The Multiple Regression Model: Hypothesis testing with Interaction Effects

Lorena G. Barberia

October 4, 2018

The Multivariate Regression Model

1. Hypothesis Testing: Additive Model

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + u_i$$

2. Hypothesis Testing: Multiplicative Models

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 X_i Z_i + u_i$$

2.1. X is a dummy and Z is a continuous variable (lecture)

2.2 X and Z are dummy variables (homework)

2.3 X and Z are continuous variables (lecture)

The Multivariate Regression Model

Population Regression Function (PRF):

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + u_i$$

Sample Regression Function -SRF:

$$\hat{y}_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \hat{u}_i$$

The Multivariate Regression Model

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \hat{u}_i$$

$\beta_o = \hat{y}$ if $x = 0$ and $z = 0$

$\beta_1 = \frac{\Delta y}{\Delta x}$ holding z constant (or the partial effect).

$\beta_2 = \frac{\Delta y}{\Delta z}$ holding x constant (or the partial effect).

The Multivariate Regression Model

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \hat{u}_i$$

$$\beta_1 = \frac{\partial y}{\partial x} = \frac{\Delta y}{\Delta x}$$

$$\beta_2 = \frac{\partial y}{\partial z} = \frac{\Delta y}{\Delta z}$$

The Multivariate Regression Model

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \beta_3 x_i z_i + \hat{u}_i$$

$$\frac{\Delta y}{\Delta x} = \beta_1 + \beta_3 z_i$$

$$\frac{\Delta y}{\Delta z} = \beta_2 + \beta_3 x_i$$

Hypothesis Testing: Additive and Interaction Models

U.S. Presidential Elections in 1996 --Bill Clinton vs. Bob Dole

Hilary Thermometer Question in ANES 1996 Pre-Election Questionnaire:

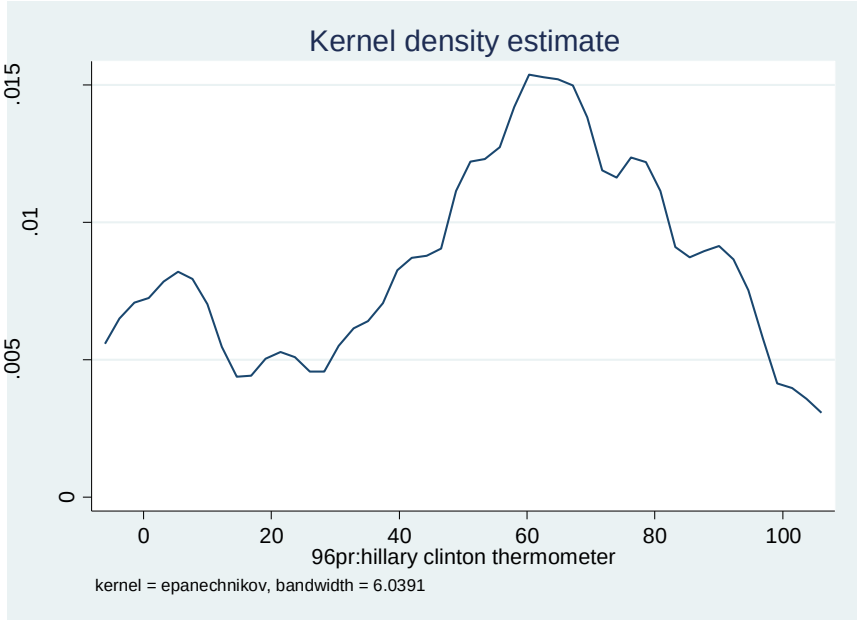
I'd like to get your feelings toward some of our political leaders and other people who are in the news these days. I'll read the name of a person and I'd like you to rate that person using something we call the feeling thermometer. Ratings between 50 degrees and 100 degrees mean that you feel favorable and warm toward the person. Ratings between 0 degrees and 50 degrees mean that you don't feel favorable toward the person and that you don't care too much for that person. You would rate the person at the 50 degree mark if you don't feel particularly warm or cold toward the person.

