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Professor Ewald Macha's contribution to the development of methods for multiaxial fatigue life estimation

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ABSTRACT

Multiaxial fatigue of machine elements and construction is still a remarkable problem in everyday life. Many accidents were consequences of inaccurate estimation of stresses and fatigue life. Despite many years of work and research on the subject, fatigue life estimation is still very important to be examined. Professor Ewald Macha was a researcher involved in the development issues of fatigue life estimation and, in particular, was interested in multiaxial fatigue failure criteria based on the critical plane approach, where he formulated the stress, strain and strain energy density criteria. Particularly noteworthy are the methods for determination of the critical plane position. In this field, Professor Macha proposed a weight function method, and then a variance method. These methods have been verified experimentally. Professor Macha also proposed a method for determination of the energy fatigue characteristics, especially for cyclically unstable materials. This paper describes the scientific achievements of Professor Macha.

1. Introduction

Estimation service life of different structural components is a very important issue for engineering. Incorrect service life estimation may result in accidents and disasters. Therefore, studies aimed to understand this phenomenon started already in the 19th century, and are being performed today. Initially, studies were limited to uniaxial and constant-amplitude issues only. With increasing knowledge on the phenomenon, the interest in multiaxial fatigue (most frequent in engineering practice) was growing, and many fatigue failure criteria were proposed. Another step was to assess the fatigue life for multiaxial random loads. This problem was explored by Professor Ewald Macha (Fig. 1) as well [1–4]. He started his work from proposing mathematical models to assess fatigue life of materials subjected to random complex stress state. These criteria are based on the critical plane approach by employing weight functions. Further studies were connected on both strain and energy criteria to determine the critical plane position. Many scientists became interested in these studies, which resulted in numerous contacts and common works, e.g. with Carpinteri et al. [5–7], Sakane et al. [8], Dragon et al. [9], Petit et al. [9], Macha and Sonsino [10], Palin-Luc et al. [11], Macha and Braun [12] and others.

Cooperation with Professor Carpinteri concerned the determination of the critical plane position according to the weight function within stress criteria. In Refs. [5,6], principal stress directions under multiaxial random loading are theoretically determined by averaging the instantaneous values of the three Euler angles through some suitable weight functions, which are assumed to take into account the main factors influencing the fatigue behaviour. The proposed algorithm was verified for experimental biaxial in- and out-of-phase stress states to assess the correlation between the expected principal stress directions and the position of the experimental fatigue fracture plane for such tests. Further remarks on these issues were presented in Ref. [7].

Fatigue tests on cruciform specimens, carried out by Będkowski, led to collaboration with Sakane, Ohnami and Itoh. In Ref. [8] the variance method to determine the fracture plane under random multiaxial stress states is presented. The fracture plane was estimated analytically by such a variance method according to three multiaxial strain criteria: the maximum normal strain in the fracture plane, the maximum shear strain in the fracture plane, the maximum shear and normal strain in the fracture plane. The estimated fracture planes were compared with experimental results obtained using cruciform plate specimens made of SUS 304 and 1Cr–1Mo–1/4V steel under random multiaxial stress states. The variance method with the maximum normal strain criterion in fracture plane gave the effective estimate of the actual fracture

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Nomenclature

a	coefficient allowing to include amplitudes below the fatigue limit in the process of fatigue damage accumulation
n'	exponent of cyclic strain curve
$r_{\sigma\tau}$	cross-correlation coefficient between normal stresses and shear stresses
E	Young's modulus
K_t	theoretical stress concentration factor
K'	coefficient of cyclic strain curve
L	the number of time instants considered
σ_y	yield strength
T_{cal}, T_{exp}	calculated and experimental fatigue life, respectively
W_a	strain energy density amplitude
W_{eq}	equivalent strain energy density parameter
W_k	weight function
ξ	angle determining the critical plane position for plane stress
ε_n	normal strain on the critical plane
ε_{ns}	shear strain on the critical plane
λ_σ	ratios of maximum stresses (τ_{max}/σ_{max})
μ_{x_1}	variance of normal stress σ_x
μ_{x_2}	variance of shear stress τ_{xy}
$\mu_{x_1x_2}$	covariance of normal σ_x and shear τ_{xy} stresses
$\mu_{\sigma eq}$	variance of equivalent stress σ_{eq}
ν	Poisson's ratio
σ_{af}	fatigue limit under fully reversed bending
σ_{max}	maximum normal stress
σ_n	normal stress on the critical plane
σ_x	stress along the longitudinal specimen axis
τ_{af}	fatigue limit under fully reversed torsion
τ_{max}	maximum shear stress
τ_{ns}	shear stress on the critical plane
τ_{xy}	shear stress in the specimen cross-section
J	J integral
K	stress intensity factor
l_i, m_i, n_i	principal direction cosines (i = 1, 2, 3 –eigenvectors of the stress tensor)
$\hat{\phi}, \hat{\theta}, \hat{\psi}$	averaged Euler's angles
α	atan (M_T/M_B) of the ratio of the torsional moment (Mode III) to the bending moment (Mode I)

Subscripts and others

a	amplitude
m	mean value
max	maximum value
min	minimum value
n	normal
pl	plastic
s	shear
t	time
Δ	range
I, II, III	crack mode I, mode II and mode III, respectively
1, 2, 3	principal axes

plane directions of cruciform specimens in high temperature biaxial low cycle fatigue.

Based on the strain measurements [9], in the rear wall of a bus and in welded fittings of a single-bucket excavator, influence of random stresses state components on calculated fatigue life was analysed. For this purpose, on the basis of the measured strain signals, covariance matrices between the stress tensor components were calculated, which were then taken into consideration in the variance method. The authors applied 8 types of covariance matrices of the stress tensor components to observe their influence on fatigue life assessment. These matrices took into consideration the influence of covariance components of the tensor on lifetime assessment. It was noted that influence of the cross-correlations between the normal stresses was greater than that between the normal and shear stresses on fatigue life assessment.

The multiaxial energy fatigue criteria were examined by Professor Macha in Ref. [10], together with Professor Sonsino. They made a review of fatigue energy criteria, divided into three groups: criteria based on elastic energy, criteria based on plastic energy, and criteria based on the total elastic and plastic energy.

Fatigue test results on 10HNAP steel subjected to random loading with a non-Gaussian probability distribution function and a wide-band frequency spectrum were used to compare the lifetime estimation by employing the models proposed by Bannantine, Fatemi–Socie, Socie, Wang-Brown, Morel, Łagoda–Macha [11]. The calculations were in time and frequency domains, and the best results were obtained using the Łagoda–Macha's model in both time and frequency domain. The model proposed by Socie, in time domain, and the Choudhury-Dover model in frequency domain gave good results.

Professor Macha also managed a team of designers of fatigue tests stands for different kinds of loads, usually bending with torsion, manufactured at Opole University of Technology [13,14].

Fig. 1. Photo of Professor Ewald Macha.

The purpose of this paper is to present academic achievements of Professor Macha.

2. Scientific activities of Professor Ewald Macha

Ewald Macha was Professor at Opole University of Technology (1992) and Head of the Department of Mechanics and Machine Design in the years 1986–2008.

He was born in 1940 in Czerwonka (Poland), passed away on July 31, 2014. He graduated from Wrocław University of Technology, Faculty of Mechanical Engineering, in 1970, where he also obtained his PhD. In 1981 he was promoted to Associate Professor at Opole University of Technology. In 1992 he became Full Professor at Opole University of Technology. There, he served in the capacity of Vice Rector for science in the years 1984–1987.

