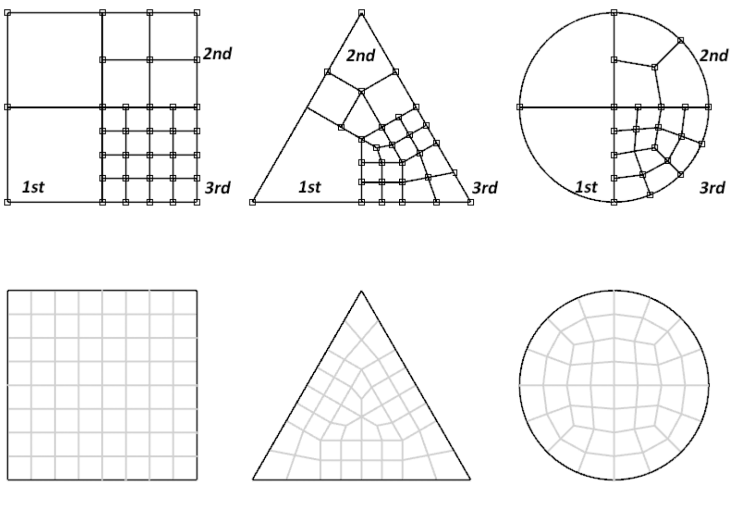


Figure 4.3. 6 Diagram of the Catmull-Clark on typical geometries

There are many meshing methods with algorithms and also plugins in design software nowadays, which help to fulfill an automated or a semi-automated process to generate the quadrilateral based tessellation of the designed surfaces. Among them most of the methods have used the subdivision schemes in computer graphics. The classic schemes such as the approximating method of Catmull-Clark (Catmull & Clark, 1978), the interpolating method of Kobbelt (Kobbelt, 1996) and the dual approximating method of the Doo-Sabin(Doo & Sabin, 1978) can be used directly on the subdivision of the surfaces. In design software for architecture nowadays, such tools are available to be used as plug-ins for certain software, such as Weaverbird in Grasshopper™. (Piacentino)

The subdivision methods for the surfaces require an initial division of the surface and work by subdividing the surface with iterations. As shown in Figure 4.32, for some initial divisions which have both quasi-triangular and quasi-quadrilateral patches, several iterations will be required to find a pure quadrilateral subdivision.

A fully-automated quadrilateral subdivision for an arbitrary surface with planar or curved forms starts with the topology of the geometry and often results in a relatively homogenous subdivision. (Figure 4.33 a) It is also possible to add some actively controlled subdivision procedures by the designer with the initial division of the surface, and this will enable the designers to get a desired network according to the design requirements. (Figure 4.33 b,c) With pre-defined initial triangular and quadrilateral patches of the surface, the subdividing process starts with all the patches and will get a pure quadrangulation after only a few iterations. However, such techniques should be cared for by the designer, as an asymmetrical and unevenly distributed tessellation will often be created when the shape and size of the initial subdivision is defined with large differences.

It should be mentioned also that similar quadrangulation methods can also be used in the 2-manifolds and also the pre-defined subdivision can be applied to control the tessellation. (Figure 4.33 d,e).

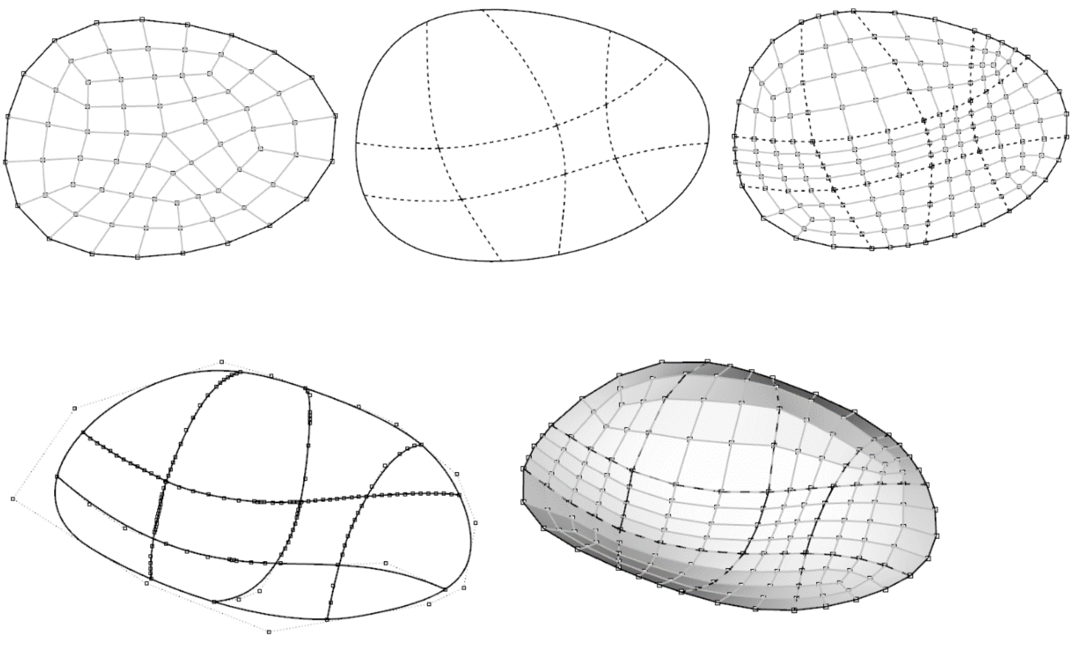


Figure 4.3. 7 Subdivision from quadrangulation

4, Subdivision from triangulation:

The most common way of making a triangulation is from the subdivision with quadrangulation, where the quadrilateral is divided into two triangles by adding a diagonal into the geometry. (Figure 4.34 a) As this would possibly result in a directed arrangement of triangles, sometimes both of the diagonals will be connected so that every quadrilateral will be divided into 4 smaller triangles. (Figure 4.34 b) However, such subdivisions from the original quadrangulation will always result in a large difference of the edge length in the triangulations. And also from the above-mentioned duality of such tessellations, the dual diagram will often consist of two types of forms which are not of similar sizes.

Another mostly used technique in meshing will be the Delaunay triangulation, where the distributed vertices will be constructed into some triangles. (Figure 4.34 c) The Delaunay triangulation algorithm will enable a maximization of the minimum angle of all the triangles in the system. By interpolating new points into the system iteratively, the average length of the system can be also controlled and relaxed in a desired range and hence a relatively homogenous triangulation can be obtained. In this research, the drawing is made with the existing algorithm from the meshing tool in the Millipede plug-in in Grasshopper.™ (Michalatos)

Analogously, a designer-based pre-defined meshing process, namely the constraint Delaunay triangulation, can also be applied in the triangulation subdivision (Figure 4.34 D,E) and the application in curved manifolds can be easily realized in a similar way. (Figure 4.34 F,G)

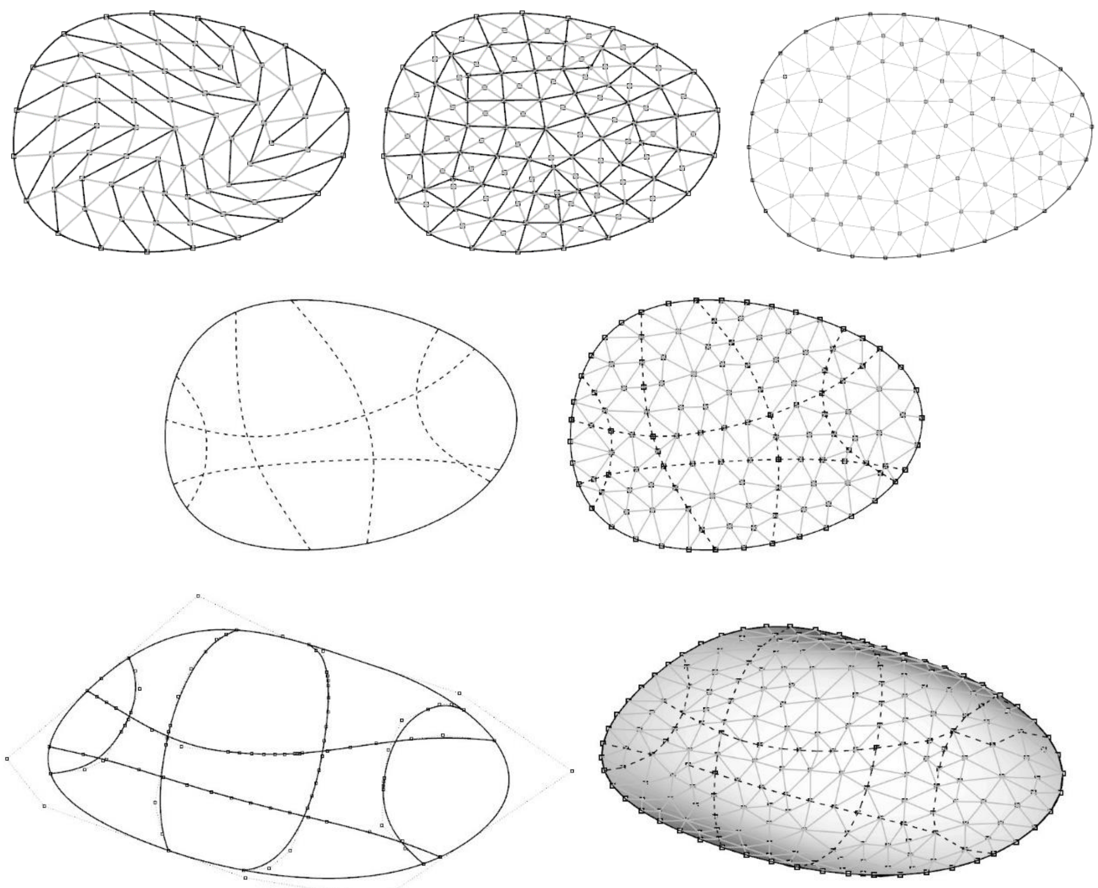


Figure 4.3. 8 Subdivision from triangulation

5, Dual operation and the construction of the 3 and 4-valence meshing:

As introduced in the above sections, the 3 and 4-valence meshes are not directly defined by the initial subdivision of the surface but from the dual operation of the mesh from the corresponding triangulation and quadrangulation. Starting with an initial triangulation or quadragulation, the dual diagram can be generated by connecting the center of adjacent polygons. (Figure 4.35 A, B) With the above-mentioned techniques such as controlling the initial subdivision of surfaces or the subdivision on curved manifolds, a similar dual diagram can also be applied with the constraint tessellation or the tessellation on a curved surface. (Figure 4.35 C, D)

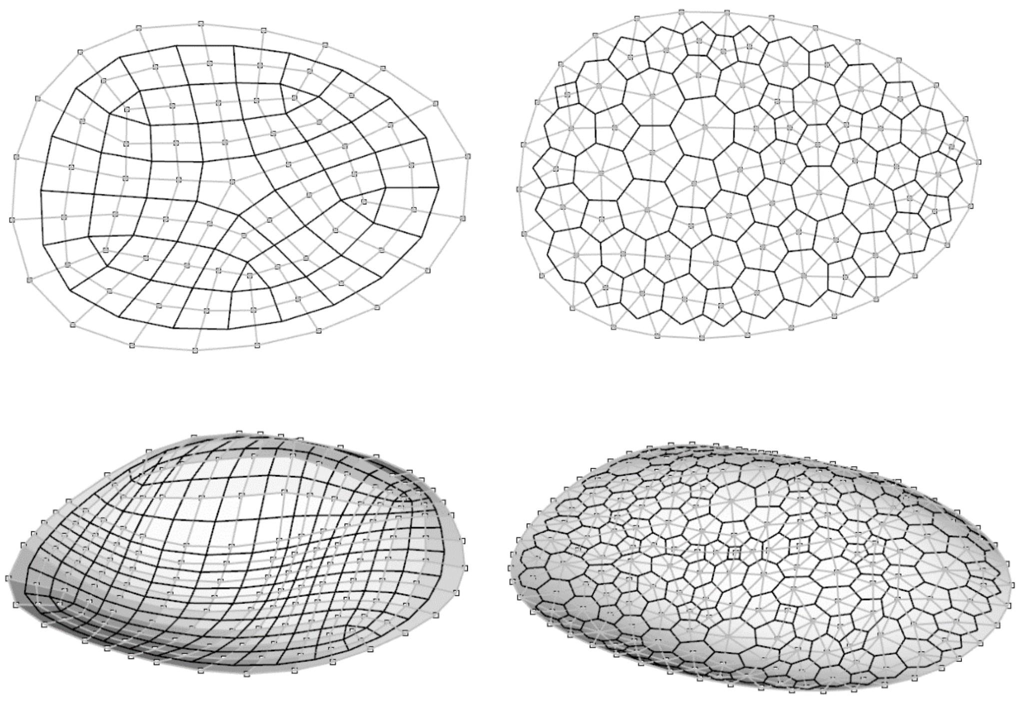


Figure 4.3. 9 Dual operation on triangulation and quadrangulation

The element size of the dual tessellation will be decided by the edge length of the initial subdivision, and if a more homogenous subdivision such as a Delaunay triangulation is used, the dual 3-valence diagram will also turn out to be a hexagon dominant tessellation. If the duality of the tessellation is considered also as the reciprocal relationship between the form diagram and the force diagram of the cellular structure, such a phenomenon will also lead to another property of the tessellation. As it is assumed that only a thin thickness of the shell space is predefined for the structure and the structure will be made in a cavity form, the mass center of every cell will lie nearly on the centroid of the polygon and the mass will be linear to the area of the polygon. In this sense, a better application with the Centroidal Voronoi Tessellation (Du, Faber, & Gunzburger, 1999) can be used to minimize the deviation between the form diagram and the equilibrium state of the force diagram.

4.3.2.2 Spatial equilibrium state with the force diagram

If an initial 2D or 3D tessellation has been created using the above-mentioned techniques, designers can also apply the modern computer-aided form finding techniques on the force diagram to create a 3D equilibrium state of the membrane force network, or to make an optimization of the initial defined geometry. About the development and application of such techniques, some discussion has been done in chapter 2. The comparison between the state of the art form finding techniques for shell structures is discussed in recent research and books. (Adriaenssens et al., 2014) In this research, multiple techniques such as the force density methods (FDM), dynamic relaxation methods (DR) and the particle spring methods (Thompson & Bonner) have been used in different cases due to their suitable application and requirements of design. Some other more powerful methods for architects and designers such as the Thrust Network Analysis (TNA) are mentioned at the end. However, due to the assumptions in this research, more free-controlled form finding and the accompanying uneven distribution of the internal forces are not the aim of this research, hence these techniques are not discussed in so much detail. Meanwhile, the possible applications of manipulating the form and forces are only shown with regular force diagrams with simple triangular and quadrilateral tessellations. And analog to traditional masonry or brick shell structures which are made out of massive and cellular forms, the expectation of the form finding of the cellular cavity structures is also a compression-only state.

1, 3D equilibrium with different scale factors:

The controlling of the 3D equilibrium of the force diagram is mainly due to the simulation of the hanging chains as it has been done since the 19th century. By defining the loads which are added on the mass points and defining the elongation of the in-between springs according to Hooke's law, the displacements can be calculated for every point in the system.

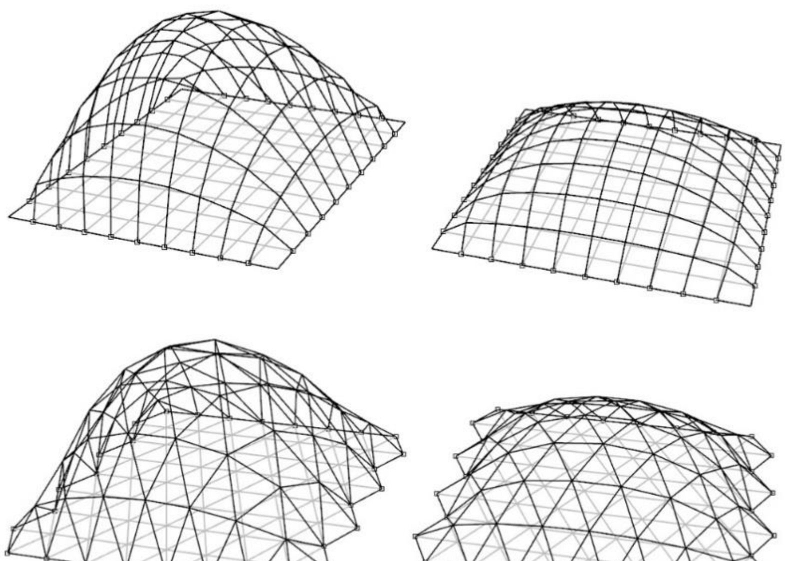


Figure 4.3. 10 Form finding of the initial force diagrams in 3D with different scale factors (FDM method used)

In all the form finding techniques, the scale of the displacement is controlled by both the load factors of the mass points and other scale factors which are defined by the users, such as the force density in the FDM methods, the axial stiffness in the DR methods and the spring stiffness in the PS methods. Figure 4.36 shows the different applications of such form findings with different load factors or scale factors. It also simulates the physical hanging process when different loads are added onto the chain system.

As it is possible to control the particular properties of the particle system or the spring system partly in the DR or the PS methods, it is also possible to control the 3D force diagrams so that a stress concentration can be led to some of the springs in the system. As shown in Figure 4.37, such form findings can be made to establish an asymmetric force diagram, which leads to evenly distributed loads mainly on to some of the springs. The result of such operations will be an unevenly distributed internal force in the cellular cavity system, which will correspond to some special cases that additional loads should be arranged on some parts of the structures. Or in some other cases, if some of the cells can be strengthened or stabilized with special methods, it is also possible to use such techniques to use such stronger elements efficiently.

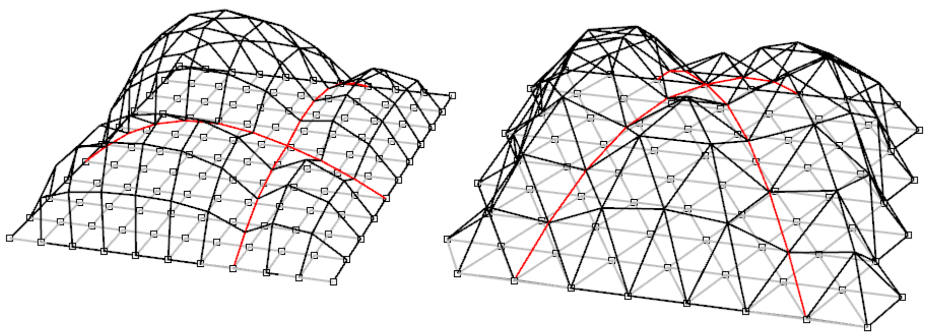


Figure 4.3. 11 Form finding of the initial force diagrams in 3D with special definition of spring stiffness (PS method used)

2, 3D equilibrium with different boundary conditions

Due to the functional requirements of the shell structures, it is common that the structure is not fully supported on every point at the boundary edges. In the form finding of the 3D force diagrams, such a requirement can be achieved through the particular definition of the boundary conditions of the system.

Figure 4.38 shows different cases of the possibilities to control the boundary condition and to create the compression-only force diagram with unsupported edges. For architects, it is a quite useful method to create more interesting forms and also to deal with problems such as the opening of the structures and so on. And also by adding additional supporting edges inside the plan of the structures (Figure 4.28 C), the complicated geometry of the equilibrium state will in return provide a more dramatic interior space for the shell structure. It should be mentioned as a characteristic of such cases, because that the unsupported edge will be under a thrust from the neighbor elements, the equilibrium will only be able to be established with an inward curved from. This is also corresponding to the discussion in chapter 2 that a common upward curvature often appears in the opening of traditional concrete shells.

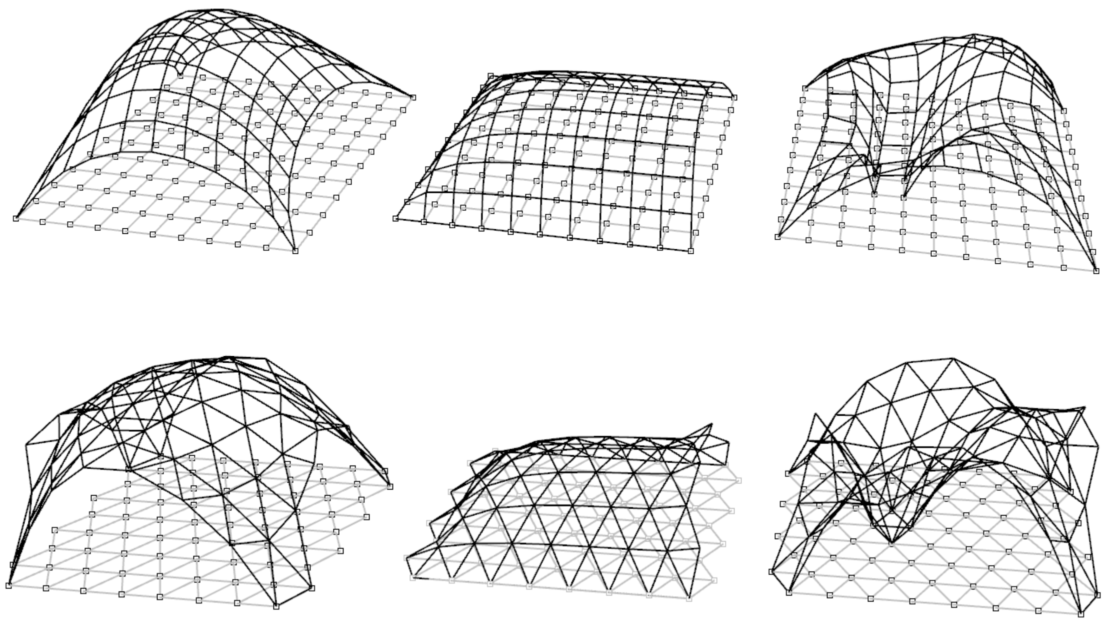


Figure 4.3. 12 Form finding of the initial force diagrams in 3D with unsupported edges (PS method used)

3, From 3D force diagram to 3D form diagram:

After finding the possible equilibrium state of the force diagram of the system in 3D, the first process of the cellular cavity structure design—the initialization of the tessellation—should be finished. However, with only the force diagram, the detailed form of the structural cells cannot be further developed. In this sense, a last post processing process should be made as the final stage of the initialization.

In this stage the 3D force diagram should be transformed to a 3D form diagram with the same mesh dual operation. By generating the polygon centers and the connecting lines between the adjacent center points, the basic contour of the cells can be defined. (Figure 4.39) These final contours will work as the input of the next stage of the design procedure and will be used as the foundations to find the reasonable forms of the rib framework.

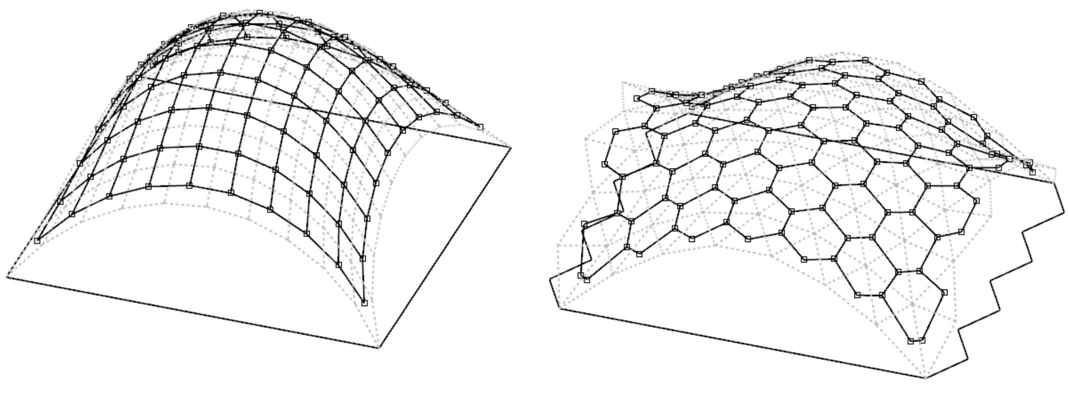


Figure 4.3. 13 Generation of the form diagrams with dual operation on the force diagram

4.3.2.3 Generating planar supporting ribs

According to the foundational assumption and the analysis of the geometric components of cellular cavity structures, the form finding of the supporting rib frameworks should be the first procedure after the generation of the initial 3D form diagrams with meshes.

As shown in Figure 4.40, the input of this form finding process should be the initial form diagram in the form of meshes. And due to the material economic consideration, the objective of this process should be the planar geometry of such a structural component, with which the fabrication can be easily achieved by the cutting techniques discussed in chapter 3.

As it is explained in the above section, the geometry of the in-between rib f_{ij} is defined by the corresponding vertices a_i, a_j, b_i and b_j from the upper and bottom meshes a and b. For such quadrilateral forms the objective is only defined as the co-planarity of the face normal $a_i b_i$ and $a_j b_j$ in this research. Although it is possible to generate other kinds of developable surfaces with the line congruence techniques as shown in recent research (J. Wang, Jiang, Bompas, Wallner, & Pottmann, 2013), it is still too complicated for the fabrication, and the flexible curved forms with small planar segments are also not welcomed for such supporting ribs.

In the analysis of this part, the discussion will be divided into three parts according to the input geometry of the meshes, the triangular mesh, quadrilateral/ 4-valence mesh and the 3-valence mesh, respectively. The discussion will be based on both the existing research and explanations with case studies. Simple proof for some theorems will be presented, but too much mathematical or algorithmic discussion in the optimization will not be restated if they are already discussed in the existing research or the recent research.

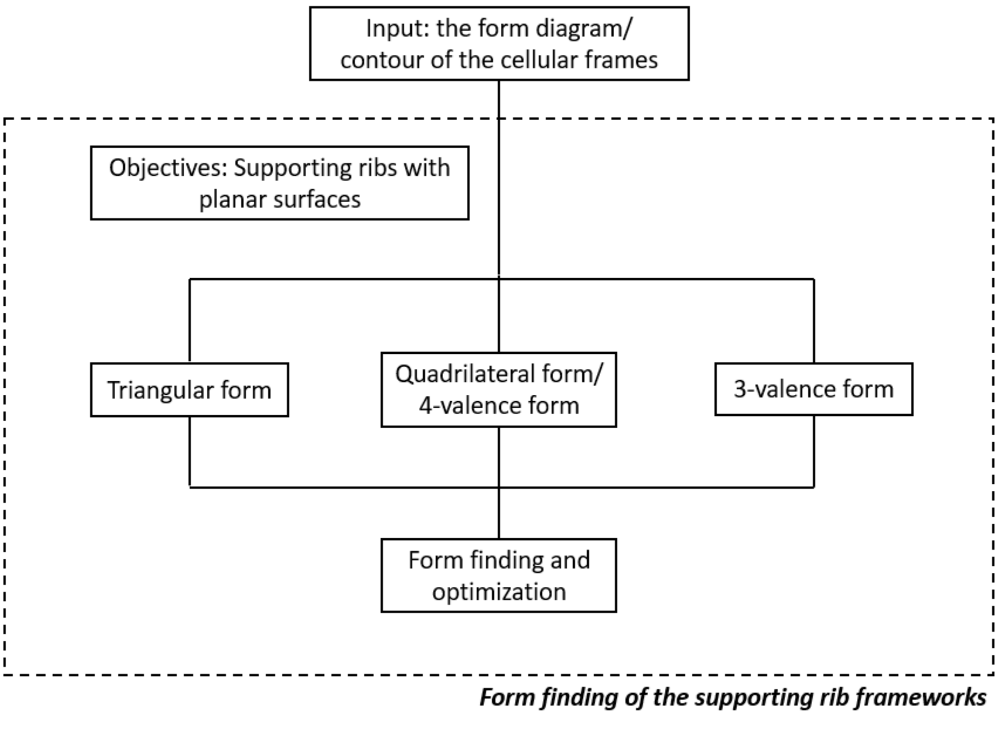


Figure 4.3. 14 Form finding work flows for the planar supporting ribs

1, Planar rib frameworks for triangular form meshes:

Trivial supporting structures from meshes:

Rib structures or supporting structures are well discussed in spatial structures, and their generation is usually started from a basic mesh and by offsetting or projecting along certain distances. The triangular meshes are also commonly used in the truss or rib system as it provides a static determined network system.