<50 =not favorable
50 = indifferent
> 50 = favorable

Hypothesis Testing: Additive and Interaction Models

U.S. Presidential Elections in 1996 --Bill Clinton vs. Bob Dole

Hilary Thermometer Question in ANES 1996 Pre-Election Questionnaire:



Hypothesis Testing: Additive and Interaction Models

Additive Model:

$$\text{Hillary Thermometer} = \beta_0 + \beta_1 \text{Female} + \beta_2 \text{Income} + u$$

Interaction Model:

$$\text{Hillary Thermometer} = \beta_0 + \beta_1 \text{Female} + \beta_2 \text{Income} + \beta_3 (\text{Female*Income}) + u$$

Hypothesis Testing: Additive and Multiplicative Models

$$\text{Hillary}=\beta_0+\beta_1\text{Income}+\beta_2\text{Female}+u$$

1. Effect of Being Female (Female==1):

intercept = $\beta_0+\beta_2$

$$\frac{\Delta y}{\Delta income}=\beta_1$$

2. Effect of Being Male (Female==0):

intercept = β_0

$$\frac{\Delta y}{\Delta income}=\beta_1$$

Hypothesis Testing: Additive and Interaction Models

Tabel 1. The Effect of Gender and Income on Hillary Thermometer Scores

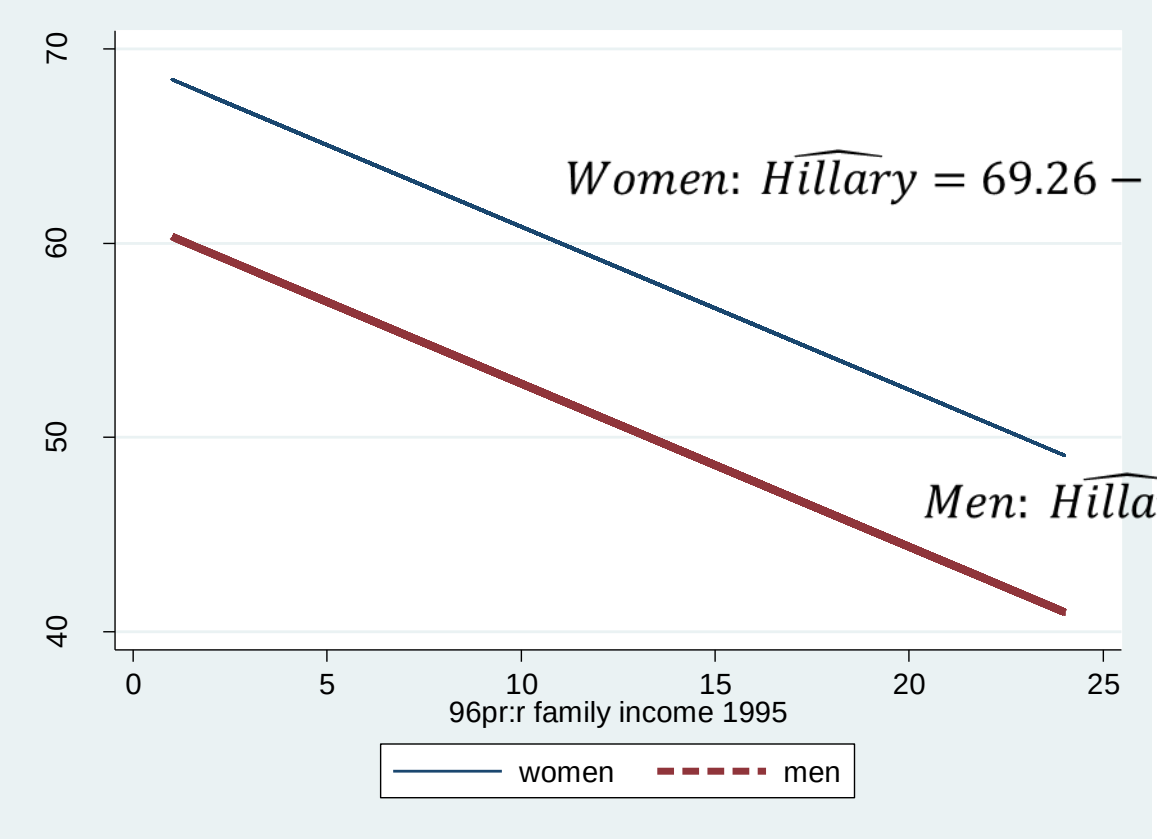
Variable	
Female	8.08*** (1.50)
Income	-0.84*** (0.12)
Intercept	61.18*** (2.22)
n	1.542
R^2	0.06

Hypothesis Testing: Additive and Interaction Models

	model_1	model_2
96pr:r family i~1995	-0.841***	-0.841***
	(0.12)	(0.12)
male	-8.081***	
	(1.50)	
female		8.081***
		(1.50)
Constant	69.262***	61.180***
	(1.92)	(2.22)
R-squared	0.059	0.059
rmse	28.68387	28.68387
N	1542	1542