He was the initiator and creator of the Scientific School "Lifetime and Reliability of Materials and Structures", by focusing on fatigue tests of structural materials under uniaxial and multiaxial loading. The range of topic includes description of multiaxial random loading, verification of methods for determination of the critical planes in metals and lifetime estimation of structural materials under multiaxial random loading, influence of mean loading values on fatigue life, description of fatigue damage accumulation under random loading and fatigue crack growth in structural components and machine elements, development of stress, strain and energy criteria for random multiaxial fatigue. These criteria form an integral part of algorithms for fatigue life assessment of machine elements and structures. The elaborated algorithms were verified and tested on fatigue machines under uniaxial and multiaxial loading stress states. In the last years, Professor Macha was actively involved in the development of a new field of studies and research – Mechatronics.

Ewald Macha is the author and co-author of more than 300 scientific publications, 18 books, monographs and textbooks, and six patents (one in several countries).

He promoted nine PhDs: two of them became associate professors and one full professor. Professor Macha was a reviewer of 8 doctoral theses, 4 post-doctoral theses and 5 applications for the title of professor.

Some of the honours and awards he received: Gold Cross of Merit, awards of the second and first degree of the Minister of Science and Higher Education, National Education Committee Medal, awards of the Rector of Opole University of Technology (OUTech), and many others.

He was also Coordinator of the Centre of Excellence CESTI (2002–2014) at Opole University of Technology and coordinator of the Scientific – Industrial Consortium for Production and Testing of Materials and Structures for Nuclear Power Plants Building (BEJ) (2010–2014).

Member of: Intersectional Team for Fatigue and Fracture Mechanics of the Committee for Machine Constructing, Polish Academy of Sciences (1987–2014), the Scientific Committee of the journal 'Engineering Machines Problems', Advisory Board of 'The Archive of Mechanical Engineering', 'Strength of Materials', 'Mechanika', European Structural Integrity Society (ESIS), American Society for Testing and Materials (ASTM), Polish Group of Fracture Mechanics, Chairman of the Polish Society of Theoretical and Applied Mechanics – Opole Branch, Chairman of Technical Committee TC3.1 – Multiaxial Fatigue in European Structure Integrity Society (ESIS), member of the Club Encyclopedia Actus Purus, a member of Jure for the nomination of the Ernst Gassner Award, Fraunhofer Institute for Structural Durability LBF, Darmstadt. Chairman of three International Conferences on Mechatronic Systems and Materials (MSM 2006, 2010, 2014) and XV International Colloquium Mechanical Fatigue of Metals (XV-ICMFM 2010), Guest Editor of the Journals Mechanical Systems and Signal Processing and International Journal of Fatigue by Elsevier.

Professor Ewald Macha will remain in our memories as an outstanding scientist, great organizer of science, inspirational personality, dedicated educator and model for a few generations of researchers.

3. Mathematical models and experimental verification

The algorithm of fatigue life calculation for multiaxial loading consists of 6 steps (Fig. 2). Professor Macha's greatest contribution was that related to the estimate of fatigue life for multiaxial and random loading for steps 2 and 3 of the presented algorithm: step 2 concerned the methods for determination of the critical plane position, and step 3 referred to the application of multiaxial fatigue criteria for transformation to uniaxial equivalent loading state.

3.1. Multiaxial fatigue stress criteria

Professor Macha, after the analysis of many existing multiaxial fatigue criteria, noticed that fatigue fracture depends on components of stress or strain in the critical plane. Such remarks allowed him to formulate detailed guidelines for the stress criterion, Eq. (1): (i) fatigue crack is caused by normal stresses, $\sigma_n(t)$, and shear stresses, $\tau_{ns}(t)$, acting in the direction $\vec{s}$ on the critical plane with normal $\vec{n}$, (ii) direction $\vec{s}$ on the plane with normal $\vec{n}$ coincides with the average direction of maximum shear stress. The above conditions are provided by the maximum value of the linear combination of stresses $\sigma_n(t)$ and $\tau_{ns}(t)$ for multiaxial random loading [1,2] in the following form:

$$\max_t [B\tau_{ns}(t) + K\sigma_n(t)] = F, \tag{1}$$

where B, K, F are constants for a given criterion version.

Originally, in this criterion the fracture plane was regarded as the critical plane. However, subsequent analyses made it possible to observe that this plane changes especially for elastic–plastic materials.

The crucial issue is to properly establish the critical plane orientation which is related to the knowledge of the principal stress axes direction. Professor Macha proposed the weight function method, which involved searching averaged positions of principal axes directions through properly selected weight functions W_k. In the case of brittle materials, the principal directions should be averaged through weight functions based on normal stresses. In case of ductile materials, weight functions should depend on shear stresses. Thus, it was necessary to determine principal stress axes, for which direction cosines were used, presented as:

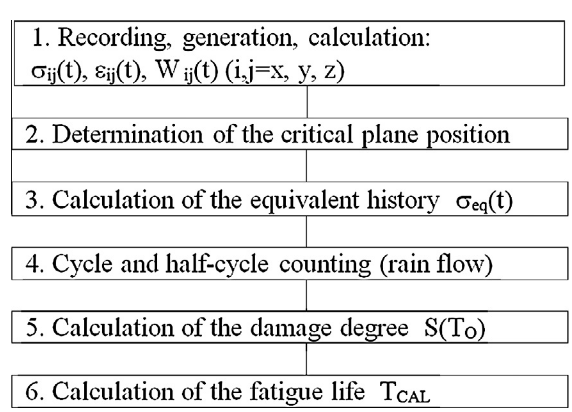

Fig. 2. Algorithm of fatigue life assessment for critical plane approach.

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$$\hat{l}_1 = \cos\frac{1}{W}\Sigma_{k=1}^{L}\alpha_{1k}W_k, \quad \hat{m}_2 = \cos\frac{1}{W}\Sigma_{k=1}^{L}\beta_{2k}W_k,$$
$$\hat{n}_3 = \cos\frac{1}{W}\Sigma_{k=1}^{L}\gamma_{3k}W_k, \tag{2}$$

where $\hat{l}_1, \hat{m}_2, \hat{n}_3$ are direction cosines, $W = \sum_{k=1}^{L} W_k$ is the sum of weights, L is the number of time instants considered, α_1, β_2, γ_3 are the angles between principal stress directions and the axes of the Cartesian reference system.

Another necessary step to determine the critical plane position is to decide the weights to be adopted. Six weight functions are presented in Ref. [1], namely:

– Weight I – $W_k = 1$, that is, each position of the principal stress axes has the same effect on the averaging;
– Weight II

$$W_k = \frac{\sigma_{1k} - \sigma_{1\,\min}}{\sigma_{1\,\max} - \sigma_{1\,\min}}, \tag{3}$$

for $k = 1, 2, \ldots, L$. This weight decreases the impact of the maximum principal stress $\sigma_1(t)$ value on the averaging;

– Weight III

$$W_k = \begin{cases} 0 & \text{for} \quad \sigma_{1k} < a \cdot \sigma_{af} \\ 1 & \text{for} \quad \sigma_{1k} \geqslant a \cdot \sigma_{af} \end{cases} \quad 0 < a \leqslant 1, \tag{4}$$

for $k = 1, 2, \ldots, L$. According to this weight, only those positions of principal axes for which maximum stress value is $\sigma_1(t) \geqslant a \cdot \sigma_{af}$ are averaged;

– Weight IV

$$W_k = \begin{cases} 0 & \text{for} \quad \sigma_{1k} < \nu \cdot \sigma_y \\ 1 & \text{for} \quad \sigma_{1k} \geqslant \nu \cdot \sigma_y \end{cases}, \tag{5}$$

for $k = 1, 2, \ldots, L$. Only those positions of principal axes for which maximum stress value $\sigma_1(t)$ is higher than the product of Poisson's ratio, ν, and yield stress, σ_y are averaged;

– Weight V

$$W_k = \begin{cases} \frac{\sigma_{1k} - a\sigma_{af}}{\sigma_{1k\,\max} - \sigma_{1\,\min}} & \text{for} \quad \sigma_{1k} \geqslant a \cdot \sigma_{af} \\ 0 & \text{for} \quad \sigma_{1k} < a \cdot \sigma_{af} \end{cases}, \tag{6}$$

for $k = 1, 2, \ldots, L$. This weight was developed as a result of combination of weights II and III;

– Weight VI

$$W_k = \begin{cases} \left(\frac{\sigma_{1k}}{a\sigma_{af}}\right)^m & \text{for} \quad \sigma_{1k} \geqslant a \cdot \sigma_{af} \\ 0 & \text{for} \quad \sigma_{1k} < a \cdot \sigma_{af} \end{cases}, \tag{7}$$

for $k = 1, 2, \ldots, L$. In this proposal, only those positions of principal axes, in which $\sigma_1(t)$ at least equal to the product of the fatigue limit and a are taken for averaging. The participation of such positions in averaging exponentially depends on the Wöhler curve slope, m.

