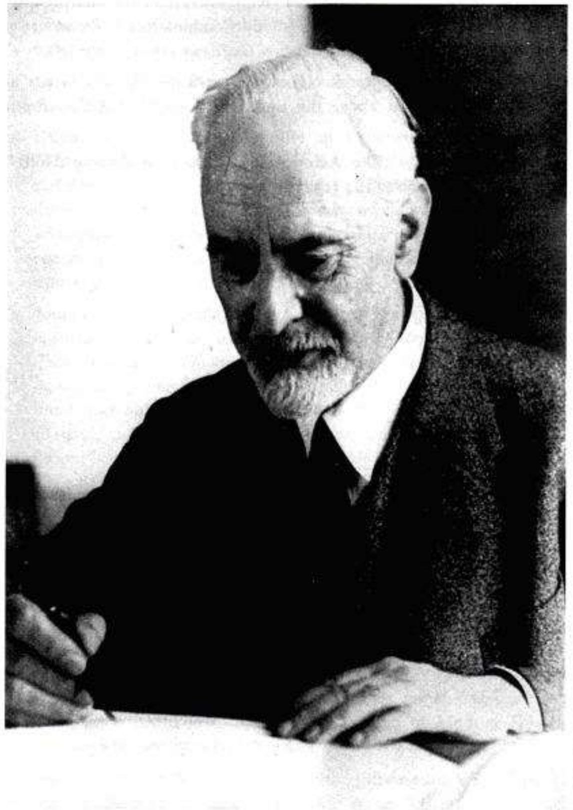
- Fenômenos de Transporte I - 2018

CAMADA LIMITE

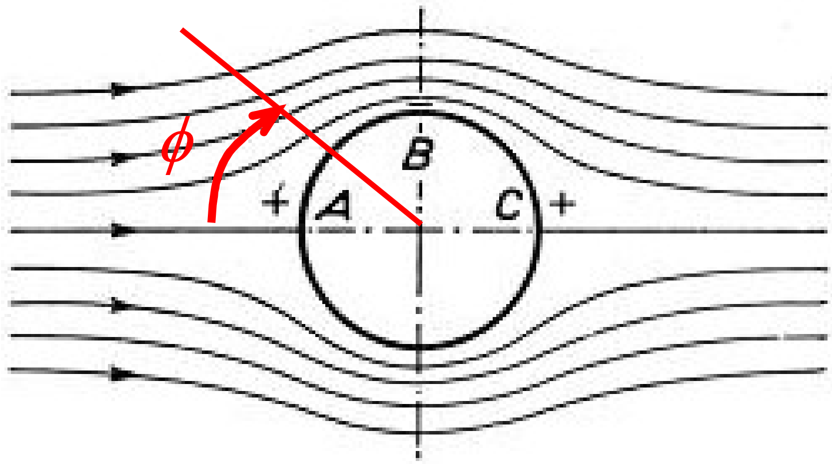
- **1904 – Prandtl (1875-1953) apresenta o artigo "Über Flüssigkeitsbewegung bei sehr kleiner Reibung" Proc. Third Internat. Math. Congress - Heidelberg – p. 484-491.**
- ***Escoamento de fluidos com cisalhamento pequeno*.**
- ***"O processo físico na camada limite entre um fluido e um corpo sólido pode ser calculado de maneira suficientemente satisfatória, assumindo-se que o fluido adere às paredes, de modo que a velocidade seja nula – ou igual à velocidade do corpo. Se a viscosidade for muito baixa e a trajetória do fluido ao longo da parede não for muito extensa, a velocidade terá o mesmo valor que muito próximo da parede. Numa fina camada de transição, a brusca variação de velocidade, apesar do pequeno coeficiente de viscosidade, produz efeitos notáveis".***

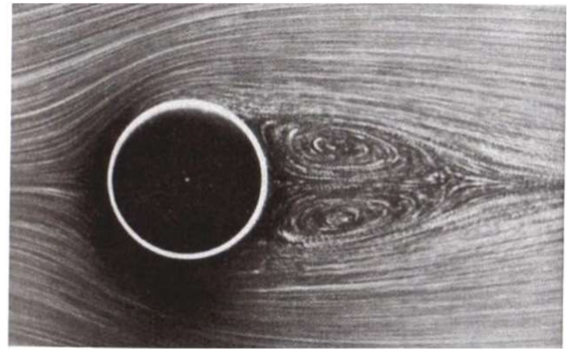


Ludwig Prandtl

Fluido ideal (invíscido) x fluido real

ideal





$$\boxed{q_\infty = \frac{\rho V^2}{2}}$$

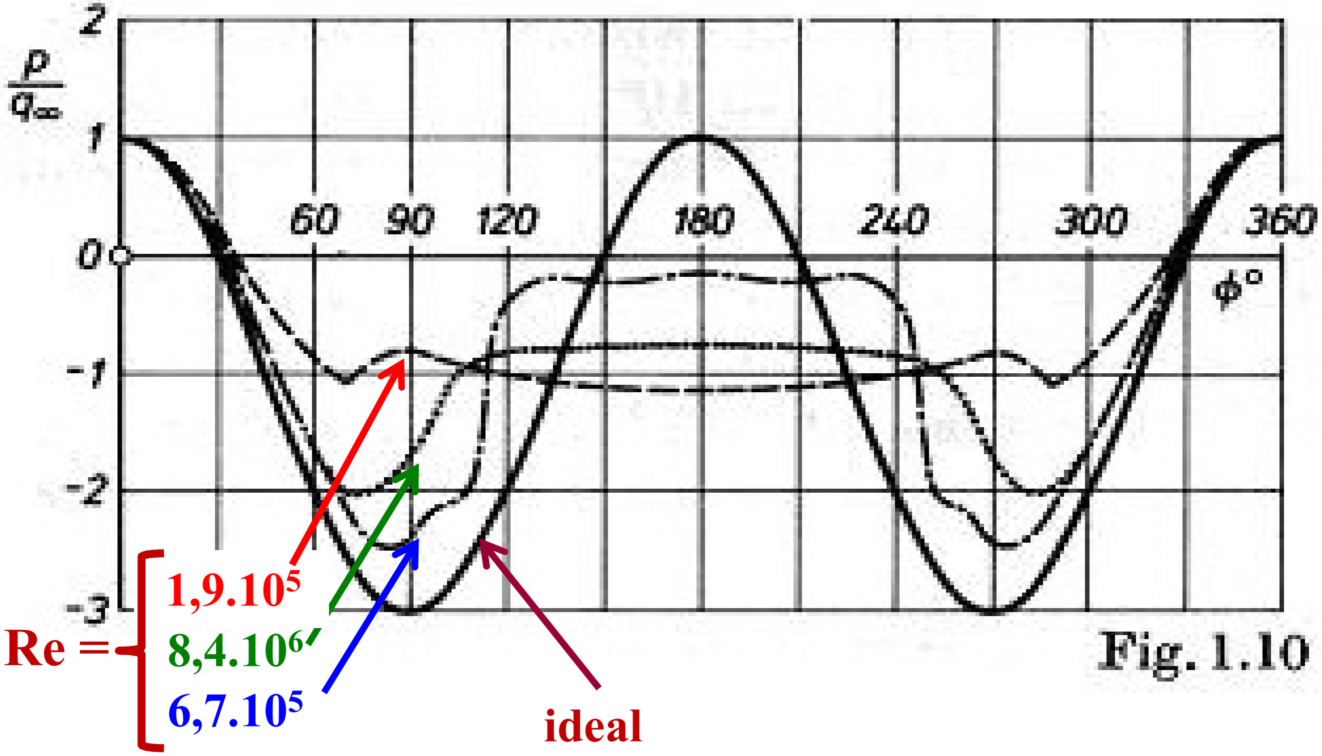
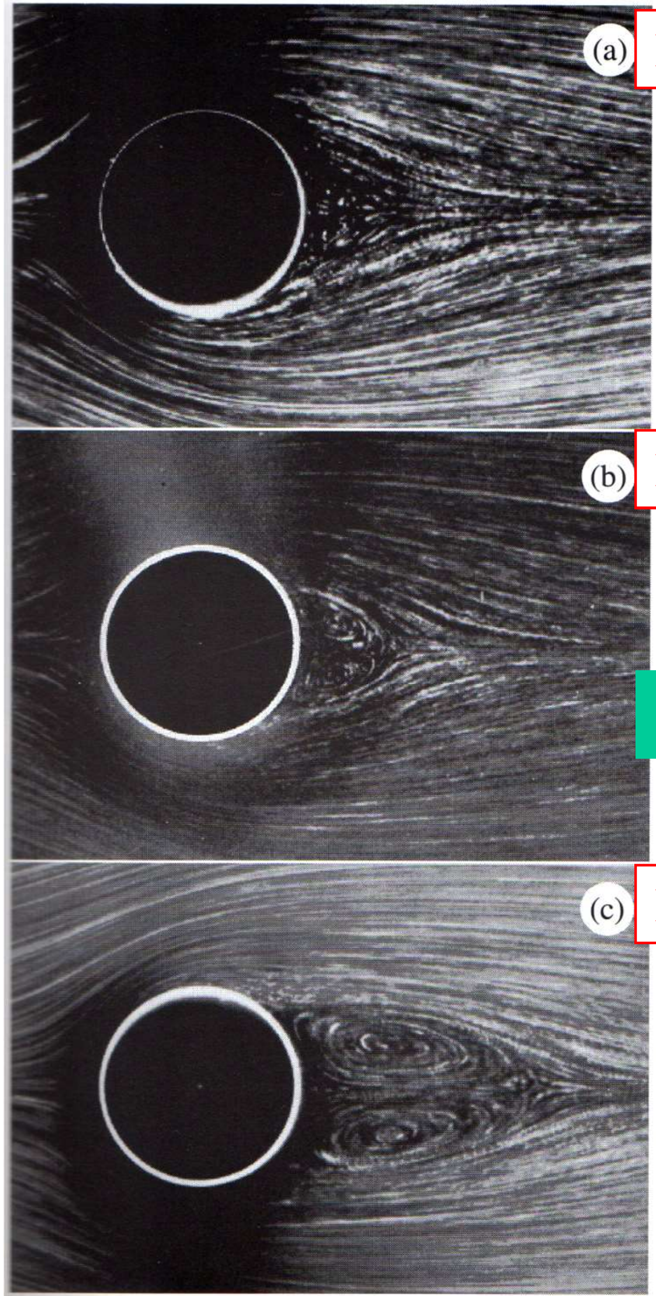


