FIELD EXPERIMENTS

Design, Analysis, and Interpretation

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CHAPTER 2

Causal Inference and Experimentation

Although the logic of experimentation is for the most part intuitive, researchers can run into trouble if they lack a firm grasp of the key assumptions that must be met in order for experiments to provide reliable assessments of cause and effect. This point applies in particular to field experimental researchers, who must frequently make real-time decisions about research design. Failure to understand core statistical principles and their practical implications may cause researchers to squander resources and experimental opportunities. It is wise, therefore, to invest time studying the formal statistical properties of experiments before launching a research project.

This chapter introduces a system of notation that will be used throughout the book. By depicting the outcomes that potentially manifest themselves depending on whether the treatment is administered to each unit, the notation clarifies a number of key concepts, such as the idea of a treatment effect. This notational system is then used to shed light on the conditions under which experiments provide persuasive evidence about cause and effect. The chapter culminates with a list of core assumptions and what they imply for experimental design. The advantage of working methodically from core principles is that a long list of design-related admonitions flows from a relatively compact set of ideas that can be stored in working memory.

2.1 Potential Outcomes

Suppose we seek to gauge the causal effect of a treatment. For concreteness, suppose we wish to study the budgetary consequences of having women, rather than men, head Indian village councils, which govern rural areas in West Bengal and Rajasthan.[1]

1 See Chattopadhyay and Duflo 2004.

What you will learn from this chapter:

1. The system of notation used to describe potential outcomes.
2. Definitions of core terms: average treatment effect, expectation, random assignment, and unbiasedness.
3. Assumptions that must be met in order for experiments to produce unbiased estimates of the average treatment effect.

Students of legislative politics have argued that women bring different policy priorities to the budgetary process in developing countries, emphasizing health issues such as providing clean drinking water. Leave aside for the time being the question of how this topic might be studied using randomly assigned treatments. For the moment, simply assume that each village either receives the treatment (a woman serves as village council head) or remains untreated (with its village council headed by a man). For each village, we also observe the share of the local council budget that is allocated to providing clean drinking water. To summarize, we observe the treatment (whether the village head is a woman or not) and the outcome (what share of the budget goes to a policy issue of special importance to women).

What we do not observe is how the budget in each village headed by a man would have been allocated if it had been headed by a woman, and vice versa. Although we do not observe these counterfactual outcomes, we can nevertheless imagine them. Taking this mental exercise one step further, we might imagine that each village has two *potential outcomes*: the budget it would enact if headed by a woman and the budget it would enact if headed by a man. The gender of the village head determines which potential budget we observe. The other budget remains imaginary or counterfactual.

Table 2.1 provides a stylized example of seven villages in order to introduce the notation that we will use throughout the book. The villages constitute the subjects in this experiment. Each subject is identified by a subscript i, which ranges from 1 to 7. The third village on the list, for example, would be designated as $i = 3$. The table imagines what would happen under two different scenarios. Let $Y_i(1)$ be the outcome if village i is exposed to the treatment (a woman as village head), and let $Y_i(0)$ be the outcome if this village is not exposed to the treatment. For example, Village 3 allocates 30% of its budget to water sanitation if headed by a woman but only 20% if headed by a man, so, $Y_3(1) = 30\%$, and $Y_3(0) = 20\%$. These are called potential outcomes because they describe what would happen if a treatment were or were not administered.

For purposes of this example, we assume that each village has just two potential outcomes, depending on whether it receives the treatment; villages are assumed to be unaffected by the treatments that other villages receive. In section 2.7, we spell out

TABLE 2.1

Illustration of potential outcomes for local budgets when village council heads are women or men. (Entries are shares of local budgets allocated to water sanitation.)

Village i	$Y_i(0)$ *Budget share if village head is male*	$Y_i(1)$ *Budget share if village head is female*	τ_i *Treatment effect*
Village 1	10	15	5
Village 2	15	15	0
Village 3	20	30	10
Village 4	20	15	−5
Village 5	10	20	10
Village 6	15	15	0
Village 7	15	30	15
Average	**15**	**20**	**5**

more precisely the assumptions that underlie the model of potential outcomes and discuss complications that arise when subjects are affected by the treatments that other subjects receive.

2.2 Average Treatment Effects

For each village, the causal effect of the treatment (τ_i) is defined as the difference between two potential outcomes:

$$\tau_i \equiv Y_i(1) - Y_i(0). \tag{2.1}$$

In other words, the treatment effect for each village is the difference between two potential states of the world, one in which the village receives the treatment and another in which it does not. For Village 3, this causal effect is $30 - 20 = 10$.

The empirical challenge that researchers typically face when observing outcomes is that at any given time one can observe $Y_i(1)$ or $Y_i(0)$ but not both. (Bear in mind that the only reason we are able to see both potential outcomes for each village in Table 2.1 is that this is a hypothetical example!) Building on the notational system introduced above, we define Y_i as the observed outcome in each village and d_i as the observed treatment that is delivered in each village. In this case, Y_i is the observed share of the budget allocated to water sanitation, and d_i equals 1 when a woman is village head and 0 otherwise.

BOX 2.1

Potential Outcomes Notation

In this system of notation, the subscript i refers to subjects 1 through N.

The variable d_i indicates whether the ith subject is treated: $d_i = 1$ means the ith subject receives the treatment, and $d_i = 0$ means the ith subject does not receive the treatment. It is assumed that d_i is observed for every subject.

$Y_i(1)$ is the potential outcome if the ith subject were treated. $Y_i(0)$ is the potential outcome if the ith subject were not treated. In general, potential outcomes may be written $Y_i(d)$, where d indexes the treatment. These potential outcomes are fixed attributes of each subject and represent the outcome that would be observed hypothetically if that subject were treated or untreated.

A schedule of potential outcomes refers to a comprehensive list of potential outcomes for all subjects. The rows of this schedule are indexed by i, and the columns are indexed by d. For example, in Table 2.1 the $Y_i(0)$ and $Y_i(1)$ potential outcomes for the fifth subject may be found in adjacent columns of the fifth row.

The connection between the observed outcome Y_i and the underlying potential outcomes is given by the equation $Y_i = d_iY_i(1) + (1 - d_i)Y_i(0)$. This equation indicates that the $Y_i(1)$ are observed for subjects who are treated, and the $Y_i(0)$ are observed for subjects who are not treated. For any given subject, we observe either $Y_i(1)$ or $Y_i(0)$, never both.

It is sometimes useful to refer to potential outcomes for a subset of all subjects. Expressions of the form $Y_i(\cdot) \mid X = x$ denote potential outcomes when the condition $X = x$ holds. For example, $Y_i(0) \mid d_i = 1$ refers to the untreated potential outcome for a subject who actually receives the treatment.

Because we often want to know about the statistical properties of a hypothetical random assignment, we distinguish between d_i, the treatment that a given subject receives (a variable that one observes in an actual dataset), and D_i, the treatment that could be administered hypothetically. D_i is a random variable, and the ith subject might be treated in one hypothetical study and not in another. For example, $Y_i(1) \mid D_i = 1$ refers to the treated potential outcome for a subject who would be treated under some hypothetical allocation of treatments.

The budget that we observe in each village may be summarized using the following expression:

$$Y_i = d_iY_i(1) + (1 - d_i)Y_i(0). \tag{2.2}$$

Because d_i is either 0 or 1, one of the terms on the right side of the equals sign will always be zero. We observe the potential outcome that results from treatment, $Y_i(1)$, if the treatment is administered ($d_i = 1$). If the treatment is not administered ($d_i = 0$), we observe the potential outcome that results when no treatment occurs, $Y_i(0)$.

The average treatment effect, or ATE, is defined as the sum of the τ_i divided by N, the number of subjects:

$$\text{ATE} \equiv \frac{1}{N}\sum_{i=1}^{N}\tau_i. \tag{2.3}$$

An equivalent way to obtain the average treatment effect is to subtract the average value of $Y_i(0)$ from the average value of $Y_i(1)$:

$$\frac{1}{N}\sum_{i=1}^{N} Y_i(1) - \frac{1}{N}\sum_{i=1}^{N} Y_i(0) = \frac{1}{N}\sum_{i=1}^{N}(Y_i(1) - Y_i(0)) = \frac{1}{N}\sum_{i=1}^{N}\tau_i. \tag{2.4}$$

The average treatment effect is an extremely important concept. Villages may have different τ_i, but the ATE indicates how outcomes would change on average if every village were to go from untreated (male village council head) to treated (female village council head).

