

Chapter 8

The Classification of Stellar Spectra

AGA 293

Slides based on the book “An Introduction to Modern Astrophysics”, unless otherwise noticed.

Classificação de estrelas: 4 classes



Tipo I: branco-azul; linhas fortes de H.
Atual classe A & F "cedo"

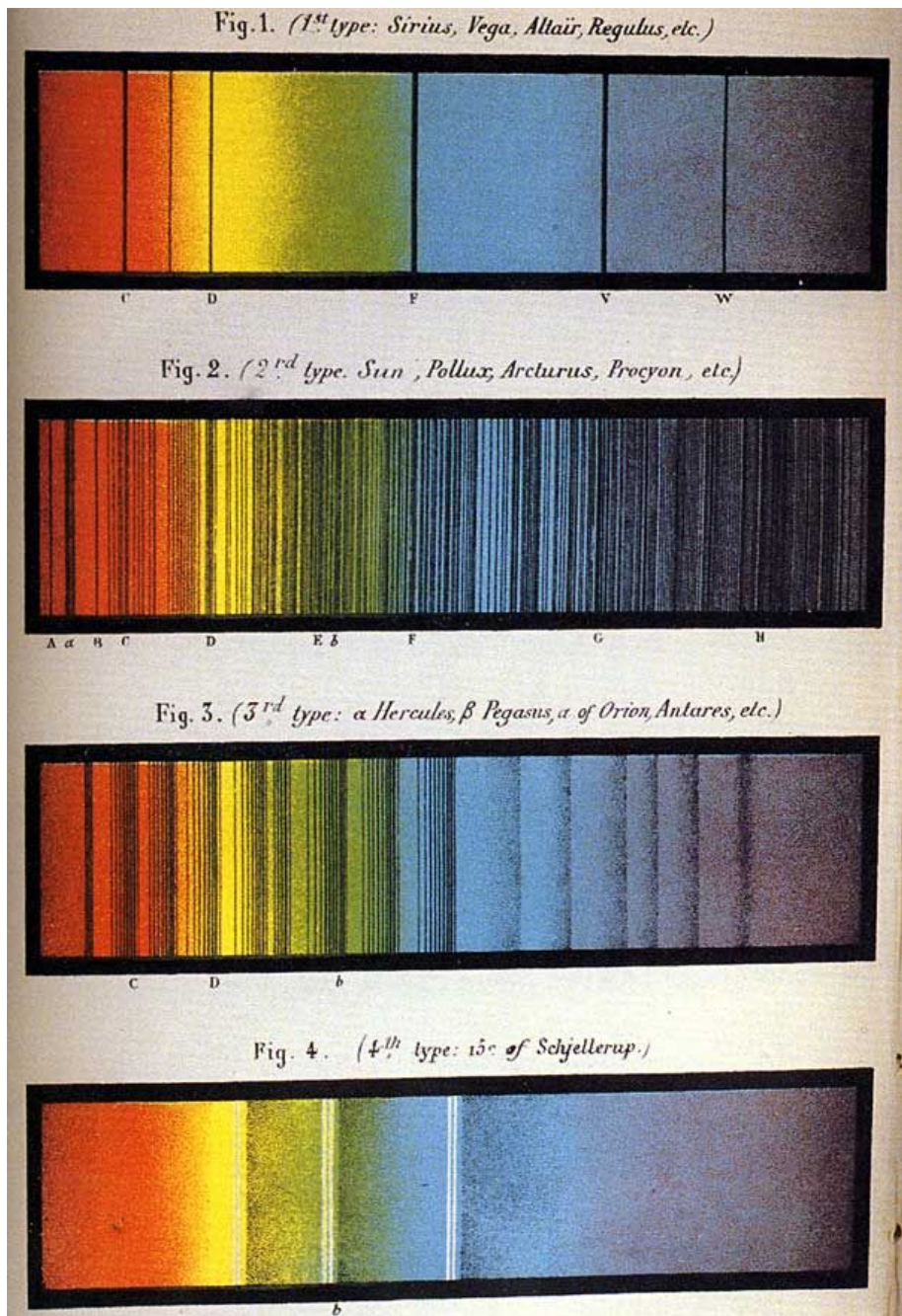
Tipo II: amarelas, de tipo solar.
Numerous metallic lines (Na, Ca, Fe), with weaker H. Atual classe G, K, F tardio

Tipo III: laranja-vermelho; metallic lines and bands.
atual classe M

Tipo IV: estrelas com linhas de emissão

Secchi's Classes of Stellar Spectra (1860-1870)

Secchi's four classes of stellar spectra, from a colored lithograph in a book published around 1870. The principal spectral lines are identified underneath by letters that Fraunhofer assigned.



Henry Draper (Mar 7, 1837 – Nov 20, 1882) pioneer of astrophotography.

Primeiro espectro estelar em 1872: Vega

**Mãe: Antonia
Coetana de Paiva
Pereira Gardner**
(filha do médico
do Imperador do
Brasil)



Williamina Fleming

(May 15, 1857 – May 21, 1911)

Edward Charles Pickering estava frustrado com seus assistentes masculinos no Harvard College Observatory e declarou que até sua empregada faria um trabalho melhor. **Em 1881 contratou sua empregada Williamina Fleming para classificar espectros.**



O sistema de classificação de Harvard

- 1890-1900s: classificação de Harvard (E. Pickering + Williamina Fleming + Antonia Maury + Annie J. Cannon):

Mulheres astrônomas @ Harvard



O B A

F G K M

L T

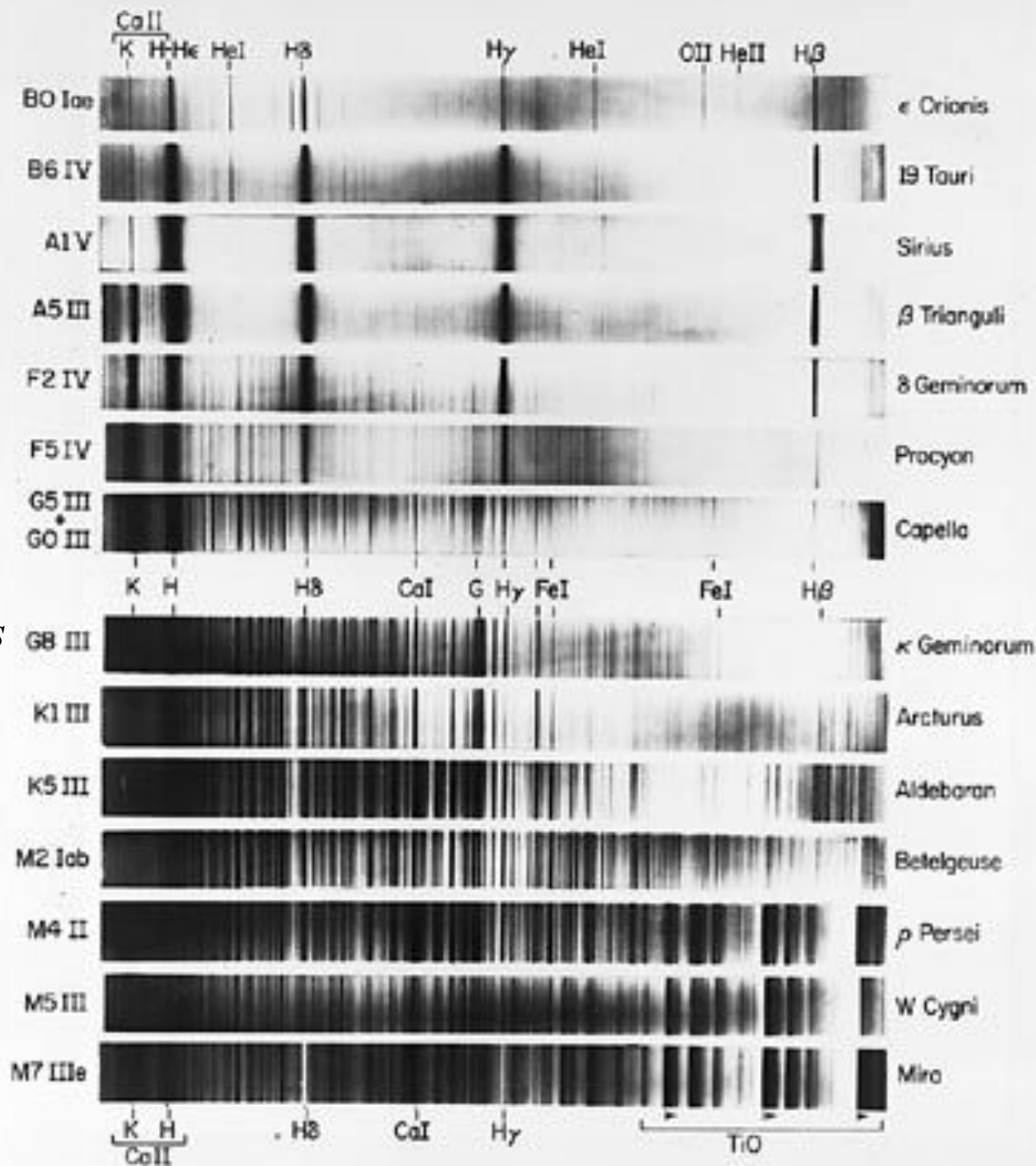
Classificação estelar:

O, B, A, F, G, K, M

O, B, A, F, G, K, M

Baseado em espectros das estações Harvard Norte (U.S.A.) & Sul (Arequipa, Peru)

Annie J. Cannon classificou mais de 250 000 espectros!



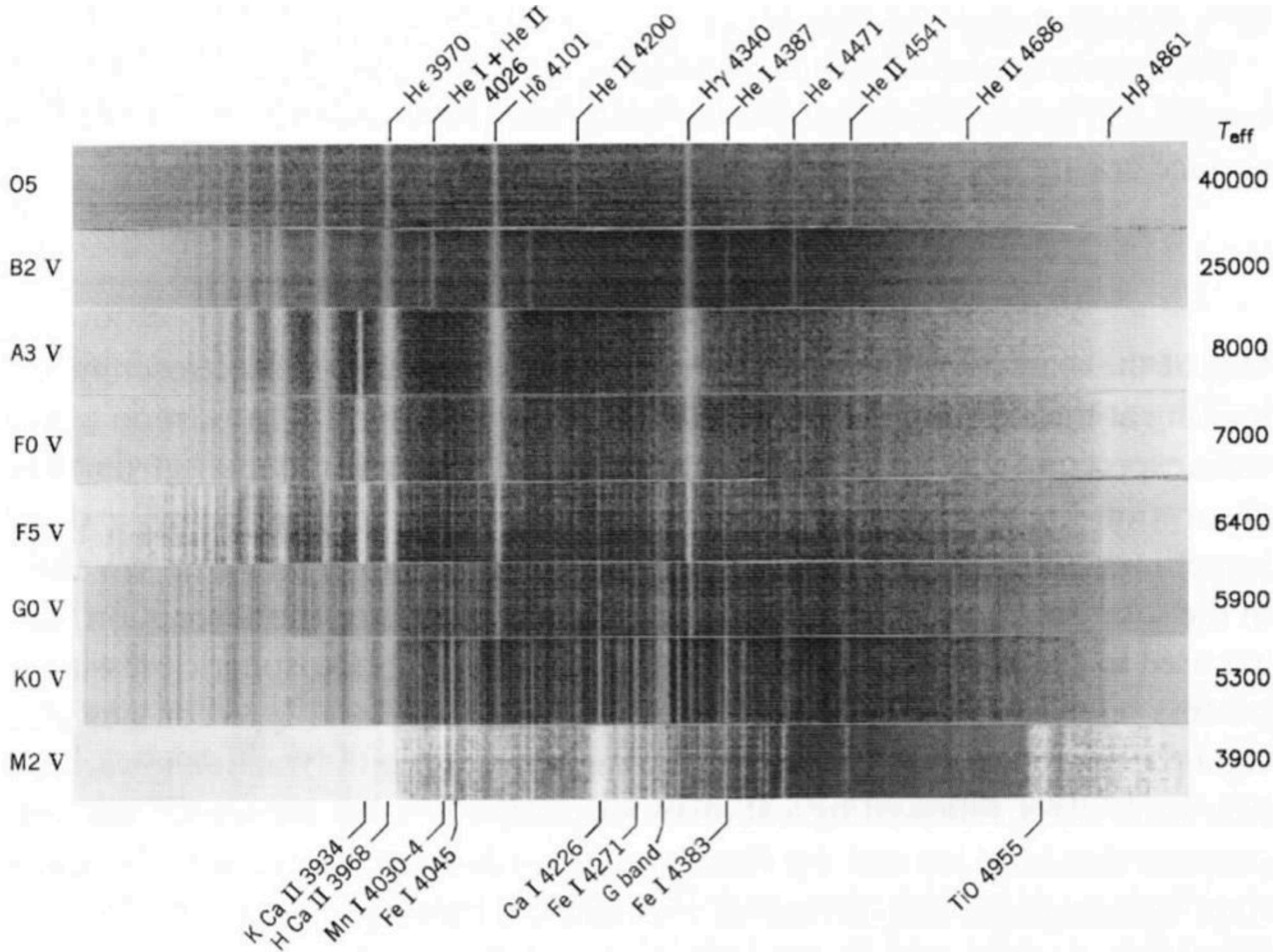
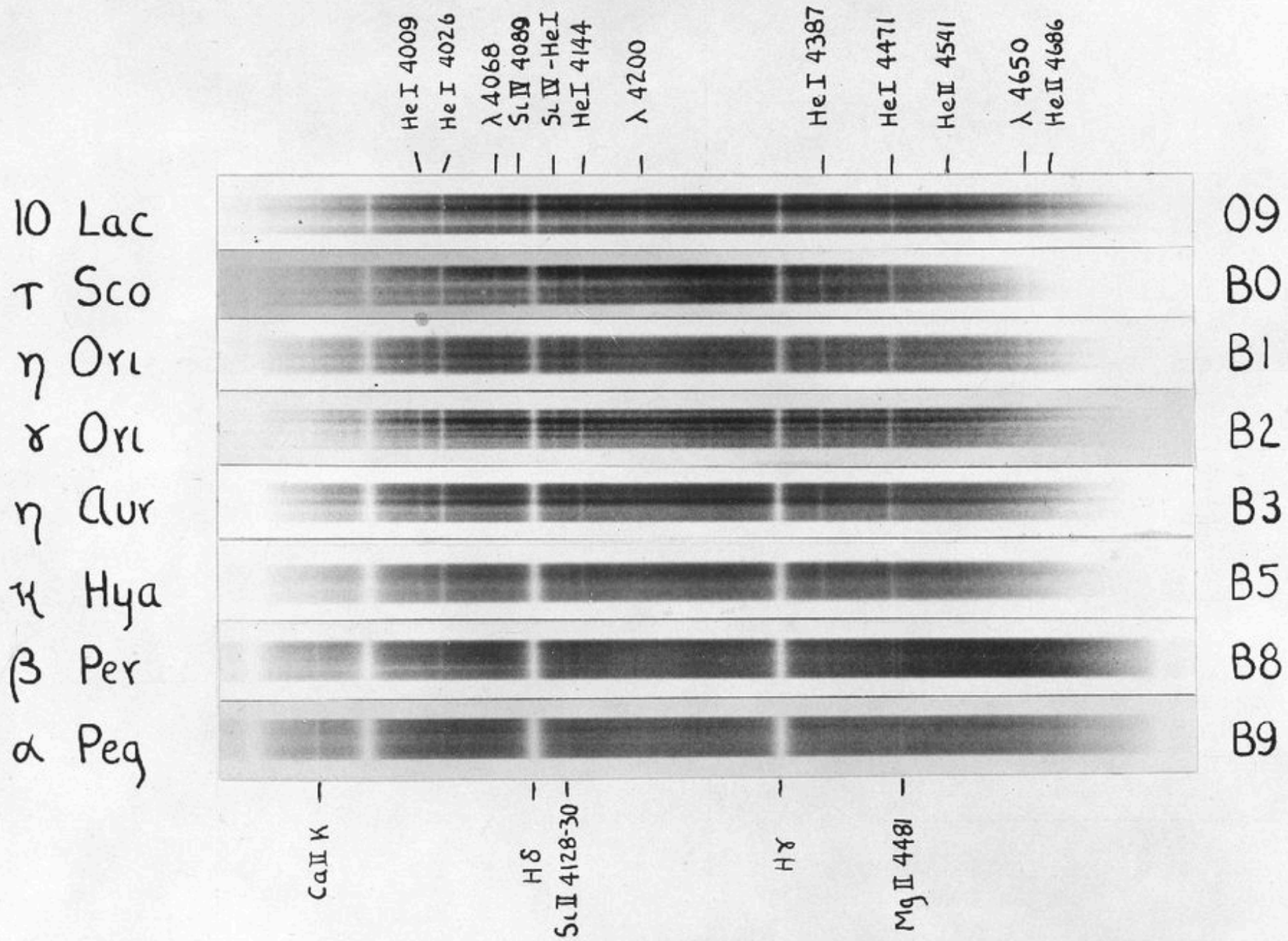
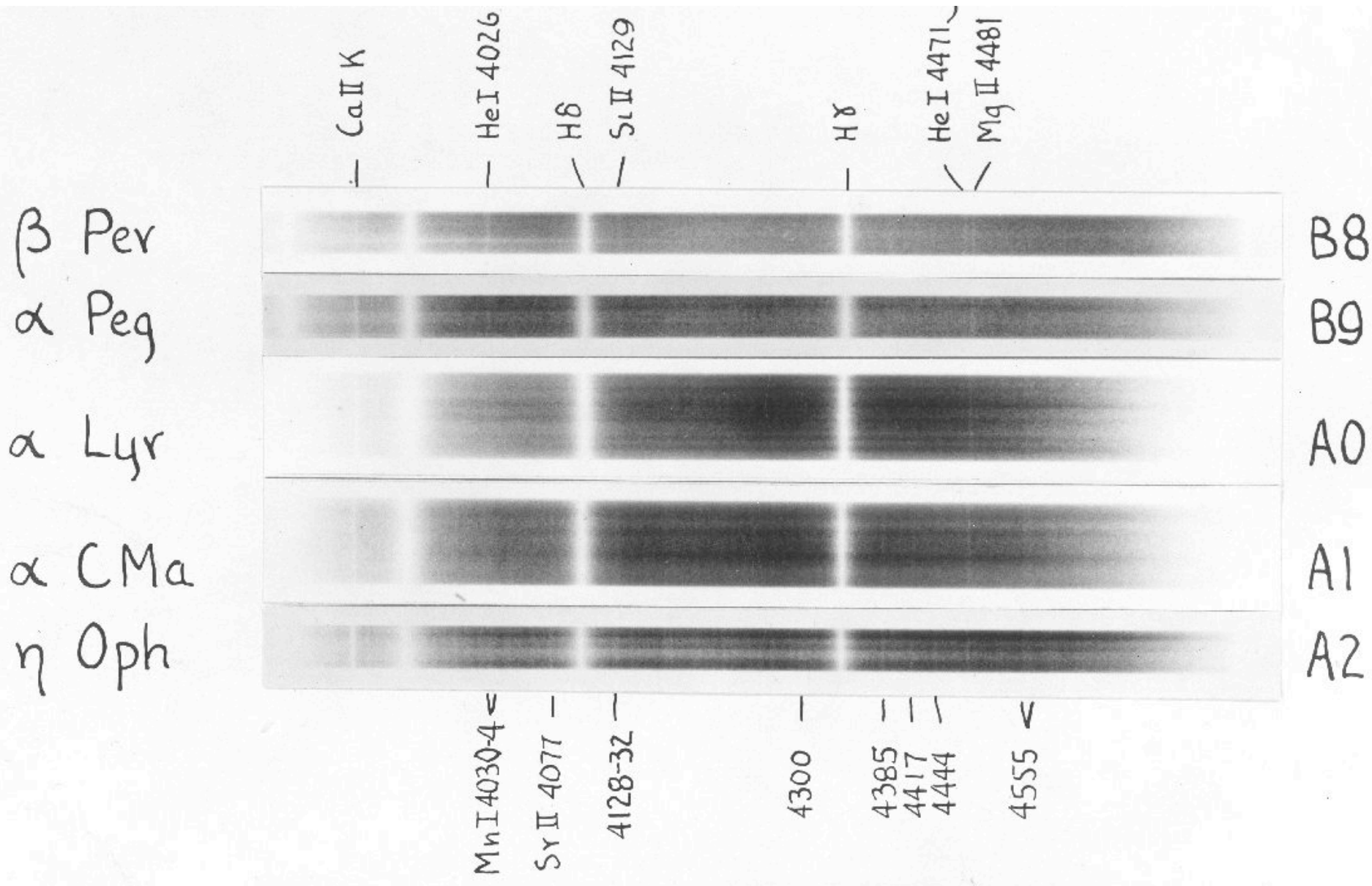


TABLE 8.1 Harvard Spectral Classification.

