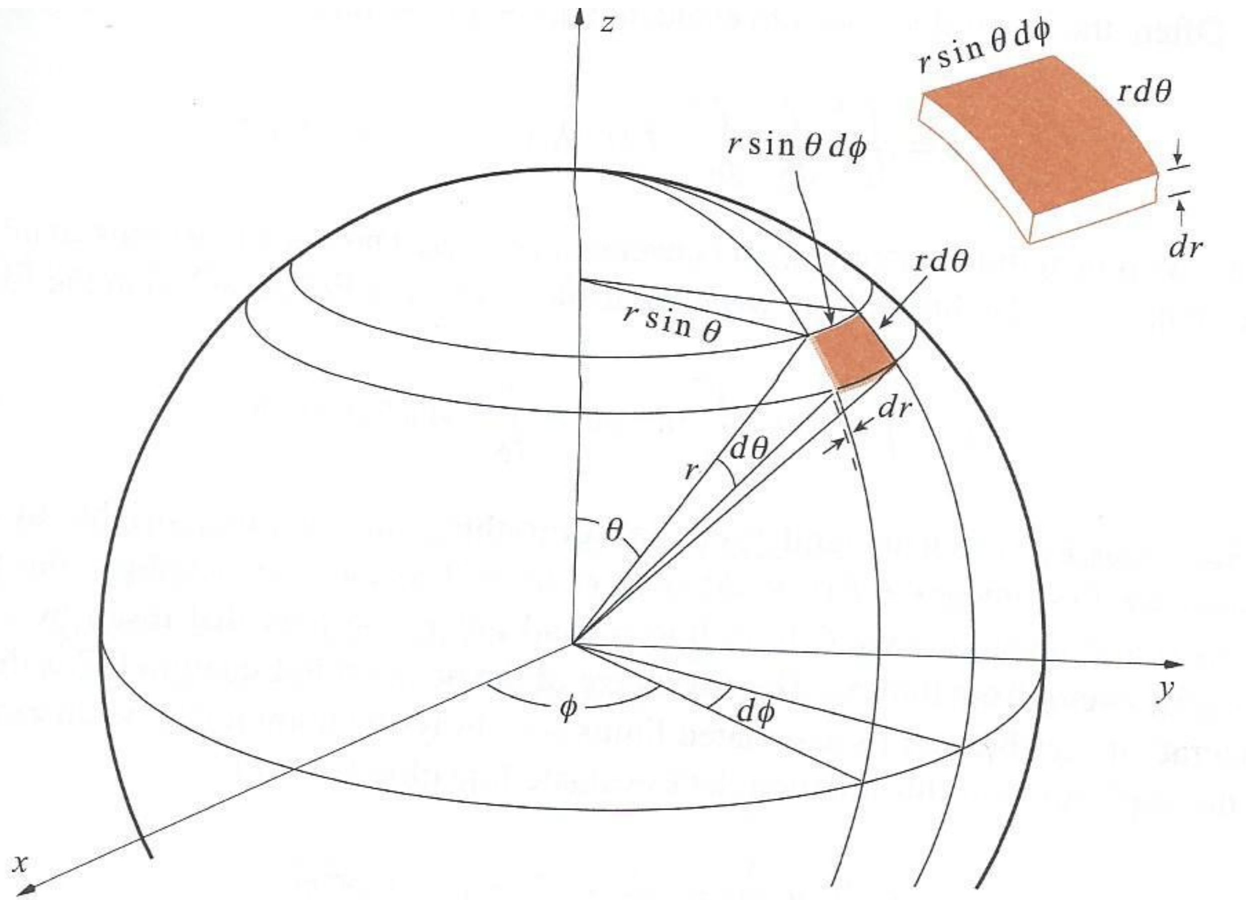


FIGURE E.2
A geometrical construction of the differential volume element in spherical coordinates.

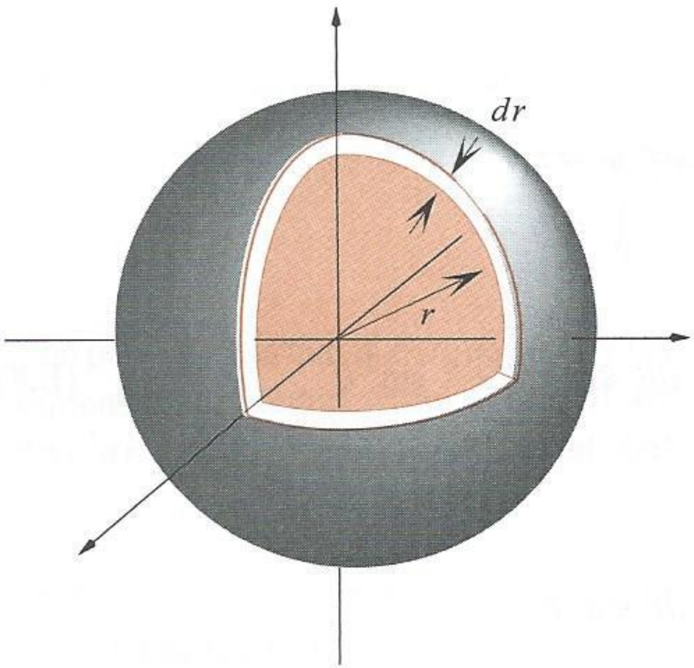


FIGURE E.3
A spherical shell of radius r and thickness dr. The volume of such a shell is $4\pi r^2 dr$, which is its area ($4\pi r^2$) times its thickness (dr).

TABLE 6.2
The first few associated Legendre functions $P_l^{|m|}(x)$

$P_0^0(x) = 1$
$P_1^0(x) = x = \cos\theta$
$P_1^1(x) = (1 - x^2)^{1/2} = \sin\theta$
$P_2^0(x) = \frac{1}{2}(3x^2 - 1) = \frac{1}{2}(3\cos^2\theta - 1)$
$P_2^1(x) = 3x(1 - x^2)^{1/2} = 3\cos\theta\sin\theta$
$P_2^2(x) = 3(1 - x^2) = 3\sin^2\theta$
$P_3^0(x) = \frac{1}{2}(5x^3 - 3x) = \frac{1}{2}(5\cos^3\theta - 3\cos\theta)$
$P_3^1(x) = \frac{3}{2}(5x^2 - 1)(1 - x^2)^{1/2} = \frac{3}{2}(5\cos^2\theta - 1)\sin\theta$
$P_3^2(x) = 15x(1 - x^2) = 15\cos\theta\sin^2\theta$
$P_3^3(x) = 15(1 - x^2)^{3/2} = 15\sin^3\theta$

TABLE 6.3

The first few spherical harmonics.

