



Escola Politécnica
Universidade de São Paulo

PSI3211

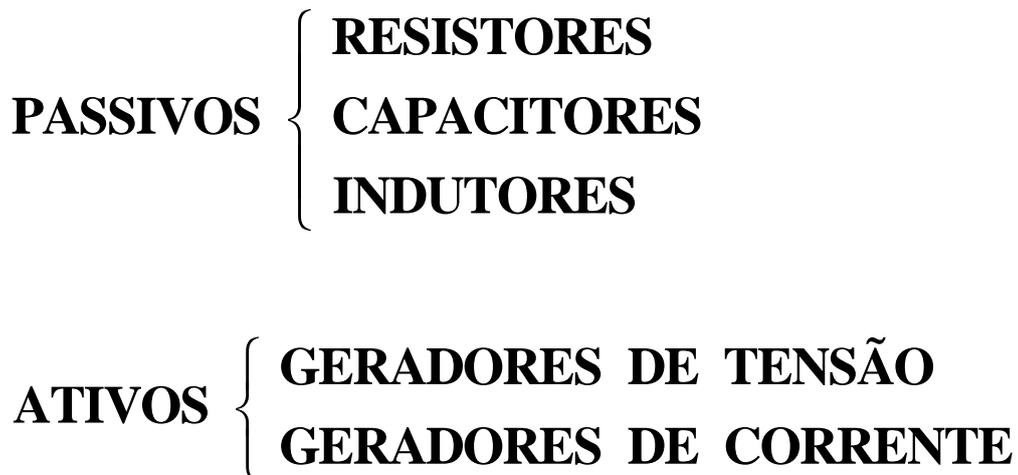
Circuitos Elétricos I

Bloco 2

Bipolos: R, L, C
Geradores Independentes e Excitações

Prof^a Denise Consonni

BIPOLOS ELEMENTARES



**CLASSIFICAÇÃO QUANTO À
RELAÇÃO CORRENTE-TENSÃO:**

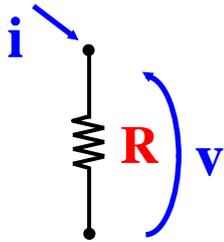
- LINEARES
- NÃO LINEARES

RESISTOR

$$\mathbf{v} = \mathbf{r}(\mathbf{i})$$

$$\mathbf{i} = \mathbf{g}(\mathbf{v})$$

1 – Linear Fixo → Ideal



$$\mathbf{v} = \mathbf{R} \mathbf{i}$$

$$\mathbf{i} = \mathbf{G} \mathbf{v}$$

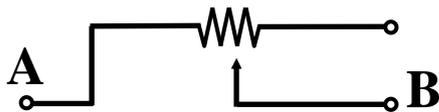
$$\mathbf{R} \rightarrow \Omega$$

$$\mathbf{G} \rightarrow \text{S}$$

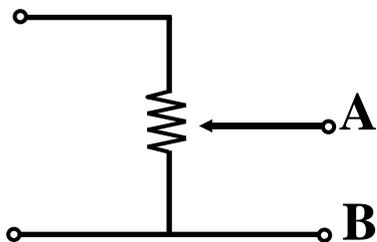
$$\mathbf{p} = \mathbf{v} \mathbf{i} = \mathbf{R} \mathbf{i}^2 = \mathbf{G} \mathbf{v}^2 = \frac{\mathbf{v}^2}{\mathbf{R}} = \frac{\mathbf{i}^2}{\mathbf{G}}$$

2 – Linear Variável

$$\mathbf{v}(\mathbf{t}) = \mathbf{R}(\mathbf{t}) \mathbf{i}(\mathbf{t})$$



reostato → controle de corrente



potenciômetro → controle de tensão

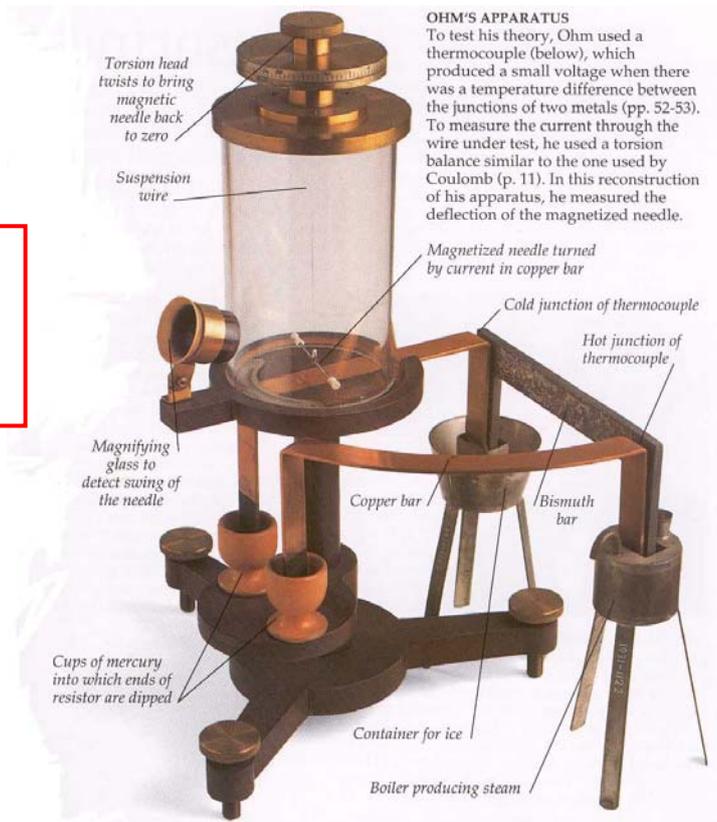
3 – Não linear

George Simon Ohm



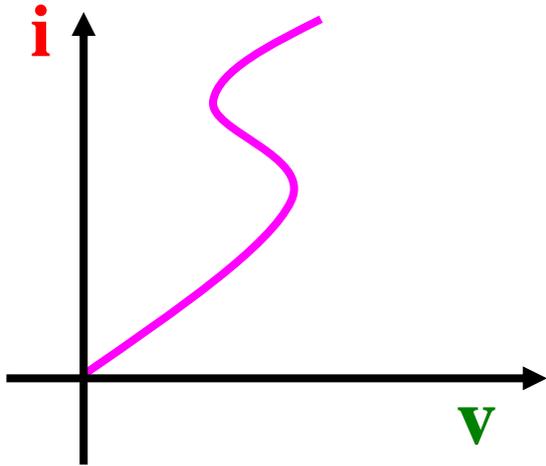
- Alemão (Erlangen, 1789; Colônia, 1854)
- Físico e Matemático
- Professor de Física, Univ. de Colônia
- 1827 Lei de Ohm (empírica) → 22 anos para ser reconhecida
- Pesquisas nas áreas de física molecular, acústica e comunicação telegráfica

$$R = \rho \cdot \frac{\ell}{A}$$



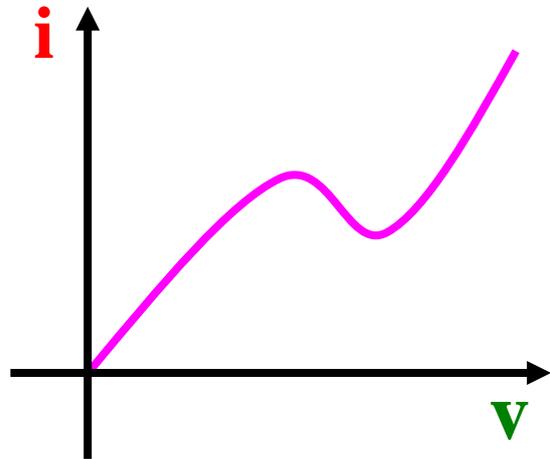
Aparato Experimental usado por Ohm

RESISTOR NÃO LINEAR



$$v = r(i)$$

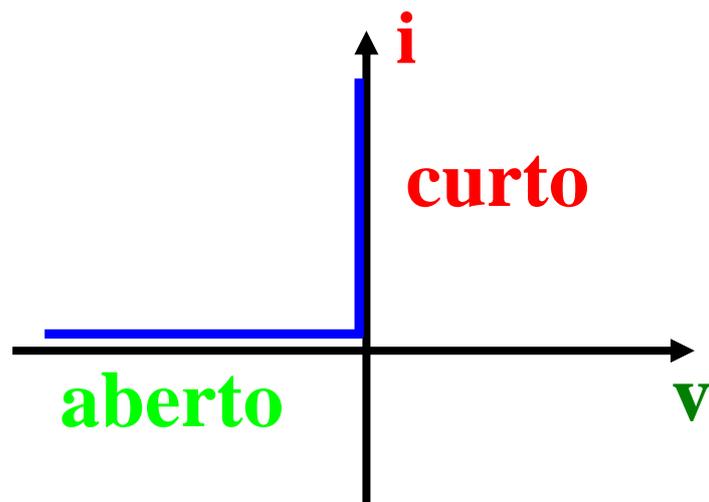
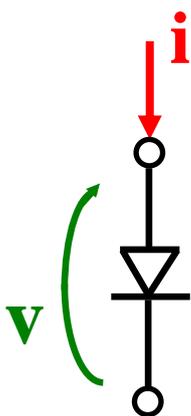
Controlado por corrente



$$i = g(v)$$

Controlado por tensão

Ex: Diodo ideal



$$\text{Diodo real: } i = g(v) = I_s (e^{\lambda v} - 1)$$

RESISTORES REAIS

1 – Carvão

Valor

Potência máxima 1/8 1/4 1/2
1 2 **watts**

Tolerância 10 % 5 % 1%
0,5 % 0,1 %

Corrente máxima:

$$I_{\text{máx}} = \sqrt{\frac{P_{\text{máx}}}{R}}$$

Tensão ↑
Frequência ↑
Umidade ↓
Temperatura ↑

Resistência ↓

2 – Fio

Potências mais elevadas

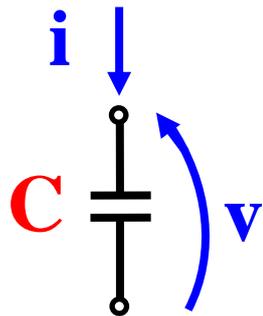
Modelo: 

