INF3580/4580 – Semantic Technologies – Spring 2018

Lecture 8: RDF and RDFS semantics

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Oblig 5

- Published today
- First delivery due 21 March
- Final delivery 2 weeks after feedback
- Extra question for INF4580 students
- "Real" semantics of RDF and RDFS
- Foundations book: Section 3.2
- Still OK to ignore some complications, see oblig text
- We provide an excerpt of Sect. 3.2 with unimportant parts removed.
- Go to group sessions!

Today's Plan

1. Why we need semantics
2. Model-theoretic semantics from a birds-eye perspective
3. Repetition: Propositional Logic
4. Simplified RDF semantics

Outline

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But RDF was supposed to be the data liberation movement

Another look at the Semantic Web cake

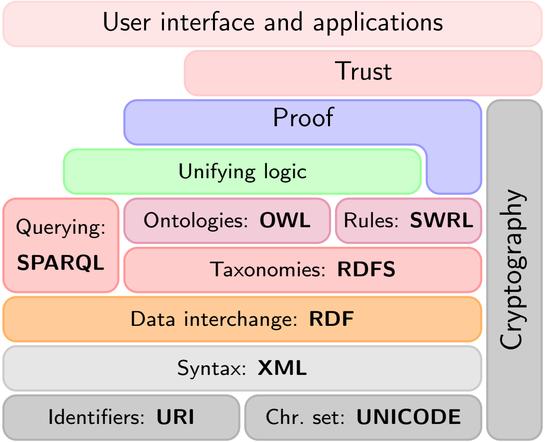

Figure: Semantic Web Stack

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- and in particular what entailment should be taken to mean.

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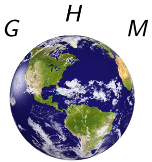
- How do we know that the inference rules are well-chosen?
- Which manipulations derive conclusions that hold in the real world?

Finding out stuff about the World

The "Real World"



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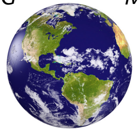

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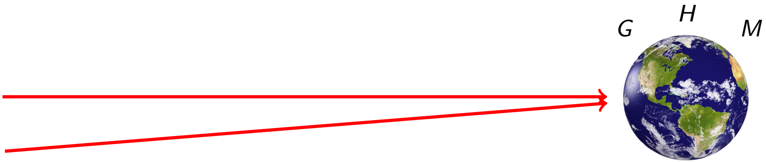

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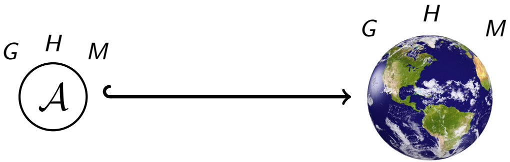

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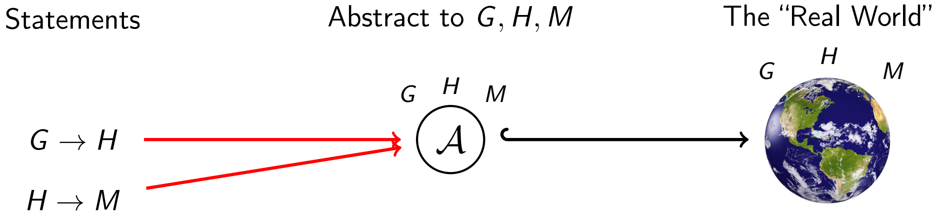

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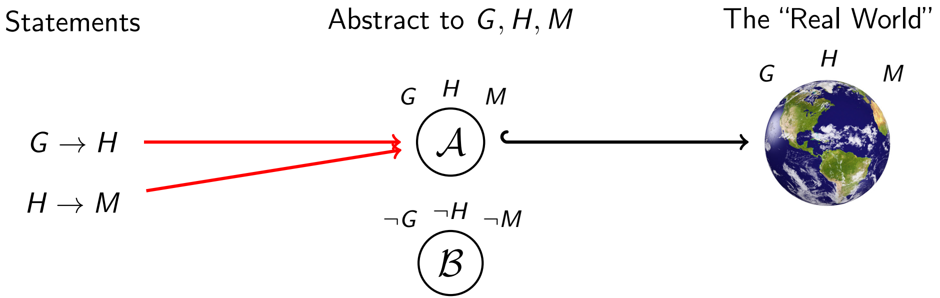