To start from a triangular mesh, some simple cases of the supporting rib forms can be presented, and this can help the understanding the other cases of the quadrilateral mesh or the 3-valence mesh. The discussion is based on the fundamental property of the meshes, which has also been similarly discussed in a very recent study in mathematics in 2016. (Pottmann & Wallner, 2016)

Theorem 4.1: A trivial supporting rib framework can be created based on any mesh either by extruding all the vertices along a direction vector v or by extruding all the vertices to a space point which doesn't lie on the mesh. (Figure 4.41)

Proof: By projecting all the vertices along a vector v, all the normal directions at every vertex will be the same and hence the face normals will be parallel to each other. In this case, all the pairs of the adjacent face normals will be co-planar and the planar quadrilateral can be found. By extruding all the vertices a_i *and* a_j *to another point p, then all the normal lines will be defined as* $a_i p$ *and* $a_j p$*. Because the two lines are connected by the point p, the adjacent normal will be also always co-planar.*

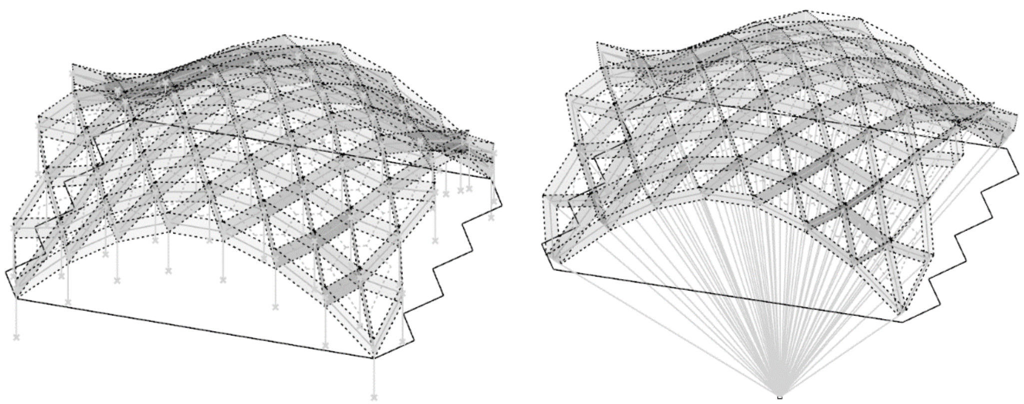


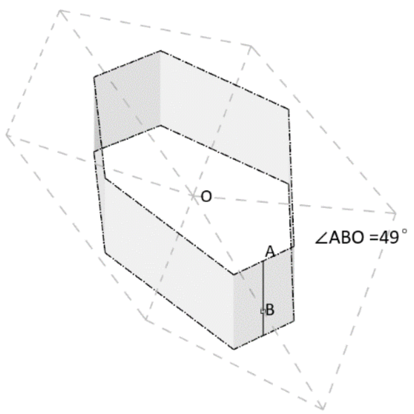
Figure 4.3. 15 Trivial supporting rib frameworks on a triangular mesh

Only trivial supporting structures for triangular meshes:

The reason that a triangular mesh is very simple and particular is that for any such kind of mesh, it is possible and only possible to construct the trivial supporting ribs framework. This will be explained by the following proof.

Theorem 4.2: For any mesh which has only triangular faces, only the trivial supporting rib frameworks can be created to guarantee that all the face normals at each edge of the original mesh are co-planar.

Proof: It can be defined for any supporting rib framework which matches the condition that the *vertices on a given triangle on the mesh named* a_i*,* a_j *and* a_k *and the pair of vertex normals from* v_i*,* v_j *and* v_k *on every edge of the triangle is co-planar. If any two of the three vertex normals are parallel to each other, it is only possible to draw another line which is also parallel to the two normals at the rest vertex to match the condition. If none of the normals are parallel to each other, it can be defined that the intersection points are* $p_{ij} = v_i \cap v_j$*,* $p_{jk} = v_j \cap v_k$ *and* $p_{ik} = v_i \cap v_k$*. With the co-planarity of a surface a definition of the plane can be made that the normals are on the planes* S_{ij}*,* S_{jk} *and* S_{ik} *respectively. Then it can be proved that* $p_{ij} = p_{jk} = p_{ik} = S_{ij} \cap S_{jk} \cap S_{ik}$*, so all the normals can only be intersected on a single point.*



According to the above theorems, the only ways to find the planar supporting rib frameworks are by a vector based or a point based projection method. But considering the relationship between the ribs and the force diagram of the cellular structure, some deviations will occur between the force vector and the rib planes. (Figure 4.42) Such deviations will make it impossible to find a perfect rib framework whose ribs are almost perpendicular to the force vectors unless a pure sphere surface is initially used. This will also lead to an "inclined" shape of the cells and hence a more detailed analysis on the stability of the ribs should be taken in the structural analysis.

2, Planar rib frameworks for quadrilateral/ 4-valence meshes :

Similar to the triangular meshes, two types of the trivial supporting rib frameworks in planar forms can be generated by projecting or extruding. (Figure 4.43) This already enables the finding of suitable rib framework, but the angle problem occurs especially when a large curvature exists in the form diagram.

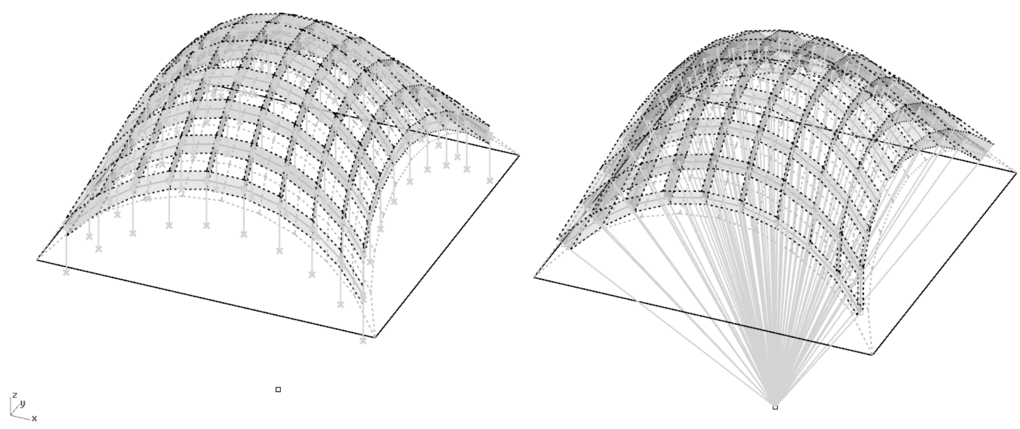


Figure 4.3. 16 Trivial supporting rib frameworks on a quadrilateral mesh

Parallelism and offset of the meshes

Other than the trivial cases of the supporting rib frameworks, the existance of other planar ribs often refers to a parallelism of the upper and bottom meshes. By definition, a mesh *M* is considered to be parallel to another mesh *M'* if *M* and *M'* are combinatorially equivalent (the list of the nodes of every face is the same and in the same sequence) and the corresponding edges are parallel. With the parallelism of every pair of edges, it is clear that all the quadrilaterals of the supporting rib frameworks are co-planar.

A more frequently discussed case of the parallelism of meshes is the offset of meshes. The definition of the offset of meshes is made in the analysis of similar problems in mathematics in 2006. (Pottmann et al., 2007) Meshes are offsets of each other if they are parallel, and in addition their distance from each other is constant throughout the mesh. However, unlike the offset of surfaces, the constant distance between meshes can be defined in mainly three ways, and hence the required properties for the available meshes also varies as:

1, vertex offset: the distance of the corresponding vertices between the parallel meshes *M* and *M'* are always constant. This often enables the use of same length rods or trusses in the linear supporting systems. A simple example of the vertex offset is the trivial supporting ribs by projecting the initial mesh along a vector. In this way, the offset mesh can be any translation of the original mesh and infinite trivial supporting rib frameworks can be constructed in this way.

However, except in cases of the trivial ways, the quadrilateral mesh which has a vertex offset can only be a quad-circular mesh, whose vertices on every quadrilateral are all laid on the same circumscribed circle. (Pottmann & Wallner, 2008)

2, edge offset: instead of the vertices, the edge offset is defined by a constant distance between the corresponding edges of the parallel meshes. In structure and constructions, such an offset will enable the possible use of materials with the same cross section. Although its geometrical meaning is simple to understand, the mathematical methods on constructing a quadrilateral mesh which has an edge offset is still an unsolved problem in recent research. The property of such meshes can only be analyzed in a complicated way: A mesh M is an edge offset mesh if and only if the edges of its Gauss image mesh *S* are tangent to S^2 , which also means it should be parallel to a Koebe mesh. (Pottmann et al., 2007) An equivalent geometric property is that all the edges around every vertex of the mesh are tangent to a local cone surface.

3, face offset: the face offset is only established on the basis that all the 4 vertices on every face of the initial mesh are co-planar. Like the edge offset mesh, a mesh *M* will be a face offset mesh or has a face offset if and only if the faces of it Gauss image *S* are tangent to S^2, or geometrically all the faces around every vertex is tangent to a local cone surface. In the existing analysis, a face offset quadrilateral mesh is also called a conical mesh. (Liu, Pottmann, Wallner, Yang, & Wang, 2006)

Approximation and optimization for different cases of offset meshes

The proof of the above-mentioned properties of the offset meshes needs to be finished in the analysis of the differential geometry. As it is not the aim to solve the mathematical problem or prove the statement, only the results and applications are used in this section. More proofs are shown in the discussion of several essays in computational geometry. (Liu et al., 2006; Pottmann et al., 2007; Pottmann & Wallner, 2008; W. Wang, Wallner, & Liu, 2007)

The most important use of the theoretical discussion in this research is to approximate a suitable mesh with a corresponding offset to a given surface or to optimize the existing mesh to a suitable offset mesh. Different techniques and disadvantages or restrictions are discussed as follows:

1, vertex offset: according to the property of the quadrilateral mesh with a vertex offset, the only suitable mesh for a non-trivial offset should be the quad-circular meshes. This gives two targets for the optimization or the approximation: firstly the mesh should be optimized to a planar quad mesh where all the vertices on every surface should be co-planar, and secondly all groups of such 4 vertices should be located on a single circumscribed circle.

For most of the input meshes in the previous process of the tessellation and the form finding it is hard to satisfy these requirements, and a change of the initial mesh should be required to make an optimization. The optimizations can be simulated with a dynamic method which is nowadays available with the use of computational physics plugins such as Kangaroo (Piker, 2013) or ShapeOp (Deuss et al., 2015) for Grasshopper™. Only with a simple hyperbolic paraboloid as an example, a test of such a optimization is made with both targets defined as edge strains and circle constrains in the ShapeOp algorithm and with boundary conditions as anchoring points on one diagonal. It can be obviously observed in Figure 4.44 that a great change of the geometry has appeared after 500 or 1000 iterations, and it needs to be mentioned that a perfect quad-circular mesh is still not found even after 1000 iterations. For the design procedure such large deformations are not welcomed because an equilibrium state of the membrane force field is set, and such a perturbation of vertices will change both the form diagram and the force diagram. It should be decided by the designer whether the tolerance can be accepted and a later reevaluation of the structural behavior should be carried out in the final verification process.

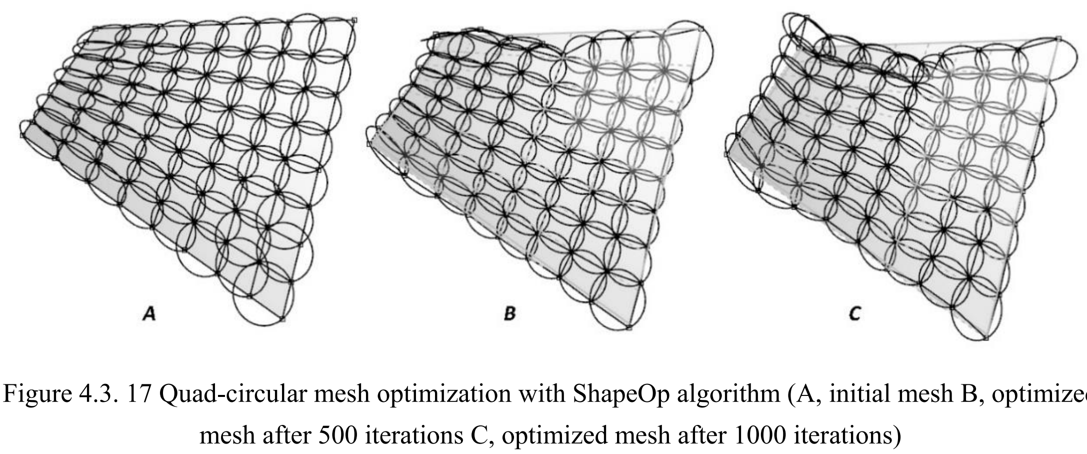


Figure 4.3. 17 Quad-circular mesh optimization with ShapeOp algorithm (A, initial mesh B, optimized mesh after 500 iterations C, optimized mesh after 1000 iterations)

2, edge offset: the method of form finding of an edge offset mesh is still a research topic in computational geometry. According to a relevant essay (Pottmann et al., 2007), it has been shown in a previous study (Bobenko, Hoffmann, & Springborn, 2006) that in the case of quad meshes, *S* is a so-called s-isothermic mesh and thus the mesh *M* is a discrete variant of a curvature line parameterization whose Gauss image is an isothermic curve network. Such "L-isothermic surfaces" have been introduced already in the 1920s (Blaschke, 1923) and have been used as a basic form to find possible edge offset meshes by applying Laguerre transformations. (Pottmann, Grohs, & Blaschitz, 2010)

Although it is possible to find some variations of forms of edge offset meshes from some specially defined initial forms, it is still unsolved for the method of doing an approximation or optimization on an arbitrary quadrilateral mesh. Hence such meshes in the quadrilateral form are still not suitable to be a goal for this process in this research. The existing limited possibilities can only be considered as some special guides for the similar designs with special goals from aesthetic considerations.

3, face offset: for the meshes discussed in this research, the property of the face offset is a favourite as it enables the same distance between the upper and bottom mesh so that an equivalent shell space with constant thickness can be achieved. The supporting rib framework should therefore as far as possible be perpendicular to the dual force diagram in return.
For quadrilateral meshes, the requirements of a face offset are equal to a conical mesh itself. The criteria of finding such meshes have been proven and suggested as an angle criteria and the algorithm to realize the optimization has been shown in relevant research. (Liu et al., 2006; W. Wang et al., 2007)

The similar problem of the optimization will occur as the form deviation in the optimization for vertex offset meshes. As shown in the existing optimization examples in previous studies, such algorithms should work with a perturbation of all the vertices on the initial mesh, and a large deformation will occur when the initial mesh has a large deviation to the angle criteria. (Figure 4.45) A small deviation is only possible if a surface is used as the initial geometry and a mesh according to its principal curves is generated to be the initial mesh. However, this is also not suitable for the design procedure of the cellular cavity structures in this research.

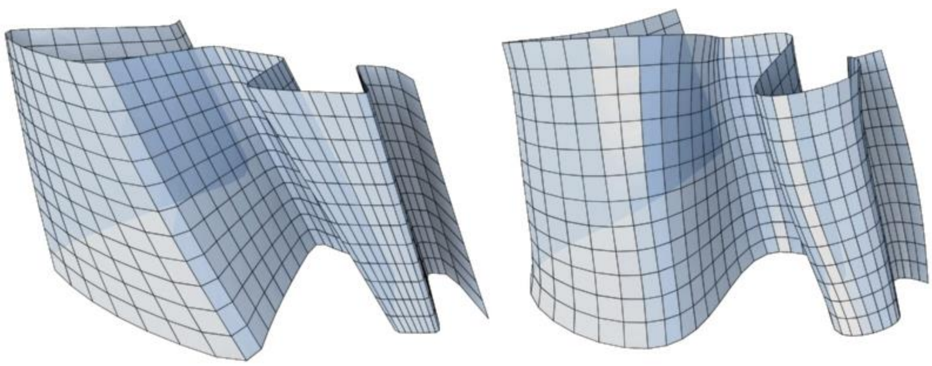


Figure 4.3. 18 Large deformations from the vertices perturbation by the conical mesh optimization (Pottmann et al., 2007)

Another method of making an optimization for conical meshes is to optimize the dual force diagram of the initial form diagram to a circular mesh. Here the duality between the conical mesh and the circular mesh is used. (Figure 4.46 A,B) An example of this technique is shown based on the optimized quasi-circular mesh with the ShapeOp algorithm (Figure 4.46 C)

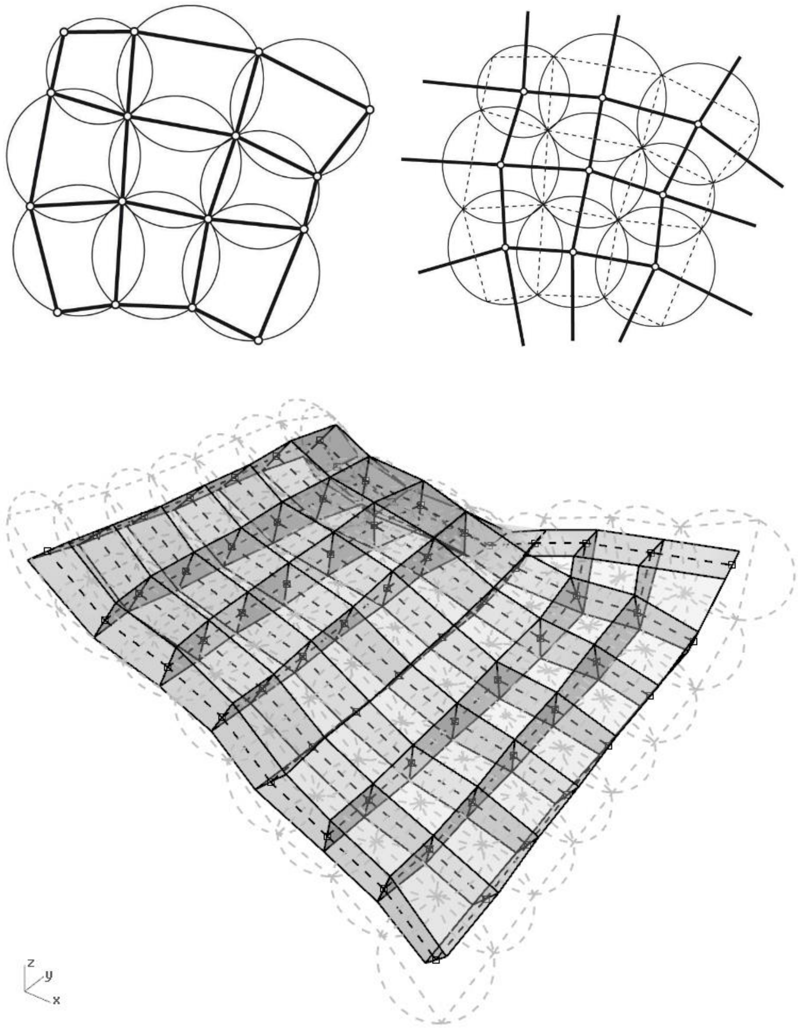


Figure 4.3. 19 A,B: Duality of a conical mesh and a quad-circular mesh(Pottmann & Liu, 2007) C: Optimization for a face offset with a quasi-conical mesh from an optimized quasi-circular mesh

3, Planar rib frameworks for 3-valence meshes:

Compared to triangular meshes and quadrilateral meshes with their dual 4-valence meshes, the 3-valence meshes, which are also the dual diagram of the triangular meshes, are with the most satisfying properties for the requirements of the ideal supporting rib frameworks in this research. After understanding the possibilities of the form finding for rib frameworks for the triangular and quadrilateral meshes, such advantages can be better explained in this section.

Theorem 4.2: For any circumcentric dual 3-valence mesh from a triangular mesh, a non-trivial supporting rib framework can be generated with only planar surfaces. This equally means that from all the 3-valence meshes a non-trivial supporting rib framework can be directly generated *or modified.*

Proof: A qualified supporting rib framework can be found in the following way: As the circumcentric dual 3-valence meshes have corresponding initial triangular meshes, where the vertices on the 3-valence meshes are the circumcenter of the corresponding triangles, it is possible to generate satisfied normal vectors by using the normals on the triangles. (Figure 4.47)

It is clear that the perpendicular line AC and AB will go through the midpoint C on the common edge CD and also that there is a relationship such that:

$AC \perp CD; AE \perp CD;\ BC \perp CD; BF \perp CD$

$so\ that: CD \perp \Delta ACE\ and\ also\ CD \perp\ \Delta BCF$

Therefore the faces ACE and BCF are co-planar and the vectors AE and BF are co-planar, too.

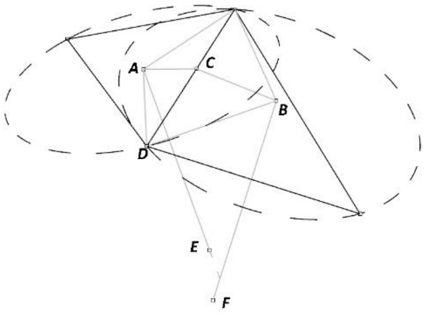


Figure 4.3. 20 Diagram for the conical property of the 3-valence mesh

The property of the circumcentric dual 3-valence meshes and their planar supporting rib frameworks is also desired by the structural requirements and a perpendicular relationship between the adjacent ribs of cells and the force direction of the cells can always be satisfied.

A test of such techniques is shown in Figure 4.48 with a hanging model of the triangular mesh and its circumcentric dual graph, which is selected as the form diagram of the structure. In Figure 4.48 B it can be observed that the direction of the normal vector is neither along the same direction nor towards a single point. It mainly reflects the changes of the curvature in the target surface and also fits the diversity of the force directions.

This method of using the circumcentric duality between the triangular meshes and the 3-valence meshes can therefore be considered the best way to satisfy the design requirements of the cellular cavity structures. As it is very similar to the relationship between the Voronoi Tessellation and the Delaunay triangulation (Figure 4.22), the Delaunay triangulation based on a homogenous distributed point cloud and a resulting Centroidal Voronoi tessellation can therefore also be considered as a powerful tool in such kinds of designs.

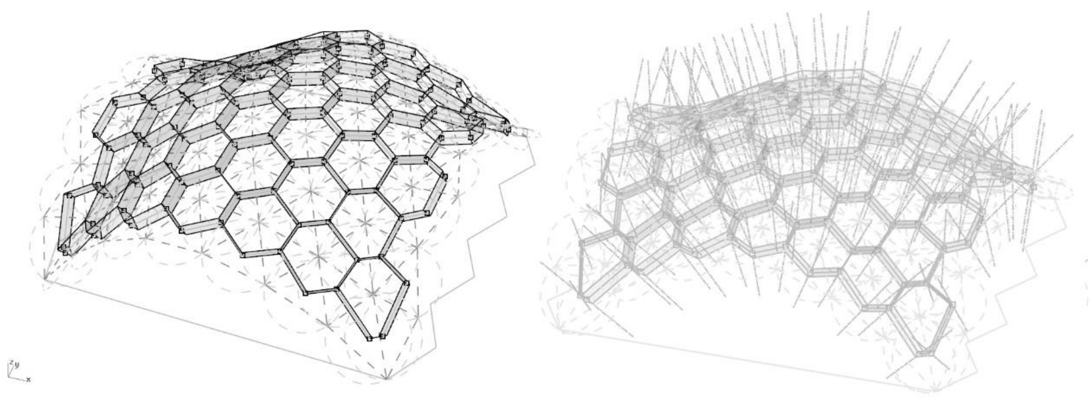


Figure 4.3. 21 A: Planar ribs framework from an arbitrary 3-valence mesh B: arrangement of the normal vectors

4.3.3 Design variations of the covering membranes

4.3.3.1 Generating 3 basic typologies

The basic form of the supporting rib framework determines the basic arrangement of the structural cells and also the global behavior of the shell structure. However, as it is discussed with the experiments of pure rib shells, the supporting rib framework itself still cannot provide enough stiffness even only under self-weight. This leads to the requirement to add additional structural components to stabilize the framework. Therefore, the design consideration of the form finding of the covering membranes in this section is firstly their effect on providing extra stiffness for the cells.

At the same time, as the shell structure often has a curved geometry, this also raises some difficulties in the prefabrication of some double curved shapes with thin sheet materials, hence the other consideration of this section will be the possibility of applying a mass customization method for the designed shape of the covering membranes.

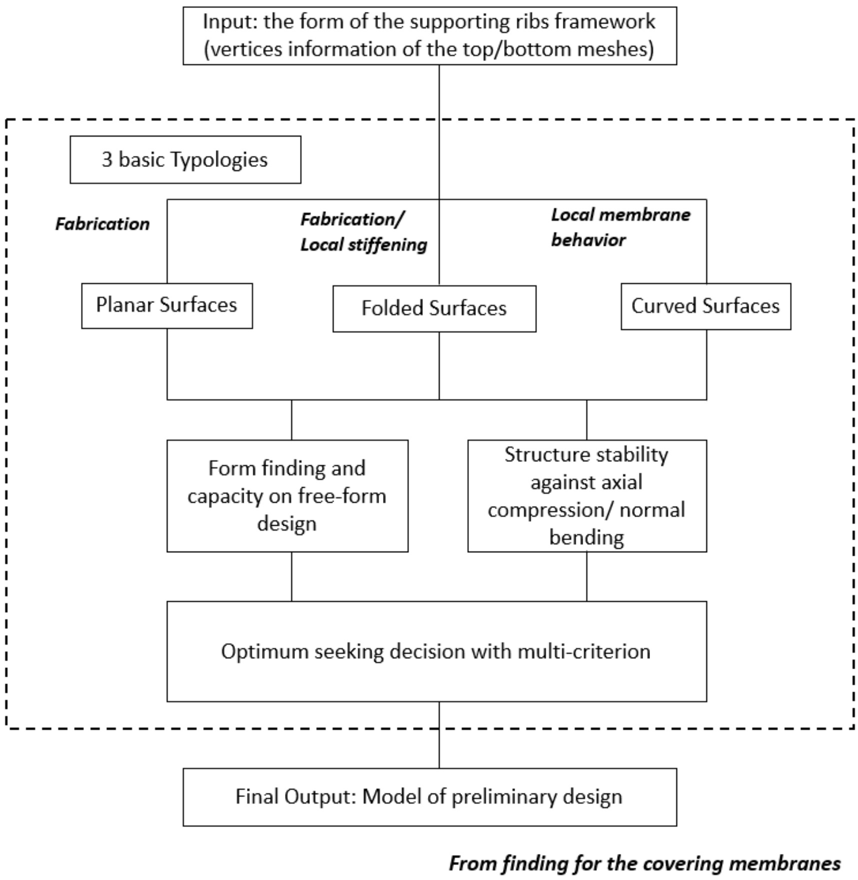


Figure 4.3. 22 Work flow and considerations of the form finding procedure of the covering membranes

A basic design flow is shown in Figure 4.49. As the geometric information of the supporting rib structures relies mainly on the vertices on both the upper and bottom meshes, such information, especially the coordinates of the vertices, should be the starting point of the design of the covering membranes.

As is defined in the general hypothesis of the structure concept, the upper and bottom meshes should both be an approximation of the external surfaces of a shell space with a certain thickness. In this way, three basic typologies of the shape for the covering surfaces can be suggested as the contents of the discussion in this section. (Figure 4.50)

1, Planar surfaces: It is clear that planar surfaces have been most popular in applications of architectural design, as they provide a very economical way for the prefabrication of the structural elements. Although the planar shapes of the covering surfaces can be different, they can be cut easily with techniques such as the laser-cutting, water-jet cutting or plasma cutting as described in chapter 3. However, for such surfaces, a requirement for the property of the outer and inner meshes is relatively restricted. In this case, only the meshes whose vertices on every face are co-planar can be directly used. (Figure 4.50 A) Otherwise, a deviation of the planar covering membranes or a demand of another optimization of the meshes should be inevitable. This will be further discussed in the following form finding part.

2, Folded planar surfaces: Although the planar surfaces would be the best choice for the fabrication of the cellular cavities, other variations are still suggested for the considerations of the local stability. An inspiration is gotten from the application of concave membranes will thin steel panels in the lightweight wall element from Hugo Junkers (Figure in Chapter 3). By extruding all the edges of the upper and bottom meshes towards the seed point of the cell, two polysurfaces can be generated and they will be bounded to each other at the seed point. (Figure 4.50 B) It is the hypothesis that such changes can help to stiffen the local stability of the cell, and will be discussed and tested in the later part of this chapter.

3, Curved developable surfaces: Based on the variation with folded planar surfaces, another variation is suggested by changing the polygons on both meshes into a continuous curve. With the same extruding methods, a developable surface can be made as a ruled surface. (Figure 4.50 C) The local behavior of the curved membranes will also be evaluated in the later part.

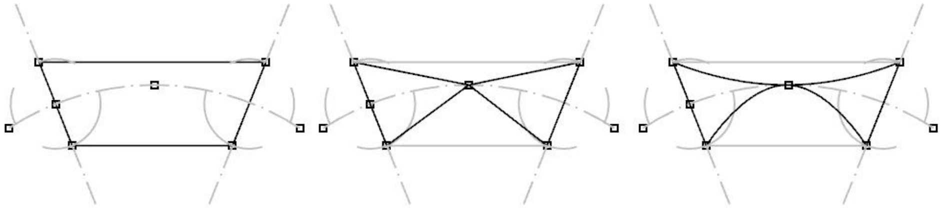


Figure 4.3. 23 Section concepts of 3 variations for covering membranes (A, planar surface; B, folded planar surface; C, curved developable surface); for case C, the section should also be straight when a developable surface (ruled surface) is constructed, the curved drawing in this diagram is only for a better differentiation

4.3.3.2 Form finding for different variations

Before going into the structural analysis of the variations of the covering membranes, the form finding methods will be discussed. Both the difficulties to find a perfect form and the possible deviation of forms will be introduced and will work as a consideration in the optimum seeking decision for the final design shapes.