$Y_0^0 = \dfrac{1}{(4\pi)^{1/2}}$	$Y_1^0 = \left(\dfrac{3}{4\pi}\right)^{1/2} \cos\theta$
$Y_1^1 = \left(\dfrac{3}{8\pi}\right)^{1/2} \sin\theta e^{i\phi}$	$Y_1^{-1} = \left(\dfrac{3}{8\pi}\right)^{1/2} \sin\theta e^{-i\phi}$
$Y_2^0 = \left(\dfrac{5}{16\pi}\right)^{1/2} (3\cos^2\theta - 1)$	$Y_2^1 = \left(\dfrac{15}{8\pi}\right)^{1/2} \sin\theta\cos\theta e^{i\phi}$
$Y_2^{-1} = \left(\dfrac{15}{8\pi}\right)^{1/2} \sin\theta\cos\theta e^{-i\phi}$	$Y_2^2 = \left(\dfrac{15}{32\pi}\right)^{1/2} \sin^2\theta e^{2i\phi}$
$Y_2^{-2} = \left(\dfrac{15}{32\pi}\right)^{1/2} \sin^2\theta e^{-2i\phi}$	

two quantum numbers n and l and are given by

$$R_{nl}(r) = -\left\{\frac{(n-l-1)!}{2n[(n+l)!]^3}\right\}^{1/2}\left(\frac{2}{na_0}\right)^{l+3/2} r^l e^{-r/na_0} L_{n+l}^{2l+1}\left(\frac{2r}{na_0}\right)$$

TABLE 6.4

The first few associated Laguerre polynomials.

$n = 1,$	$l = 0$	$L_1^1(x) = -1$
$n = 2,$	$l = 0$	$L_2^1(x) = -2!(2 - x)$
	$l = 1$	$L_3^3(x) = -3!$
$n = 3,$	$l = 0$	$L_3^1(x) = -3!(3 - 3x + \frac{1}{2}x^2)$
	$l = 1$	$L_4^3(x) = -4!(4 - x)$
	$l = 2$	$L_5^5(x) = -5!$
$n = 4,$	$l = 0$	$L_4^1(x) = -4!(4 - 6x + 2x^2 - \frac{1}{6}x^3)$
	$l = 1$	$L_5^3(x) = -5!(10 - 5x + \frac{1}{2}x^2)$
	$l = 2$	$L_6^5(x) = -6!(6 - x)$
	$l = 3$	$L_7^7(x) = -7!$

TABLE 6.5

The complete hydrogenlike atomic wave functions for $n = 1, 2,$ and 3. The quantity Z is the atomic number of the nucleus, and $\sigma = Zr/a_0$, where a_0 is the Bohr radius.

$n = 1,$	$l = 0,$	$m = 0$	$\psi_{100} = \dfrac{1}{\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} e^{-\sigma}$
$n = 2,$	$l = 0,$	$m = 0$	$\psi_{200} = \dfrac{1}{\sqrt{32\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} (2-\sigma)e^{-\sigma/2}$
	$l = 1,$	$m = 0$	$\psi_{210} = \dfrac{1}{\sqrt{32\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2}\cos\theta$
	$l = 1,$	$m = \pm 1$	$\psi_{21\pm1} = \dfrac{1}{\sqrt{64\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2}\sin\theta\, e^{\pm i\phi}$
$n = 3,$	$l = 0,$	$m = 0$	$\psi_{300} = \dfrac{1}{81\sqrt{3\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} (27 - 18\sigma + 2\sigma^2)e^{-\sigma/3}$
	$l = 1,$	$m = 0$	$\psi_{310} = \dfrac{1}{81}\left(\dfrac{2}{\pi}\right)^{1/2}\left(\dfrac{Z}{a_0}\right)^{3/2} (6\sigma - \sigma^2)e^{-\sigma/3}\cos\theta$
	$l = 1,$	$m = \pm 1$	$\psi_{31\pm1} = \dfrac{1}{81\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} (6\sigma - \sigma^2)e^{-\sigma/3}\sin\theta\, e^{\pm i\phi}$
	$l = 2,$	$m = 0$	$\psi_{320} = \dfrac{1}{81\sqrt{6\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}(3\cos^2\theta - 1)$
	$l = 2,$	$m = \pm 1$	$\psi_{32\pm1} = \dfrac{1}{81\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}\sin\theta\cos\theta\, e^{\pm i\phi}$
	$l = 2,$	$m = \pm 2$	$\psi_{32\pm2} = \dfrac{1}{162\sqrt{\pi}}\left(\dfrac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}\sin^2\theta\, e^{\pm 2i\phi}$

TABLE 6.6

The complete hydrogenlike atomic wave functions expressed as real functions for $n = 1$, 2, and 3. The quantity Z is the atomic number of the nucleus and $\sigma = Zr/a_0$, where a_0 is the Bohr radius.

$n = 1,$	$l = 0,$	$m = 0$	$\psi_{1s} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-\sigma}$
$n = 2,$	$l = 0,$	$m = 0$	$\psi_{2s} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} (2 - \sigma)e^{-\sigma/2}$
	$l = 1,$	$m = 0$	$\psi_{2p_z} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2} \cos\theta$
	$l = 1,$	$m = \pm 1$	$\psi_{2p_x} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2} \sin\theta \cos\phi$
			$\psi_{2p_y} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2} \sin\theta \sin\phi$
$n = 3,$	$l = 0,$	$m = 0$	$\psi_{3s} = \frac{1}{81\sqrt{3\pi}}\left(\frac{Z}{a_0}\right)^{3/2} (27 - 18\sigma + 2\sigma^2)e^{-\sigma/3}$
	$l = 1,$	$m = 0$	$\psi_{3p_z} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma(6 - \sigma)e^{-\sigma/3} \cos\theta$
	$l = 1,$	$m = \pm 1$	$\psi_{3p_x} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma(6 - \sigma)e^{-\sigma/3} \sin\theta \cos\phi$
			$\psi_{3p_z} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma(6 - \sigma)e^{-\sigma/3} \sin\theta \sin\phi$
	$l = 2,$	$m = 0$	$\psi_{3d_{z^2}} = \frac{1}{81\sqrt{6\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}(3\cos^2\theta - 1)$
	$l = 2,$	$m = \pm 1$	$\psi_{3d_{xy}} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3} \sin\theta \cos\theta \cos\phi$
			$\psi_{3d_{yz}} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3} \sin\theta \cos\theta \sin\phi$
	$l = 2,$	$m = \pm 2$	$\psi_{3d_{x^2-y^2}} = \frac{1}{81\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3} \sin^2\theta \cos 2\phi$
			$\psi_{3d_{xy}} = \frac{1}{81\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3} \sin^2\theta \sin 2\phi$