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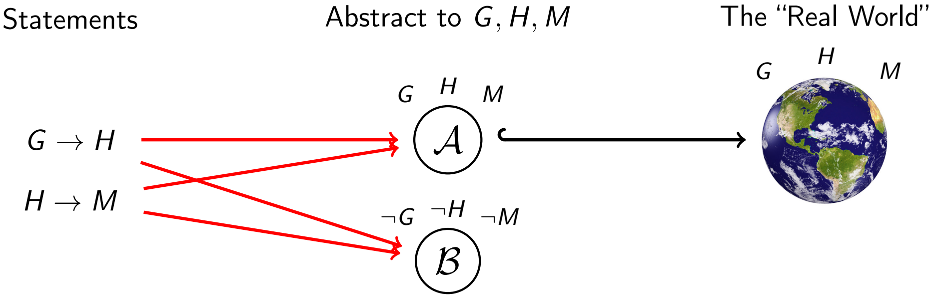

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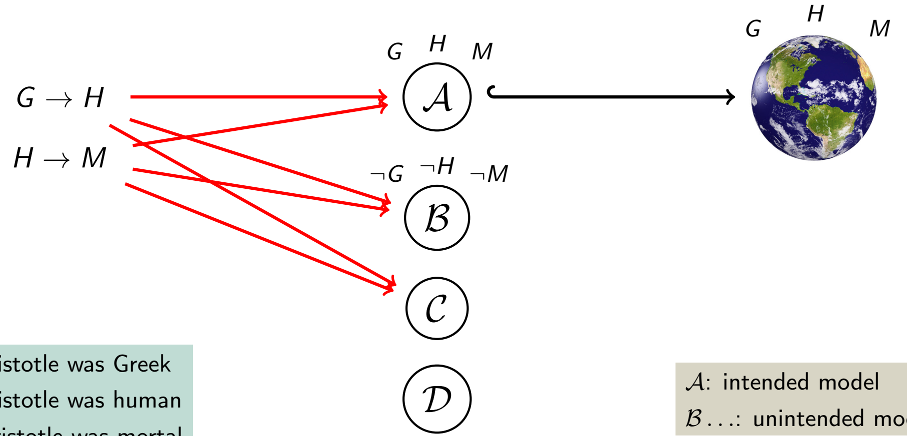

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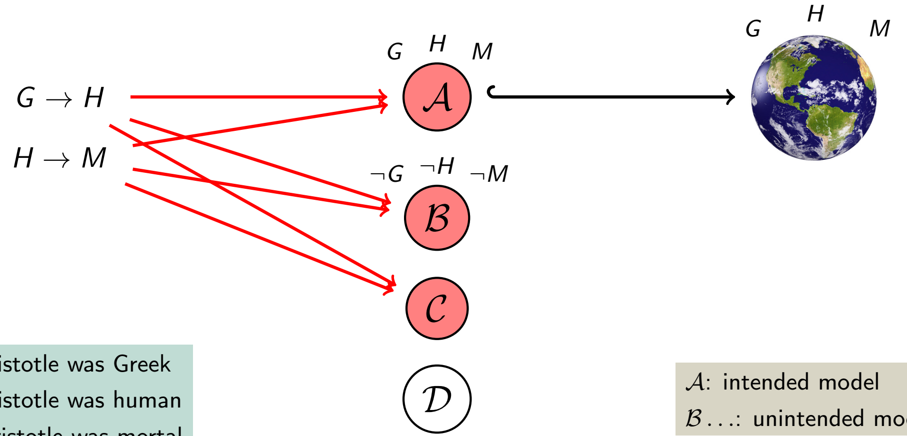