1. Planar surfaces: The planar requirements of the covering membranes equal an additional restriction on the form finding of the supporting rib frameworks. For triangular meshes, there is no need to do any optimization. But for quadrilateral meshes or the 3-valence mesh, it is only possible to try a new form optimization with similar techniques as for the supporting ribs, or by allowing a deviation of the surfaces between different cells. For an optimization, popular methods such as the planarization in Kangaroo can be used, but it is possible that in most cases there will no solution for the shapes. For the deviation, larger ones will occur especially if the meshes have large curvatures. (Figure 4.51 A) For some relatively "flat" meshes such tolerances can be accepted in the building process. (Figure 4.51 B)

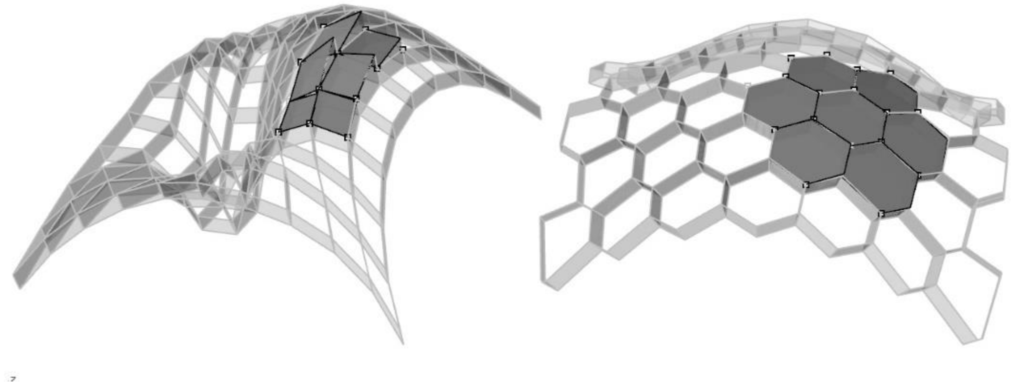


Figure 4.3. 24 Deviations and tolerance by planar covering membrane design

2. Folded planar surfaces: The folded planar polysurface can be directly found by extruding the polygons on both meshes to the seed point. (Figure 4.52 A) In this case, there is no deviation between the covering membranes and the mesh edges.

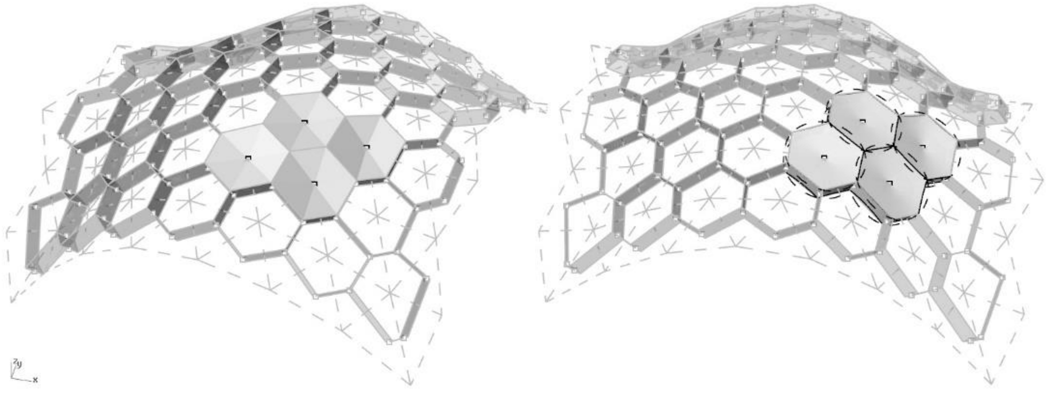


Figure 4.3. 25 Form finding of the covering membranes for quadrilateral and 3-valence meshes

3. Curved developable surfaces: A developable surface in curved forms can also be generated by first creating an interpolating curve along all the corresponding vertices in both meshes and then extruding these curves to the seed point. (Figure 4.52 B) There will also be a small deviation between the curved surfaces with the mesh edges. But the thickness of shell space will be defined as smaller than the edge length. This is sometimes also ignorable in the designs.

4.4 Stability analysis and comparison of the cellular cavity structures

4.4.1 Hypothesis on the differences of stability

The consideration of the stability problem of the 3 variations in the last section is based on the stiffening effects with the change of geometry (discussed in chapter 3) and only for the assumed load cases by the equilibrium state according to the force diagram.

The planar surface is considered to have the lowest stability due to the possibility of the buckling behavior under the in-plane stresses. And in the prefabrication process, the initial imperfection of the surface can also easily be caused by even a small unnoticed bending of the surface.

Compared to the planar case, the folded planar surface would show a better stability as it divides the surfaces into several small parts of planar surfaces, hence the risk of a local buckling is relatively decreased. A quasi membrane behavior will be in this case activated through the continuity of the folded creases. The creases will in this sense transfer the in-plane stress, but the discontinuity of curvature will also make such a part hard to maintain at a pure membrane behavior. At the same time, as both the surfaces are placed towards each other and will be stabilized in the center part of the structural cell, this will lead to a possible stress concentration in the middle part of the surfaces. For such a problem, a local stabilization with extra elements should be required.

Among the three variations of typologies, the curved one is supposed to be the best choice from the consideration of the stability. With the curved geometry, a local membrane behavior will be fully activated and the covering membranes will also work like a thin shell. This will give both the covering membranes a better stiffness against buckling and also bending from the external forces on the system. A simple test against bending is easy to make and it can be obviously observed that even under a very small external force perpendicular to the membrane surface, the planar surface will be bent and the deformations can be observed. However, the folded surface and the curved surface have shown much better stiffness in this case. And for the stability under the supposed compressive loads, theoretical and physical tests in FEM analysis and also with full-scale structural cells will be shown in the next part. The results will be used only for a simple comparison of the three variations and as a suggestion for the choice of designs.

4.4.2 Theoretical analysis through FEM simulation

A test with full-scaled cells was designed to compare and evaluate the different stabilities of the geometries under a similar compressive load. For the shell structures which have, simultaneously, internal stresses with compression, tension and bending moments, the simulation in a physical test is not considered because of the difficulty. With the consideration of a better establishment of the physical test, the cell form is designed as a unified 250*250*125mm block in rectangular form. The applied material is 1mm press-paperboard and the anisotropy of the materials is considered in the FEM analysis with a quasi-isotropic material parameter as discussed in chapter 3. In the real physical tests such an approximation is also made by a 90° rotation of the bottom surface based on the original upper surface, which makes the cell similar to a quasi-isotropic composite material.

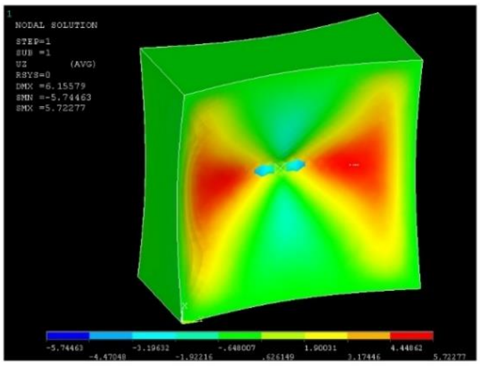
In a pre-research the tests are simulated in the ANSYS FEM software, where a linear eigenvalue buckling test is made. As in the real shell, every cell is equilibrated under the in-plane stresses from every direction which are perpendicular to its supporting ribs. A different load case with only a uniaxial compression is used in the test due to the restriction of the testing machine in the physical tests. For a more accurate simulation to the real physical test, two cases are also compared by defining the unloaded ribs as supported or unsupported.

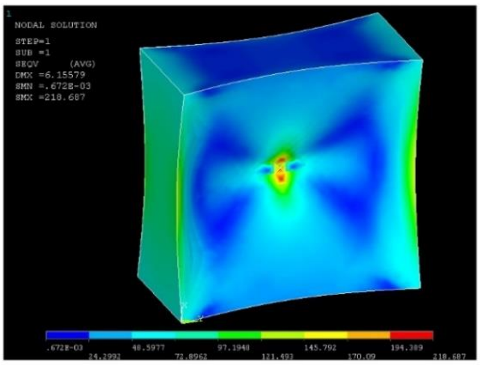
The analysis for the three typologies of surfaces all show a similar result (complete description and setting is ANSYS see appendix 1):

For both the curved surfaces and the folded surface, the unsupported ribs will remarkably decrease the critical buckling load of the systems. But for the planar surfaces there is almost no difference caused by the unsupported ribs. At the same time, no matter in which cases, the curved geometry shows a better stiffness, and is followed by the folded surface. The planar surface provides in both cases the worst stiffness against the buckling deformation. (Table 4.1)

Shape and supports	Critical load factor (compressive load)	Deformed shape with normal displacement/Von Mises Stress
Curved side-supported	0.57882(20648N)	Figure 4.53 A/ Figure 4.53 B
Curved side-unsupported	0.045436(1620N)	Figure 4.53 C/ Figure 4.53 D
Folded side-supported	0.078823(3464N)	Figure 4.54 A/ Figure 4.54 B
Folded side-unsupported	0.032308(1419N)	Figure 4.54 C/ Figure 4.54 D
Planar side-supported	0.014442(634N)	Figure 4.55 A/ Figure 4.55 B
Planar side-unsupported	0.014699(646N)	Figure 4.55 C/ Figure 4.55 D

Tabel 4.2 Critical load factors in FEA analysis results





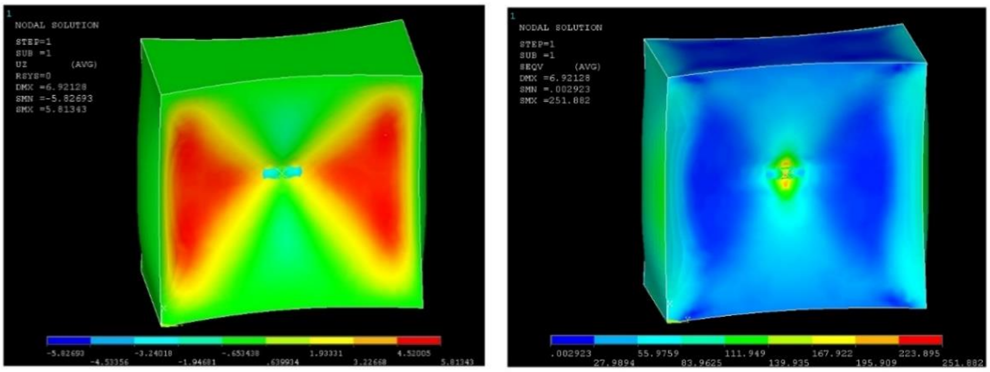


Figure 4.4. 1 FEA analysis on cells with curved covering membranes

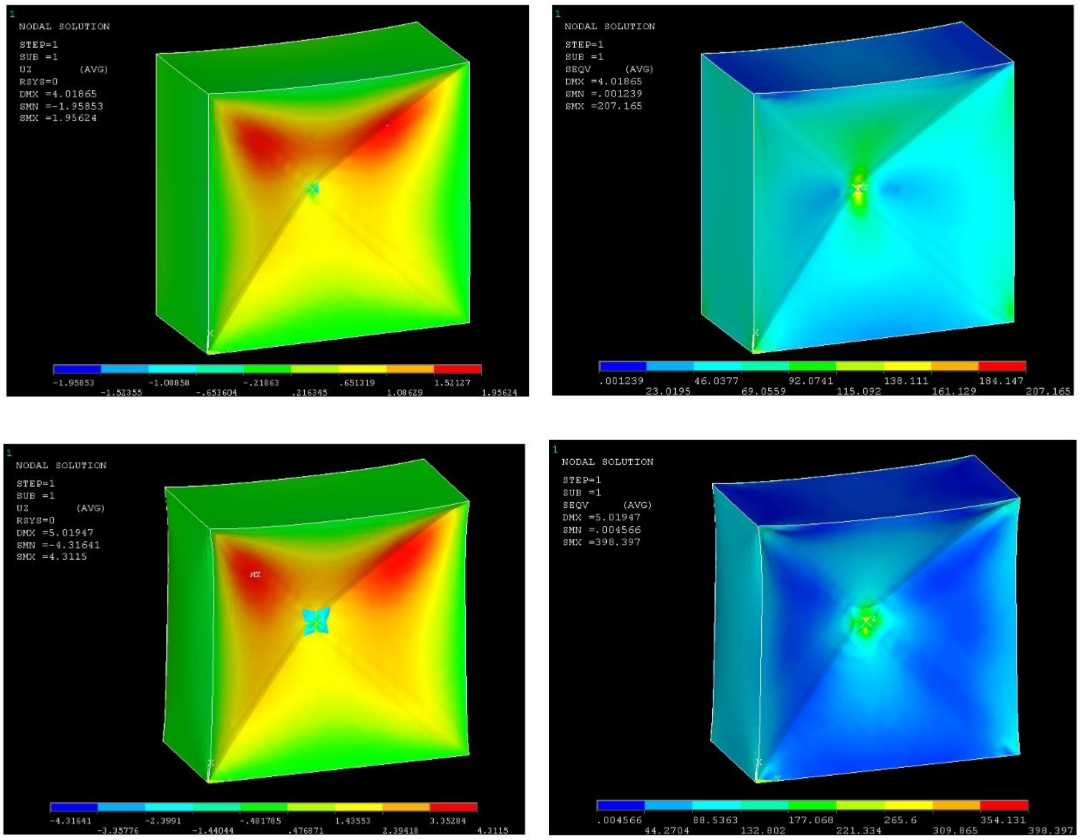
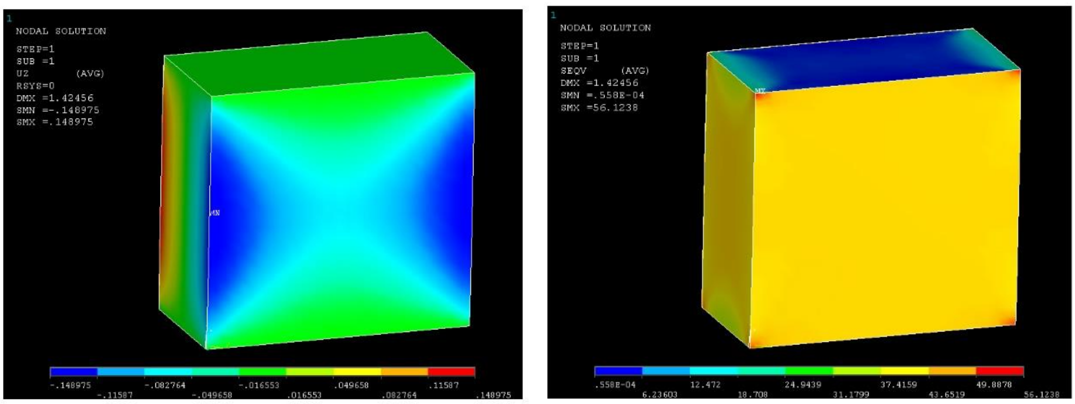


Figure 4.4. 2 FEA analysis on cells with folded planar covering membranes



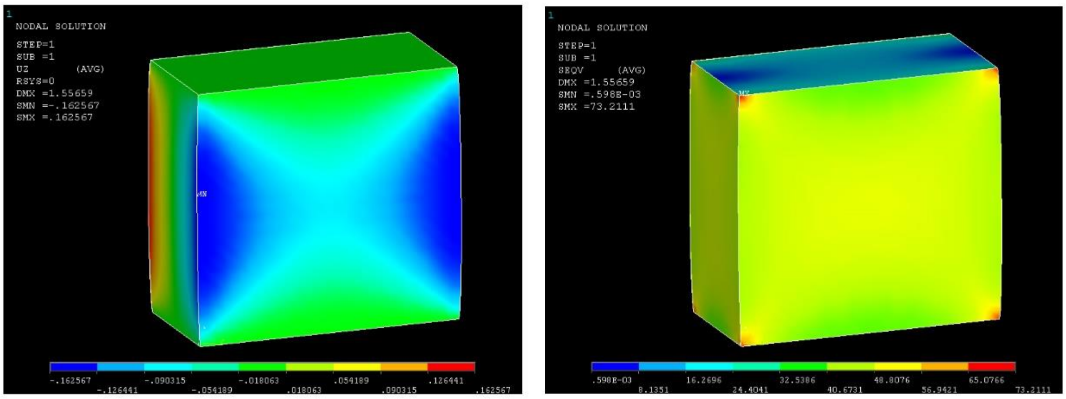


Figure 4.4. 3 FEA analysis on cells with planar covering membranes

The result of the linear eigenvalue buckling analysis can be explained by the expectation and hypothesis: as the planar surface will activate no local membrane behavior of the structure, the covering membranes are much weaker even than their supporting ribs at the side. In this case, the structure is more dangerous in the part of the covering membranes, and when the buckling of the membranes happens, the whole structure will be dangerous because no resistance against the vertical deformation can be added on the supporting ribs in both cases. However, in both other typologies with the curved and folded covering membranes, as the geometry stiffness is enhanced, the covering membranes will in return provide a better resistance against the deformation and the local buckling. In such cases, the supporting ribs will help to improve the stiffness of the covering membranes and will provide a horizontal stress other than the uniaxial load. This resistance will help the covering membranes to remain in their form and hence the critical load factor can be enhanced by the supporting ribs.

In the real cellular cavity system, such supporting conditions can also be achieved or approximated by connecting the supporting ribs of the adjacent cells. As the connections will double the thickness of the supporting ribs and also transfer the lateral force to the adjacent cell which is also stabilized by its covering membranes, a similar effect will be activated. In this sense, the lateral supports are also considered to be necessary in the real physical tests.

4.4.3 Structural details and the physical tests

4.4.3.1 Pre-test and the experience on the construction details

Apart from the theoretical buckling analysis which is based on perfect geometry and perfect connection details, the real behavior of the cells should also be affected by the initial imperfections of the surface geometries and also the imperfections from the construction details. This will also raise a discussion about the constructions of the cellular cavity structures. The detail diagrams will also be shown and discussed in the next chapter with the built pavilion.

In this test, a pre-test is made with the typical building techniques. Because a single unrolled surface cannot be made in the cases of the curved and folded membranes, a divided system is used. The supporting ribs are first built and fabricated. Some small folded flanges are created by a simple folding and will work as a hinged connection to transfer the load from the ribs to the covering membranes (Figure 4.56 A). A supporting frame is then glued to the ribs to fix the basic form of the cell (Figure 4.56 B) and offer a continuous surface for the gluing with the covering membranes. Finally, the covering membranes will be glued onto the supporting frames and then be stabilized in the center with a 3D-printed stabilizer. (Figure 4.56 C)

Figure 4.4. 4 Construction details of the cells in pre-test

The procedure and results of the pre-test have shown the great influence of the lateral supports and the connecting details to the critical load. As the connections between the ribs and covering membranes are mainly achieved by gluing the flanges and the surfaces, its density and the strength will be important to the result. The tests without the lateral supports are shown to have a great influence from the imperfections of the connecting details and have often unpredictable buckling forms due to a sudden break of the glued flanges. (Figure 4.57 A, B) This will directly lead to an improvement of the construction details with a higher density of the flanges. At the same time, the result of the lateral supported cells shows a more desired deformed shape which can be better compared with the results of the FEM analysis. (Figure 4.57 C)

Figure 4.4. 5 Deformations in the pre-tests

Based on the knowledge from the pre-test, improvements have been made in the final tests. The flanges are made in a higher density and no free gaps are left between the flanges so that a stronger connection can be made with the same technique. (Figure 4.58 A, B, C) At the same time, the lateral support is also determined to be used in the pressing test. As there is only a limited restriction against the lateral bending or buckling in the real shells (the lateral bending or buckling of the supporting ribs can also happen when a large load is added on the structure), a similar lateral support is also made in the test. Two MDF plates are stuck together with the side ribs of the cell with Velcro® Tapes and are also slightly clamped by C-clamps horizontally. In this way, a limited restriction can be made with only little pre-stressing from the lateral side and the side ribs can also bend inwards when a large bending moment occurs. More detailed setting of the final tests is shown in Figure 4.59 A, B.

Figure 4.4. 6 Improvement of construction details for the cellular structure

4.4.3.2 Pressing tests and the results of ultimate loads

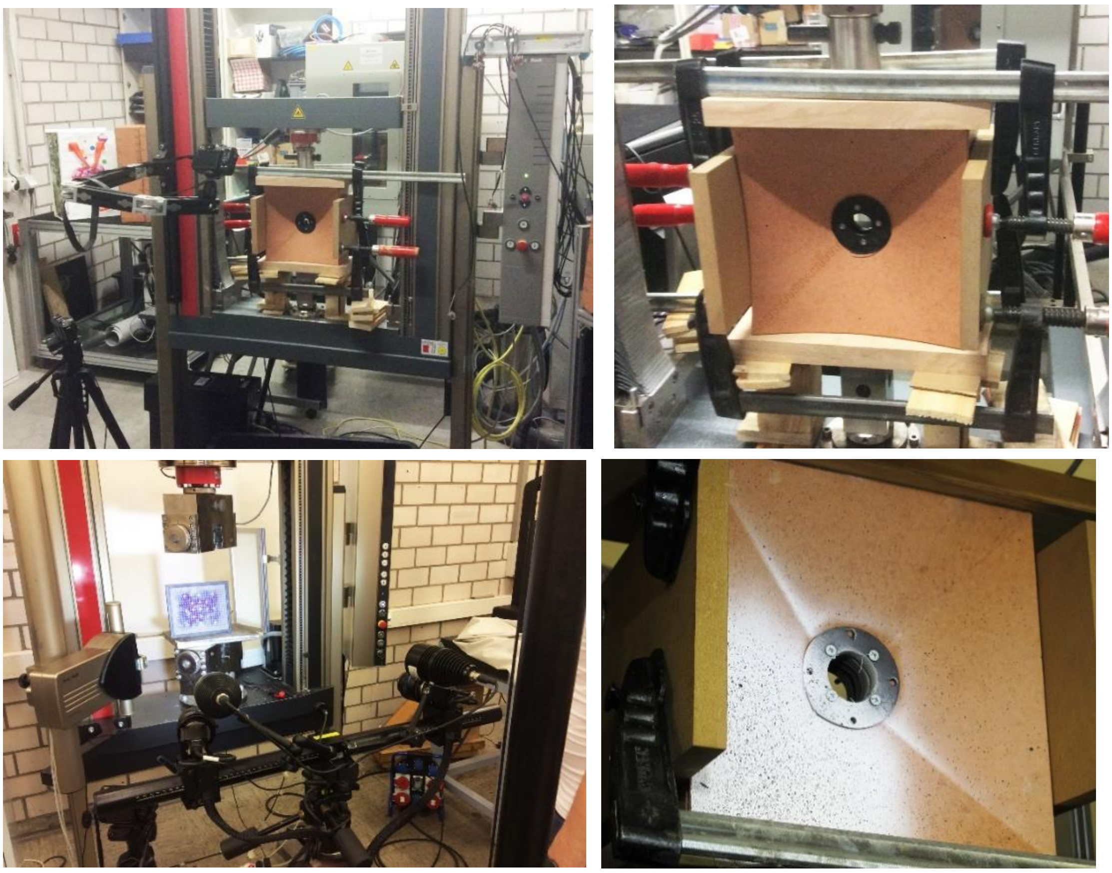


Figure 4.4. 7 Settings of the final tests (A, B: Normal pressing test with lateral support, only the camera system are used to record the deformation; C, D: Tests with DIC analysis, the detailed 3D deformation of the surfaces is measured in a quarter range.)

5 groups of specimens are tested in the final pressing test and each group has three cells in the different typology with different covering membranes. A force-based pressing test is made by the testing machines Z050 and Z100 from ZWICK Roell. For the even distributed surface load, two thick wood panels are used in the test. The lateral support is achieved or simply simulated with 4 large C-clamps and 2 MDF plates. Only little pre-stress is given to hold the position of

the supporting MDF plates. The software for the evaluation is *Zwick-Roell testXpert II*.

As there is no equipment available to realize a simple measurement of the slight deformation of the membranes due to the limit of budget and accessibility to the corresponding equipment, a DIC (Digital Image Correlation) analysis is carried out with only the last group in the test. Due to the limited area of measurement with less than 150*150mm, only a quarter of the cell is measured and covered by a spray to fulfill the DIC test. (Figure 4.59) The spray here used has a small amount constituent of water and has also shown the effect of humidity on the limit capacity of the cellular structure which is made of paper products. Detailed results are shown in the later tables.

Groups:	Type/ Number of specimens	Weight (g)	Ultimate Load F_{max} (N)	Remarks
Group 1 Normal pressing	Curved/ No. 1	492	5665	
	Folded/ No. 1	534	3958	
	Planar/ No. 1	437	4433	
Group 2 Normal pressing	Curved/ No. 2	489	5967	
	Folded/ No. 2	544	4071	
	Planar/ No. 2	434	3836	
Group 3 Normal pressing	Curved/No. 3	484	4963	Lateral support failed
	Folded/ No. 3	507	4332	
	Planar/ No. 3	441	4497	
Group 4 Normal pressing	Curved/ No. 4	493	5586	
	Folded/ No. 4	545	5086	Interrupted and continued
	Planar/ No. 4	439	4212	
Group 5 Normal pressing + DIC analysis	Curved/ No. 5	488	4471	Water-contained spray used Little asymmetric deformation
	Folded/ No. 5	548	3412	Water-contained spray used Observable asymmetric deformation
	Planar/ No. 5	434	2555	Water-contained spray used Initial outward bending (caused by an interrupt)

Table 4.2 Test series and results with critical load

Apart from the DIC groups, 2 tests by specimens "Curved/ No. 3" and "Folded/ No. 4" are disturbed by the sudden failure of the lateral supports or the failure of the testing machine. The results of these two specimens are either too low (4963 to the average 5739) or too high (5086 to the average 4120) to the average of other specimens and are hence marked as invalid results.

Among all the tested specimens, an average ultimate load is calculated for each group. (Curved: 5739N --- 0.161N/mm^2; Folded: 4120N --- 0.094N/mm^2; Planar: 4245N --- 0.097N/mm^2) The results have shown a clear higher stiffness of the curved surfaces than the folded and planar ones, hence the stiffening effect with the local membrane behavior is therefore testified.

With the Load-Displacement-diagram the similarity of the deformation can be shown for each typology of cells. (Figure 4.4.8) The deformation curves of cells with different surfaces are clearly divided into three different groups. However, a similar ultimate bearing capacity is shown by both the folded surfaces and the planar surfaces. It can also be obviously observed that the maximum vertical displacement of the planar cells (about 4mm) is much smaller than the folded ones or the curved ones (about 10mm). And from the video record of the tests, the normal displacement of the planar surfaces can be observed very quickly after the start of the test. In comparison, such observable normal deformations can be seen only a little before the collapse of the curved and folded cells.

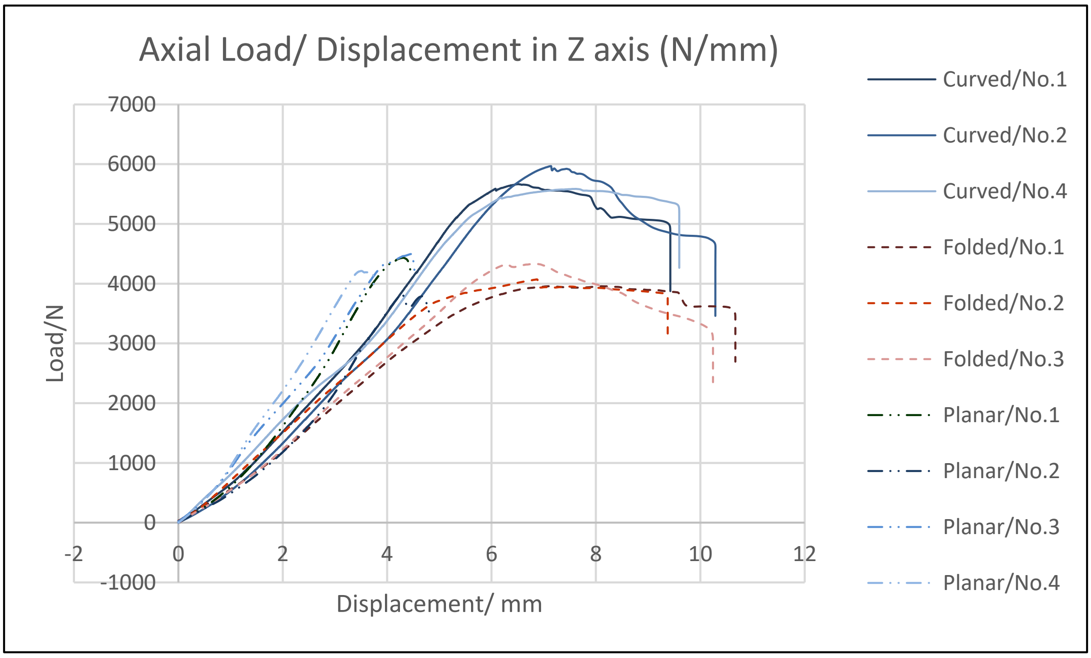


Figure 4.4. 8 Load-Displacement-diagram of the valid non-DIC groups

As it is hard to measure the small deformations on the membranes with normal techniques, only another hypothesis of the post-buckling behavior can be made to explain this difference. This hypothesis is based on the theoretical analysis of the buckling behavior and the post-buckling behavior of the plate and shells. In the planar cells or even the folded cells, it can be assumed that the buckling of the structure happens already at a very early stage in the test and a small deformation is caused. After this buckling point, a post-buckling behavior is soon activated as the deformed curved geometry can still provide a sufficient stiffness for the structure. However, for the curved surface itself such an effect of the post-buckling is not as positive as the planar or folded ones. The reason is already discussed in chapter 3, that the buckling for a shell is "ultra-catastrophic" and the buckling for a plate is still "benign" for its ultimate load capacity

For a better understanding of the small deformations on the surface, a DIC analysis is used on the last group of the test and the results are shown in Figure 4.4.9. Even with the similar testing machines and the same configuration, the results are all a little lower than the other 4 groups. Although all groups have a lower ultimate load, a linear relationship between the non-DIC groups and the DIC group is found in the curved and folded ones. (Curved: 5739: 4471 = 1.28:1; Folded: 4120:3412= 1.21:1) The planar cell in the DIC group is disturbed due to a sudden failure of the testing machine and restarted with an initial geometrical imperfection. As a result, the final ultimate load is much smaller than the folded cell. In a later investigation of the test, the water constituent is found in the spray for the coating process. The linear reduction of the test result is hence considered as a linear reduction of material property caused by the humidity.