From the rightmost column of Table 2.1, we can calculate the ATE for the seven villages. The average treatment effect in this example is 5 percentage points: if all villages were headed by men, they would on average spend 15% of their budgets on water sanitation, whereas if all villages were headed by women, this figure would rise to 20%.

BOX 2.2

Definition: Average Treatment Effect

The average treatment effect (ATE) is the sum of the subject-level treatment effects, $Y_i(1) - Y_i(0)$, divided by the total number of subjects. An equivalent way to express the ATE is to say that it equals $\mu_{Y(1)} - \mu_{Y(0)}$, where $\mu_{Y(1)}$ is the average value of $Y_i(1)$ for all subjects and $\mu_{Y(0)}$ is the average value of $Y_i(0)$ for all subjects.

2.3 Random Sampling and Expectations

Suppose that instead of calculating the average potential outcome for all villages, we drew a random sample of villages and calculated the average among the villages we sampled. By *random sample*, we mean a selection procedure in which v villages are selected from the list of N villages, and every possible set of v villages is equally likely to be selected. For example, if we select one village at random from a list of seven villages, seven possible samples are equally likely. If we select three villages at random from a list of seven villages,

$$\frac{N!}{v!(N-v)!} = \frac{7!}{3!4!} = \frac{7\times6\times5\times4\times3\times2\times1}{(3\times2\times1)(4\times3\times2\times1)} = 35 \tag{2.5}$$

possible samples are equally likely. If potential outcomes vary from one village to the next, the average potential outcome in the villages we sample will vary, depending on which of the possible samples we happen to select. The sample average may be characterized as a *random variable*, a quantity that varies from sample to sample.

The term *expected value* refers to the average outcome of a random variable. (See Box 2.3.) In our example, the random variable is the number we obtain when we sample villages at random and calculate their average outcome. Recall from introductory statistics that under random sampling, the expected value of a sample average is equal to the average of the population from which the sample is drawn.[2] This principle may be illustrated using the population of villages depicted in Table 2.1. Recall that the average value of $Y_i(0)$ among all villages in Table 2.1 is 15. Suppose we sample two villages at random from the list of seven villages and calculate the average value of $Y_i(0)$ for the two selected villages. There are

$$\frac{N!}{v!(N-v)!} = \frac{7!}{2!5!} = 21 \tag{2.6}$$

possible ways of sampling two villages at random from a list of seven, and each sample is equally likely to be drawn. Any given sample of two villages might contain an average value of $Y_i(0)$ that is higher or lower than the true average of 15, but the expected value refers to what we would obtain on average if we were to examine all 21 possible samples, for each one calculating the average value of $Y_i(0)$:

$$\{10, 12.5, 12.5, 12.5, 12.5, 12.5, 12.5, 15, 15, 15, 15, 15, 15, 15, 15, 17.5, 17.5, 17.5, 17.5, 17.5, 17.5, 20\}. \tag{2.7}$$

2 The easiest way to see the intuition behind this principle is to consider the case in which we randomly sample just one village. Each village is equally likely to be sampled. The average over all seven possible samples is identical to the average for the entire population of seven villages. This logic generalizes to samples where $v > 1$ because each village appears in exactly $v/7$ of all possible samples.

BOX 2.3

The expectation of a discrete random variable X is defined as

$$E[X] = \sum x \Pr[X = x],$$

where $\Pr[X = x]$ denotes the probability that X takes on the value x, and where the summation is taken over all possible values of x.

For example, what is the expected value of a randomly selected value of τ_i from Table 2.1?

$$\begin{aligned} E[\tau_i] &= \sum \tau \Pr[\tau_i = \tau] \\ &= (-5)\left(\frac{1}{7}\right) + (0)\left(\frac{2}{7}\right) + (5)\left(\frac{1}{7}\right) + (10)\left(\frac{2}{7}\right) + (15)\left(\frac{1}{7}\right) = 5. \end{aligned}$$

Properties of Expectations

The expectation of the constant α is itself: $E[\alpha] = \alpha$.

For a random variable X and constants α and β, $E[\alpha + \beta X] = \alpha + \beta E[X]$.

The expectation of a sum of two random variables, X and Y, is the sum of their expectations: $E[X + Y] = E[X] + E[Y]$.

The expectation of the product of two random variables, X and Y, is the product of their expectations plus the covariance between them: $E[XY] = E[X]E[Y] + E[(X - E[X])(Y - E[Y])]$.

The average of these 21 numbers is 15. In other words, the expected value of the average $Y_i(0)$ obtained from a random sample of two villages is 15.

The concept of expectations plays an important role in the discussion that follows. Because we will refer to expectations so often, a bit more notation is helpful. The notation $E[X]$ refers to the expectation of a random variable X. (See Box 2.3.) The expression "the expected value of $Y_i(0)$ when one subject is sampled at random" will be written compactly as $E[Y_i(0)]$. When a term like $Y_i(0)$ appears in conjunction with an expectations operator, it should be read not as the value of $Y_i(0)$ for subject i but instead as a random variable that is equal to the value of $Y_i(0)$ for a randomly selected subject. When the expression $E[Y_i(0)]$ is applied to values in Table 2.1, the random variable is the random selection of a $Y_i(0)$ from the list of all $Y_i(0)$; since there are seven possible random selections, the average of which is 15, it follows that $E[Y_i(0)] = 15$.

Sometimes attention is focused on the expected value of a random variable within a subgroup. *Conditional expectations* refer to subgroup averages. In terms of notation, the logical conditions following the | symbol indicate the criteria that define the subgroup. For example, the expression "the expectation of $Y_i(1)$ when one village is selected at random from those villages that were treated" is written $E[Y_i(1) \mid d_i = 1]$. The idea of a conditional expectation is straightforward when working with quantities that are in principle observable. More mind-bending are expressions like $E[Y_i(1) \mid d_i = 0]$, which denotes "the expectation of $Y_i(1)$ when one village is selected at random from those villages that were not treated." In the course of conducting research, we will never actually see $Y_i(1)$ for an untreated village, nor will we see $Y_i(0)$ for a treated village. These potential outcomes can be imagined but not observed.

One special type of conditional expectation arises when the subgroup is defined by the outcome of a random process. In that case, the conditional expectation may vary depending on which subjects happened to meet the condition in any particular realization of the random process. For example, suppose that a random process, such as a coin flip, determines which subjects are treated. For a given treatment assignment d_i, we could calculate $E[Y_i(1) \mid d_i = 0]$, but this expectation might have been different had the coin flips come out differently. Suppose we want to know the expected conditional expectation, or how the conditional expectation would come out, on average, across all possible ways that d_i could have been allocated. Let D_i be a random variable that indicates whether each subject would be treated in a hypothetical experiment. The conditional expectation $E[Y_i(1) \mid D_i = 0]$ is calculated by considering all possible realizations of D_i (all the possible ways that N coins could have been flipped) in order to form the joint probability distribution function for $Y_i(1)$ and D_i. As long as we know the joint probability of observing each paired set of values $\{Y(1), D\}$, we can calculate the conditional expectation using the formula in Box 2.4.[3]

With this basic system of notation in place, we may now describe the connection between expected potential outcomes and the average treatment effect (ATE):

$$\begin{aligned} E[Y_i(1) - Y_i(0)] &= E[Y_i(1)] - E[Y_i(0)] \\ &= \frac{1}{N}\sum_{i=1}^{N} Y_i(1) - \frac{1}{N}\sum_{i=1}^{N} Y_i(0) \\ &= \frac{1}{N}\sum_{i=1}^{N}[Y_i(1) - Y_i(0)] \equiv \text{ATE}. \end{aligned} \tag{2.8}$$

3 The notation $E[Y_i(1) \mid D_i = 0]$ may be regarded as shorthand for $E[E[Y_i(1) \mid d_i = 0, \boldsymbol{d}]]$, where $\boldsymbol{d}$ refers to a vector of treatment assignments and d_i refers its ith element. Given $\boldsymbol{d}$, we may calculate the probability distribution function for all $\{Y(1), d\}$ pairs and the expectation given this set of assignments. Then we may take the expectation of this expected value by summing over all possible $\boldsymbol{d}$ vectors.

BOX 2.4

Definition: Conditional Expectation

For discrete random variables Y and X, the conditional expectation of Y given that X takes on the value x is

$$E[Y \mid X = x] = \sum y \Pr[Y = y \mid X = x] = \sum y \frac{\Pr[Y = y, X = x]}{\Pr[X = x]},$$

where $\Pr[Y = y, X = x]$ denotes the joint probability of $Y = y$ and $X = x$, and where the summation is taken over all possible values of y.

