USING A PERSONAL COMPUTER TO TEACH POWER SYSTEM TRANSIENTS

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ABSTRACT – This paper presents a state variable technique for teaching power system transients using a personal computer. Transmission lines are divided into a series of pi–section segments. Each segment consists of a series resistance and inductance and a shunt conductance and capacitance. Using this line representation, a state model is formulated for the power system using the capacitor voltages and inductor currents as the state variables. The state equations describing the system are transformed to a set of linear difference equations through the use of trapezoidal integration. The state variables are updated by solving this set of equations. Nonlinear elements such as surge arresters may be included in the analysis. The technique presented here utilizes the student's knowledge of network theory and is easily implemented on a personal computer.

KEYWORDS: education, personal computer, state variables, power system transients

INTRODUCTION

Electromagnetic transients in power systems may be produced by switching actions, faults, or lightning strikes. Students may investigate the effects of transients by using a transient network analyzer or a digital computer. A transient network analyzer requires the construction of a power system analog. Computer–based transient analysis techniques necessitate developing a mathematical model to describe the transient behavior of the power system. These techniques are especially attractive for student investigation of transient phenomena because of the availability of low cost personal computers. This paper presents a state variable technique which can be used to teach a senior elective course in power system transients using the personal computer.

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One popular computer program utilized by the electric utility industry is the Electromagnetics Transients Program (EMTP) [1], which is based on the ideas of Dommel [2–4]. Simple equivalent networks are derived for all components in the system. Nodal equations are formulated for the system represented by the equivalent networks. The transient is calculated through repeat solutions of these nodal equations. Since EMTP is not user friendly, the student can easily become frustrated with it. In addition, EMTP requires an extensive background in power system analysis and will overwhelm the beginning student. Glover and Sarma [5] present a technique similar to that used in EMTP for the solution of transient problems. Their computer program is much more user friendly than EMTP and, therefore, is much easier for the student to use. The classical lattice diagram technique [5,6,10] may be used in a study of transients. This technique uses an approximation to the travelling wave equation. The student needs to understand the concepts of reflection coefficients and impedance matching, which are not network concepts. In addition, it is very difficult to incorporate resistive effects. The lattice technique appears to be more difficult to implement on a computer than the concepts behind EMTP [3].

The technique presented in this paper utilizes the student's background in network theory. It requires the formulation of a state model for the power system. Therefore, the student only needs a course in network theory as a prerequisite. Many network textbooks such as [7] present techniques for deriving state models for networks. The state variables are selected to be the capacitor voltages and inductor currents in the network. The state equations describing the network can be written in the form

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{1}$$

where $\underline{x}$ is a vector of state variables, $\underline{u}$ is a vector of inputs, and A and B are matrices. Equation (1) may be solved using many techniques. Trapezoidal integration [7] is employed here to transform the state equations to a set of linear difference equations [8]

$$A'\underline{x}[(k+1)T] = \underline{b}'[kT] \quad (2)$$

where $\underline{b}'[kT] = \underline{f}(\underline{x}[kT], \underline{u}[kT], \underline{u}[(k+1)T])$. A' is a sparse matrix which is constant for a fixed time step T. The vector $\underline{x}[(k+1)T]$ is solved for by either finding the inverse of A' or performing an LU factorization of A' to preserve the sparsity. The latter is utilized in this paper because of the special structure of the A' matrix. Since A' is constant for a fixed time step, it is only factored once at the beginning of the simulation. The right hand side of (2) is recalculated at each time step.

Nonlinear devices such as surge arresters are a part of electromagnetic transient studies. In [4], an iterative procedure is utilized to find a solution when a nonlinear device is present in the system. In this paper the nonlinear device is treated as a piecewise linear resistance. The A matrix is formulated in symbolic form and is evaluated for the different values of the piecewise linear resistance. All A and A' matrices are evaluated and factored or inverted before simulation begins.

This state variable technique is easily implemented on the personal computer. The student only needs a routine to solve (2). Sparsity techniques can be utilized in the LU factorization of A' to reduce memory requirements. Since A' only needs to be factored once and iterative procedures are unnecessary, the transient may be calculated quickly on a personal computer. This technique is illustrated with a simple example.