Fig. 1.10

Von Karman



Circular cylinder at $R = 9.6$ (a), $R = 13.1$ (b) and $R = 26$ (c) (Van Dyke
otograph S. Taneda.

Fig. 1.6. Kármán vortex street behind a circular cylinder at $R = 140$ (Van Dyke 1982). Photograph S. Taneda.

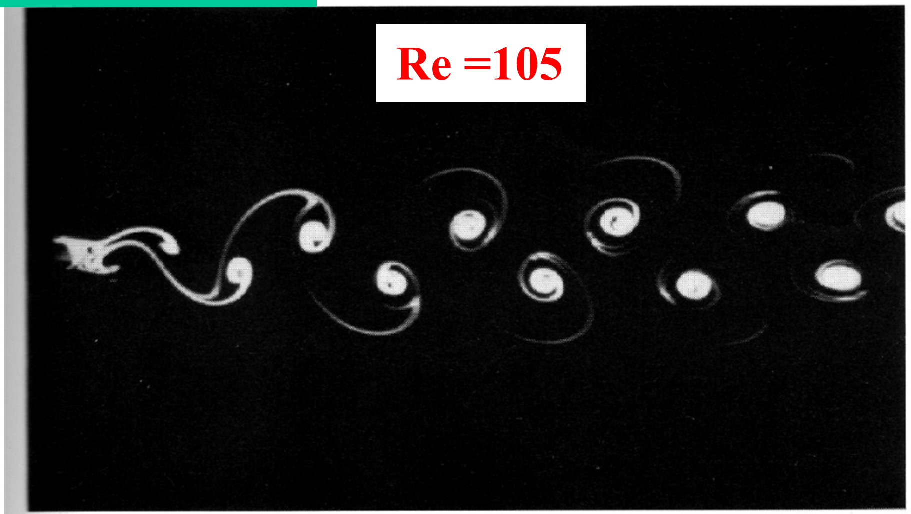
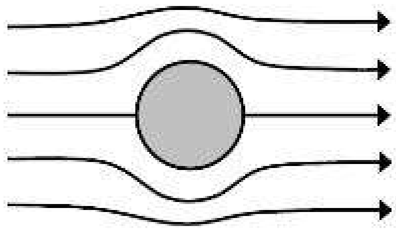
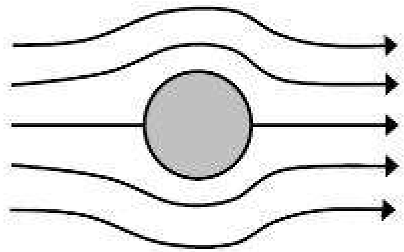


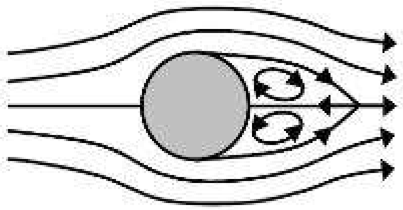
Fig. 1.7. Kármán vortex street behind a circular cylinder at $R = 105$ (Van Dyke 1982). Photograph S. Taneda.