* p<0.05, ** p<0.01, *** p<0.001

Hypothesis Testing: Additive and Interaction Models



$$Women: \widehat{Hillary} = 69.26 - 0.84 * Income$$

$$Men: \widehat{Hillary} = 61.18 - 0.84 * Income$$

Hypothesis Testing: Additive and Multiplicative Models

$$\text{Hillary}=\beta_0+\beta_1\text{Income}+\beta_2\text{Female}+\beta_3(\text{Female*Income})+u$$

1. Effect of Being Female (Female==1):

intercept = $\beta_0+\beta_2$

$$\frac{\Delta y}{\Delta Income}=(\beta_1+\beta_3)$$

2. Effect of Being Male (Female==0):

intercept = β_0

$$\frac{\Delta y}{\Delta Income}=\beta_1$$

Hypothesis Testing: Additive and Interaction Models

```
.
. esttab m1 m2, se
```

	(1) hillary	(2) hillary
female	8.081***	3.367
	(1.495)	(4.027)
faminc	-0.841***	-1.026***
	(0.118)	(0.189)
fem_faminc		0.304
		(0.242)
_cons	61.18***	64.23***
	(2.220)	(3.282)
N	1542	1542

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Hypothesis Testing: Additive and Multiplicative Models

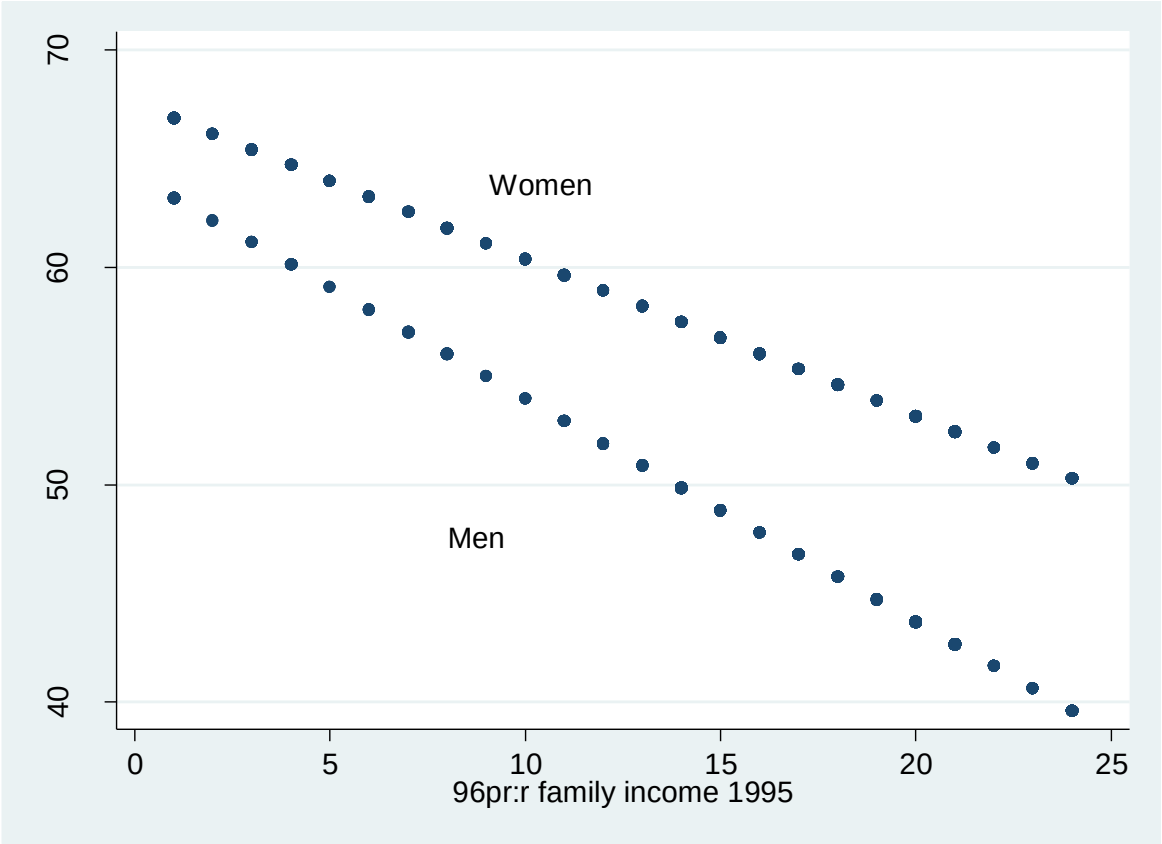
	Female=0	Female=1
Intercept	β_0	$\beta_0 + \beta_2$
Marginal Effect of Income	β_1 $\sqrt{\text{var}\,\beta_1}$	$\beta_1 + \beta_3$ $\sqrt{\text{var}(\beta_1) + Z^2\,\text{var}(\beta_3) + 2Z\,\text{cov}(\beta_1\beta_3)}$