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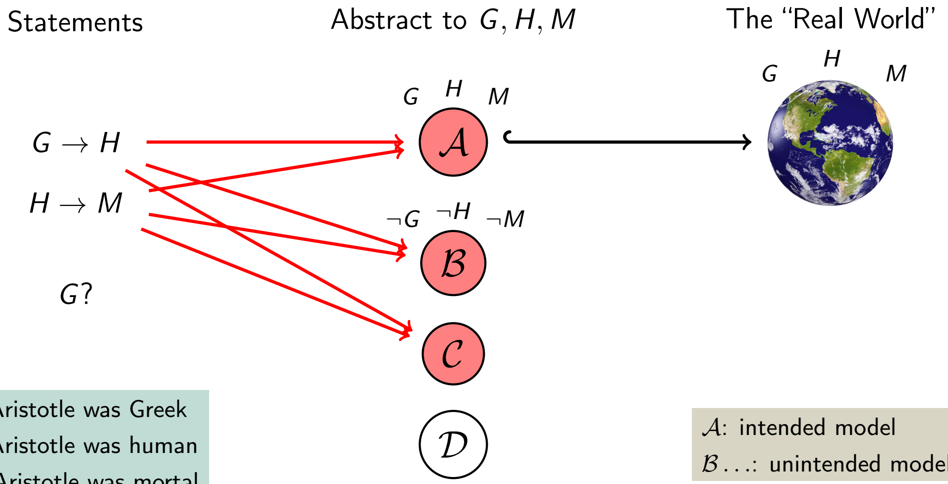

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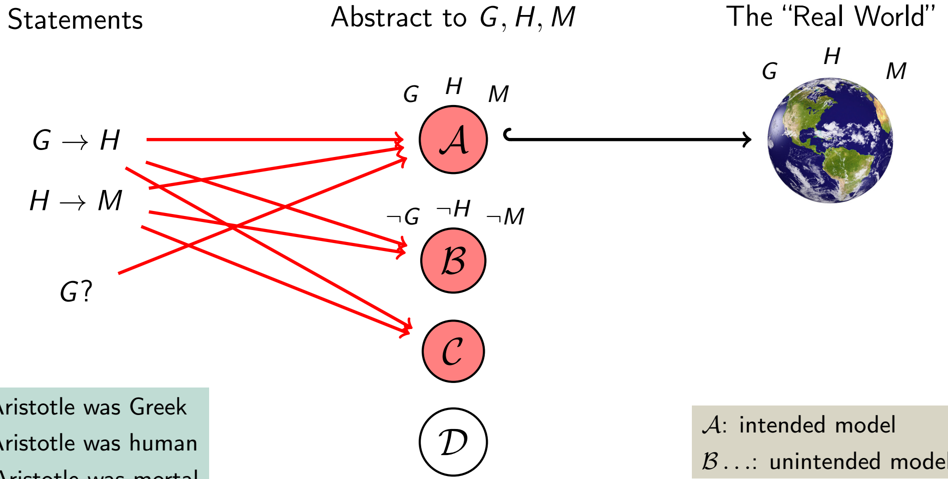

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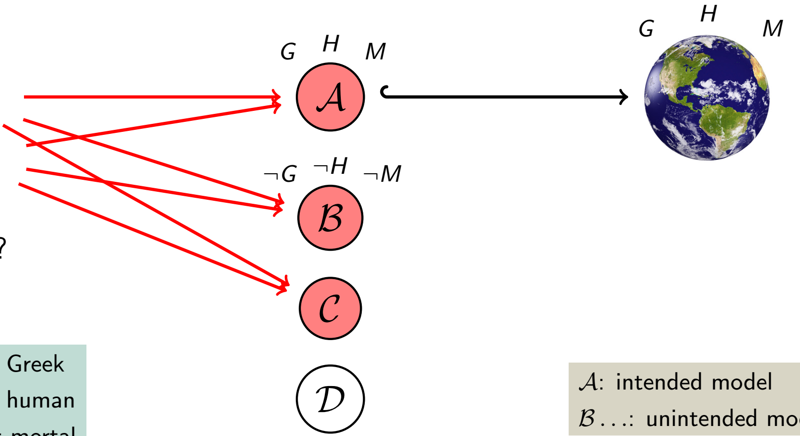

