

# Couplers

Splitters/combiners/couplers can be classified according to numerous characteristics. The most common ones are: distributed/ lumped-element (or a combination of both), number of ports, equal/unequal power division, fixed/tunable power division, bandwidth, and phase difference between the outputs. Figure 1 shows the main planar dividers/ combiners/ couplers classified according to the outputs phase difference ( $\Delta_\varphi$ ). Outputs can be in phase ( $\Delta_\varphi = 0^\circ$ ), in quadrature ( $\Delta_\varphi = 90^\circ$ ) or out-of-phase ( $\Delta_\varphi = 180^\circ$ ). Highlighted, the most used in microwave systems.

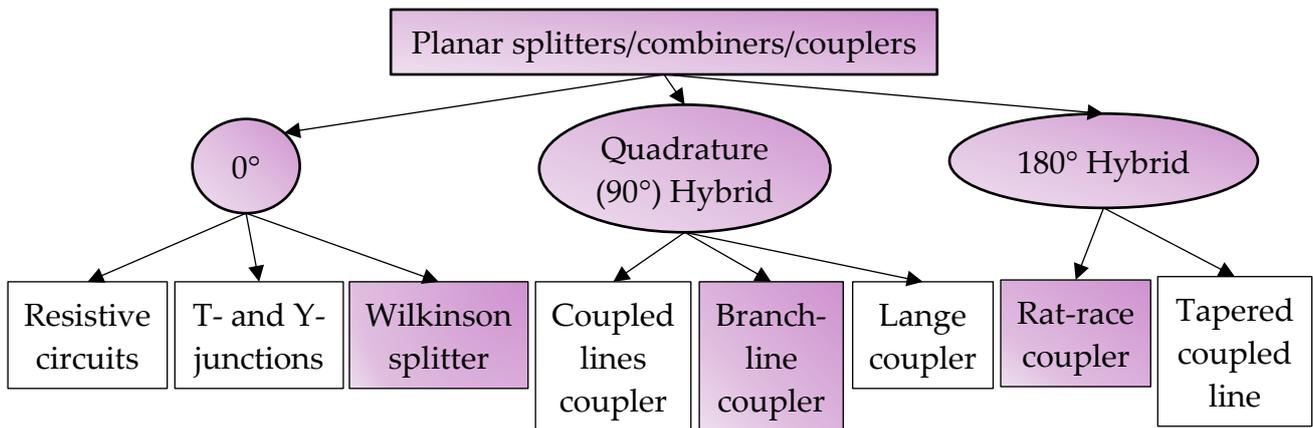


FIGURE 1. MAIN TYPES OF PLANAR DIVIDERS/COMBINERS/COUPLERS.

## Rat-race coupler

### Introduction

Ring hybrids are often referred to as Rat-race couplers. Considering the  $180^\circ$  hybrid symbol in Figure 2, a signal applied to port ① will be equally split into two in-phase components at ports ② and ③, and port ④ will be isolated. In this case the  $180^\circ$  hybrid is a simple power splitter. If the input is applied to port ④, it will be equally split into two components with a  $180^\circ$  phase difference at ports ② and ③, and port ① will be isolated. In that case the  $180^\circ$  hybrid acts as a balun with a differential signal between ports ② and ③. When operated as a combiner, with input signals applied at ports ② and ③, the sum of the inputs will be formed at port ① while the difference will be formed at port ④. Hence, ports ① and ④ are referred to as **the sum and difference ports**, respectively. This component is thus also called a  $\Sigma \Delta$  device.

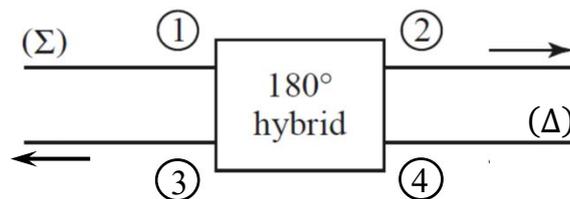


FIGURE 2.  $180^\circ$  HYBRID  $\Sigma \Delta$  SYMBOL.