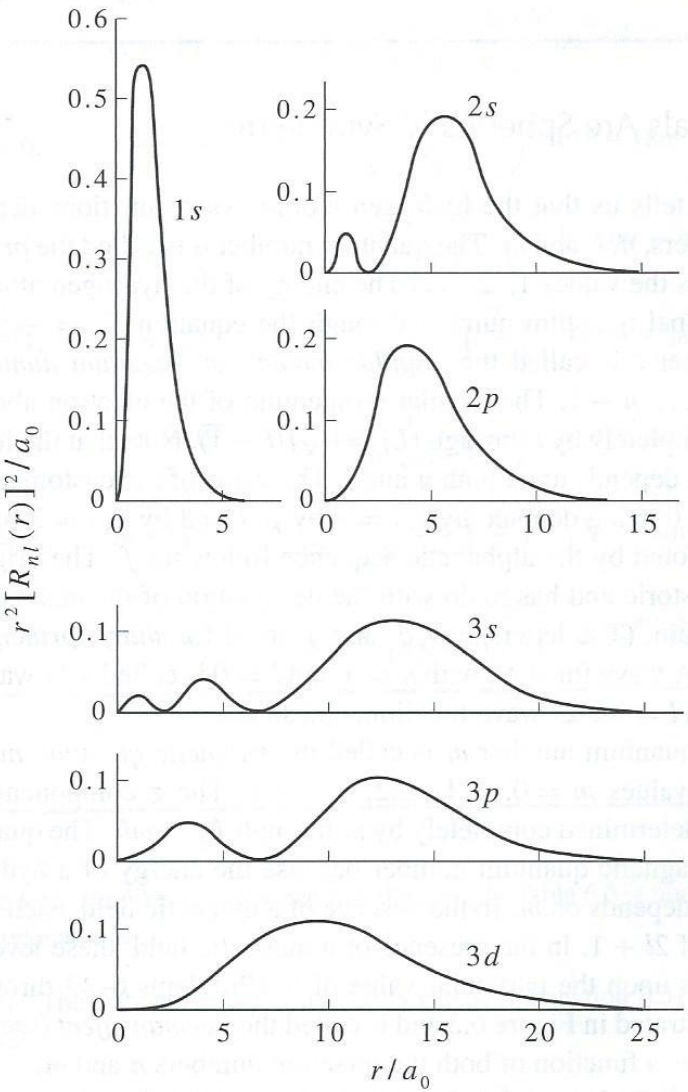


FIGURE 6.3
The probability densities $r^2[R_{nl}(r)]^2$ associated with the radial parts of the hydrogen atomic wave functions.

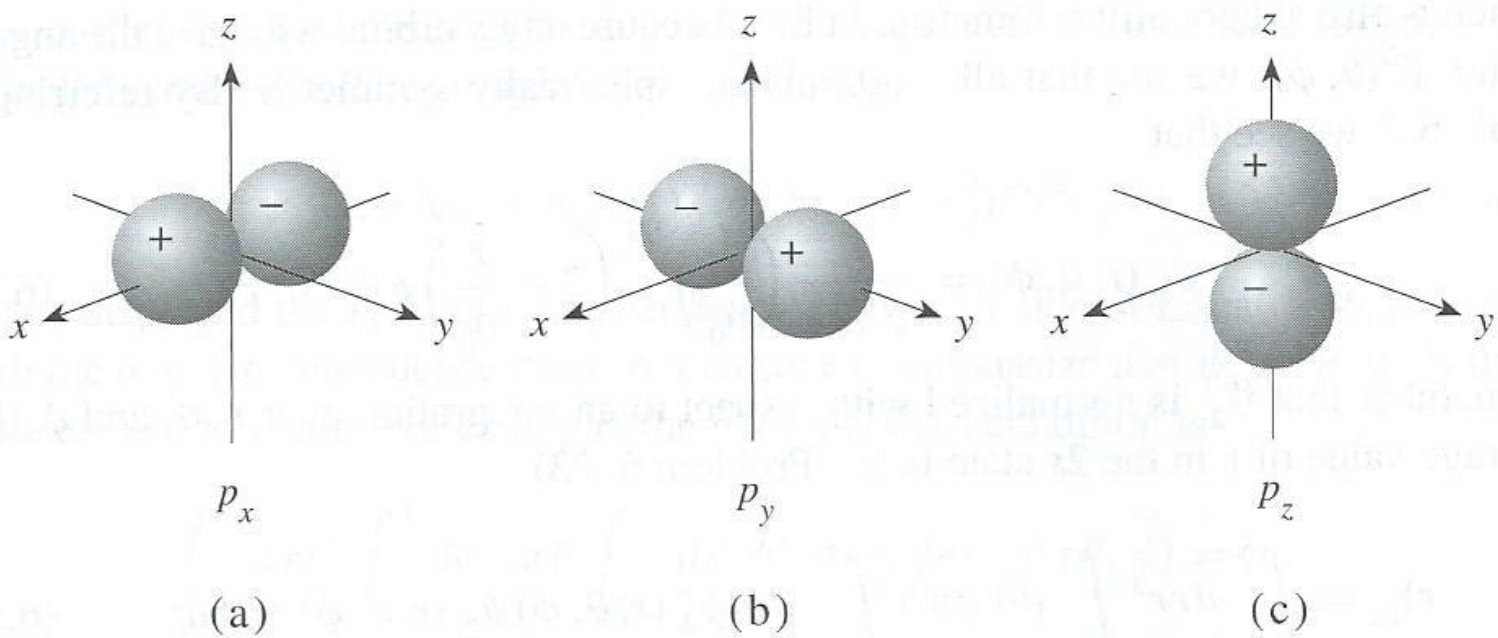


FIGURE 6.4
Three-dimensional polar plots of the angular part of the real representation of the hydrogen atomic wave functions for $l = 1$ (see Equations 6.62 for real representations of p_x and p_y.)