3 – Filme Metálico: **Circuitos integrados**

CAPACITOR

$$q(t) = C(v)$$

1- Linear , Fixo \rightarrow Ideal



$$q = C v$$

$$i = C \frac{dv}{dt}$$

$$v = \frac{1}{C} \int_{t_0}^t i dt + v(t_0)$$

$$p = \frac{1}{2} C \frac{dv^2}{dt}$$

$$W = \frac{1}{2} C (v^2 - \cancel{v^2(t_0)}) = \boxed{\frac{1}{2} \frac{q^2}{C}}$$

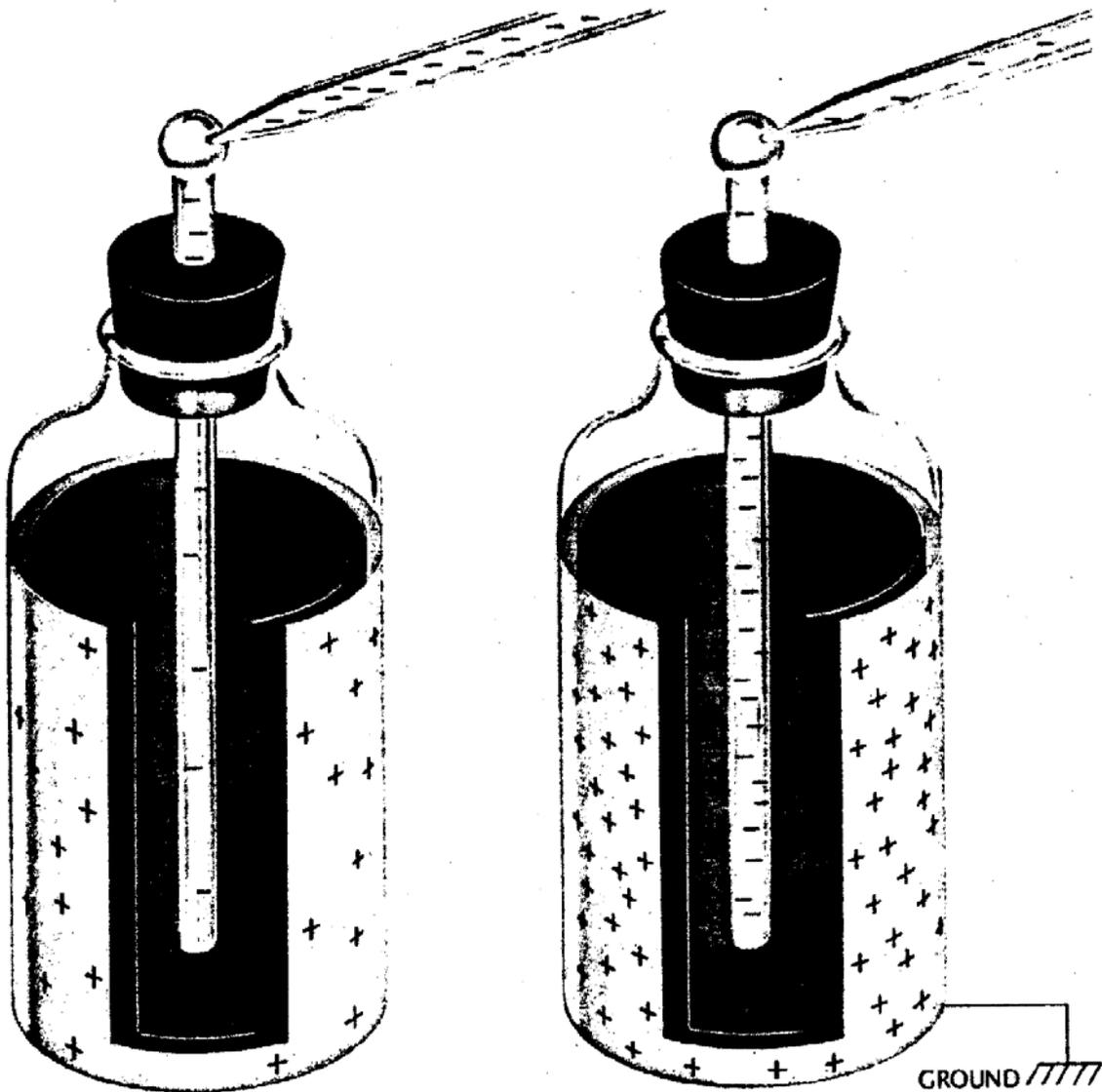
2 - Linear , Variável

$$q(t) = C(t) v(t)$$

$$i = C(t) \frac{dv}{dt} + v \frac{dC(t)}{dt}$$

3 - Não linear

$$\text{Ex.: } q = C(v) \cdot v$$



Garrafa de Leyden

Universidade de Leyden (Holanda)
1746

$$C \uparrow \quad \begin{matrix} A \uparrow \\ d \downarrow \end{matrix}$$

$$C = \epsilon \frac{A}{d}$$

CAPACITORES REAIS

Valores: $\mu\text{F} \rightarrow \text{pF}$

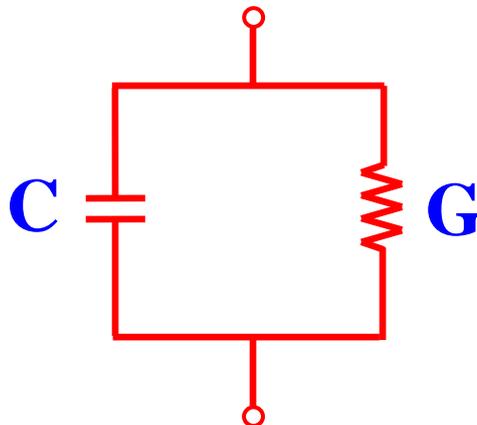
Especificações: **Ex.:** 100 nF / 500V

↑
tensão de ruptura
do dielétrico

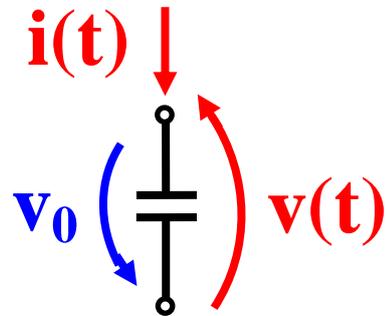
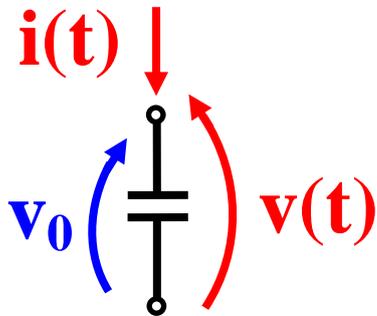
Tipos: de acordo com o dielétrico

- cerâmica
- mylar
- poliestireno
- eletrolítico
- tântalo

Modelo:



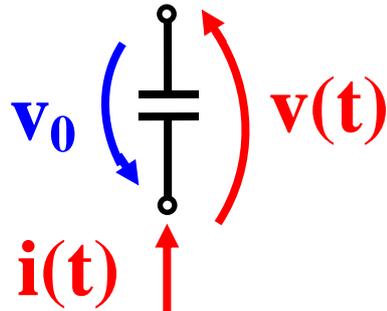
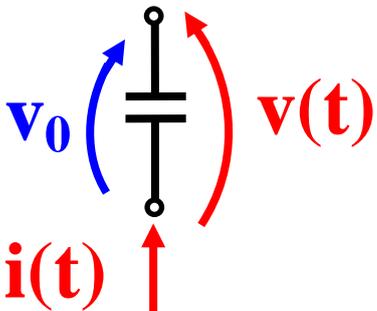
CAPACITOR E CONVENÇÕES



$$i = C \frac{dv}{dt}$$

$$v = \frac{1}{C} \int i dt + v_0$$

$$v = \frac{1}{C} \int i dt - v_0$$



$$i = -C \frac{dv}{dt}$$

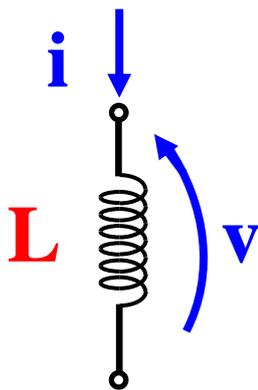
$$v = -\frac{1}{C} \int i dt + v_0$$

$$v = -\frac{1}{C} \int i dt - v_0$$

INDUTOR

$$\psi = L(i)$$

1 – Linear , Fixo → Ideal



$$\psi = L \cdot i$$

$$v = \frac{d\psi}{dt} = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int_{t_0}^t v dt + i(t_0)$$

$$p = \frac{1}{2} L \frac{di^2}{dt}$$

$$w = \frac{1}{2} Li^2 - \frac{1}{2} L i_0^2$$

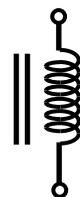
2 – Linear, Variável

$$\psi = L(t) i(t)$$

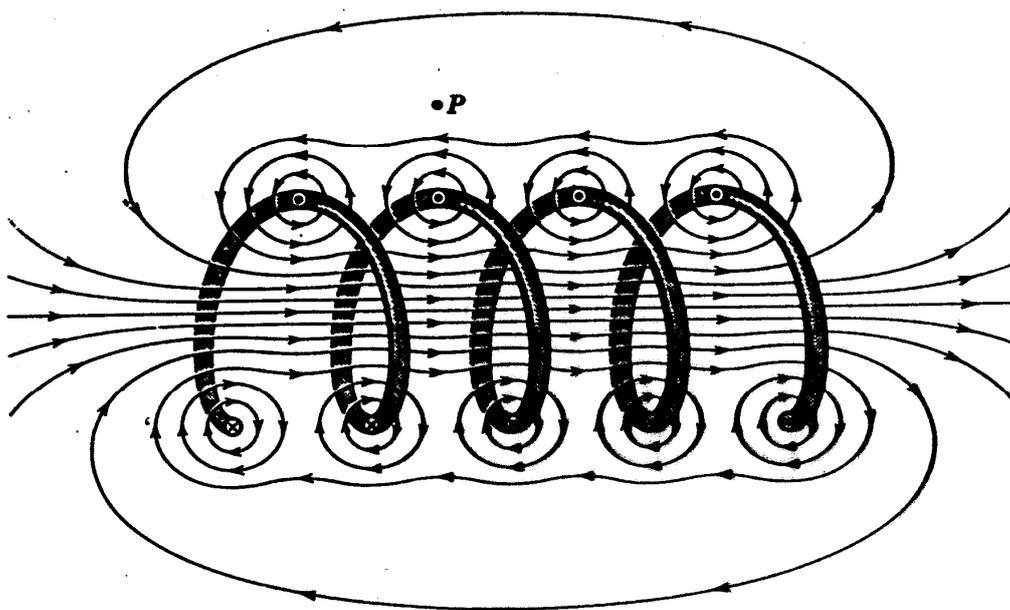
$$v = L(t) \frac{di}{dt} + i \frac{dL(t)}{dt}$$