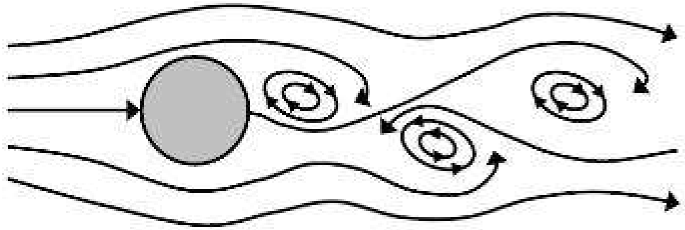
Inviscid flow: $Re = \infty$



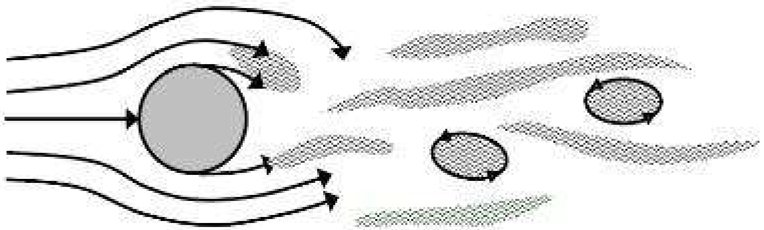
$Re \approx 0.01$



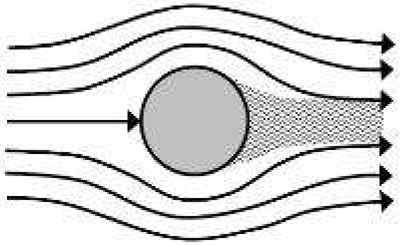
$Re \approx 20$



$Re \approx 100$



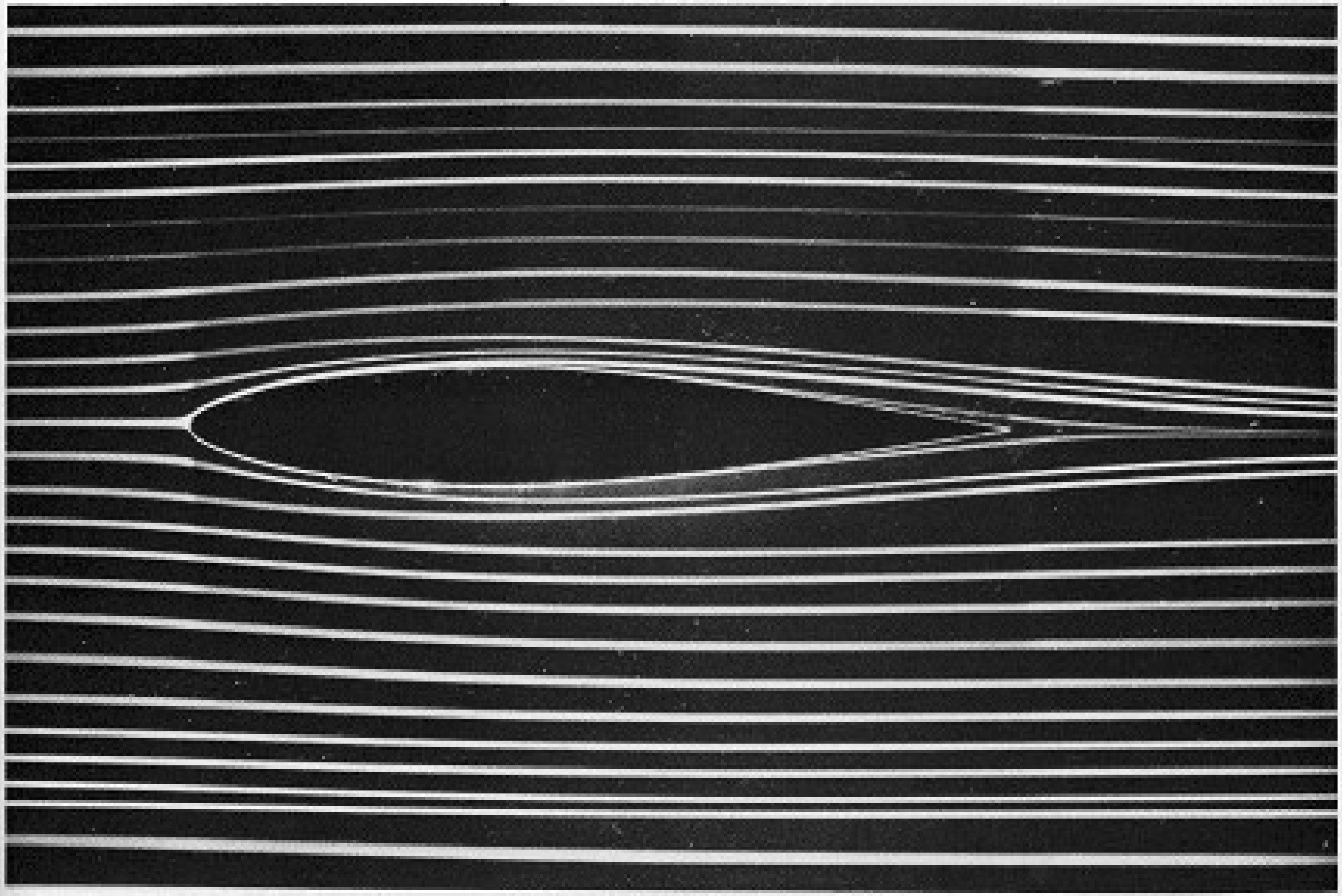
$Re \approx 10\,000$



$Re \approx 10\,000\,000$

cilindros

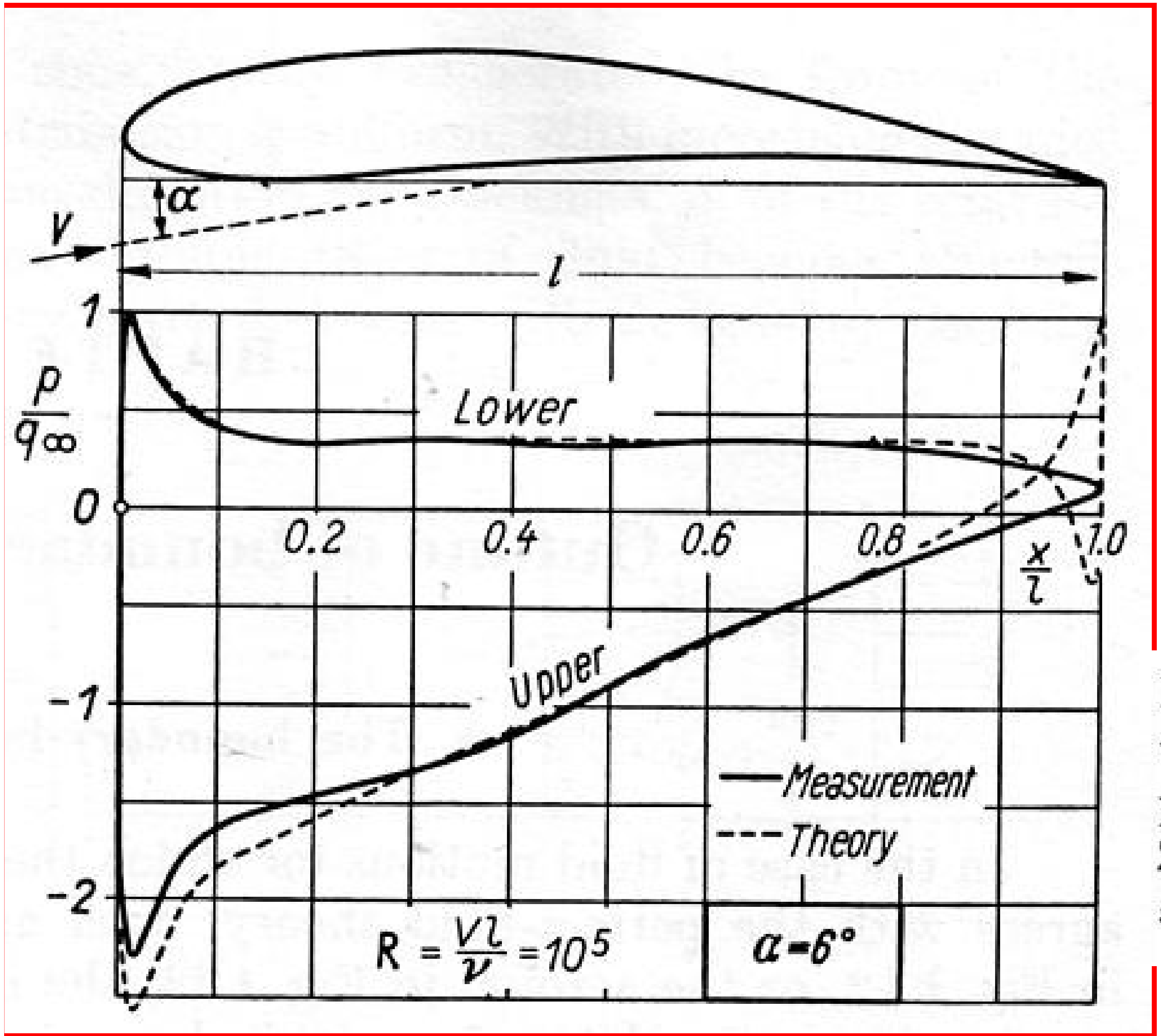
Aerofólio



23. Symmetric plane flow past an airfoil. An NACA 64A015 profile is at zero incidence in a water tunnel. The Reynolds number is 7000 based on the chordlength. Streamlines are shown by colored fluid introduced upstream. The flow is evidently laminar and appears to be unseparated, though one might anticipate a small separated region near the trailing edge. *ONERA photograph, Werlé 1974*

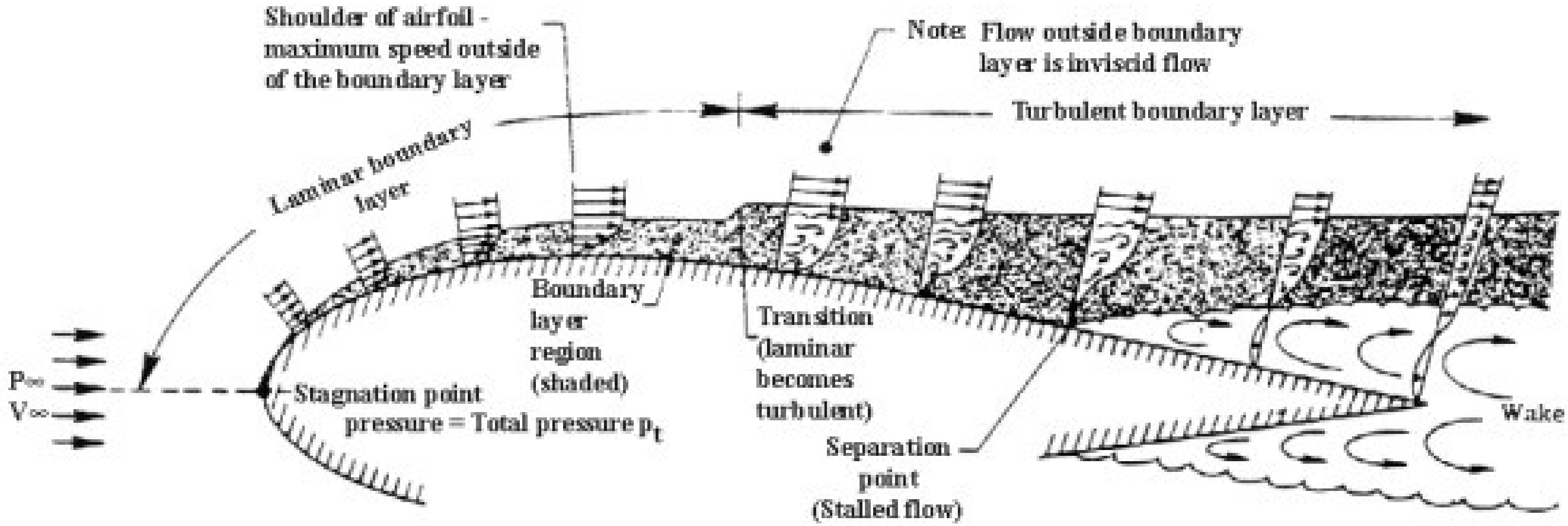
Re = 7000

Escoamento Invíscido - Pressão

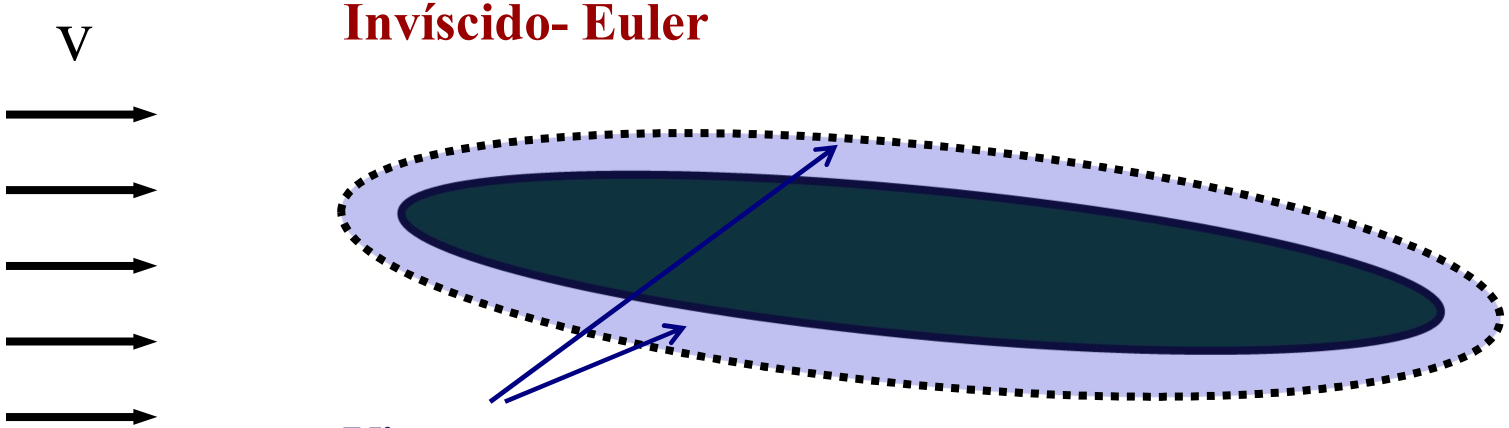


$$q_\infty = \frac{\rho V^2}{2}$$

Fig. 1.14. Comparison between the theoretical and measured pressure distribution for a Zhukovskii profile at equal lifts, after A. Betz [2]