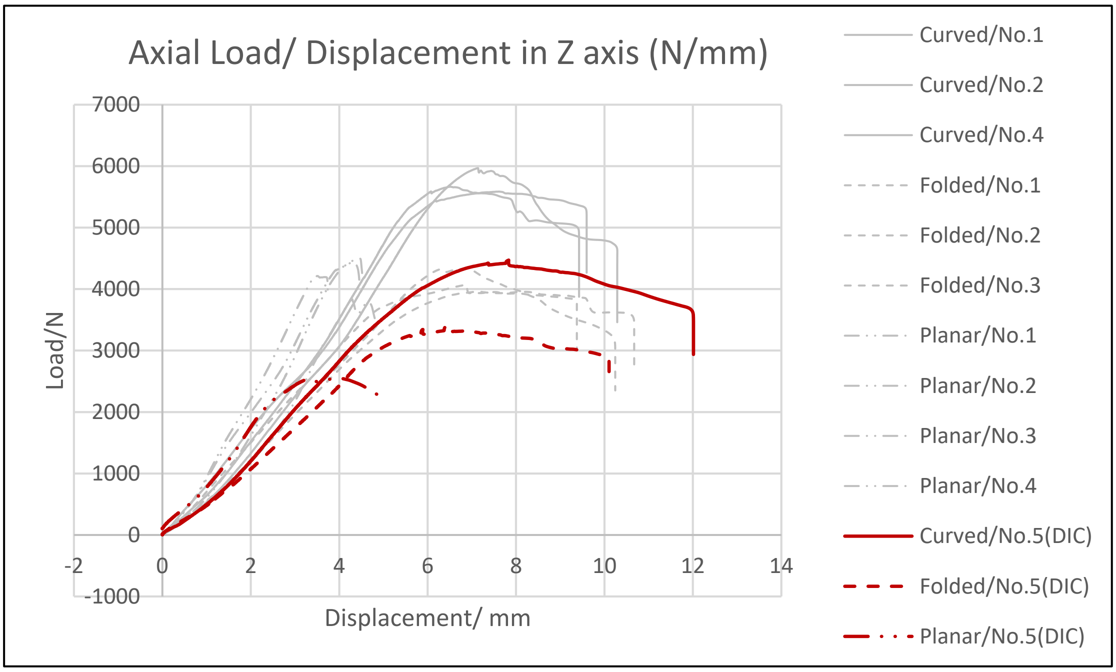


Figure 4.4. 9 Load-Displacement-diagram of the DIC groups

As a conclusion of the physical test, the most important finding is the positive local stiffening effect of the curved covering membranes. For the structural cells which weigh only about 500g, the maximum load bearing capacity of 5739N is almost 1170 times of its self-weight. In this case, it can be considered as very safe for its stability in the lightweight cellular cavity shells.

4.4.3.3 Comparison with the theoretical analysis about the buckling

For a better understanding of the deformations and the buckling behavior, a comparison is also made between the test results and the theoretical analysis in ANSYS.

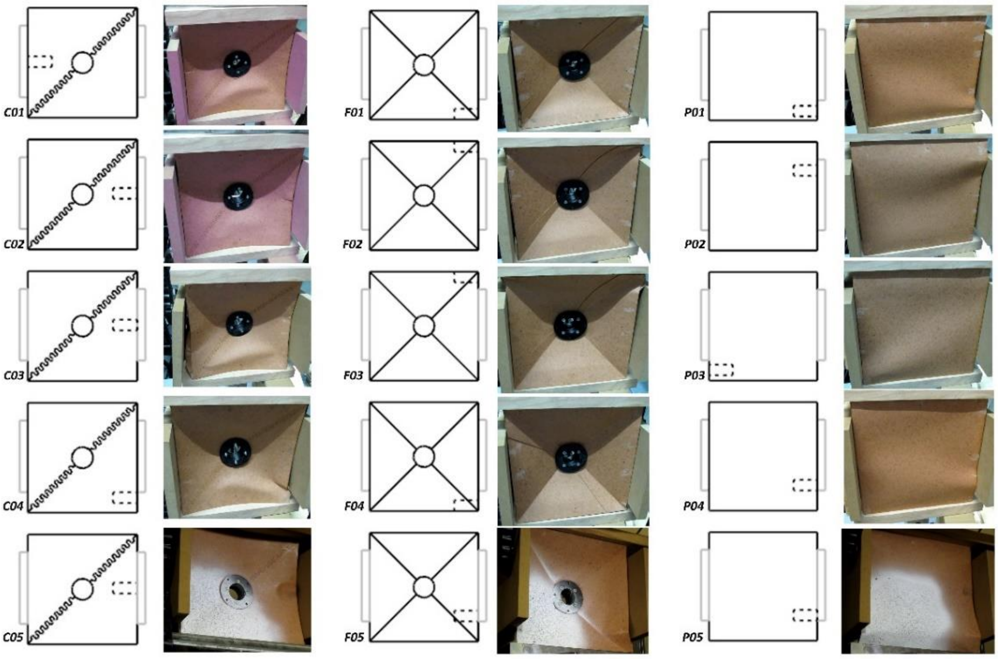
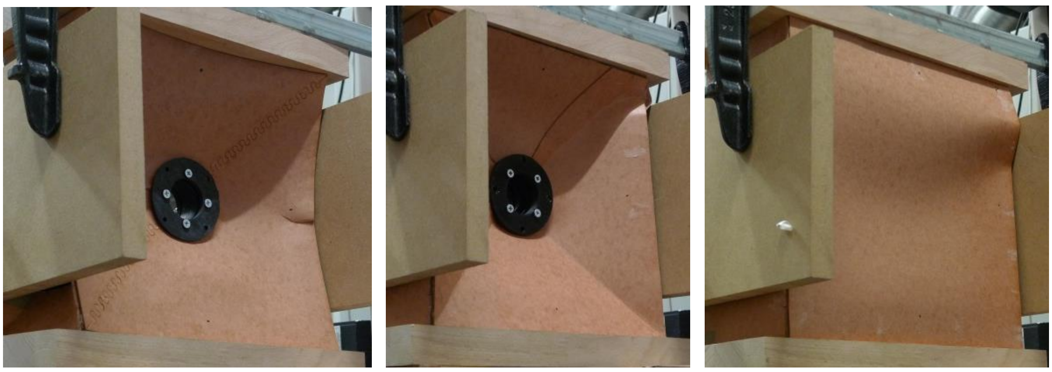


Figure 4.4. 10 Final deformation and the knick point of the surfaces

As shown in Figure 4.4.10, the final deformations and the knick points of the surfaces are mapped with diagrams. According to this, a rule can also be found with different groups. For the curved surface, the deformations are similar to the result of the FEM analysis (Figure 4.4.1) and are mainly outward bulges in the top and bottom parts and also inward bulges in the left and right parts. (Figure 4.4.10) The final knick points are mostly on the center point on the left or right edge of the surface and the center point of the surface. (Figure 4.4.11) For the folded surface, the deformations are mainly the inward bending in the center part of the surface and the slight outward bulge on the edge areas. (Figure 4.4.11) The knick points are mainly located on the four corners of the surface. For the planar surface, a typical buckling form with outward bulges on the top and bottom areas and inward bulge in the middle area can be observed (Figure 4.4.11) and the knick points are also located near the corners of the surfaces.



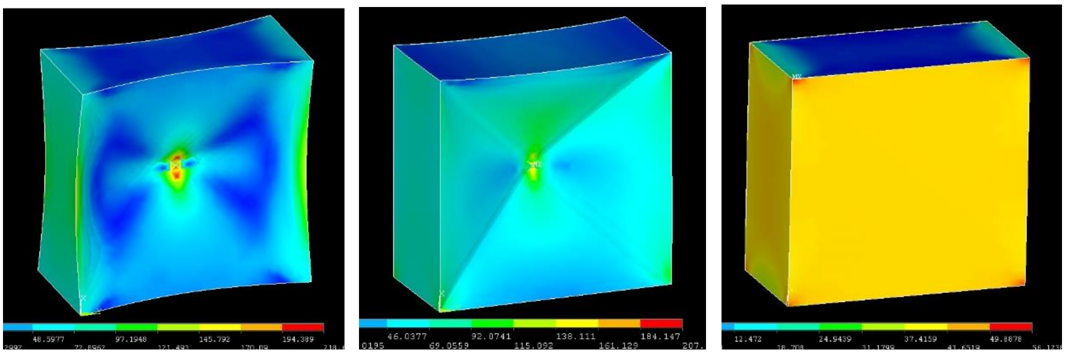


Figure 4.4. 11 Typical knick points of the surfaces and the analysis of Von Mises stresses in ANSYS analysis

Not only similar to the buckling forms in the ANSYS analysis, the deformation of the real tests can also be explained by the analysis of the stress distribution from the FEM analysis. As shown in figure 4.4.11, due to the activation of the membrane behaviors, a large stress has been seen just on the mid-area of the side edges and also on the middle point of the curved surfaces. However, in the folded surface the stresses are mainly concentrated on the corner points and the middle point of the surface. And for the planar surface, only the four corners have shown a local stress concentration.

As a conclusion, the result of the deformation and the different positions of the knick points in the real test have to some extent fit the FEA analysis and also testify the validation of the theoretical research with the perfect models. The corresponding deformation has also shown the feasibility of the construction details with the gluing techniques.

The specific critical buckling load for different surfaces is hard to determine because the criteria for the deformation are hard to establish, and the measurement is impossible for a normal group. A very small normal displacement has been shown in the FEA analysis of both the folded and planar cases. For a better understanding of the appearance of the buckling form, the DIC analysis of the deformation in the normal direction is used to be compared to the FEA analysis.

Three stages have been chosen as the appearance of the buckling form when a main deformation concentration appears at the first time on the surfaces. The results of the corresponding stages of the deformation are shown in Figure 4.4.12-14. For more detailed information of the result of the DIC analysis see the appendix 2.

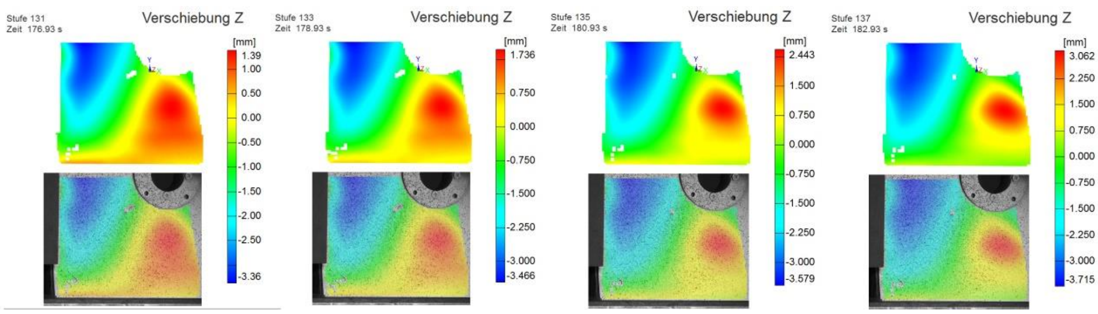


Figure 4.4. 12 Sudden change of deformation on the curved surface at a load about 3300N

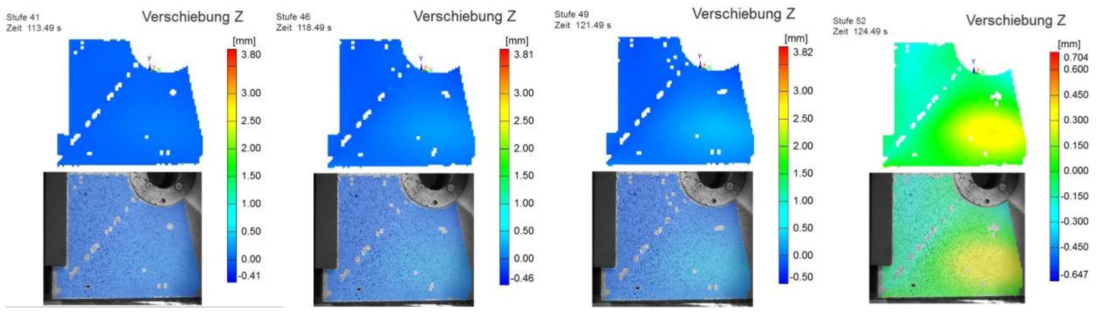


Figure 4.4. 13 Deformation from 3s to 14s of the folded surface (load under 400N)

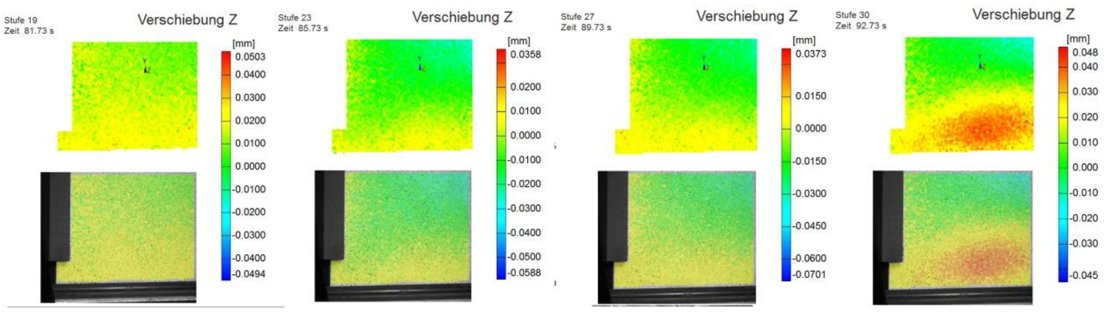


Figure 4.4. 14 Deformation from 4s to 15s of the planar surface (load under 300N)

As shown in Figure 4.4.12, the deformation stages have shown an obvious membrane behavior of the curved surface. A quasi-linear deformation in the whole area of the surface remains until the appearance of the concentrated bulge on the bottom part of the surface. In this stage, the normal displacement in the concentrated area has arrived 3mm from 1.4mm in only 6s. The stage is therefore considered as the appearance of the buckling and happens at a relatively late stage of the test (selected stage at almost 115s and the cell collapses at 150s). Compared to the corresponding Z-displacement in the FEA analysis, a comparable deformation also exists in the real physical test. (Normal displacement in the DIC analysis in the concentrated area is almost 3mm and the corresponding displacement in the FEA analysis by the buckling point is 2-3mm.) And the load value at such a stage is about 3300N.

Similarly, as shown in Figure 4.4.13 and 14, the corresponding buckling stages are both in the very early stage of the test (folded: 3 to 14s compared to the total 120s; planar: 4 to 14s compared to the total 84s) and only under a very small axial load (folded: under 400N; planar: under 300N). The deformation of such stages in the real test (folded: 0.21mm; planar: 0.02mm) is also corresponding to the result of the FEA analysis (folded: 0.3mm; planar: 0.04mm).

The result of the DIC analysis has testified the previous hypothesis about the post-buckling behavior of the planar and the folded surfaces. As it is a too early stage with a slight deformation and load, it makes no sense here to go too deep to find an exact time of the buckling point. However, it is sure that the curved surface shows its better performance with a higher ultimate load due to the stiffening effects from the curved geometry and the accompanied local membrane behavior.

4.4.4 Conclusion of the stability and design difficulties

The conclusion of the stability analysis works mainly as a guide for the decision-making processes for the type selection of the covering membranes.

Although the FEA analysis shows a large difference of the critical buckling loads for the 3 cases and has a correct prediction of the behavior which fits the result from the tests, it is from the final results that only an obvious advantage has been shown by the curved membranes and that the planar and folded surfaces provide a similar load bearing capacity due to the post-buckling behavior. In the decision-making process, such a difference of the maximum load bearing capacity is considered at the first place in the comparison.

Apart from the static consideration, the difficulties in the form finding of the variations of covering membranes should also be taken in consideration. As is discussed in section 4.3.3, the planar surfaces are the simplest variation for the fabrication process, but the form-finding for such cells requires the structural optimization of the initial meshes and the supporting rib framework. Sometimes it is even inevitable to have some deviations between the covering membranes of the adjacent cell. This complicated shape optimization process can be removed in the form-finding of the folded surfaces because a perfect form can be found by simply extruding the mesh polygon to the seed point of the cell. However, this change of geometry cannot provide a much better structural behavior to the cell as the crease by the folding cannot be arranged perfectly against the applied load. Among the three variations, the curved surface shows not only a novel shape of the structure but also a great improvement of the ultimate load with the curved geometry, and hence is most welcomed from the aspect of architectural design. However, to make such a curved surface, a complicated prefabrication process is required. For a large shell with many different shapes of cells, a predefined curved surface with a constant curvature is also favored for an efficient mass customization, and this will also lead to a relatively large deviation of the surfaces of adjacent cells. If the curved surfaces are made manually, the center point of the surface should also be cut out due to the large curvature and a too large bending radius, at the same time, a local stiffening against the stress concentration should also be considered.

The architectural design of the cellular cavity structure is a complex process and the final design should be made by a comprehensive balancing of the requirements and capacity for the building project. A compromising process should also be made between the advantages and disadvantages of the structural variations. As a conclusion of the stability and form analysis of the variations, the difference is summarized in table 4.3.

Typology	Advantages	Disadvantages
Planar Surface	Easy to fabricate	Lower stability/ Complicate form finding (optimization required)
Folded Surface	Easy to find a best form	Not much improvement in the stability problem
Curved Surface	Highest stability/ Local membrane behavior	Deviations in shapes/ Surfaces hard to fabricate/ Constant curvature required for mass customization/ Middle point of surface should be cut out

Table 4.3 Comparison between the design variations of the cellular cavity structures

4.5 Comprehensive design procedure and the considerations

After the discussion of the form-finding techniques and the consideration of the stability problems of the cellular cavity structure and its design requirements, a final summarization of the whole design procedure of the structure system is made as the conclusion of the analysis for the structure concept.

According to the abstract of design procedure in Figure 4.3.3, a fully structural analysis process should also be made for a complete design procedure with the consideration of different load cases, design of cross sections (thickness) and the detailed stability analysis. As a full discussion of this process will lead to a too complicated review of the basic structural analysis process nowadays, it will be skipped over and be shown in the next chapter with an analysis with a built pavilion.

The method of making a final summarization of the total design process is according to the analysis method by the Ph.D. research by Tessmann about the collaboration design procedure between architects and engineers (Tessmann, 2008), where a design methodology of the negotiation between form and forces and the iteration loops of the design procedure are introduced from both the architectural and structural realms. According to the abstract and assumption in the generation of the structure concept, a complete description of the workflow is illustrated in Figure 4.5.1 with detailed information.

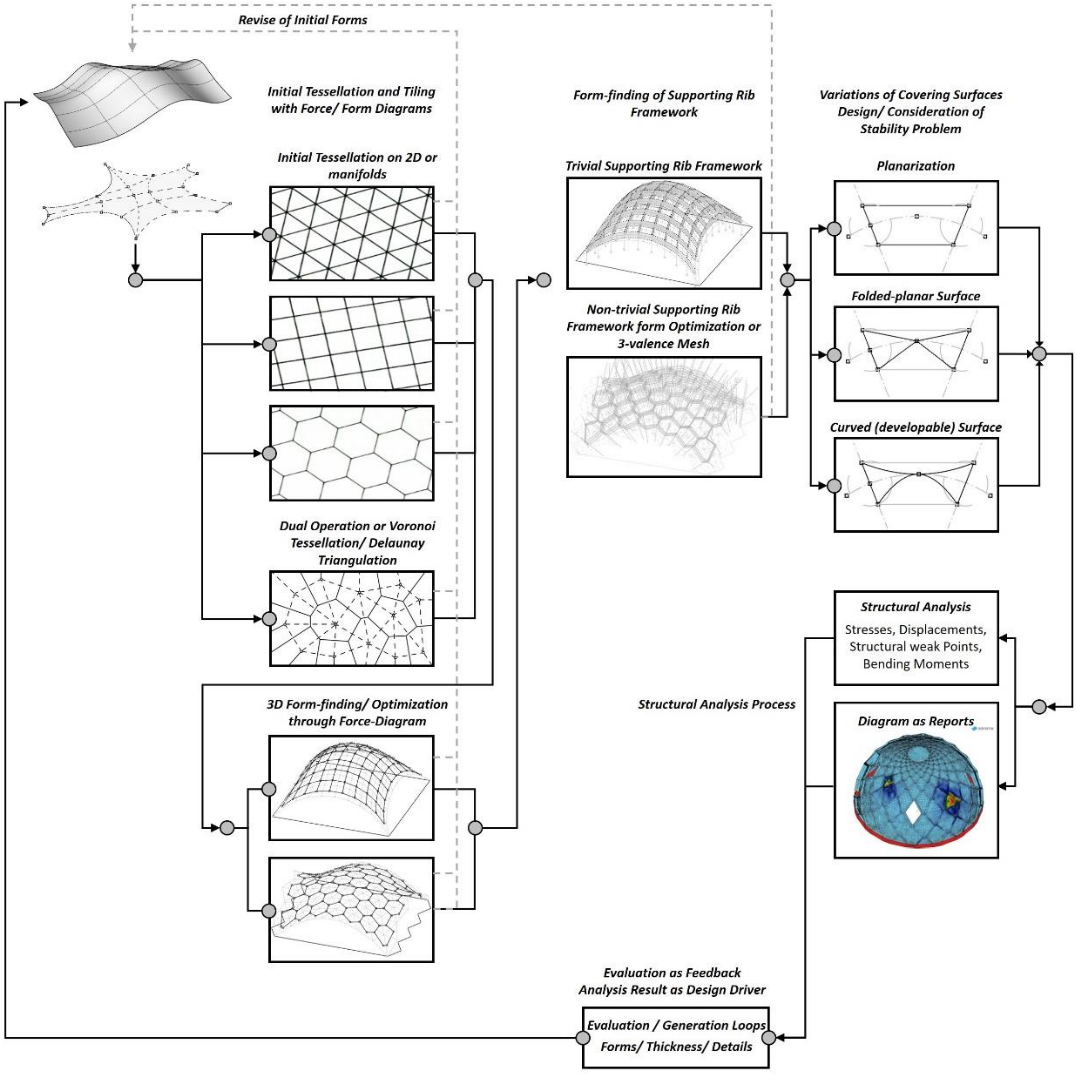


Figure 4.5. 1 Comprehensive design loops of the cellular cavity structure

5 Building experiment and construction details

As a demonstrator of the structure concept, a pavilion is built to testify the feasibility of the cellular cavity structure and also to find the potential problems for the construction details and the building techniques. In this chapter, the whole design and building process of the pavilion, from the geometric design to the final building process, will be shown, and the problems including their solutions will be discussed as a suggestion for the development of the structure concept in the future.

5.1 Introduction of the building project

5.1.1 Background and the aim of the building project

In September 2013, a research and building project for a shell-structure pavilion was started with a cooperation design process between the departments of Architecture and of Civil Engineering at the Technische Universität Darmstadt. The major works done in this research are to establish and build a prototype of a structure concept for shell structures with thin sheet materials. At the same time, an assistant structural analysis (FEA analysis with simplified models in software SOFiSTiK) of the final pavilion is done by the master student Roland Werth with his master thesis "Design, calculation and constructions of a shell structure with paper and thin glass" (Werth, 2015b). At the same time, a cooperation has been done with the help of the Institute of Paper Fabrication and the mechanical process technique. Some contacts of the material manufacturer and the technical data sheets are supplied from this cooperation.

The aim of the building project is to help the verification of the structure concept and also the building techniques of this research. In the framework of the research plan, the target of this experiment contains mainly three aspects as following:

1. Geometric Form-finding and Architectural Design: From the architectural side, the whole design process and the involved parametrical design methods are tested and evaluated. The geometric form finding of the structural components as well as the basic concept of the building system is generated.

2. Structural Analysis and the Evaluation of the Structure: Different simplifications of the pavilion are made and evaluated with the FEM analysis. The capacity of the structure is testified. Weak points and the stiffening techniques are analyzed. At the same time, the thickness of the materials is determined.

3. Design of the Construction Details and the Research of the Building Techniques: With the full scale model of a prototype with several cells the construction details and the building procedure are tested. With the final pavilion, a group building technique and also the building procedure are also tested and optimized.

5.1.2 Content and the organization of the project and designation of works

The research of the building experiment with the full scale pavilion is mainly divided into two parts: the general research of the structure concept and the specific FEM analysis of the final pavilion. Apart from the general research as shown in the above chapter, the design of the

construction details as well as the research of the building techniques and the final building process are also done separately and independently in this research.
The FEM analysis is done in July 2015 in the master thesis by Roland Werth and the relevant result will also be cited and used in this dissertation.

5.1.3 Public exhibitions and the relative roles in other research projects

Between August 2015 and August 2016, the pavilion or part of the pavilion was assembled and shown in some exhibitions. These include:

1. August 2015- October 2015: IASS (International Association of Shell and spatial Structures) EXPO 2015, team Technische Universität Darmstadt

2. February 2016: Exhibition for the application of the research “Löwe Schwerpunkt—Bauen mit Papier” (Building with papers); Shown as the on-going research and also as examples for the application of paper products in building experiments

3. August 2016- October 2016: Final assembly indoors and outdoors. Shown as the final built pavilion of this Ph.D. research.

5.2 Material selection and tensile testing

5.2.1 Material selection and the investigation of its mechanical property

As explained in section 5.1.1, the material selection was first determined due to the limited budget and the availability to find a suitable supplier.

Due to the cheap price and its infrequent application in architecture design, especially the shell structure designs, paper products and thin transparent material such as thin glass were selected as the target materials for the experimental pavilion.

Figure 5.2. 1 Selected materials and the form of standard sheets

After the investigation of the available paper products in the market, pressed paperboard was selected as the final raw material for the paper pavilion due to its higher strength. The pressed paperboard used in this design is the electronic PSP (Pressspan) 3022, which is supplied by the wholesaler ISM GmbH in Germany. The materials are supplied and transported in sheet forms with standard size about 1300*1000mm. (Figure 5.2.1) The total amount of the materials used here is 400kg. A data sheet of the mechanical properties is first gotten from the supplier (ISM_GmbH, 2014) and are shown in Table 5.1.

Property	Unit	Measured Value	Standard Value
Thickness Tolerance	%	10	Not mentioned
Density	g/cm^3	1.2	1.0-1.2
Tensile Strength (Manufacturing Direction)	MPa	85	65
Fracture Strain (MD)	%	5	2.5
Tensile Strength (Cross Direction)	MPa	50	35
Fracture Strain (Coxeter)	%	11	6.5

Tabel 5.1 Mechanic properties of pressed paperboard from supplier

For the transparent thin sheet materials the VIVAK® PET-G was used as an alternative to thin glass due to the problem of finding a supplier. Because the PET-G has a very good bending capacity and only a small amount of such materials are used in the building project, the details of the materials property is not mentioned in this dissertation. (More details of the materials are documented in the datasheet. (Plaskolite, 2014))

5.2.2 Material proof test and the parameters for the geometric design

1. Material proof test

Because the provided data sheet only showed some limited properties of the selected materials, a proof test was made to examine the more detailed material propertyproperties. The test was made in May 2015 by Roland Werth after his master study with the provided specimens from this research. The goal was to ascertain the materials' behavior and the parameters which were used in the FEM analysis. As the results are also used in the determination of the surface curvature in the geometric design, only a brief description is introduced here. For more details, the test procedures and results are documented in the unpublished test report by Werth. (Werth, 2015c)

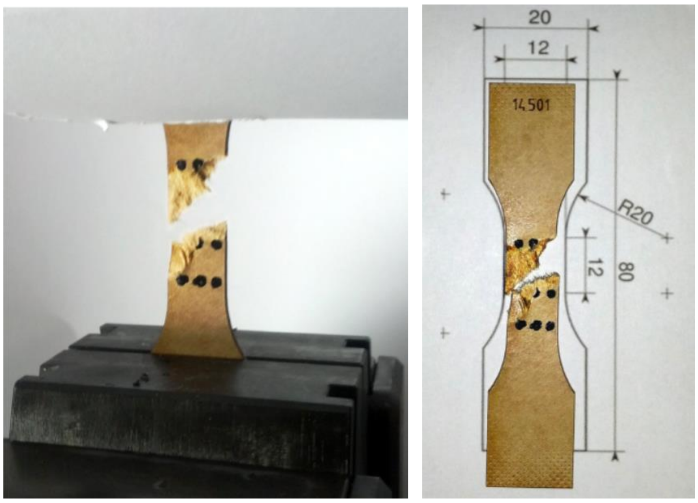


Figure 5.2. 2 Material test result of Specimen 12 (14501)

The proof tests were made in form of a tensile test, where the same testing machine as in section 4.4.3.2 is used. With 25 specimens from the manufacturing direction, the cross direction and also the diagonal directions are tested due to the anisotropic property of the material. The 24 valid results are documented and shown as in Figure 5.2.3.