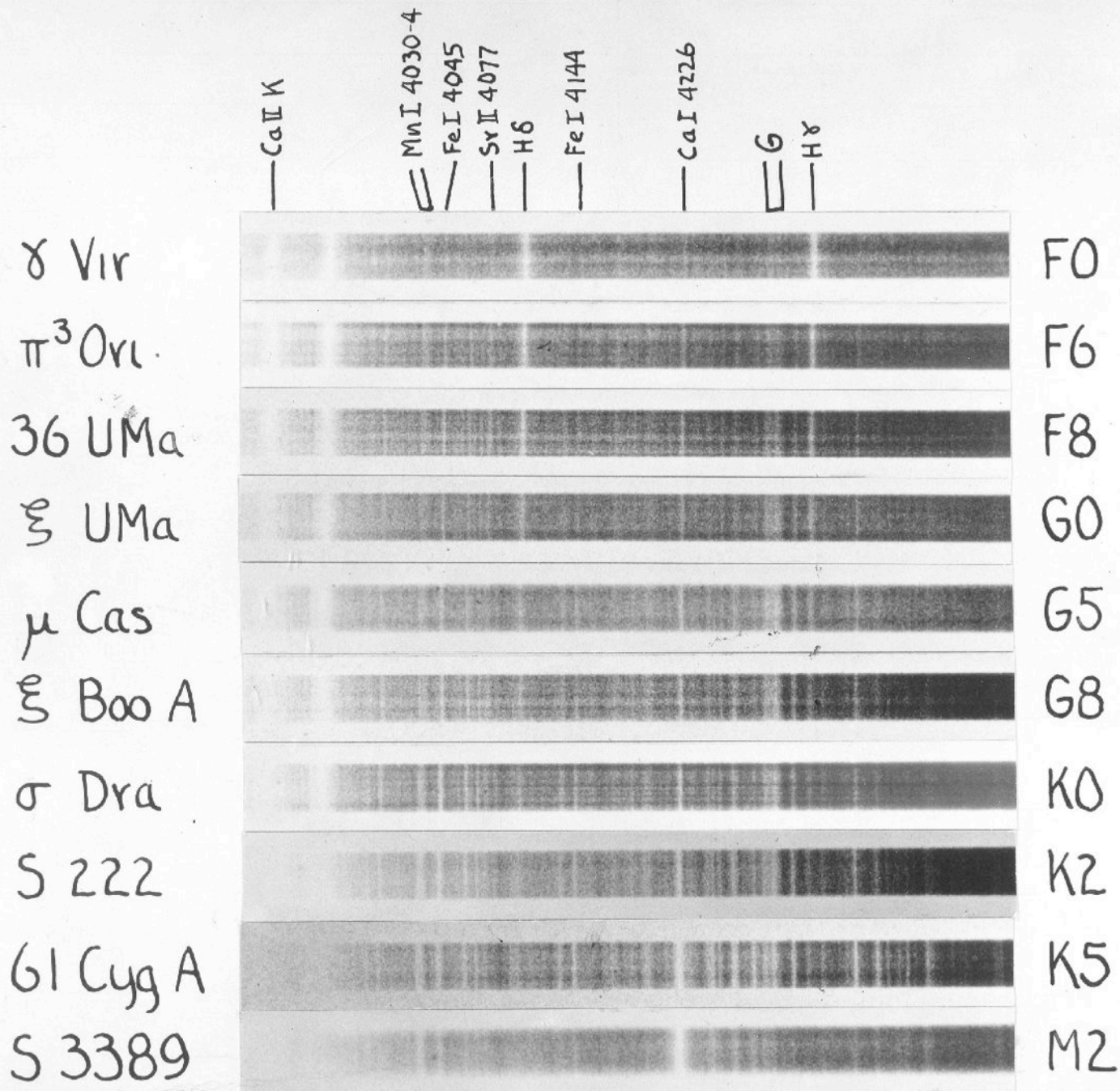
Spectral Type	Characteristics
O	Hottest blue-white stars with few lines Strong He II absorption (sometimes emission) lines. He I absorption lines becoming stronger.
B	Hot blue-white He I absorption lines strongest at B2. H I (Balmer) absorption lines becoming stronger.
A	White Balmer absorption lines strongest at A0, becoming weaker later. Ca II absorption lines becoming stronger.
F	Yellow-white Ca II lines continue to strengthen as Balmer lines continue to weaken. Neutral metal absorption lines (Fe I, Cr I).



An Atlas of Stellar Spectra with an Outline of Spectral Classification. Morgan, Keenan & Kellman, 1943



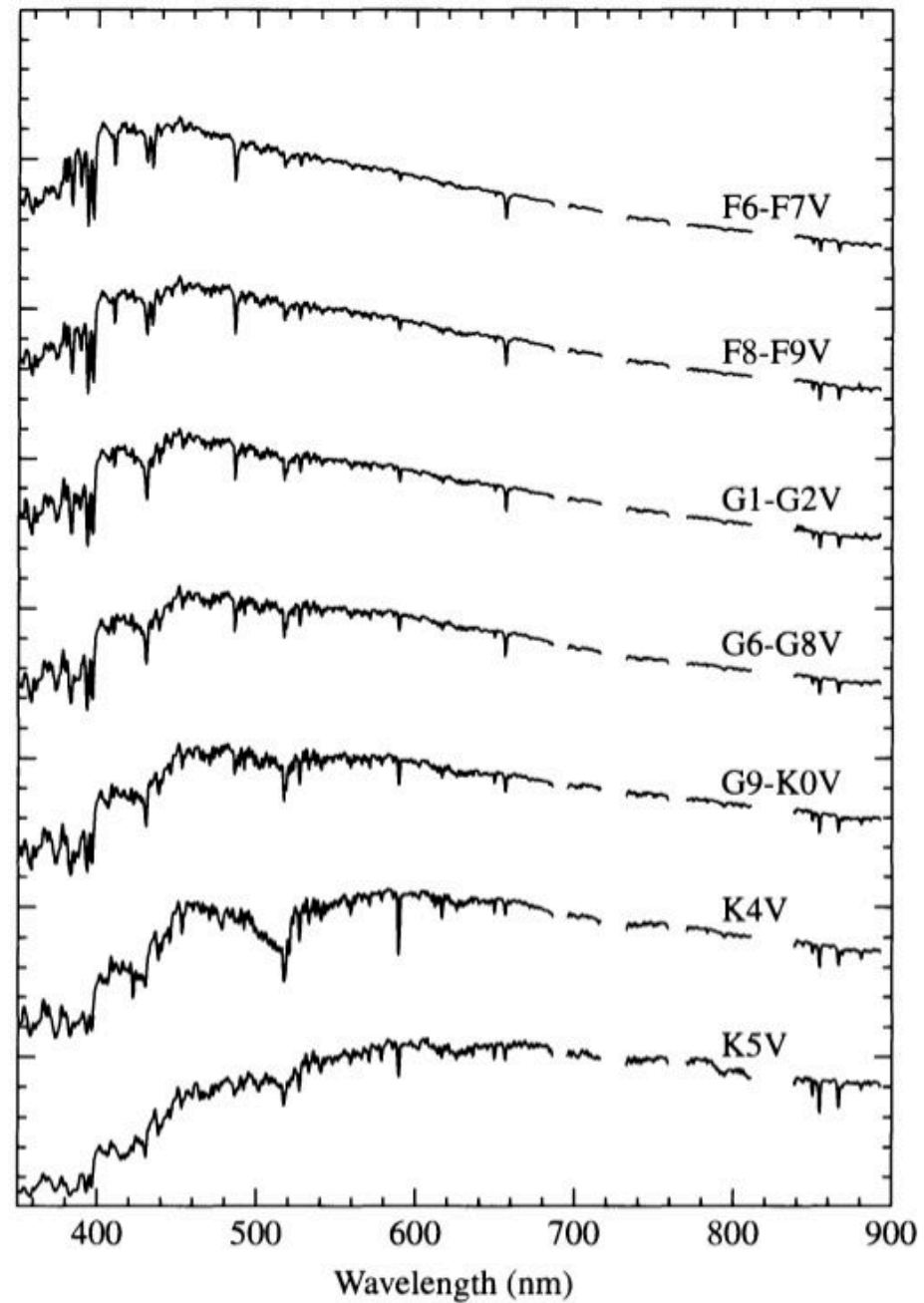
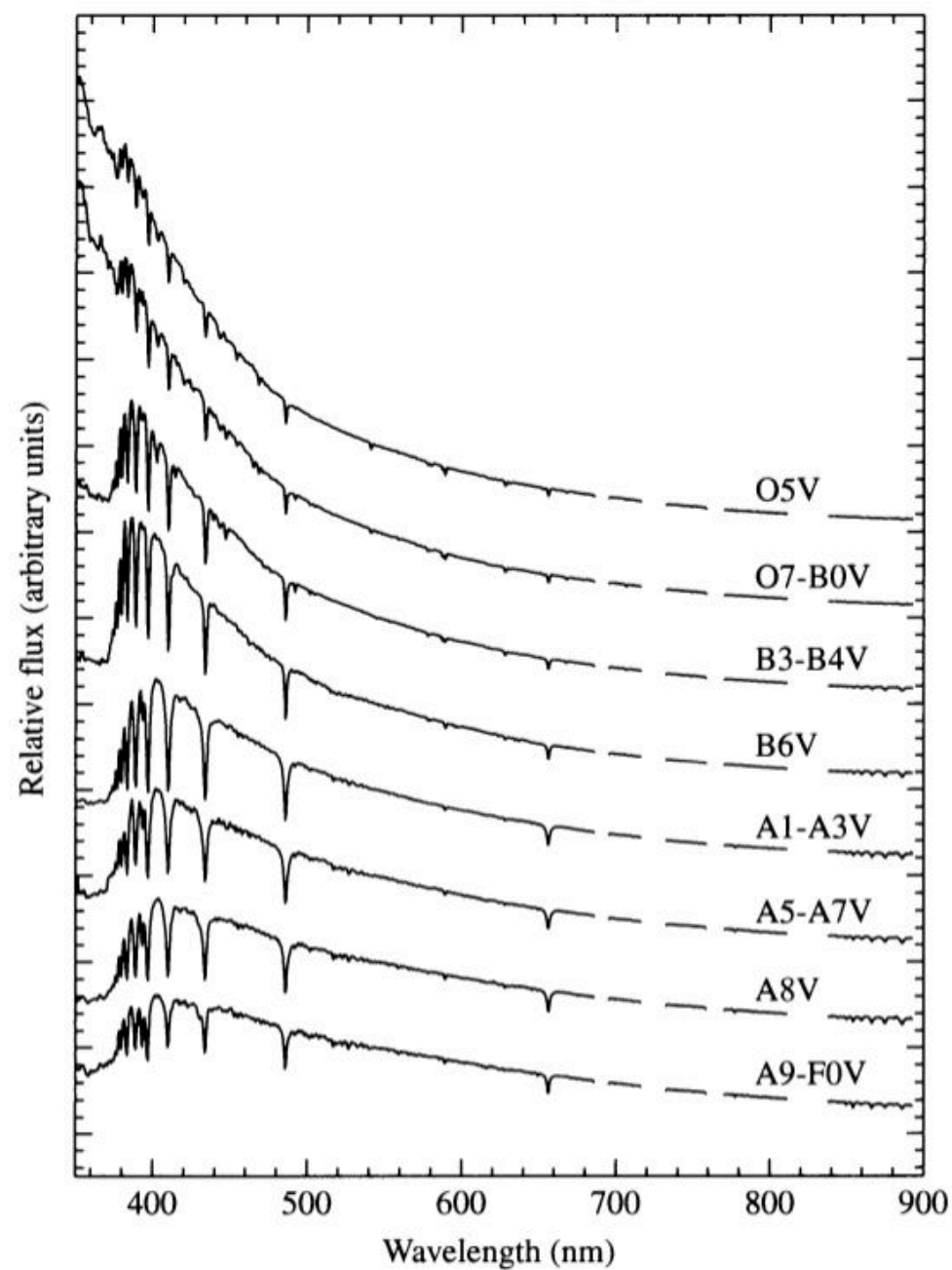
- G Yellow
Solar-type spectra.
Ca II lines continue becoming stronger.
Fe I, other neutral metal lines becoming stronger.
- K Cool orange
Ca II H and K lines strongest at K0, becoming weaker later.
Spectra dominated by metal absorption lines.
- M Cool red
Spectra dominated by molecular absorption bands,
 especially titanium oxide (TiO) and vanadium oxide (VO).
Neutral metal absorption lines remain strong.



An Atlas of Stellar Spectra with an Outline of Spectral Classification. Morgan, Keenan & Kellman, 1943

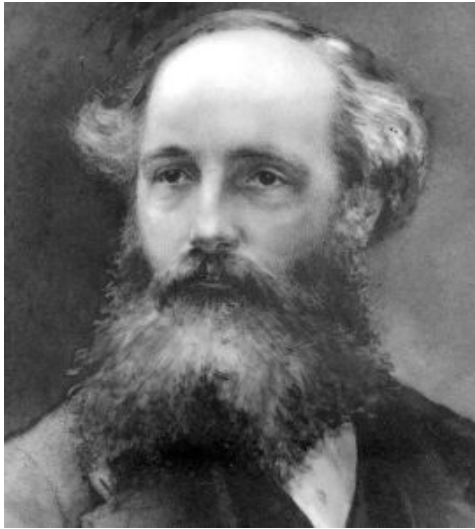
- M Cool red
Spectra dominated by molecular absorption bands,
especially titanium oxide (TiO) and vanadium oxide (VO).
Neutral metal absorption lines remain strong.
- L Very cool, dark red
Stronger in infrared than visible.
Strong molecular absorption bands of metal hydrides (CrH, FeH), water
(H₂O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs).
TiO and VO are weakening.
- T Coolest, Infrared
Strong methane (CH₄) bands but weakening CO bands.

S and C spectral types for evolved giant stars are discussed on page 466.



Como explicar os espectros observados?

Com as equações da física estatística



James Clerk Maxwell
1831-1879



Ludwig Boltzmann
1844-1906



Meghnad Saha
1893-1956

Meghnad Saha. Índia, 1893-1956

- Desenvolveu a equação de Saha, fundamental para entender os espectros de estrelas
- Família pobre
- Boa parte das suas descobertas feitas na Índia
- Membro da *Royal Society* em 1927
- “Ivory tower” até 1930 → Política e planejamento, criação de instituições científicas (e.g., *The National Academy of Sciences, India*), desenvolvimento do país.



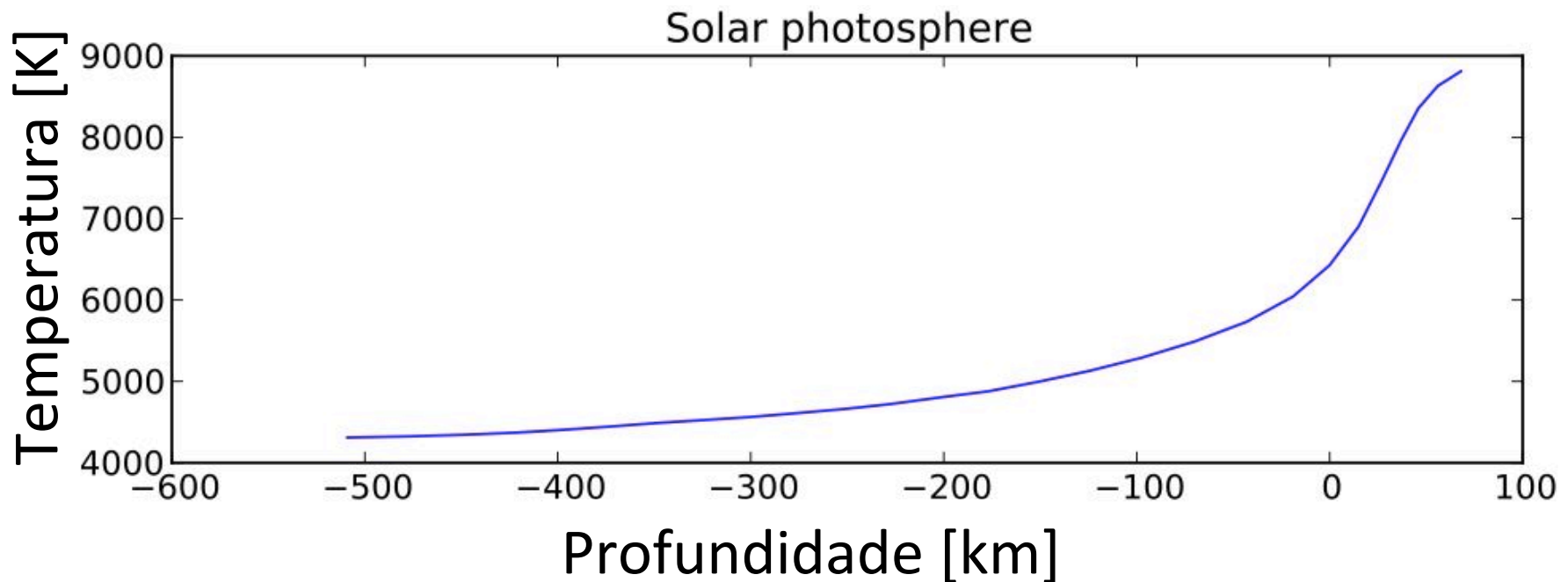
Tipos de astrônomos

- Investigator (Universidade, Instituto, Observatório)
- Operational (p.ex., Observatório; apoio a astrônomos)
- Applied (p.ex., construir instrumentos, lentes, filtros, etc.)
- Business/marketing (p.ex., CCDs, telescópios, instrumentos, filtros para observar o Sol)
- Entrepreneur (p.ex., viagens espaciais)
- Communicator (p.ex., divulgação em planetários ou institutos)
- Teacher (escola, universidade, treinamentos)
- Management (p.ex., diretores de institutos)
- Policy maker (e.g., agências de fomento, governo)

Simplificações nos modelos de atmosferas:

Equilíbrio Termodinâmico Local (ETL)

- Distribuição de velocidades: Maxwell
- Excitação e Ionização: Boltzmann e Saha
- Distribuição de fótons: Planck (corpo negro)



Distribuição de velocidades (Maxwell-Boltzmann)

Maxwell–Boltzmann velocity distribution function⁵ describes the fraction of particles having a given range of speeds. The number of gas particles per unit volume having speeds between v and $v + dv$ is given by

$$n_v dv = n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 dv,$$

- n : total number density
- $n_v = dn/dv$
- m : massa
- k : Boltzmann's constant
- T : temperature in K