However, the selection of proper angles for averaging creates problems, and there are no physical guidelines to decide, which angles should be averaged. The issue of averaging proper angles is discussed in Ref. [7], where direction cosines are dependent on the Euler angles. Matrix of direction cosines defined in this way is expressed in the following form

$$\begin{bmatrix} \cos\phi\cos\theta\cos\psi - \sin\phi\sin\psi & -\cos\phi\cos\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta \\ \sin\phi\cos\theta\cos\psi + \cos\phi\sin\psi & -\sin\phi\cos\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta \\ -\sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta \end{bmatrix}, \tag{8}$$

where ϕ, θ, ψ are Euler's angles (Fig. 3).

Nevertheless, some transformations are required in order to obtain the values of Euler angles. The first step involves calculation of the quantity:

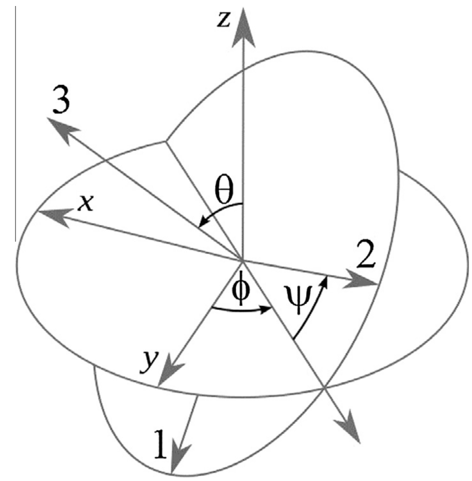

Fig. 3. Description of principal stress directions 1, 2, 3 through the Euler angles ϕ, θ, ψ .

$$\chi = \arccos\frac{1}{2}(l_1 + m_2 + n_3 - 1), \quad u_1 = \frac{m_3 - n_2}{2\sin\chi}, \quad u_2 = \frac{n_1 - l_3}{2\sin\chi},$$
$$u_3 = \frac{l_2 - m_1}{2\sin\chi}, \tag{9}$$

where l_1, m_2, and n_3 are principal direction cosines.

Then, Euler–Rodriguez parameters are used:

$$\lambda = u_1\sin\frac{\chi}{2}, \quad \mu = u_2\sin\frac{\chi}{2}, \quad \kappa = u_3\sin\frac{\chi}{2}, \quad \rho = \cos\frac{\chi}{2} \tag{10}$$

to determine the values of angles:

$$\phi = \mathrm{arctg}\left(\frac{\kappa}{\rho}\right) - \mathrm{arctg}\left(\frac{\lambda}{\mu}\right), \quad \theta = \arcsin\left(\frac{m_3}{\sin\phi}\right),$$
$$\psi = \mathrm{arctg}\left(\frac{\kappa}{\rho}\right) + \mathrm{arctg}\left(\frac{\lambda}{\mu}\right). \tag{11}$$

Euler angles calculated in this way are averaged using the following relationships:

$$\hat{\phi} = \frac{1}{W}\Sigma_{k=1}^{L}\phi(k)W(k), \quad \hat{\theta} = \frac{1}{W}\Sigma_{k=1}^{L}\theta(k)W(k),$$
$$\hat{\psi} = \frac{1}{W}\Sigma_{k=1}^{L}\psi(k)W(k), \tag{12}$$

where W is the sum of weights.Experimental and numerical verification of critical plane position using presented methods were carried out many times, for example by Carpinteri et al. [6,7] and Karolczuk and Macha [15].

Another method to determine the critical plane position, proposed and developed by Macha and Będkowski [16], is the variance method. In this method, the critical plane is assumed to be the plane for which the variance of equivalent stress for the selected criterion reached maximum. Experimental verification of the variance method was carried out in Ref. [17], where results of fatigue life and critical plane position were compared with results determined by the damage accumulation method.

The method of damage accumulation proposed by Professor Macha [15] seems to be the most interesting due to its close relation to the idea of critical plane. In this method, the selected fatigue failure criterion is applied for searching the maximum damage plane, i.e. the plane of the minimum fatigue life. This is an iterative method, so that the search of the critical plane requires multiple repetitions of all the calculation algorithm. In order to compare both methods, three multiaxial fatigue criteria were taken into account:

(a) the criterion of maximum normal stress on the critical plane,

$$\sigma_{eq}(t) = \sigma_x(t) \cdot l_1^2 + \sigma_y(t) \cdot m_1^2 + \sigma_z(t) \cdot n_1^2 + 2\tau_{xy}(t) \cdot l_1 \cdot m_1 + 2\tau_{xz}(t) \cdot l_1 \cdot n_1 + 2\tau_{yz}(t) \cdot m_1 \cdot n_1, \quad (13)$$

where σ_x is the stress along the longitudinal specimen axis, τ_{xy} is the shear stress in the specimen cross-section and l_1, m_1, n_1 are the principal direction cosines,

(b) the criterion of maximum shear stress on the critical plane,

$$\sigma_{eq}(t) = \sigma_x(t) \cdot (l_1^2 - l_3^2) + \sigma_y(t) \cdot (m_1^2 - m_3^2) + \sigma_z(t) \cdot (n_1^2 - n_3^2) + 2 \cdot \tau_{xy}(t)(l_1 \cdot m_1 - l_3 \cdot m_3) + 2 \cdot \tau_{xz}(t) \cdot (l_1 \cdot n_1 - l_3 \cdot n_3) + 2 \cdot \tau_{yz}(t) \cdot (m_1 \cdot n_1 - m_3 \cdot n_3), \quad (14)$$

where l_3, m_3, n_3 are the principal direction cosines

(c) the criterion of maximum normal and shear stress on the critical plane

$$\begin{aligned}\sigma_{eq}(t) = {} & \sigma_x(t) \cdot \left[l_1^2 \cdot \left(2 - \frac{B}{2}\right) - \frac{B \cdot l_3^2}{2}\right] + \sigma_y(t) \\ & \cdot \left[m_1^2 \cdot \left(2 - \frac{B}{2}\right) - \frac{B \cdot m_3^2}{2}\right] + \sigma_z(t) \\ & \cdot \left[n_1^2 \cdot \left(2 - \frac{B}{2}\right) - \frac{B \cdot n_3^2}{2}\right] + \tau_{xy}(t) \\ & \cdot [4 \cdot l_1 \cdot m_1 - B \cdot (l_1 \cdot m_1 + l_3 \cdot m_3)] + \tau_{xz}(t) \\ & \cdot [4 \cdot l_1 \cdot n_1 - B \cdot (l_1 \cdot n_1 + l_3 \cdot n_3)] + \tau_{yz}(t) \\ & \cdot [4 \cdot m_1 \cdot n_1 - B \cdot (m_1 \cdot n_1 + m_3 \cdot n_3)], \end{aligned} \quad (15)$$

where $B = \frac{\sigma_{af}(N_f)}{\tau_{af}(N_f)}$, σ_{af} is the fatigue limit under fully reversed bending, τ_{af} is the fatigue limit under fully reversed torsion, N_f is the number of loading cycles to failure.