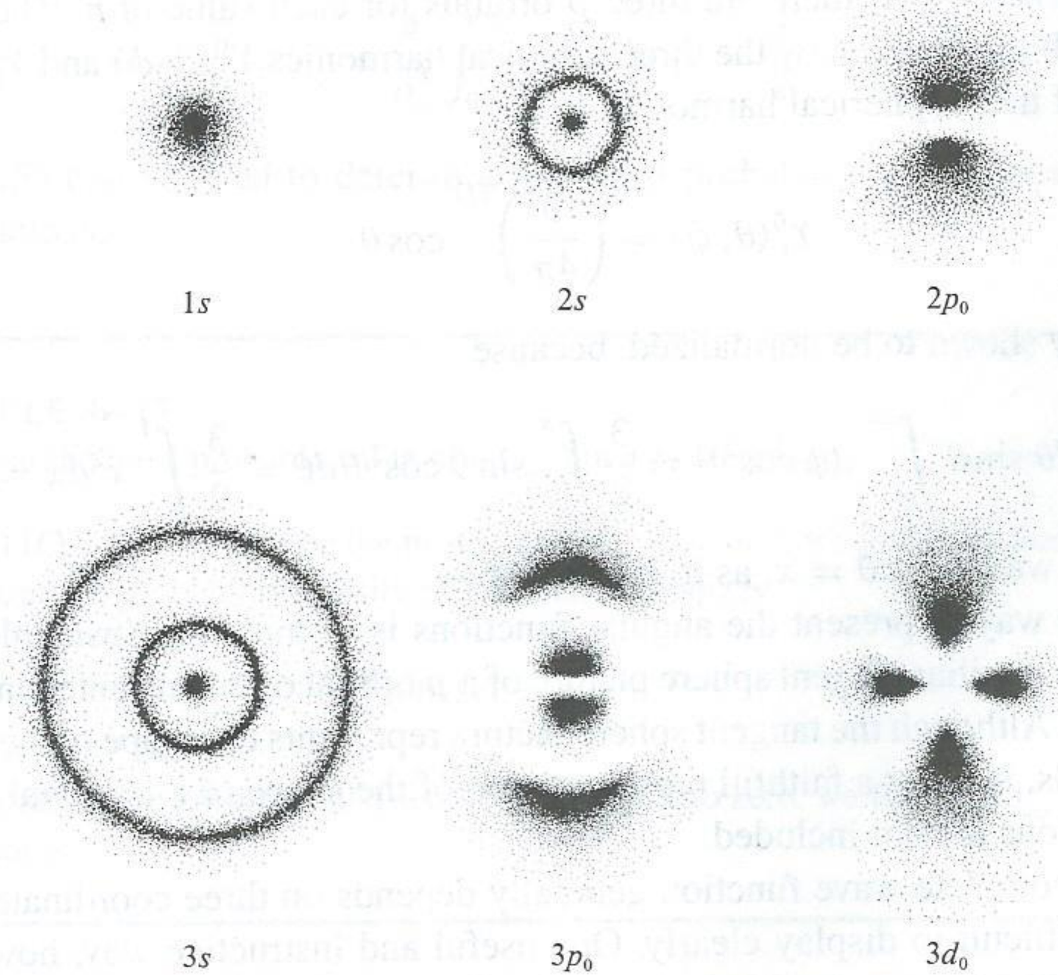


FIGURE 6.5
Probability density plots of some hydrogen atomic orbitals. The density of the dots is proportional to the probability of finding the electron in that region.