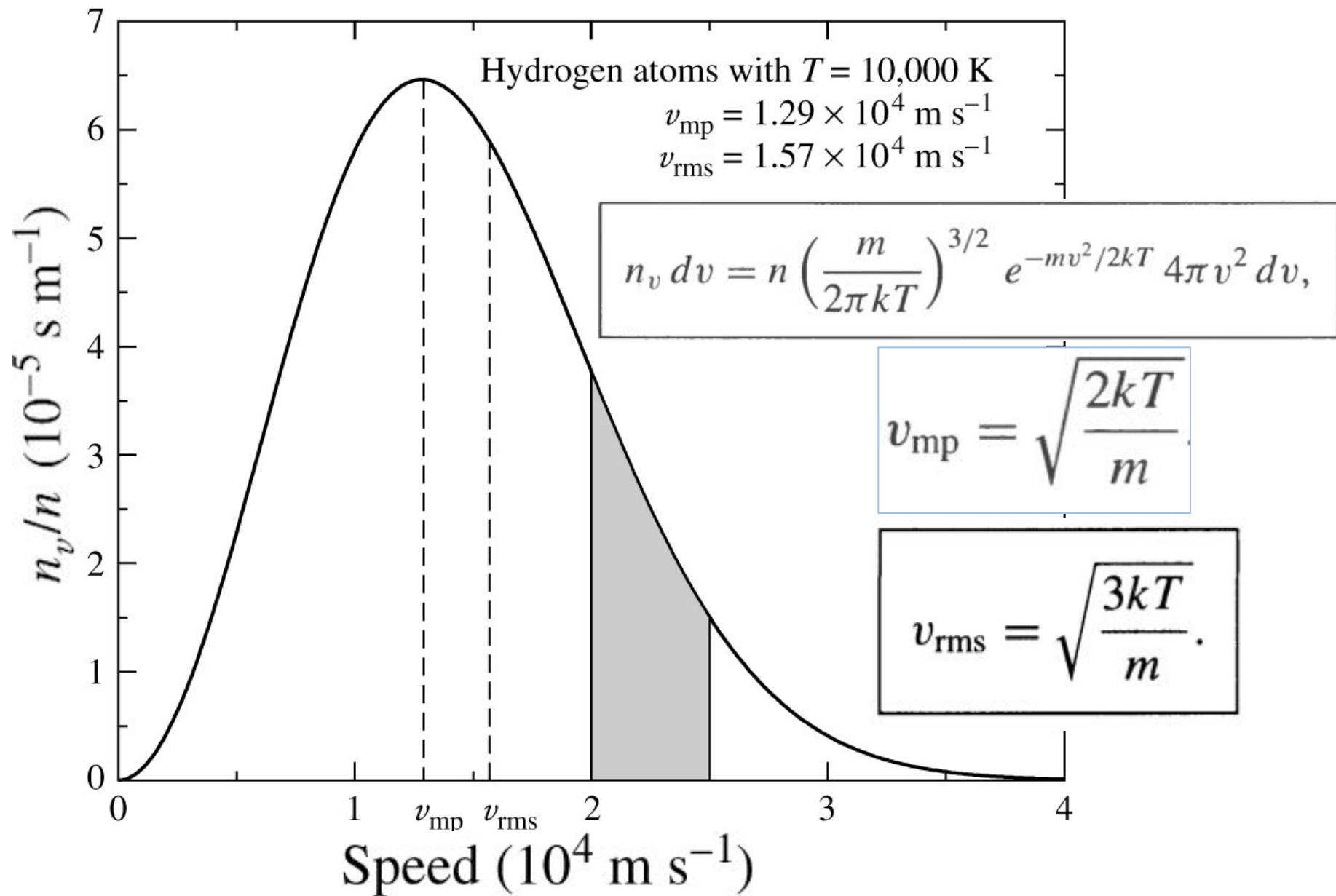


FIGURE 8.6 Maxwell–Boltzmann distribution function, n_v/n , for hydrogen atoms at a temperature of 10,000 K. The fraction of hydrogen atoms in the gas having velocities between $2 \times 10^4 \text{ m s}^{-1}$ and $2.5 \times 10^4 \text{ m s}^{-1}$ is the shaded area under the curve between those two velocities; see Example 8.1.1.

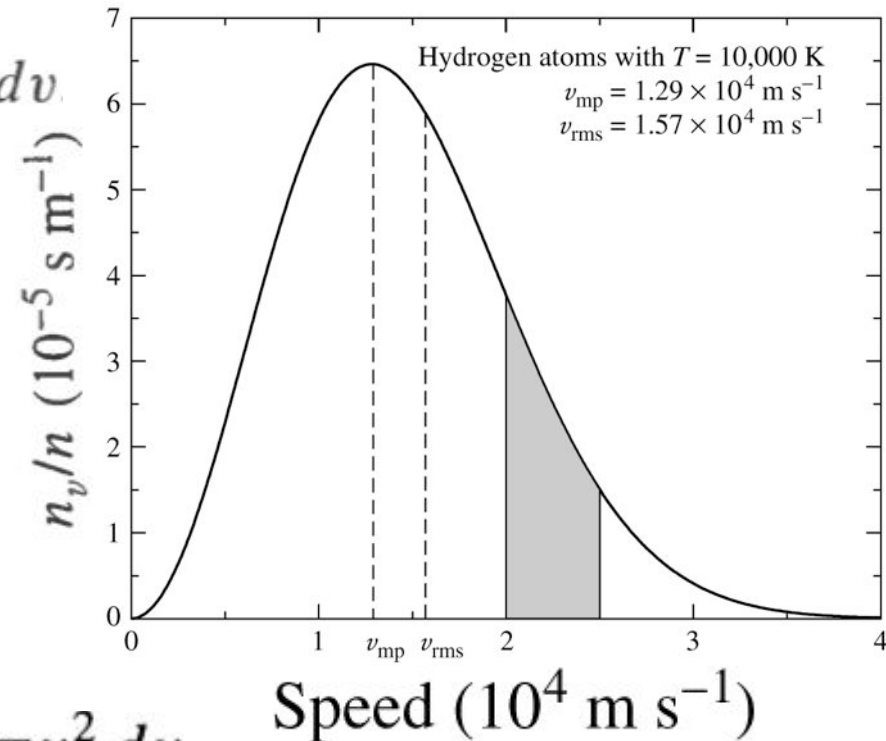
Exemplo 8.1.1. Determinar a fração de átomos de hidrogênio em um gás com $T = 10\,000\text{K}$, para velocidades entre $v_1 = 2 \times 10^4\text{ m/s}$ e $v_2 = 2,5 \times 10^4\text{ m/s}$.

$$n_v dv = n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 dv$$

Para obter a fração entre v_1 e v_2 , integramos:

$$N/N_{\text{total}} = \frac{1}{n} \int_{v_1}^{v_2} n_v dv$$

$$= \left(\frac{m}{2\pi kT} \right)^{3/2} \int_{v_1}^{v_2} e^{-mv^2/2kT} 4\pi v^2 dv$$



Precisamos integrar numericamente, ou aproximar:

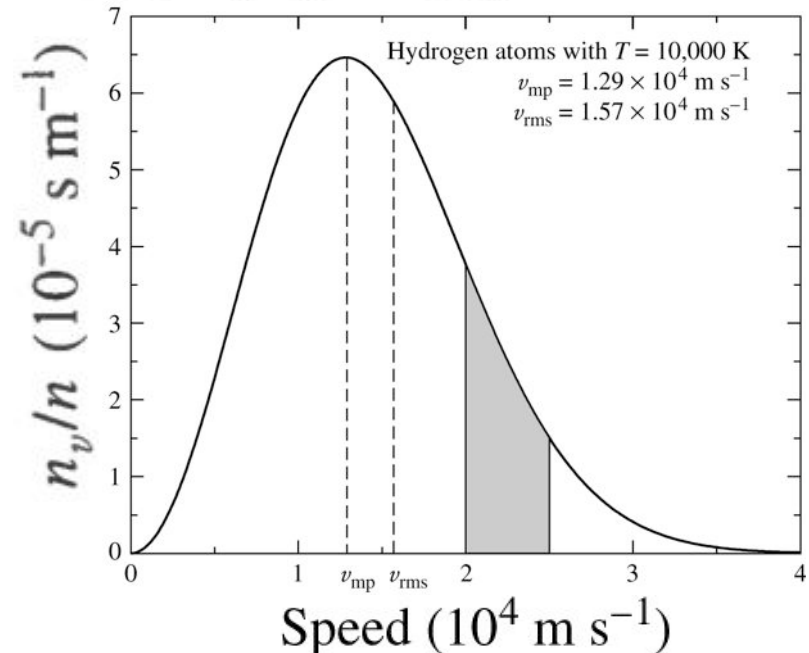
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Precisamos integrar numericamente, ou aproximar:

$$N/N_{\text{total}} = \frac{1}{n} \int_{v_1}^{v_2} n_v(v) dv \simeq \frac{1}{n} n_v(\bar{v}) (v_2 - v_1)$$

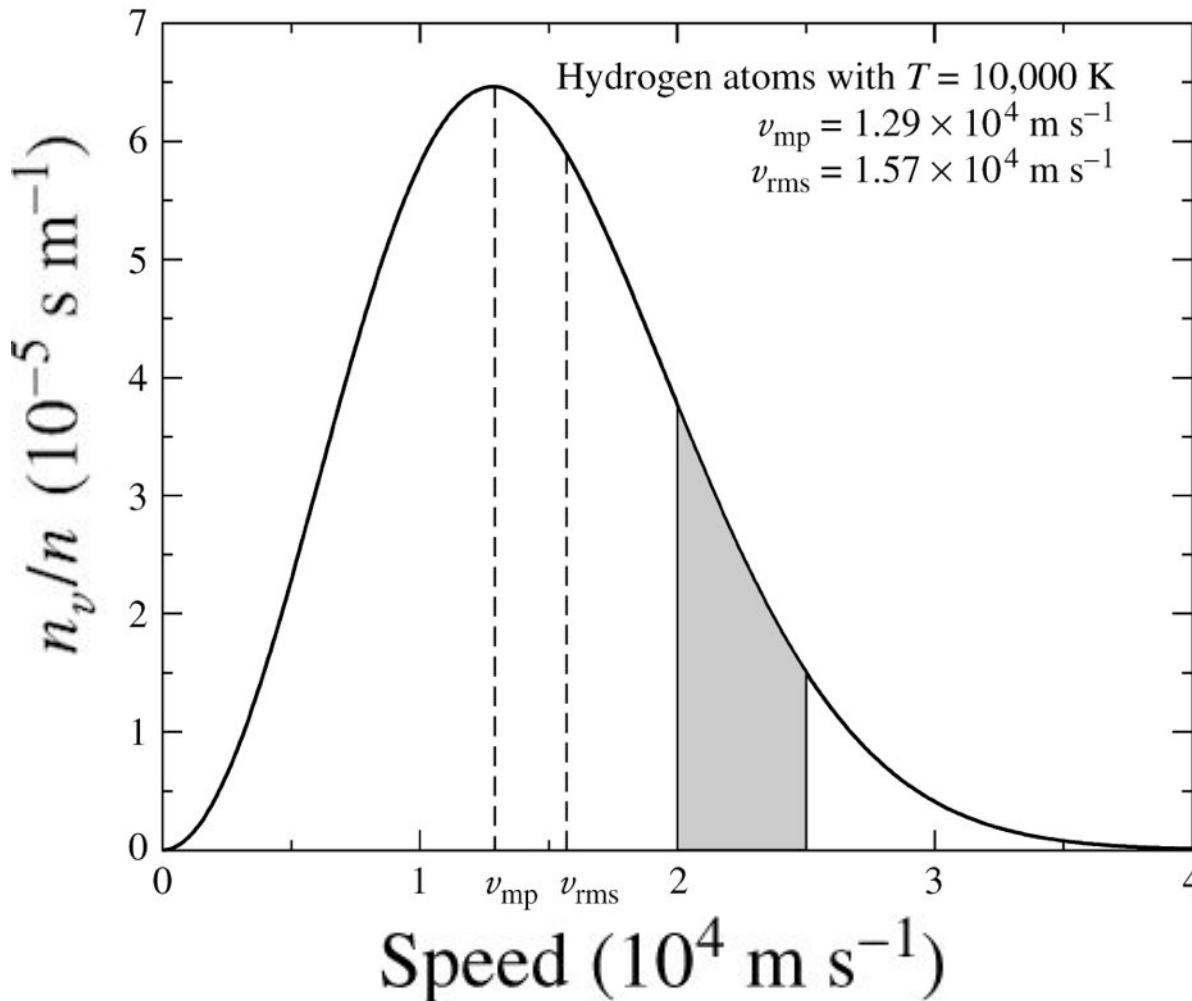
$$N/N_{\text{total}} \simeq \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-m\bar{v}^2/2kT} 4\pi \bar{v}^2 (v_2 - v_1) \simeq 0.125$$

where $\bar{v} \equiv (v_1 + v_2)/2$.



Exemplo 8.1.1. Determinar a fração de átomos de hidrogênio em um gás com $T = 10\,000\text{K}$, entre $v_1 = 2 \times 10^4\text{ m/s}$ e $v_2 = 2,5 \times 10^4\text{ m/s}$.

Aproximação:
$$N/N_{\text{total}} = \frac{1}{n} \int_{v_1}^{v_2} n_v(v) dv \simeq \frac{1}{n} n_v(\bar{v}) (v_2 - v_1) \simeq 0.125.$$



Aproximadamente 12,5% dos átomos de hidrogênio a 10000 K têm velocidades entre $2 \times 10^4 - 2,5 \times 10^4\text{ m/s}$

A integração numérica resulta em 12,75%

Linhas do espectro dependem do estado de **excitação (Boltzmann)** e **ionização (Saha)** na fotosfera estelar

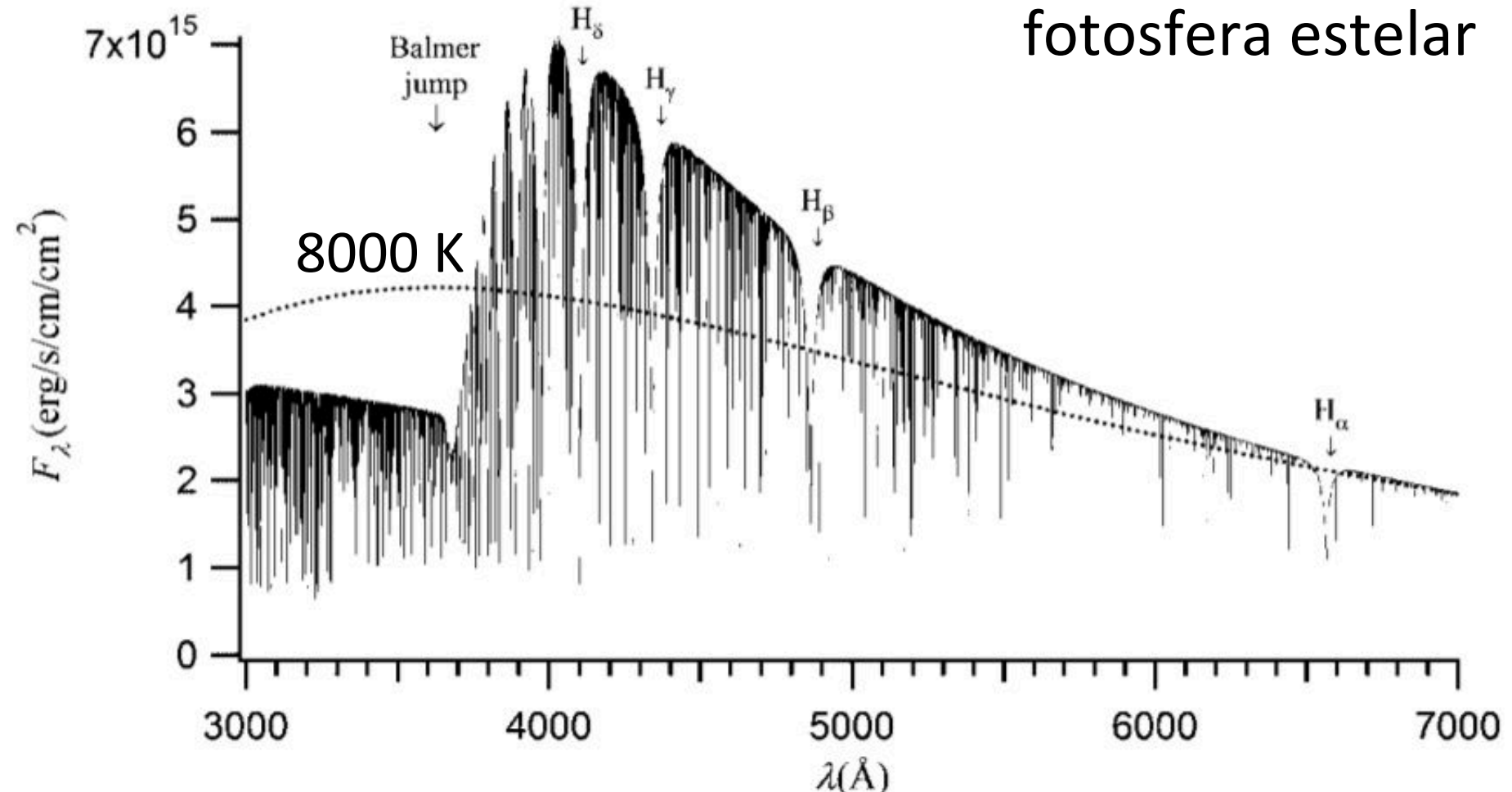
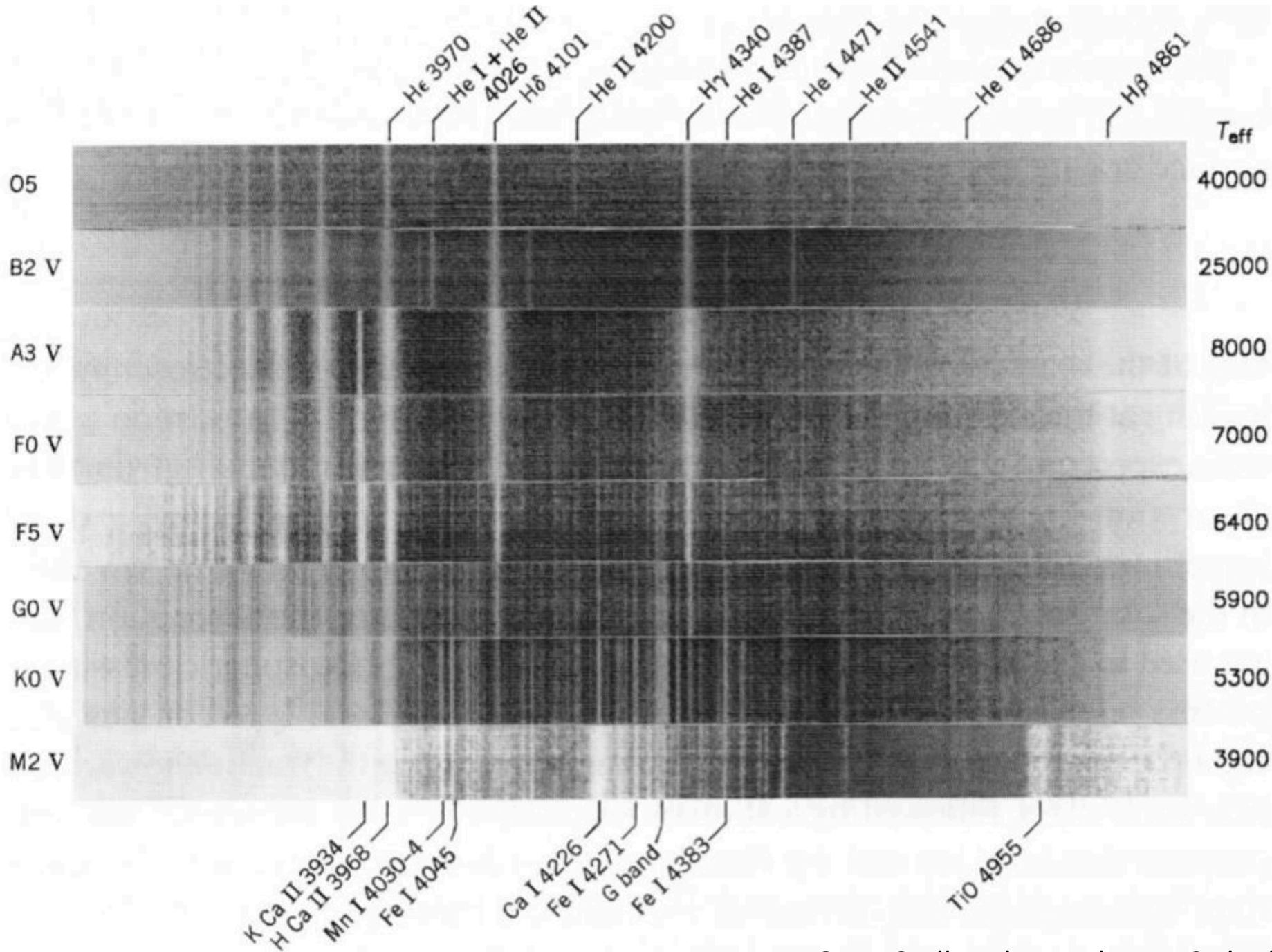
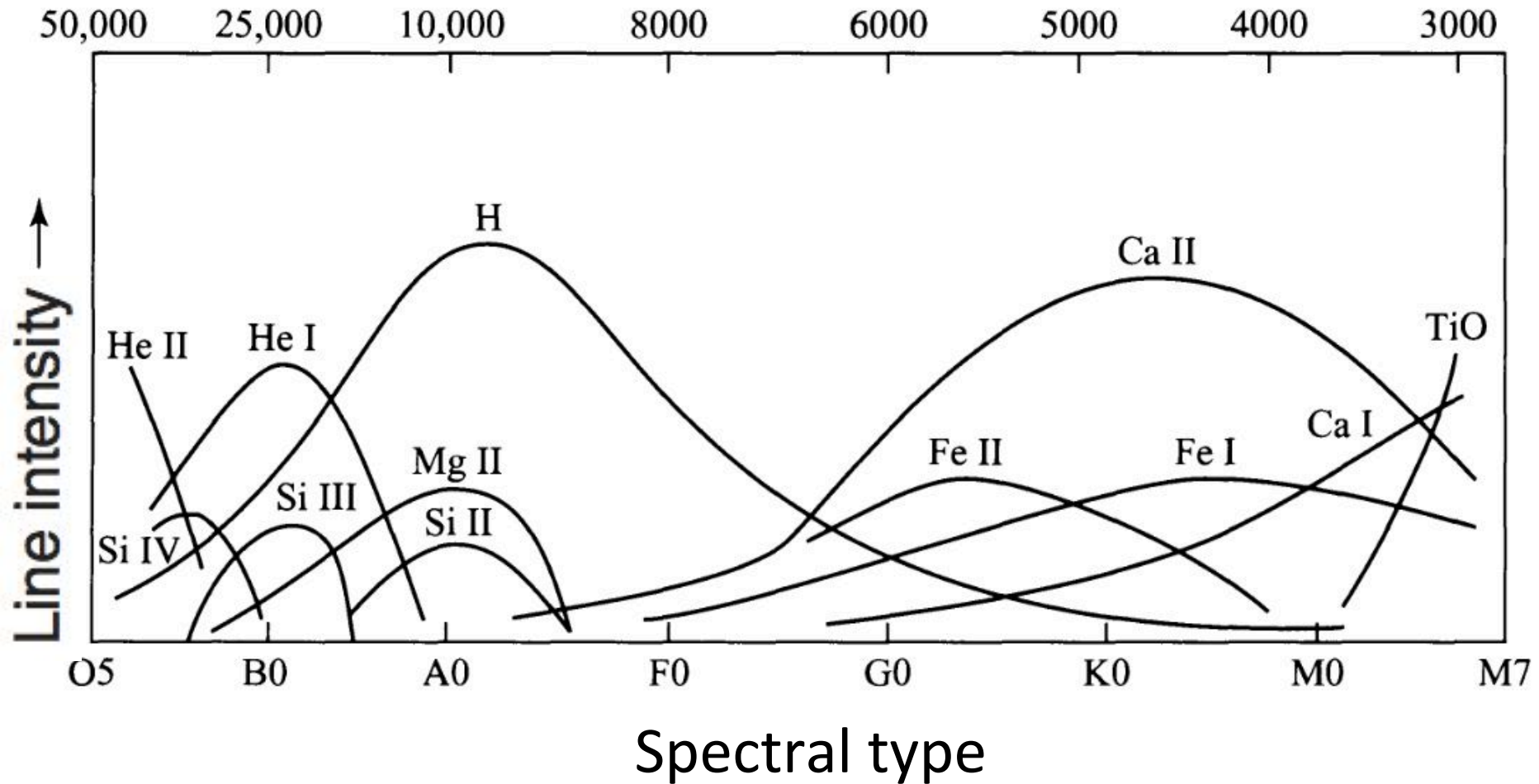