The procedure for application of variance methods to assess fatigue life and the critical plane position in the case of the maximum shear stress on critical plane is shown below. According to this criterion, the equivalent stress $\sigma_{eq}(t)$ takes the following form under a biaxial (bending with torsion) loading condition:

$$\sigma_{eq}(t) = \sigma_x(t) \cdot \sin(2\xi) - 2\tau_{xy}(t) \cdot \cos(2\xi), \quad (16)$$

where ξ is the angle determining the critical plane position for plane stress.

Since the equivalent stress $\sigma_{eq}(t)$ (Eq. (16)) is linearly dependent on the stress state components, $\sigma_x(t)$, and, $\tau_{xy}(t)$, it can be also expressed as follows:

$$\sigma_{eq} = \sum_{j=1}^{L} a_j x_j = a_1 x_1 + a_2 x_2, \quad (17)$$

where $a_1 = \sin(2\xi)$, $a_2 = 2\cos(2\xi)$, $x_1 = \sigma_x$, $x_2 = \tau_{xy}$.

From the theory of probability [18] results that the variance of a random variable is a linear function of some random variables, and is expressed by the following formula:

$$\mu_{\sigma eq} = \sum_{j=1}^{L} a_j^2 \mu_{xj} + 2\sum_{j,k=1}^{L} a_j a_k \mu_{xjk} = a_1^2 \mu_{x_1} + a_2^2 \mu_{x_2} + 2a_1 a_2 \mu_{x_1 x_2}, \quad (18)$$

where μ_{x_1} is the variance of the normal stress σ_x, μ_{x_2} is variance of the shear stress τ_{xy}, $\mu_{x_1x_2}$ is the covariance of normal σ_x and shear τ_{xy} stresses, and a_1, a_2 are constants.

Under biaxial random stationary and ergodic stress state, the variances μ_{x_1}, μ_{x_2} and the covariance, $\mu_{x_1x_2}$, are constant.

In the variance method to determination of the critical plane position, the maximum of the function (Eq. (18)) is searched in relation to the angle ξ on which the coefficients a_1 and a_2 depend. After transformation, the variance of the equivalent stress $\mu_{\sigma eq}$ against the angle ξ can be written as follows:

$$\mu_{\sigma eq} = \sin^2(2\xi)\mu_{x_1} + 4\cos^2(2\xi)\mu_{x_2} + 2\sin(4\xi)\mu_{x_1x_2}. \quad (19)$$

An exemplary assessment of the critical plane position for variable-amplitude loading combination of bending with torsion [17] obtained using the variance and damage accumulation methods is shown in Fig. 4. Fig. 4a presents relation between the normalized value of equivalent stress variance and the critical plane angle ξ, and Fig. 4b presents relation between the normalized value of the damage degree and the critical plane angle ξ.

Fig. 5 compares calculated life values and experimental values for variance (Fig. 5a) and damage accumulation (Fig. 5b) method [17], for pseudo-random loading of bending combined with torsion with cross correlation coefficient $r_{\sigma\tau} = 0.16$.

3.2. Multiaxial fatigue strain criteria

Professor Macha formulated a multiaxial fatigue criterion for strains on the critical plane [3] based on the assumption that fatigue fracture is caused by normal strain $\varepsilon_n(t)$ and shear strain $\varepsilon_{ns}(t)$ acting in the direction $\vec{s}$ on the critical plane with normal $\vec{n}$, direction $\vec{s}$ in the fracture plane being coincident with the averaged direction of maximum shear strain. The above conditions are provided by the maximum value of the linear combination of strain $\varepsilon_n(t)$ and $\varepsilon_{ns}(t)$ for multiaxial random loading in the following form:

$$\max_t [b\varepsilon_{ns}(t) + k\varepsilon_n(t)] = q \quad (20)$$

where b, k and q are constants used to select a particular form of Eq. (20).

From a practical point of view, sometimes it is convenient to estimate fatigue life in frequency domain, especially for offshore structures. For this purpose spectral methods are applied, in which the power spectral density of a random signal describes its general frequency structure by means of the spectral density mean square value of this signal. Macha et al. [19] attempted to apply a spectral method to solve this kind of problems. Studies on these issues were continued further in cooperation with Niesłony [20]. A generalised spectral method was formulated for determining fatigue life of materials subjected to multiaxial loading, using the power spectral density function of stress or strain in the frequency domain. The multiaxial stress state is reduced to a uniaxial equivalent stress state, and accumulation of damage is carried out. The accuracy of the results in terms of fatigue life assessed using the proposed spectral method is the same as that employing the time domain – based method (critical plane approach). In both cases the fatigue damage accumulation was performed using Palmgren–Miner hypothesis. Further, determination of expected critical plane position using variance method for the time domain – based method gave results equivalent to those by the spectral method.

3.3. Strain energy density fatigue criteria

Professor Macha also focused his attention on the stress distribution in notches. Similary to Neuber's [21] and Molski–Glinka's [22] relationship, Łagoda–Macha [23] proposed an energy equation to determine the stress notch

$$W_{LM} = \frac{\sigma_a^2}{2E} + \frac{1 - n'}{1 + n'}\sigma_a \left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}}, \quad (21)$$

where W_{LM} is Łagoda–Macha strain energy density, σ_a is stress amplitude, E is Young's modulus, n' is cyclic strain hardening exponent, K' is cyclic strength coefficient.

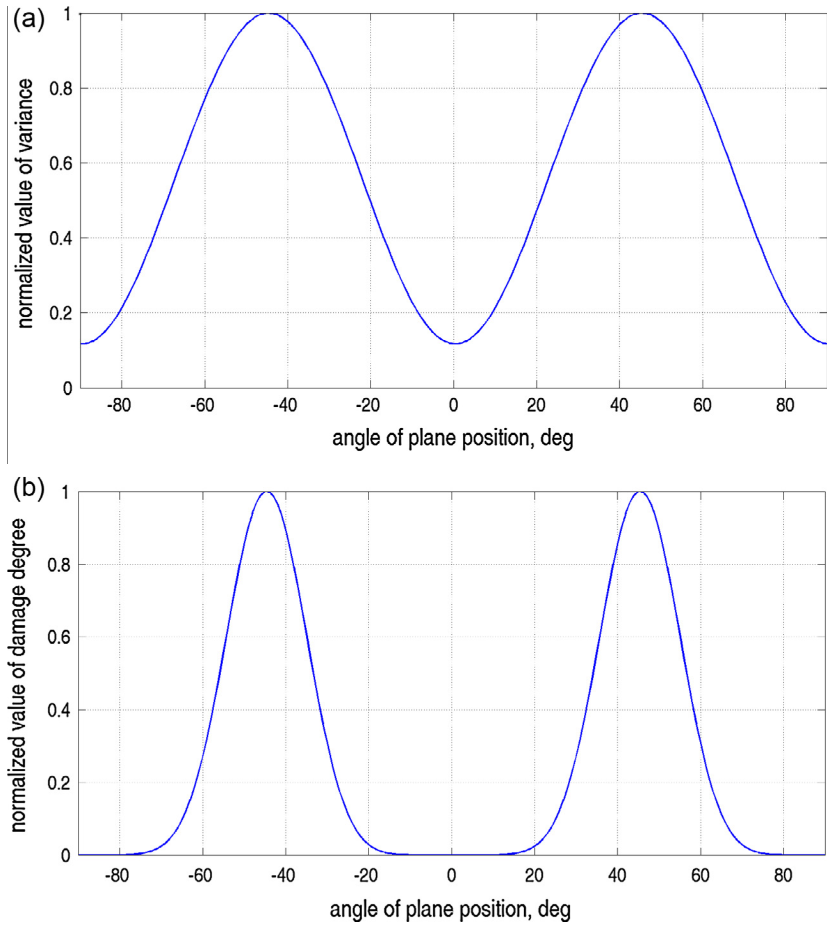

Fig. 4. Dependence of the normalized value of: (a) variance and (b) damage accumulation on the angle ξ of critical plane position.