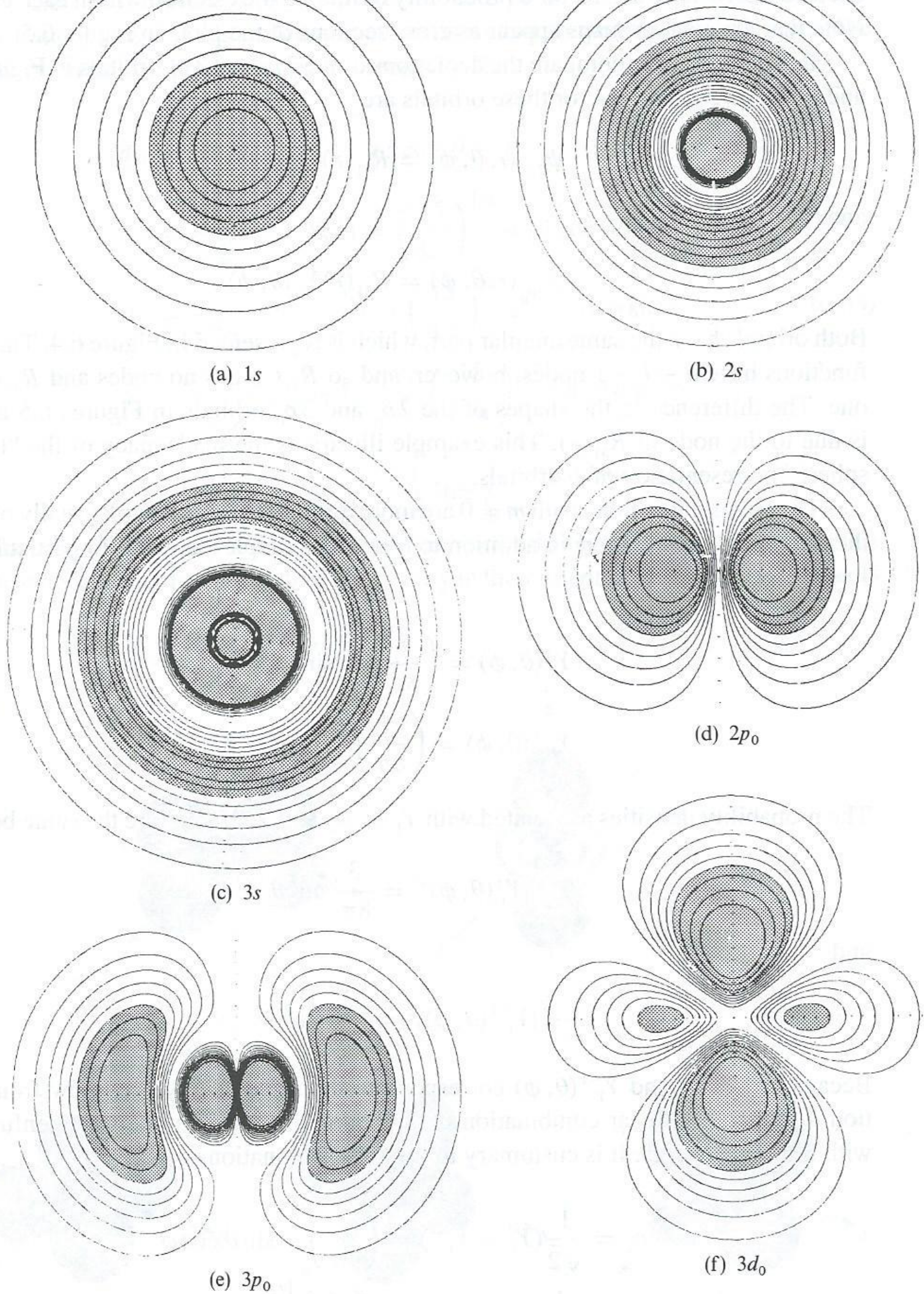


FIGURE 6.6
Probability contour maps for the hydrogen atomic orbitals. The nine contours shown in each case enclose the 10%, 20%, . . ., 90% probability of finding the electron within each

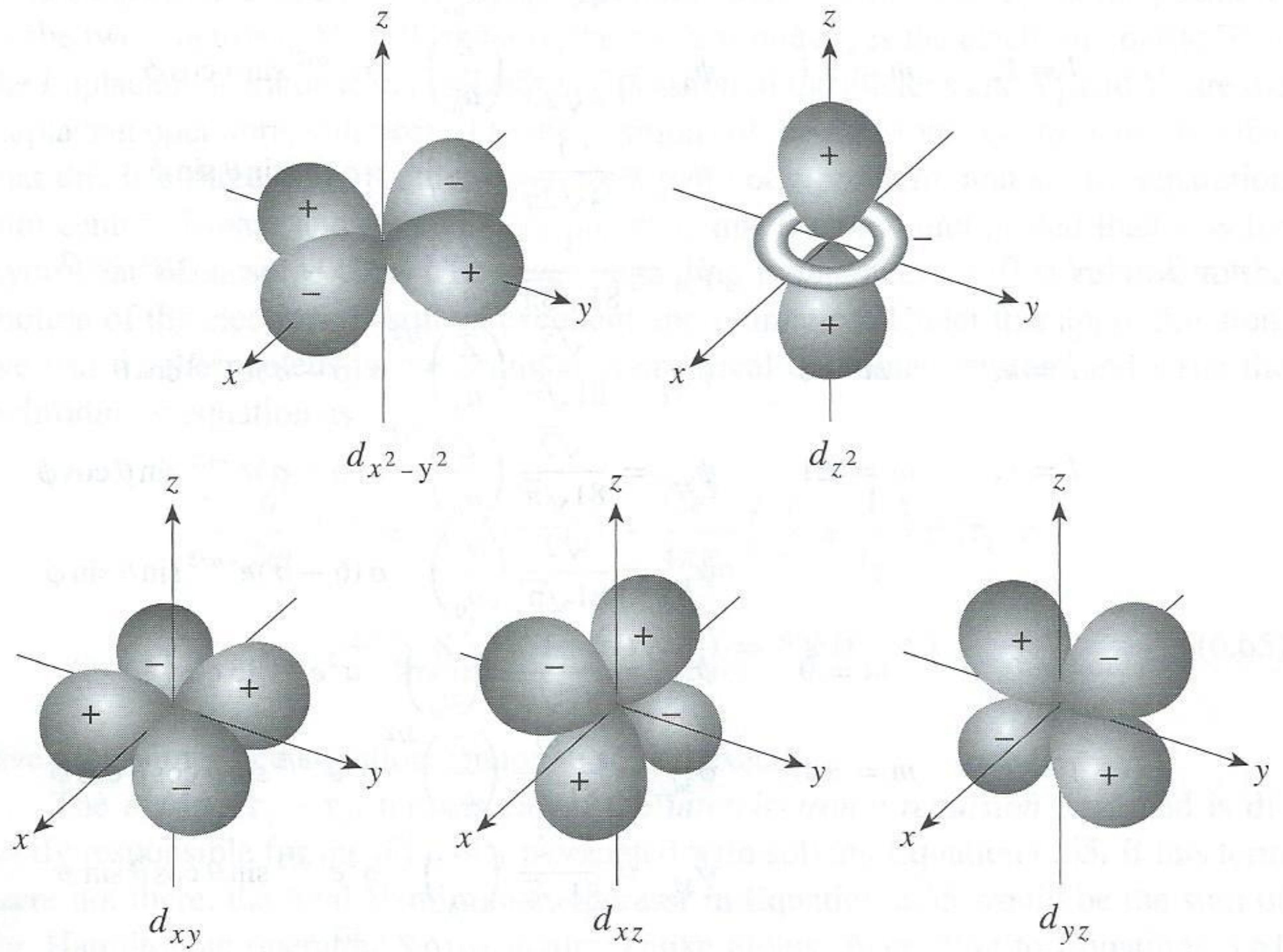


FIGURE 6.7

Three-dimensional plots of the angular part of the real representation of the hydrogen atomic wave functions for $l = 2$. Such plots show the directional character of these orbitals but are not good representations of the shape of these orbitals because the radial functions are not included.

$$d_{z^2} = Y_2^0 = \left(\frac{5}{16\pi}\right)^{1/2} (3\cos^2\theta - 1)$$

$$d_{xz} = \frac{1}{\sqrt{2}}(Y_2^1 + Y_2^{-1}) = \left(\frac{15}{4\pi}\right)^{1/2} \sin\theta\cos\theta\cos\phi$$

$$d_{yz} = \frac{1}{\sqrt{2}i}(Y_2^1 - Y_2^{-1}) = \left(\frac{15}{4\pi}\right)^{1/2} \sin\theta\cos\theta\sin\phi$$