Figure 1.8 Theoretical monochromatic flux emerging from an A type star with $T_{\text{eff}} = 8000$ K. The first four Balmer absorption lines, as well as the Balmer jump, are identified in this figure. Thousands of other absorption atomic lines can also be seen. This theoretical flux was obtained with the Phoenix stellar atmosphere code (Hauschildt, P.H., Allard, F. and Baron, E., *The Astrophysical Journal*, 512, 377 (1999)) while using the elemental abundances found in the Sun. The flux at the surface of a blackbody with $T = 8000$ K (dotted curve) is also shown.

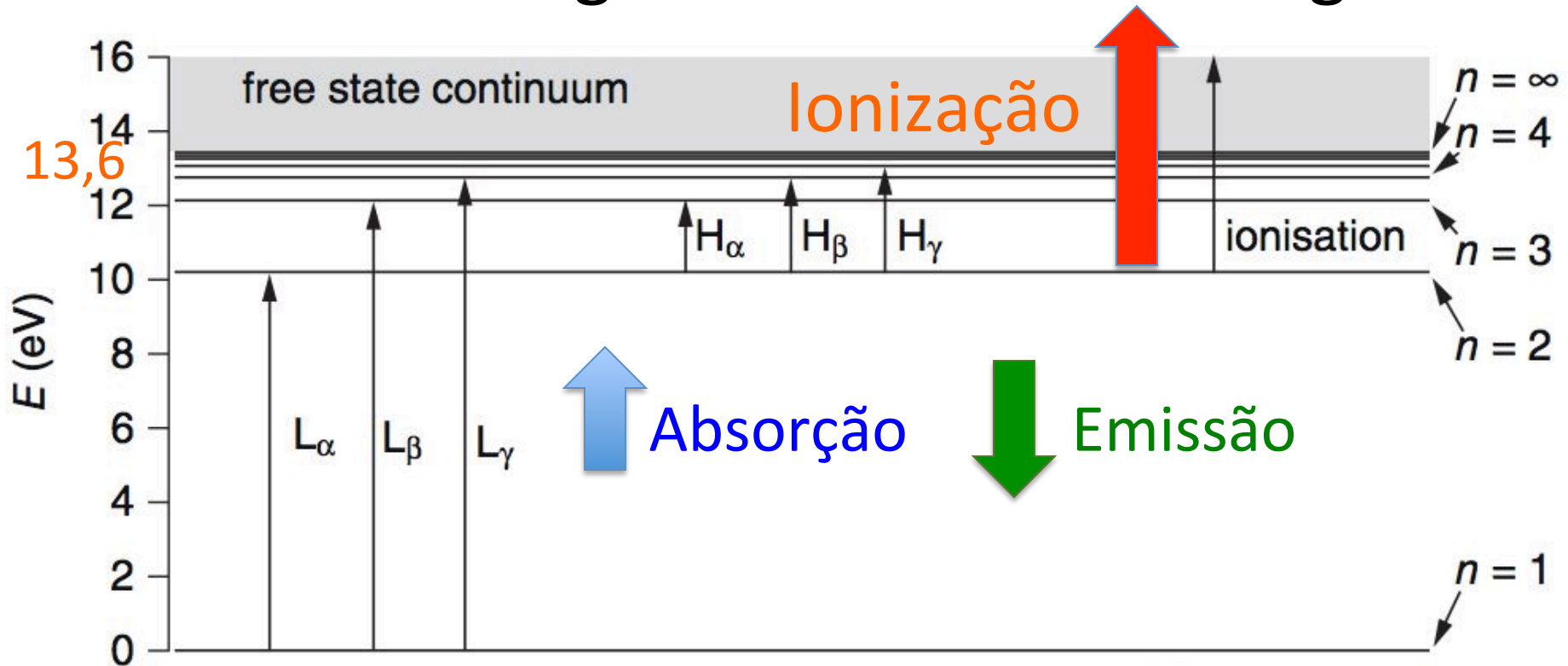


Intensidade das linhas vs. T_{eff}

Temperature [K]



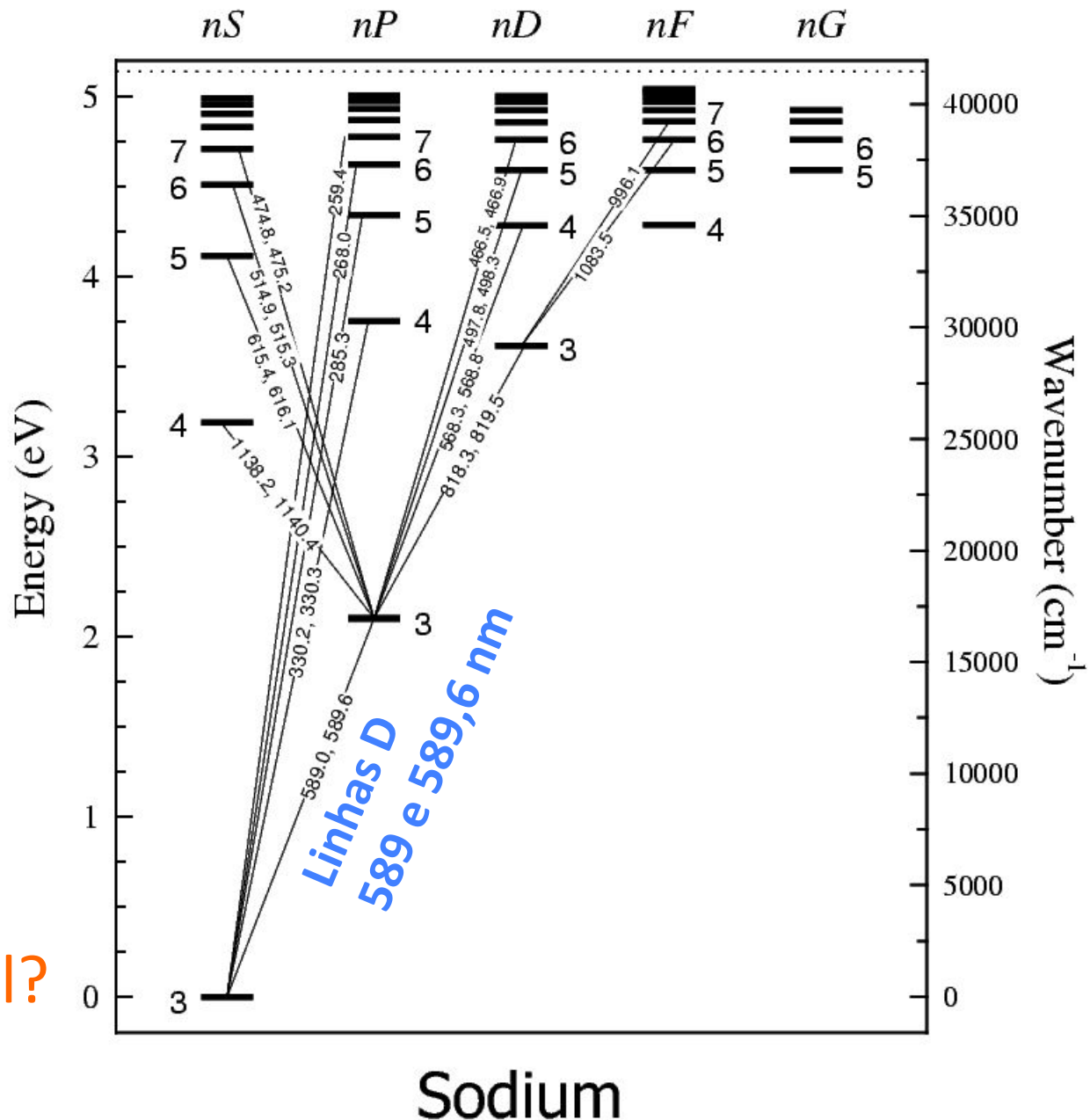
Níveis de energia no átomo de hidrogênio



Energy levels of hydrogen in eV:
$$E_n = 13.6 \left[1 - \frac{1}{n^2} \right]$$

Diagrama de níveis de energia (term ou Grotrian diagram) para o Na I

Qual a chance de popular determinado nível?



Sodium

http://128.104.164.100/data/e_sodium.gif

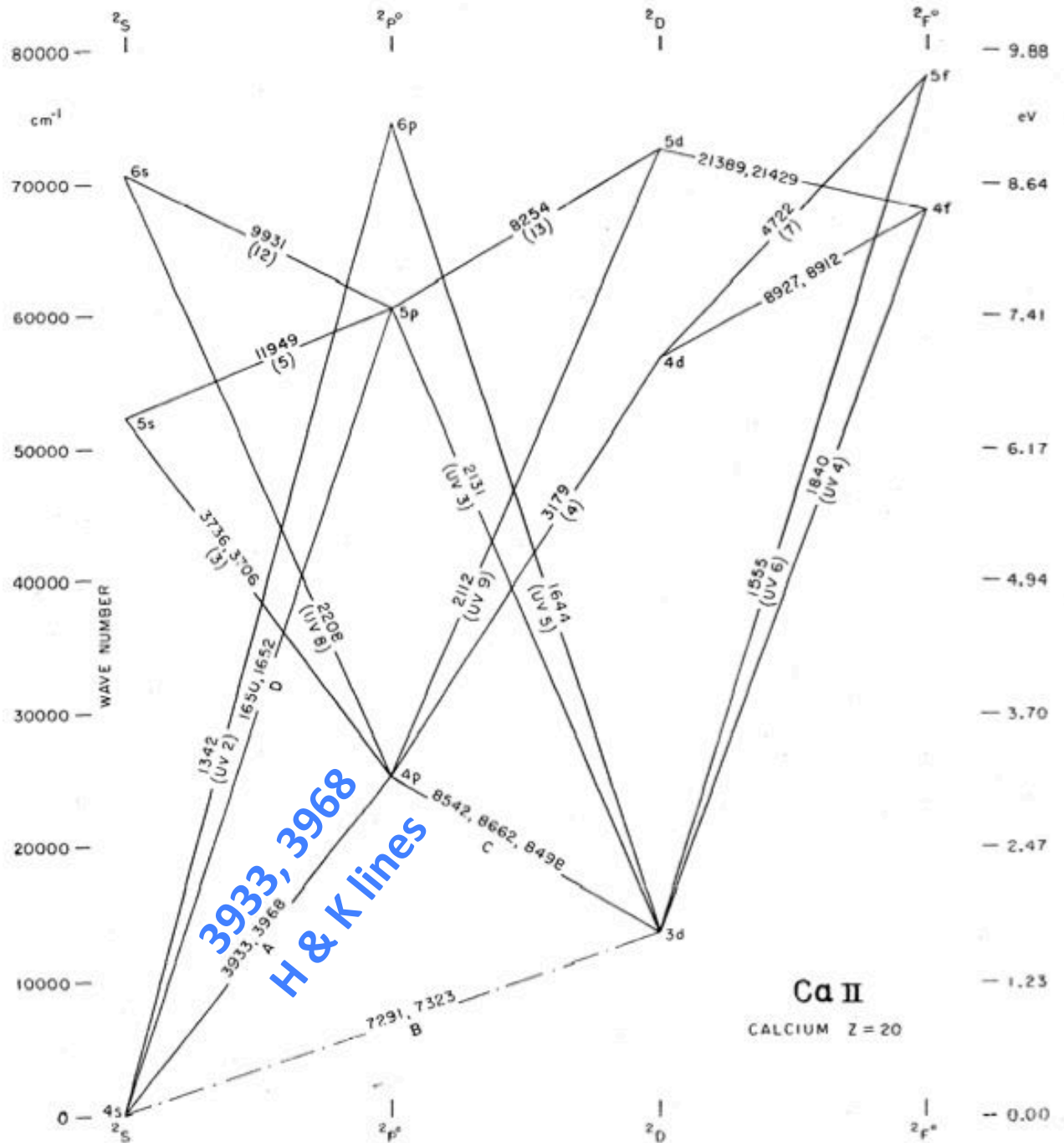
Z : 11

Ioniz. Pot. : 5.138 eV

ground state : $1s^2 2s^2 2p^6 3s$

Diagrama de níveis de energia (term ou Grotrian diagram) para o Ca II

Grotrian diagrams of Astrophysical Interest (Moore+Merrill)
<http://www.nist.gov/data/nsrds/NSRDS-NBS-23.pdf>



População entre dois níveis de energia a e b
(Equação de Boltzmann):

$$\frac{N_b}{N_a} = \frac{g_b e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

- N_a, N_b : número de átomos com energia E_a e E_b
- g_a : peso estatístico: número total de possíveis estados quânticos com energia E_a
- Para hidrogênio, $g = 2n^2$

$$k = 8.6173423 \times 10^{-5} \text{ eV K}^{-1}$$

Exemplo 8.1.2. Peso estatístico e números quânticos para o hidrogênio

Peso estatístico g_a :
 número total de
 possíveis estados
 quânticos com
 energia E_a

Hidrogênio: $g = 2n^2$

$$l = 0, \dots, n-1$$

$$m_l = -l, \dots, 0, \dots, +l$$

$$m_s = -1/2, +1/2$$

Ground States s_1				Energy E_1
n	l	m_l	m_s	(eV)
1	0	0	+1/2	-13.6
1	0	0	-1/2	-13.6

} 2

First Excited States s_2				Energy E_2
n	l	m_l	m_s	(eV)
2	0	0	+1/2	-3.40
2	0	0	-1/2	-3.40
2	1	1	+1/2	-3.40
2	1	1	-1/2	-3.40
2	1	0	+1/2	-3.40
2	1	0	-1/2	-3.40
2	1	-1	+1/2	-3.40
2	1	-1	-1/2	-3.40

} 8

Exemplo 8.1.3. Para um gás neutro de átomos de hidrogênio, qual seria a temperatura necessária para termos o mesmo número de elétrons no estado base ($n = 1$) e no primeiro estado excitado ($n = 2$) ?

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

Condição do problema: $N_b = N_a$

Para hidrogênio, $g = 2n^2$

$$E_n = -13,6 \frac{1}{n^2} \text{ eV}$$

$$1 = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

$$1 = \frac{2(2)^2}{2(1)^2} e^{-[(-13.6 \text{ eV}/2^2) - (-13.6 \text{ eV}/1^2)]/kT}$$

$$k = 8.6173423 \times 10^{-5} \text{ eV K}^{-1}$$

Exemplo 8.1.3. Para um gás neutro de átomos de hidrogênio, qual seria a temperatura necessária para termos o mesmo número de elétrons no estado base ($n = 1$) e no primeiro estado excitado ($n = 2$) ?