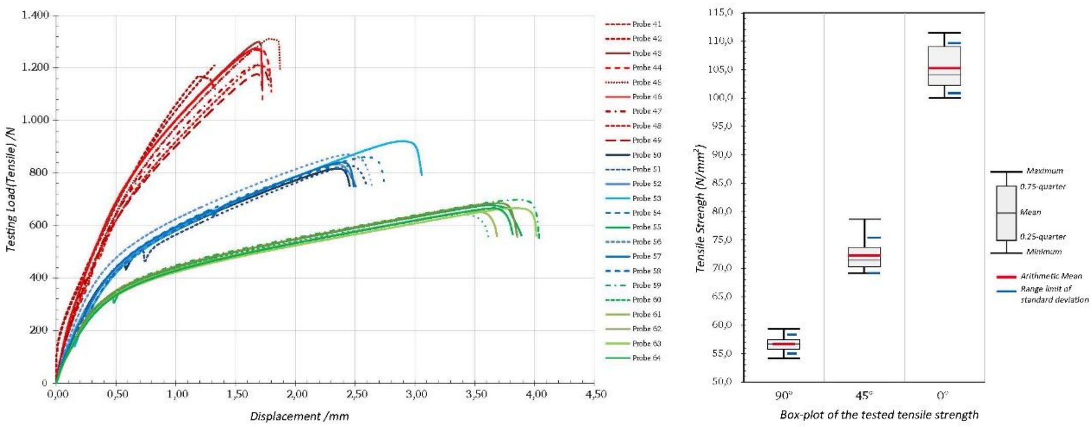


Figure 5.2. 3 Result of the material proof test

2. Post-processing and determining the material parameters

In the post-processing process a comparison between the test reports and the expectation value from the supplier's data sheet is made and shown in Table 5.2. Similar values have been shown as a proper application as the parameters of the materials in both the designs and analysis.

		Manufacturing Direction 0°	Diagonal Direction 45°	Cross Direction 90°
Number of specimens	n	9	8	7
Mean value of specimens	$\bar{x}$	105.27N/mm^2	72.31 N/mm^2	56.7 N/mm^2
Median value of specimens	M	104.10N/mm^2	71.40 N/mm^2	56.7 N/mm^2
Standard deviation of specimens	SD	4.2	2.9	1.5
Expected value from data sheet	E	105.3N/mm^2	72.3 N/mm^2	56.7 N/mm^2

5%-quarter value	$R_{p0.05}$	96.8 N/mm^2	66.5 N/mm^2	53.3 N/mm^2
2%-quarter value	$R_{p0.02}$	94.8 N/mm^2	65.2 N/mm^2	52.4 N/mm^2

Table 5.2 Comparison from the test result and the expected value from data sheet

With a following analysis of the Stress-Strain-Curve of the test result, an analysis of the non-linear E-modulus model of the material is made based on the approximation with the Taylor polynomial.

$$p(x) = k1\,x + k2\,x^2 + k3\,x^3 + k4\,x^4 + k5\,x^5 + k6\,x^6 + k7\,x^7 + k8\,x^8$$

And with a simplification of the formula a possible form was used as the description of the E-modulus:

$$\mathrm{p(x)} = \sum_{i=1}^{n} (-1)^i \cdot (-\exp(-a \cdot i^3 + b \cdot i^2 + c \cdot i + d) \cdot x^i)$$

where for MD, CD and diagonal direction:

$$a_{0°} = -0.0136 \qquad a_{45°} = -0.0125 \qquad a_{90°} = -0.0090$$
$$b_{0°} = -0.1029 \qquad b_{45°} = -0.1152 \qquad b_{90°} = -0.1454$$
$$c_{0°} = 5.5355 \qquad c_{45°} = 5.6965 \qquad c_{90°} = 4.3946$$
$$d_{0°} = 3.7379 \qquad d_{45°} = 3.6682 \qquad d_{90°} = 4.0742$$

So that a tangent E-modulus at the point $\sigma, \varepsilon = 0$ can be generated as:

$$E_{0,0°} = 9.622GPa; \quad E_{0,45°} = 4.512GPa; \quad E_{0,90°} = 3.987GPa$$

For an ideal linear approximation of the E-modulus, a secant modulus for the safe situation of the material and structures was generated with comparison to tangent E-modulus by:

$$1 \le \frac{E_{tan}}{E_{sek}} \le \frac{1}{\eta_M \, \gamma_M}$$

With a selected safety factor $\gamma_M = 1.50$ and the scale factor $\eta_M = 0.85$ a linear approximation of the material can also be generated and used as:

$$E_{0°,linear} = 5.571GPa; \quad E_{45°,linear} = 3.566GPa; \quad E_{90°,linear} = 3.041GPa$$

Hence the linear tensile strength are therefore generated as:

$$f_{t.0,linear} = 24.49MPa; \quad f_{t,45,linear} = 19.32MPa; \quad f_{t,90,linear} = 16.45MPa$$

5.3 Geometric design and the form finding

This section shows the detailed process of the geometric form finding process of the final pavilion according to the comprehensive design process in section 4.5. A full parametric and automated process is made to test the possibility for an efficient and logical design process. As the first experiment of the structure system and also a typical demonstrator for the application of the cellular cavity structure in shells, the classic form—the dome in a half-sphere shape—is selected as the initial and ideal form of the structure.

5.3.1 Generation of the initial form/ force diagrams

5.3.1.1 2D tessellation from Phyllotaxis pattern with Voronoi diagram

The initial design of the pavilion is based on the foregoing experiments of the cellular rib structure illustrated by Figure 4.1.6- D, E where an application of the Phyllotaxis pattern was used in a half-sphere surface. For the final pavilion, the asymmetric arrangement of the pattern is abandoned due to the unwanted global behavior of the structure. To make a fully automated and controllable design process, a definition is made in Grasshopper™ in Rhinoceros® with several sliders to control the parameter of the pattern and is shown in Figure 5.3.1.

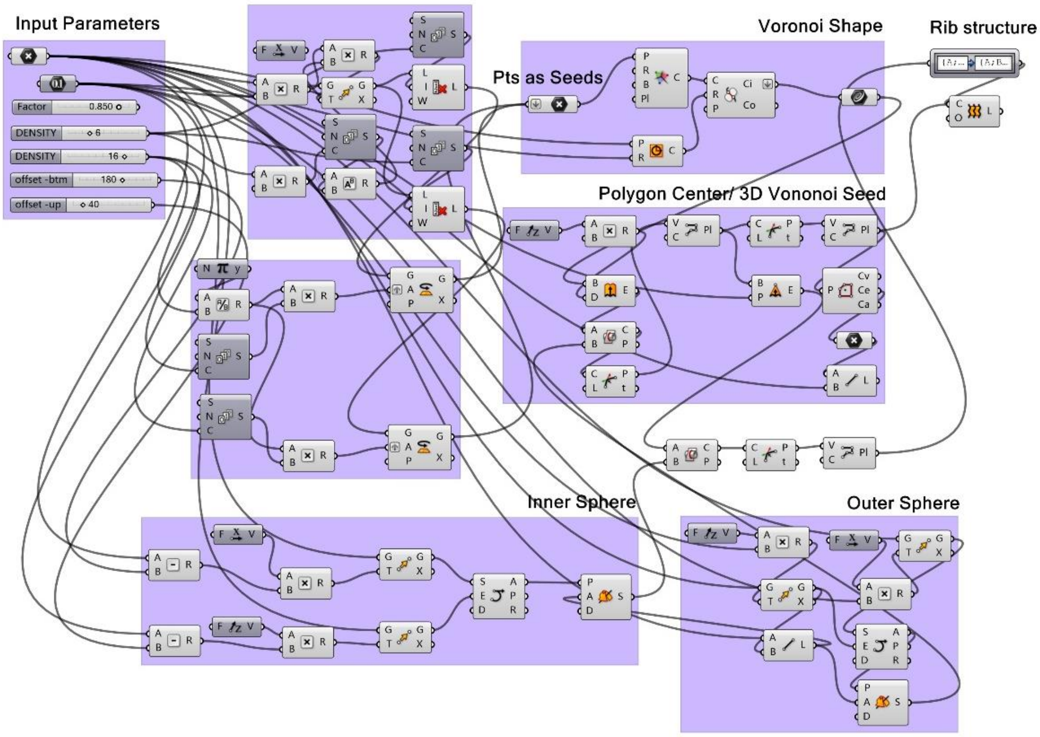


Figure 5.3. 1 Grasshopper definition for the generation of the planar pattern and the original supporting rib framework

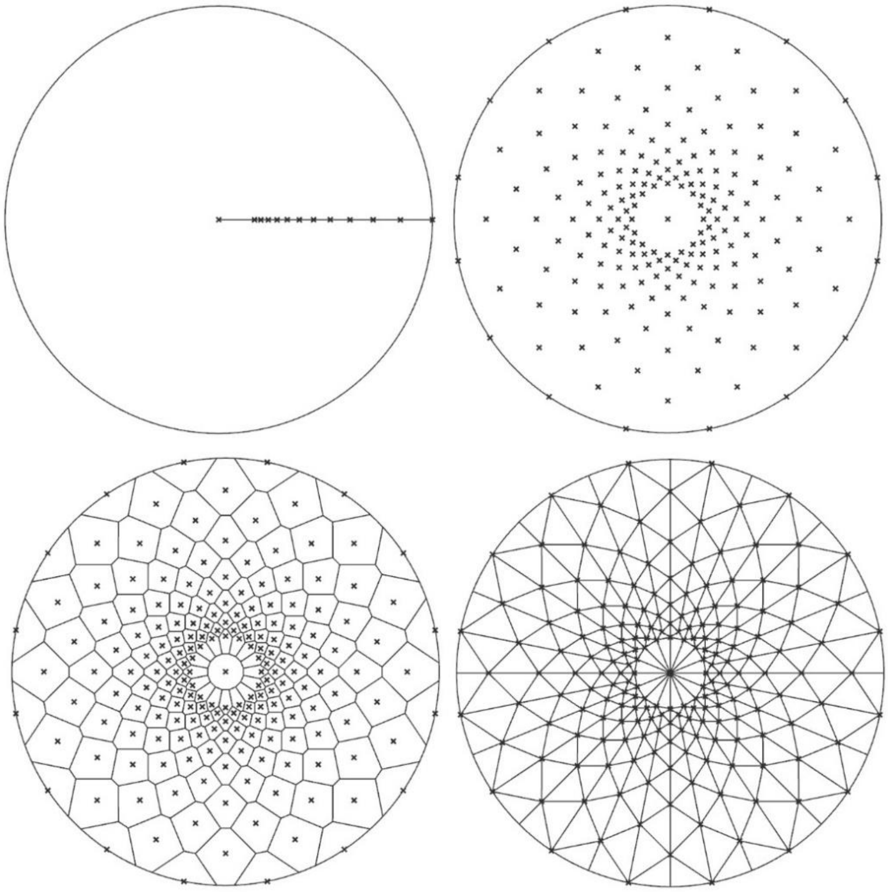


Figure 5.3. 2 Steps of the 2D pattern form-finding and the generation of the force diagram

The steps of the form finding of the 2D pattern are shown in Figure 5.3.2, where the tessellation is mainly based on the seed points whose distances between each other are following a geometric sequence. (Figure 5.3.2 A) To avoid an asymmetric arrangement of the cells according to the Phyllotactic spirals which is defined by a Fibonacci sequence, a variation is made by simply making a staggered position and a polar array of the seed points. (Figure 5.3.3 B) By applying a Voronoi tessellation and Delaunay triangulation algorithm the planar form diagram and the corresponding force diagram can be generated respectively.

5.3.1.2 3D equilibrium state of the global force diagram and the negotiation in form

The form finding of the 3D form/ force diagram is a negotiation and decision making process in this design because an initial preference to the pure half-sphere geometry is set up at the beginning of the design. This means that, with the aesthetic and architectural requirements an imperfect static behavior of the shell with unevenly distributed internal stresses and a non-compression-only situation is allowed with also a certain amount of shearing forces and bending moments in the shell.

However, a form-finding process with the simulation of the hanging techniques is also made with the Particle-Spring (Thompson & Bonner) method by using the Kangaroo 2 Plugin in Grasshopper™ (GH) for Rhinoceros®. As in the real cellular structure with a 3D curved geometry, the weight of every cell is proportional to the weight of its covering surfaces, or in other words for the area of every face in the 3D form diagram, an unequal distribution of the nodal weights is also defined (as the initial diagram is only in 2D, the original weight is defined by the area of each Voroni cell in the 2D plan). The GH definition is shown in Figure 5.3.3.

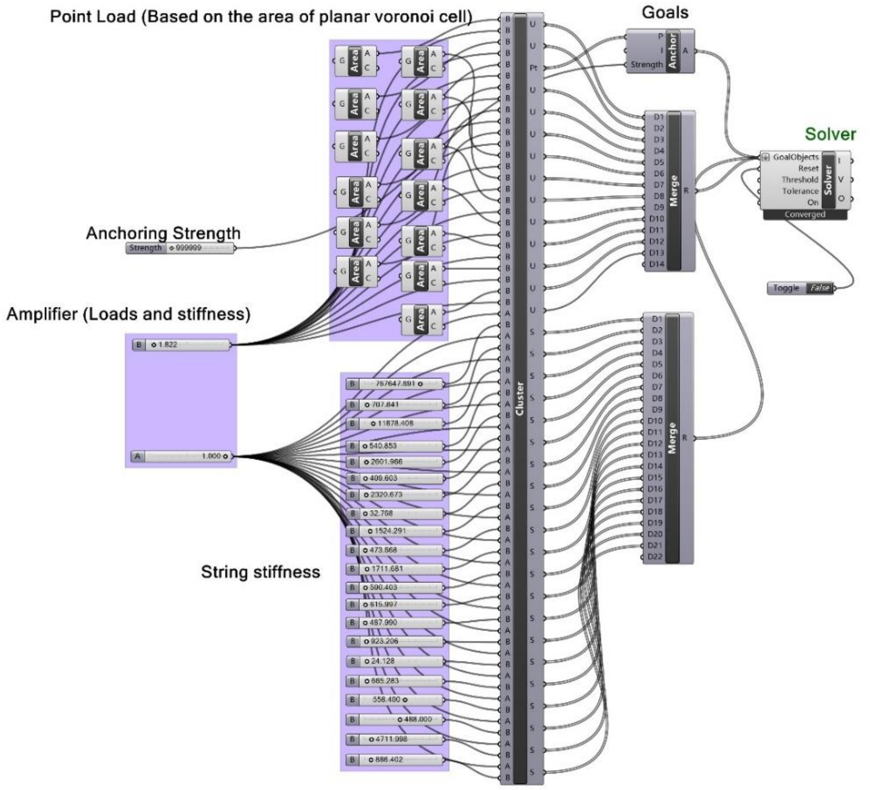


Figure 5.3. 3 GH definition of the 3D form finding tests with the Particle Spring Method

In the form finding process with the PS method, a manipulation of the variations of the final form can be reached by either changing the definition of the nodal loads or the stiffness of the in-between springs. The latter method enables unevenly distributed stresses in the shell by allowing a pre-defined stress concentration on certain selected cells so that a strategy of a specific local stiffening can be managed. In the simulation, the springs which lie in a same horizontal hoop are defined to have a same controllable stiffness so that an axial-symmetric form can be maintained. Apart from that, a general factor is also defined to control the amount of displacement of the nodes in the vertical direction.

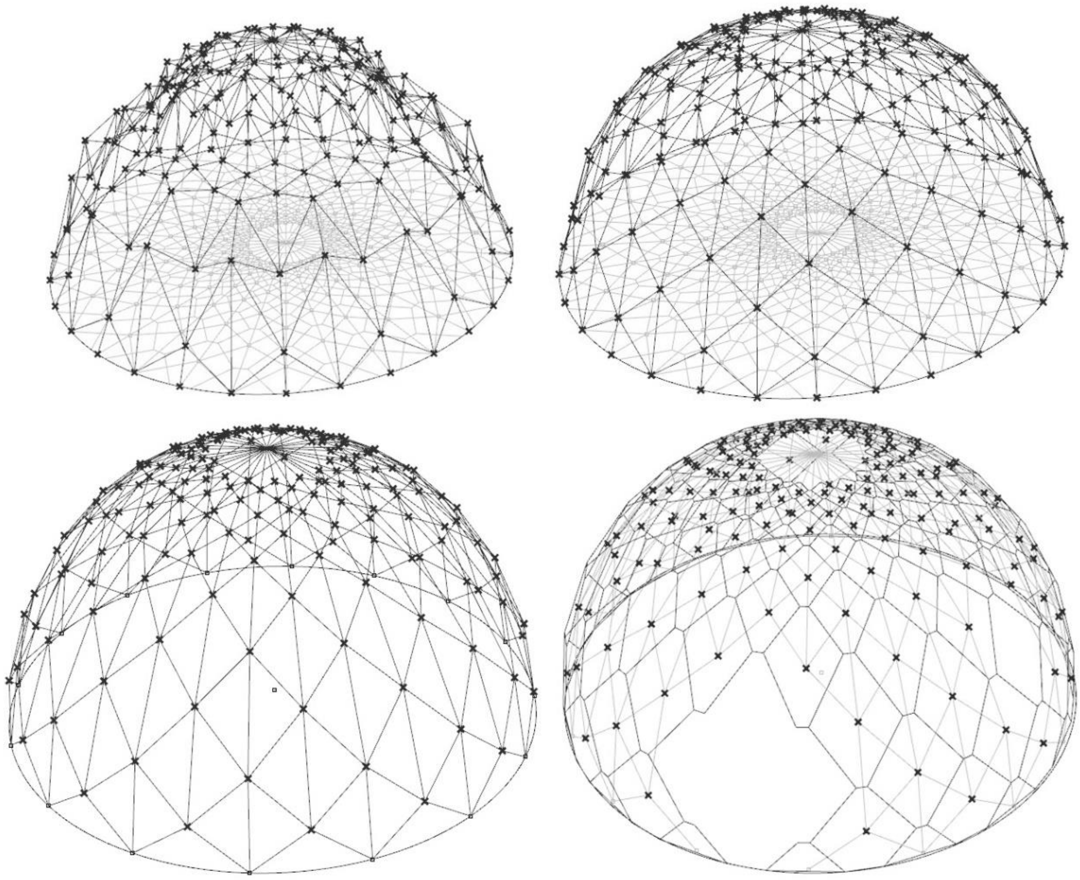


Figure 5.3. 4 Form finding with PS method and the compromising final forms as a pure half-sphere dome

As shown in Figure 5.3.4 A and B, different possible forms can be gotten with different definitions of the spring stiffness. With a combination of the manual control of the general factor and the separate spring stiffness, a quasi-spherical shape can also be generated as in Figure 5.3.4 B. With the settings of the stiffness, it can be known that the stresses are mainly distributed along the meridian direction and also function as the compressive load. The largest cells at the top and bottom of the structure will be under the most stress compared to the smaller elements in the center of the shell surfaces. In the horizontal direction, the hoop force will be much smaller and be evenly distributed on the whole shell surface. This also ascertains the membrane behavior in the initial expectation of the shell design.

However, a pure half-sphere form cannot be found when only the compressions are allowed in the shell. According to both the analysis of the spherical shell and the form finding experience, it is only possible to generate such a form by allowing the horizontal tension in the bottom part of the structure. By some cells, both the existence of the compression and tension will also cause different deformations of the supporting rib and will hence cause a bending moment on the covering membranes.

After the negotiation process and the consideration of the different problems, the initial design concept with a half-sphere surface is still selected as a compromise under the multiple criteria. (Figure 5.3.4 C) However, this also creates another objective in the test with the pavilion. As the cellular cavity structure is able to provide a stiffness against tension or the local bending of the structure, it will also show a wider range of application of such a structure concept. And with the dual operation to the final force diagram, a quasi-Voronoi tessellation can be also gotten as the form-diagram of the final design. (Figure 5.3.4 D)

5.3.2 Generation of the supporting ribs and the covering membranes

5.3.2.1 Trivial supporting rib framework with a 3-valence form diagram

The selection of the pure spherical form provides another advantage for the generation of the supporting rib framework. As a sphere center can be found for all the cells, a trivial rib system can be easily found by extruding all the edges on the form diagram to the center point. (Figure 5.3.5 A, B) At the same time, because all the lines to the center point will be perpendicular to the shell surface, this will also enable a best position of the rib system. However, due to the consideration from the aesthetic aspect, a circumcentric dual operation between the force and form diagrams as discussed in section 4.3.2.3 is revised a little to control the 3D pattern more as expected. This also causes a small inclination between the force vectors and the corresponding ribs so that an effect of the lateral deformation will be raised in the whole structure. This will also be borne by the cellular structure with its stiffness against the small bending moments and will also help to demonstrate the capacity of the structure concept.

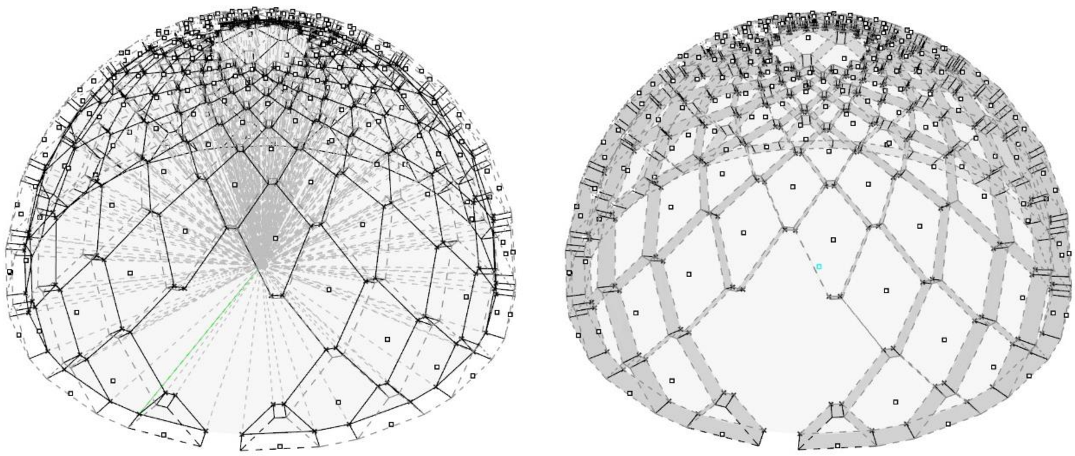


Figure 5.3. 5 Form finding of the supporting rib framework

For the opening of the structure, some cells are removed from the whole system. A new form finding is not made and a solution with a local stiffening with stronger material is chosen with the aesthetic consideration.

5.3.2.2 Conical surface for covering membranes with constant curvature

To maintain the form from more changes from the geometric optimization and also to apply an innovational form to the pavilion, the curved surface is selected as the style of the final design. According to the stability analysis in section 4.4, this variation will also enhance the stability of the cellular structures so that the 1mm thin paperboard can be used in the final pavilion.

With the consideration of a mass prefabrication of the covering membranes, a conical surface with a uniquely designed curvature is determined as the form for all the cells. Being a developable surface, the curved surface can be built efficiently by laser-cutting with the standard sheets and then by a simple fabrication process with only one type of mould.

According to the analysis of the minimal bending radius in section 3., a large stress will occur in the center part of the cone because the curvature becomes larger in the center area. A hole is cut out of the surface and the upper and bottom surfaces are hence connected together with a stiffener made either by a thick paper rope or by 3D-printed structural elements.

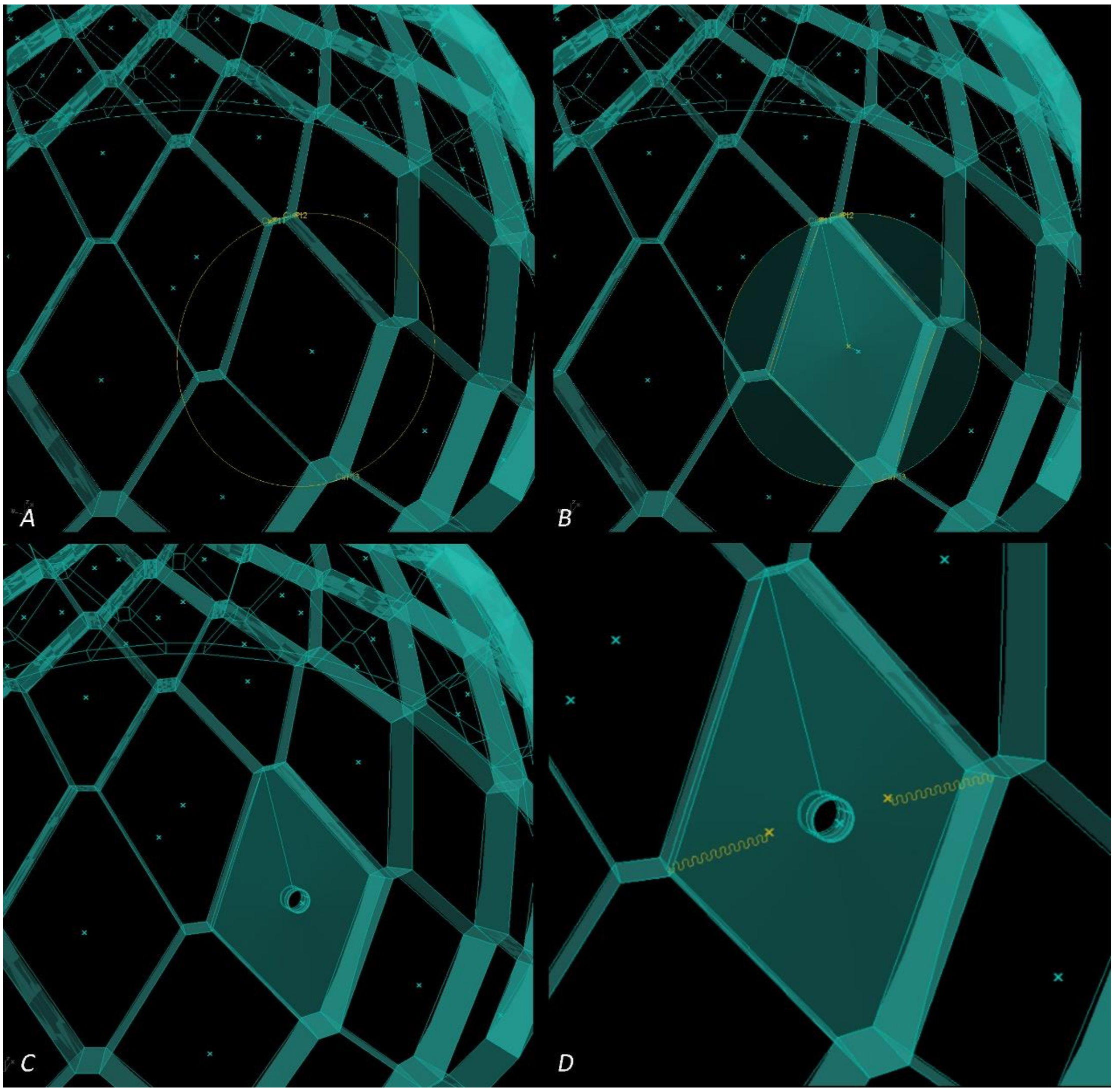


Figure 5.3. 6 Form finding of conical surface with constant curvature and its connection details (A, defining the circumscribed circle, B, calculating the curvature; C, cut out the large stress area; D, designing the connection details)

A detailed illustration of the digital form finding process of the covering membranes is shown in Figure 5.3.6. For the unique conical surface, a circumscribed circle is first defined by the 4 vertices of the upper and bottom side of the mesh face. (Figure 5.3.6 A) Because a unique curvature of the conical surface is desired, a calculation of the curvature is also made to determine the position of the cone apex according to the analysis in section 3 and also with the material's parameters in section 5.2.2. (Figure 5.3.7) After a trim with supporting ribs a small deviation will occur between the two curves on the same surface and this will also be mentioned in the later section about the construction details.