Shoulder of airfoil -
maximum speed outside
of the boundary layer
Note: Flow outside boundary
layer is inviscid flow
Turbulent boundary layer
Laminar boundary
layer
Boundary
layer
region
(shaded)
Transition
(laminar
becomes
turbulent)
P∞
V∞
Stagnation point
pressure = Total pressure p_t
Separation
point
(Stalled flow)
Wake

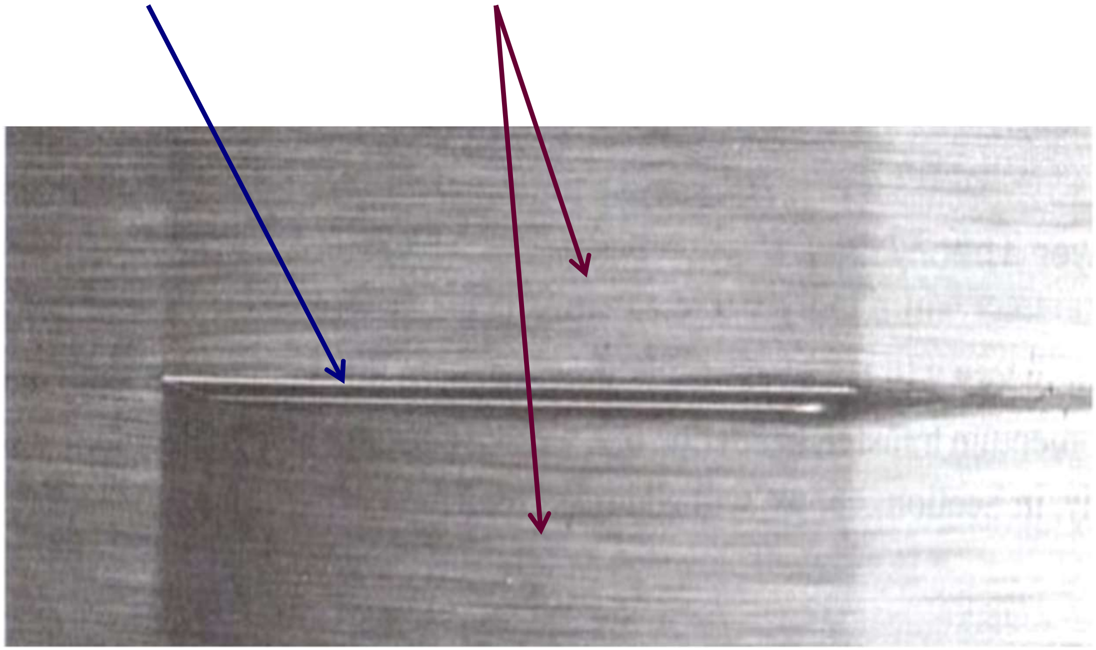
Modelo - Prandtl



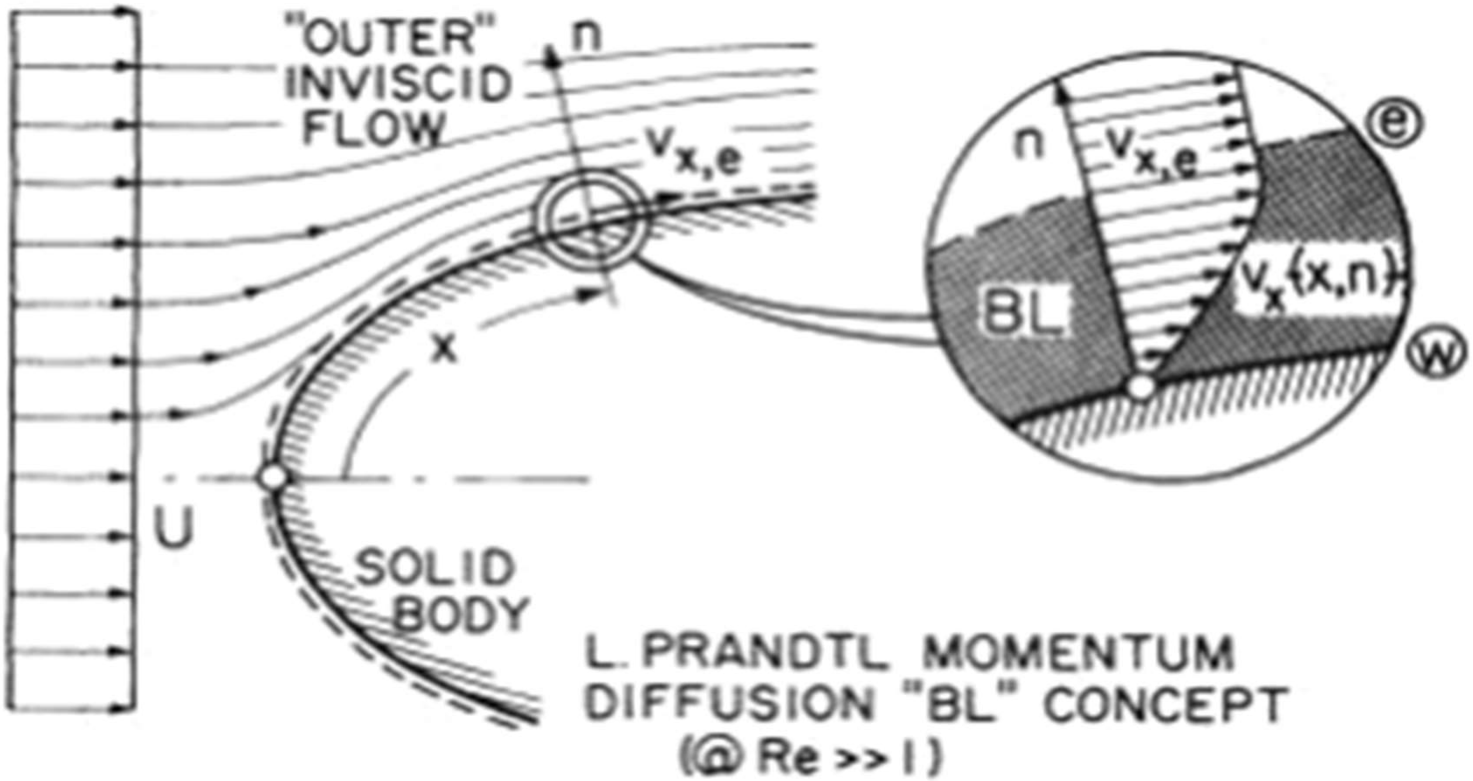
Viscoso –
Navier Stokes

Invíscido- Euler

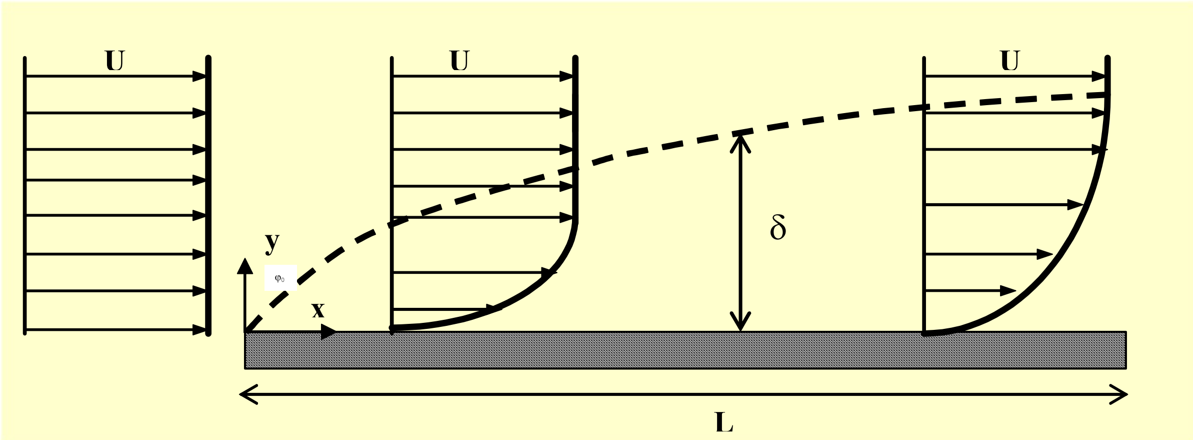
Placa Plana



Modelo - Prandtl



Camada Limite Laminar Hidrodinâmica – Bidimensional – Placa Plana



Continuidade

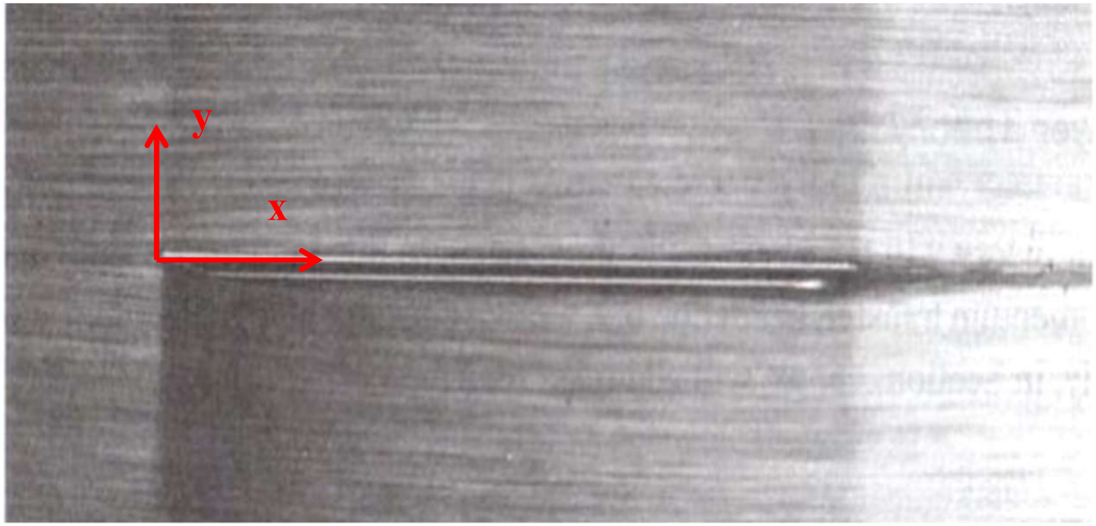
$$\frac{\partial \rho}{\partial t} = -\operatorname{div} \rho \vec{v} \quad \Rightarrow \quad \begin{array}{l} \dfrac{\partial \rho}{\partial t} = 0 \\ \operatorname{div} \vec{v} = 0 \end{array} \quad \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Navier Stokes - bidimensional

$$\rho \frac{D\vec{v}}{Dt} = \rho \frac{\partial \vec{v}}{\partial t} + \rho\, \vec{v}.\vec{\operatorname{grad}}\, \vec{v} = \rho \vec{g} - \vec{\operatorname{grad}}\, p + \mu \operatorname{lap} \vec{v}$$

$$\begin{cases} \rho \left(u \dfrac{\partial u}{\partial x} + v \dfrac{\partial u}{\partial y} \right) = -\dfrac{\partial p}{\partial x} + \mu \left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} \right) \\ \\ \rho \left(u \dfrac{\partial v}{\partial x} + v \dfrac{\partial v}{\partial y} \right) = -\dfrac{\partial p}{\partial y} + \mu \left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} \right) \end{cases}$$

Scaling



Escalas de adimensionalização:

- $x \rightarrow L \;,\; y \rightarrow \delta$
- $u \rightarrow U \;,\; v \rightarrow V$