For example, in Table 2.1 what is the conditional expectation of a randomly selected value of τ_i, for villages where $Y_i(0) > 10$? This question requires us to describe the joint probability distribution function for the variables τ_i and $Y_i(0)$ so that we can calculate $\Pr[\tau_i = \tau, Y_i(0) > 10]$. Table 2.1 indicates that the $\{\tau, Y(0)\}$ pair $\{0, 15\}$ occurs with probability $2/7$, while the other pairs $\{5, 10\}$, $\{10, 20\}$, $\{-5, 20\}$, $\{10, 10\}$, and $\{15, 15\}$ each occur with probability $1/7$. The marginal distribution of $Y_i(0)$ reveals that 5 of the 7 $Y_i(0)$ are greater than 10, so $\Pr[Y_i(0) > 10] = 5/7$.

$$\begin{aligned} E[\tau_i \mid Y_i(0) > 10] &= \sum \tau \frac{\Pr[\tau_i = \tau, Y_i(0) > 10]}{\Pr[Y_i(0) > 10]} \\ &= (-5)\frac{\frac{1}{7}}{\frac{5}{7}} + (0)\frac{\frac{2}{7}}{\frac{5}{7}} + (5)\frac{0}{\frac{5}{7}} + (10)\frac{\frac{1}{7}}{\frac{5}{7}} + (15)\frac{\frac{1}{7}}{\frac{5}{7}} = 4. \end{aligned}$$

In order to illustrate the idea of a conditional expectation when conditioning on the outcome of a random process, suppose we randomly assign one of the observations in Table 2.1 to treatment ($D_i = 1$) and the remaining six observations to control ($D_i = 0$). If each of the seven possible assignments occurs with probability $1/7$, what is the expected value of a randomly selected τ_i given that $D_i = 1$? Again, we start with the joint probability density function for τ_i and D_i and consider all possible pairings of these two variables' values. The $\{\tau, D\}$ pairings $\{-5, 1\}$, $\{5, 1\}$, and $\{15, 1\}$ occur with probability $1/49$, while the pairings $\{0, 1\}$ and $\{10, 1\}$ occur with probability $2/49$; the remaining $\{\tau, D\}$ pairings are instances in which τ is paired with 0. The marginal distribution $\Pr[D_i = 1] = 3(1/49) + 2(2/49) = 1/7$.

$$\begin{aligned} E[\tau_i \mid D_i = 1] &= \sum \tau \frac{\Pr[\tau_i = \tau, D_i = 1]}{\Pr[D_i = 1]} \\ &= (-5)\frac{\frac{1}{49}}{\frac{1}{7}} + (0)\frac{\frac{2}{49}}{\frac{1}{7}} + (5)\frac{\frac{1}{49}}{\frac{1}{7}} + (10)\frac{\frac{2}{49}}{\frac{1}{7}} + (15)\frac{\frac{1}{49}}{\frac{1}{7}} = 5. \end{aligned}$$

The first line of equation (2.8) expresses the fact that when a village is selected at random from the list of villages, its expected treatment effect is equal to the difference between the expected value of a randomly selected treated potential outcome and the expected value of a randomly selected untreated potential outcome. The second equality in equation (2.8) indicates that the expected value of a randomly selected $Y_i(1)$ equals the average of all $Y_i(1)$ values, and that the expected value of a randomly selected $Y_i(0)$ equals the average of all $Y_i(0)$ values. The third equality reflects the fact that the difference between the two averages in the second line of equation (2.8) can be expressed as the average difference in potential outcomes. The final equality notes that the average difference in potential outcomes is the definition of the average treatment effect. In sum, the difference in expectations equals the difference in average potential outcomes for the entire list of villages, or the ATE.[4]

This relationship is apparent from the schedule of potential outcomes in Table 2.1. The column of numbers representing the treatment effect (τ_i) is, on average, 5. If we were to select villages at random from this list, we would expect their average treatment effect to be 5. We get the same result if we subtract the expected value of a randomly selected $Y_i(0)$ from the expected value of a randomly selected $Y_i(1)$.

2.4 Random Assignment and Unbiased Inference

The challenge of estimating the average treatment effect is that at a given point in time each village is either treated or not: either $Y_i(1)$ or $Y_i(0)$ is observed, but not both. To illustrate the problem, Table 2.2 shows what outcomes would be observed if Village 1 and Village 7 were treated, while the remaining villages were not. We observe $Y_i(1)$ for Villages 1 and 7 but not $Y_i(0)$. For Villages 2, 3, 4, 5, and 6, we observe $Y_i(0)$ but not $Y_i(1)$. The unobserved or "missing" values in Table 2.2 are indicated with a "?".

4 The notation used here is just one way to explicate the link between expectations and the ATE. Samii and Aronow (2012) suggest an alternative formalization. Their model envisions a finite population U consisting of units j in 1, 2, . . . , N, each of which has an associated triple $(y_j(1), y_j(0), D_j')$ such that $y_j(1)$ and $y_j(0)$ are fixed potential outcomes and D_j' is a random variable indicating the treatment status of unit j. Reassign a random index ordering i in 1, 2, . . . , N. Then, for an arbitrary unit i, there exists an associated triple of random variables $(Y_i(1), Y_i(0), D_i)$ such that the random variable $Y_i = D_iY_i(1) + (1 - D_i)Y_i(0)$. It follows that for equation (2.8):

$$E[Y_i(1)] - E[Y_i(0)] = \frac{1}{N}\sum_{j=1}^{N} y_j(1) - \frac{1}{N}\sum_{j=1}^{N} y_j(0) = \text{ATE}.$$

Statistical operators such as expectations or independence refer to random variables associated with an arbitrary index i. Looking ahead to later chapters, one might expand this system to include other unit-level attributes, such as covariates or missingness, by attaching them to the triple indexed by j before reassigning the ordering.

TABLE 2.2

Illustration of observed outcomes for local budgets when two village councils are headed by women.

Village i	*Y(0) Budget share if village head is male*	*Y(1) Budget share if village head is female*	*τ_i Treatment effect*
Village 1	?	15	?
Village 2	15	?	?
Village 3	20	?	?
Village 4	20	?	?
Village 5	10	?	?
Village 6	15	?	?
Village 7	?	30	?
Estimated average based on observed data	**16**	**22.5**	**6.5**

Note: The observed outcomes in this table are based on the potential outcomes listed in Table 2.1.

Random assignment addresses the "missing data" problem by creating two groups of observations that are, in expectation, identical prior to application of the treatment. When treatments are allocated randomly, the treatment group is a random sample of all villages, and therefore the expected potential outcomes among villages in the treatment group are identical to the average potential outcomes among all villages. The same is true for villages in the control group. The control group's expected potential outcomes are also identical to the average potential outcomes among all villages. Therefore, in expectation, the treatment group's potential outcomes are the same as the control group's. Although any given random allocation of villages to treatment and control groups may produce groups of villages that have different average potential outcomes, this procedure is fair in the sense that it does not tend to give one group a higher set of potential outcomes than the other.

As Chattopadhyay and Duflo point out, random assignment is in fact used in rural India to assign women to head one-third of the local village councils.[5] Ordinarily, men would head the village councils, but Indian law mandates that selected

5 Chattopadhyay and Duflo 2004. A lottery is used to assign council positions to women in Rajasthan. In West Bengal, a near-random assignment procedure is used whereby villagers are assigned according to their serial numbers.

villages install a female representative as head of the council. For purposes of illustration, suppose that our collection of seven villages were subject to this law, and that two villages will be randomly assigned female council heads. Consider the statistical implications of this arrangement. This random assignment procedure implies that every village has the same probability of receiving the treatment; assignment bears no systematic relationship to villages' observed or unobserved attributes.

Let's take a closer look at the formal implications of this form of random assignment. When villages are assigned such that every village has the same probability of receiving the treatment, the villages that are randomly chosen for treatment are a random subset of the entire set of villages. Therefore, the expected $Y_i(1)$ potential outcome among treated villages is the same as the expected $Y_i(1)$ potential outcome for the entire set of villages:

$$E[Y_i(1) \mid D_i = 1] = E[Y_i(1)]. \tag{2.9}$$

BOX 2.5

Two Commonly Used Forms of Random Assignment

Random assignment refers to a procedure that allocates treatments with known probabilities that are greater than zero and less than one.

The most basic forms of random assignment allocate treatments such that every subject has the same probability of being treated. Let N be the number of subjects, and let m be the number of subjects who are assigned to the treatment group. Assume that N and m are integers such that $0 < m < N$. Simple random assignment refers to a procedure whereby each subject is allocated to the treatment group with probability m/N. Complete random assignment refers to a procedure that allocates exactly m units to treatment.