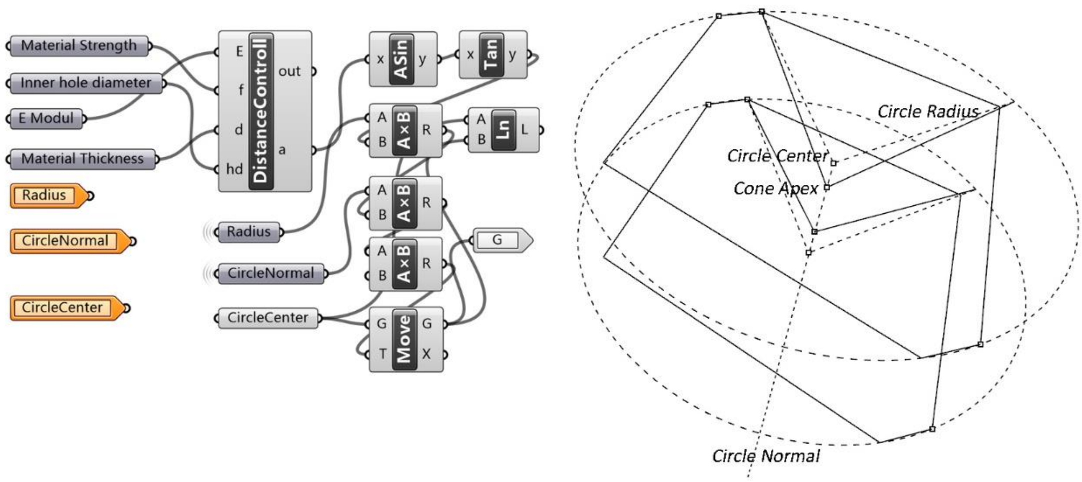


Figure 5.3. 7 Control of the surface curvature

For the prefabrication of the covering membranes, every conical surface is divided into two parts connected by a finger connection. This enables the feasibility to unroll the curved surface onto a plane without a large deformation so that it can be cut from a planar standard sheet. With the finger connection the deformation caused by the in-plane tension can be restricted and the out-plane bending can also be restricted by gluing a strip on the back side of the surface. With the definition of a parametric tool to do the calculation, such a geometric optimization can also be finished through automation for every cell in the structure.

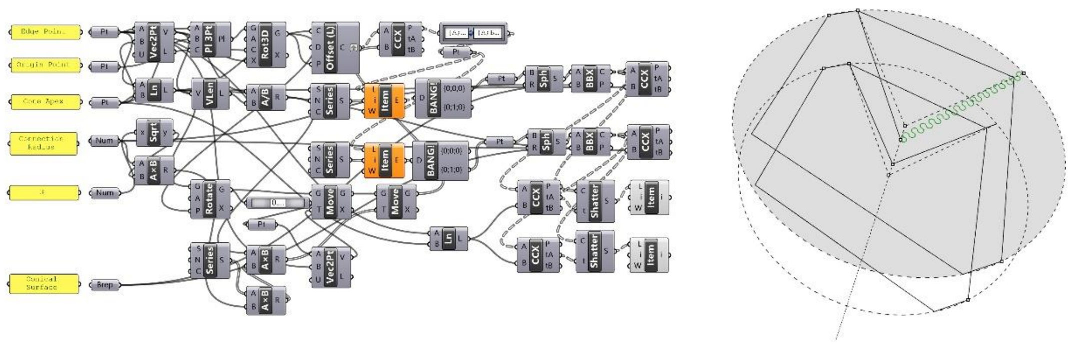


Figure 5.3. 8 Generating the finger connection details

5.3.2.3 Modification with other structural and constructional considerations

Based on the general form finding process according to the design principles in section 4.3, some specific modifications of the structural form should also be added with the consideration of the construction detail. Such modifications are both done in the initial design process as shown in Figure 5.3.9 and 5.4.10 in this section and also in the following section of the construction details.

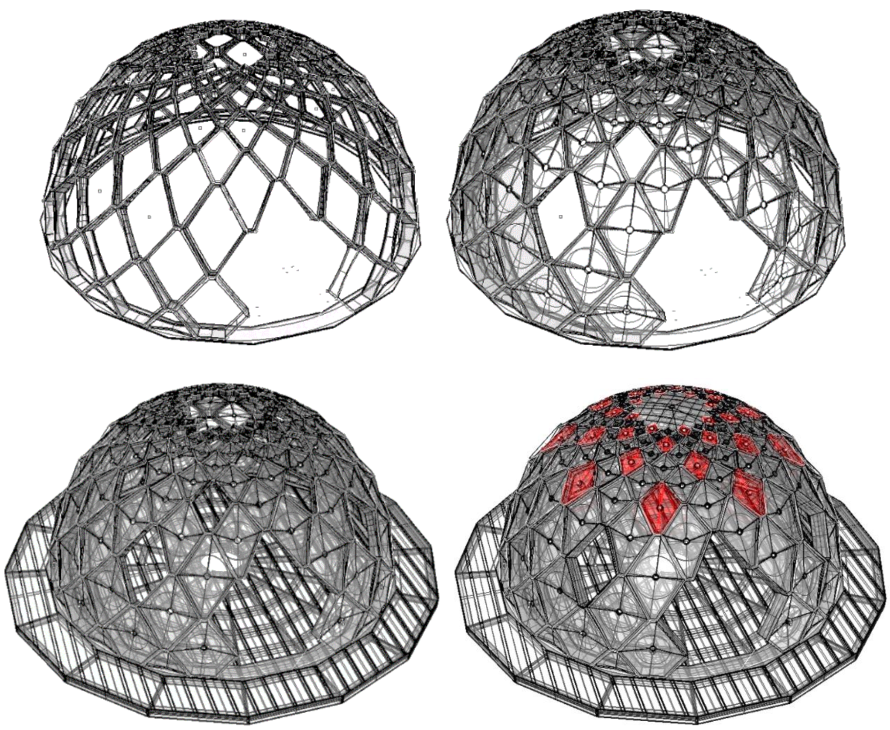


Figure 5.3. 9 Form finding of some construction details and the design of the support and foundation (A, Flanges as supporting frame for gluing; B, Tension ring at the bottom part; C, wood foundation; D, Top element and transparent cells)

As it is a low-budget project, a stable wood foundation with keels and the surfaces plats are designed as the solution (not finally built). According to the discussion in section 2.1.3.4, a tension-ring is added in the bottom part of the structure to restrict the deformation caused by the tension in the bottom areas. The tension ring is connected with the foundation with screws as pin joints. With the same method, the bottom cells are also connected to the tension ring with similar pin joints. For the opening in the shell, three variations are made and the original one is selected as the final form due to the consideration of the difficulty to make the cells which are cut by the local stiffening components. (Figure 5.3.10)

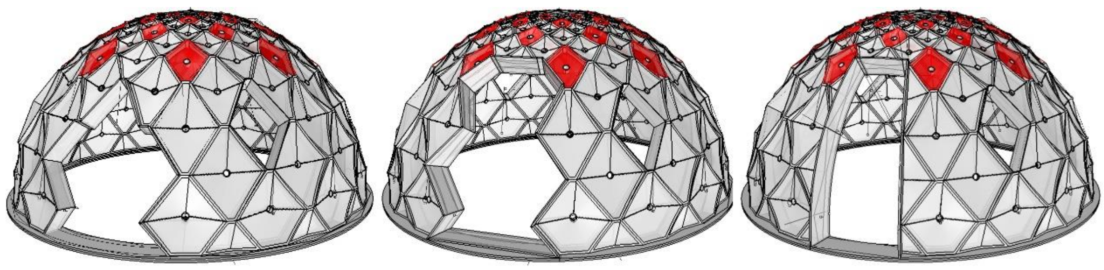


Figure 5.3. 10 Variations of the openings of the pavilion

Figure 5.3. 11 Rendering and illustrations of the whole structure and its components

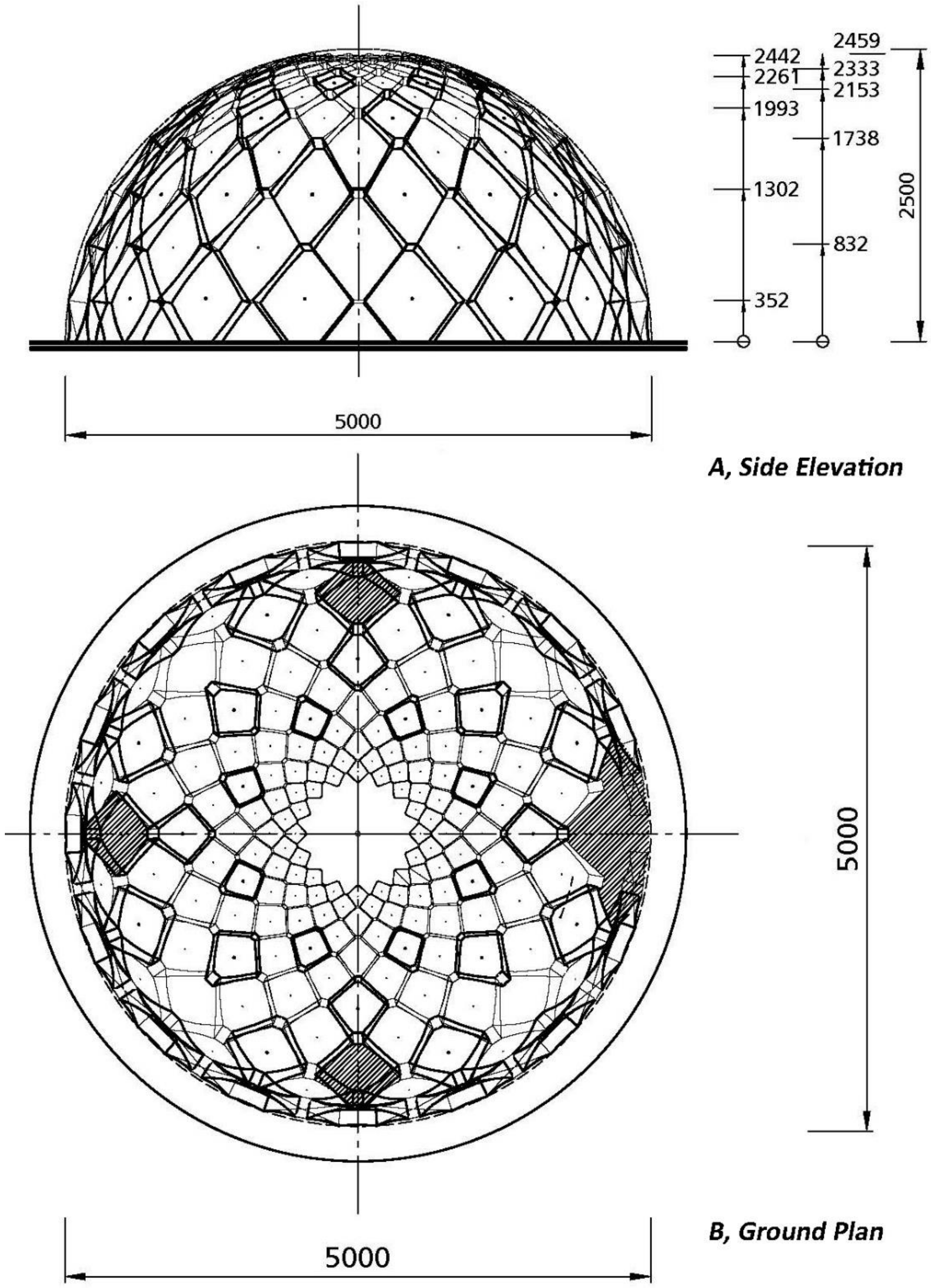


Figure 5.3. 12 Technical drawing of the design of the final pavilion

5.4 Structural analysis and comparison with different models

As introduced in section 5.1, a structural analysis and evaluation of the final pavilion are done by Roland Werth in the cooperation project with his master thesis in 2015 (Werth, 2015b). In this chapter, the result and the comparisons will be introduced to show the feasibility of the final structure and also the comparison between different structure systems. As it is out of the range of this research to fulfill a well-rounded structural analysis of the final structure, such results will also work as a feedback of the structural analysis process in the structural design to provide some advice or inspiration for the revision of the structure details. For a more detailed discussion from the structural side and the full result of the FEM analysis with SOFiSTiK, reference the final report of the structure study by Roland. (Werth, 2015a, 2015b)

5.4.1 Content of analysis with different grades of simplification

As the form of the designed pavilion is too complicated for the traditional structure analysis and the model cannot fully be well applied or rebuilt in the FEM software, a particular framework and its workflow are also designed in the comprehensive analysis process of the structural FEM analysis.

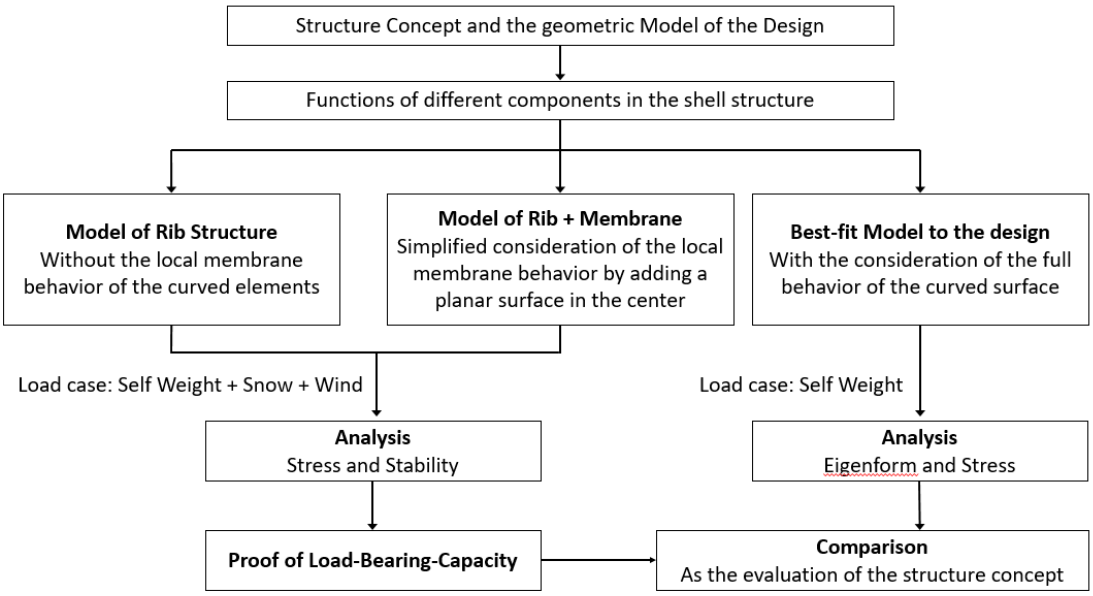


Figure 5.4. 1 Framework and workflow of the structural analysis

The simplifications and analysis of the structure are based on several grades of simplifications of the structure. In the first and simplest way, the structure is simplified as a rib structure system, where the local stability is only enhanced by some short flanges in planar forms. This enables a full analysis of the rib systems of the structure and will give a first feedback of the global behavior of the structure. The first simplification has clearly not considered the membrane behavior of the thin sheets as covering membranes. For this problem, a second grade of simplification is made to take such behavior into consideration. By inserting a planar surface in the center part of the ribs, a rib + membrane system is hence created and the membrane's thickness is defined with double the thickness of the used material. This simplification is then directly used to provide a proof of the load-bearing capacity of the structure as it is the best approximation model which can be used in an analysis with complicated load cases. The result of this simplification is also compared to the result of the rib structure so that the effect of the covering membranes on the whole structure can be analyzed. At the final step, a detailed full model of the designed form is also created to test the real behavior of the double curved membranes. Due to the feasibility with the FEA software, only the self-weight can be selected as the load case for this group and due to the huge model data, only a quarter of the structure is applied in the analysis.

5.4.2 Analysis result of the rib structure

The modelling of the rib structure is also based on the form diagram of the designed pavilion and a flange system in a double-C-form is added onto the rib framework. (Figure 5.4.2) This is mainly based on the tested building methods and the construction details to use such flange supporting frames for the gluing of the covering membranes.

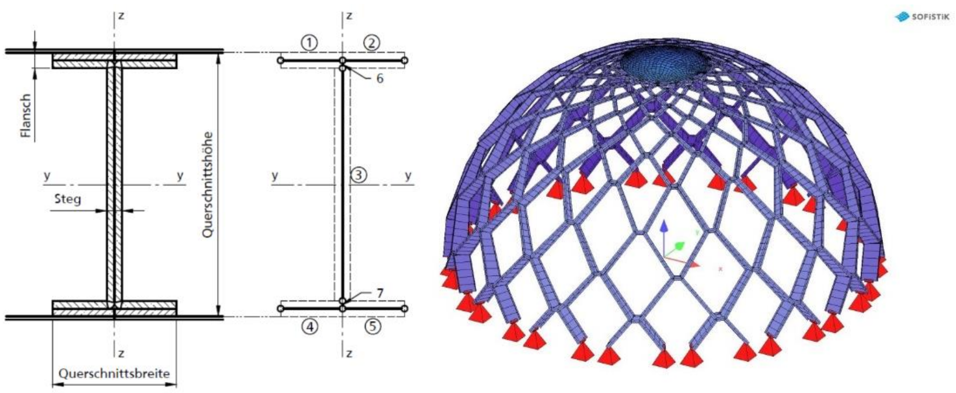


Figure 5.4. 2 Model details of the rib structure in SOFiSTiK (Werth, 2015b)

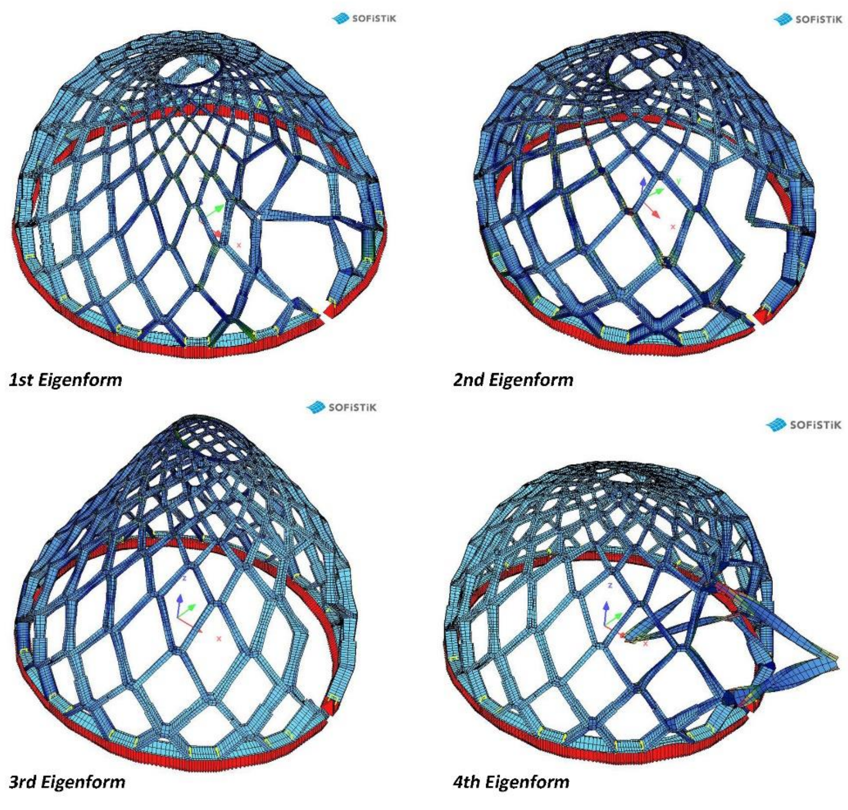


Figure 5.4. 3 1st-4th eigenform of the rib structure (Werth, 2015b)

The stability analysis starts first with the eigenform analysis of the structure, here all the 1st to the 4th eigenforms show that the opening area creates a weak point for the whole structure. The flexible thin sheets cannot provide enough stiffness when a large load or shake is added on the structure, hence local stiffening should be added on the structure as an advice for the construction details.

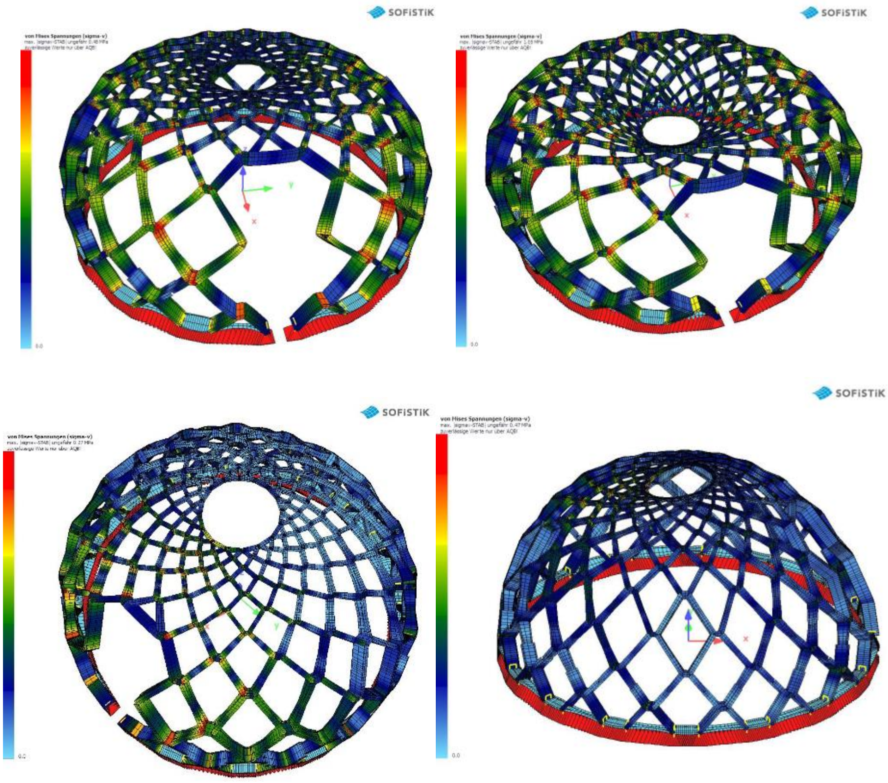


Figure 5.4. 4 VonMises Stresses in different load cases in a linear analysis (A, self-weight; B, self-weight+ wind; C, self-weight+ wind (changed direction)) (Werth, 2015a)

The structural analysis with different load cases shows the marginal load-bearing capacity of the rib structure, where in the symmetric load case an evenly distributed stress and in the load case with wind a stress concentration can be observed around the opening areas. The normal stresses are transferred along the ribs to the supports and the larger stresses are distributed in the very top area and also the bottom areas.

The analysis of the rib structure provides a basic understanding of the global behavior of the structure and also testifies the expected load-bearing behavior of the rib structures.

5.4.3 Analysis result of the grid-shell with membrane behavior

A simplified hybrid structure is modelled as a more precise approximation after the understanding of the global behavior of the rib structure. (Figure 5.4.5) By adding a planar surface into the middle area of the rib structure, a grid-shell with a membrane behavior is therefore generated. This enables the evaluation of the effect of the membrane behavior on the global or even the local areas of the whole structure and a more precise analysis of the weak points in the structure..

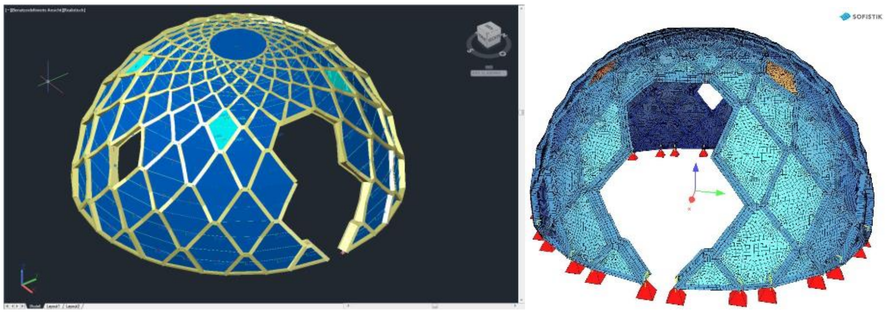


Figure 5.4. 5 Modelling of the simplified membrane structure by inserting a planar surface in the center part of the rib structure (Werth, 2015b)

The eigenform and the first order structural analysis don't show a great difference of the global structure's behavior but only the activation of a large proportion of the membrane forces in the planar surface. A decrease of the maximal stresses have been shown in the ribs when it is compared to the pure rib structure. However, as the additional membranes also lead to an increase of the structure's total weight, the stress ratio is only decreased by a small amount.

In the grid-shell with membranes, the stresses in the membrane plane are much more than the stresses in the supporting ribs. This also caused the failure of the partial membranes, especially in the top and bottom parts of the structure. The large proportion of the membrane forces testifies the expectation of the local membrane behavior and its effects on the stress distribution in the supporting ribs. The bad result of some of the membranes is also acceptable as the effect of the double curved membranes is obviously underestimated here. As the data of the proofs of stresses is too huge, only the results are shown in this dissertation and more detailed proofs can be found in the total report of the structural analysis of the pavilion. (Werth, 2015a)

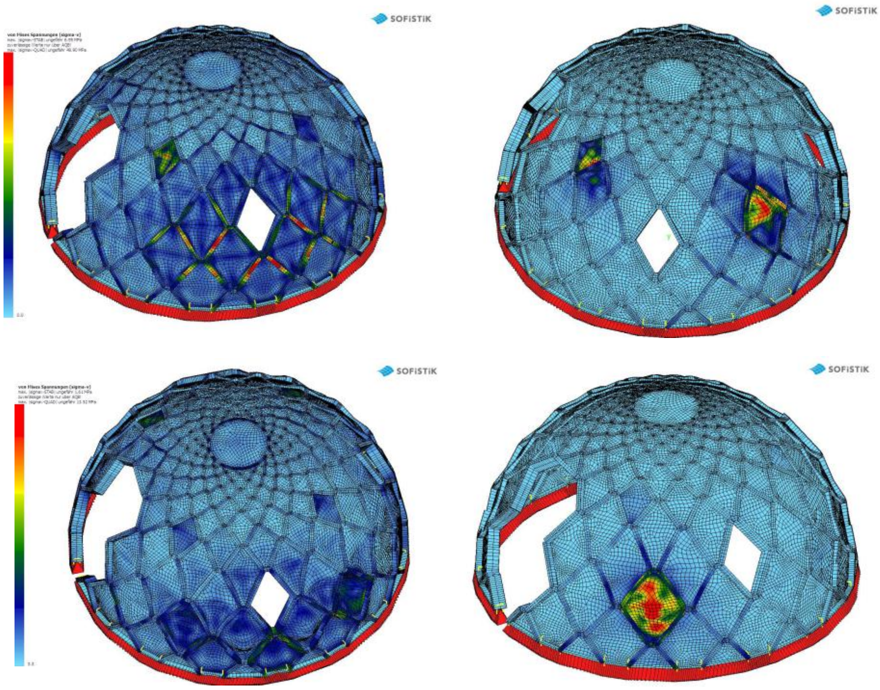


Figure 5.4. 6 Analysis with load case Self-weight + Wind and the buckling figure. Top: without imperfection, Load iteration η_T =16.25, Buckling factor η_b =1.07; Bottom: with imperfection $\eta_T = 3.7616.25$, η_b =1.08 (Werth, 2015a)

The stability problem is also analyzed with the 2nd as well as the 3rd order structural analysis with the consideration of an imperfection which is defined by the 1st eigenform of the structure. With a load iteration, the stability of the structure is tested. (Figure 5.4.6) The result also shows the effect of the imperfection to the ultimate load of the whole structure, and also figures out the importance to enhance the local stability of the membranes as well as the opening areas.

5.4.4 Analysis result of the full structure with curved membranes

As the function and effect of the double layered covering membranes are too underestimated by both the simplified structural analyses, a more detailed model is made and tested with the self-weight to make a comparison. As the modelling process is too complicated and the calculation task is too huge for the structure, only a quarter of the full structure is made according to the axial-symmetry and centro-symmetry properties of the structure (Figure 5.4.7)

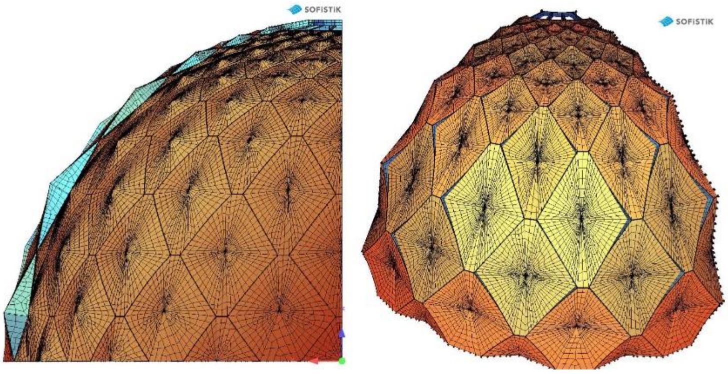


Figure 5.4. 7 FEM model of the full structure with double curved membranes (Werth, 2015b)

The eigenform analysis shows the weak point of the structures first by the foot points of the structure, which also fits the result of the global behavior of the rib shell and the simplified shell. The buckling of surface starts at first form the center point of the cell possibly due to a stress concentration and then rapidly expands to the whole foot area of the structure. The behavior fits also the experience from the physical pressing test with the full-scale cells.