- $\delta << L$ **(observação experimental)**

$$\hat{x} = \frac{x}{L} \qquad \hat{y} = \frac{y}{\delta} \qquad \hat{u} = \frac{u}{U} \qquad \hat{v} = \frac{v}{V}$$

SCALING =>
ordem de grandeza das variáveis e das variações ~ 1

“Scaling”

Continuidade:

$$\Rightarrow \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

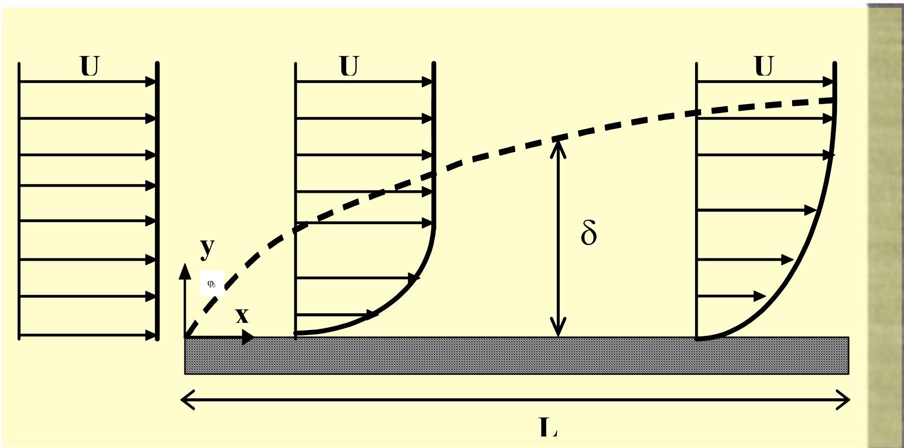
$$\frac{U}{L}\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{V}{\delta}\frac{\partial \hat{v}}{\partial \hat{y}} = 0 \qquad \Rightarrow \left(\frac{U\delta}{VL}\right)\underbrace{\frac{\partial \hat{u}}{\partial \hat{x}}}_{\sim 1} + \underbrace{\frac{\partial \hat{v}}{\partial \hat{y}}}_{\sim 1} = 0$$

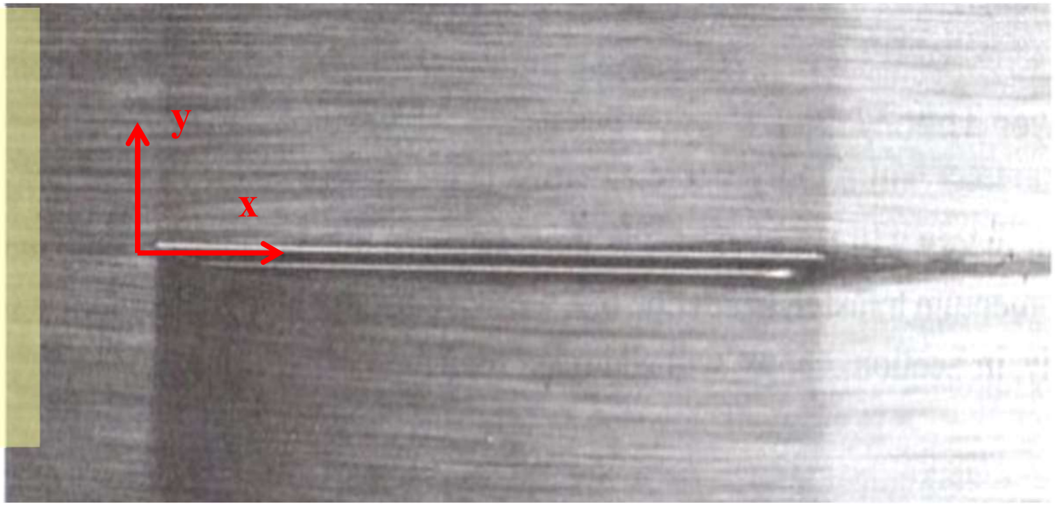
SCALING => ordem de grandeza das variáveis ~ 1

$$\frac{U\delta}{VL} = 1 \qquad \Rightarrow \quad V = \frac{U\delta}{L}$$

V << U

Camada Limite – Placa Plana





$$\boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0}$$

v << U

$$\begin{cases}\rho\left(u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}\right)=-\dfrac{\partial p}{\partial x}+\mu\left(\dfrac{\partial^2 u}{\partial x^2}+\cancel{\dfrac{\partial^2 u}{\partial y^2}}\right) & \boxed{\sim\dfrac{\rho U^2}{L}}\\[2em] \xcancel{\rho\left(u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}\right)=-\dfrac{\partial p}{\partial y}+\mu\left(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}\right)} & \boxed{\sim\dfrac{\rho U^2}{L}\dfrac{\delta}{L}}\end{cases}$$

Equações – Camada Limite Laminar – Placa Plana

$$p = p(x)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \boxed{-\frac{dp}{dx}} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right)$$

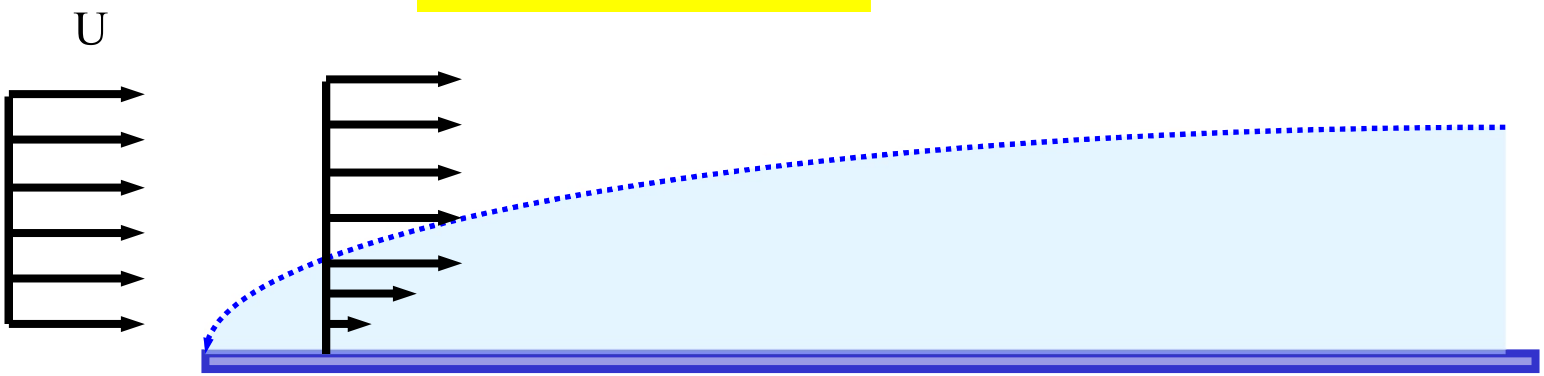
$$\frac{\partial p}{\partial y} \cong 0$$