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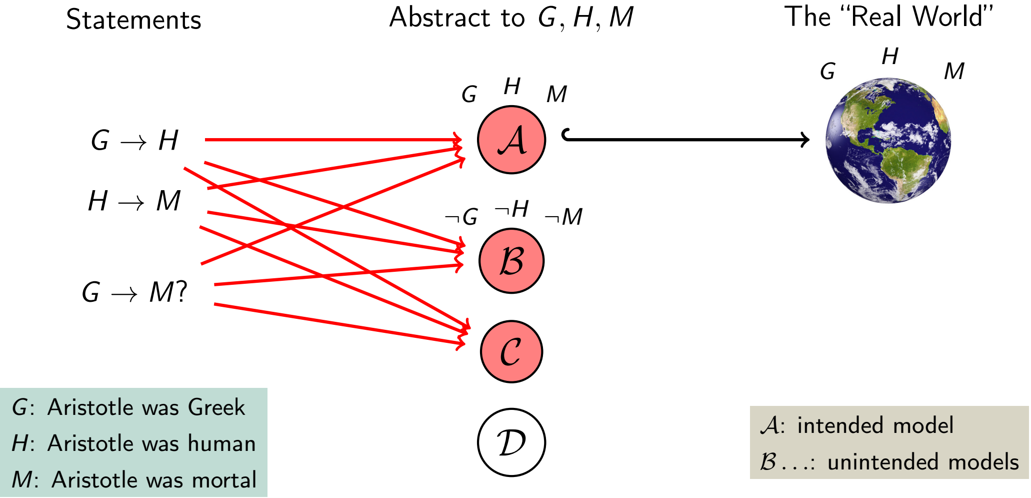

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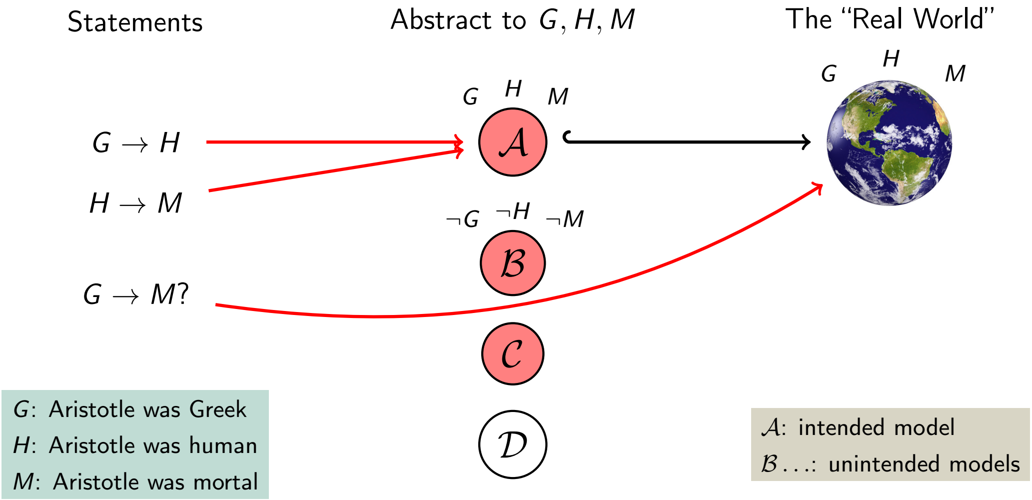

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- Whatever these models all share can be said to be entailed by those features.

Outline

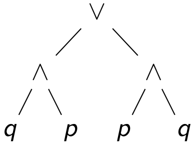
Propositional Logic: Formulas

- Formulas are defined "by induction" or "recursively":

1. Any letter p, q, r,... is a formula
2. *if* A and B are formulas, *then*
   - $(A \wedge B)$ is also a formula (read: "A and B")
   - $(A \vee B)$ is also a formula (read: "A or B")
   - $\neg A$ is also a formula (read: "not A")