3 – Não linear

Ex.:

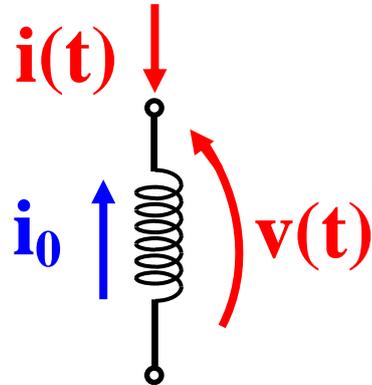
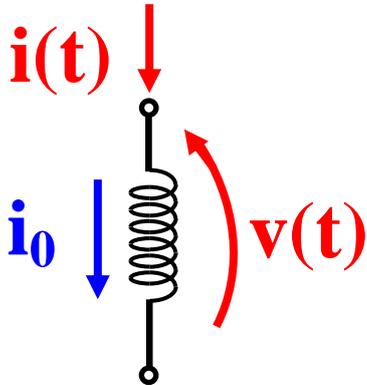


INDUTOR



Solenóide com espiras bem afastadas, mostrando as linhas de indução magnética, resultantes da soma vetorial do campo criado por cada espira.

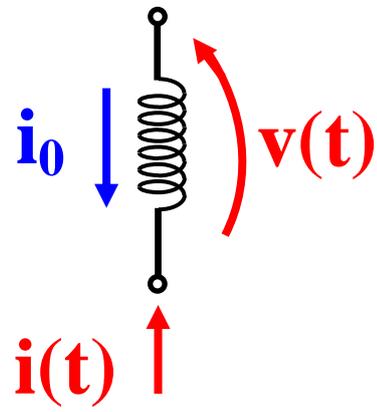
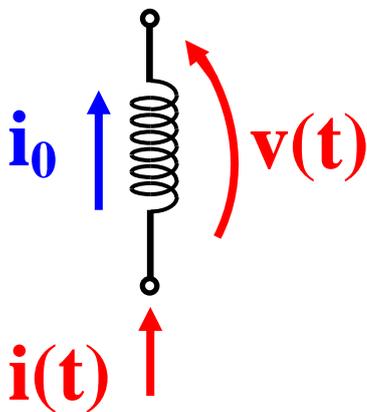
INDUTOR E CONVENÇÕES



$$v = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int v dt + i_0$$

$$i = \frac{1}{L} \int v dt - i_0$$

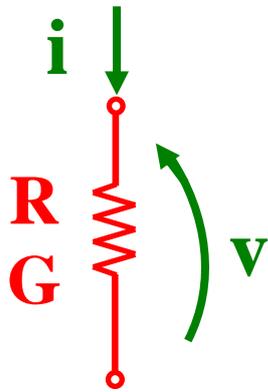


$$v = -L \frac{di}{dt}$$

$$i = \frac{-1}{L} \int v dt + i_0$$

$$i = \frac{-1}{L} \int v dt - i_0$$

RESISTOR

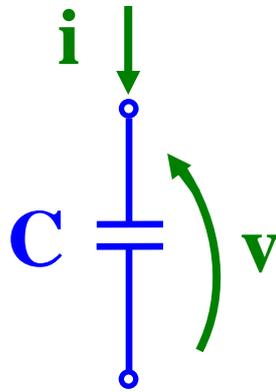


$$v = Ri$$

$$i = Gv$$

$$p = Ri^2$$
$$Gv^2$$
$$v^2/R$$
$$i^2/G$$

CAPACITOR



$$q = Cv$$

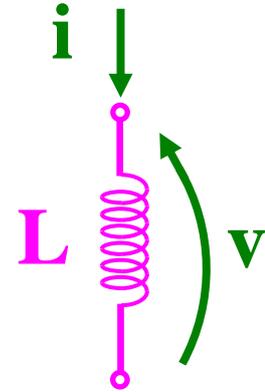
$$v = \frac{1}{C} \int idt + v_0$$

$$i = C \frac{dv}{dt}$$

$$p = \frac{1}{2} C \frac{dv^2}{dt}$$

$$w = \frac{1}{2} Cv^2$$

INDUTOR



$$\psi = Li$$

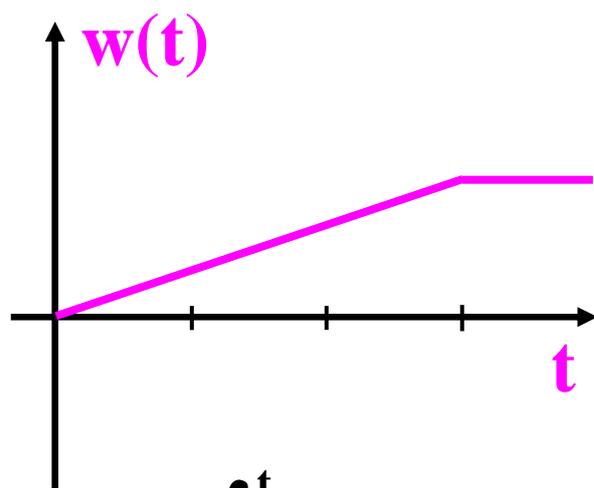
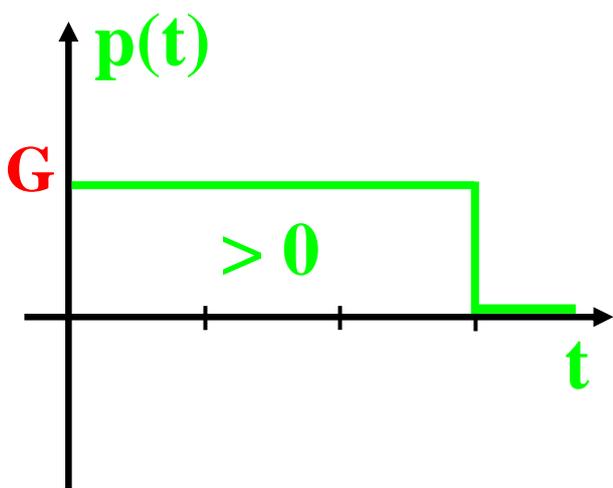
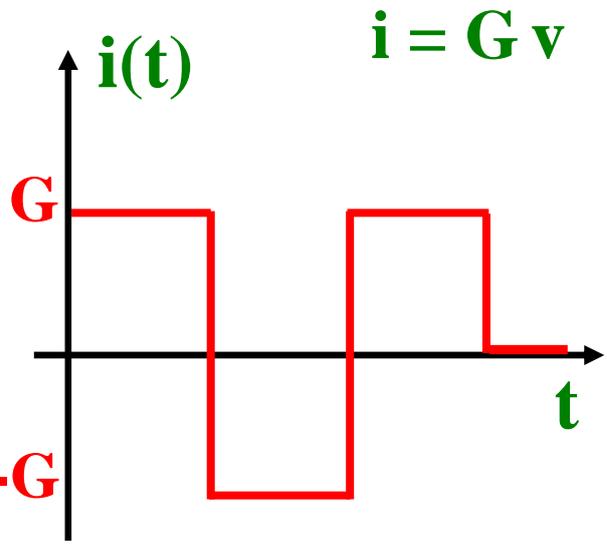
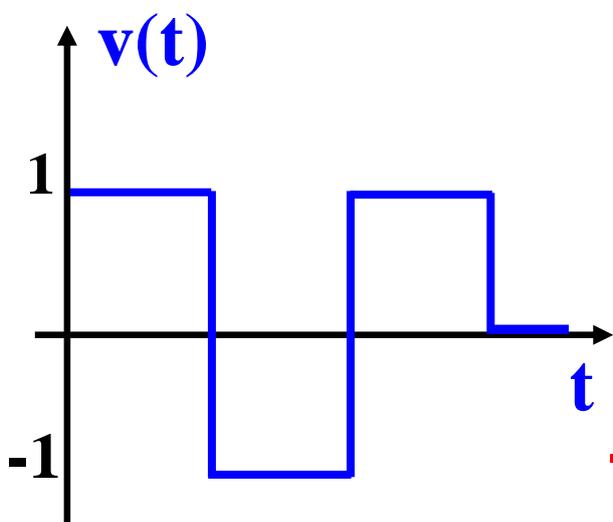
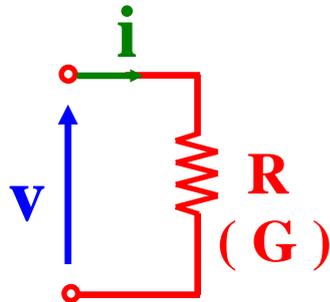
$$v = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int vdt + i_0$$