TRANSMISSION LINE MODEL

The state variable technique discussed here requires an equivalent network from which state equations can be written. The transmission line is modeled as an interconnection of n pi–networks. Each pi–network contains a series resistance and inductance and a shunt conductance and capacitance as seen in Figure 1. R', L', C', and G' are the resistance, inductance, conductance, and capacitance per unit length, respectively, of the transmission line. The resistance and inductance for each pi–network are determined by dividing the total resistance and inductance for the line by the number of pi–networks n. The shunt conductance and capacitance for each pi–network are determined in the same manner. One half of the calculated values is assigned to each end of the pi–network in Figure 1.

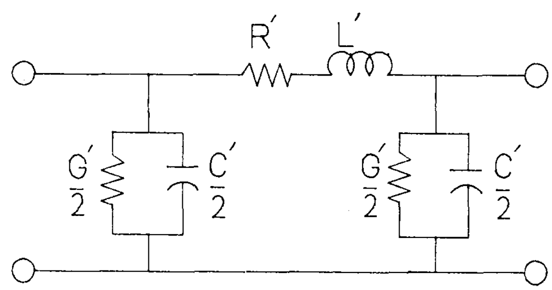

Figure 1. Pi–network section for transmission line

The n pi–networks are connected in series as shown in Figure 2 to form the transmission line model. Note that the shunt elements of adjacent pi–networks are combined in parallel. The capacitor voltages and inductor currents are designated as the state variables. Examination of Figure 2 indicates that the state equations for the transmission line have a special structure. The state for each of the shunt capacitors is affected only by the two inductors connected to it and the capacitor itself. This will be demonstrated more clearly with an example system in a later section.

SOLUTION PROCEDURE

The model of the transmission line presented in the previous section is combined with other component models. State equations are formulated for the system and can be written in the form of (1). This set of linear ordinary differential equations is transformed into a set of linear difference equations using trapezoidal integration [8]. If T is the time step, equation (1) becomes

$$\underline{x}[k+1] = \underline{x}[k] + \frac{T}{2}\Big[A\underline{x}[k+1] + B\underline{u}[k+1] + A\underline{x}[k] + B\underline{u}[k]\Big] \quad (3)$$

The indices k and k+1 are utilized to indicate the values of $\underline{x}$ or $\underline{u}$ at time t=kT or t=(k+1)T, respectively. Rearranging (3) yields

$$\left[I - \frac{T}{2}A\right]\underline{x}[k+1] = \left[I + \frac{T}{2}A\right]\underline{x}[k] + \frac{T}{2}B\left[\underline{u}[k] + \underline{u}[k+1]\right] \quad (4)$$

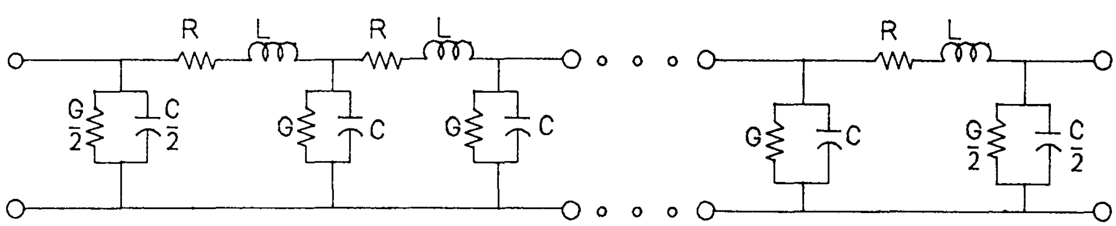

Figure 2. Transmission line model

If the inputs $\underline{u}$ are known for all discrete points in time $t=kT$ for $k = 0, 1, 2 \ldots, \underline{x}[k+1]$ can be determined using (4). Define

$$A' = \left[I - \frac{T}{2}A\right] \qquad A'' = \left[I + \frac{T}{2}A\right] \qquad B' = \frac{T}{2}B \tag{5}$$

$$\underline{b}'[k] = A''\underline{x}[k] + B'(\underline{u}[k] + \underline{u}[k+1]) \tag{6}$$

All matrices in (5) are constant for a fixed time step; they are evaluated before simulation begins. An LU factorization is also performed on A' before simulation begins. The vector $\underline{b}'[k]$ must be evaluated at each discrete point in time.