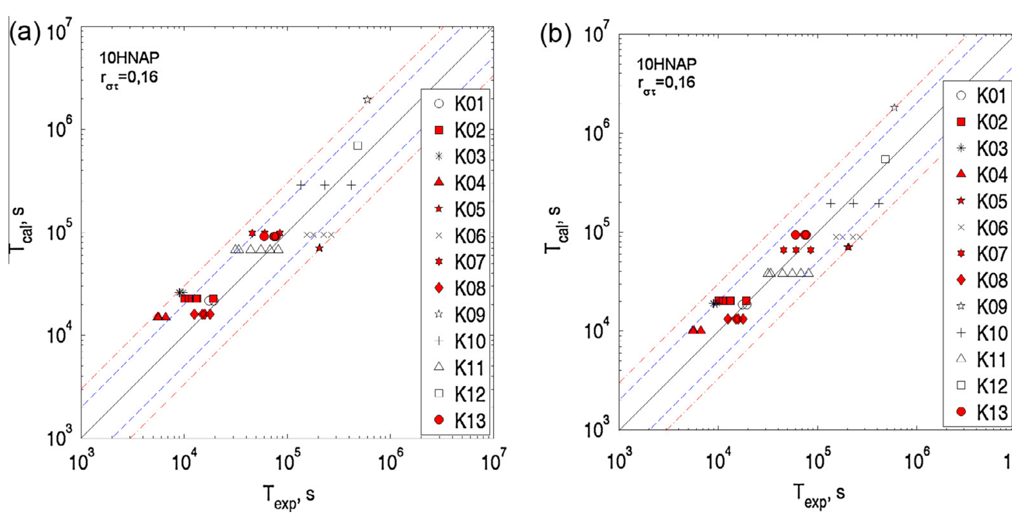

Fig. 5. Comparison of calculated and experimental fatigue lives, with critical planes determined according to: (a) variance and (b) damage accumulation methods; where K01 – λ_σ = 0.189, K02 – λ_σ = 0.274, K03 – λ_σ = 0.358, K04 – λ_σ = 0.442, K05 – λ_σ = 0.214, K06 – λ_σ = 0.309, K07 – λ_σ = 0.405, K08 – λ_σ = 0.500, K09 – λ_σ = 0.394, K10 – λ_σ = 0.515, K11 – λ_σ = 0.636, K12 – λ_σ = 0.68, K13 – λ_σ = 0.84.

Experimental verification proved that the values obtained through this expression are close to the results obtained using Neuber and Molski–Glinka relations. A modified version of Eq. (21) is used to calculate values of maximum stress in case of non-zero mean stress ($\sigma_{max} = \sigma_a + \sigma_m$), and results for all three relations are presented in Table 1 [24].

Professor Macha was most interested in application of energy criteria to lifetime estimation under multiaxial random fatigue. In this field, in cooperation with Łagoda, he proposed a generalised criterion of the normal and shear strain energy density in the critical plane [23,25]:

Table 1
The dependence σ_{max} on K_t for strain energy density models.

K_t	Model		
	Neuber σ_{max} (MPa)	Molski–Glinka σ_{max} (MPa)	Macha–Łagoda σ_{max} (MPa)
9.61	421	386	405
4.30	329	302	316
3.23	294	270	282
1.85	233	215	224

$$\max_t[\beta W_{ns}(t) + \kappa W_n(t)] = Q \text{ or } \max_t[W_{eq}(t)] = Q, \tag{22}$$

where β, κ, Q are constants to be selected for a particular criterion version. Further, W_{ns} is shear strain energy density, W_n is normal strain energy density, W_{eq} is equivalent strain energy density.

Guidelines of the proposed criterion are as follows [15]:

(a) The portion of strain energy density is responsible for fatigue crack, which corresponds to the work of normal stress $\sigma_n(t)$ on normal strain $\varepsilon_n(t)$, that is $W_n(t)$, and work of shear stress $\tau_{ns}(t)$ on shear strain $\varepsilon_{ns}(t)$ in the direction s on critical plane with normal n, that is $W_{ns}(t)$.
(b) The direction s in the critical plane coincides with the average direction, in which shear strain energy density parameter is maximum. Direction of the maximum shear strain energy density parameter is assumed to be the average direction from all directions of $\max_s[W_{ns}(t)]$ occurring in the analyzed time.
(c) For a given fatigue life, strain energy density is determined by the maximum value of linear combination of energy parameters $W_n(t)$ and $W_{ns}(t)$.

For uniaxial stress state, strain energy density parameter is expressed as follows:

$$\begin{aligned} W(t) &= 0.5\sigma(t)\varepsilon(t)\mathrm{sgn}[\sigma(t),\varepsilon(t)] \\ &= 0.5\sigma(t)\varepsilon(t)\frac{\mathrm{sgn}[\sigma(t)] + \mathrm{sgn}[\varepsilon(t)]}{2}, \end{aligned} \tag{23}$$

where sgn(x,y) is two-argument logical function, sensitive to signs of the variables, defined as

$$\mathrm{sgn}[x,y] = \frac{\mathrm{sgn}[x] + \mathrm{sgn}[y]}{2} = \begin{cases} 1 & \text{when } \mathrm{sgn}(x) = \mathrm{sgn}(y) = 1 \\ 0.5 & \text{when } (x = 0 \text{ and } \mathrm{sgn}(y) = 1) \text{ or } (\mathrm{sgn}(x) = 1 \text{ and } y = 0) \\ 0 & \text{when } \mathrm{sgn}(x) = -\mathrm{sgn}(y) \\ -0.5 & \text{when } (x = 0 \text{ and } \mathrm{sgn}(y) = -1) \text{ or } (\mathrm{sgn}(x) = -1 \text{ and } y = 0) \\ -1 & \text{when } \mathrm{sgn}(x) = \mathrm{sgn}(y) = -1 \end{cases} \tag{24}$$

For multiaxial stress state, the course of equivalent strain energy density parameter is calculated in the critical plane with normal n and shear direction s as follows:

$$\begin{aligned} W_{eq}(t) &= \beta W_{ns}(t) + \kappa W_n(t) \\ &= 0.5\kappa\sigma_n(t)\varepsilon_n(t)\mathrm{sgn}[\sigma_n(t),\varepsilon_n(t)] \\ &\quad + 0.5\beta\tau_{ns}(t)\varepsilon_{ns}(t)\mathrm{sgn}[\tau_{ns}(t),\varepsilon_{ns}(t)]. \end{aligned} \tag{25}$$

where W_n, W_{ns} normal and shear strain energy density on the critical plane, respectively.

The proposed energy criterion in the critical plane is applicable for cyclic and random loads for small and large number of loading cycles. Depending on the coefficients chosen, different criteria are obtained:

– when $\beta = 0$ and $\kappa = 1$, the maximum normal strain energy density criterion in the critical plane,
– when $\beta = 1$ and $\kappa = 0$, the maximum shear strain energy density criterion in the critical plane,
– when $\beta = 1$ and $\kappa = 1$, the maximum normal and shear strain energy density criterion in the critical plane.