Under simple or complete random assignment, the probability of being assigned to the treatment group is identical for all subjects; therefore treatment status is statistically independent of the subjects' potential outcomes and their background attributes ($\boldsymbol{X}$):

$$Y_i(0), Y_i(1), \boldsymbol{X} \perp\!\!\!\perp D_i,$$

where the symbol $\perp\!\!\!\perp$ means "is independent of." For example, if a die roll is used to assign subjects to treatment with probability $1/6$, knowing whether a subject is treated provides no information about the subject's potential outcomes or background attributes. Therefore, the expected value of $Y_i(0)$, $Y_i(1)$, and X_i is the same in treatment and control groups.

When we randomly select villages into the treatment group, the villages we leave behind for the control group are also a random sample of all villages. The expected $Y_i(1)$ in the control group ($D_i = 0$) is therefore equal to the expected $Y_i(1)$ for the entire set of villages:

$$E[Y_i(1) \mid D_i = 0] = E[Y_i(1)]. \tag{2.10}$$

Putting equations (2.9) and (2.10) together, we see that under random assignment the treatment and control groups have the same expected potential outcome:

$$E[Y_i(1) \mid D_i = 1] = E[Y_i(1) \mid D_i = 0]. \tag{2.11}$$

Equation (2.11) also underscores the distinction between realized and unrealized potential outcomes. On the left side of the equation is the expected treated potential outcome among villages that receive the treatment. The treatment causes this potential outcome to become observable. On the right side of the equation is the expected treated potential outcome among villages that do not receive the treatment. Here, the lack of treatment means that the treated potential outcome remains unobserved for these subjects.

The same logic applies to the control group. Villages that do not receive the treatment ($D_i = 0$) have the same expected untreated potential outcome $Y_i(0)$ that the treatment group ($D_i = 1$) would have if it were untreated:

$$E[Y_i(0) \mid D_i = 0] = E[Y_i(0) \mid D_i = 1] = E[Y_i(0)]. \tag{2.12}$$

Equations (2.11) and (2.12) follow from random assignment: D_i conveys no information whatsoever about the potential values of $Y_i(1)$ or $Y_i(0)$. The randomly assigned values of D_i determine which value of Y_i we actually *observe*, but they are nevertheless statistically independent of the *potential* outcomes $Y_i(1)$ and $Y_i(0)$. (See Box 2.5 for discussion of the term *independence*.)

When treatments are assigned randomly, we may rearrange equations (2.8), (2.11), and (2.12) in order to express the average treatment effect as

$$\text{ATE} = E[Y_i(1) \mid D_i = 1] - E[Y_i(0) \mid D_i = 0]. \tag{2.13}$$

This equation suggests an empirical strategy for estimating the average treatment effect. The terms $E[Y_i(1) \mid (D_i = 1)]$ and $E[Y_i(0) \mid (D_i = 0)]$ may be estimated using experimental data. We do not observe the $Y_i(1)$ potential outcomes for all observations, but we do observe them for the random sample of observations that receive the treatment. Similarly, we do not observe the $Y_i(0)$ potential outcomes for all observations, but we do observe them for the random sample of observations in the control group. If we want to estimate the average treatment effect, equation (2.13) suggests that we should take the difference between two sample means: the average

outcome in the treatment group minus the average outcome in the control group. Ideas that enable researchers to use observable quantities (e.g., sample averages) to reveal parameters of interest (e.g., average treatment effects) are termed *identification* strategies.

Statistical procedures used to make guesses about parameters such as the average treatment effect are called *estimators*. In this example, the estimator is very simple, just a difference between two sample averages. Before applying an estimator to actual data, a researcher should reflect on its statistical properties. One especially important property is *unbiasedness*. An estimator is unbiased if it generates the right answer, on average. In other words, if the experiment were replicated an infinite number of times under identical conditions, the average *estimate* would equal the true parameter. Some guesses may be too high and others too low, but the average guess will be correct. In practice, we will not be able to perform an infinite number of experiments. In fact, we might just perform one experiment and leave it at that. Nevertheless, in theory we can analyze the properties of our estimation procedure to see whether, on average, it recovers the right answer. (In the next chapter, we consider another property of estimators: how precisely they estimate the parameter of interest.)

In sum, when treatments are administered using a procedure that gives every subject the same probability of being treated, potential outcomes are independent of the treatments that subjects receive. This property suggests an identification strategy for estimating average treatment effects using experimental data.

The remaining task is to demonstrate that the proposed estimator—the difference between the average outcome in the treatment group and the average outcome in the control group—is an unbiased estimator of the ATE when all subjects have the same probability of being treated. The proof is straightforward. Because the units assigned to the control group are a random sample of all units, the average of the control group outcomes is an unbiased estimator of the average value of $Y_i(0)$

BOX 2.6

Definition: Estimator and Estimate

An estimator is a procedure or formula for generating guesses about parameters such as the average treatment effect. The guess that an estimator generates based on a particular experiment is called an estimate. Estimates are denoted using a "hat" notation. The estimate of the parameter θ is written $\hat{\theta}$.

among all units. The same goes for the treatment group: the average outcome among units that receive the treatment is an unbiased estimator of the average value of $Y_i(1)$ among all units. Formally, if we randomly shuffle the villages and place the first m subjects in the treatment group and the remaining $N - m$ subjects in the control group, we can analyze the expected, or average, outcome over all possible random assignments:

$$
\begin{aligned}
E\left[\frac{\sum_1^m Y_i}{m} - \frac{\sum_{m+1}^N Y_i}{N-m}\right] &= \overbrace{E\left[\frac{\sum_1^m Y_i}{m}\right]}^{\text{Average outcome among treated units}} - \overbrace{E\left[\frac{\sum_{m+1}^N Y_i}{N-m}\right]}^{\text{Average outcome among untreated units}} \\
&= \frac{E[Y_1] + E[Y_2] + \cdots + E[Y_m]}{m} - \frac{E[Y_{m+1}] + E[Y_{m+2}] + \cdots + E[Y_N]}{N-m} \\
&= E[Y_i(1) \mid D_i = 1] - E[Y_i(0) \mid D_i = 0] \\
&= E[Y_i(1)] - E[Y_i(0)] = E[\tau_i] = \text{ATE}. \qquad (2.14)
\end{aligned}
$$

Equation (2.14) conveys a simple but extremely useful idea. When units are randomly assigned, a comparison of average outcomes in treatment and control groups (the so-called *difference-in-means estimator*) is an unbiased estimator of the average treatment effect.

BOX 2.7

Definition: Unbiased Estimator

An estimator is unbiased if the expected value of the estimates it produces is equal to the true parameter of interest. Call θ the parameter we seek to estimate, such as the ATE. Let $\hat{\theta}$ represent an estimator, or procedure for generating estimates. For example, $\hat{\theta}$ may represent the difference in average outcomes between treatment and control groups. The expected value of this estimator is the average estimate we would obtain if we apply this estimator to all possible realizations of a given experiment or observational study. We say that $\hat{\theta}$ is unbiased if $E(\hat{\theta}) = \theta$; in words, the estimator $\hat{\theta}$ is unbiased if the expected value of this estimator is θ, the parameter of interest. Although unbiasedness is a property of estimators and not estimates, we refer to the estimates generated by an unbiased estimator as "unbiased estimates."

2.5 The Mechanics of Random Assignment

The result in equation (2.14) hinges on random assignment, and so it is important to be clear about what constitutes random assignment. *Simple random assignment* is a term of art, referring to a procedure—a die roll or coin toss—that gives each subject an identical probability of being assigned to the treatment group. The practical drawback of simple random assignment is that when N is small, random chance can create a treatment group that is larger or smaller than what the researcher intended. For example, you could flip a coin to assign each of 10 subjects to the treatment condition, but there is only a 24.6% chance of ending up with exactly 5 subjects in treatment and 5 in control. A useful special case of simple random assignment is *complete random assignment*, where exactly m of N units are assigned to the treatment group with equal probability.[6]

The procedure used to conduct complete random assignment can take any of three equivalent forms. Suppose one has N subjects and seeks to assign treatments to m of them. The first method is to select one subject at random, then select another at random from the remaining units, and so forth until you have selected m subjects into the treatment group. A second method is to enumerate all of the possible ways that m subjects may be selected from a list of N subjects, and randomly select one of the possible allocation schemes. A third method is to randomly permute the order of all N subjects and label the first m subjects as the treatment group.[7]

Beware of the fact that *random* is a word that is used loosely in common parlance to refer to procedures that are arbitrary, haphazard, or unplanned. The problem is that arbitrary, haphazard, or unplanned treatments may follow systematic patterns that go unnoticed. Procedures such as alternation are risky because there may be systematic reasons why certain types of subjects might alternate in a sequence, and indeed, some early medical experiments ran into exactly this problem.[8] We use the term *random* in a more exacting sense. The physical or electronic procedure by which randomization is conducted ensures that assignment to the treatment group is statistically independent of all observed or unobserved variables.