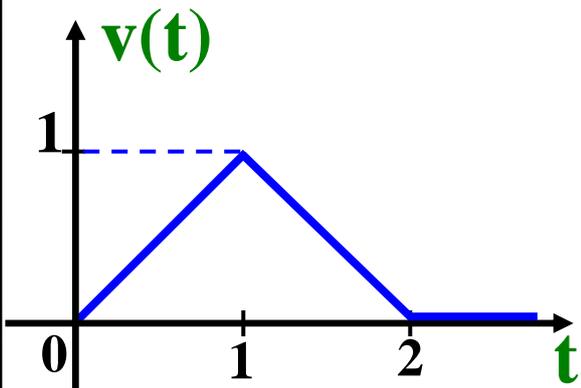
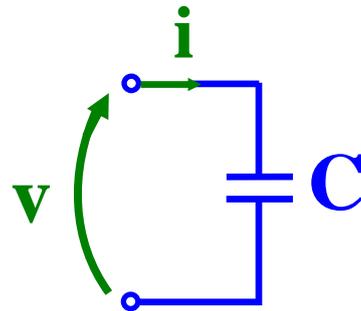
$$p = \frac{1}{2} L \frac{di^2}{dt}$$

$$w = \frac{1}{2} Li^2$$

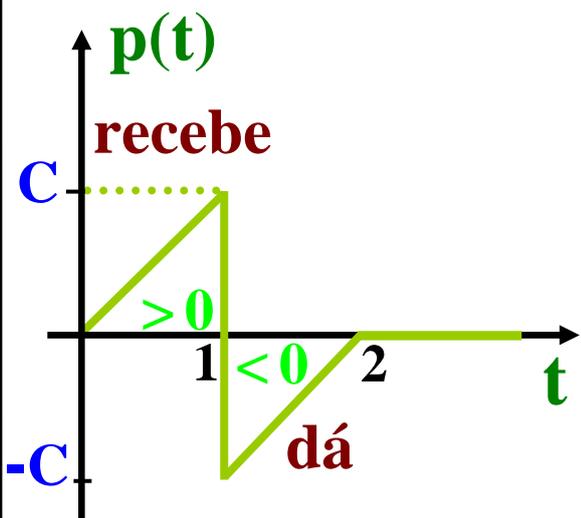
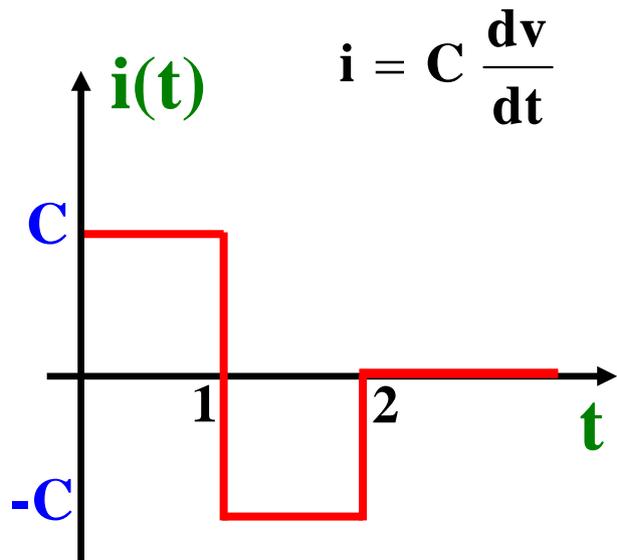
FORMAS DE ONDA EM RESISTOR



FORMAS DE ONDA EM UM CAPACITOR



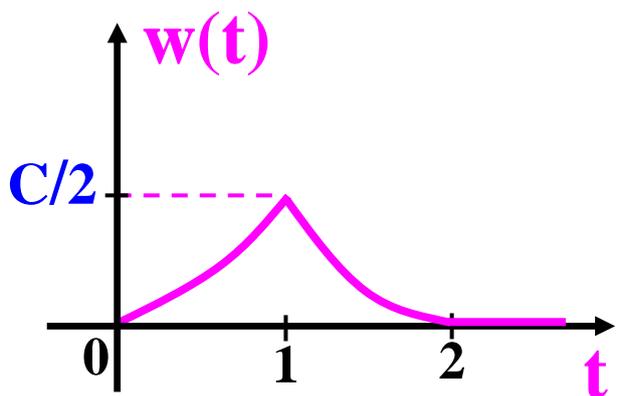
$$v(t) = \frac{1}{C} \int_{t_0}^t i(\lambda) d\lambda + v(t_0)$$



$$p = v i$$

$$v(t_0) = 0$$

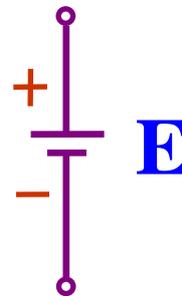
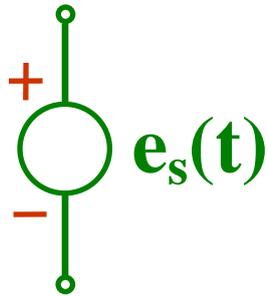
$$t_0 = 0$$



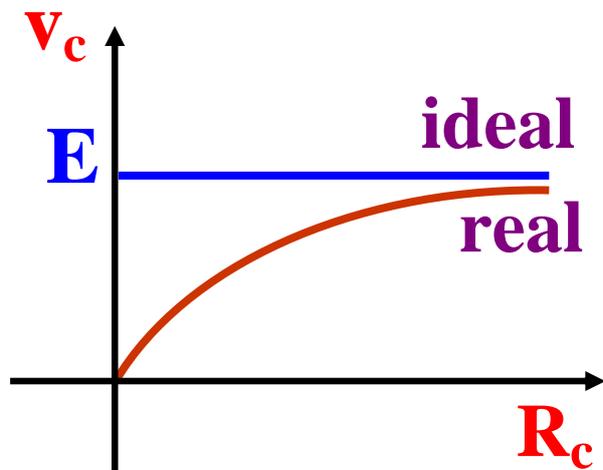
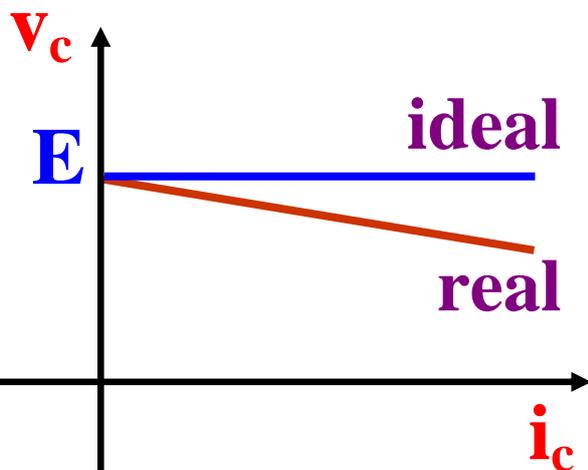
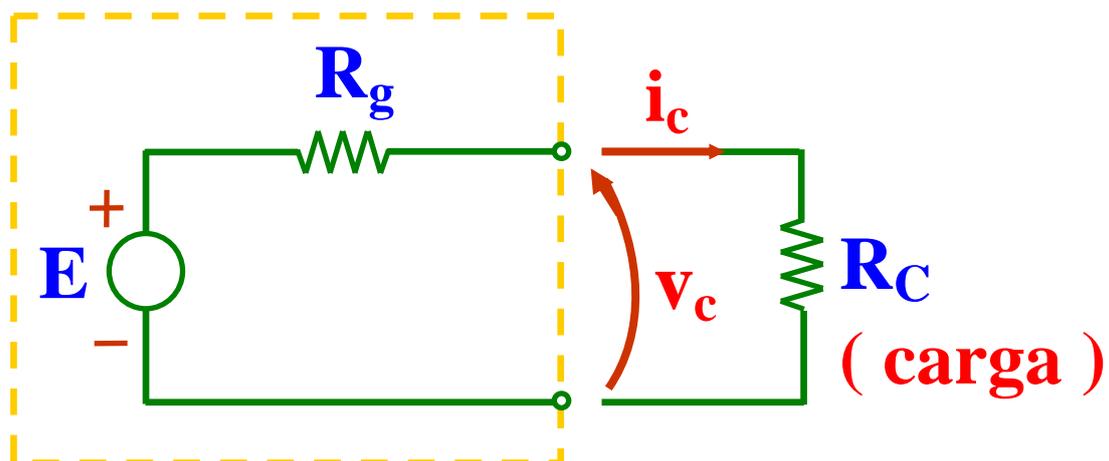
$$w = \frac{1}{2} C v^2$$

$W > 0$ passivo
(convenção receptor)

GERADORES DE TENSÃO



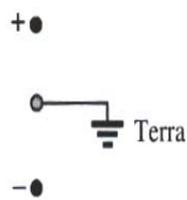
Gerador Real:



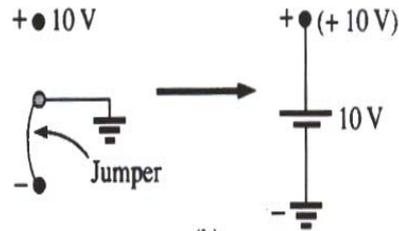
FONTES DE ALIMENTAÇÃO AC/DC

Tensão AC → Retificação e Filtragem

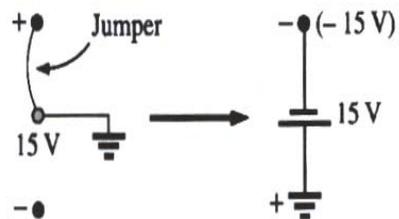
→ Tensão DC



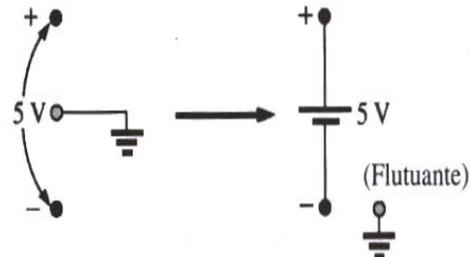
(a)



(b)