For more detailed discussion, see:
http://mattgolder.com/wp-content/uploads/2015/05/standarderrors1.png

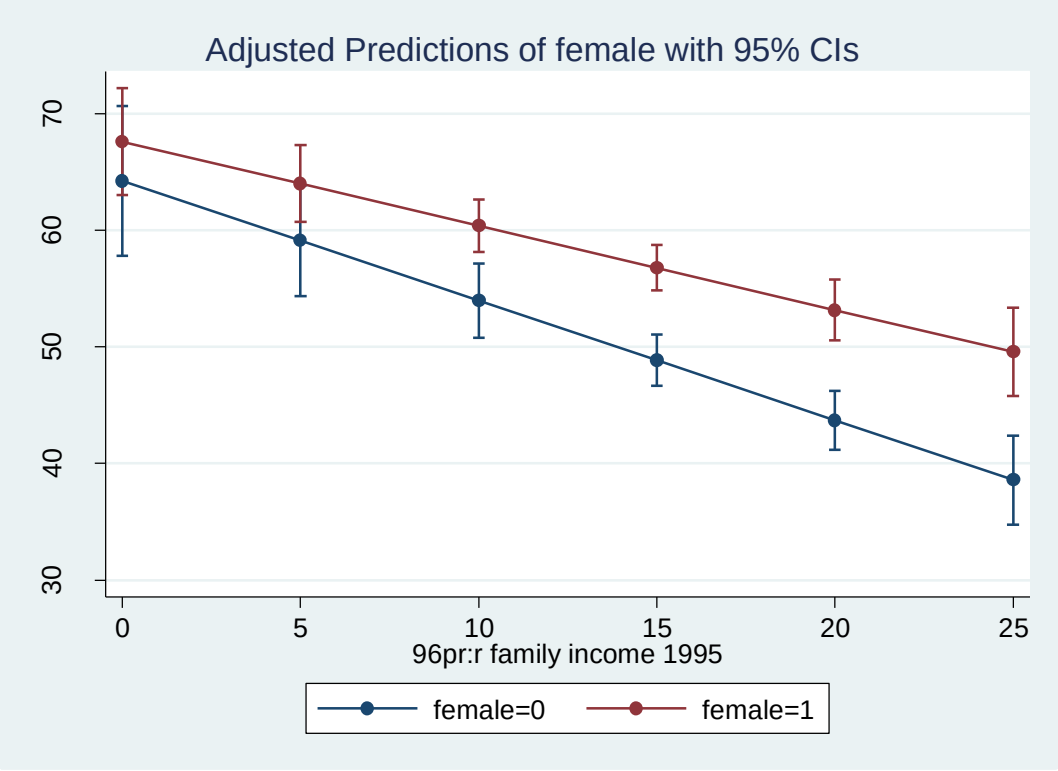
Hypothesis Testing: Additive and Interaction Models

	Men	Women
Intercept	64.23*** (3.282)	67.60*** (2.33)
Intercept 95% CI	[57.790, 70.665]	[63.018 , 72.173]
Marginal Effect of Income	-1.026*** (0.189)	-0.722*** (0.151)
Marginal Effect of Income 95% CI	[-1.396, -0.657]	[-1.018,-0.426]