Borda da camada limite u ≈ U:

$$\rho\left(U\frac{\partial U}{\partial x} + \cancel{v\frac{\partial U}{\partial y}}\right) = -\frac{dp}{dx} + \cancel{\mu\left(\frac{\partial^2 U}{\partial y^2}\right)}$$

EULER - INVÍSCIDO

$$\Rightarrow \quad \rho U\frac{\partial U}{\partial x} = -\frac{dp}{dx}$$



Equações – Camada Limite Laminar – Placa Plana

$$\frac{dp}{dy} \cong 0$$

dp/dx dentro da camada = dp/dx fora da camada

$$\rho U \frac{dU}{dx} = -\frac{dp}{dx}$$

Dentro da camada limite:

VISCOSO

$$p = p(x)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho U\frac{dU}{dx} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right)$$

Placa Plana U = cte: $\rho U \frac{dU}{dx} = 0 = -\frac{dp}{dx}$ $\Rightarrow$ **P cte**

$$\boxed{\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\left(\frac{\partial^2 u}{\partial y^2}\right)}$$

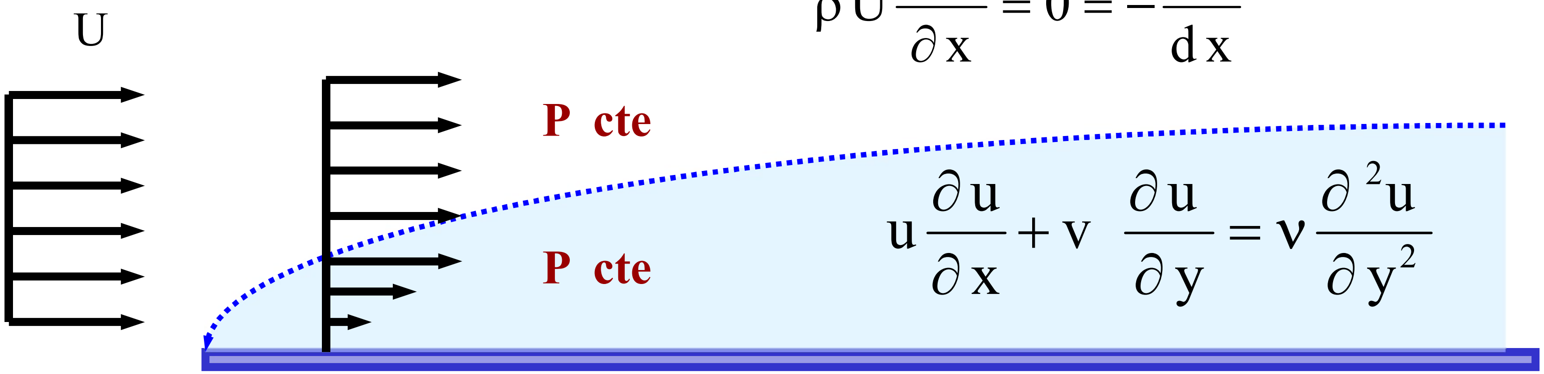
Equações – Camada Limite Laminar – Placa Plana

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

$$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}}$$

$$\boxed{P \quad cte}$$

$$\rho U \frac{\partial U}{\partial x} = 0 = -\frac{dp}{dx}$$



$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Camada Limite Laminar – Placa Plana – Equações Aproximadas e Condições de Contorno

Continuidade:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Quantidade de Movimento:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Condições de Contorno:

parede, $y = 0 \rightarrow u = 0$, $v = 0$

borda, $y = \infty \rightarrow u = U$

Solução de Blasius

$$u = \frac{\partial \Psi}{\partial y} \quad ; \quad v = -\frac{\partial \Psi}{\partial x}$$

$$\eta = y\sqrt{\frac{U}{\nu x}}$$

$$f(\eta) = \frac{\Psi}{\sqrt{x \nu U}}$$

$$f(\eta)\frac{d^2 f(\eta)}{d\eta^2} + 2\frac{d^3 f(\eta)}{d\eta^3} = 0$$

parede, $\eta = 0 \rightarrow f = 0$ e $f' = 0$

borda, $\eta = \infty \rightarrow f' = 1$

Solução de Blasius

Solução :

$$u = U\,\frac{d\,f(\eta)}{d\eta}$$

$$v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta\,\frac{d\,f(\eta(}{d\eta} - f(\eta(\right)$$

$$\frac{u}{U} = 0{,}99 = \frac{df}{d\eta} = 0{,}99 \Rightarrow \eta = 5 = \delta\sqrt{\frac{U}{\nu x}}$$

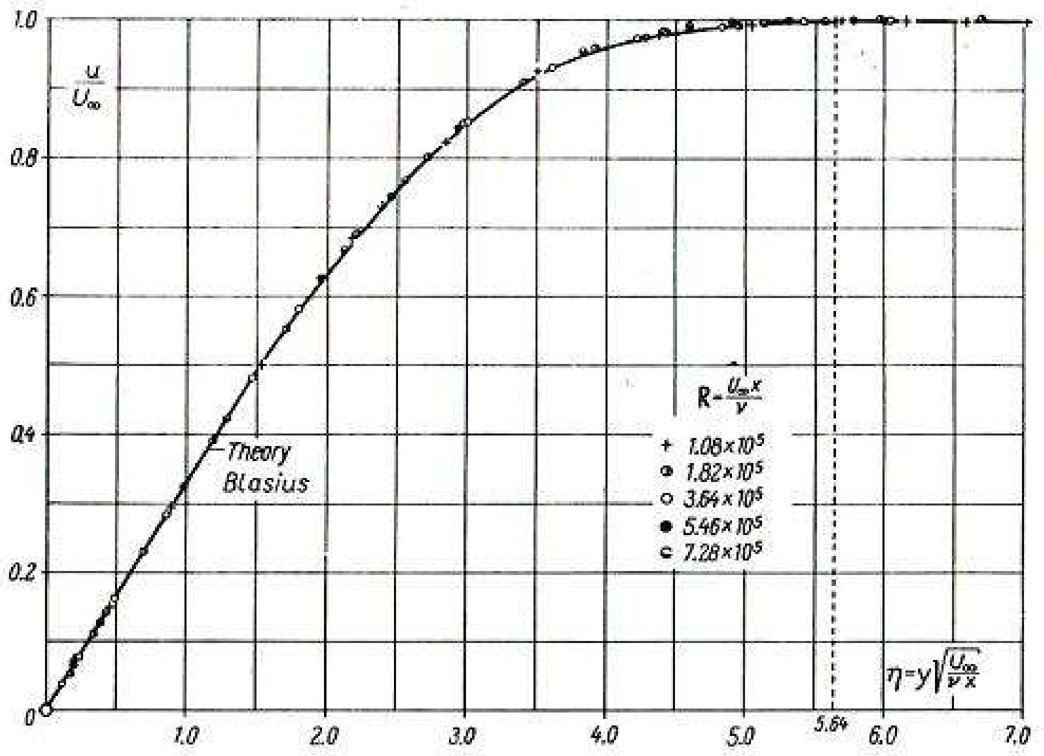
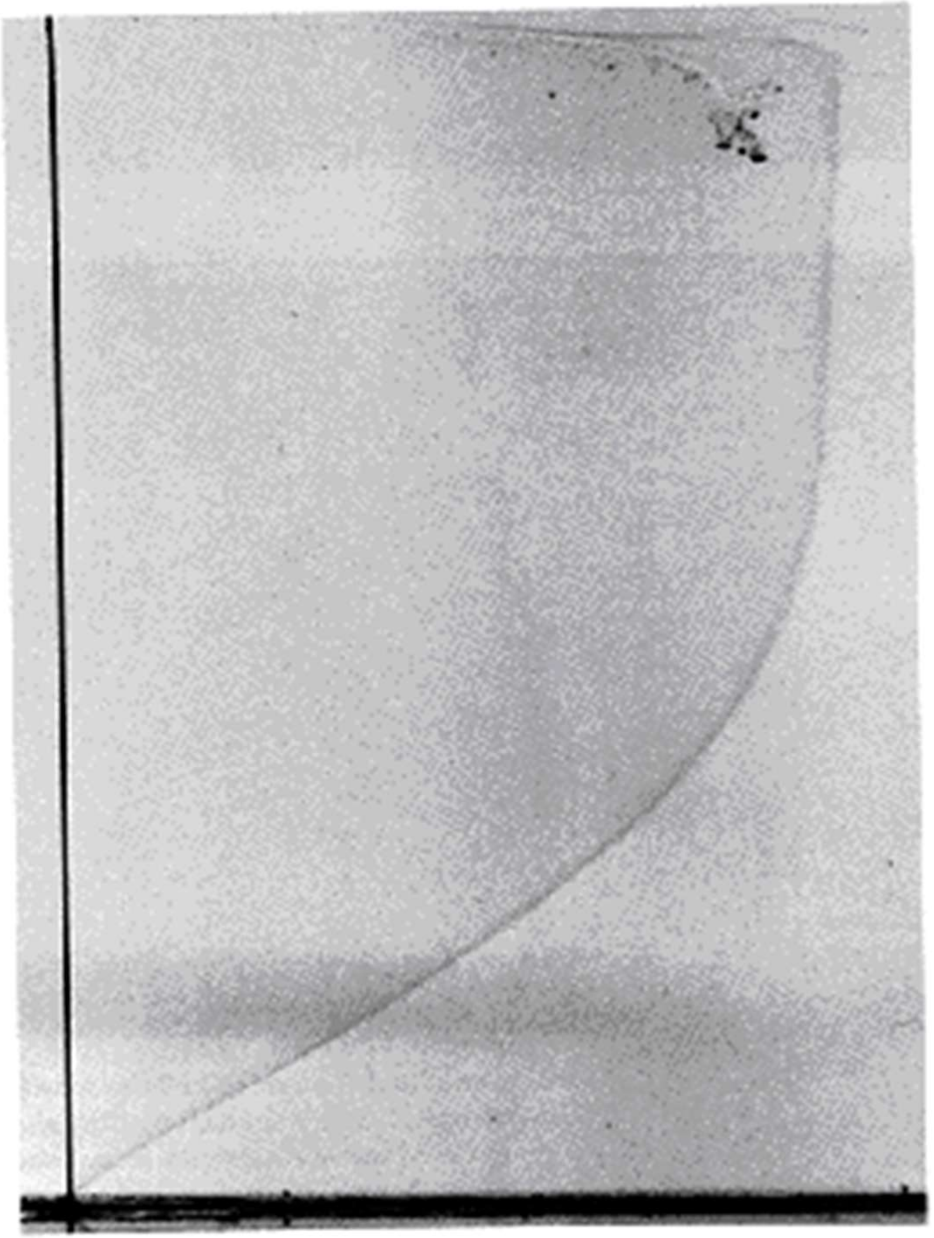