6 In Chapters 3 and 4, we discuss other frequently used methods of random assignment: clustered random assignment, where groups of subjects are randomly assigned to treatment and control, and block random assignment (also called stratified random assignment), where individuals are first divided into blocks, and then random assignment is performed within each block. Box 2.5 notes that a defining feature of complete (as opposed to clustered or blocked) random assignment is that all possible assignments of N subjects to a treatment group of size m are equally likely.

7 Cox and Reid 2000, p. 20. The term *complete randomization* is a bit awkward, as the word *complete* does not convey the requirement that exactly m units are allocated to treatment, but this terminology has become standard (see Rosenbaum 2002, pp. 25–26).

8 Hróbjartsson, Gøtzsche, and Gluud 1998.

In practical terms, random assignment is best done using statistical software. Here is an easy procedure for implementing complete random assignment. First, determine N, the number of subjects in your experiment, and m, the number of subjects who will be allocated to the treatment group. Second, set a random number "seed" using a statistics package, so that your random numbers may be reproduced by anyone who cares to replicate your work. Third, generate a random number for each subject. Fourth, sort the subjects by their random numbers in ascending order. Finally, classify the first m observations as the treatment group. Example programs using R may be found at http://isps.research.yale.edu/FEDAI.

Generating random numbers is just the first step in implementing random assignment. After the numbers are generated, one must take pains to preserve the integrity of the assignment process. A deficiency of alternation and many other arbitrary procedures is that they allow those administering the allocation to foresee who will be assigned to which experimental group. If a receptionist seeks to get the sickest patients into the experimental treatment group and knows that the pattern of assignments alternates, he can reorder the patients in such a way as to shuttle the sickest subjects into the treatment group.[9] The same concern arises even when a random sequence of numbers is used to assign incoming patients: random allocation may be undone if the receptionist knows the order of assignments ahead of time, because that enables him to position patients so that they will be assigned to a certain experimental group. In order to guard against potential threats to the integrity of random assignment, researchers should build extra procedural safeguards into their experimental designs, such as blinding those administering the experiment to the subjects' assigned experimental groups.

2.6 The Threat of Selection Bias When Random Assignment Is Not Used

Without random assignment, the identification strategy derived from equation (2.14) unravels. The treatment and control groups are no longer random subsets of all units in the sample. Instead, we confront what is known as a *selection problem*: receiving treatment may be systematically related to potential outcomes. For example, absent random assignment, villages determine whether their councils are headed by women. The villages that end up with female council heads may not be a random subset of all villages.

9 For examples of experiments in which random assignment was subverted, see Torgerson and Torgerson 2008.

To see how nonrandom selection jeopardizes the identification strategy of comparing average outcomes in the treatment and control groups, rewrite the expected difference in outcomes from equation (2.13) by subtracting and adding $E[Y_i(0)\,|\,D_i = 1]$:

$$\underbrace{E[Y_i(1)\,|\,D_i = 1] - E[Y_i(0)\,|\,D_i = 0]}_{\text{Expected difference between treated and untreated outcomes}}$$

$$= \underbrace{E[Y_i(1) - Y_i(0)\,|\,D_i = 1]}_{\text{ATE among the treated}} + \underbrace{E[Y_i(0)\,|\,D_i = 1] - E[Y_i(0)\,|\,D_i = 0]}_{\text{Selection bias}}. \quad (2.15)$$

Under random assignment, the selection bias term is zero, and the ATE among the (randomly) treated villages is the same as the ATE among all villages. In the absence of random assignment, equation (2.15) warns that the apparent treatment effect is a mixture of selection bias and the ATE for a subset of villages.

In order to appreciate the implications of equation (2.15), consider the following scenario. Suppose that instead of randomly selecting villages to receive the treatment, our procedure were to let villages decide whether to take the treatment. Refer back to Table 2.1 and imagine that, if left to their own devices, Village 5 and Village 7 always elect a woman due to villagers' pent-up demand for water sanitation, while the remaining villages always elect a man.[10] Self-selection in this case leads to an exaggerated estimate of the ATE because receiving the treatment is associated with lower-than-average values of $Y_i(0)$ and higher-than-average values of $Y_i(1)$. The average outcome in the treatment group is 25, and the average outcome in the control group is 16. The estimated ATE is therefore 9, whereas the actual ATE is 5. Referring to equation (2.15) we see that in this case the ATE among the treated is not equal to the ATE for the entire subject pool, nor is the selection bias term equal to zero. The broader point is that it is risky to compare villages that choose to receive the treatment with villages that choose not to. In this example, self-selection is related to potential outcomes; as a result, the comparison of treated and untreated villages recovers neither the ATE for the sample as a whole nor the ATE among those villages that receive treatment.

The beauty of experimentation is that the randomization procedure generates a schedule of treatment and control assignments that are statistically independent of

10 When taking expectations over hypothetical replications of an experiment, we consider all possible random assignments. In our example of non-random allocation, however, nature makes the assignment. When taking expectations, we must therefore consider the average of all possible natural assignments. Rather than make up an assortment of possible assignments and stipulate the probability that each scenario occurs, we have kept the example as simple as possible and assumed that the villages "always" elect the same type of candidate. In effect, we are taking expectations over just one possible assignment that occurs with probability 1.

potential outcomes. In other words, the assumptions underlying equations (2.9) to (2.13) are justified by reference to the *procedure* of random assignment, not substantive arguments about the comparability of potential outcomes in the treatment and control groups.

The preceding discussion should not be taken to imply that experimentation invokes no substantive assumptions. The unbiasedness of the difference-in-means estimator hinges not only on random assignment but also on two assumptions about potential outcomes, the plausibility of which will vary depending on the application. The next section spells out these important assumptions.

2.7 Two Core Assumptions about Potential Outcomes

To this point, our characterization of potential outcomes has glossed over two important details. In order to ease readers into the framework of potential outcomes, we simply stipulated that each subject has two potential outcomes, $Y_i(1)$ if treated and $Y_i(0)$ if not treated. To be more precise, each potential outcome depends *solely* on whether the subject *itself* receives the treatment. When writing potential outcomes in this way, we are assuming that potential outcomes respond only to the treatment and not some other feature of the experiment, such as the way the experimenter assigns treatments or measures outcomes. Furthermore, potential outcomes are defined over the set of treatments that the subject itself receives, not the treatments assigned to other subjects. In technical parlance, the "solely" assumption is termed *excludability* and the "itself" assumption is termed *non-interference*.

2.7.1 Excludability

When we define two, and only two, potential outcomes based on whether the treatment is administered, we implicitly assume that the only relevant causal agent is receipt of the treatment. Because the point of an experiment is to isolate the causal effect of the treatment, our schedule of potential outcomes excludes from consideration factors other than the treatment. When conducting an experiment, therefore, we must define the treatment and distinguish it from other factors with which it may be correlated. Specifically, we must distinguish between d_i, the treatment, and z_i, a variable that indicates which observations have been allocated to treatment or control. We seek to estimate the effect of d_i, and we assume that the treatment assignment z_i has no effect on outcomes except insofar as it affects the value of d_i.

The term *exclusion restriction* or *excludability* refers to the assumption that z_i can be omitted from the schedule of potential outcomes for $Y_i(1)$ and $Y_i(0)$. Formally, this

assumption may be written as follows. Let $Y_i(z, d)$ be the potential outcome when $z_i = z$ and $d_i = d$, for $z \in (0, 1)$ and for $d \in (0, 1)$. For example, if $z_i = 1$ and $d_i = 1$, the subject is assigned to the treatment group and receives the treatment. We can also envision other combinations. For example, if $z_i = 1$ and $d_i = 0$, the subject is assigned to the treatment group but for some reason does not receive the treatment. The exclusion restriction assumption is that $Y_i(1, d) = Y_i(0, d)$. In other words, potential outcomes respond only to the input from d_i; the value of z_i is irrelevant. Unfortunately, this assumption cannot be verified empirically because we never observe both $Y_i(1, d)$ and $Y_i(0, d)$ for the same subject.