Hypothesis Testing: Additive and Interaction Models



Hypothesis Testing: Additive and Interaction Models



Hypothesis Testing: Additive and Interaction Models

U.S. Presidential Elections in 1996-- Bill Clinton vs. Bob Dole

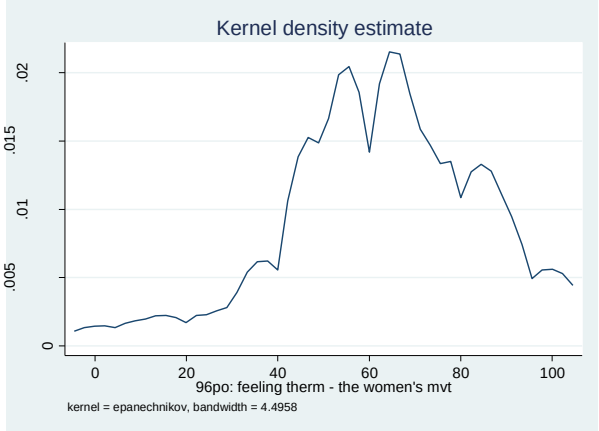
Question: Conditional on Income, how does the feeling of voters toward the feminist movement (Feminist Thermometer) impact their feeling toward Hillary Clinton (Hillary Thermometer)?

Hypothesis Testing: Additive and Interaction Models

U.S. Presidential Elections in 1996--Bill Clinton vs. Bob Dole

Women's Rights Question in ANES 1996 Pre-Election Questionnaire:

Recently there has been a lot of talk about women's rights. Some people feel that women should have an equal role with men in running business, industry, and government. Others feel that a woman's place is in the home. Where would you place yourself on this scale, or haven't you thought much about this?



Hypothesis Testing: Additive and Interaction Models

Interaction Model:

$$\text{Hillary Thermometer} = \alpha + \beta_1 \text{Income} + \beta_2 \text{Women's Movement Thermometer} + \beta_3 (\text{Income*Women`s Movement}) + u$$

Hypothesis Testing: Additive and Interaction Models

Interaction Model:

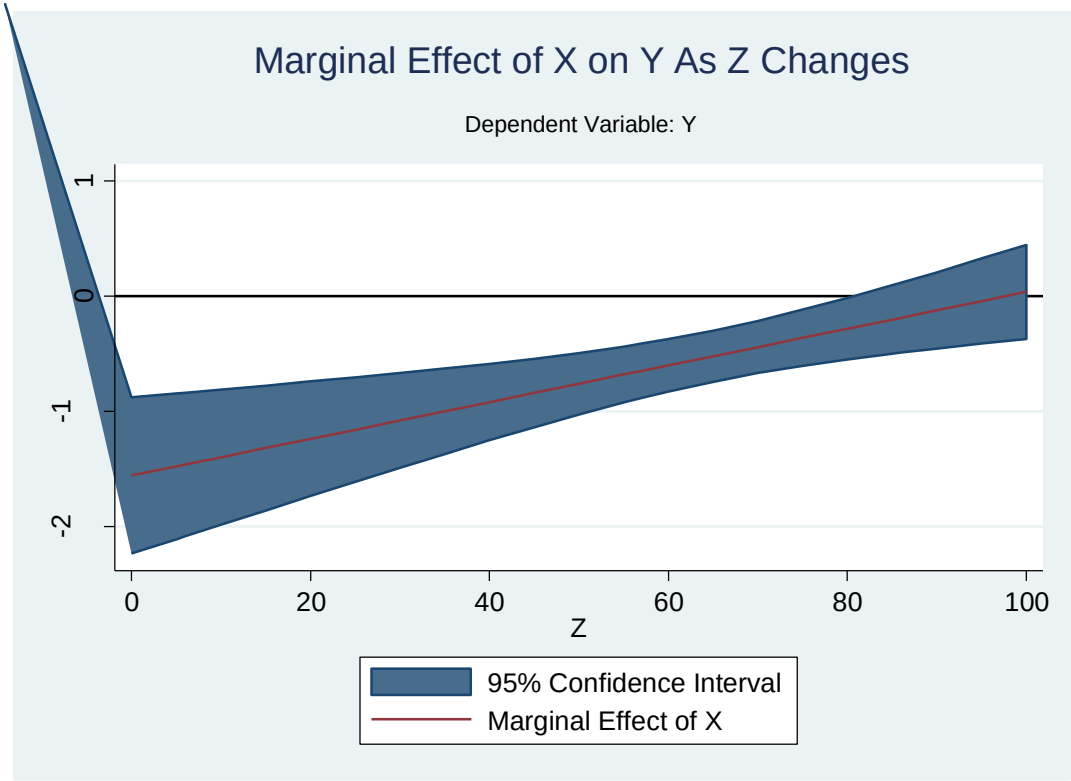
$$\text{Hillary Thermometer} = \alpha + \beta_1 \text{Income} + \beta_2 \text{Women's Movement Thermometer} + \beta_3 (\text{Income*Women`s Movement}) + u$$