Fig. 7.9. Velocity distribution in the laminar boundary layer on a flat plate at zero incidence, as measured by Nikuradse [20]

$$\boxed{\delta = 5\sqrt{\frac{\nu x}{U}} = \frac{5x}{\sqrt{Re_x}}}$$

$$\boxed{\mathbf{Re_{crítico} = 5.10^5}}$$

Perfil de Velocidades- Camada Limite



30. Blasius boundary-layer profile on a flat plate. The tangential velocity profile in the laminar boundary layer on a flat plate, discovered by Prandtl and calculated accurately by Blasius, is made visible by tellurium. Water is flowing at 9 cm/s. The Reynolds number is 500 based on distance from the leading edge, and the displacement thickness is about 5 mm. A fine tellurium wire perpendicular to the plate at the left is subjected to an electrical impulse of a few milliseconds duration. A chemical reaction produces a slender colloidal cloud, which drifts with the stream and is photographed a moment later to define the velocity profile. Photograph by F. X. Wortmann

Solução de Blasius

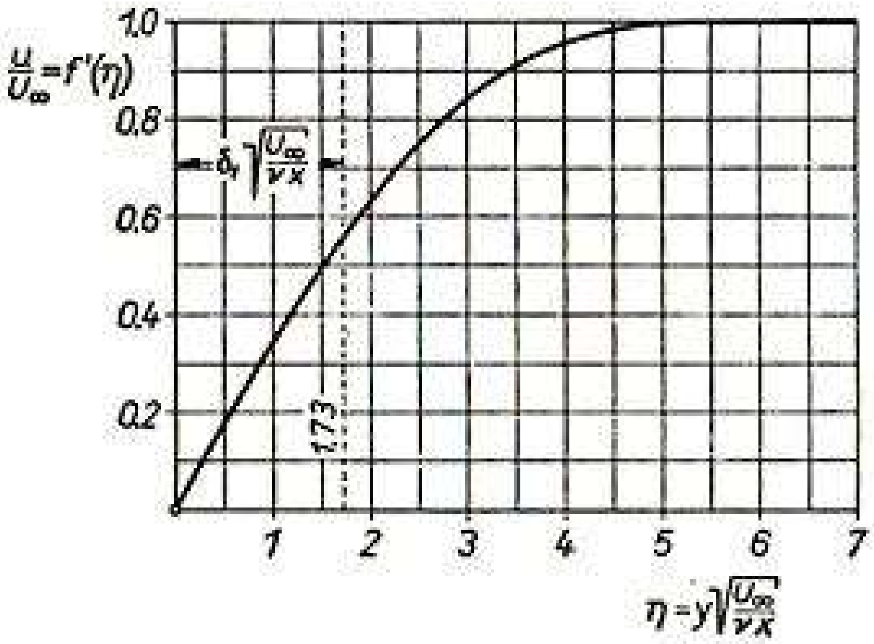


Fig. 7.7. Velocity distribution in the boundary layer along a flat plate, after Blasius [2]

$$u = U \frac{d\,f(\eta)}{d\eta}$$

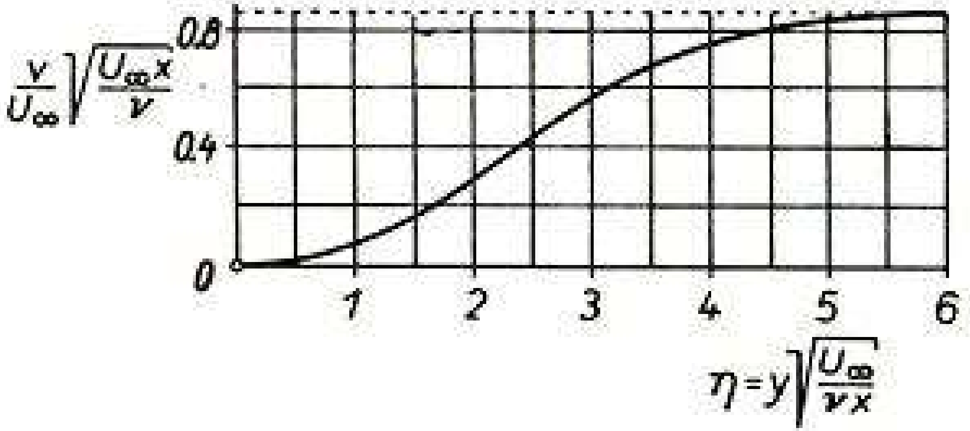


Fig. 7.8. The transverse velocity component in the boundary layer along a flat plate

$$v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta \frac{d\,f(\eta(}{d\eta} - f(\eta(\right)$$

Solução de Blasius

$\eta = y\sqrt{\frac{U_\infty}{\nu x}}$	f	$f' = \frac{u}{U_\infty}$	f''
0	0	0	0·33206
0·2	0·00664	0·06641	0·33199
0·4	0·02656	0·13277	0·33147
0·6	0·05974	0·19894	0·33008
0·8	0·10611	0·26471	0·32739
1·0	0·16557	0·32979	0·32301
1·2	0·23795	0·39378	0·31659
1·4	0·32298	0·45627	0·30787
1·6	0·42032	0·51676	0·29667
1·8	0·52952	0·57477	0·28293
2·0	0·65003	0·62977	0·26675
2·2	0·78120	0·68132	0·24835
2·4	0·92230	0·72899	0·22809
2·6	1·07252	0·77246	0·20646
2·8	1·23099	0·81152	0·18401
3·0	1·39682	0·84605	0·16136
3·2	1·56911	0·87609	0·13913
3·4	1·74696	0·90177	0·11788
3·6	1·92954	0·92333	0·09809
3·8	2·11605	0·94112	0·08013
4·0	2·30576	0·95552	0·06424
4·2	2·49806	0·96696	0·05052
4·4	2·69238	0·97587	0·03897
4·6	2·88826	0·98269	0·02948
4·8	3·08534	0·98779	0·02187
5·0	3·28329	0·99155	0·01591
5·2	3·48189	0·99425	0·01134
5·4	3·68094	0·99616	0·00793
5·6	3·88031	0·99748	0·00543
5·8	4·07990	0·99838	0·00365
6·0	4·27964	0·99898	0·00240
6·2	4·47948	0·99937	0·00155
6·4	4·67938	0·99961	0·00098
6·6	4·87931	0·99977	0·00061
6·8	5·07928	0·99987	0·00037
7·0	5·27926	0·99992	0·00022
7·2	5·47925	0·99996	0·00013
7·4	5·67924	0·99998	0·00007
7·6	5·87924	0·99999	0·00004
7·8	6·07923	1·00000	0·00002
8·0	6·27923	1·00000	0·00001
8·2	6·47923	1·00000	0·00001
8·4	6·67923	1·00000	0·00000
8·6	6·87923	1·00000	0·00000
8·8	7·07923	1·00000	0·00000