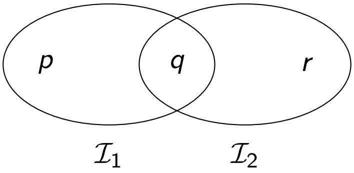
- Nothing else is. Only what rules [1] and [2] say is a formula.
- Examples of formulae: $p \quad (p \wedge \neg r) \quad (q \wedge \neg q) \quad ((p \vee \neg q) \wedge \neg p)$
- Formulas are just a kind of strings until now:
  - no meaning
  - but every formula can be "parsed" uniquely.

$$((q \wedge p) \vee (p \wedge q))$$



Interpretations

- Logic is about truth and falsity
- Truth of compound formulas depends on truth of letters.
- Idea: put all letters that are "true" into a set!
- Define: An *interpretation* $\mathcal{I}$ is a set of letters.
- Letter p is true in interpretation $\mathcal{I}$ if $p \in \mathcal{I}$.
- E.g., in $\mathcal{I}_1 = \{p, q\}$, p is true, but r is false.



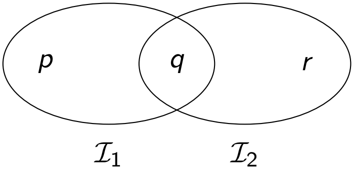
- But in $\mathcal{I}_2 = \{q, r\}$, p is false, but r is true.

Semantic Validity $\models$

- To say that p is true in $\mathcal{I}$, write

$$\mathcal{I} \models p$$

- For instance



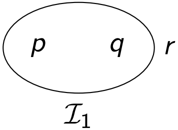
$$\mathcal{I}_1 \models p \qquad \mathcal{I}_2 \not\models p$$

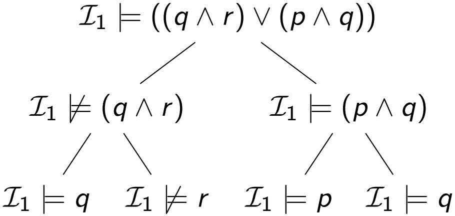
- In other words, for all letters p:

$$\mathcal{I} \models p \qquad \text{if and only if} \qquad p \in \mathcal{I}$$

Validity of Compound Formulas

- Is $((q \land r) \lor (p \land q))$ true in $\mathcal{I}$?
- Idea: apply our rule recursively
- For any formulas A and B,...
- ...and any interpretation $\mathcal{I}$,...
  - ...$\mathcal{I} \models A \land B$ if and only if $\mathcal{I} \models A$ and $\mathcal{I} \models B$
  - ...$\mathcal{I} \models A \lor B$ if and only if $\mathcal{I} \models A$ or $\mathcal{I} \models B$ (or both)
  - ...$\mathcal{I} \models \neg A$ if and only if $\mathcal{I} \not\models A$.
- For instance





Truth Table

- Semantics of $\neg$, $\wedge$, $\vee$ often given as *truth table*:

A	B	$\neg A$	$A \wedge B$	$A \vee B$
f	f	t	f	f
f	t	t	f	t
t	f	f	f	t
t	t	f	t	t

Tautologies

- A formula A that is true in *all* interpretations is called a *tautology*
- also *logically valid*
- also a *theorem* (of propositional logic)
- written:

$$\models A$$

- $(p \vee \neg p)$ is a tautology
- True whatever p means:
  - The sky is blue or the sky is not blue.
  - P.N. will win the 50km in 2016 or P.N. will not win the 50km in 2016.
  - The slithy toves gyre or the slithy toves do not gyre.
- Possible to derive true statements mechanically. . .
- . . . without understanding their meaning!
- . . . e.g. using truth tables for small cases.

Entailment

- Tautologies are true in all interpretations
- Some formulas are true only under certain assumptions
- A *entails* B, written $A \models B$ if

  $\mathcal{I} \models B$

  for all interpretations $\mathcal{I}$ with $\mathcal{I} \models A$
- Also: "B is a logical consequence of A"
- Whenever A holds, also B holds
- For instance:

$$p \wedge q \models p$$

- Independent of meaning of p and q:
  - If it rains and the sky is blue, then it rains
  - If P.N. wins the race and the world ends, then P.N. wins the race
  - If 'tis brillig and the slythy toves do gyre, then 'tis brillig
- Also entailment can be checked mechanically, without knowing the meaning of words.

