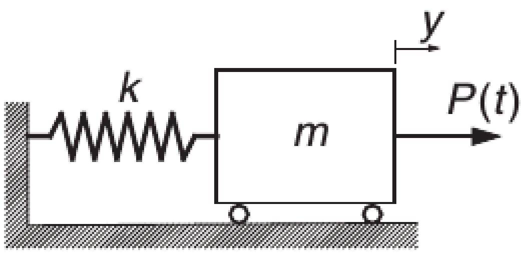
University of São Paulo
São Carlos School of Engineering
Department of Aeronautical Engineering

Initial value problem

Ricardo Afonso Angélico
raa@sc.usp.br

Problems

- 

The spring–mass system is at rest when the force $P(t)$ is applied, where

$$P(t) = \begin{cases} 10t\,\text{N} & \text{when } t < 2\text{ s} \\ 20\,\text{N} & \text{when } t \geq 2\text{ s} \end{cases}$$

The differential equation of the ensuing motion is

$$\ddot{y} = \frac{P(t)}{m} - \frac{k}{m}y$$

Determine the maximum displacement of the mass. Use $m = 2.5$ kg and $k = 75$ N/m.

Problems

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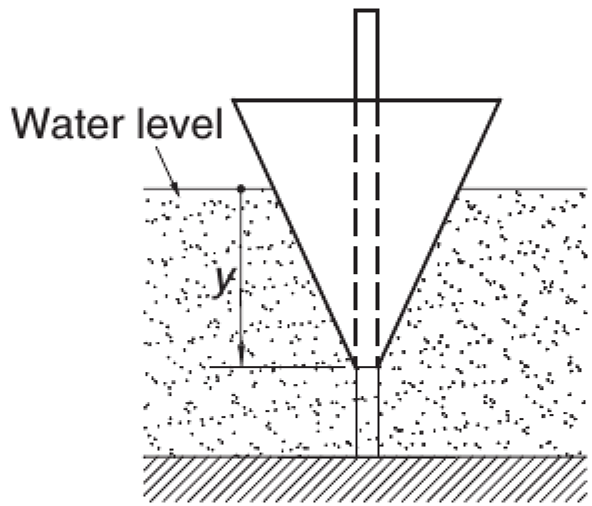

The conical float is free to slide on a vertical rod. When the float is disturbed from its equilibrium position, it undergoes oscillating motion described by the differential equation

$$\ddot{y} = g\left(1 - ay^3\right)$$

where $a = 16\ \mathrm{m}^{-3}$ (determined by the density and dimensions of the float) and $g = 9.80665\ \mathrm{m/s}^2$. If the float is raised to the position $y = 0.1$ m and released, determine the period and the amplitude of the oscillations.

Problems

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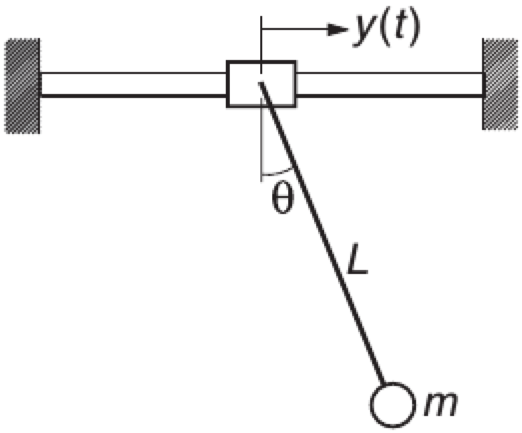
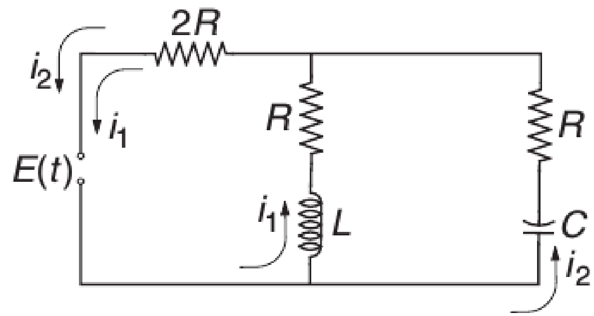


The pendulum is suspended from a sliding collar. The system is at rest when the oscillating motion $y(t) = Y \sin \omega t$ is imposed on the collar, starting at $t = 0$. The differential equation describing the motion of the pendulum is

$$\ddot{\theta} = -\frac{g}{L} \sin \theta + \frac{\omega^2}{L} Y \cos \theta \sin \omega t$$

Plot θ vs. t from $t = 0$ to 10 s and determine the largest θ during this period. Use $g = 9.80665$ m/s^2, $L = 1.0$ m, $Y = 0.25$ m and $\omega = 2.5$ rad/s.

Problems



Kirchoff's equations for the circuit shown are

$$L\frac{di_1}{dt} + Ri_1 + 2R(i_1 + i_2) = E(t) \quad \text{(a)}$$

$$\frac{q_2}{C} + Ri_2 + 2R(i_2 + i_1) = E(t) \quad \text{(b)}$$

where i_1 and i_2 are the loop currents, and q_2 is the charge of the condenser. Differentiating Eq. (b) and substituting the charge–current relationship $dq_2/dt = i_2$, we get

$$\frac{di_1}{dt} = \frac{-3Ri_1 - 2Ri_2 + E(t)}{L} \quad \text{(c)}$$

$$\frac{di_2}{dt} = -\frac{2}{3}\frac{di_1}{dt} - \frac{i_2}{3RC} + \frac{1}{3R}\frac{dE}{dt} \quad \text{(d)}$$

We could substitute di_1/dt from Eq. (c) into Eq. (d), so that the latter would assume the usual form $di_2/dt = f(t, i_1, i_2)$, but it is more convenient to leave the equations as they are. Assuming that the voltage source is turned on at time $t = 0$, plot the loop currents i_1 and i_2 from $t = 0$ to 0.05 s. Use $E(t) = 240\sin(120\pi t)$ V, $R = 1.0\,\Omega$, $L = 0.2 \times 10^{-3}$ H and $C = 3.5 \times 10^{-3}$ F.