4. The energy parameter for cyclic unstable materials

Fatigue properties of materials are necessary to calculate the fatigue life of structural components or structures. Such properties are described by formulas for stress (Wöhler, Basquin) and strain (Coffin–Manson), employing conventional techniques of fitting curves to experimental results. In the case of an energy parameter, the Coffin–Manson relationship is used [26–28]. Professor Macha and Słowik proposed a new model to determine energy fatigue characteristics directly from experimental tests. This model (normal strain energy density parameter) is described in Ref. [29]

– for random loading history

$$W(t) = 0.5 \cdot \sigma(t) \cdot |\varepsilon(t) - \varepsilon_i^{pl}|, \tag{26}$$

where $\varepsilon_i^{pl} = \varepsilon(t_i)$ for $\sigma(t_i) = 0$, and $i = 1, 2, 3, \ldots$. are successive numbers of load reversal,
– for constant amplitude loading

$$W_a = 0.5 \cdot \sigma_{exti} \cdot |\varepsilon_{exti} - \varepsilon_i^{pl}|, \tag{27}$$

where ε_{exti}, σ_{exti} are extreme values of strain and stress, respectively, $\varepsilon_i^{pl} = \varepsilon(t_i)$ for $\sigma(t_i) = 0$, and $i = 1, 2, 3, \ldots$. are successive numbers of load reversal.

In order to conduct the tests at a controlled amplitude of strain energy density, W_a, according to Eq. (27), half of the product of the extreme stress (σ_{exti}) and the absolute difference of the extreme strain (ε_{exti}) and the plastic strain (ε_i^{pl}) must be maintained at each load reversal.

Hysteresis loops are shown in Fig. 6, and a procedure to determine the amplitude of the strain energy density parameter can be discussed. The analysis of this procedure begins from point A_1 (ε_{ext1}, σ_{ext1}, Fig. 6). During the first unloading, when the value of the stress is equal to zero (point B_1, Fig. 6), the value of the plastic strain equals $\varepsilon_1^{pl} = \varepsilon_{B1}$. Further unloading causes that the extreme value of strain energy density at point C_1 is achieved, and the stress, total strain and plastic strain are equal to $\varepsilon_{ext1} = \varepsilon_{C1}$, $\sigma_{ext1} = \sigma_{C1}$, $\varepsilon_1^{pl} = \varepsilon_{B1}$, respectively. Then, during loading, the values of stresses and strains increase, and at point B_2 they reach the values $\sigma_{B2} = 0$, $\varepsilon_2^{pl} = \varepsilon_{B2}$. Further loading causes that the extreme value

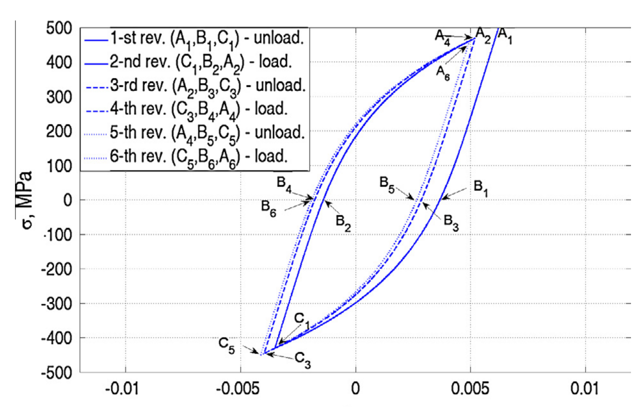

Fig. 6. Example of hysteresis loops (σ–ε) for test with strain energy density amplitude W_a controlled (Eq. (27)).

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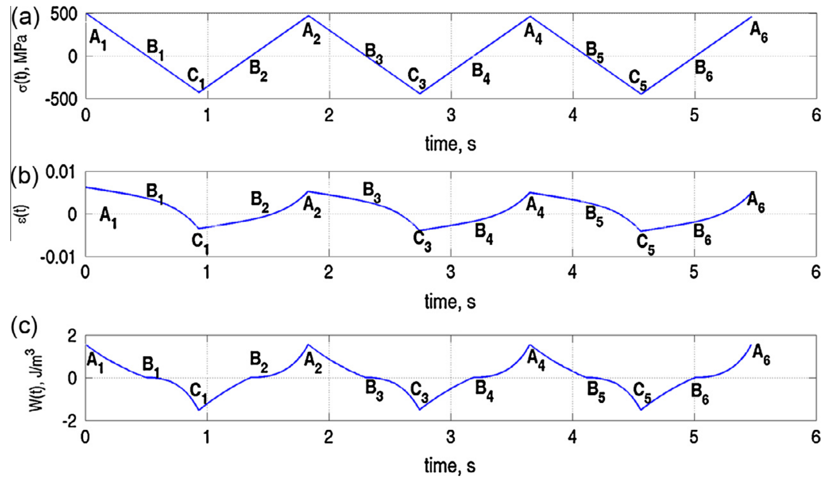

Fig. 7. Example of: stress courses (a), strain courses (b), energy parameter courses (c) for test with controlled strain energy density amplitude W_a (Eq. (27)).

is reached at point A_2, where $\varepsilon_{ext2} = \varepsilon_{A2}$, $\sigma_{ext2} = \sigma_{A2}$, a $\varepsilon_2^{pl} = \varepsilon_{B2}$, etc. Exemplary time histories obtained according to the above description are shown in Fig. 7: $\sigma(t)$ (Fig. 7a), $\varepsilon(t)$ (Fig. 7b) and $W(t)$ (Fig. 7c).

Because the standard fatigue tests stands do not have appropriate control system procedures, Professor Macha created a team to build a mechatronic system that could meet the research needs resulting from Eq. (26) [30]. Energy fatigue characteristics were determined for structural materials (steel and aluminium alloy) on the basis of the proposed model (27). Using the experimentally obtained points (W_a–N_f), the linear approximation of the results was defined by employing a conventional technique, and exemplary results for C45 steel are shown in Fig. 8 [31].

The tests performed at five levels W_a = 0.2, 0.27, 0.3, 0.4, 0.5 MJ/m^3. The energy fatigue characteristic obtained from tests at a controlled strain density amplitude of the energy parameter (Fig. 9, line No. 1) was then compared to a standard fatigue characteristic at a controlled amplitude of the bending moment (Fig. 9, line No. 2). In order to compare both of the characteristics in W_a–N_f reference system, the amplitude of the bending moment was converted by linear elastic model of solid on the amplitude of elastic energy, which corresponded to W_a = 0.25, 0.28, 0.32, 0.36 MJ/m^3. These characteristics are shown in Fig. 9.

A comparison of the obtained characteristics showed differences in fatigue life, particularly for high values of amplitudes. The energy fatigue characteristic obtained directly from tests in the range from 10^4 up to $7 \cdot 10^6$ number of loading cycles to failure proved longer fatigue life than characteristic based on the standard procedures. From the comparison of both characteristics, it can be observed that, in the case of C45 steel, the characteristics converge at low energy parameter value (approx. W_a = 0.25 MJ/m^3), because they have different slope coefficients of fatigue curves. The reason of such a behaviour is not considering plastic strains in tests with controlled amplitude of bending moment.

In the case of random or variable-amplitude loading, the energy parameter is calculated based on stress and strain courses. Fig. 10 illustrates the change in stress (Fig. 10b), strain (Fig. 10c), hysteresis loops (Fig. 10a) and the energy parameter courses (Fig. 10d).