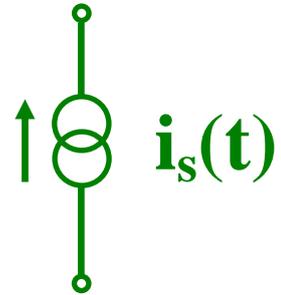
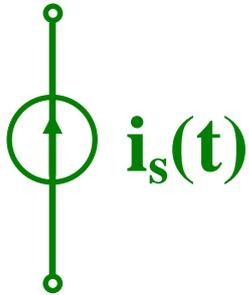
(c)



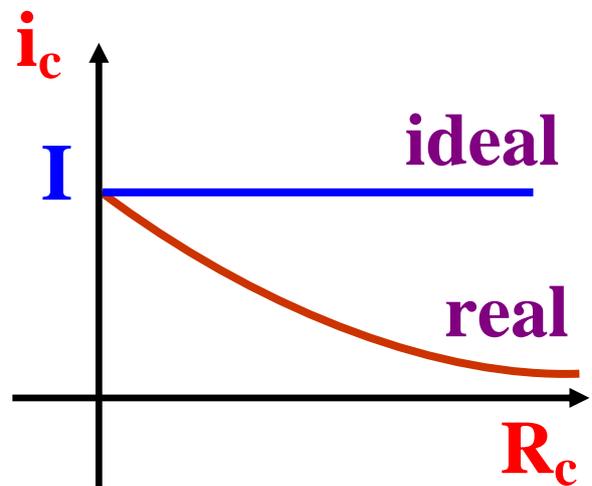
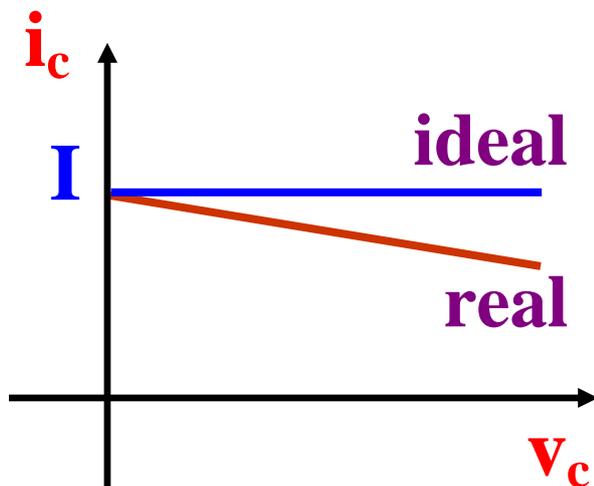
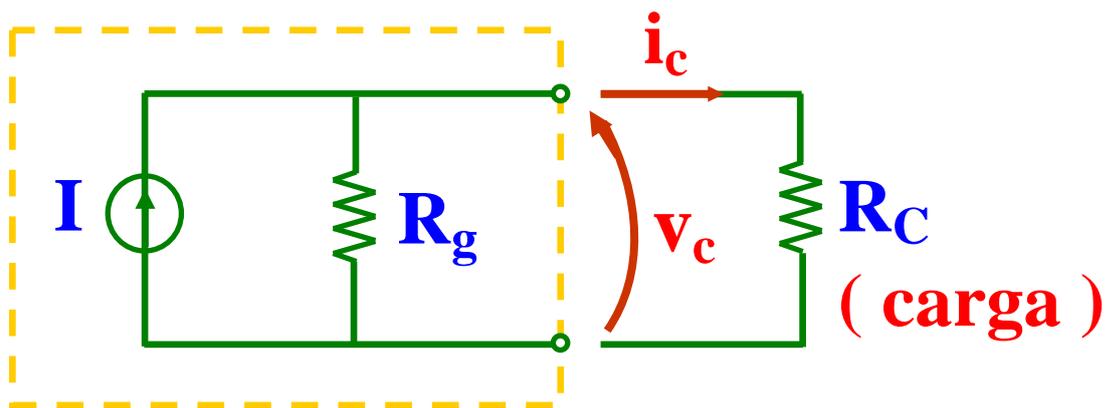
(d)

- a) Terminais disponíveis
- b) Tensão positiva em relação ao terra
- c) Tensão negativa em relação ao terra
- d) Tensão flutuante

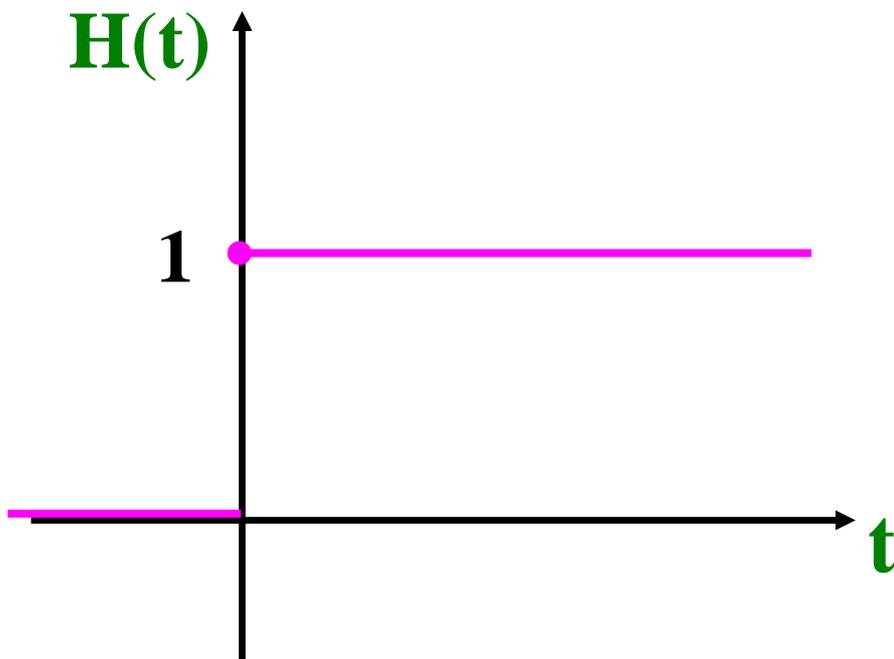
Geradores de Corrente



Gerador Real



Função Degrau Unitário ou Função de Heaviside



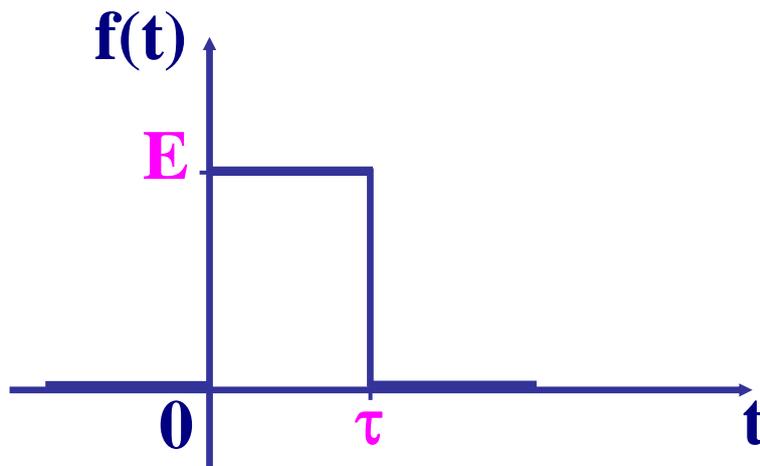
$$H(t) = 1(t) =$$

$$\begin{cases} \mathbf{0} & \text{para } t < \mathbf{0} \\ \mathbf{1} & \text{para } t \geq \mathbf{0} \end{cases}$$

Funções Obtidas da Função Degrau

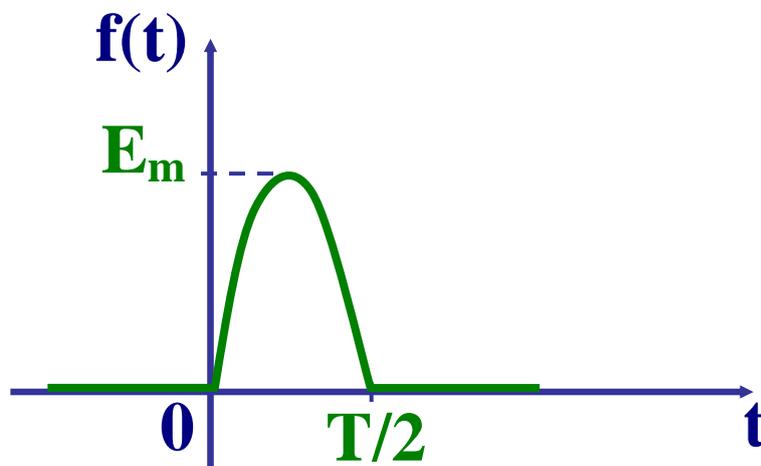
Pulso retangular de duração τ :

$$f(t) = E [H(t) - H(t - \tau)]$$

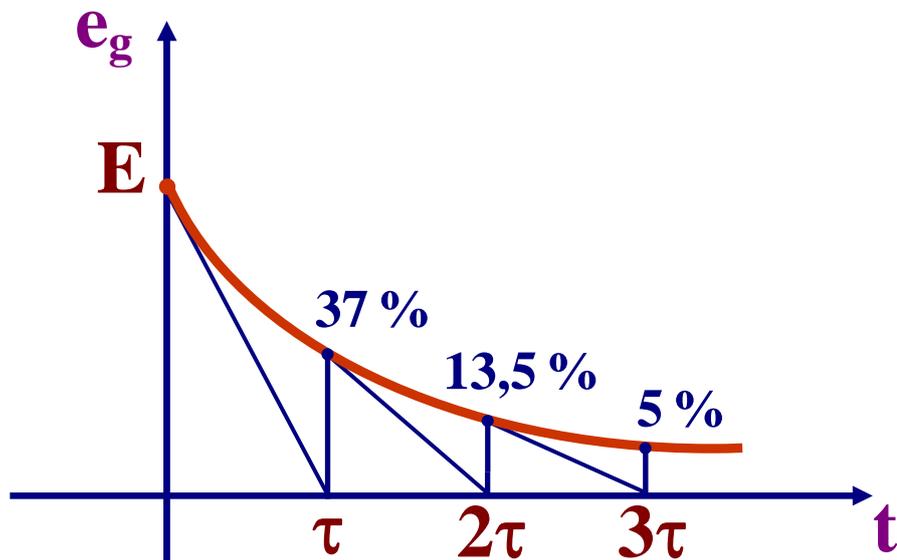


Pulso senoidal :

$$f(t) = E_m \sin\left(\frac{2\pi}{T} \cdot t\right) \cdot \left[H(t) - H\left(t - \frac{T}{2}\right) \right]$$