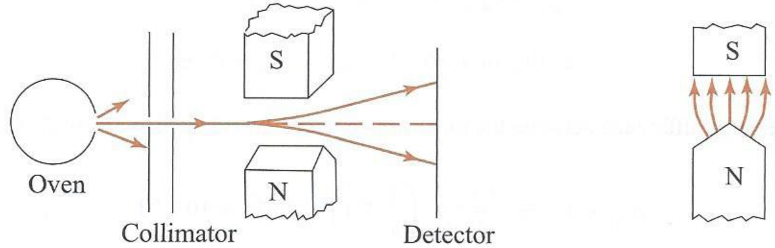
$$d_{x^2-y^2} = \frac{1}{\sqrt{2}}(Y_2^2 + Y_2^{-2}) = \left(\frac{15}{16\pi}\right)^{1/2} \sin^2\theta\cos 2\phi$$

$$d_{xy} = \frac{1}{\sqrt{2}i}(Y_2^2 - Y_2^{-2}) = \left(\frac{15}{16\pi}\right)^{1/2} \sin^2\theta\sin 2\phi$$

The angular parts of the five d orbitals are shown in Figure 6.7. Note that

TABLE 6.6
The complete hydrogenlike atomic wave functions expressed as real functions for $n = 1, 2,$ and 3. The quantity Z is the atomic number of the nucleus and $\sigma = Zr/a_0$, where a_0 is the Bohr radius.

$n = 1,$	$l = 0,$	$m = 0$	$\psi_{1s} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-\sigma}$
$n = 2,$	$l = 0,$	$m = 0$	$\psi_{2s} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} (2-\sigma)e^{-\sigma/2}$
	$l = 1,$	$m = 0$	$\psi_{2p_z} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2}\cos\theta$
	$l = 1,$	$m = \pm 1$	$\psi_{2p_x} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2}\sin\theta\cos\phi$
			$\psi_{2p_y} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma e^{-\sigma/2}\sin\theta\sin\phi$
$n = 3,$	$l = 0,$	$m = 0$	$\psi_{3s} = \frac{1}{81\sqrt{3\pi}}\left(\frac{Z}{a_0}\right)^{3/2} (27 - 18\sigma + 2\sigma^2)e^{-\sigma/3}$
	$l = 1,$	$m = 0$	$\psi_{3p_z} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma(6-\sigma)e^{-\sigma/3}\cos\theta$
	$l = 1,$	$m = \pm 1$	$\psi_{3p_x} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma(6-\sigma)e^{-\sigma/3}\sin\theta\cos\phi$
			$\psi_{3p_z} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma(6-\sigma)e^{-\sigma/3}\sin\theta\sin\phi$
	$l = 2,$	$m = 0$	$\psi_{3d_{z^2}} = \frac{1}{81\sqrt{6\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}(3\cos^2\theta - 1)$
	$l = 2,$	$m = \pm 1$	$\psi_{3d_{xy}} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}\sin\theta\cos\theta\cos\phi$
			$\psi_{3d_{yz}} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}\sin\theta\cos\theta\sin\phi$
	$l = 2,$	$m = \pm 2$	$\psi_{3d_{x^2-y^2}} = \frac{1}{81\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}\sin^2\theta\cos 2\phi$
			$\psi_{3d_{xy}} = \frac{1}{81\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 e^{-\sigma/3}\sin^2\theta\sin 2\phi$



(a) (b)

FIGURE 7.10
(a) A schematic diagram of the Stern–Gerlach experiment. (b) A cross-sectional view of the pole pieces of the magnet depicting the inhomogeneous magnetic field that they produce.

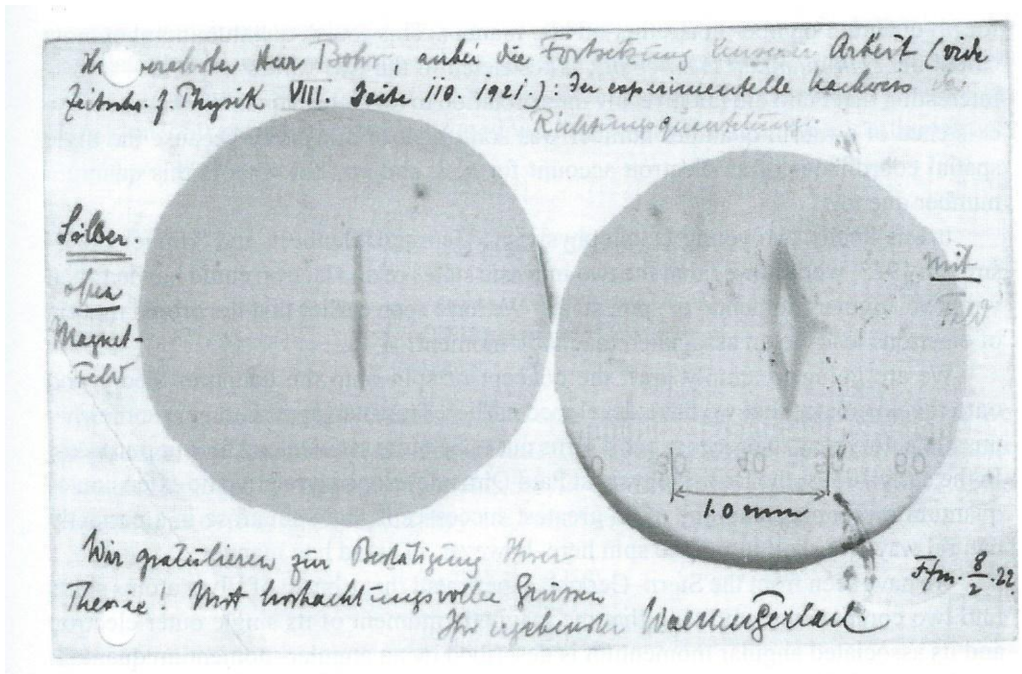


FIGURE 7.11
A postcard from Walther Gerlach to Niels Bohr, dated February 8, 1922. The left side shows the pattern of the beam of silver atoms without a magnetic field, and the right side shows the pattern with the inhomogeneous magnetic field. Reproduced courtesy of Niels Bohr Archive, Copenhagen, Denmark.