Initially (point 0), all values of stress, strain and the energy parameter are equal to 0 ($\sigma(t_0) = 0$, $\varepsilon(t_0) = 0$, $\varepsilon_0^{pl} = 0$, $W(t_0) = 0$). During loading the parameters increase until they reach maximum values at point A: $\sigma(t_A) = \sigma_A$, $\varepsilon(t_A) = \varepsilon_A$, $\varepsilon_A^{pl} = \varepsilon_0^{pl} = 0$, $W(t_A) = 0.5 \cdot \sigma_A|\varepsilon_A - \varepsilon_0^{pl}| = 0.5 \cdot \sigma_A \cdot \varepsilon_A$. Then during unloading, at point B, stress is equal to 0 and other values are as follows: $\varepsilon(t_B) = \varepsilon_B$, $\varepsilon_B^{pl} = \varepsilon_B$, $W(t_B) = 0.5 \cdot \sigma_B \cdot |\varepsilon_B - \varepsilon_B^{pl}| = 0$. Further unloading leads to point C, where $\sigma(t_C) = \sigma_C$, $\varepsilon(t_C) = \varepsilon_C$, $\varepsilon_C^{pl} = \varepsilon_B^{pl}$, $W(t_C) = 0.5 \cdot \sigma_C \cdot |\varepsilon_C - \varepsilon_B^{pl}|$. Next, in the course of reloading at point D, particular parameters take the following values: $\sigma(t_D) = \sigma_D = 0$, $\varepsilon(t_D) = \varepsilon_D = 0$, $\varepsilon_D^{pl} = \varepsilon_B^{pl}$, $W(t_D) = 0.5 \cdot \sigma_D \cdot |\varepsilon_D - \varepsilon_D^{pl}| = 0$, and at point E $\sigma(t_E) = \sigma_E$, $\varepsilon(t_E) = \varepsilon_E$, $\varepsilon_E^{pl} = \varepsilon_D^{pl}$, $W(t_E) = 0.5 \cdot \sigma_E \cdot |\varepsilon_E - \varepsilon_D^{pl}|$, etc.

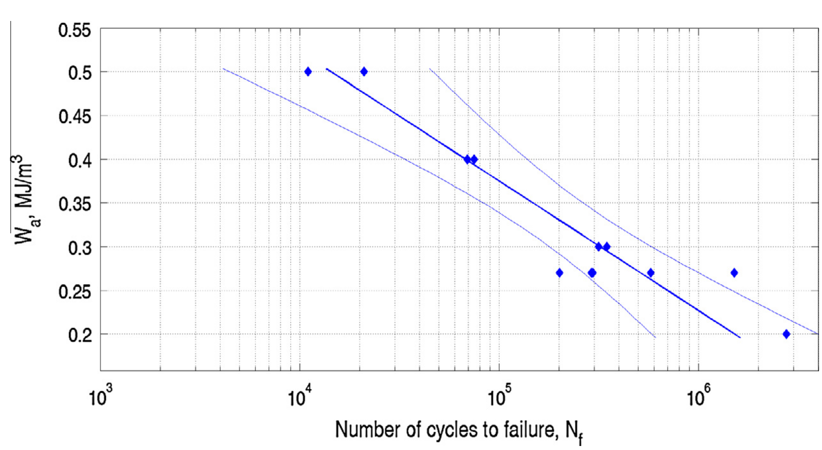

Fig. 8. The energy fatigue characteristic for C45 steel.

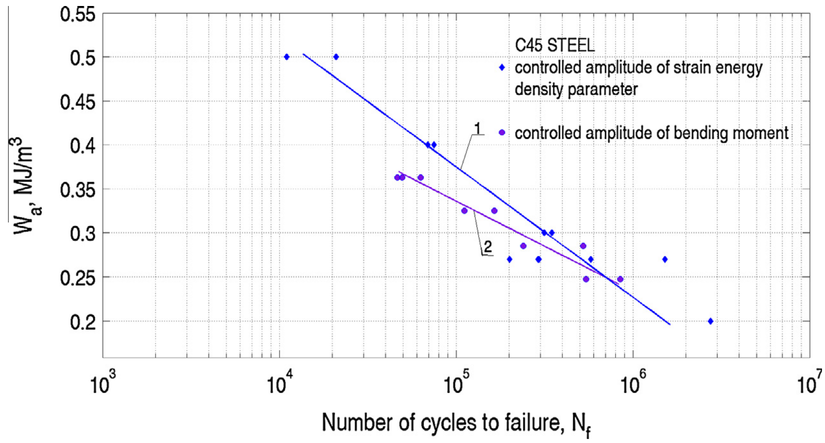

Fig. 9. The fatigue characteristic of C45 steel subjected to bending with a controlled amplitude of the ◆ – energy parameter with Eq. (26), ● – bending moment (elastic energy).

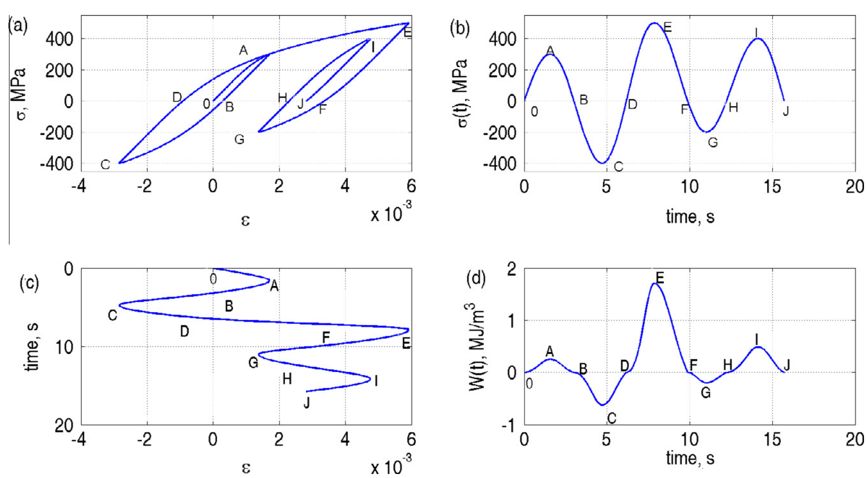

Fig. 10. Example of: (a) hysteresis loops, (b) stress courses, (c) strain courses, and (d) energy parameter courses.

5. Multiaxial fatigue fracture model

As regards development of fatigue cracks, Rozumek and Macha proposed an energy criterion based on parameter J for three crack modes [32,33]. This criterion was verified for Mode I and Mode III [32].

$$\left(\frac{\Delta J_{\mathrm{I}}}{J_{\mathrm{Ic}}}\right)^2 + \left(\frac{\Delta J_{\mathrm{II}}}{J_{\mathrm{IIc}}}\right)^2 + \left(\frac{\Delta J_{\mathrm{III}}}{J_{\mathrm{IIIc}}}\right)^2 = 1, \tag{28}$$

where J_{Ic}, J_{IIc}, J_{IIIc} are critical values for Modes I, II and III.

The criterion (28) was successfully verified by performing tests on several steels and aluminium alloy. The plane specimens of square cross-section made of 18G2A steel and AlCuMg1 aluminium alloy were tested. Different bending (Mode I) to torsion (Mode III) ratio in steel 18G2A is shown in Fig. 11 [34]. It provides grounds to observe shift of experimental points towards increasing value of parameter ΔJ_{I}, except the angle $\alpha = 60°$, where a decrease in these values was confirmed. Considerable increase of ΔJ_{I} and ΔJ_{III} values was observed with rising fatigue crack growth rate (Fig. 11, curves 1 and 2).

The diagrams 1 and 2 shown in Fig. 11 concern fatigue crack growth rates: $\mathrm{d}a/\mathrm{d}N = 1.68 \cdot 10^{-8}$ m/cycle and $\mathrm{d}a/\mathrm{d}N = 4.23 \cdot 10^{-8}$ m/cycle, respectively.