Because of the transmission line model employed here, A is a tridiagonal matrix. Therefore, A' and A'' are also tridiagonal. Only the nonzero elements of A'' and B' are stored to decrease memory requirements and execution time. In order to retain the sparsity of A, Crout reduction [9] is utilized to obtain an LU factorization of A' considering its tridiagonal nature. Crout reduction may be applied to a system of linear equations if the diagonal entries of L are nonzero [9]. For a set of n equations in n unknowns, Crout reduction requires only $(5n - 4)$ multiplications/divisions and $(3n - 3)$ additions/subtractions to solve the set of equations. This can result in considerable savings in execution time for large n.

The simulation procedure will now be summarized:

1. Calculate A', A'', and B' from (5).
2. Compute LU factorization of A'.
3. Set $k = 0$.
4. Calculate $\underline{b}'[k]$ from (6).
5. Solve for $\underline{x}[k+1]$ using (4).
6. $k = k + 1$.
7. Repeat 4, 5, and 6 until end of simulation.

EXAMPLE SYSTEM

The state variable technique presented in this paper is illustrated using an example from [5]. In this example, a lightning strike occurs at the center of a 20 kV, 10 km line which has $R' = 0.05\ \Omega/\text{km}$, $G' = 0.556\ \mu\text{S/km}$, $L' = 1$ mH/km, and $C' = 11.11$ nF/km. A transformer is connected to one end of the line which is modeled by a capacitance to ground of $C_T = 6$ nF. The other end of the line is terminated in an open circuit. The lightning strike is modeled by an ideal square wave current source connected to a node in the middle of the line; the source has a magnitude of 20 kA for 20 μs. The transformer is protected by a surge arrester whose v–i curve is approximated as: $R_A = 2\ \text{M}\Omega$ for $V_A < 55$ kV and $R_A = 4.5\ \Omega$ for $V_A \geq 55$ kV.

Since the surge arrester and transformer are connected to one end of the transmission line, the shunt conductance and capacitance at the end of the line in Figure 2 become $(G/2 + 1/R_A)$ and $(C/2 + C_T)$, respectively. If the state equations are formulated such that the odd–numbered state variables correspond to capacitor voltages and the even–numbered state variables correspond to inductor currents, the following A matrix results

$$A = \begin{bmatrix} -\frac{G}{C} & -\frac{2}{C} & & & & \\ \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} & & & \\ & \frac{1}{C} & -\frac{G}{C} & -\frac{1}{C} & & \\ & & \ddots & \ddots & \ddots & \\ & & & \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} \\ & & & & \frac{2}{C+2C_T} & -\left[\frac{G+2/R_A}{C+2C_T}\right] \end{bmatrix} \tag{7}$$

Only the nonzero entries of A are shown in (7). Note that the A matrix is tridiagonal for this system as was previously stated. With the exception of the first and last rows, the elements of the superdiagonal alternate between $(-1/L)$ and $(-1/C)$, and the elements on the subdiagonal alternate between $(1/L)$ and $(1/C)$.

For the n section line in this example, there are 2n+1 state variables. Since the square wave current source is the only source in this example, only one entry in B is nonzero. This current source is connected across one of the capacitors; therefore, this entry must have an odd subscript because the capacitor voltages are the odd–numbered state variables. The nonzero entry in B corresponding to the current source will be designated b_{n+1} and has a value of $(1/C)$.