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$$1 = \frac{2(2)^2}{2(1)^2} e^{-[(-13.6 \text{ eV}/2^2) - (-13.6 \text{ eV}/1^2)]/kT}$$

Fazendo as contas: $\frac{10.2 \text{ eV}}{kT} = \ln(4)$

$$T = \frac{10.2 \text{ eV}}{k \ln(4)} = 8.54 \times 10^4 \text{ K}$$

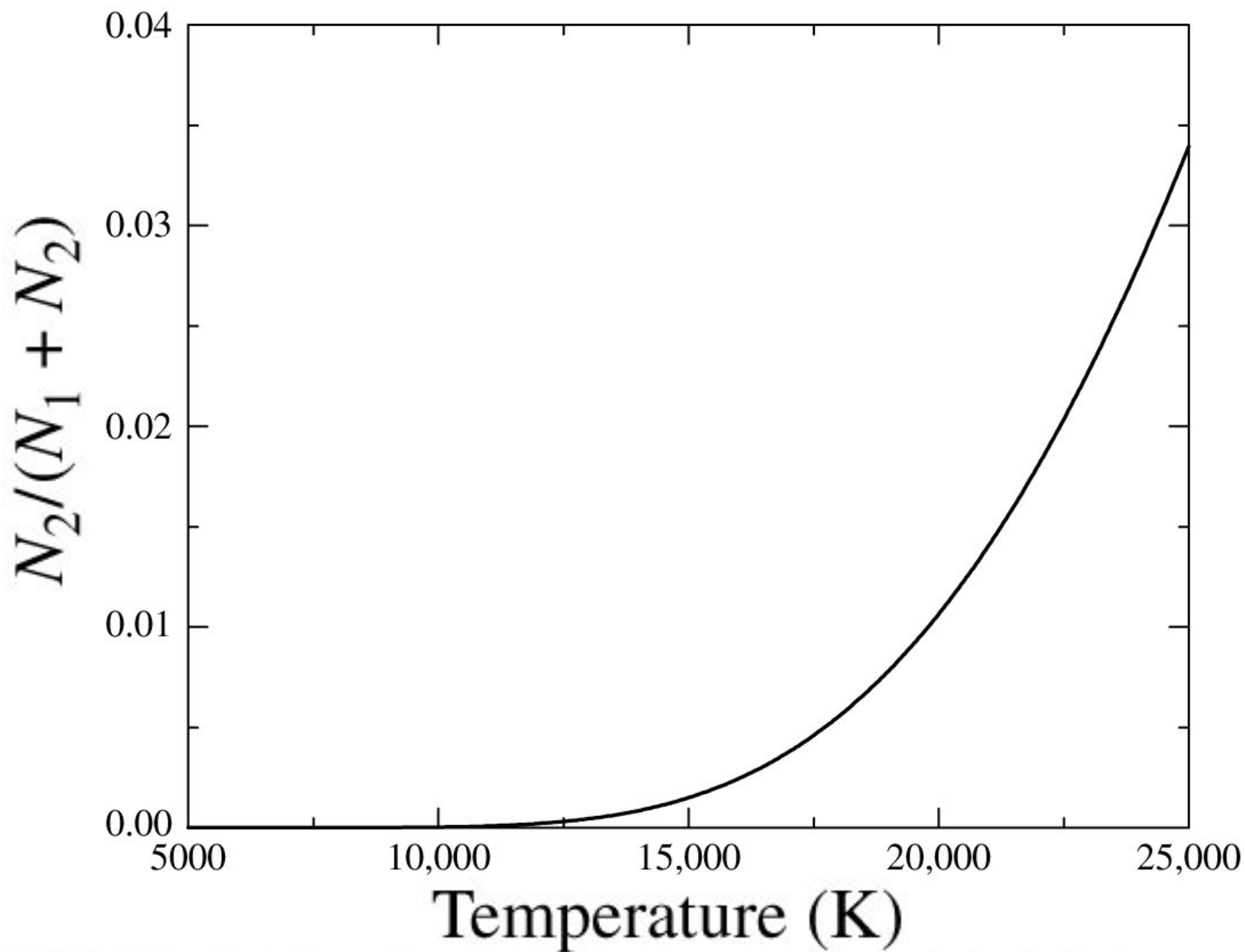
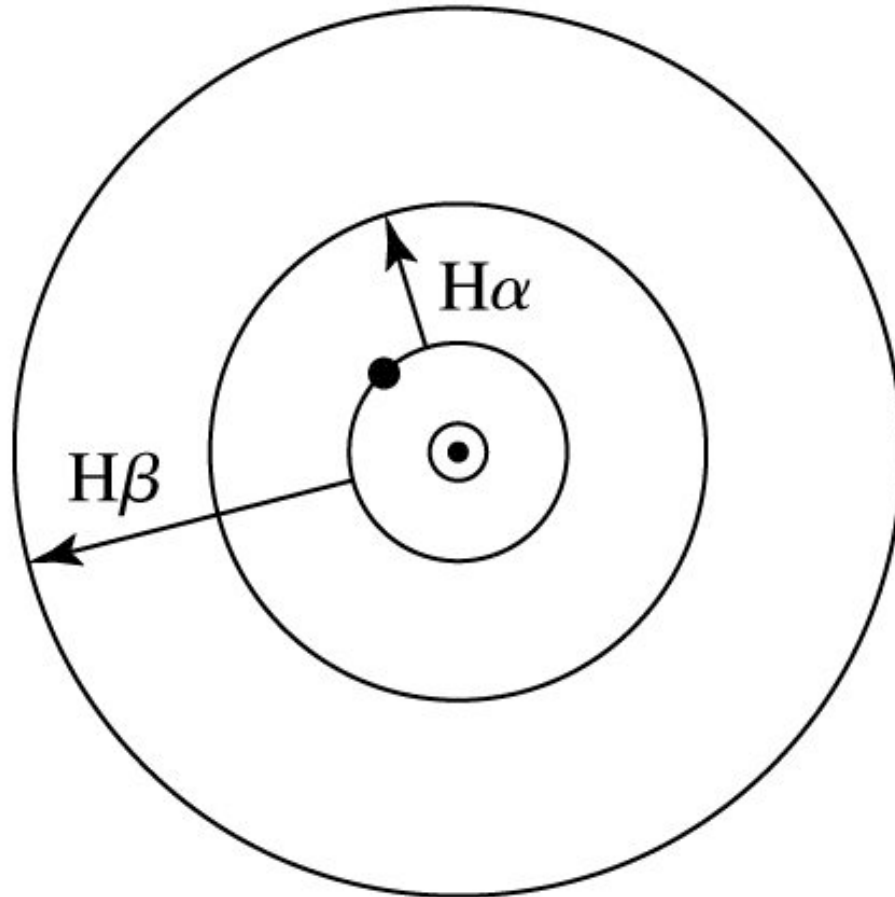


FIGURE 8.7 $N_2/(N_1 + N_2)$ for the hydrogen atom obtained via the Boltzmann equation

Como as linhas da série de Balmer
($n = 2$) são mais intensas em estrelas
A0 ($T \sim 10\,000\text{ K}$) ?



Ionização: Notação

- Hidrogênio neutro: H , H^0 ou $H\text{ I}$
- Hidrogênio ionizado: H^+ ou $H\text{ II}$
- Ferro neutro: Fe , Fe^0 ou $Fe\text{ I}$
- Ferro ionizado: Fe^+ ou $Fe\text{ II}$
- Ferro três vezes ionizado: Fe^{3+} , $Fe\text{ IV}$

Populações:

- N_n : elétron no nível de excitação n
- N_{I} : neutro; N_{II} : ionizado, N_{III} : 2 vezes ionizado
- N_n^{III} : nível de excitação n , 2 vezes ionizado

Energias de ionização

Table D.1. *Atomic weights and ionization potentials.*

No.	Element	Symbol	Weight	I_1	I_2	I_3
1	Hydrogen	H	1.008	13.598	–	–
2	Helium	He	4.003	24.587	54.418	–
3	Lithium	Li	6.941	5.392	75.640	122.454
4	Beryllium	Be	9.012	9.323	18.211	153.897
5	Boron	B	10.811	8.298	25.155	37.931
6	Carbon	C	12.011	11.260	24.383	47.888
7	Nitrogen	N	14.007	14.543	29.601	47.449
8	Oxygen	O	15.994	13.618	35.117	54.936
9	Fluorine	F	18.998	17.423	34.971	62.708
10	Neon	Ne	20.179	21.565	40.963	63.45
11	Sodium	Na	22.990	5.139	47.286	71.620
12	Magnesium	Mg	24.305	7.646	15.035	80.144
13	Aluminum	Al	26.982	5.986	18.829	28.448
14	Silicon	Si	28.086	8.152	16.346	33.493
15	Phosphorus	P	30.974	10.487	19.769	30.203
16	Sulfur	S	32.06	10.360	23.338	34.79
17	Chlorine	Cl	35.45	12.968	23.814	39.61
18	Argon	Ar	39.95	15.760	27.63	40.74
19	Potassium	K	39.10	4.341	31.63	45.806
20	Calcium	Ca	40.08	6.113	11.872	50.913

21	Scandium	Sc	44.96	6.561	12.800	24.757
22	Titanium	Ti	47.90	6.828	13.578	27.492
23	Vanadium	V	50.941	6.746	14.66	29.311

No.	Element	Symbol	Weight	I ₁	I ₂	I ₃
24	Chromium	Cr	51.996	6.767	16.486	30.96
25	Magnesium	Mn	54.938	7.434	15.640	33.668
26	Iron	Fe	55.847	7.902	16.188	30.652
27	Cobalt	Co	58.933	7.881	17.083	33.50
28	Nickel	Ni	58.71	7.640	18.169	35.19
29	Copper	Cu	63.646	7.726	20.292	36.841
30	Zinc	Zn	65.37	9.394	17.964	39.723
31	Gallium	Ga	69.72	5.999	20.514	30.71
32	Germanium	Ge	72.59	7.900	15.935	34.224
33	Arsenic	As	74.922	9.815	18.633	28.351
34	Selenium	Se	78.96	9.752	21.19	30.820
35	Bromine	Br	79.904	11.814	21.8	36
36	Krypton	Kr	83.80	14.000	24.360	36.95
37	Rubidium	Rb	85.468	4.177	27.285	40
38	Strontium	Sr	87.62	5.695	11.030	42.89
39	Yttrium	Y	88.906	6.217	12.24	20.52
40	Zirconium	Zr	91.22	6.634	13.13	22.99

Função de partição Z_i

Representa a soma pesada (peso = peso estatístico) pelas diferentes maneiras como um átomo pode arranjar seus elétrons da mesma energia

$$Z_i = \sum_{j=1}^{\infty} g_j e^{-(E_j - E_1)/kT}$$

A equação de Saha

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT}$$

Meghnad Saha (1920)

Representa a população relativa entre os átomos do nível de ionização i (N_i) e os de ionização $i+1$ (N_{i+1})

Z_i : função de partição do nível de ionização i

χ_i : energia de ionização

n_e : densidade eletrônica

A equação de Saha

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT}$$

Como

$$P_e = n_e kT$$

Então:

$$\frac{N_{i+1}}{N_i} = \frac{2kT Z_{i+1}}{P_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT}$$

$P_e = 0.1 \text{ Nm}^{-2}$ em estrelas frias a 100 Nm^{-2} em quentes.

N_i : População de átomos no nível de ionização i

Z_i : função de partição do nível de ionização i

χ_i : energia de ionização

Combinando as equações de Saha e Boltzmann

Exemplo 8.1.4. Considere o grau de ionização de uma atmosfera composta de apenas hidrogênio. Por simplicidade assumir $P_e = 20 \text{ Nm}^{-2}$

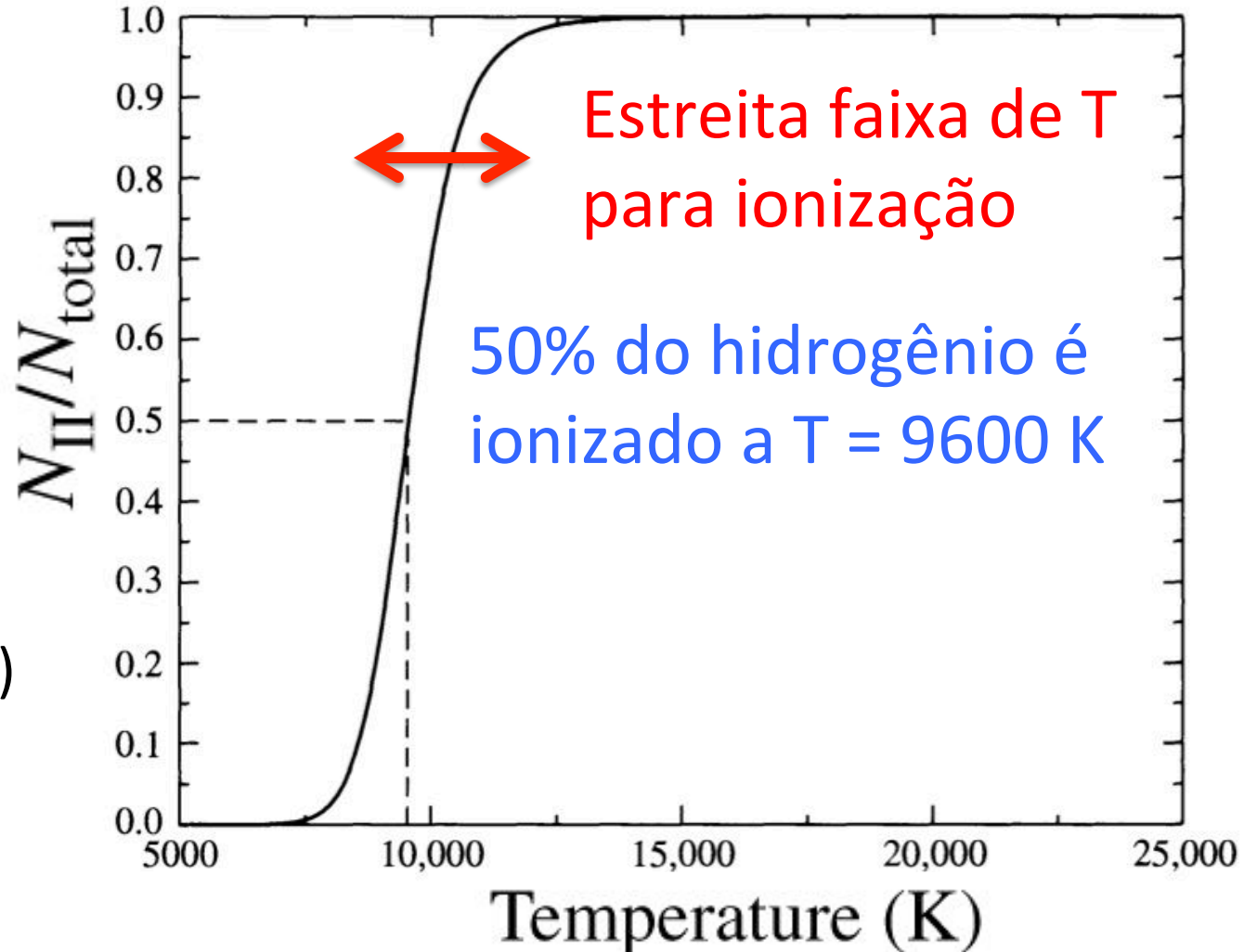
$$\frac{N_{i+1}}{N_i} = \frac{2kT Z_{i+1}}{P_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT}$$

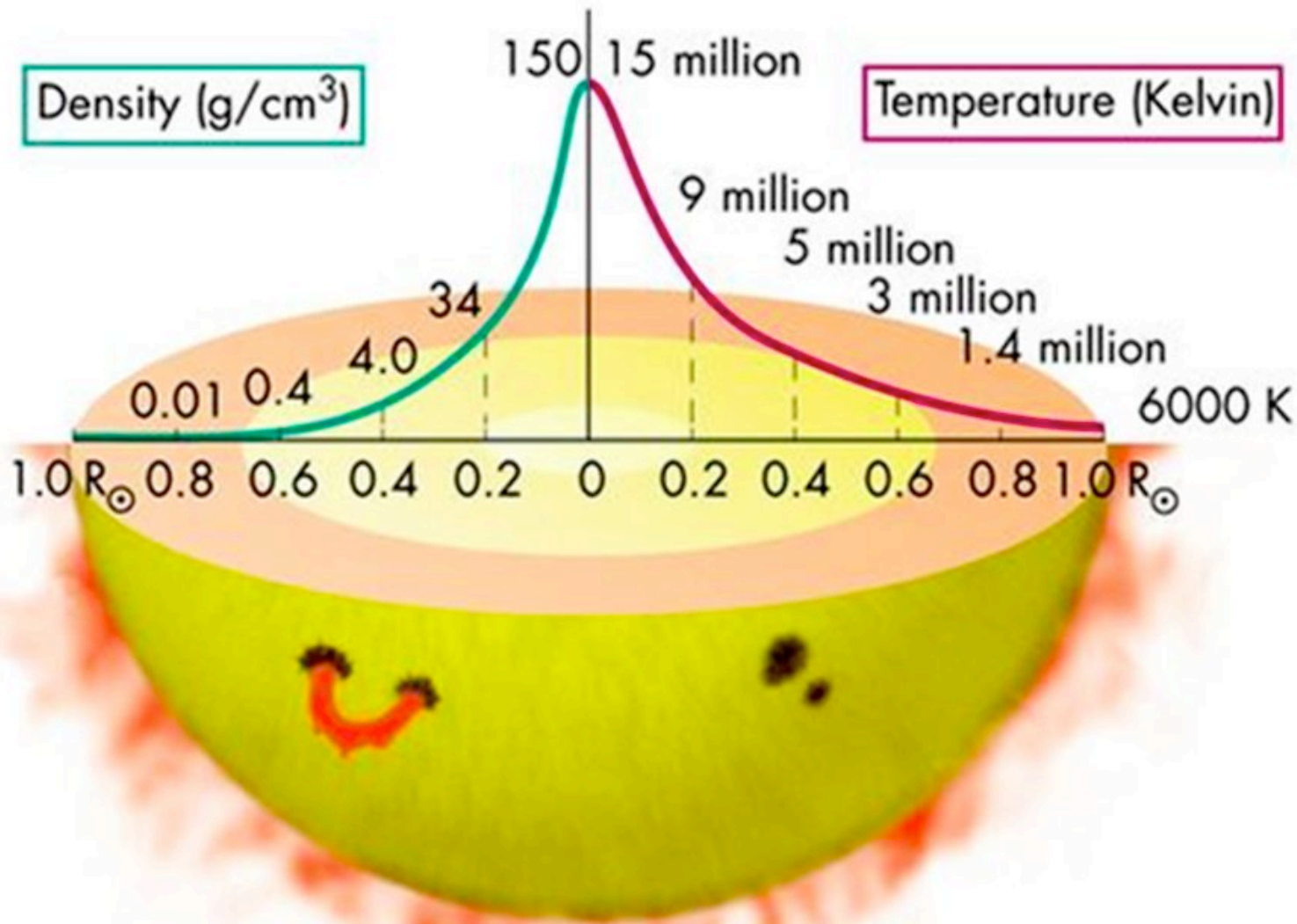
The Saha equation (8.9) will be used to calculate the fraction of atoms that are ionized, $N_{\text{II}}/N_{\text{total}} = N_{\text{II}}/(N_{\text{I}} + N_{\text{II}})$, as the temperature T varies between 5000 K and 25,000 K. However, the partition functions Z_{I} and Z_{II} must be determined first. A hydrogen ion is just a proton and so has no degeneracy; thus $Z_{\text{II}} = 1$. The energy of the first excited state of hydrogen is $E_2 - E_1 = 10.2 \text{ eV}$ above the ground state energy. Because $10.2 \text{ eV} \gg kT$ for the temperature regime under consideration, the Boltzmann factor $e^{-(E_2 - E_1)/kT} \ll 1$. Nearly all of the H I atoms are therefore in the ground state (recall the previous example), so Eq. (8.7) for the partition function simplifies to $Z_{\text{I}} \simeq g_1 = 2(1)^2 = 2$.

$$Z_{\text{I}} = 2 \quad Z_{\text{II}} = 1 \quad \chi_{\text{I}} = 13.6 \text{ eV}$$

$$\frac{N_{\text{II}}}{N_{\text{total}}} = \frac{N_{\text{II}}}{N_{\text{I}} + N_{\text{II}}} = \frac{N_{\text{II}}/N_{\text{I}}}{1 + N_{\text{II}}/N_{\text{I}}}$$