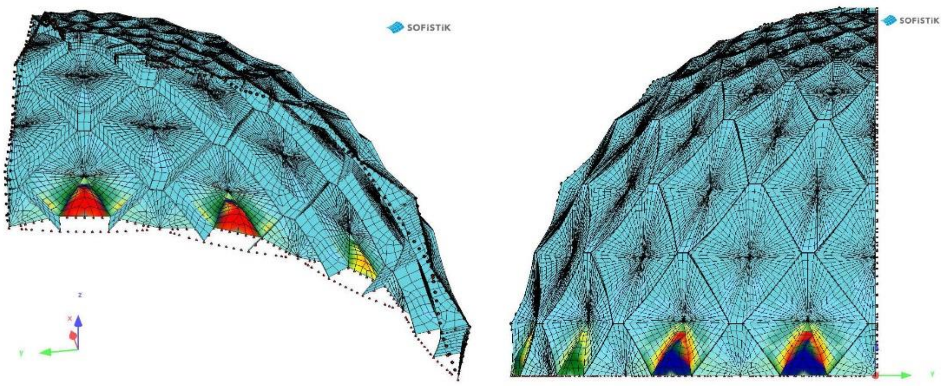


Figure 5.4. 8 1st eigenform of the full structure (Werth, 2015b)

The structural analysis of the full structure under its self-weight shows the expected result of the design and the assumption. Both the global and local distribution of the stresses fit the typical behavior of the half-sphere shell (stresses are mainly distributed along the meridian direction as compression, and also along the horizontal direction as compression in the top parts as well as tension in the bottom parts. More stresses are shown by the larger cells at the top part and the bottom part of the structure.). Compared to both the rib structure and the simplified membrane structure, the stresses in the full structure are significantly smaller for both the ribs and the covering membranes. In this sense, a very small stress ratio appears and a reserve of the load-bearing capacity remains in the structure. The lower amount of stresses in the supporting ribs also fits the theoretical analysis by Flügge (Flügge, 2013) about the double-layered shells.

The analysis also shows an expected local stress concentration in the central part of every cell as well as a very small lateral deformation of the supporting ribs. This result also fits the design concept with two membranes placed against each other and bonded in the center. At the same time, it shows the necessity to design a local stiffening component also in the center of every cell.

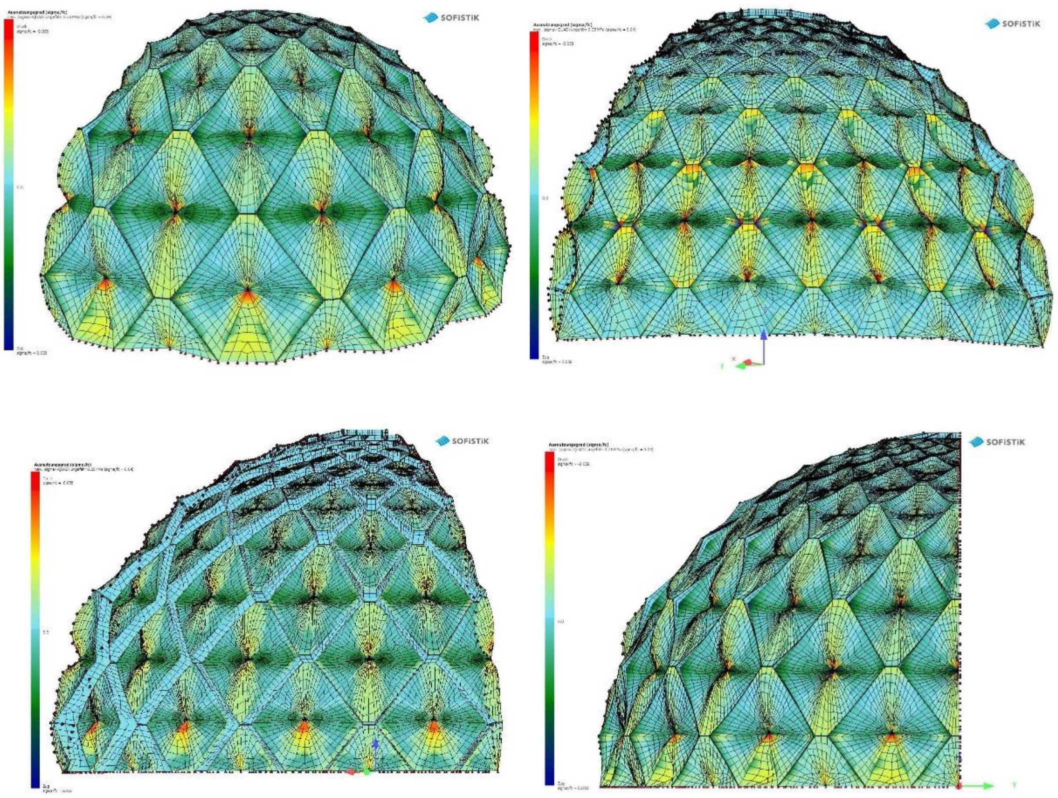


Figure 5.4. 9 Stress ratio of the full structure under its self-weight (Werth, 2015b)

Nevertheless, the full model with a quarter of the structure cannot be applied to any asymmetric load case with wind or an external extra load. Meanwhile, it also shows a disadvantage that a very intensive calculation process is required for even the most simple load case with the self-weight. The construction details such as the gluing connections as well as the connection between cells cannot be modelled or analyzed in the structural analysis because it is too difficult to finish a proper modelling or make a definition of an efficient calculation method. Therefore, the analysis only provides a basic understanding and evaluation of the feasibility of the structure concept.

5.4.5 Conclusion and the suggestion of the construction details

As a conclusion or feedback of the structural analysis, the basic behavior of the structure is shown to the final revision of the structure. As the global behavior doesn't show too much difference and the stress ratio is also very low in the full structure, the basic form of the structure and initial definition of the thickness of the materials remain the same. However, as all of the analyses show the similar weak points of the structure, a local stiffening strategy is made with some suggestions of the construction details.

1, Local stiffening for top element and opening

As it is shown by the analysis of the simplified model with membranes and also the full structure, the top element of the structure is under a big compression and also a relatively large bending moment. As a simple solution with the use of the cavity space in the cell, a stiffening method is made by an interpolating core made from 6mm corrugated cardboard. (Figure 5.4.10 A)

For the openings in the structure, a passive method is used with a local stiffening component with 25mm honeycomb panels. (Figure 5.4.10 B) As the panels have a strong stiffness against bending, this is considered the most efficient solution for the local stability problems.

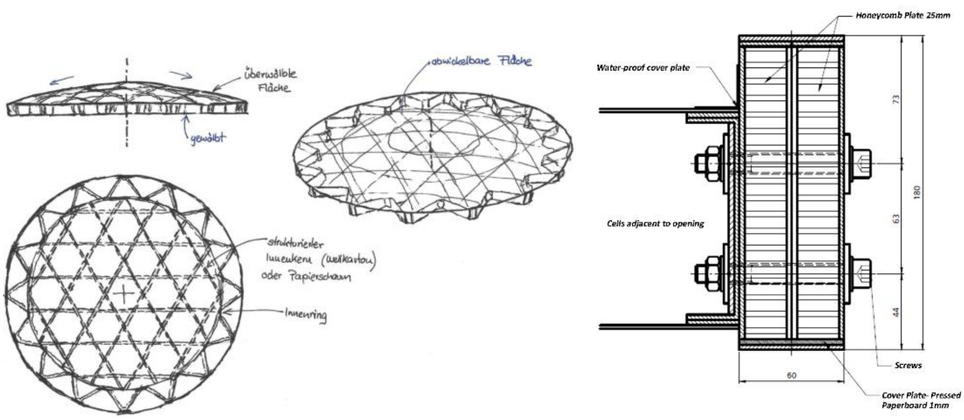


Figure 5.4. 10 Constructional suggestion for the top element (A) and the openings (B) (Werth, 2015a)

2, Local stiffening for bottom elements

For the bottom cells, a similarly large stress and bending moment also appears under a principle meridian compression. For the construction details, 2 variations are suggested as the solution similar to the top element. (Figure 5.4.11)

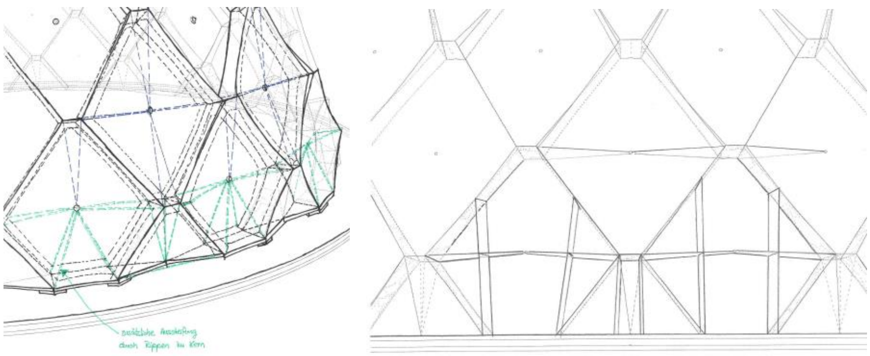


Figure 5.4. 11 Constructional suggestions for the bottom elements (Werth, 2015a)

5.5 Construction details and prefabrication

For the introduction of the experimented construction details and the prefabrication of the structure, both the improvements in designs and the full-scale building process with the prototypes are illustrated in this section. It starts first from the prefabrication of the structural elements of each cell, and then comes to the fabrication process of cell groups and also a following building method with large structural components with a large group of cells.

5.5.1 Prefabrication from standard thin sheets

1, Geometric post-processing with cells

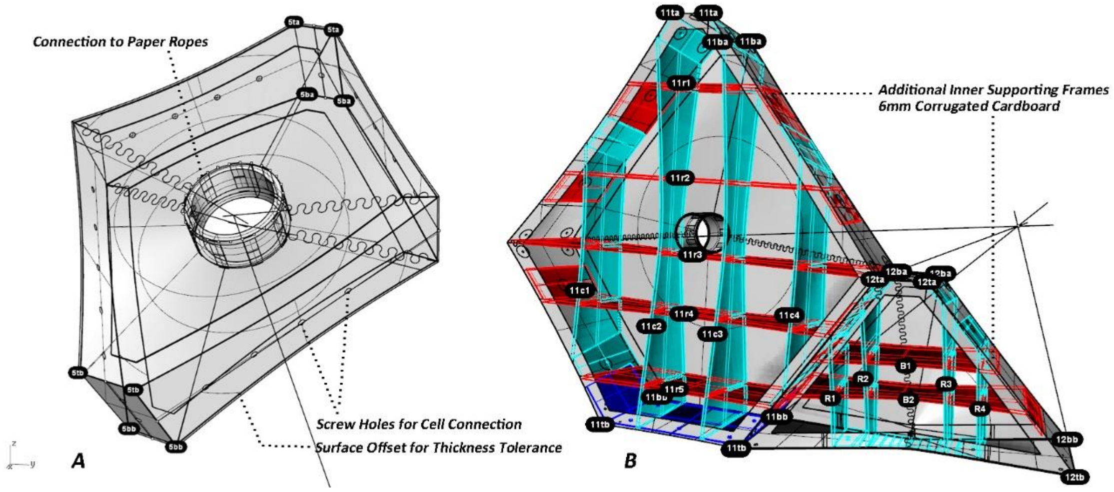


Figure 5.5. 1 Geometric post-processing on the construction details for each cell

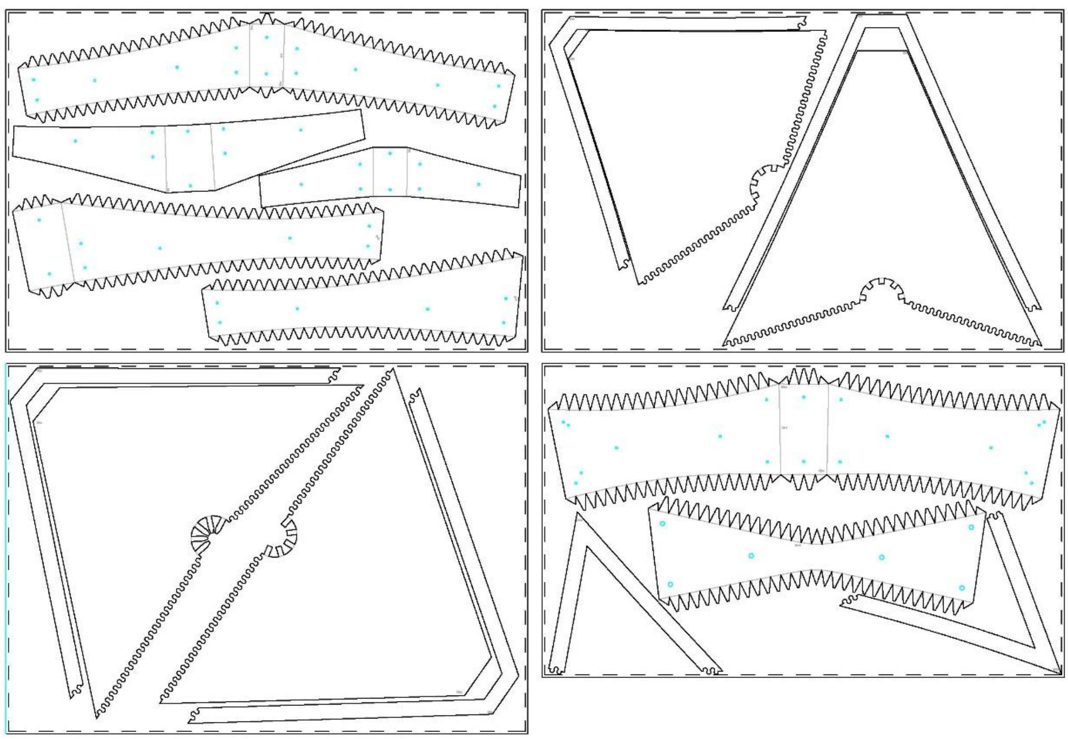


Figure 5.5. 2 Layout for the lasering of the standard sheets

As shown in Figure 5.5.1 A, an improvement in the geometric design is made based on the initial geometry of the cells in the pavilion. With the decision of the material thickness after the structural analysis, the geometric tolerance is solved in the design by offsetting the original curved surface according to the fabrication sequence. Meanwhile, the connection details are also made after the selection of the paper ropes as the local stiffening elements.
For the bottom cells, as it is suggested in the structural analysis that extra supporting components should be used to guarantee the stiffness of the cells under supposed snow load or external concentrated loads, a detailed form finding of such supporting components is made so that they can be simply prefabricated with e 6mm corrugated cardboard by interpolating the horizontal/ vertical plates on to each other and a simple gluing with the cell ribs. (Figure 5.5.1 B)

The most important goal of the geometric form finding process of cells is to make all of the structural elements developable and able to be cut from the standard sheets. As a result, the final layouts for the lasering job are made out (Figure 5.5.2). The ribs are in this process oriented all along the manufacturing direction of the sheets. However, due to the economic nesting and the large size of the covering membranes, a rotation is frequently used in the final layouts.

2, Prefabrication of the cell elements

The prefabrication of the cell elements is mainly achieved by the cutting technique with a CO2-laser cutter, where a minimal tolerance can be guaranteed for the complicated geometry. (Figure 5.5.3) According to the building process of the testing cells as shown in Figure 4.4.6 in section 4.4.3, the forming process of the supporting ribs and the gluing of the supporting flanges can be easily made by the simple folding and gluing technique.

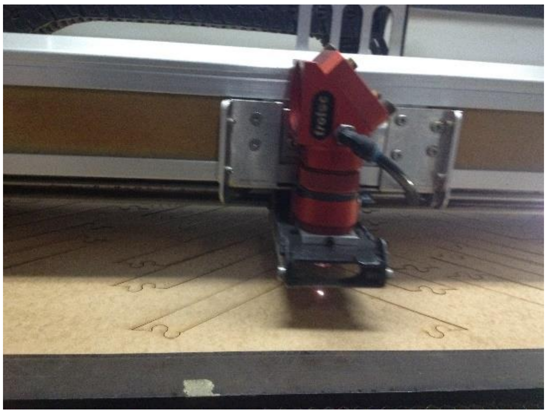



Figure 5.5. 3 Fabrication of the basic element- lasering with standard sheets

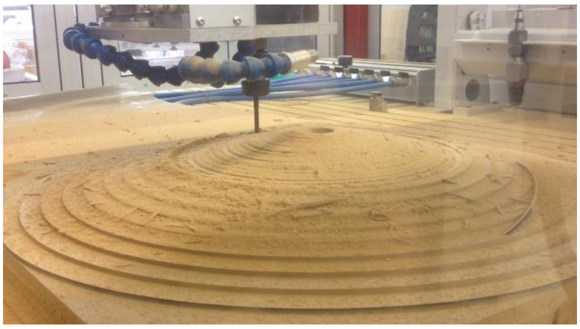



Figure 5.5. 4 Fabrication of the covering membranes- CNC milled mould (A) and the fabrication (B)

The conical surface of the covering membranes is the most complicated part of the prefabrication of the structural elements, because a development of the curved surface cannot be achieved without a break in the continuous surface. In this way, a connection technique is required for the prefabrication of such membranes. With the previous consideration of the mass fabrication of the membranes by using a unique pre-calculated curvature, a mould can be made with the 3-axis CNC milling machine and helps the forming process of the conical membranes. (Figure 5.5.4) Unlike the prefabricating process of the cells in the pressing test, only some adhesive tapes are used for the fixing of the positions and the resistance against the rebounding deformations. This technique has been shown as a marginal technique in the later building tests and also the lasting exhibition process of the pavilion. A final discussion of a possible improvement will be introduced the last chapter as a suggestion for the technical improvements and also for the future study of this research.

The prefabrication of the inner supporting core element is shown in Figure 5.5.5, a time-saving process can be achieved well with the assumed fabrication techniques. Such additional inner supports have shown a good result where the pure cavity structure is improved into a stiffer sandwich structure. It provides a better stiffness against the local bending or the external forces normal to the surface. At the same time, for the huge cellular elements, the inner support core also helps to fix the form as only the bounding ribs are too flexible when a large size is used. The inspiration of such an additional structural element will also be discussed in the final chapter as the inspirations from the building experiments.

Figure 5.5. 5 Prefabrication of the inner supporting core elements in the bottom area

5.5.2 Prototypes of individual cell and cell groups

As the designed cellular cavity structure has only closed structural cells, it is necessary to make a plan of the fabrication sequence of the structural cells. At the same time, as there are almost 200 cells in the whole structure, a building method with different groups of cells is also considered as a typical method for a better distribution of human power.

1, Structural components and the fabrication sequence with a prototype

The plan of the fabrication sequence of the structure cells and the cell groups is simply according to the hierarchy of the structural elements in the cellular structure. The supporting ribs should be fabricated in an approximate form in order to define the shape of the cells. As it is hard to glue the covering membranes directly onto the small finger strips on the ribs, two assisting supporting flanges should then be glued with the ribs. The flanges will help to establish a quasi-hinged connection between the ribs and membranes and also to provide an initial stability of the cellular structure so that the final shape of the cell can be adjusted and guaranteed with minimal deviation. After this process, the covering membranes should be glued onto the flanges and be hold in the position until the finish of the solidification of adhesives. As the final stage, the paper rope should be inserted and glued with the flanges of the covering membranes so that the local stiffening can be achieved.

An illustration of the fabricating process of the cell groups is shown in Figure 5.5.6 as following:

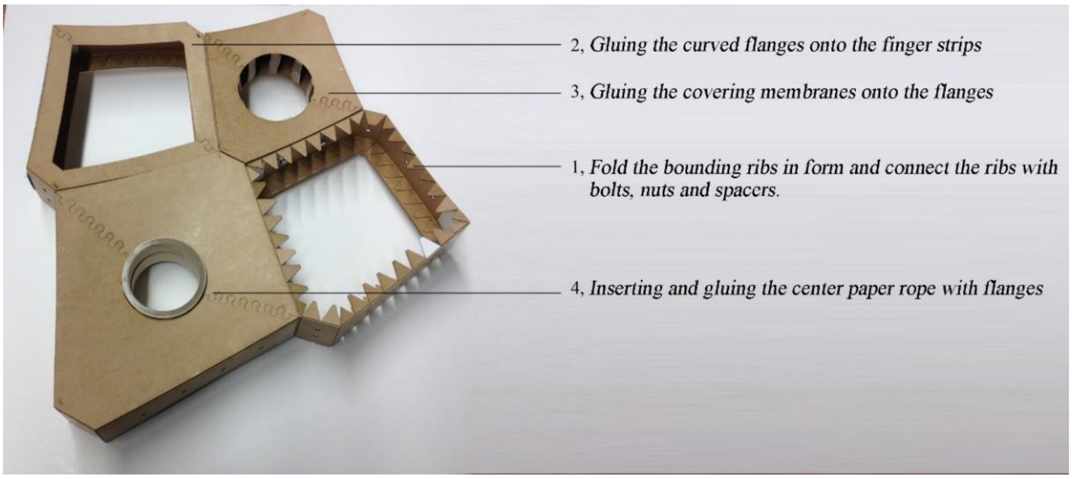


Figure 5.5. 6 Illustration of the fabricating sequence of the structural cells and the cell groups

2, Prototypes for variations of details and materials

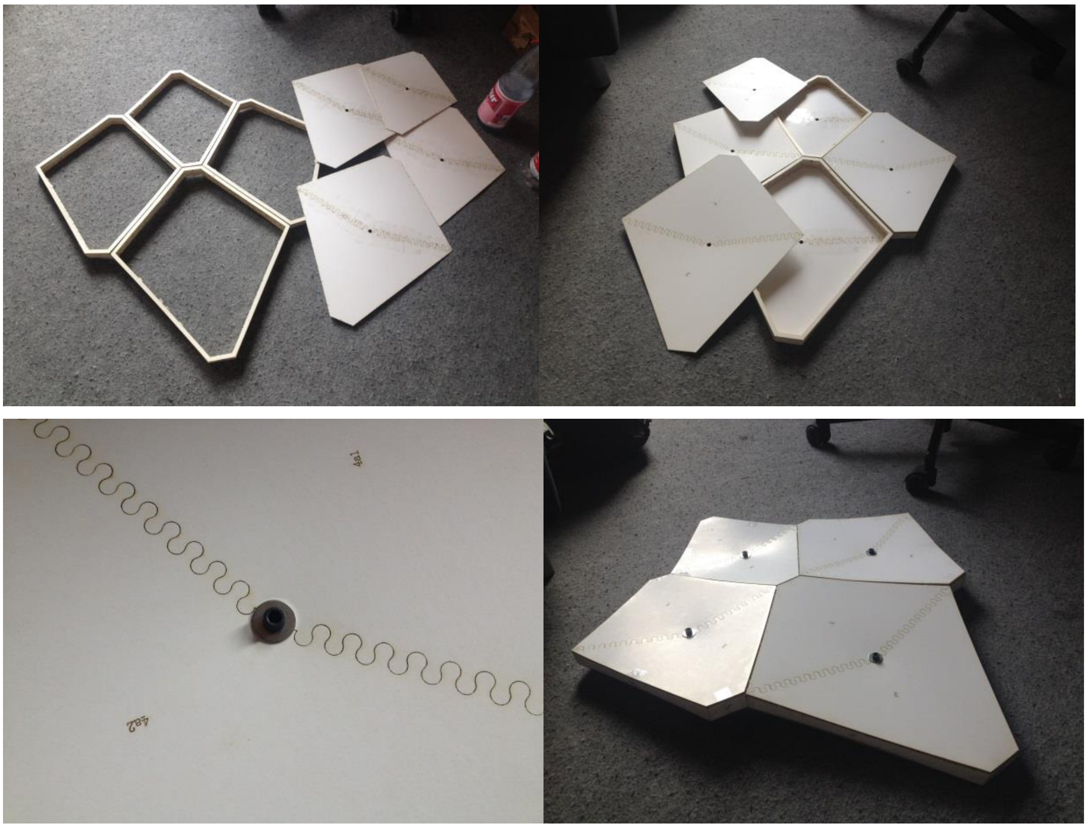


Figure 5.5. 7 Prototypes of cells with small curvature and simplified details in the center

Apart from the prototype with the selected materials, other prototypes are made also with other common paperboards and also with other variations of the construction details.

Figure 5.5.7 shows an experiment of a prototype for a similar cellular cavity structure with a smaller curvature. Because the bending radius becomes larger than the original prototype, a simplified stiffening construction is designed with a screw and two spacers to fix the center points of the curved membranes. For a temporary structure or a similar structure with flatter surfaces, such techniques are very helpful to save time in the prefabrication process.

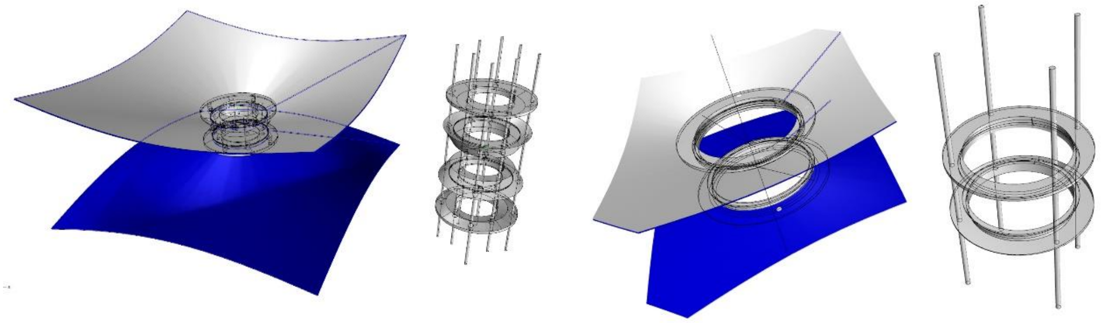


Figure 5.5. 8 Details of the center stiffening components for transparent PET-G elements

Because it is too hard to make similar flanges with the transparent PET-G plate, a better solution of the stiffening at the center point of the cell is made. As shown in Figure 5.5.8, two prototypes are designed according to the geometry of the membranes. The stiffening components are finally made by 3D-printing and used respectively in the cells for the pressing tests with paperboard (Figure 4.4.7 B, D) and the PET-G cells in the pavilion (Figure 5.5.9).

Figure 5.5. 9 Prototypes and details in the PET-G elements

3, Fabrication process of the top cell group

For a final test of the feasibility of the fabrication techniques in the prototypes, a final fabrication of the top cell group is built at first by 5 people.
The total fabrication time of the 33 cells took about 2 days. All the steps are proved to be reasonable for a collaborative work process and the results of both the time cost and the quality of the finished building groups has already fit the original expectation.

Figure 5.5. 10 Fabrication process of the top cell group

5.5.3 Subdivision and prefabrication plan of the building groups

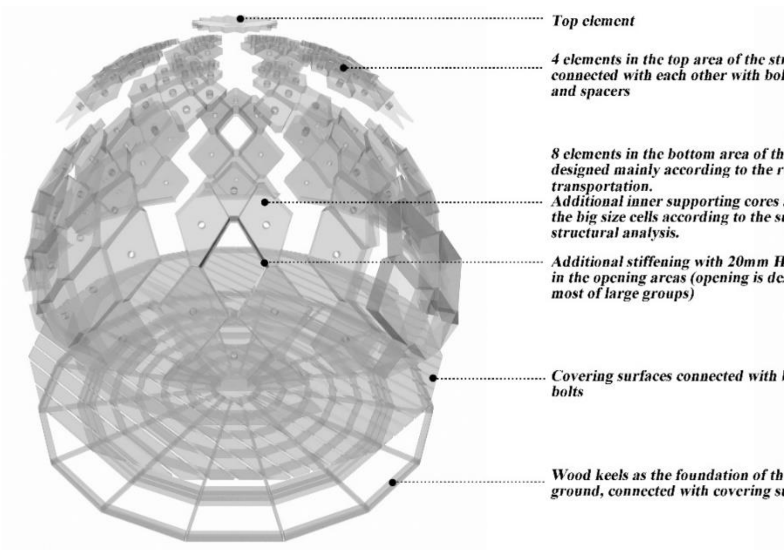


Figure 5.5. 11 Plan of the subdivision of the whole structure into 13 small building groups

With a success of the fabrication test with the top cell group, a building method with the large groups as building components is set up as the basic methods for this pavilion. As shown in Figure 5.5.11, the whole structure is divided into 3 vertical layers and then subdivided into 14 building groups with different numbers of cells. This building method is also mainly due to the requirements of different exhibitions with the pavilion and the sizes of the groups are determined mainly with the consideration of the convenience for transportation and also the possibility to assemble and dismount the structure by a few people in a relatively short time.

The openings and entrance of the structure are therefore arranged all at the center part of every building group, so that the surrounding cells will help to resist a possible deformation of the building groups. The tension ring is also divided into 8 parts and assembled together with the bottom parts.

The connection between different cell groups is designed to be achieved by the heavy duty hook and loop tape from Velcro®. The tensile strength of the tapes is tested with a previous simulation with 2 groups of prototype cells. However, the application of the tapes also lead to an unexpected problem in the final assembly, where a small gap in millimeters comes out at the boundary of the adjacent groups. The result is a small tolerance in the final shape of the shell, which will be shown in the section 5.7.