Two simulations are performed to illustrate the state variable technique. The transmission line is divided into 50 pi–networks and a time step of 0.05 μs is employed. The example system is first simulated without the surge arrester. The simulation results are given in Figure 3. A second simulation is performed with the surge arrester included. Two A' matrices, one for $R_A = 2\ \text{M}\Omega$ and another for $R_A = 4.5\ \Omega$, are evaluated and factored for this simulation. When the voltage across the surge arrester exceeds 55 kV, its resistance is changed from 2 MΩ to 4.5 Ω. The A' matrix corresponding to the lower resistance is now used in the simulation until the

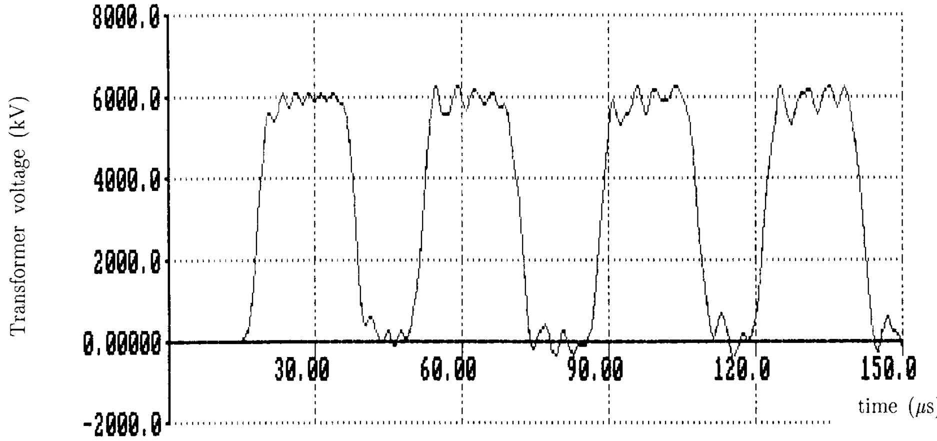

Figure 3. Transformer voltage without surge arrester

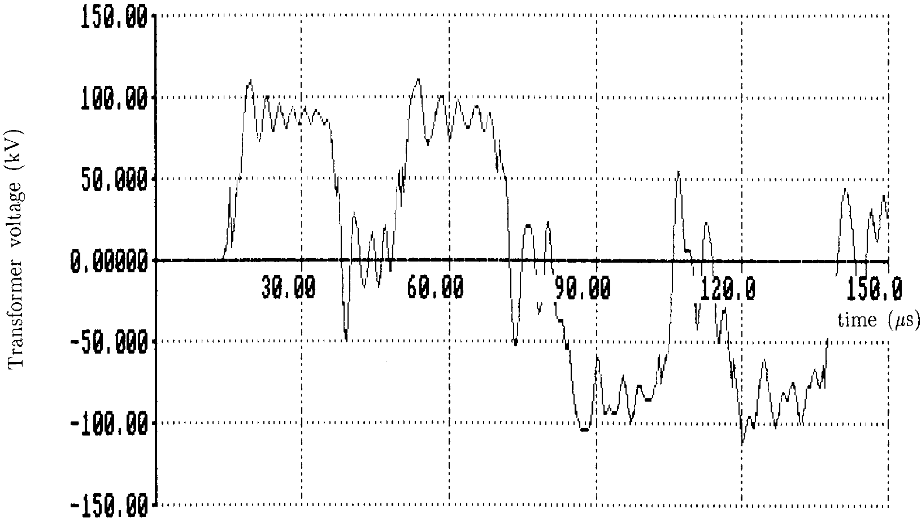

Figure 4. Transformer voltage with surge arrester

surge arrester voltage drops below 55 kV. Figure 4 shows the results of this simulation. Figures 3 and 4 compare favorably with the results given in [5] and with results obtained from EMTP.

CONCLUSION

This paper has presented a state variable technique for teaching power system transients. Transmission lines are modeled as a cascade connection of pi–networks. Using this line model and other component models, the student can formulate a state model for the power system using elementary network theory. The state equations are converted to a set of linear difference equations using trapezoidal integration. If the inputs are known for all time, the response of the power system can be determined by repetitively solving this set of linear equations. This technique can be easily implemented by the student on a personal computer. A subroutine to solve a set of linear equations is all the student needs. The technique can also be extended to include nonlinear elements such as surge arresters.

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