### Principle

The most suitable distributed way to realize a  $\Sigma \Delta$  is the *rat-race*, shown in Figure 3. The *rat-race* is a four-port circuit. The name *rat-race* was given because if one considers rats entering through port ① into a dark tunnel, the rats will separate equally in the direction of

port ② and ③, without any rat sorting through port ④. It is the same if one considers TLines and power entering through port ①. No power will flow through port ④.

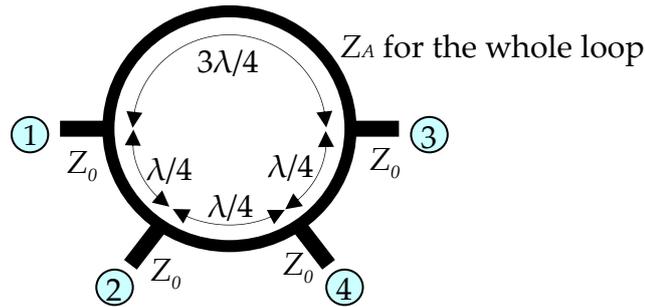


FIGURE 3. RAT-RACE COUPLER.

Let's consider a signal (a "wave")  $a_1$  incoming through port ①. The characteristic impedance of the loop being equal to  $\sqrt{2}Z_0$  for the whole loop, and hence for the two TLines between ports ① and ② and ports ① and ③, the signal  $a_1$  separates equally. The magnitude is null at port ④ since the signals coming from the two paths are out of phase. The signal arriving from port ③ path has a phase difference of  $2\pi(\lambda)$  whereas the one arriving from port ② path has a phase difference of  $\pi(\lambda/2)$ .

We can demonstrate that a loop characteristic impedance ( $Z_A$ ) equal to  $\sqrt{2}Z_0$  leads to a perfect matching ( $Z_0$ ) of port ①, if ports ② and ③ are also matched. For that, consider Figure 4. Since the magnitude of the signal is null at port ④, a short circuit (SC) can be connected there without changing the behavior of the circuit. The two  $\lambda/4$  TLines situated between ports ④ and ② and between ports ④ and ③ transform the SC into an open-circuit (OC).

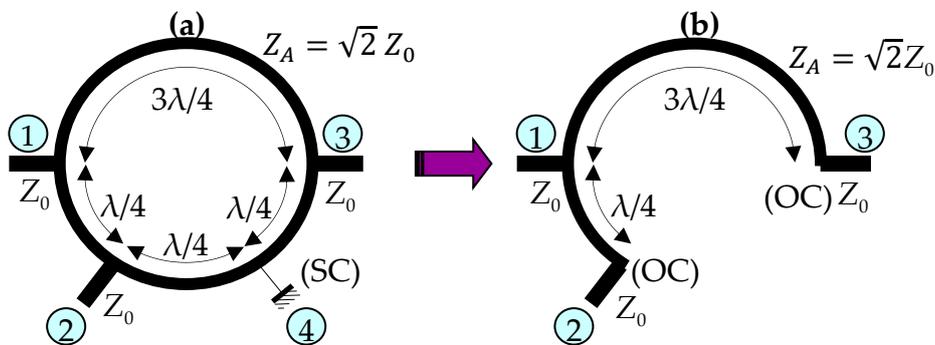


FIGURE 4. RAT-RACE. (A) SHORT-CIRCUIT AT PORT ④. (B) EQUIVALENT OPEN-CIRCUITS AT PORTS ② AND ③ AND (C) EQUIVALENT ELECTRICAL CIRCUIT.