Excitação Exponencial



$$e_g(t) = E e^{s t} \quad E, s \text{ reais}$$

$$s = -\sigma \quad E > 0 \quad s < 0$$

$$e_g(t) = E e^{-\sigma t} = E e^{-t/\tau}$$

$\sigma \rightarrow$ frequência neperiana (Np/s)

$\tau = \frac{1}{\sigma} \rightarrow$ constante de tempo (s)

Para $t = \tau \rightarrow e_g = E/e$

EXCITAÇÃO COSSENOIDAL

- Derivada e Integral → Senoides

Circuito em Regime Permanente Senoidal

- Dispositivos Reais →
geram excitação senoidal
- Soma de senoides de mesma frequência =
senoide
- Análise de Fourier → \forall função periódica =
soma de senoides

$$f(t) = A_m \cos(\omega t + \theta)$$

A_m = amplitude ou valor máximo ou valor de pico (real e > 0)

ω = frequência angular (real, rd/s)

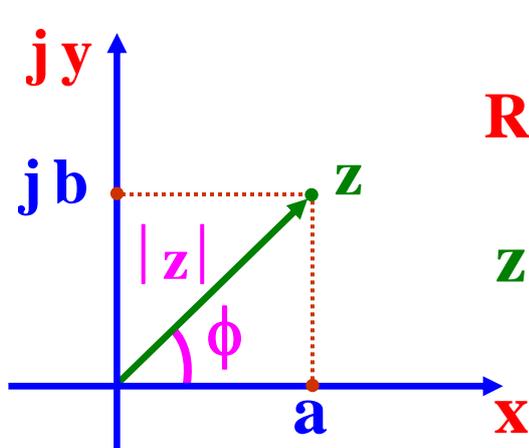
θ = defasagem (real, $^\circ$ ou rd)

f = frequência (real, Hz ou ciclos/s)

T = período (real, s) = $1 / f$

$$\omega = 2\pi / T$$

NÚMEROS COMPLEXOS



$$z = a + jb$$

Retangular ou Cartesiana

$$z = |z| e^{j\phi} = |z| \angle \phi$$

Polar

Fórmula de Euler : $e^{j\phi} = \cos \phi + j \sen \phi$

Séries de Mac Laurin:

$$\sen x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$e^{jx} = \cos x + j \sen x = 1 + jx + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} + \dots$$

$$z = |z| \cos \phi + j |z| \sen \phi = |z| (\cos \phi + j \sen \phi) = |z| e^{j\phi}$$

Identidade de Euler

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Seja $B = \cos\theta + j\sin\theta$

$$\begin{aligned}\frac{dB}{d\theta} &= -\sin\theta + j\cos\theta \\ &= j(\cos\theta + j\sin\theta)\end{aligned}$$

ou $\frac{dB}{d\theta} = jB$

$$\frac{dB}{B} = j d\theta$$

Integrando :

$$\ln B = j\theta + C \leftarrow \text{constante}$$

Para $\theta = 0 \rightarrow B = 1 \rightarrow \ln B = 0$

$$\Rightarrow C = 0 \Rightarrow B = e^{j\theta}$$

$$\Rightarrow e^{j\theta} = \cos\theta + j\sin\theta$$

Números Complexos

Fórmula de Euler :

$$e^{j\phi} = \cos \phi + j \operatorname{sen} \phi$$

$$e^{-j\phi} = \cos \phi - j \operatorname{sen} \phi$$

Forma Cartesiana: $z = a + jb$

Forma Polar : $z = |z| e^{j\phi}$

$$\begin{cases} a = |z| \cos \phi \\ b = |z| \operatorname{sen} \phi \end{cases}$$

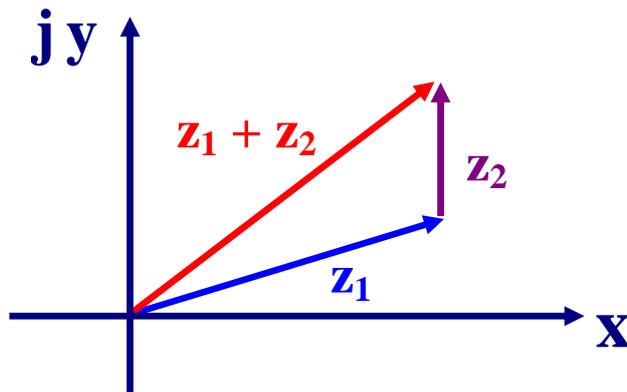
$$\begin{cases} |z| = \sqrt{a^2 + b^2} \\ \phi = \operatorname{arctg} b/a \end{cases}$$

Operações com Complexos

1 – Soma e Subtração → Forma Retangular ou Cartesiana

$$z_1 = a_1 + j b_1 \quad z_2 = a_2 + j b_2$$

$$z_1 \pm z_2 = (a_1 \pm a_2) + j (b_1 \pm b_2)$$



2 – Multiplicação e Divisão → Forma Polar

$$z_1 = c_1 e^{j\phi_1} \quad z_2 = c_2 e^{j\phi_2}$$

$$z_1 z_2 = c_1 c_2 e^{j(\phi_1 + \phi_2)}$$

$$z_1 / z_2 = \frac{c_1}{c_2} e^{j(\phi_1 - \phi_2)}$$

Números Complexos

Propriedades :

$$z = a + j b = |z| e^{j\phi}$$

$$z^* = a - j b = |z| e^{-j\phi}$$

$$z + z^* = 2a = 2 \operatorname{Re}(z)$$

$$|e^{j\phi}| = 1$$

$$e^{\pm j\pi} = 1 \angle \pm \pi = -1$$

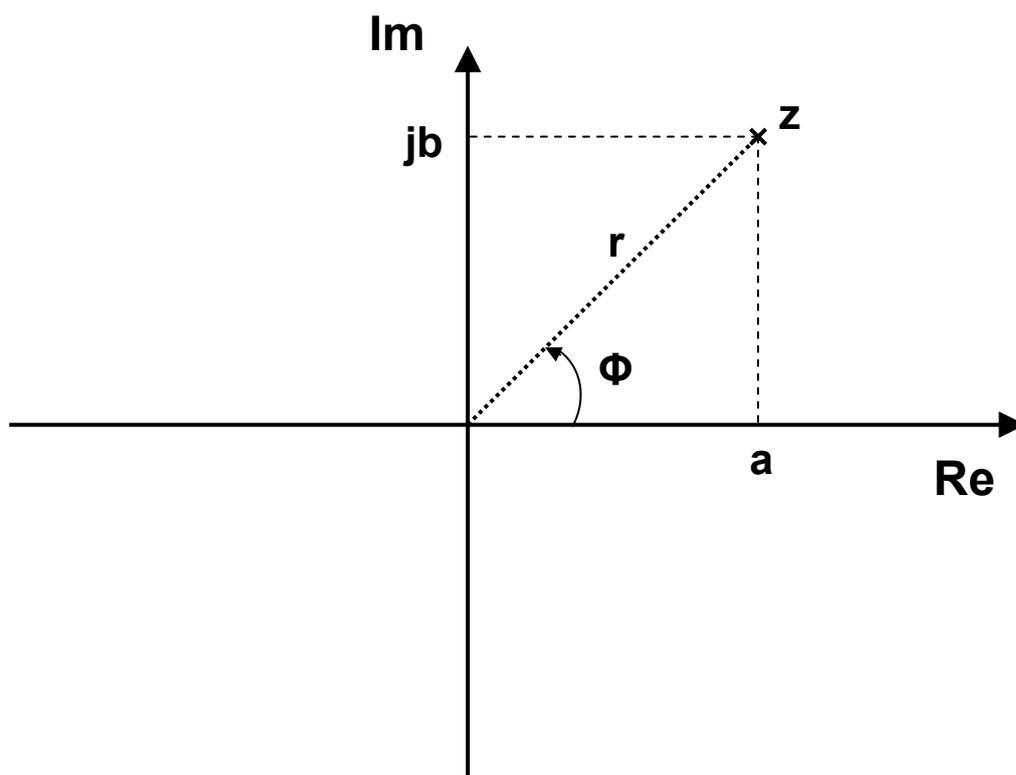
$$e^{\pm j\pi/2} = 1 \angle \pm \pi/2 = \pm j 1$$

Fórmulas de Moivre :

$$\cos \omega t = \frac{1}{2} \left(e^{j\omega t} + e^{-j\omega t} \right)$$

$$\operatorname{sen} \omega t = \frac{1}{2j} \left(e^{j\omega t} - e^{-j\omega t} \right)$$