A fração de hidrogênio ionizado é a fração de N_{II} sobre o total (neutro e ionizado)

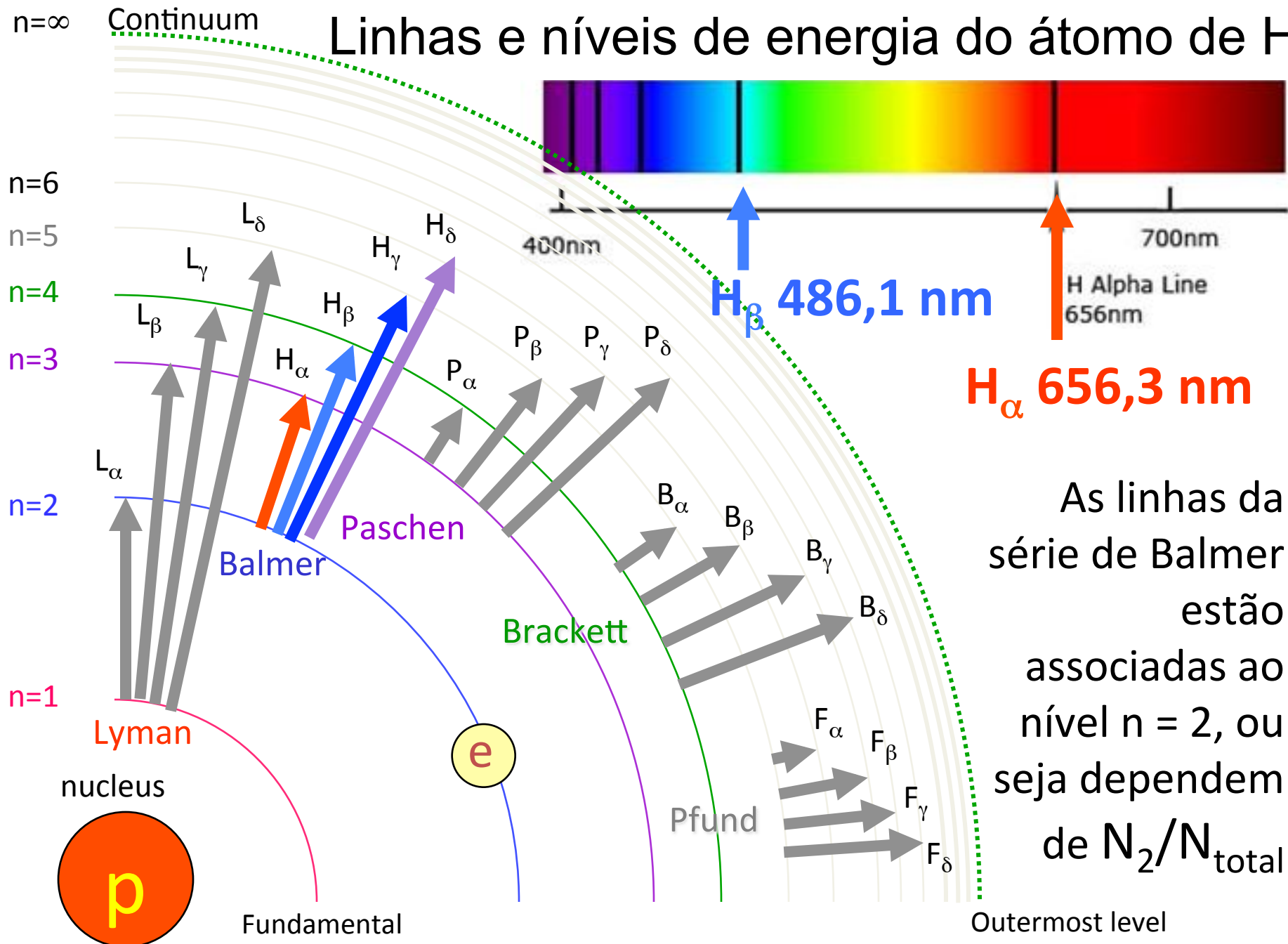




The Standard Solar Model

A região estreita onde o H é parcialmente ionizado é chamada de zona de ionização parcial

Linhas e níveis de energia do átomo de H



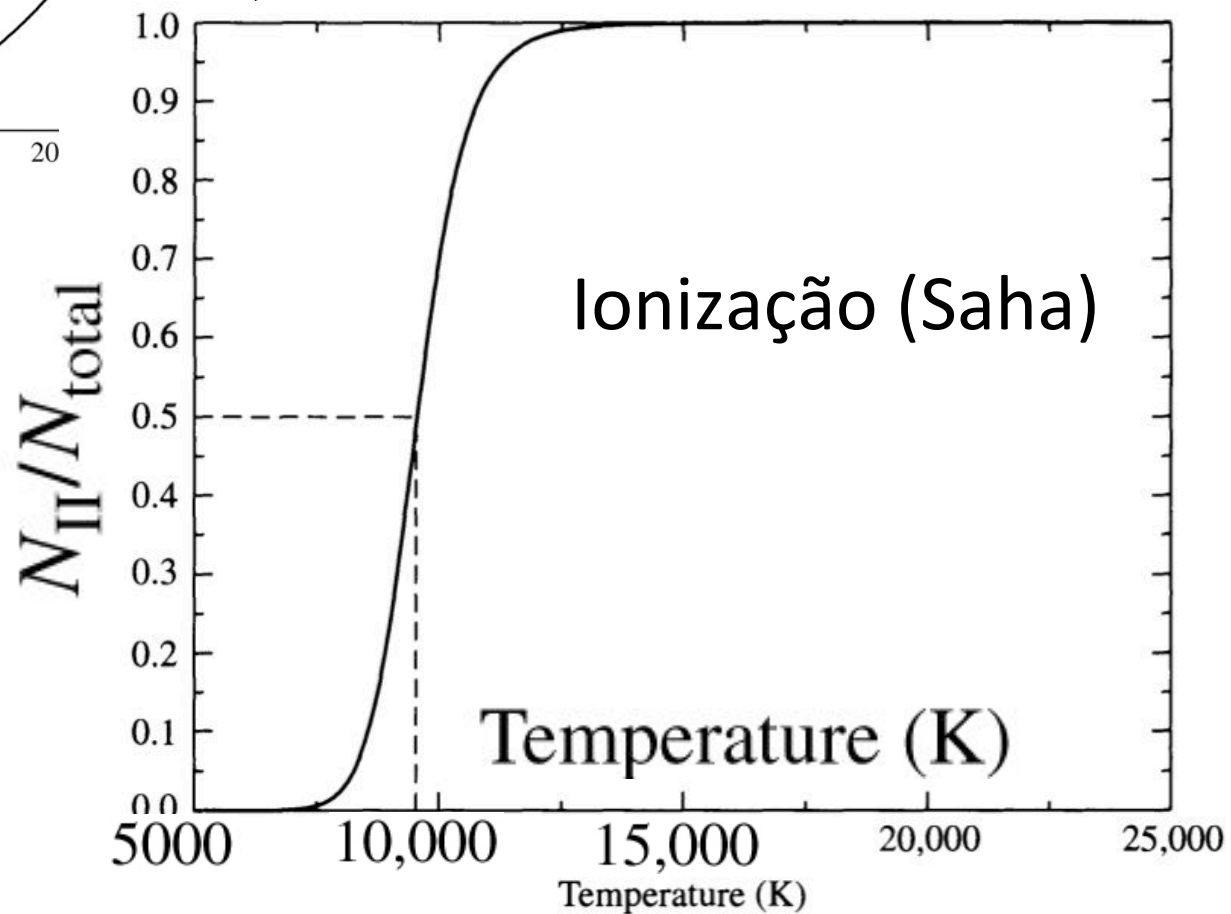
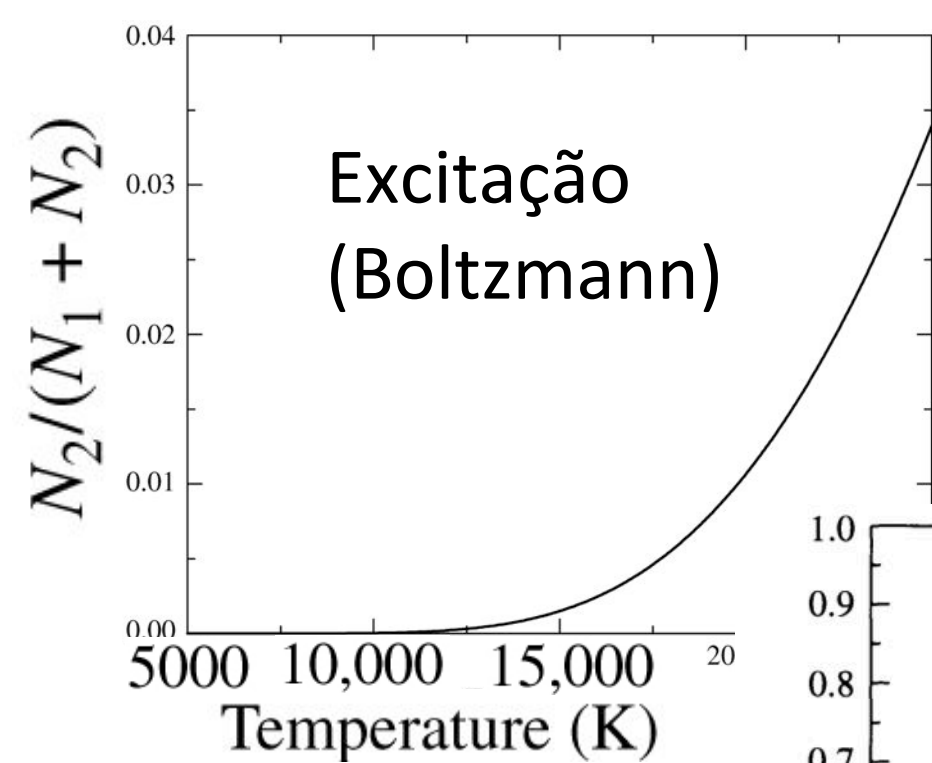
Combinando as equações de Saha e Boltzmann

A maioria de e- do H se encontram no estado base (N_1) ou no primeiro nível de excitação (N_2). Ou seja o total $N_{\text{total}} = N_1 + N_2$

$$\frac{N_2}{N_{\text{total}}} = \left(\frac{N_2}{N_1 + N_2} \right) \left(\frac{N_1}{N_{\text{total}}} \right) = \left(\frac{N_2/N_1}{1 + N_2/N_1} \right) \left(\frac{1}{1 + N_{\text{II}}/N_1} \right)$$

Boltzmann

Saha



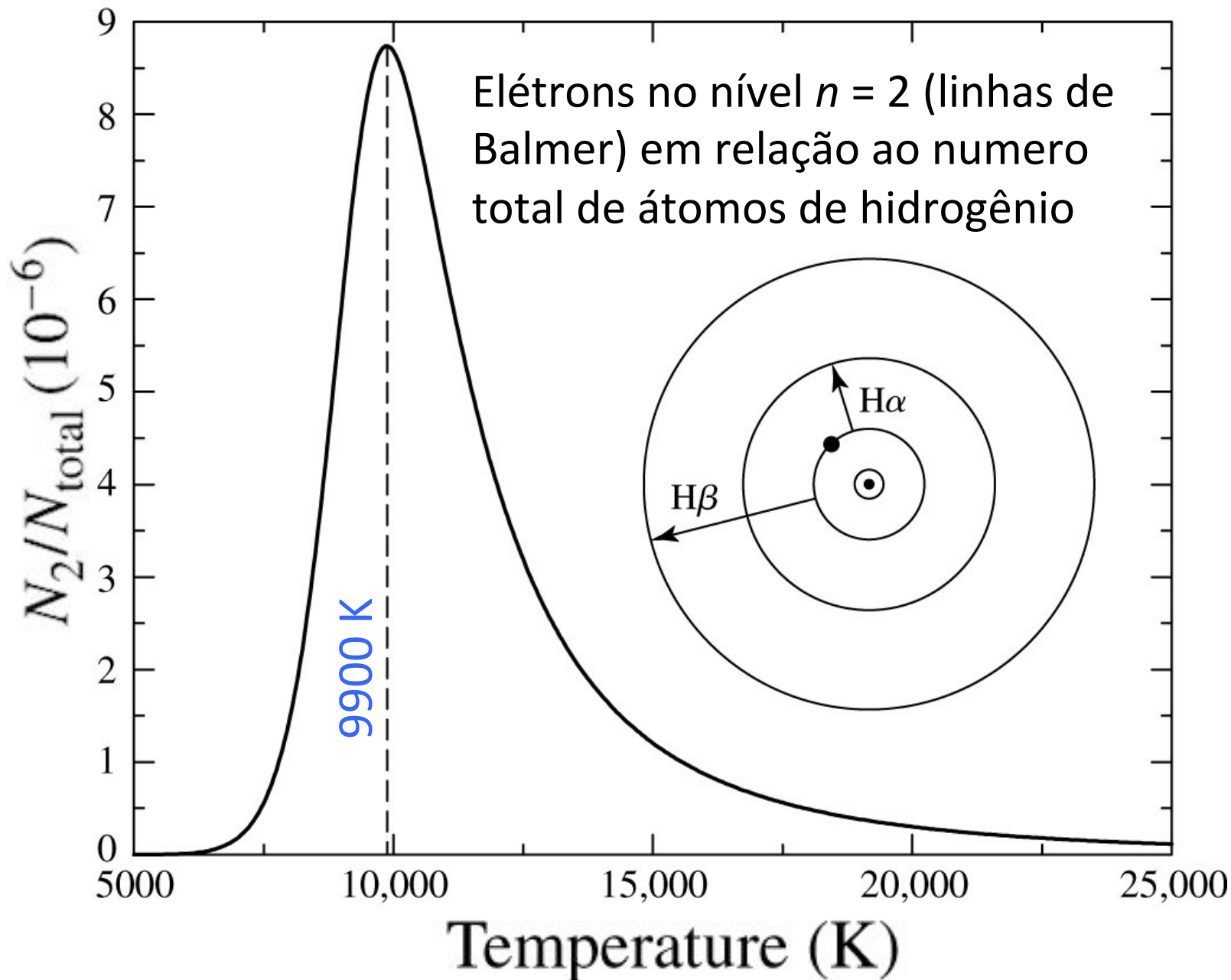


FIGURE 8.9 N_2/N_{total} for hydrogen from the Boltzmann and Saha equations, assuming $P_e = 20 \text{ N m}^{-2}$. The peak occurs at approximately 9900 K.

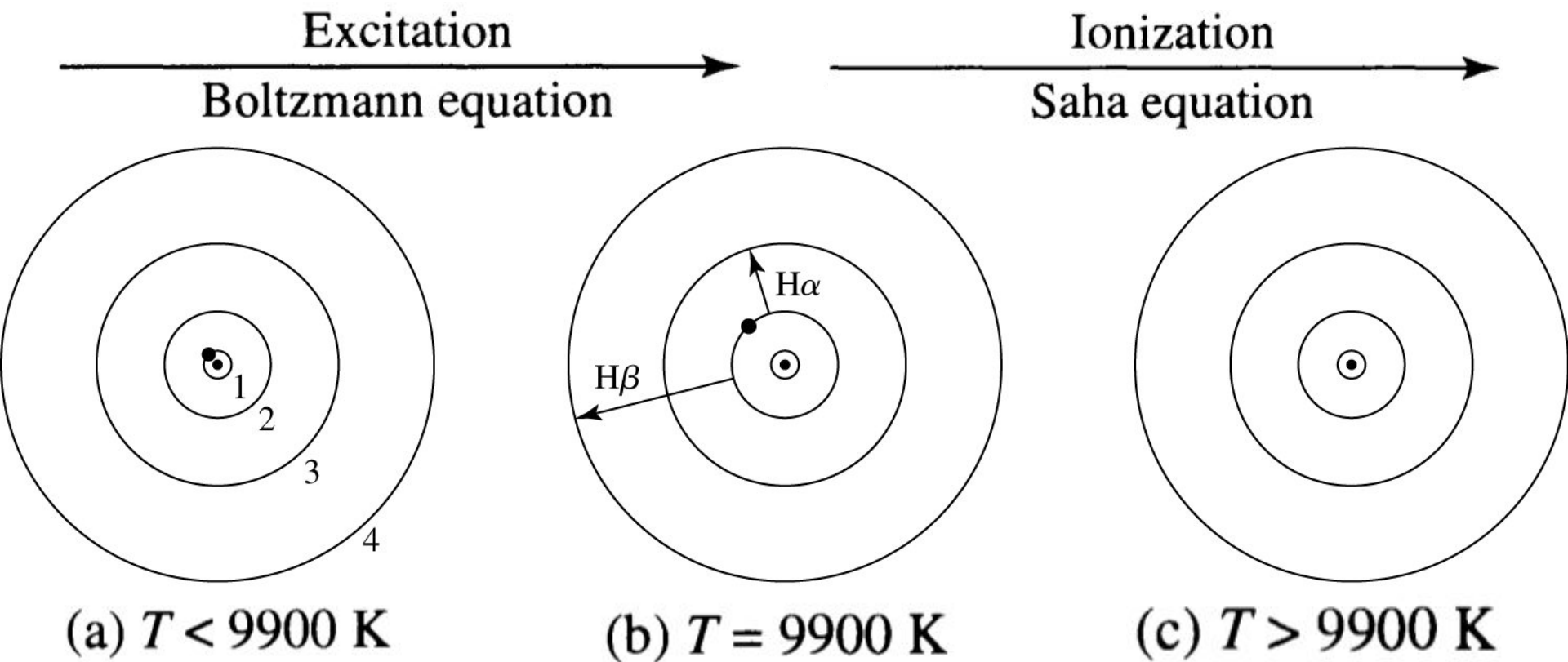
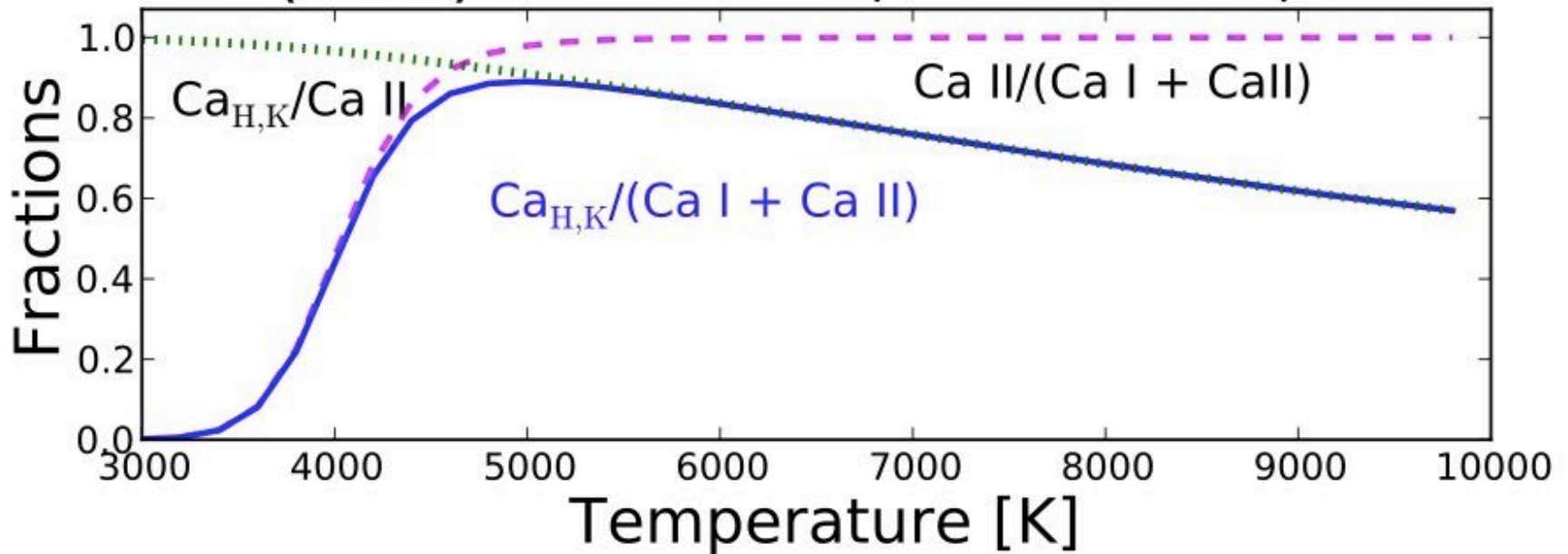


FIGURE 8.10 The electron's position in the hydrogen atom at different temperatures. In (a), the electron is in the ground state. Balmer absorption lines are produced only when the electron is initially in the first excited state, as shown in (b). In (c), the atom has been ionized.