The exclusion restriction breaks down when random assignment sets in motion causes of Y_i other than the treatment d_i. Suppose the treatment in our running example were defined as whether or not a woman council head presides over deliberations about village priorities. Our ability to estimate the effect of this treatment would be jeopardized if nongovernmental aid organizations, sensing that newly elected women will prioritize clean water, were to redirect their efforts to promote water sanitation to male-led villages. If outside aid flows to male-led villages, obviating the need for male village council leaders to allocate their budgets to water sanitation, the apparent difference between water sanitation budgets in councils led by women and councils led by men will exaggerate the true effect of the treatment, as defined above.[11] Even if it were the case that women council leaders have no effect on their own villages' budgets, the behavior of the NGOs could generate different average budgets in male- and female-led villages.

Asymmetries in measurement represent another threat to the excludability assumption. Suppose, for example, that in our study of Indian villages, we were to dispatch one group of research assistants to measure budgets in the treatment group and a different group of assistants to measure budgets in the control group. Each group of assistants may apply a different standard when determining what expenditures are to be classified as contributing to water sanitation. Suppose the research assistants in the treatment group were to use a more generous accounting standard—they tend to exaggerate the amount of money that the village allocates to water sanitation. When we compare average budgets in the treatment and control groups, the estimated treatment effect will be a combination of the true effect of female village heads on budgets and accounting procedures that exaggerate the amount of money spent on water sanitation in those villages. Presumably, when we envisioned the experiment and what we might learn from it, we sought to estimate only the first of these two effects. We wanted to know the effect of female leadership on budgets using a consistent standard of accounting.

11 Whether an excludability violation occurs depends on how a treatment effect is defined. If one were to define the effect of electing a woman to include the compensatory behavior of NGOs, this assumption would no longer be violated.

To illustrate the consequences of measurement asymmetry, we may write out a simple model in which outcomes are measured with error. Under this scenario, the usual schedule of potential outcomes expands to reflect the fact that outcomes are influenced not only by d_i, but also by z_i, which determines which set of research assistants measure the outcome. Suppose that among untreated units we observe $Y_i(0)^* = Y_i(0) + e_{i0}$, where e_{i0} is the error that is made when measuring the potential outcome if an observation is assigned to the control group. For treated units, let $Y_i(1)^* = Y_i(1) + e_{i1}$. What happens if we compare average outcomes among treated and untreated units? The expected value of the difference-in-means estimator from equation (2.14) is

$$E\left[\frac{\sum_1^m Y_i}{m} - \frac{\sum_{m+1}^N Y_i}{N-m}\right] = E[Y_i(1)^* \mid D_i = 1] - E[Y_i(0)^* \mid D_i = 0]$$

$$= E[Y_i(1) \mid D_i = 1] + E[e_{i1} \mid D_i = 1] - E[Y_i(0) \mid D_i = 0] - E[e_{i0} \mid D_i = 0]. \quad (2.16)$$

Comparing equation (2.16) to equation (2.14) reveals that the difference-in-means estimator is biased when the measurement errors in the treated and untreated groups have different expected values:

$$E[e_{i1} \mid D_i = 1] \neq E[e_{i0} \mid D_i = 0]. \quad (2.17)$$

In this book, when we speak of a "breakdown in symmetry," we have in mind procedures that may distort the expected difference between treatment and control outcomes.

What kinds of experimental procedures bolster the plausibility of the excludability assumption? The broad answer is anything that helps ensure uniform handling of treatment and control groups. One type of procedure is double-blindness—neither the subjects nor the researchers charged with measuring outcomes are aware of which treatments the subjects receive, so that they cannot consciously or unconsciously distort the results. Another procedure is parallelism in the administration of an experiment: the same questionnaires and survey interviewers should be used to assess outcomes in both treatment and control groups, and both groups' outcomes should be gathered at approximately the same time and under similar conditions. If outcomes for the control group are gathered in October, but outcomes in the treatment group are gathered in November, symmetry may be jeopardized.

The exclusion restriction cannot be evaluated unless the researcher has stated precisely what sort of treatment effect the experiment is intended to measure and designed the experiment accordingly. Depending on the researcher's objective, the control group may receive a special type of treatment so that the treatment vs. control comparison isolates a particular aspect of the treatment. A classic example of a research design that attempts to isolate a specific cause is a pharmaceutical trial in

which an experimental pill is administered to the treatment group while an identical sugar pill is administered to the control group. The aim of administering a pill to both groups is to isolate the pharmacological effects of the ingredients, holding constant the effect of merely taking some sort of pill. In the village council example, a researcher may wish to distinguish the effects of female leadership of local councils from the effects of merely appointing non-incumbents to the headship. In principle, one could compare districts with randomly assigned women heads to districts with randomly assigned term limits, a policy that has the effect of bringing non-incumbents into leadership roles. This approach to isolating causal mechanisms is revisited again in Chapter 10, where we discuss designs that attempt to differentiate the active ingredients in a multifaceted treatment.

Protecting the theoretical integrity of the treatment vs. control comparison is of paramount importance in experimental design. In the case of the village budget study, the aim is to estimate the budgetary consequences of having a randomly allocated female village head, not the consequences of using a different measurement standard to evaluate outcomes in treatment and control villages. The same argument goes for other aspects of research activity that might be correlated with treatment assignment. For example, if the aim is to measure the effect of female leadership on budgets per se, bias may be introduced if one sends a delegation of researchers to monitor village council deliberations in women-headed villages only. Now the observed treatment effect is a combination of the effect of female leadership and the effect of research observers. Whether one regards the presence of the research delegation as a distortion of measurement or an unintended pathway by which assignment to treatment affects the outcome, the formal structure of the problem remains the same. The expected outcome of the experiment no longer reveals the causal effect we set out to estimate.

The symmetry requirement does not rule out cross-cutting treatments. For example, one could imagine a version of India's reservation policy that randomly assigned some village council seats to women, others to people from lower castes, and still others to women from lower castes. When we discuss factorial designs in Chapter 9, we will stress what can be learned from deploying several treatments in combination with one another. The point of these more complex designs is to learn about combinations of treatments while still preserving symmetry: randomly assigning treatments both alone and in combination with one another allows the researcher to distinguish empirically between having a female village head and having a female village head who is also from a lower caste.

Finally, let's revisit the case in which other actors intervene in response to your treatment assignments. For example, suppose that in anticipation of greater spending on water sanitation, interest groups devote special attention to lobbying village councils headed by women. Or it may go the other way: interest groups focus greater efforts on villages headed by men because they believe that's where they will meet the most resistance from budget makers. Whether interest group interference violated

the assumption of excludability depends on how we define the treatment effect. Interest group activity presents no threat to the exclusion restriction if we define the effect of installing a female council head to include all of the indirect repercussions that it could have on interest group activity. If, however, we seek to estimate the specific effect of having female council heads without any interference by interest groups, our experimental design may be inadequate unless we can find a way to prevent interest groups from responding strategically. These kinds of scenarios again underscore the importance of clearly stating the experimental objectives so that researchers and readers can assess the plausibility of the exclusion restriction.

2.7.2 Non-Interference

For ease of presentation, the above discussion only briefly mentioned an assumption that plays an important role in the definition and estimation of causal effects. This assumption is sometimes dubbed the Stable Unit Treatment Value Assumption, or SUTVA, but we refer to it by a more accessible name, non-interference.[12] In the notation used above, expressions such as $Y_i(d)$ are written as though the value of the potential outcome for unit i depends only upon whether or not the unit itself gets the treatment (whether d equals one or zero). A more complete notation would express a more extensive schedule of potential outcomes depending on which treatments are administered to other units. For example, for Village 1 we could write down all of the potential outcomes if only Village 1 is treated, if only Village 2 is treated, if Villages 1 and 2 are treated, and so forth. This schedule of potential outcomes quickly gets out of hand. Suppose we listed all of the potential outcomes if exactly two of the seven villages are treated: there would now be 21 potential outcomes for each village. Clearly, if our study involves just seven villages, we have no hope of saying anything meaningful about this complex array of causal effects unless we make some simplifying assumptions.

The non-interference assumption cuts through this complexity by ignoring the potential outcomes that would arise if subject i were affected by the treatment of other subjects. Formally, we reduce the schedule of potential outcomes $Y_i(\boldsymbol{d})$, where $\boldsymbol{d}$ describes all of the treatments administered to all subjects, to a much simpler schedule $Y_i(d)$, where d refers to the treatment administered to subject i.[13] In the context of our example, non-interference implies that the sanitation budget in one village is unaffected by the gender of the council heads in other villages. Non-interference is an assumption common to both experimental and observational studies.