Question

Given the letters

P – Ola answers none of the questions correctly

Q – Ola fails the exam

Which of the following are tautologies of propositional logic?

1. Q
2. $\neg Q$
3. $P \rightarrow Q$
4. $Q \rightarrow (P \rightarrow Q)$

Outline

Taking the structure of triples into account

Unlike propositions, triples have parts, namely:

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- predicates, and
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Triples are true or false on the basis of what each part refers to.

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The semantics of typed and language tagged literals is considerably more complex.

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- Forget blank nodes and literals for a while!

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`indi prop indi .`	$r(i_1, i_2)$
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- Used much in particular for OWL

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- DL syntax, without namespaces:

$$loves(romeo, juliet)$$
$$Lady(juliet)$$

$$Lady \sqsubseteq Person$$
$$loves \sqsubseteq knows$$
$$\mathrm{dom}(loves, Lover)$$
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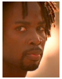
  - For each property URI r, a relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- Given these, it will be possible to say whether a triple holds or not.

An example "intended" interpretation

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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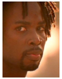
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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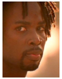
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
- $romeo^{\mathcal{I}_1} =$  $juliet^{\mathcal{I}_1} =$ 
- $Lady^{\mathcal{I}_1} = \{$  $\}$ $Person^{\mathcal{I}_1} = \Delta^{\mathcal{I}_1}$

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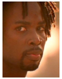
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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- $Lady^{\mathcal{I}_2} = \{2^n \mid n \in \mathbb{N}\} = \{2, 4, 8, 16, 32, \ldots\}$
  $Person^{\mathcal{I}_2} = \{2n \mid n \in \mathbb{N}\} = \{2, 4, 6, 8, 10, \ldots\}$
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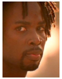
    $\mathit{Lover}^{\mathcal{I}_1} = \{$ ,  $\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \{$ , ,  $\}$

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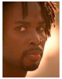
    $\mathit{Lover}^{\mathcal{I}_1} = \{$ ,  $\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \{$ , ,  $\}$
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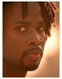
    $\mathit{Lover}^{\mathcal{I}_1} = \{$ ,  $\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \{$ , ,  $\}$
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Finding out stuff about Romeo and Juliet

Statements | Interpretations | The "Real World"

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Lady(*juliet*)
Lady $\sqsubseteq$ *Person*
loves $\sqsubseteq$ *knows*
dom(*loves*, *Lover*)
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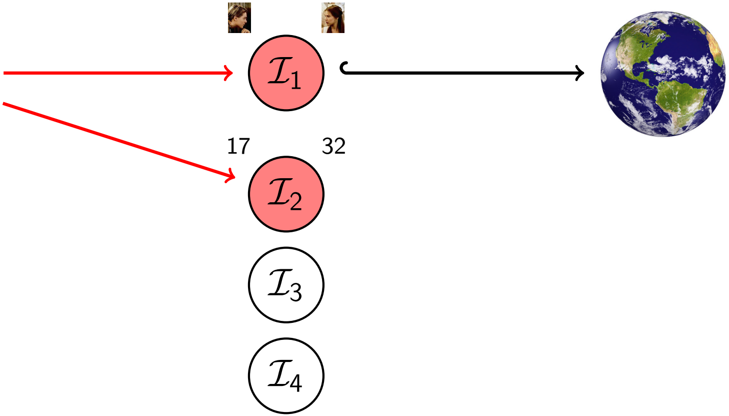

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Interpretations

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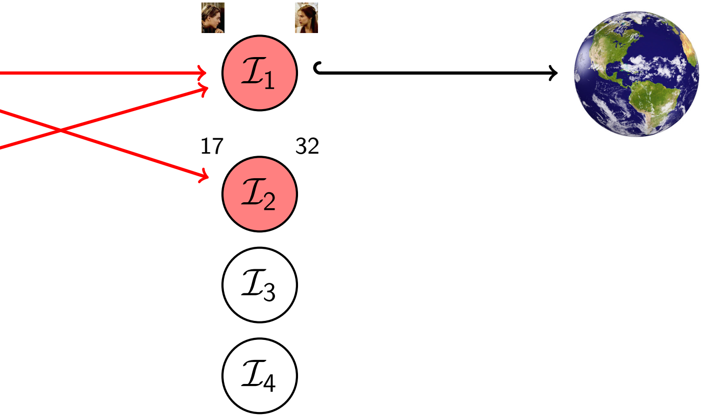