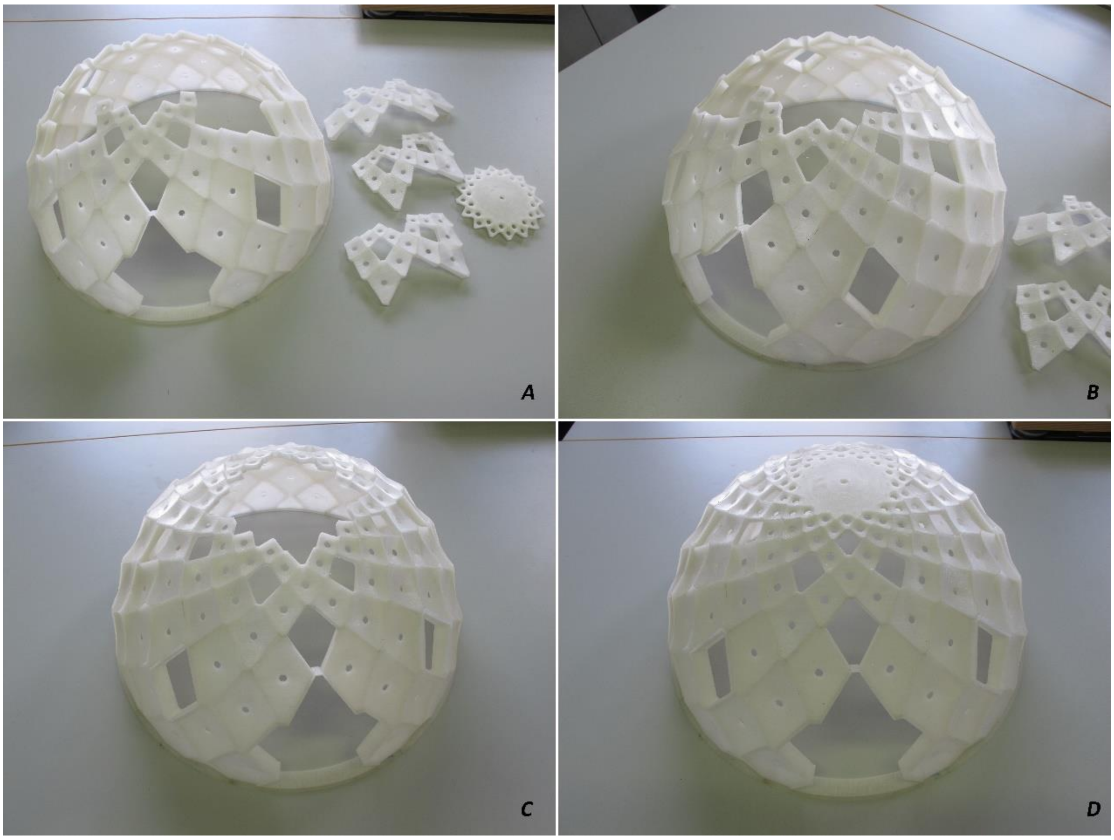


Figure 5.5. 12 Simulation of the assemly process with 3d-printed 1:12.5 Model

To simulate the final assembly process of the pavilion, a 3D-printed model with a scale of 1:12.5 is build according to the subdivision of the structure. The assembly sequence is then tested and the possibility to build without a large scaffold is testified. (Figure 5.5.12) With the model of the whole structure, the tolerance problem is also researched and the load-bearing capacity is tested with a manual pressing test.

5.5.4 Fabrication process and the collaborative building method

The large middle and bottom cell groups are made in the final stage of the fabrication, where team work is required because of the large size of the cells (almost 0.6*1.0 m). (Figure 5.5.13) The inner supporting core has shown to be a positive help for the fabrication process because the ribs with only 1mm materials are too flexible to be fixed in position. It is only possible to build the large cells with thicker ribs or with a larger assistant flange. During the building process is unnecessary to follow the steps in the building process of the prototypes strictly and a single bottom cell group with 9-10 large cells can be fabricated in two days by two people.

Figure 5.5. 13 Fabrication process of one cell groups in the bottom area by two people

The local stiffening at the opening areas in the pavilion is also finished in the fabrication process of the building groups. As shown in Figure 5.5.14, the 20mm honeycomb paperboard is first cut into the right form and folded as the supporting frame of the entrances. The connection points are meanwhile strengthened by the 3D-printed components. After the fabrication of the supporting frame, the cells are assembled together into a full cell group. A similar fabricating methods can be later applied similarly to the other cell groups.

Figure 5.5. 14 Local stiffening for the entrance of the pavilion

As the whole structure is divided into 13 parts, a collaborative fabrication process is also tested with 3 cell groups at the bottom part of the pavilion. With a collaboration between 7-8 people, the three group can also be finished in a short time of two to three days.

From the building experience of the simultaneous fabrication of the cell groups, a better building method is found as an inspiration of the building techniques. When the supporting ribs of different cell groups are connected together, a bigger framework will be hence be constructed. Because the self-weight of the ribs and the inner supporting cores is very low, the framework itself will compose a self-supporting structure and the extra support for the fabrication process will not be needed. When a complete loop of the cell groups is firstly built together, the bending deformation caused by the self-weight will be therefore be restricted by the tension in the frameworks. This will help to minimize the tolerance caused in the fabrication process and will lead to a better result for the final structure.

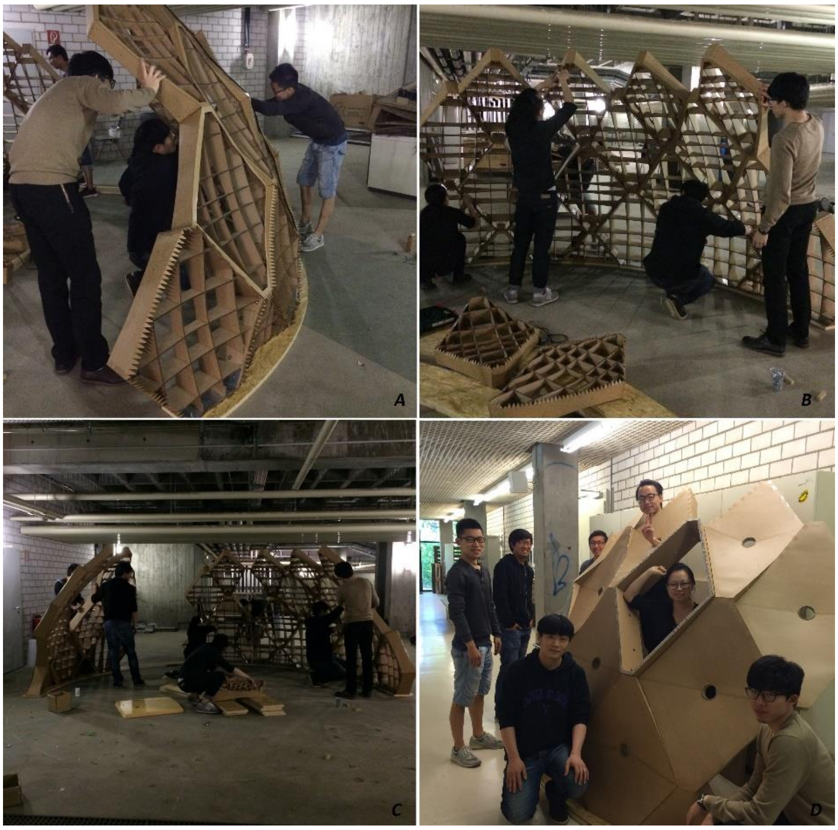


Figure 5.5. 15 Collaborative fabrication with different cell groups

5.6 Building and disassemble process

The pavilion has been assembled several times before the final assembly in August 2016. The previous assemblies are only with part of the structure, so that will only be introduced in the next section. For the final building process, the two processes including the indoor and outdoor assembly will be shown in this section.

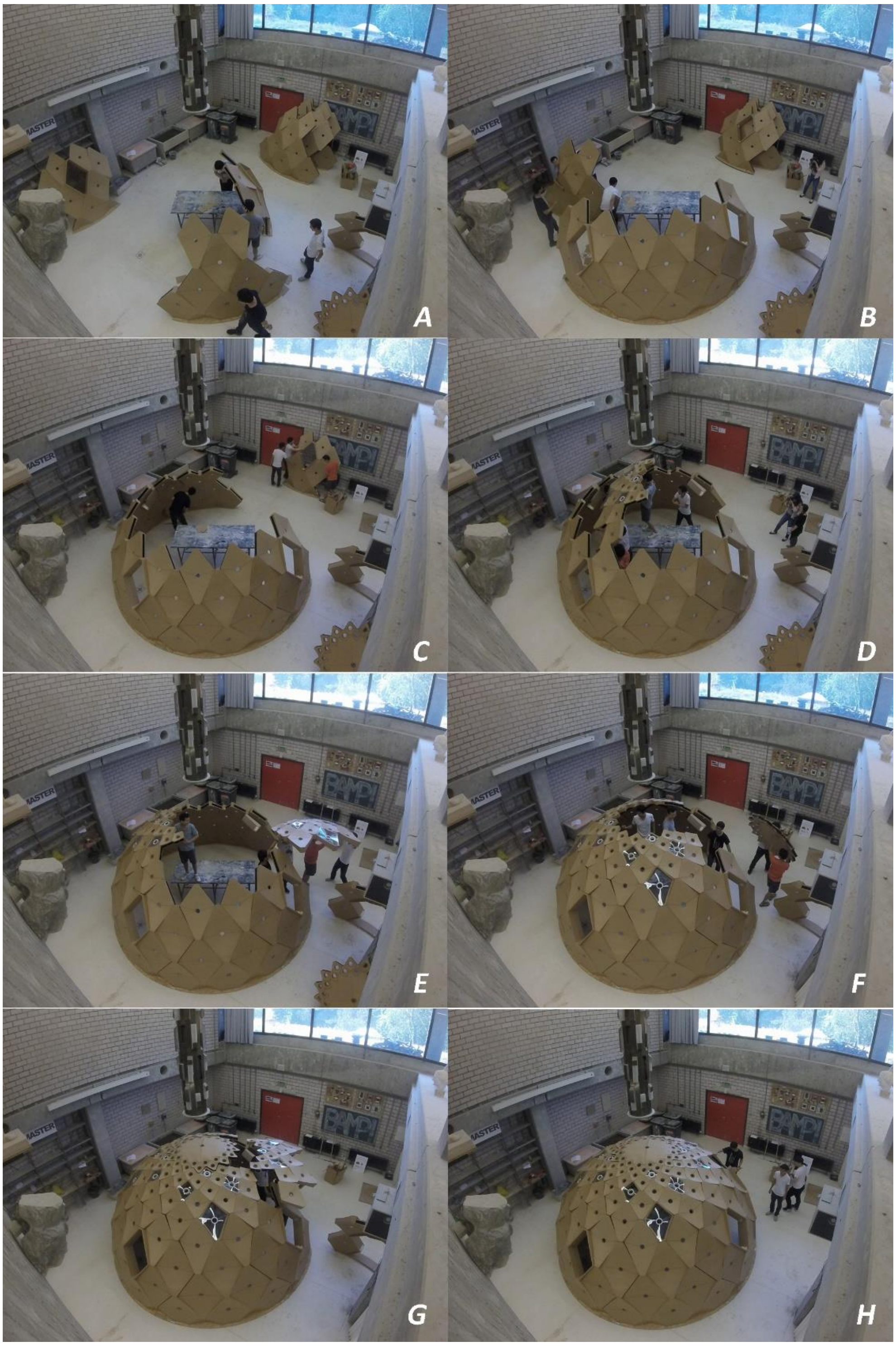


Figure 5.6. 1 The indoor final assembly

Figure 5.6.1 shows the first indoor assembly process of the pavilion. The whole procedure is finished by 8 people with a total working period of 2 hours. As the weight of every cell group is only less than 30kg, it is possible to transport and hold the position of the group with only two people.

The building sequence is most important in the experience of the final assembly process. Because the subdivision of the structure enables a bottom loop which consists of 8 cell groups in the bottom area of the structure, a stable part of the structure can be first built as a foundation to the top 5 cell groups. (Figure 5.6.1 E and 5.6.2 D) After finishing the assembly of the bottom loop, the tension rings should be connected to each other with stronger iron connectors. Together with the strength of the Velcro® tapes, a very stable substructure can be built without the requirement of a costly scaffold or building machines.

The final process with the top groups also requires a designed sequence of the placing of different groups. As it is not possible to hold the top group from above, it should be first fixed into the structure. (Figure 5.6.1 G and Figure 5.6.2 E)

Due to the limited budget of the project and the change of plan to build the pavilion only as a temporary structure, the building of the structure's basement is given up in the final building procedure. As the wind load of the indoor and outdoor site is very low, no problem appears in the final standing period of the pavilion.

Figure 5.6. 2 The outdoor final assembly

5.7 Lasting performance and the discussion of the building method

The pavilion has been built 4 times in the research period between 2014 and 2016. In this section, the lasting performance will be summarily introduced as the final discussion of the builing experiment.

5.7.1 Final form of the pavilion and simple strength/ stability test

The final form of the structure has shown its high-level accuracy to the original design. In all the indoor standing tests and the exhibitions, the structure shows a stable behavior with both a high strength and stiffness.

Simple tests have been made to test the stability of the pavilion as shown in Figure 5.7.1 A and B. As the total weight of the pavilion is only slightly more than 300kg, the successful bearing of a human's weight at about 70-90kg shows a certain capacity of the cellular cavity structure.

Figure 5.7. 1 Final form of the built pavilion and some stability tests

5.7.2 Participated public exhibition and the last standing performance

The pavilion has been invited into two public exhibitions in the time period between August 2015 to October 2015 and also in February 2016. Being the on-going research project of shell structures at Techinische Universiät Darmstadt, the structure has to stand for more than one month in the national music hall in Amsterdam, Netherlands. During this exhibition—the EXPO 2015 for International Association of Shell and spatial Structure (IASS) —the pavilion kept its form well. In the later exhibition in February 2016, the pavilion has been shown as an ongoing-research project of the application of paper products in architectural design for the application of a future research project at TU Darmstadt (Löwe Schwerpunkt- Bauen mit Papier). In the rebuilding process within 2 hours, the capacity of the structure for a multiple assembly has been shown with the good performance of the structure.

Figure 5.7. 2 Participated exhibitions of the pavilion (A: Exhibition of the on-going researches for the application of the research topic "Löwe Schwerpunkt- Bauen mit Papier"; B: IASS EXPO 2015)

5.7.3 Damage of the structure and the experiences for construction details

After the outdoor assembly of the pavilion in September 2016, the damage of the structure due to the outdoor humidity changes is also tested and proved as a main threat to the structures from paperboard.

Figure 5.7.3 A shows the great change of humidity in the outdoor environment especially in the morning and the night. This has also caused the obvious damage of the covering surfaces from the weak connections which are only finished with some adhesive tapes at the back side. The experience also brings a direction for the future research to improve the construction details of the structure.

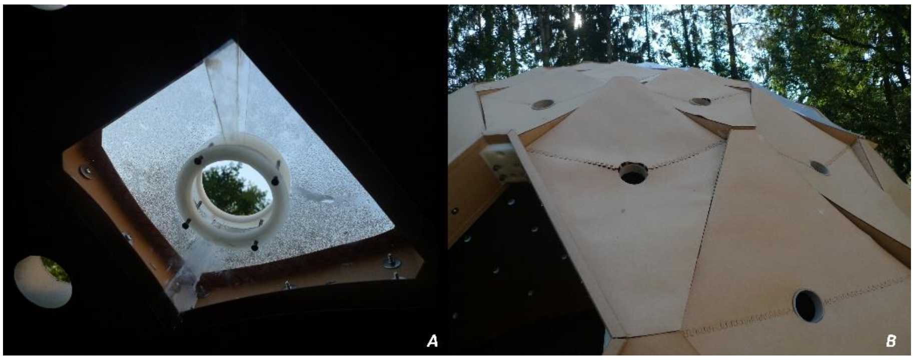


Figure 5.7. 3 Humidity in the outdoor environment and the damage of the structure

6 Conclusion and future works

This dissertation has argued for a possible efficient structure concept for shell structures with a certain kind of modern efficient materials—thin sheet materials. It presented an innovational use of curved surfaces as the main structural element with the activation of the local membrane behavior in the shell structure. The various chapters provided the motivation and logical basis of the problem with a comprehensive revisit of the development of shell structure design from a special point of view and also reviewed the related background literature for the shell theory and materials' performance. They also introduced the comprehensive analysis of the structure concept and the detailed form finding techniques as well as the consideration and the physical tests for the stability problem. With the builing experiment of a full-scale pavilion as a demonstrator, the fabrication techniques and building methods are illustrated.

This final chapter presents the concluding summarization by mentioning the contributions of this research and the on-going parallel research targeting similar problems as well as the different emphasis. Finally, the limitation of this research and the key points for future works are laid out.

6.1 Contributions and conclusions

This dissertation contributes to the field of experimental shell structure design through the development of a novel structure concept which is specially generated for a special group of modern efficient materials. This primary contribution enables the possible designs of new kinds of shells or lightweight structures with a new range of materials. It enables a new way to build more efficient structures by using lightweight, cheaper and more industrialized materials such as metal, plastic and paper sheets. The form finding analysis enables various designs with different starting points and objectives. More free-form structures have been enabled to be designed with a reasonable design procedure. The shown case study with the design procedure of the built pavilion also shows a way for a collaborative design between both architects and structural engineers. The solution of the fabricating process and the construction details also contribute to the experimental uses of paper products in architectural and structural design.

6.1.1 Contribution related to the form finding techniques

This form finding technique of the cellular cavity structure in this research is based on the theory of the 2D/ 2-manifold tessellation theory and the state-of-the-art discretization methods in computational graphics. By defining the coupling system of the supporting rib framework and the covering membranes, a general classification of the cell types is generated for the definition of the structure morphology.

The relationship between the form and forces in the discussion of shell structure is related with the duality of graphs in graph theory. This enables a simplified analysis method for balancing the desired form and the reasonable force flows in the preliminary design procedure. By using the discretization methods on a continuous surface, the popular form finding techniques nowadays can also be borrowed and used in the design process.

The discussion of the form finding of the supporting rib frameworks introduced several recent front research results in computational geometry to generate or optimize several meshes with specific characteristics. The examples of the detailed form finding techniques and the application in different types of meshes also give an easy workable method which can be achieved with the popular architectural design software and hence enables geometric innovations in architectural forms.

6.1.2 Contribution related to the materials application and fabrication

The structure concept and the comprehensive analysis gets it root from a new trend of shell designs which are based on the materials' performance. The whole procedure of the establishment of the structure system contributes to the new design methods of such material-driven designs.

The built pavilion contributes to the various experiments of the shell structure designs based on the dome geometry. The application of the 1mm paperboard shows a possible application of thin materials in architectural designs. The construction details in the final pavilion also contribute to the applications of developable surfaces in a hybrid-hierarchical structure system with free-form geometry.

The variations of the covering surfaces contribute to the capacity of thin sheet materials as the load-bearing components in a structural system. It provides a rethinking of the old conservative roles of such materials in the existing structure typologies. Although the physical tests and the structural analysis still cannot provide a thorough and perfect analysis of the detailed behavior of the real structure or every component in the structure, the comparison has shown the advantages of this novel structure concept.

The digital design and the fabrication processes contribute to the design methodology for the new shell structures and the digital fabrication methods based on the individuation and mass customization. The discussion of the design procedures helps to generate a universal framework for similar structure concepts. The discussion of the detailed fabrication process also contributes to establish a basic framework for a computer-aided or a robot-aided building process of such structures in future research.

6.2 On-going related parallel research and building experiments

Such a research topic of shell structure design is also a popular field in many subjects such as architecture design, art design, mathematics and structural engineering. Since 2012, some on-going parallel research has also been done in similar fields or to solve similar problems. With an introduction of the parallel research, the contribution of this research can be well evaluated and future work can also be better defined.

1, 3D form finding of shell structures and funicular design

Among the shell structure research in the field of structural engineering, a very popular research topic is the form finding techniques and their applications in structural design of new shells. As introduced in section 2.1.3, many studies of the form finding methods were developed in the past decades.

With their great impact on the design methods and procedures of structure designers, the interactive design methods are also being researched to enable more possibilities of the forms for shell structures.

The graphic statics is nowadays a powerful tool for the designer to create energetic compression-only shells. The numerous on-going research and projects by the Block Research Group (BRG) at the Institute of Technology in Architecture at ETH Zurich also concentrate on such interactive design methods for architectural designers to design shell structures. With the recently built masonry shell (P. Block et al., 2016) and the recent finished Ph.D. research by M. Rippmann (Matthias Rippmann, 2016) , the form finding methods with the tool RhinoVAULT in Rhinoceros® have been illustrated with a well-designed vault structure with 399 masonry blocks with a minimal thickness of 5cm. (Figure 6.1)

Figure 6. 1 Project Armadillo Vault—a building experiment of shells from an interactive form finding technique (P. Block et al., 2016)

Such research is based on the foregoing research of the Thrust Network Analysis (TNA) method by Block (P. P. C. V. Block, 2009), which expands the use of graphic statics and provides an analysis method of the three-dimensional equilibrium with only a 2D graphic analysis. The applications of the design methods are mainly the shell structures will massive masonry and concrete blocks which can be cut and milled with CNC milling machines. For the materiality of the structural elements and the possibility to use the cavity structures in this research, a blind spot still needs to be researched.

The research in this dissertation shares the same design philosophy and the same future vision of the design of shell structures. At the same time, the impact of the materials' behavior is also considered an important factor of the design of the structure's forms. With the use of super thin materials in the structure concept, a possible cellular structure with much lighter cavity form is discussed. It expands the application of this popular form finding technique in the shell structure and also adds a new form typology into the family of the recent related building experiments.

2, Geometric analysis of the cellular structure

With the cited research of the computational research of the specific meshes in section 4.3.2, the research group Geometric Modeling and Industrial Geometry in the Institute of Discrete Mathematics and Geometry at TU Wien are also doing some related geometric research in similar fields of architectural geometry. In a recent publication of a paper about the "cell-packing structure" in 2015 (Pottmann et al., 2015), a similar definition of the cellular structure is made to define a basic type of polyhedral cell structures.

However, due to the interests on the interesting geometric properties of the quadrilateral meshes, the research focuses itself mainly on the quad-dominant supporting structures. The global stability problem of the quad-based tessellation and the static analysis are in consideration as a secondary problem and considered as a separate process in the structural design.
In the conclusion of this Ph.D. research, only the foregoing research which has been cited in section 4.3.2 is considered as the most related research and used as some of the possible techniques in the geometric form finding methods. Based on the own logic from the aspect of architectural design, more comprehensive discussions of the form and the structure behavior of the structure concept are analyzed and a more complicated design procedure with multiple criteria is generated.

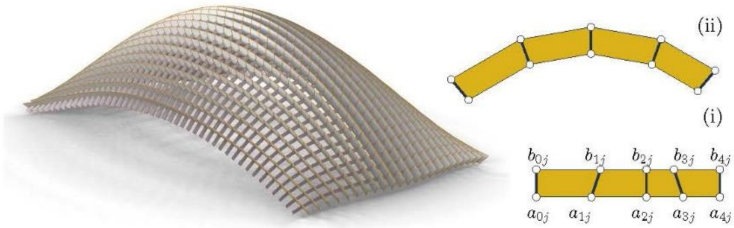


Figure 6. 2 A quadrilateral supporting structure and its basic geometric analysis (Pottmann et al., 2015)

3, Building experiments with cellular and volumetric elements

Recently, numerous designers and researchers have also shown their interest and experiments in the designs of new lightweight structures with innovational structural concept.

Among them, many prototypes have also gotten their inspirations from natural structures and shown their similarities with cellular and volumetric elements.

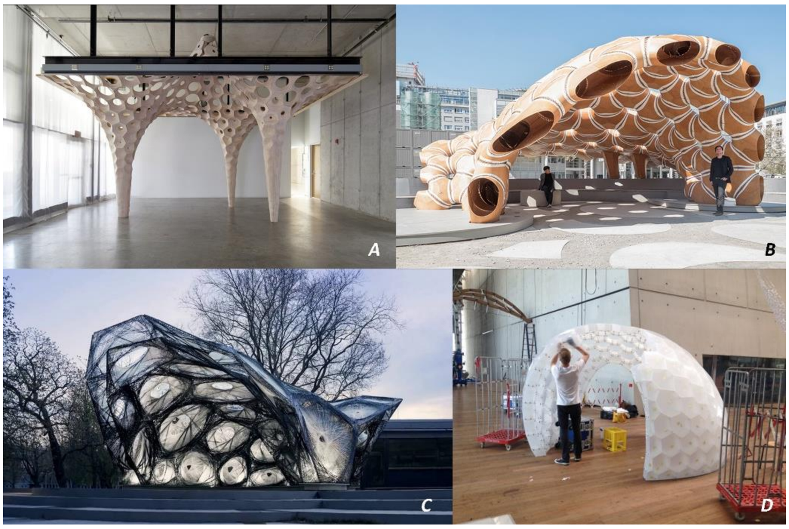


Figure 6. 3 Experimental structures in recent research projects (Clifford & McGee, 2014; Doerstelmann, Knippers, Koslowski, et al., 2015; Kao, 2016)

Cell forms from nature have always inspired the architects' creation in structures. In recent designs some experimental structures such as the structure "la voûte de LeFevre" in 2014 (Clifford & McGee, 2014) and the ICD/ILKE research pavilion 2013-2014 (Doerstelmann, Knippers, Menges, et al., 2015) are built with the similar cellular forms. Starting from different design bases such as the application of the Particle-Spring form finding methods or the biological morphologic research of the double-layered shell forms in the bionics of animal constructions, a similar cellular structure form is designed with inward concave shapes and also with a 3-valence arrangement of cells. Although the fabrication and the final structure have sometimes been shown as a waste of material (for the Levre structure the cells are milled from a massive wood block laminated by plenty of wood plates) or the experiment of some uncommon materials with the existing structure forms (in the ICD/ILEK pavilion the cell is built mainly with linear frameworks from steels and the covering surface made of fibers), such structures also share the same ideas to generate more efficient shells with cellular forms. The building experiments have also inspired the future study of this research and also provide a lot of advising experiences.

At the same time, the use of economic developable surfaces and thin sheet materials have also been applied in many parallel design projects of free-form structures. Based on the inspiration of the bending-active structures from the Ph.D. research by Lienhard (Lienhard & Knippers, 2014) and also the application of thin sheets of plywood (Kao, 2016), a modular structure with the bent form in developable surfaces is built as an experimental project of the new timber shell. The form finding method is to some extent similar as the aforementioned design of pavilions as shown in section 4.1.3.1, but provides a much more interesting and active geometry of the structure. The research of such pioneering projects is also the future goal of this research project. With the deeper research of the forming technique of thin sheet materials, many more new forms are also a major task of the future research.

In the same exhibition of the IASS EXPO 2015, another similar shell structure was built by Søren Jensen consulting engineers with 1mm polypropylene panels (Figure 6.3 D). In the design process, the form finding is based on planarization optimization of a convex surface with a hexagon-dominant mesh by using the Kangaroo engine in Grasshopper™, and only one layer of covering surfaces is added onto the structure to offer an outside surface to the structure. The pavilion introduced in this dissertation here is considered a similar example as the above-mentioned project but is based on a more detailed establishment of the design procedure and with a full consideration of the form finding problems and the structural stability problems. Nevertheless, both parallel pavilions shown in the same exhibition provide a same vision for the development of the shell structures with new kinds of efficient materials.

All the on-going parallel research mentioned here has shown the related interest and works in similar field as the research in this dissertation. They are also a good example for future works of this research and have provided fruitful inspiration for the further development of the cellular cavity structure with thin sheet materials.

6.3 Limitations and future works

The developed structure concept, the form finding techniques and the construction details of the pavilion can be used in the design of similar cellular cavity structures in different forms. However, the establishment and the reasoning of the design logic and the design procedure are mainly based on typical materials which share a certain similarity.

The criterion of the form finding technique for the supporting rib framework and the variations of the covering surfaces are due to the most economic forming techniques nowadays. Plenty of other forming techniques such as vacuum forming and tensile forming as shown in section 3.3.1 are not fully researched and experimented.

The construction details such as the flanges and the gluing techniques are a particular solution for paperboard. Its applicability in cellular cavity structures with other thin sheets should also be tested and evaluated. The use of the assisting flanges is also the most convenient solution for the manual fabricating process, for the forms with planar and folded surface, simpler construction details can still be developed and used in the fabrication.

The form of the final pavilion is due to the multiple objectives of the demonstrator determined with a classic half-sphere dome. The complicated form-finding techniques in a free-form structure design are still not tested with a real built full-scale structure. The structural analysis is due to the feasibility with FEM analysis software only carried out with some simplified models for all the load cases. The capacity of the construction details cannot be fully analyzed either.

Based on the limitations of the research in this Ph.D. study, the following aspects are considered as the key points of future work.

1. Establishment of a fully parametrical design procedure or the related design software/ algorithms based on the framework of the design procedure in this dissertation

2. Experiments with more types of thin sheet materials and the evaluation of the feasibility of the construction details

3. More experiments with free-form architectural design and the evaluation of the typical framework of the design procedure

4. Establishment of a more industrialized or robot-aided fabrication process and assembly process

5. More detailed structural analysis with pressing/ tensile/ bending tests with more full-scale models

As a final conclusion, this research provides small improvements and innovations for the shell structure design with the analysis and experiments of the cellular cavity structure concept. Through the small seeds of the new development of the shell structure design, more novel ideas, concepts and experiments are expected towards a new design culture.

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