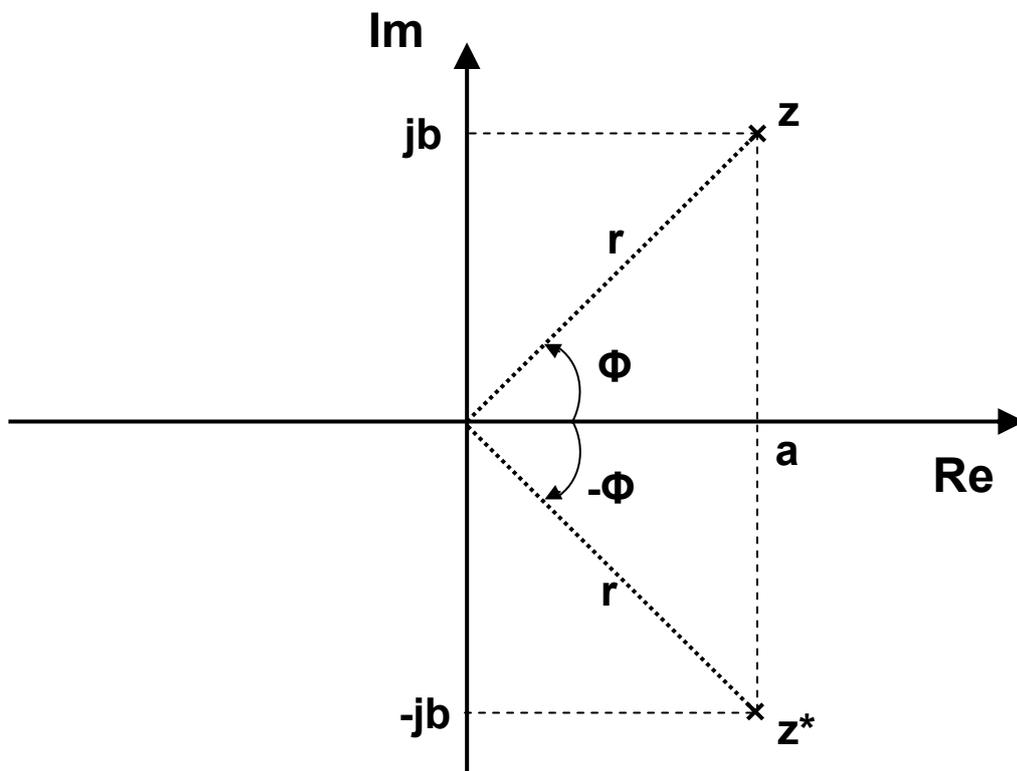
Plano Complexo



Coordenadas Retangulares: a, b

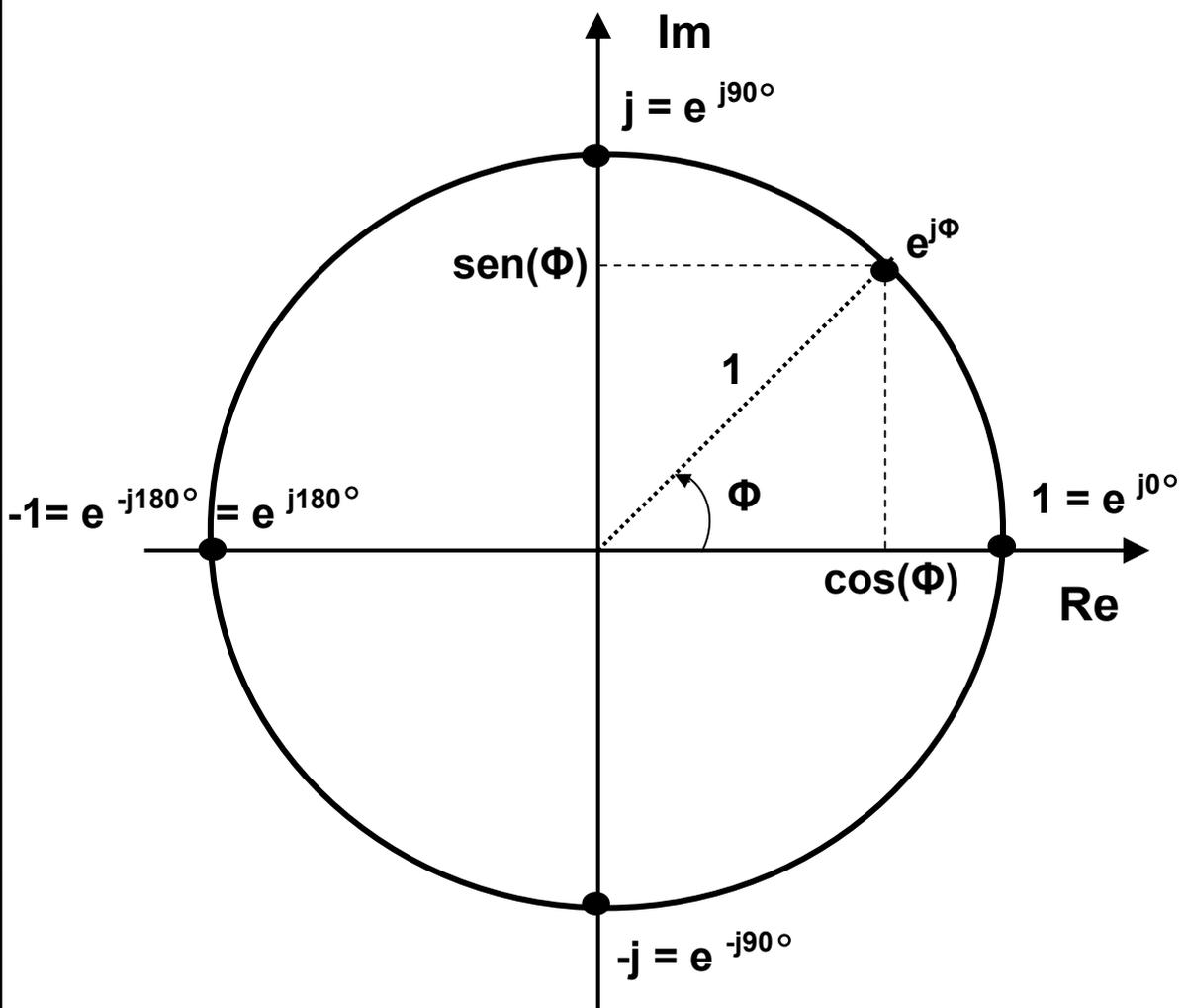
Coordenadas Polares: r, Φ

Plano Complexo



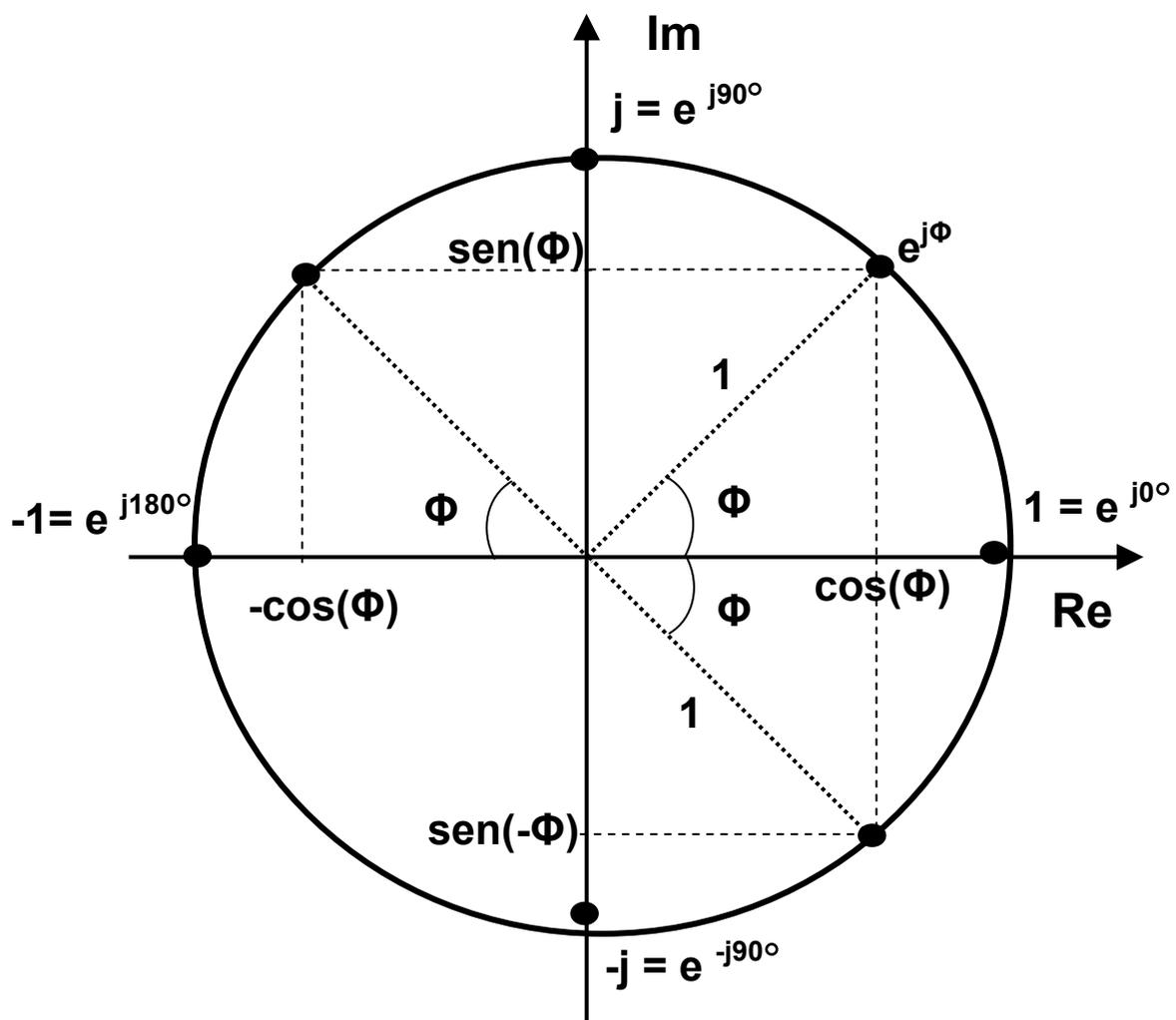
Conjugados

Plano Complexo



Círculo Unitário

Plano Complexo



Círculo Unitário

FASORES

$$A_m \cos(\omega t + \theta) =$$

$$\left\{ \begin{array}{l} \frac{1}{2} \left(\hat{A}_m e^{j\omega t} + \hat{A}_m^* e^{-j\omega t} \right) \\ \operatorname{Re} \left[\hat{A}_m e^{j\omega t} \right] \end{array} \right.$$