Remember that ports ② and ③ are loaded with a port impedance equal to  $Z_0$ . Impedance  $Z_{13}$  seen from port ① when looking to port ③ can be written as  $Z_{13} = Z_A \frac{Z_0 + jZ_A \tan(\beta l)}{Z_A + jZ_0 \tan(\beta l)}$  with  $l = \frac{3\lambda}{4}$ , leading to  $Z_{13} = \frac{Z_A^2}{Z_0}$ . If  $Z_A = \sqrt{2}Z_0$ , then  $Z_{13} = 2Z_0$ .

Similarly, it can be shown that  $Z_{12} = 2Z_0$ . Hence, the circuit in Figure 4 (c) is equivalent to that in Figure 4 (b). Since  $Z_{13}$  and  $Z_{12}$  are connected in parallel to port ①, the input impedance seen at port ① is equal to  $Z_0$ , thus port ① is matched.

#### Properties summary and S-matrix

We consider a signal  $a_1$  entering through port ①, the other ports being matched:

- Port ① is matched.

- Ports ① and ④ are uncoupled.
- The power incoming through port ① is distributed equally to the two ports ② and ③. Then  $b_3 = ja_1/\sqrt{2}$  (phase shift  $-3\pi/2$  or  $+\pi/2$ ) and  $b_2 = -ja_1/\sqrt{2}$  (phase shift  $-\pi/2$ ).
- S-Matrix : by symmetry, a signal incoming through port ③ is distributed equally to the two ports ① and ④, port ② being uncoupled.

Let's consider now a signal  $a_2$  entering through port ②, the other ports being matched.

The same approach can be used to show that the power incoming by port ② is distributed equally to the two ports ① and ④, port ③ being uncoupled. Hence,  $b_3 = 0$ ,  $b_1 = -ja_2/\sqrt{2}$  and  $b_4 = -ja_2/\sqrt{2}$ , and in this case,  $b_1$  and  $b_4$  are in phase.

The S-Matrix of an ideal rat-race coupler is given by:  $\frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

### Electrical performance and dimensions

The electrical performance of a *rat-race* considering the four characteristics above when a signal  $a_1$  is incoming through port ① is evaluated by:

- the matching of each port given by  $S_{11}, S_{22}, S_{33}$  and  $S_{44}$ .
- the insertion loss given by  $S_{21}$  and  $S_{31}$ .
- the isolation between the two output ports given by  $S_{23}$ .
- the imbalance between the two output ports, given by the magnitude imbalance  $S_{21\text{ dB}} - S_{31\text{ dB}}$ , and the phase imbalance  $\varphi(S_{21}) - \varphi(S_{31})$ .
- the covered bandwidth.

An ideal *rat-race* without any losses and bandwidth limitations is defined by the S-matrix above. As the power incoming through port ① is distributed equally to the ports ② and ③, each output receives half of the power:  $S_{21\text{ ideal}} = S_{31\text{ ideal}} = -3\text{ dB}$ .

Due to the  $\lambda/2$  length difference between ports ② and ③, the phase difference is  $\varphi(S_{21}) - \varphi(S_{31}) = 180^\circ$ .

In practical life, things are not so ideal, and hence:

- $S_{21\text{ real}}$  and  $S_{31\text{ real}} < -3\text{ dB}$ .
- The TLines length being not equal between ports ① and ②, and ① and ③, respectively, the output ports are inherently unbalanced in terms of magnitude.
- Isolation depends on the correct design of the lengths and widths of the TLines. Some parasitic couplings may also appear between TLines. For most of modern applications, a 20-dB isolation corresponds to an acceptable performance. Hence  $S_{23} < -20\text{ dB}$  is mandatory.
- Like isolation, matching at each port is dependent on the right design of the lengths and widths of the TLines. As well some parasitic couplings may appear between TLines. A 10-dB matching can sometimes be sufficient, for example in the case of antennas array. In other cases matching must be higher than 20 dB.

The principle described here is only valid at the working frequency, where the TLines electrical lengths are equal to  $90^\circ(\lambda/4)$  or  $270^\circ(3\lambda/4)$ . When frequency deviates from the working frequency, performance degrades.