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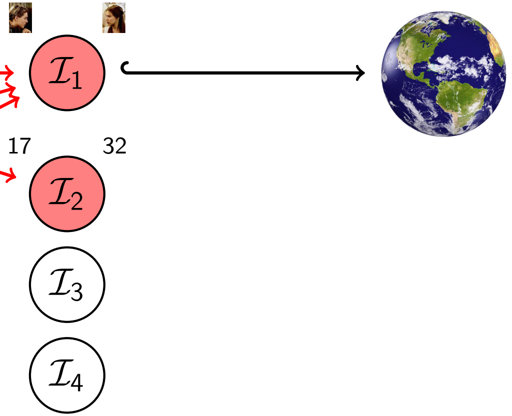

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- Then $\mathcal{I}_1 \models \mathcal{A}$ and $\mathcal{I}_2 \models \mathcal{A}$

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- And a further triple T (also any kind)
- T is entailed by $\mathcal{A}$, written $\mathcal{A} \models T$
- iff
  - For any interpretation $\mathcal{I}$ with $\mathcal{I} \models \mathcal{A}$
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- $\mathcal{A} \models \mathcal{B}$ iff $\mathcal{I} \models \mathcal{B}$ for all $\mathcal{I}$ with $\mathcal{I} \models \mathcal{A}$
- Example:
- $\mathcal{A} = \{\ldots, \mathit{Lady}(\mathit{juliet}),\ \mathit{Lady} \sqsubseteq \mathit{Person}, \ldots\}$ as before
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Finding out stuff about Romeo and Juliet

Statements

$loves(romeo, juliet)$
$Lady(juliet)$
$Lady \sqsubseteq Person$
$loves \sqsubseteq knows$
$\mathbf{dom}(loves, Lover)$
$\mathbf{rg}(loves, Beloved)$

Interpretations


$\mathcal{I}_1$

17 32

$\mathcal{I}_2$

$\mathcal{I}_3$

$\mathcal{I}_4$

The “Real World”

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Lady ⊑ *Person*
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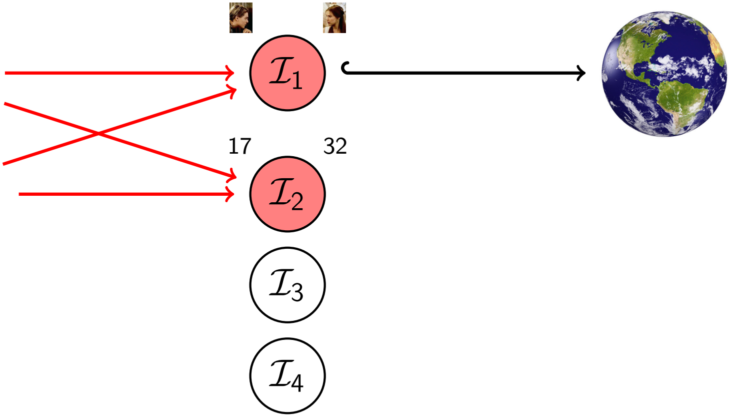
Interpretations

The "Real World"



Countermodels

- If $\mathcal{A} \not\models T$,...
- then there is an $\mathcal{I}$ with
  - $\mathcal{I} \models \mathcal{A}$
  - $\mathcal{I} \not\models T$
- Vice-versa: if $\mathcal{I} \models \mathcal{A}$ and $\mathcal{I} \not\models T$, then $\mathcal{A} \not\models T$
- Such an $\mathcal{I}$ is called a *counter-model* (for the assumption that $\mathcal{A}$ entails T)
- To show that $\mathcal{A} \models T$ does *not* hold:
  - Describe an interpretation $\mathcal{I}$ (using your fantasy)
  - Prove that $\mathcal{I} \models \mathcal{A}$ (using the semantics)
  - Prove that $\mathcal{I} \not\models T$ (using the semantics)