Linhas H e K em espectros estelares

De maneira similar, podemos estimar a intensidade das linhas de Ca H e K (são linhas de Ca ionizado: Ca II)

H and K (Ca II) lines. Saha, Boltzmann, combined



Example 8.1.5. The Sun's "surface" is a thin layer of the solar atmosphere called the *photosphere*; see Section 11.2. The characteristic temperature of the photosphere is $T = T_e = 5777$ K, and it has about 500,000 hydrogen atoms for each calcium atom with an electron pressure of about 1.5 N m^{-2} .¹⁴ From this information and knowledge of the appropriate statistical weights and partition functions, the Saha and Boltzmann equations can be used to estimate the relative strengths of the absorption lines due to hydrogen (the Balmer lines) and those due to calcium (the Ca II H and K lines).

We must compare the number of neutral hydrogen atoms with electrons in the first excited state (which produce the Balmer lines) to the number of singly ionized calcium atoms with electrons in the ground state (which produce the Ca II H and K lines). As in Example 8.1.4, we will use the Saha equation to determine the degree of ionization and will use the Boltzmann equation to reveal the distribution of electrons between the ground and first excited states.

Como se compara a absorção das linhas de Balmer de H com as linhas de Ca H & K (linhas de Ca II no estado base)?

Let's consider hydrogen first. If we substitute the partition functions found in Example 8.1.4 into the Saha equation (8.9), the ratio of ionized to neutral hydrogen is

$$\left[\frac{N_{\text{II}}}{N_{\text{I}}} \right]_{\text{H}} = \frac{2kT Z_{i+1}}{P_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT} = 7.70 \times 10^{-5} \simeq \frac{1}{13,000}$$

Thus there is only one hydrogen ion (H II) for every 13,000 neutral hydrogen atoms (H I) at the Sun's surface. Almost none of the hydrogen is ionized.

Apenas 1 H II (H neutro) a cada 13 000 H I (neutro)

The Boltzmann equation (8.6) reveals how many of these neutral hydrogen atoms are in the first excited state. Using $g_n = 2n^2$ for hydrogen (implying $g_1 = 2$ and $g_2 = 8$), we have

$$\left[\frac{N_2}{N_1} \right]_{\text{H I}} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT} = 5.06 \times 10^{-9} \simeq \frac{1}{198,000,000}$$

The result is that only one of every 200 million hydrogen atoms is in the first excited state and capable of producing Balmer absorption lines:

1 em cada 200 milhões de átomos de H está em n=2

$$\frac{N_2}{N_{\text{total}}} = \left(\frac{N_2}{N_1 + N_2} \right) \left(\frac{N_1}{N_{\text{total}}} \right) = 5.06 \times 10^{-9}$$

Os que de fato produzem linhas de Balmer apenas 1 em 10^9 !!!

We now turn to the calcium atoms. The ionization energy χ_I of Ca I is 6.11 eV, about half of the 13.6 eV ionization energy of hydrogen. We will soon see, however, that this small difference has a great effect on the ionization state of the atoms. Note that the Saha equation is very sensitive to the ionization energy because χ/kT appears as an *exponent* and $kT \approx 0.5 \text{ eV} \ll \chi$. Thus a difference of several electron volts in the ionization energy produces a change of many powers of e in the Saha equation.

Evaluating the partition functions Z_I and Z_{II} for calcium is a bit more complicated than for hydrogen, and the results have been tabulated elsewhere:¹⁵ $Z_I = 1.32$ and $Z_{II} = 2.30$. Thus the ratio of ionized to un-ionized calcium is

$$\left[\frac{N_{II}}{N_I} \right]_{\text{Ca}} = \frac{2kT Z_{II}}{P_e Z_I} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_I/kT} = 918$$

A cada 918 Ca II, apenas 1 é neutro (Ca I)

Practically all of the calcium atoms are in the form of Ca II; only one atom out of 900 remains neutral. Now we can use the Boltzmann equation to estimate how many of these calcium ions are in the ground state, capable of forming the Ca II H and K absorption lines. The next calculation will consider the K ($\lambda = 393.3 \text{ nm}$) line; the results for the H ($\lambda = 396.8 \text{ nm}$) line are similar. The first excited state of Ca II is $E_2 - E_1 = 3.12 \text{ eV}$ above the ground state. The degeneracies for these states are $g_1 = 2$ and $g_2 = 4$. Thus the ratio of the number of Ca II ions in the first excited state to those in the ground state is

$$\left[\frac{N_2}{N_1} \right]_{\text{Ca II}} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT} = 3.79 \times 10^{-3} = \frac{1}{264}$$

Quase tudo o Ca II se encontra no nível base n=1

Out of every 265 Ca II ions, all but one are in the ground state and are capable of producing the Ca II K line. This implies that nearly *all* of the calcium atoms in the Sun's photosphere are singly ionized and in the ground state,¹⁶ so that almost all of the calcium atoms are available for forming the H and K lines of calcium:

$$\begin{aligned} \left[\frac{N_1}{N_{\text{total}}} \right]_{\text{Ca II}} &\approx \left[\frac{N_1}{N_1 + N_2} \right]_{\text{Ca II}} \left[\frac{N_{\text{II}}}{N_{\text{total}}} \right]_{\text{Ca}} \\ &= \left(\frac{1}{1 + [N_2/N_1]_{\text{Ca II}}} \right) \left(\frac{[N_{\text{II}}/N_{\text{I}}]_{\text{Ca}}}{1 + [N_{\text{II}}/N_{\text{I}}]_{\text{Ca}}} \right) \\ &= \left(\frac{1}{1 + 3.79 \times 10^{-3}} \right) \left(\frac{918}{1 + 918} \right) \\ &= 0.995. \end{aligned}$$

Quase tudo o Ca pode produzir linhas de H e K

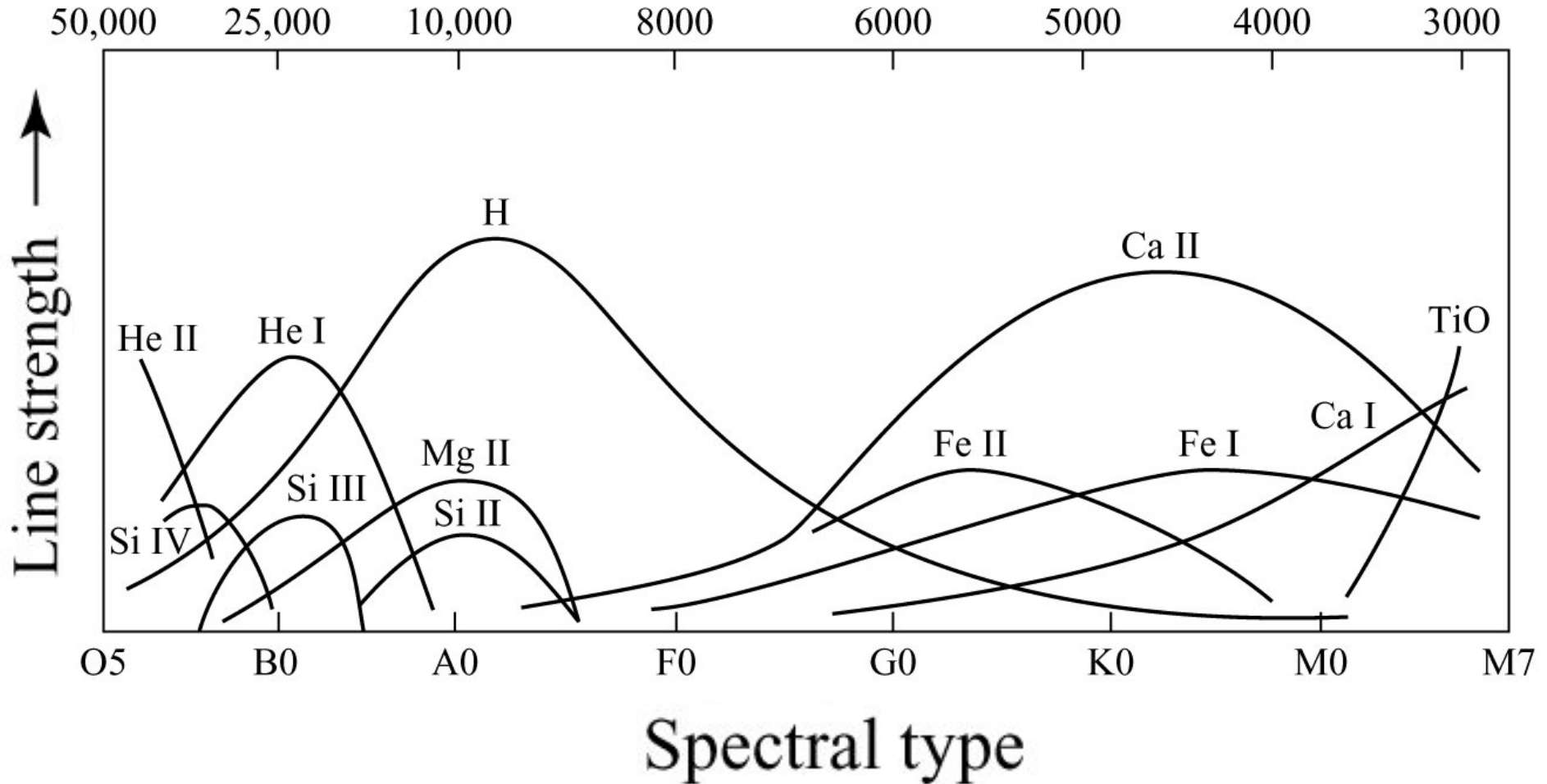
Now it becomes clear why the Ca II H and K lines are so much stronger in the Sun's spectrum than are the Balmer lines. There are 500,000 hydrogen atoms for every calcium atom in the solar photosphere, but only an extremely small fraction, 5.06×10^{-9} , of these hydrogen atoms are un-ionized and in the first excited state, capable of producing a Balmer line. Multiplying these two factors,

$$(500,000) \times (5.06 \times 10^{-9}) \approx 0.00253 = \frac{1}{395},$$

reveals that there are approximately 400 times more Ca II ions with electrons in the ground state (to produce the Ca II H and K lines) than there are neutral hydrogen atoms with electrons in the first excited state (to produce the Balmer lines). The strength of the H and K lines is *not* due to a greater abundance of calcium in the Sun. Rather, the strength of these Ca II lines reflects the sensitive temperature dependence of the atomic states of excitation and ionization.

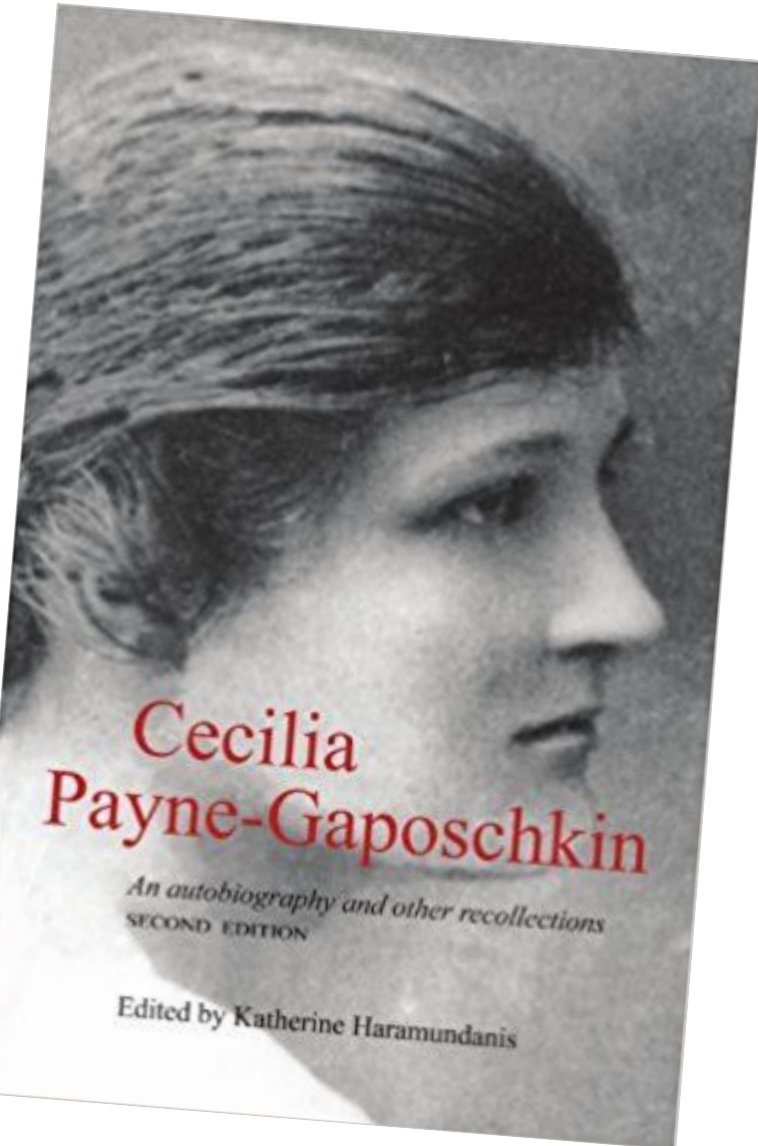
Intensidade das linhas vs. T_{eff}

Temperature (K)



Cecilia Payne-Gaposchkin

(May 10, 1900 – December 7, 1979)



1919: Botânica, Física e Química em Cambridge (UK)

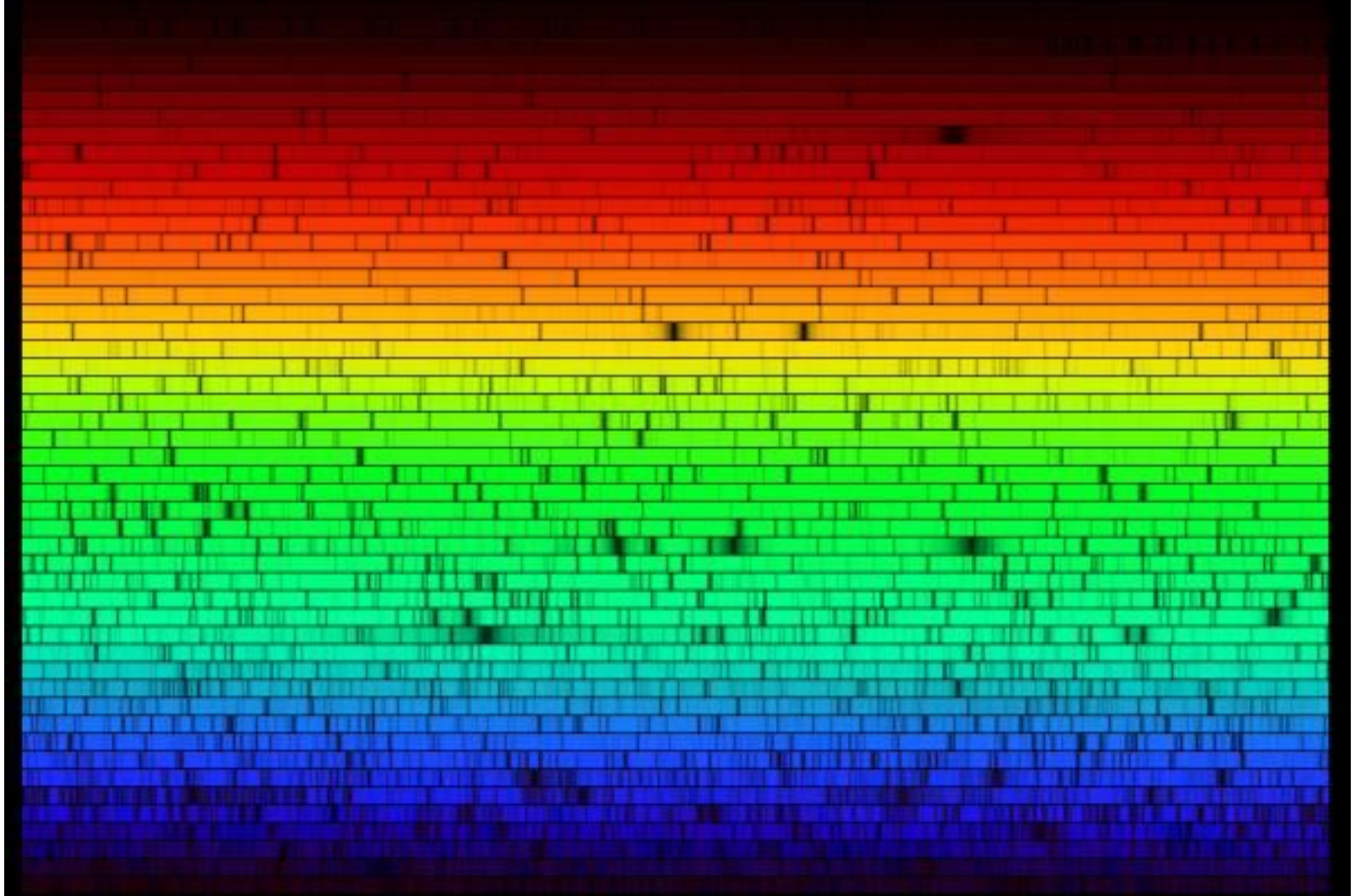
1922: inicia estudos de astronomia nos Estados Unidos (Harvard)

1925: Tese de doutorado

H e He são os elementos mais abundantes em estrelas

Cecilia Payne Gaposchkin

Mas a maioria de linhas no espectro de estrelas é devida ao ferro ...