$$\frac{\Delta y}{\Delta income} = \beta_1 + \beta_3 \text{Women`s Movement}$$

$$\frac{\Delta y}{\Delta womenmovt} = \beta_2 + \beta_3 \text{Income}$$

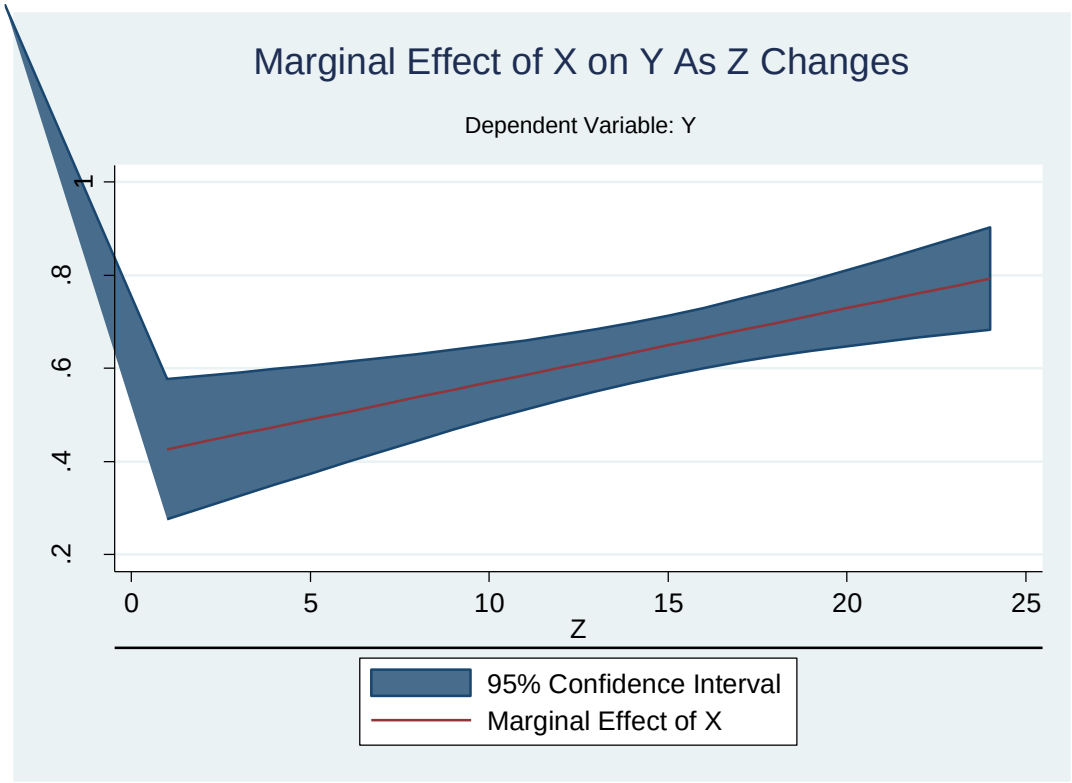
Hypothesis Testing: Additive and Interaction Models

Conditional on the women's movement, what is the marginal effect of changes in income ?



Hypothesis Testing: Additive and Interaction Models

Conditional on income, what is the marginal effect of being more in favor of women's rights?



Asymmetric Hypotheses

Clark, William, Michael Gilligan & Matt Golder. 2006. A Simple Multivariate Test for Asymmetric Hypotheses. *Political Analysis* 14: 311-331.

Table 3: All Logically Possible Combinations of Coefficients from a Simple Interaction Model and their Interpretation

Combination	β_1	β_2	β_3	*Valid conclusion about X*	*Valid conclusion about Z*
1	=0	=0	>0	necessary for Y	necessary for Y
2	>0	=0	>0	necessary and sufficient for Y	conditionally independent from Y
3	=0	>0	>0	conditionally independent from Y	necessary and sufficient for Y
4	>0	>0	>0	sufficient for Y and reinforcing	sufficient for Y and reinforcing
5	=0	=0	=0	unconditionally independent from Y	unconditionally independent from Y
6	>0	=0	=0	necessary and sufficient for Y	unconditionally independent from Y
7	=0	>0	=0	unconditionally independent from Y	necessary and sufficient for Y
8	>0	>0	=0	sufficient for Y	sufficient for Y

Notes: We are assuming only non-negative coefficients (for ease of presentation)