Countermodel Example

- $\mathcal{A}$ as before:

$$\mathcal{A} = \{\mathit{loves}(\mathit{romeo}, \mathit{juliet}),\ \mathit{Lady}(\mathit{juliet}),\ \mathit{Lady} \sqsubseteq \mathit{Person},\\ \mathit{loves} \sqsubseteq \mathit{knows},\ \mathrm{dom}(\mathit{loves}, \mathit{Lover}),\ \mathrm{rg}(\mathit{loves}, \mathit{Beloved})\}$$

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- Holds in $\mathcal{I}_1$ and $\mathcal{I}_2$.

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- With $Lover^{\mathcal{I}} = \{a\}$ and $Beloved^{\mathcal{I}} = \{b\}$, $\mathcal{I} \not\models Lover \sqsubseteq Beloved$!
- Choose

$$loves^{\mathcal{I}} = knows^{\mathcal{I}} = \{\langle a, b \rangle\} \qquad Lady^{\mathcal{I}} = Person^{\mathcal{I}} = \{b\}$$

  to complete the counter-model while satisfying $\mathcal{I} \models \mathcal{A}$

Countermodels about Romeo and Juliet

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Lady $\sqsubseteq$ *Person*
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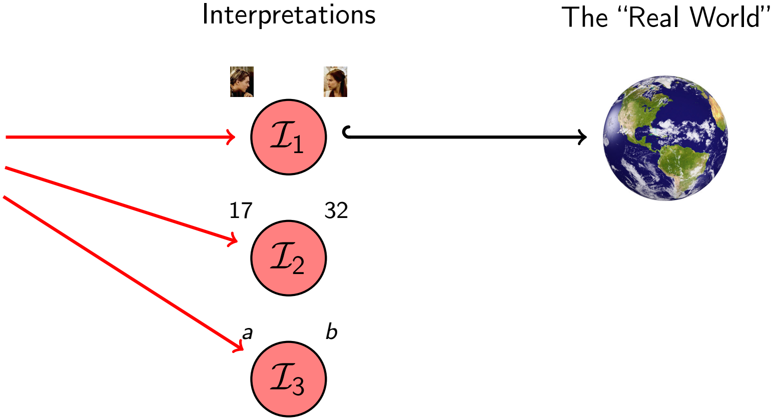
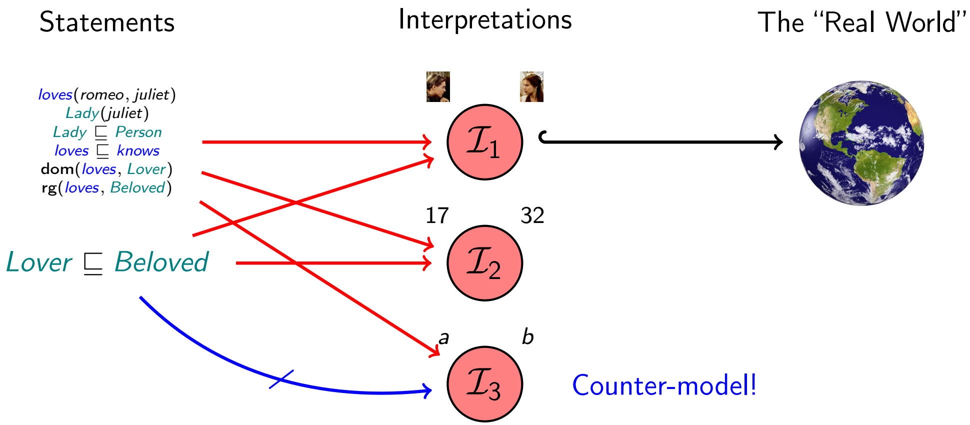


Countermodels about Romeo and Juliet



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Supplementary reading on RDF and RDFS semantics:

- `http://www.w3.org/TR/rdf-mt/`
- Section 3.2 in Foundations of SW Technologies