TABLE 7.7
A Screen Shot of the First Few Electronic States of Atomic Hydrogen

H I 167 Levels Found (Page 1 of 10)

Data on Lande factors and level compositions are not available for this ion

Configuration	Term	J	Level (cm^{-1})
1s	^2S	$1/2$	0
2p	$^2P°$	$1/2$	*82 258.9206*
		$3/2$	*82 259.2865*
2s	^2S	$1/2$	82 258.9559
3p	$^2P°$	$1/2$	*97 492.2130*
		$3/2$	*97 492.3214*
3s	^2S	$1/2$	97 492.2235
3d	^2D	$3/2$	97 492.3212
		$5/2$	97 492.3574
4p	$^2P°$	$1/2$	*102 823.8505*
		$3/2$	*102 823.8962*
4s	^2S	$1/2$	102 823.8549
4d	^2D	$3/2$	102 823.8961
		$5/2$	102 823.9114
4f	$^2F°$	$5/2$	*102 823.9113*
		$7/2$	*102 823.9190*
5p	$^2P°$	$1/2$	*105 291.6306*
		$3/2$	*105 291.6540*

Source: http://physics.nist.gov/PhysRefData/ASD/levels_form.html

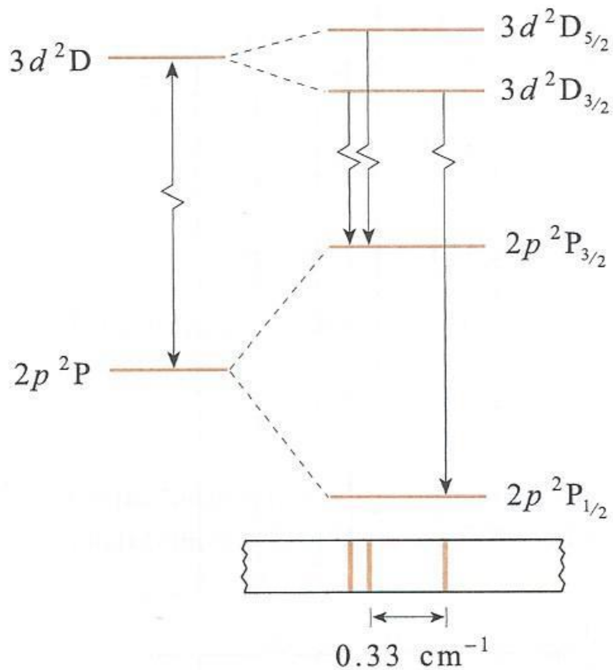


FIGURE 7.12
The fine structure of the spectral line associated with the $3d\,^2D \rightarrow 2p\,^2P$ transition in atomic hydrogen.

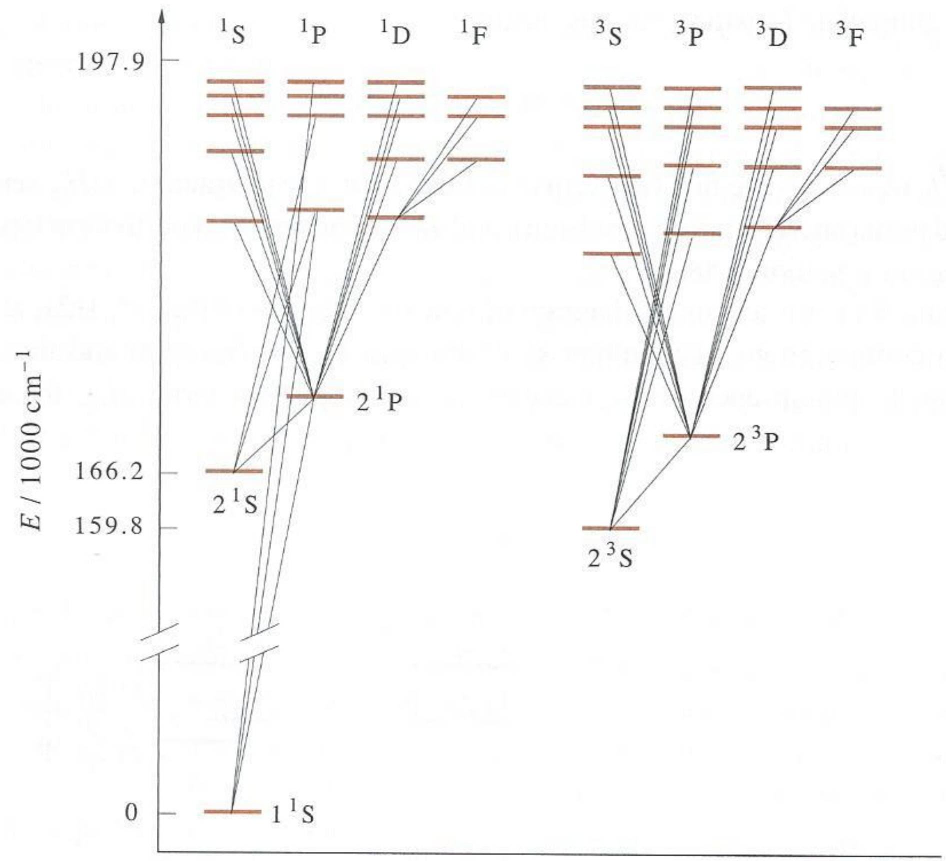


FIGURE 9.10
The energy-level diagram of a helium atom, showing the two separate sets of singlet and triplet states.