Fator de atrito

Coeficiente de arraste (fator de atrito) na placa:

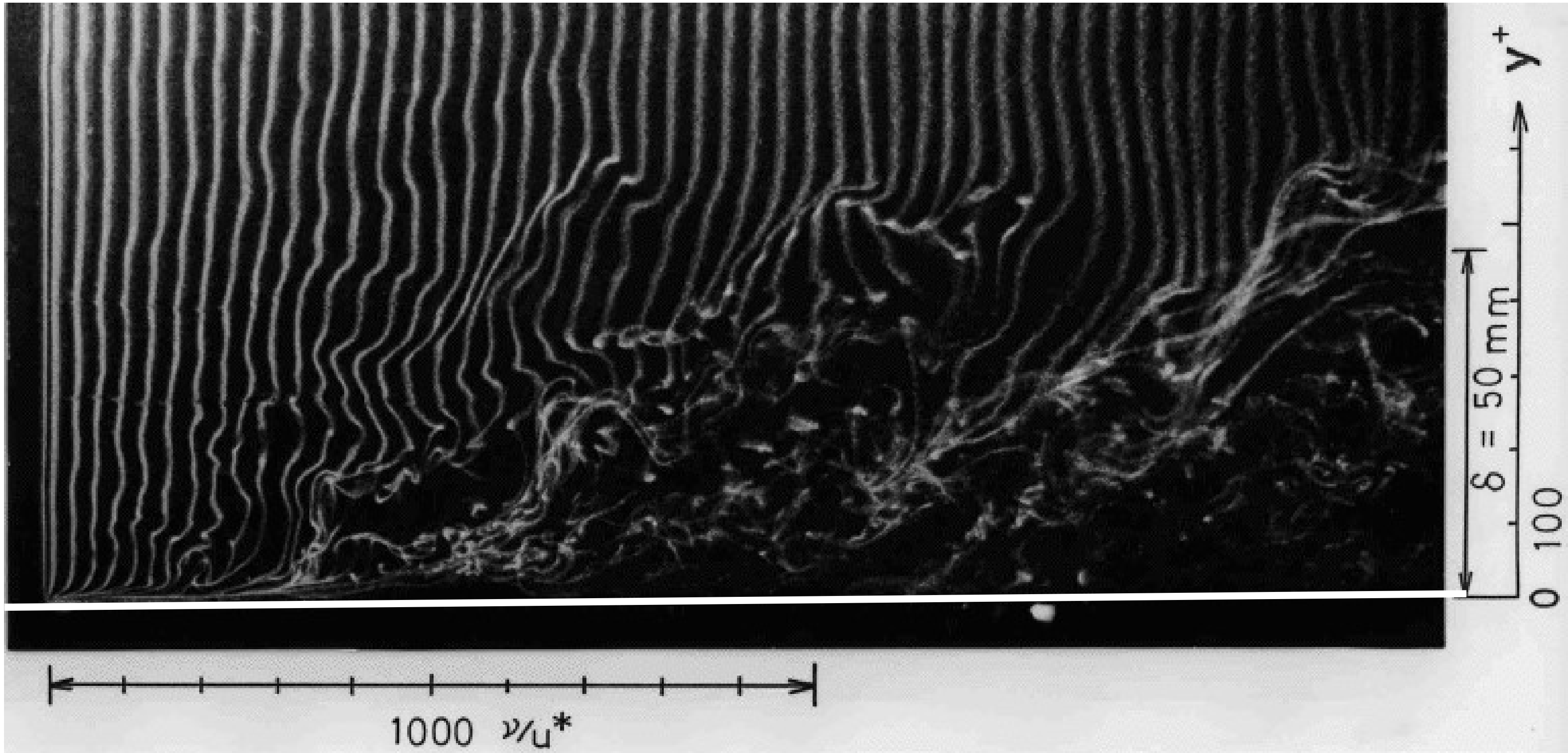
$$C_D = \frac{\left(\tau_{yx}\right)_{y=0}}{\frac{1}{2}\rho U^2}$$

$$\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$C_D = \frac{\mu}{\frac{1}{2}\rho U^2}\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right)_{y=0} = \frac{\mu}{\frac{1}{2}\rho U^2}\, U \sqrt{\frac{U}{x\,\nu}}\, \frac{d^2 f}{d\eta^2}(0)$$

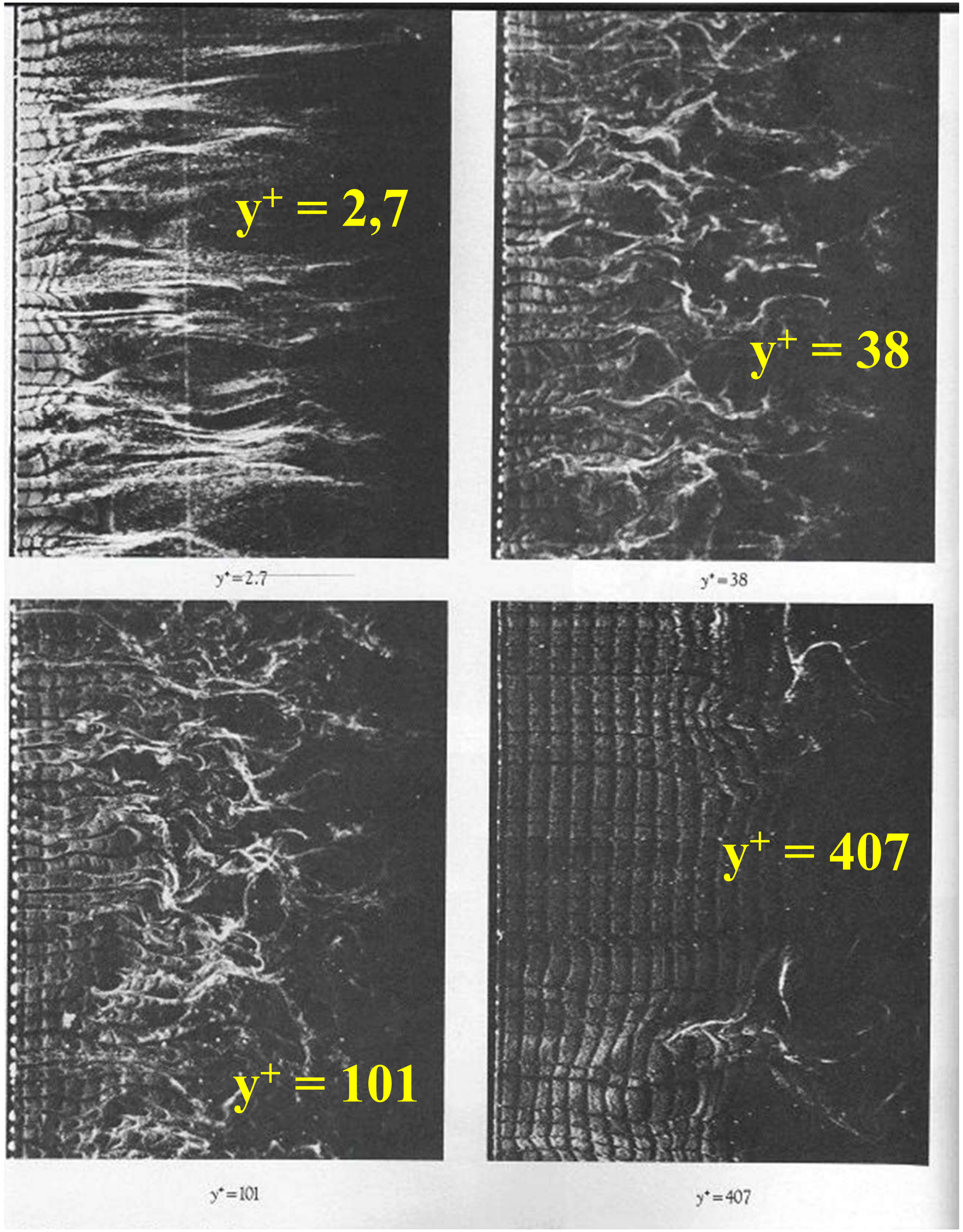
Para y = 0 (η=0) ⇒ f″(0) = 0,332, resulta:

$$C_D = 0{,}664\sqrt{\frac{\nu}{U\,x}} \Rightarrow \boxed{C_D = 0{,}664\ \mathrm{Re}_x^{-1/2}}$$



y+
δ = 50 mm
0 100
1000 ν/u*

161. Structure of a turbulent boundary layer. Successive layers of the flow near a flat plate in a water channel are shown by tiny hydrogen bubbles released periodically from a thin platinum wire seen at the left. The height $y^+ = y\,u_\tau/\nu$ of the wire above the plate is shown in wall variables, where $u_\tau = (\tau_w/\varrho)^{1/2}$ is the friction velocity. The characteristic low- and high-speed streaks shown in the viscous sublayer at $y^+ = 2.7$ become less noticeable farther away, and have disappeared in the logarithmic region at $y^+ = 101$. In the wake region at $y^+ = 407$ the turbulence is seen to be intermittent and of larger scale. *Kline, Reynolds, Schraub & Runstadler 1967*



y+