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Departamento de Ciências Exatas
Disciplina: LCE0220 Cálculo II - 11/09/2018

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Lista de exercícios - Integrais VI
Funções gama e beta[1]

Resumo

1. Função gama
   (a) Definição:
   $\boxed{\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}\mathrm{e}^{-x}dx}$, para $\alpha > 0$.
   (b) Propriedades:
      i. $\boxed{\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)}$, para $\alpha > 1$
      ii. $\boxed{\Gamma(n) = (n-1)!}$, para n inteiro positivo.
   (c) Observações:
      i. $\Gamma(1) = (1-1)! = 0! = 1$
      ii. $\Gamma(\frac{1}{2}) = \sqrt{\pi}$
2. Função beta
   (a) Definições:
      i. $\boxed{B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx}$, para $m > 0$ e $n > 0$
      ii. $\boxed{B(m,n) = 2\int_0^{\frac{\pi}{2}} \mathrm{sen}^{2m-1}t.\cos^{2n-1}t \;\; dt}$.
   (b) Relação entre as funções gama e beta:
   $\boxed{B(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}}$

Exercícios

1. Resolva as seguintes integrais:
   (a) $\int_0^\infty x^5\mathrm{e}^{-x}dx$
   (b) $\int_0^\infty x\ \mathrm{e}^{-x^2}dx$
   (c) $\int_0^\infty x^{\frac{3}{2}}\ \mathrm{e}^{-\frac{x}{2}}dx$
   (d) $\int_{-\infty}^0 x^2\ \mathrm{e}^{3x}dx$
   (e) $\int_{-\infty}^\infty x^6\ \mathrm{e}^{-x^2}dx$
   (f) $\int_0^1 x^3(1-x)^2dx$
   (g) $\int_0^{\frac{1}{5}} 4x^3(1-5x)^5dx$
   (h) $\int_1^2 (x-1)^5(2-x)^3dx$
   (i) $\int_0^{\frac{\pi}{2}} \mathrm{sen}^3x\cos x\ dx$
   (j) $\int_0^2 \frac{x^3}{\sqrt{4-x^2}}dx$

   Respostas: (a) 120; (b) $\frac{1}{2}$; (c) $3\sqrt{2\pi}$; (d) $\frac{2}{27}$; (e) $\frac{15}{8}\sqrt{\pi}$; (f) $\frac{1}{60}$; (g) $\frac{1}{78750}$; (h) $\frac{1}{504}$; (i) $\frac{1}{4}$; (j) $\frac{16}{3}$.

2. Resolva as seguintes integrais:
   (a) $\int_0^\infty \sqrt{x}\ \mathrm{e}^{-\frac{x}{3}}dx$
   (b) $\int_0^\infty \sqrt{x}\ \mathrm{e}^{-\frac{\sqrt{x}}{2}}dx$
   (c) $\int_{-\infty}^\infty x^2\ \mathrm{e}^{-x^2}dx$
   (d) $\int_{-\infty}^\infty x^3\ \mathrm{e}^{-x^2}dx$
   (e) $\int_0^1 \frac{x\ \mathrm{e}^{-\frac{x}{1-x}}}{(1-x)^3}\ dx$
   (f) $\int_0^{\frac{\pi}{2}} \sec^3 x\ \mathrm{tg}x\ \mathrm{e}^{1-\sec x}dx$
   (g) $\int_{\frac{1}{2}}^\infty \frac{[\ln(2x)]^3\ \mathrm{e}^{-\ln(2x)}}{x}\ dx$
   (h) $\int_{-\infty}^\infty \frac{1}{\sigma\sqrt{2\pi}}\ \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\ dx \quad (\sigma > 0)$
   (i) $\int_0^3 \frac{1}{\sqrt{3x-x^2}}dx$
   (j) $\int_{-\infty}^0 x^2\ \mathrm{e}^x dx$
   (k) $\int_0^1 \sqrt{1-x}\ dx$
   (l) $\int_0^1 \sqrt{x(1-x)^3}\ dx$
   (m) $\int_0^3 x^4\sqrt{9-x^2}\ dx$
   (n) $\int_1^{\mathrm{e}} \frac{(\ln x)^4(1-\ln x)}{5x}dx$
   (o) $\int_0^{\frac{\pi}{2}} \frac{\mathrm{sen}^3x\ \mathrm{e}^{-\mathrm{tg}x}}{\cos^5 x}dx$
   (p) $\int_0^{\frac{\pi}{2}} \mathrm{sen}^4x\ dx$
   (q) $\int_0^{\frac{\pi}{4}} \mathrm{sen}^3(2x)\ \cos^5(2x)\ dx$
   (r) $\int_0^\pi \mathrm{sen}\frac{x}{2}\ dx$
   (s) $\int_0^{\frac{\pi}{2a}} \cos^2(ax)\ dx \quad (a > 0)$
   (t) $2\int_0^{\frac{\pi}{3}} \mathrm{sen}^4\frac{3x}{2}\ \cos^2\frac{3x}{2}\ dx$

   Respostas: (a) $\frac{3}{2}\sqrt{3\pi}$; (b) 32; (c) $\frac{\sqrt{\pi}}{2}$; (d) 0; (e) 1; (f) 5; (g) 6; (h) 1; (i) π; (j) 2; (k) $\frac{2}{3}$; (l) $\frac{\pi}{16}$; (m) $\frac{729}{32}\pi$; (n) $\frac{1}{150}$; (o) 6; (p) $\frac{3\pi}{16}$; (q) $\frac{1}{48}$; (r) 2; (s) $\frac{\pi}{4a}$; (t) $\frac{\pi}{24}$.

3. Resolva as integrais do exercício (1) usando um pacote matemático, por exemplo, o WolframAlpha. Ex. comando para resolver o item (a):

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integrate x^5*exp(-x)dx, from 0 to infinity
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[1] Godoi *et al.* (1979, seção 11). **Análise Matemática II. Exercícios**. Publicado pelo Departamento de Matemática e Estatística da ESALQ - USP. 189p.