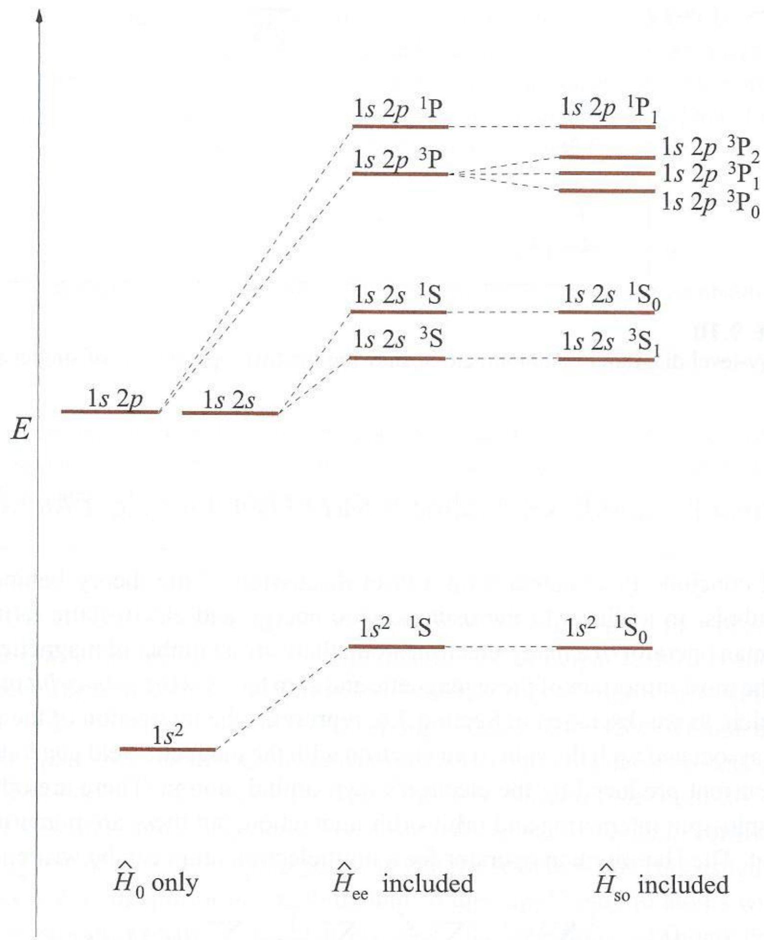


FIGURE 9.11
A schematic diagram of how the energies of the $1s^2$, $1s2s$, and $1s2p$ electron configurations of a helium atom are split by the electron-electron repulsion term and the spin-orbit interaction term. The energies in this figure are not to scale, the splitting due to spin-orbit coupling is much smaller than that due to electron-electron repulsion.

TABLE 9.5
A Screen Shot of the First Few Electronic States of Atomic Sodium [a]

Na I 409 Levels Found

Data on Lande factors and level compositions are not available for this ion

Configuration	Term	J	Level (cm^{-1})
$2p^63s$	^2S	$^1/_2$	0
$2p^63p$	$^2P°$	$^1/_2$	16 956.172
		$^3/_2$	16 973.368
$2p^64s$	^2S	$^1/_2$	25 739.991
$2p^63d$	^2D	$^5/_2$	29 172.839
		$^3/_2$	29 172.889
$2p^64p$	$^2P°$	$^1/_2$	30 266.99
		$^3/_2$	30 272.58
$2p^65s$	^2S	$^1/_2$	33 200.675
$2p^64d$	^2D	$^5/_2$	34 548.731
		$^3/_2$	34 548.766

a. *Source:* The NIST website *http://physics.nist.gov/PhysRefData/ASD/levels_form.html*

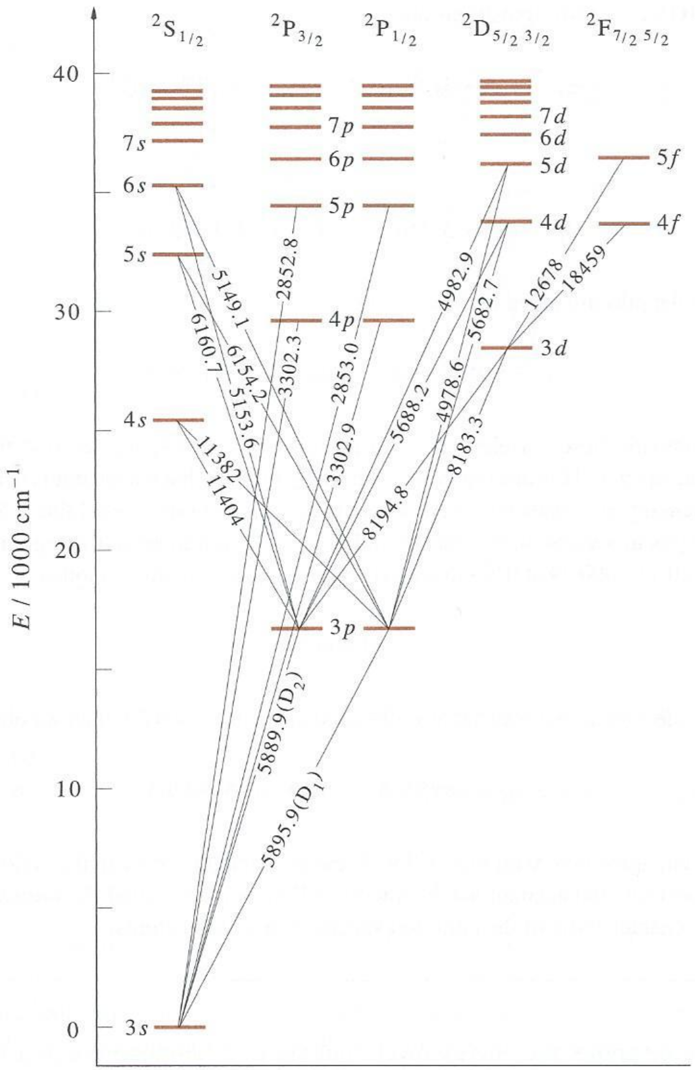


FIGURE 9.9
An energy-level diagram of atomic sodium.

TABLE 9.4
The Possible Term Symbols (Excluding the J Subscript) for Various Electron Configuration for Equivalent Electrons

Electron configuration	Term symbol (excluding the J subscript)
s^1	^{2}S
p^1	^{2}P
p^2, p^4	^{1}S, ^{1}D, ^{3}P
p^3	^{2}P, ^{2}D, ^{4}S
p^1, p^5	^{2}P
d^1, d^9	^{2}D
d^2, d^8	^{1}S, ^{1}D, ^{1}G, ^{3}P, ^{3}F
d^3, d^7	^{2}P, ^{2}D (twice), ^{2}F, ^{2}G, ^{2}H, ^{4}P, ^{4}F
d^4, d^6	^{1}S (twice), ^{1}D (twice), ^{1}F, ^{1}G (twice), ^{1}I, ^{3}P (twice), ^{3}D, ^{3}F (twice), ^{3}G, ^{3}H,
d^5	^{2}S, ^{2}P, ^{2}D (thrice), ^{2}F (twice), ^{2}G (twice), ^{2}H, ^{2}I, ^{4}P, ^{4}D, ^{4}F, ^{4}G, ^{6}S