12 The term "stable" in SUTVA refers to the stipulation that the potential outcomes for a given village remain stable regardless of which other villages happen to be treated. The technical aspects of this term are discussed in Rubin 1980 and Rubin 1986.

13 Implicit in this formulation of potential outcomes is the assumption that potential outcomes are unaffected by the overall pattern of actual or assigned treatments. In other words, $Y_i(z, \boldsymbol{d}) = Y_i(z, d)$.

Is non-interference realistic in this example? It is difficult to say without more detailed information about communication between villages and the degree to which their budget allocations are interdependent. If the collection of villages were dispersed geographically, it might be plausible to assume that the gender of the village head in one village has no consequences for outcomes in other villages. On the other hand, if villages were adjacent, the presence of a woman council head in one village might encourage women in other villages to express their policy demands more forcefully. Proximal villages might also have interdependent budgets; the more one village spends on water sanitation, the less the neighboring village needs to spend in order to maintain its own water quality.

The estimation problems that interference introduces are potentially quite complicated and unpredictable. Untreated villages that are affected by the treatments that nearby villages receive no longer constitute an untreated control group. If women council heads set an example of water sanitation spending that is then copied by neighboring villages headed by men, a comparison between average outcomes in treatment villages and (semi-treated) control villages will tend to understate the average treatment effect as defined in equation (2.3), which is usually understood to refer to the contrast between treated potential outcomes and completely untreated potential outcomes. On the other hand, if female council heads cause neighboring villages headed by men to free ride on water sanitation projects and allocate less of their budget to it, the apparent difference in average budget allocations will exaggerate the average treatment effect. Given the vagaries of estimation in the face of interference, researchers often try to design experiments in ways that minimize interference between units by spreading them out temporally or geographically. Another approach, discussed at length in Chapter 8, is to design experiments in ways that allow the researcher to detect spillover between units. Instead of treating interference as a nuisance, these more complex experimental designs aim to detect evidence of communication or strategic interaction among units.

SUMMARY

This chapter has limited its purview to a class of randomized experiments in which treatments are deployed exactly as assigned and outcomes are observed for all of the assigned subjects. This class of studies is a natural starting point for discussing core assumptions and what they imply for research design. The chapters that follow will introduce further assumptions in order to handle the complications that arise due to noncompliance (Chapters 5 and 6) and attrition (Chapter 7).

We began by defining a causal effect as the difference between two potential outcomes, one in which a subject receives treatment and the other in which the subject does not receive treatment. The causal effect for any given subject is not directly observable. However, experiments provide unbiased estimates of the average treatment effect (ATE) among all subjects when certain assumptions are met. The three assumptions invoked in this chapter are random assignment, excludability, and non-interference.

1. Random assignment: Treatments are allocated such that all units have a known probability between 0 and 1 of being placed into the treatment group. Simple random assignment or complete random assignment implies that treatment assignments are statistically independent of the subjects' potential outcomes.

   This assumption is satisfied when all treatment assignments are determined by the same random procedure, such as the flip of a coin. Because random assignment may be compromised by those allocating treatments or assisting subjects, steps should be taken to minimize the role of discretion.
2. Excludability: Potential outcomes respond solely to receipt of the treatment, not to the random assignment of the treatment or any indirect by-products of random assignment. The treatment must be defined clearly so that one can assess whether subjects are exposed to the intended treatment or something else.

   This assumption is jeopardized when (i) different procedures are used to measure outcomes in the treatment and control groups and (ii) research activities, other treatments, or third-party interventions other than the treatment of interest differentially affect the treatment and control groups.
3. Non-interference: Potential outcomes for observation i reflect only the treatment or control status of observation i and not the treatment or control status of other observations. No matter which subjects the random assignment allocates to treatment or control, a given subject's potential outcomes remain the same.

   This assumption is jeopardized when (i) subjects are aware of the treatments that other subjects receive, (ii) treatments may be transmitted from treated to untreated subjects, or (iii) resources used to treat one set of subjects diminish resources that would otherwise be available to other subjects. See Chapter 10 for a more extensive list of examples.

Random assignment is different from the other two assumptions in that it refers to a procedure and the manner in which researchers carry it out. Excludability and non-interference, on the other hand, are substantive assumptions about the ways in which subjects respond to the allocation of treatments. When assessing excludability and non-interference in the context of a particular experiment, the first step is to carefully consider how the causal effect is defined. Do we seek to study the effect of electing women to village council positions or rather the effect of electing women from a pool of candidates that consists only of women? When defining the treatment effect of installing a female village council head, is the appropriate comparison a village with male leadership, or a male-led village with no neighboring female-led villages? Attending to these subtleties encourages a researcher to design more exacting experimental comparisons and to interpret the results with greater precision.

Attentiveness to these core assumptions also helps guide experimental investigation, urging researchers to explore the empirical consequences of different research designs. A series of experiments in a particular domain may be required before a researcher can gauge whether subjects seem to be affected by the random assignment over and above the treatment (a violation of excludability) or by the treatments administered to other units (interference).

SUGGESTED READINGS

Holland (1986) and Rubin (2008) provide non-technical introductions to potential outcomes notation. Fisher (1935) and Cox (1958) are two classic books on experimental design and analysis; Dean and Voss (1999) and Kuehl (1999) offer more modern treatments. See Rosenbaum and Rubin (1984) on the distinctive statistical properties of randomly assigned treatments.

EXERCISES: CHAPTER 2

1. Potential outcomes notation:
   (a) Explain the notation "$Y_i(0)$."
   (b) Explain the notation "$Y_i(0) \mid D_i = 1$" and contrast it with the notation "$Y_i(0) \mid d_i = 1$."
   (c) Contrast the meaning of "$Y_i(0)$" with the meaning of "$Y_i(0) \mid D_i = 0$."
   (d) Contrast the meaning of "$Y_i(0) \mid D_i = 1$" with the meaning of "$Y_i(0) \mid D_i = 0$."
   (e) Contrast the meaning of "$E[Y_i(0)]$" with the meaning of "$E[Y_i(0) \mid D_i = 1]$."
   (f) Explain why the "selection bias" term in equation (2.15), $E[Y_i(0) \mid D_i = 1] - E[Y_i(0) \mid D_i = 0]$, is zero when D_i is randomly assigned.
2. Use the values depicted in Table 2.1 to illustrate that $E[Y_i(0)] - E[Y_i(1)] = E[Y_i(0) - Y_i(1)]$.
3. Use the values depicted in Table 2.1 to complete the table below.
   (a) Fill in the number of observations in each of the nine cells.
   (b) Indicate the percentage of all subjects that fall into each of the nine cells. (These cells represent what is known as the joint frequency distribution of $Y_i(0)$ and $Y_i(1)$.)
   (c) At the bottom of the table, indicate the proportion of subjects falling into each category of $Y_i(1)$. (These cells represent what is known as the marginal distribution of $Y_i(1)$.)
   (d) At the right of the table, indicate the proportion of subjects falling into each category of $Y_i(0)$ (i.e., the marginal distribution of $Y_i(0)$).
   (e) Use the table to calculate the conditional expectation that $E[Y_i(0) \mid Y_i(1) > 15]$. (Hint: This expression refers to the expected value of $Y_i(0)$ given that $Y_i(1)$ is greater than 15.)
   (f) Use the table to calculate the conditional expectation that $E[Y_i(1) \mid Y_i(0) > 15]$.