Tests were performed for different ratios of torsional to bending moments ($M_{\mathrm{T}}(t)/M_{\mathrm{B}}(t) = \tan\alpha$). Fig. 12 shows an example of fatigue

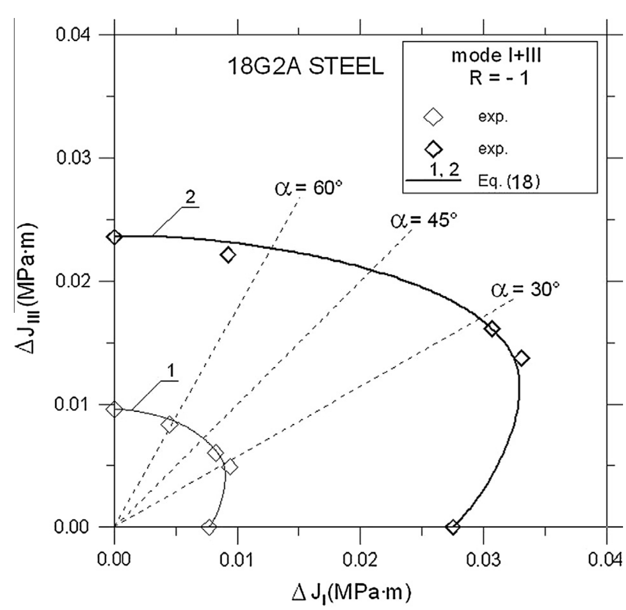

Fig. 11. Comparison between experimental results for different bending to torsion ratios and those calculated according to Eq. (28) for 18G2A steel.

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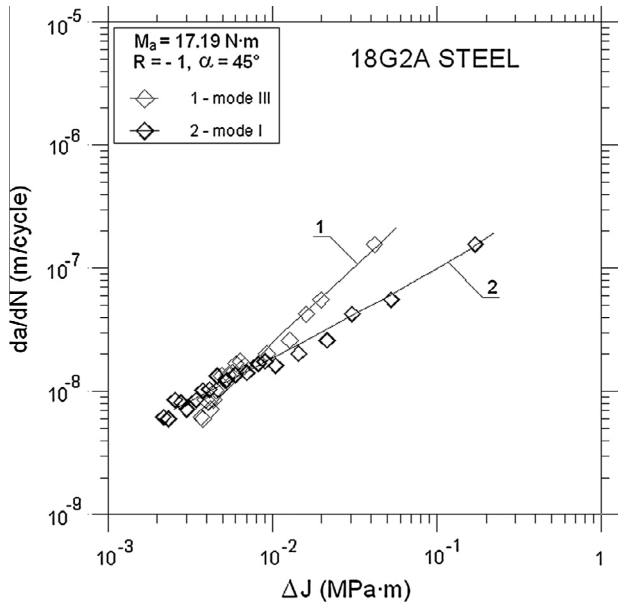

Fig. 12. Fatigue crack growth rate da/dN versus ΔJ, for 18G2A steel and $M_T(t)/M_B(t)$ = 1.

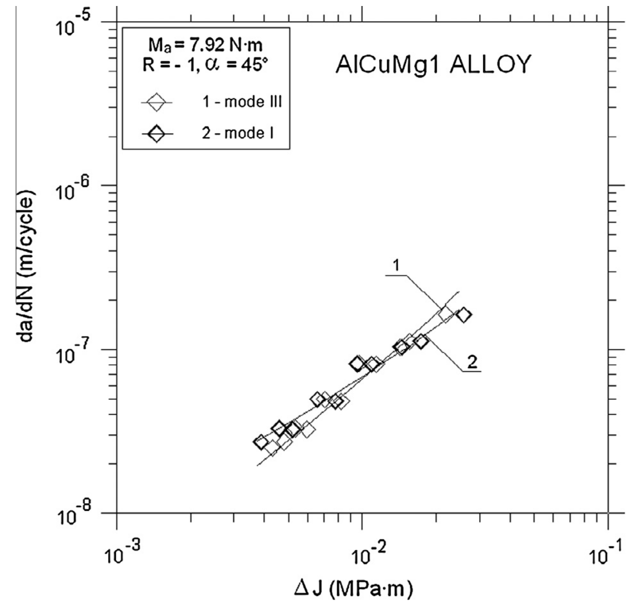

Fig. 14. Fatigue crack growth rate da/dN versus ΔJ, for AlCuMg1 alloy and α = 45°.

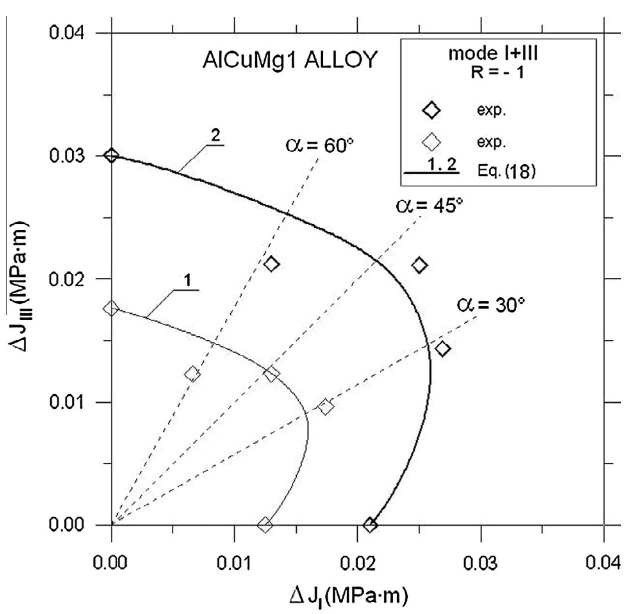

Fig. 13. Comparison between experimental results for different bending to torsion ratios and those calculated according to Eq. (28) for AlCuMg1 alloy.

crack growth rates da/dN versus ΔJ parameter range, for 18G2A steel and mixed modes I + III loading, which was separated in Mode I and Mode III. The crack growth rate for the same value ΔJ and angle α = 45°, is higher for Mode III than for Mode I from the value $\Delta J > 5.97 \cdot 10^{-3}$ MPa m.

Different bending (Mode I) to torsion (Mode III) ratio in AlCuMg1 alloy is shown in Fig. 13 [33], which provides grounds to observe shift of experimental points towards increasing values of parameter ΔJ_I – these increment values were lower than those for steel 18G2A. Experimental results of interdependences between Mode I and Mode III, for constant da/dN ratio value, were defined by Eq. (28). The diagrams 1 and 2 shown in Fig. 12 concern fatigue crack growth rates da/d$N = 7.64 \cdot 10^{-8}$ m/cycle and da/d$N = 1.41 \cdot 10^{-7}$ m/cycle, respectively. In Ref. [35] the experimental relationship between Mode I and Mode III described through the ranges of ΔK is presented. The used parameter ΔK shows a linear relationship between bending and torsion as well as differences in comparison with ΔJ range.

Fig. 14 shows fatigue crack growth rates da/dN versus ΔJ range for AlCuMg1 alloy, load ratio $R = -1$ and mixed modes I + III loading.

For α = 45°, a higher crack growth rate was found for Mode I than for Mode III to the value da/d$N = 8 \cdot 10^{-8}$ m/cycle.

6. Summary

This description of Professor Macha's activity proves his wide interests and great influence on progress regarding the assessment of the fatigue life of machine components and structures. During his academic career, Macha with colleagues proposed many fatigue failure criteria concerning stress, strain and strain energy density parameter in both time and frequency domain. Professor Macha's interests covered initiation stage and propagation of fatigue cracks. These criteria were verified in the case of various load conditions for different materials, and the results were presented during many scientific conferences.

Braun and Macha [12] reviewed and published articles from mechatronics in the journal "Mechanical Systems and Signal Processing". The edition of the journal concerned current research highlighting interdisciplinary aspects of piezoelectric technologies in integrated systems.

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