Valor instantâneo do sinal →

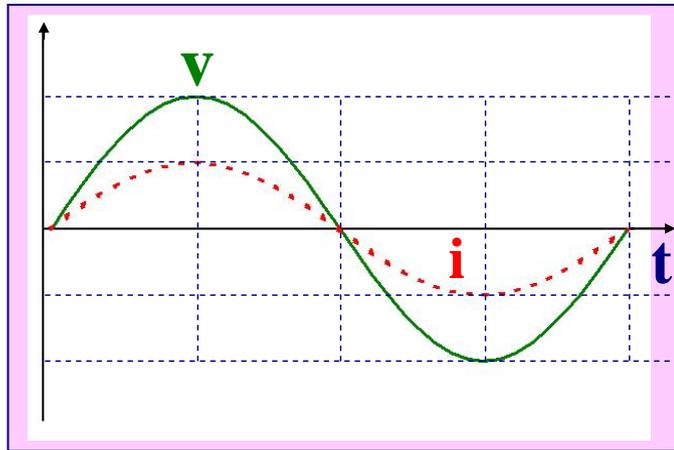
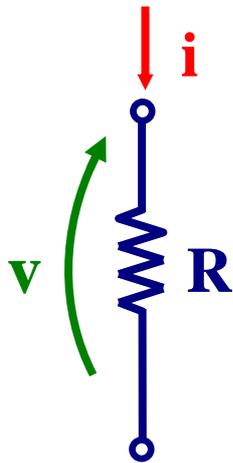
Domínio do tempo →

$$s(t) = A_m \cos(\omega t + \theta)$$

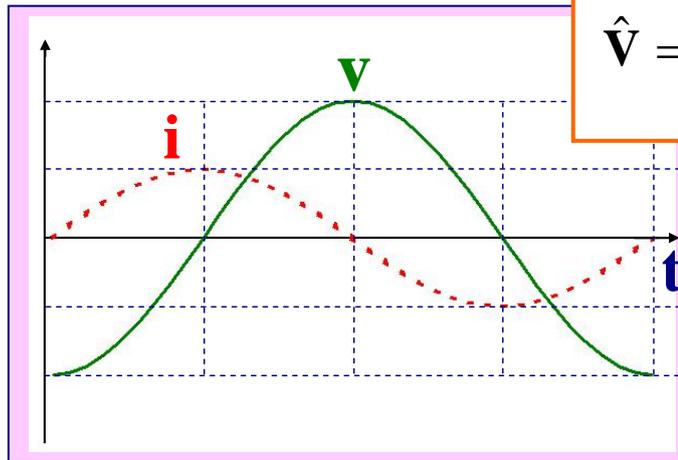
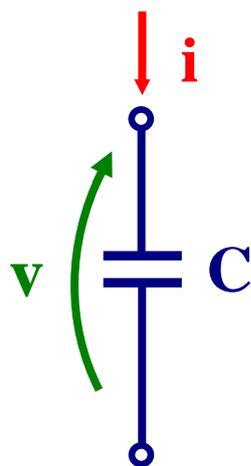
Fasor associado a sinal senoidal:

$$\hat{S} = A_m e^{j\theta} = A_m \underline{\angle \theta}$$

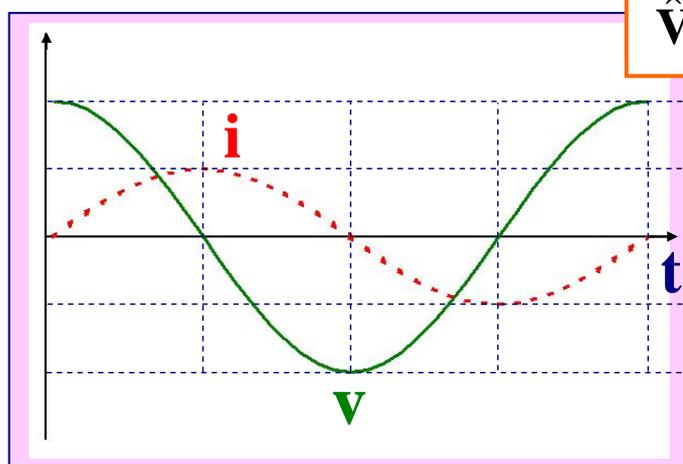
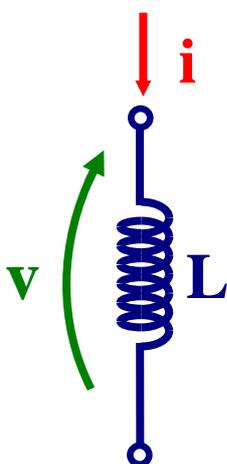
Relações v - i senoidais nos bipolos ideais



$$\hat{V} = R\hat{I}$$



$$\hat{V} = \frac{1}{j\omega C} \hat{I}$$

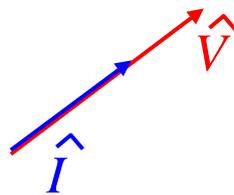
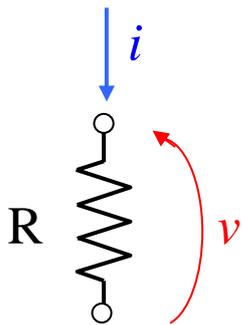


$$\hat{V} = j\omega L\hat{I}$$

DIAGRAMAS FASORIAIS NOS ELEMENTOS BÁSICOS DE CIRCUITOS

Resistências

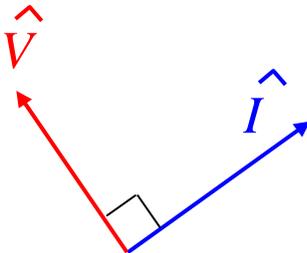
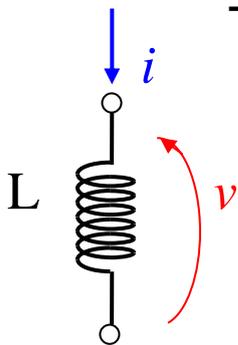
- corrente e tensão **em fase**



$$\hat{V} = R \hat{I}$$

Indutâncias

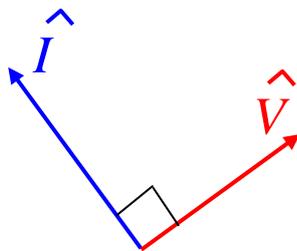
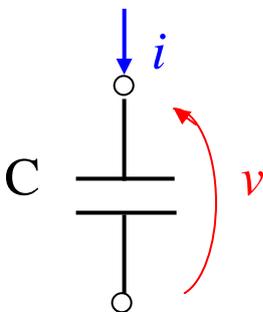
- corrente **atrasada** de $\pi/2$



$$\hat{V} = j \omega L \hat{I}$$

Capacitâncias

- corrente **adiantada** de $\pi/2$



$$\hat{V} = -j \hat{I} / (\omega C)$$

Relações Fasoriais \hat{V} - \hat{I}

Resistor

$$\hat{V} = R\hat{I}$$

$$\hat{I} = G\hat{V}$$

Capacitor

$$\hat{V} = -j\frac{1}{\omega C}\hat{I}$$

$$\hat{I} = j\omega C\hat{V}$$

Indutor

$$\hat{V} = j\omega L\hat{I}$$

$$\hat{I} = -j\frac{1}{\omega L}\hat{V}$$

Impedância: $Z = \hat{V}/\hat{I}$
Admitância: $Y = \hat{I}/\hat{V}$

Resistor

$$Z = R$$

$$Y = G$$

Capacitor

$$Z = \frac{1}{j\omega C}$$

$$Y = j\omega C$$

Indutor

$$Z = j\omega L$$

$$Y = \frac{1}{j\omega L}$$

Excitação Cossenooidal

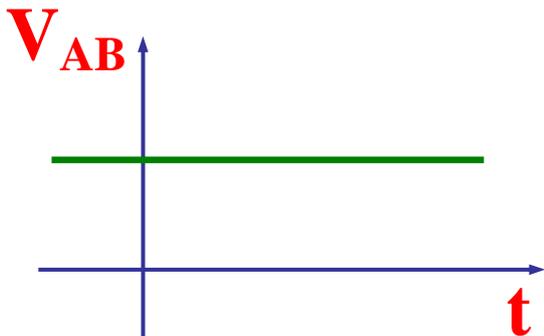
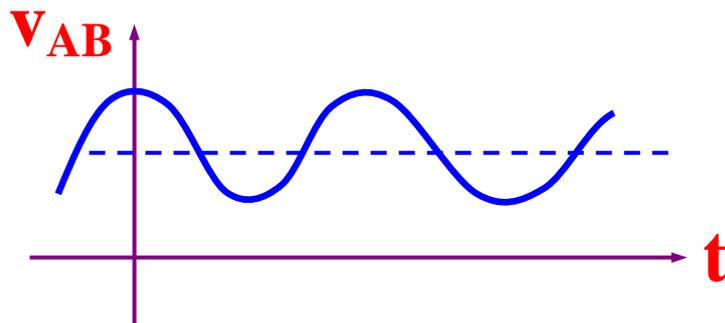
$$f(t) = A_m \text{sen}(\omega t + \phi) = A_m \text{cos}(\omega t + \phi - 90^\circ)$$

$$\text{sen } a = \text{cos}(a - 90^\circ) \quad *$$

$$\text{sen } a = \text{cos}(90^\circ - a)$$

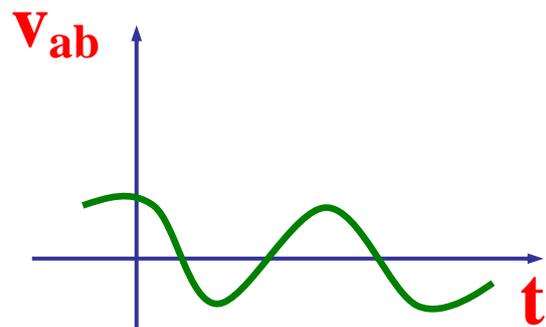
$$a = \omega t + \phi$$

Cossenoide + DC →



Componente Contínua
DC

+



Componente incremental
AC (alternativa)

Valor Médio

$$V_{AB} = \frac{1}{T} \int_0^T v_{AB} dt$$