	$Y_i(1)$			Marginal distribution
$Y_i(0)$	15	20	30	of $Y_i(0)$
10				
15				
20				
Marginal distribution of $Y_i(1)$				1.0

4. Suppose that the treatment indicator d_i is either 1 (treated) or 0 (untreated). Define the average treatment effect among the treated, or ATT for short, as $\sum_1^N \tau_i d_i / \sum_1^N d_i$. Using the equations in this chapter, prove the following claim: "When treatments are allocated using complete random assignment, the ATT is, in expectation, equal to the ATE. In other words, taking expectations over all possible random assignments, $E[\tau_i \mid D_i = 1] = E[\tau_i]$, where τ_i is a randomly selected observation's treatment effect.
5. A researcher plans to ask six subjects to donate time to an adult literacy program. Each subject will be asked to donate either 30 or 60 minutes. The researcher is considering three methods for randomizing the treatment. One method is to flip a coin before talking to each person and to ask for a 30-minute donation if the coin comes up heads or a 60-minute donation if it comes up tails. The second method is to write "30" and "60" on three playing cards each, and then shuffle the six cards. The first subject would be assigned the number on the first card, the second subject would be assigned the number on the second card, and so on. A third method is to write each number on three different slips of paper, seal the six slips into envelopes, and shuffle the six envelopes before talking to the first subject. The first subject would be assigned the first envelope, the second subject would be assigned the second envelope, and so on.
   (a) Discuss the strengths and weaknesses of each approach.
   (b) In what ways would your answer to (a) change if the number of subjects were 600 instead of 6?
   (c) What is the expected value of D_i (the assigned number of minutes) if the coin toss method is used? What is the expected value of D_i if the sealed envelope method is used?
6. Many programs strive to help students prepare for college entrance exams, such as the SAT. In an effort to study the effectiveness of these preparatory programs, a researcher draws a random sample of students attending public high school in the United States, and compares the SAT scores of those who took a preparatory class to those who did not. Is this an experiment or an observational study? Why?
7. Suppose that an experiment were performed on the villages in Table 2.1, such that two villages are allocated to the treatment group and the other five villages to the control group. Suppose that an experimenter randomly selects Villages 3 and 7 from the set of seven villages and places them into the treatment group. Table 2.1 shows that these villages have unusually high potential outcomes.
   (a) Define the term *unbiased estimator*.
   (b) Does this allocation procedure produce upwardly biased estimates? Why or why not?
   (c) Suppose that instead of using random assignment, the researcher placed Villages 3 and 7 into the treatment group because the treatment could be administered inexpensively in those villages. Explain why this procedure is prone to bias.
8. Peisakhin and Pinto[14] report the results of an experiment in India designed to test the effectiveness of a policy called the Right to Information Act (RTIA), which allows citizens to inquire about the status of a pending request from government officials. In their study, the researchers hired confederates, slum dwellers who sought to obtain ration cards (which permit the purchase of food at low cost). Applicants for such cards must fill out a

14 Peisakhin and Pinto 2010.

form and have their residence and income verified by a government agent. Slum dwellers widely believe that the only way to obtain a ration card is to pay a bribe. The researchers instructed the confederates to apply for ration cards in one of four ways, specified by the researchers. The control group submitted an application form at a government office; the RTIA group submitted a form and followed it up with an official Right to Information request; the NGO group submitted a letter of support from a local nongovernmental organization (NGO) along with the application form; and finally, a bribe group submitted an application and paid a small fee to a person who is known to facilitate the processing of forms.

	Bribe	RTIA	NGO	Control
Number of confederates in the study	24	23	18	21
Number of confederates who had residence verification	24	23	18	20
Median number of days to residence verification	17	37	37	37
Number of confederates who received a ration card within one year	24	20	3	5

(a) Interpret the apparent effects of the treatments on the proportion of applicants who have their residence verified and the speed with which verification occurred.
(b) Interpret the apparent effects of the treatments on the proportion of applicants who actually received a ration card.
(c) What do these results seem to suggest about the effectiveness of the Right to Information Act as a way of helping slum dwellers obtain ration cards?

9. A researcher wants to know how winning large sums of money in a national lottery affects people's views about the estate tax. The researcher interviews a random sample of adults and compares the attitudes of those who report winning more than $10,000 in the lottery to those who claim to have won little or nothing. The researcher reasons that the lottery chooses winners at random, and therefore the amount that people report having won is random.
   (a) Critically evaluate this assumption. (Hint: are the potential outcomes of those who report winning more than $10,000 identical, in expectation, to those who report winning little or nothing?)
   (b) Suppose the researcher were to restrict the sample to people who had played the lottery at least once during the past year. Is it now safe to assume that the potential outcomes of those who report winning more than $10,000 are identical, in expectation, to those who report winning little or nothing?
10. Suppose researchers seek to assess the effect of receiving a free newspaper subscription on students' interest in politics. A list of student dorm rooms is drawn up and sorted randomly. Dorm rooms in the first half of the randomly sorted list receive a newspaper at their door each morning for two months; dorm rooms in the second half of the list do not receive a paper.
   (a) University researchers are sometimes required to disclose to subjects that they are participating in an experiment. Suppose that prior to the experiment, researchers distributed a letter informing students in the treatment group that they would be receiving a newspaper as part of a study to see if newspapers make students more interested in politics. Explain (in words and using potential outcomes notation) how this disclosure may jeopardize the excludability assumption.
   (b) Suppose that students in the treatment group carry their newspapers to the cafeteria where they may be read by others. Explain (in words and using potential outcomes notation) how this may jeopardize the non-interference assumption.

11. Several randomized experiments have assessed the effects of drivers' training classes on the likelihood that a student will be involved in a traffic accident or receive a ticket for a moving violation.[15] A complication arises because students who take drivers' training courses typically obtain their licenses faster than students who do not take a course.[16] (The reason is unknown but may reflect the fact that those who take the training are better prepared for the licensing examination.) If students in the control group on average start driving much later, the proportion of students who have an accident or receive a ticket could well turn out to be higher in the treatment group. Suppose a researcher were to compare the treatment and control group in terms of the number of accidents that occur within three years of obtaining a license.
   (a) Does this measurement approach maintain symmetry between treatment and control groups?
   (b) Would symmetry be maintained if the outcome measure were the number of accidents per mile of driving?
   (c) Suppose researchers were to measure outcomes over a period of three years starting the moment at which students were randomly assigned to be trained or not. Would this measurement strategy maintain symmetry? Are there drawbacks to this approach?
12. A researcher studying 1,000 prison inmates noticed that prisoners who spend at least three hours per day reading are less likely to have violent encounters with prison staff. The researcher therefore recommends that all prisoners be required to spend at least three hours reading each day. Let d_i be 0 when prisoners read less than three hours each day and 1 when prisoners read more than three hours each day. Let $Y_i(0)$ be each prisoner's potential number of violent encounters with prison staff when reading less than three hours per day, and let $Y_i(1)$ be each prisoner's potential number of violent encounters when reading more than three hours per day.
   (a) In this study, nature has assigned a particular realization of d_i to each subject. When assessing this study, why might one be hesitant to assume that $E[Y_i(0) \mid D_i = 0] = E[Y_i(0) \mid D_i = 1]$ and $E[Y_i(1) \mid D_i = 0] = E[Y_i(1) \mid D_i = 1]$?
   (b) Suppose that researchers were to test this researcher's hypothesis by randomly assigning 10 prisoners to a treatment group. Prisoners in this group are required to go to the prison library and read in specially designated carrels for three hours each day for one week; the other prisoners, who make up the control group, go about their usual routines. Suppose, for the sake of argument, that all prisoners in the treatment group in fact read for three hours each day and that none of the prisoners

15 See Roberts and Kwan 2001.
16 Vernick et al. 1999.

in the control group read at all during the week of the study. Critically evaluate the excludability assumption as it applies to this experiment.

(c) State the assumption of non-interference as it applies to this experiment.

(d) Suppose that the results of this experiment were to indicate that the reading treatment sharply reduces violent confrontations with prison staff. How does the non-interference assumption come into play if the aim is to evaluate the effects of a policy whereby all prisoners are required to read for three hours?

CHAPTER 3

Sampling Distributions, Statistical Inference, and Hypothesis Testing

Rigorous quantification of uncertainty is a hallmark of scientific inquiry. When analyzing experimental data, the aim is not only to generate unbiased estimates of the average treatment effect but also to draw inferences about the uncertainty surrounding these estimates. Among the most attractive features of experimentation is that random allocation of treatments is a reproducible procedure. Reproducibility allows us to assess the sampling distribution, or collection of estimated ATEs that could have come about under different random assignments in order to better understand the uncertainty associated with the experiment we conducted. One objective of this chapter is to explain how experimental design affects the sampling distribution. We consider ways of designing experiments so as to reduce sampling variability, and we call attention to the fact that the sampling distribution may change markedly depending on the procedures used to randomly allocate subjects to treatment and control conditions.

A second objective is to guide the reader through the calculation and interpretation of key statistical results. When analyzing an experiment, you should consider both the estimated ATE and the uncertainty with which it is estimated. Unless you have prior information about the value of the ATE, the experimental estimate is one's best guess of the true treatment effect, but this guess may be close to or far from the true average causal effect. Statisticians commonly assess uncertainty in two ways. One method is to investigate whether the experimental results are sufficiently informative to refute a determined skeptic who insists that there is no treatment effect whatsoever. Another approach is to identify a range of values that probably bracket the true average treatment effect. This chapter introduces a flexible set of statistical techniques that may be used to assess uncertainty across a wide array of different experimental designs.