**Shapley: diretor do Harvard observatory
(supervisor da Cecilia Payne)**



Cecilia foi obrigada a mencionar na tese de doutorado que seus resultados sobre a alta abundância de H e He poderiam estar errados

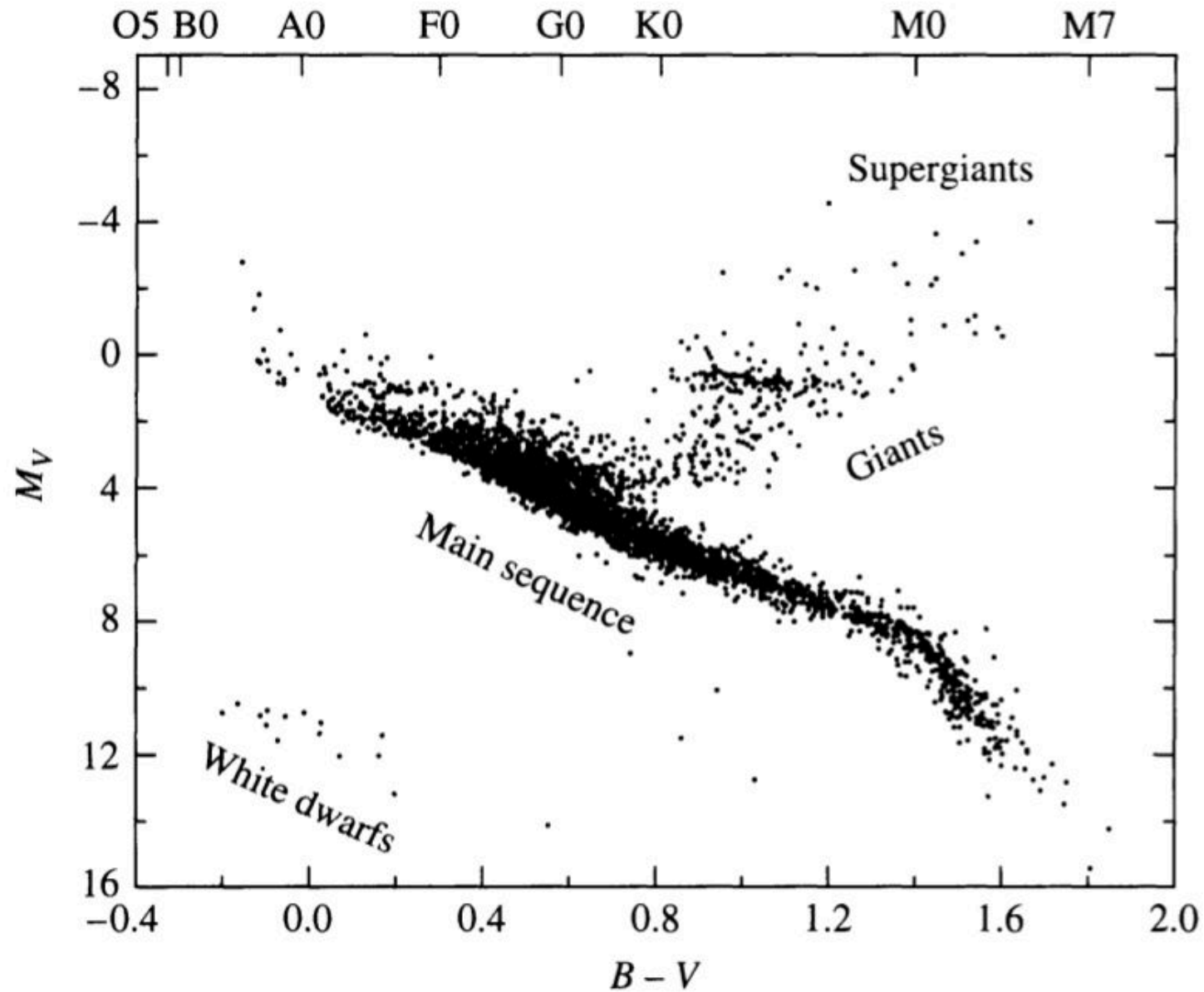
Professor Russell (Princeton)



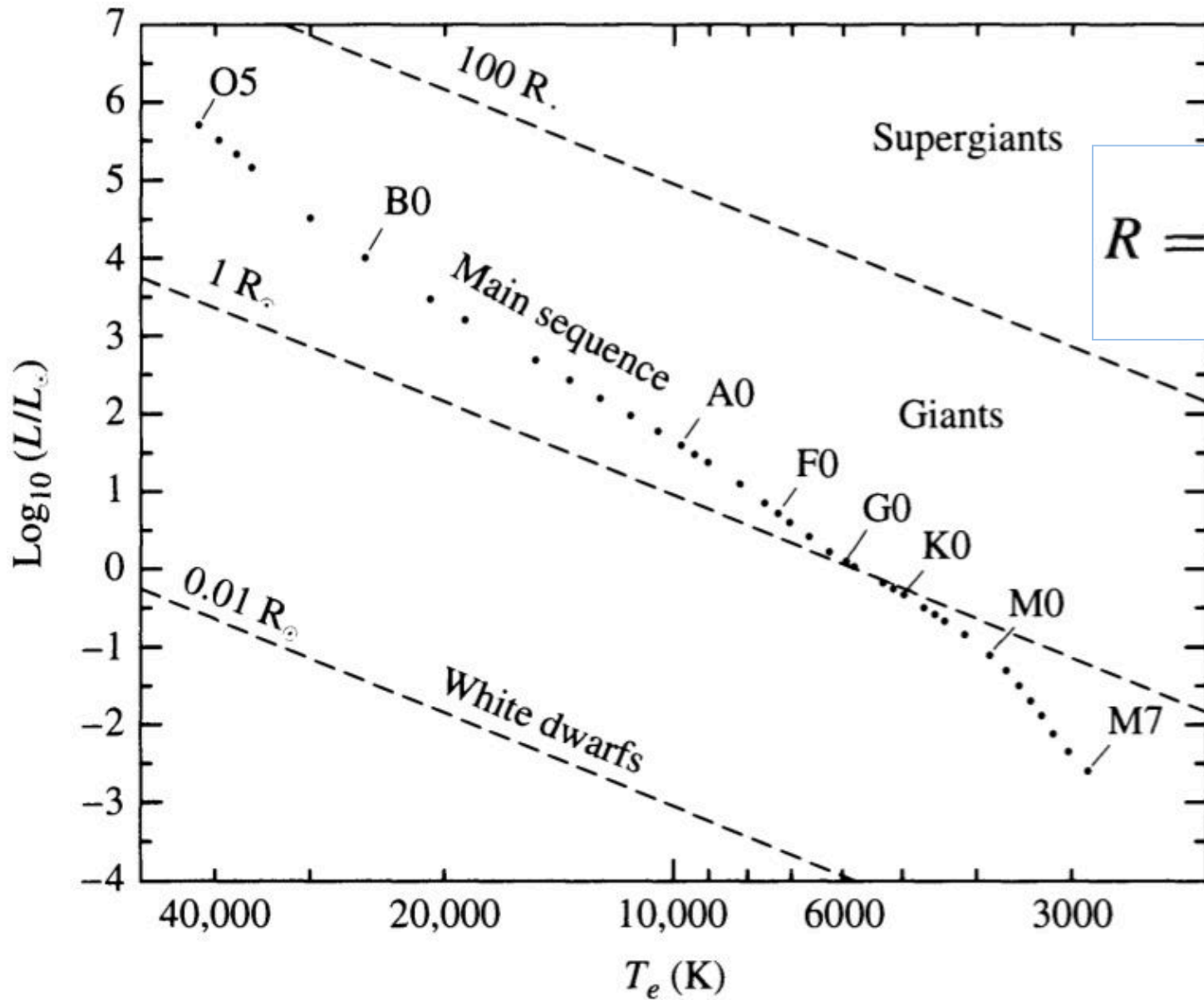
*É impossível !,
o Sol NÃO é composto
maiormente de H e He*

4 anos depois Russell publicou um artigo anunciando que o Sol é maiormente H ...

The H-R diagram: observational



The H-R diagram: theoretical



$$R = \frac{1}{T_e^2} \sqrt{\frac{L}{4\pi\sigma}}$$

Example 8.2.1. The Sun, a G2 main-sequence star, has a mass of $M_{\odot} = 1.9891 \times 10^{30}$ kg and a radius of $R_{\odot} = 6.95508 \times 10^8$ m. Its average density is thus

$$\bar{\rho}_{\odot} = \frac{M_{\odot}}{\frac{4}{3} \pi R_{\odot}^3} = 1410 \text{ kg m}^{-3}.$$

Sirius, the brightest-appearing star in the sky, is classified as an A1 main sequence star with a mass of $2.2 M_{\odot}$ and a radius of $1.6 R_{\odot}$. The average density of Sirius is

$$\bar{\rho} = \frac{2.2 M_{\odot}}{\frac{4}{3} \pi (1.6 R_{\odot})^3} = 760 \text{ kg m}^{-3} = 0.54 \bar{\rho}_{\odot},$$

which is about 76 percent of the density of water. However, this is enormously dense compared to a giant or supergiant star. The mass of Betelgeuse is estimated to lie between 10 and 15 M_{\odot} ; we will adopt 10 M_{\odot} here. For illustration, if we take the maximum radius of this pulsating star to be about 1000 R_{\odot} , then the average density of Betelgeuse (at maximum size) is roughly

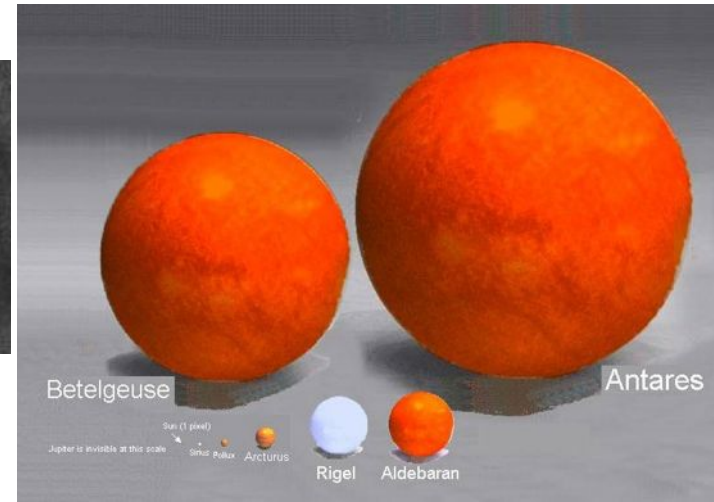
$$\bar{\rho} = \frac{10 M_{\odot}}{\frac{4}{3} \pi (1000 R_{\odot})^3} = 10^{-8} \bar{\rho}_{\odot}!$$

Classe de luminosidade

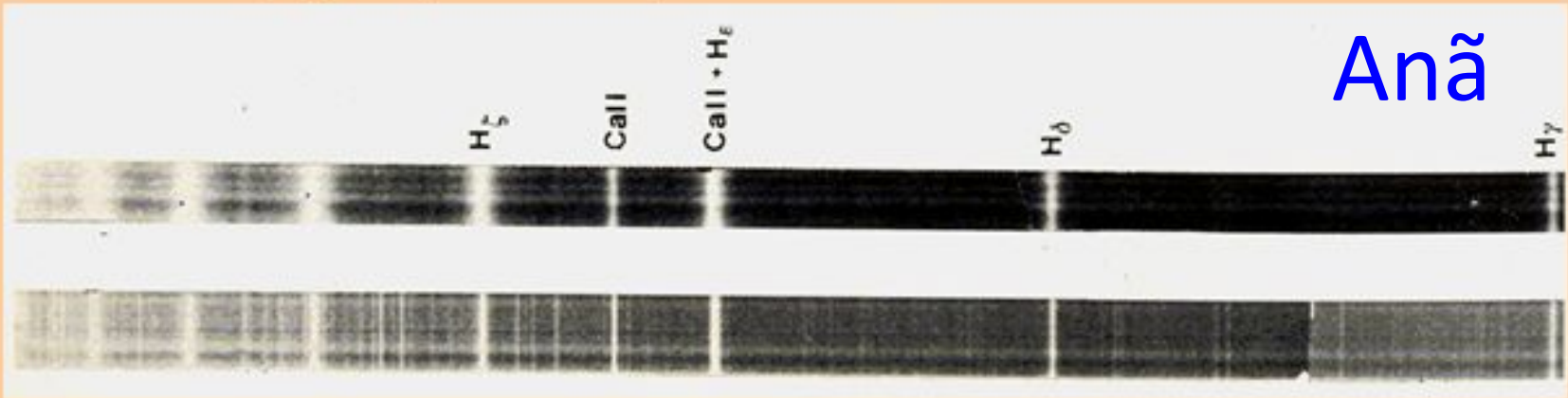


Antonia
Caetana
de Paiva
Pereira
Maury

Antonia Maury : foi contratada em 1888 por E. Pickering (Harvard) para classificar espectros. Ela propôs um novo sistema de classificação levando em conta tb a forma das linhas, mas foi ignorado por Pickering.



Dwarf and Supergiant spectra in comparison

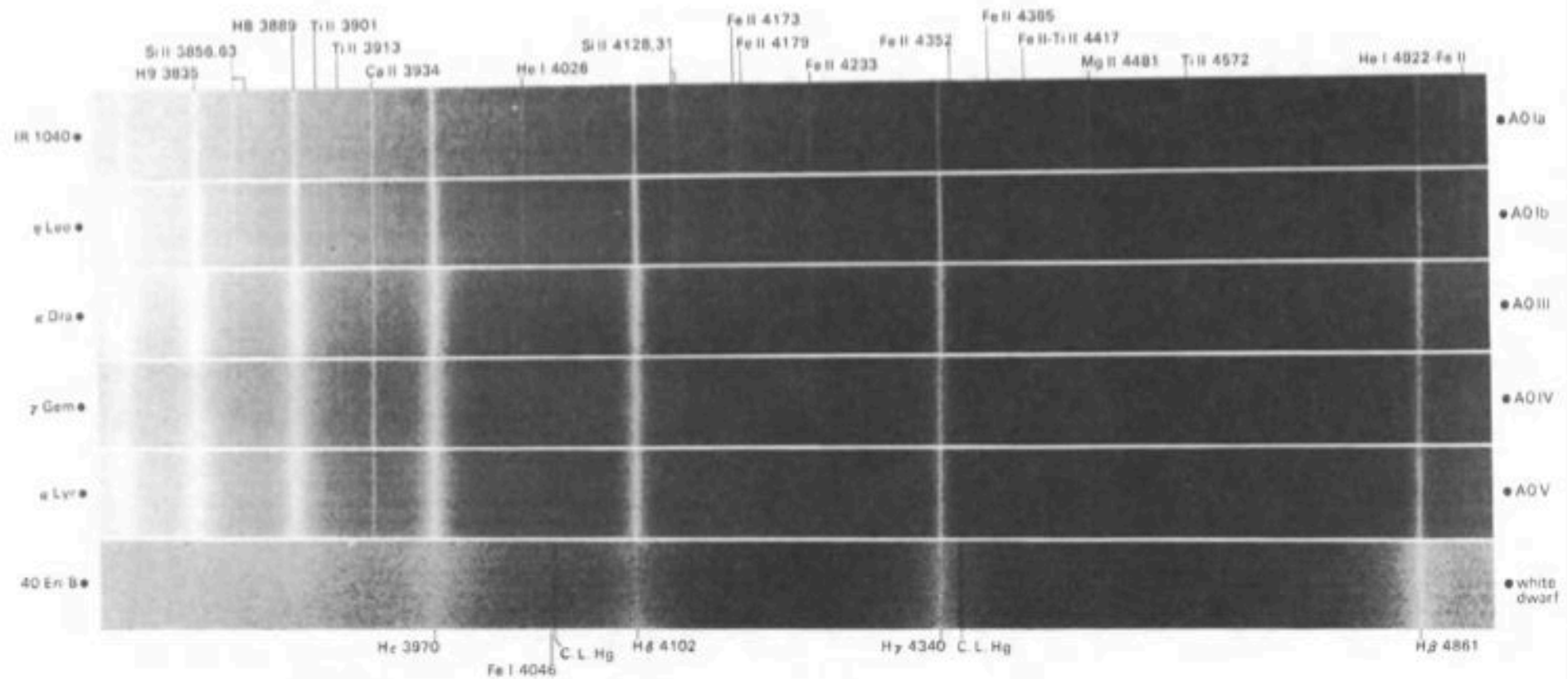


Above: normal star
Below: supergiant star

Supergigante

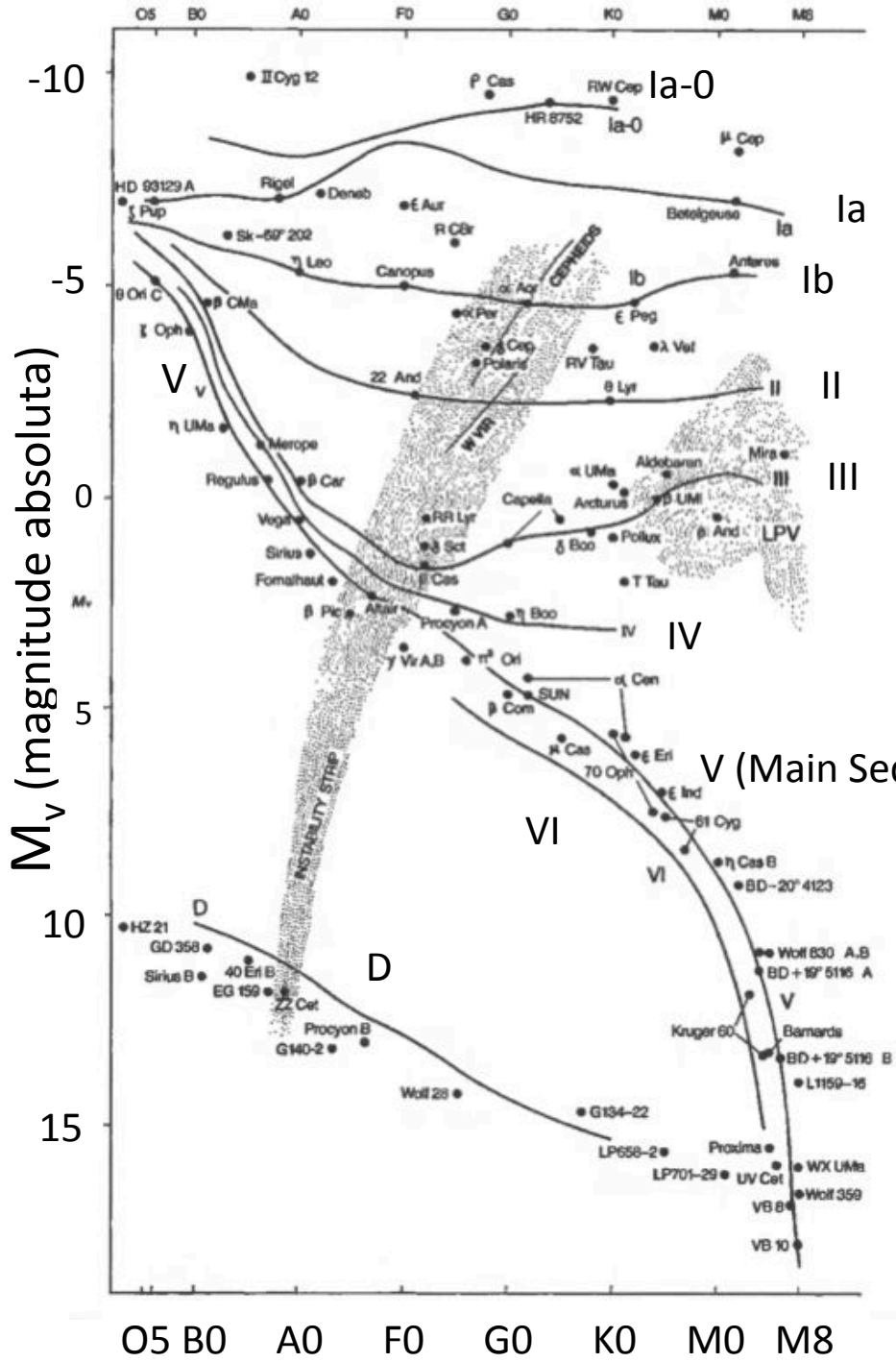
Note wide and diffuse hydrogen and calcium lines in normal stars atmosphere, against the extreme sharpness of the same lines in the supergiant atmosphere.

1943: Morgan-Keenan Luminosity Classes



Morgan–Keenan Luminosity Classes.

Class	Type of Star
Ia-O	Extreme, luminous supergiants
Ia	Luminous supergiants
Ib	Less luminous supergiants
II	Bright giants
III	Normal giants
IV	Subgiants
V	Main-sequence (dwarf) stars
VI, sd	Subdwarfs
D	White dwarfs



Spectroscopic distance:

$$d = 10^{(m - M + 5)/5}$$

Slides from previous years

- Não entra nas provinhas (ou avaliação oral)

População entre dois níveis de energia a e b
(Boltzmann):

$$\frac{N_b}{N_a} = \frac{g_b e^{-E_b/kT}}{g_a e^{-E_a/kT}} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

- N_a, N_b : número de átomos com energia E_a e E_b
- Peso estatístico $g = 2J + 1$.

J : angular momentum number

$$J = L + S$$

Designação de níveis: $(2S+1)L_J$

Por ex. nível base de nitrogênio: ${}^4S_{3/2} \rightarrow g = 4$

Exemplo para CaII (linhas H & K)

http://physics.nist.gov/PhysRefData/ASD/lines_form.html

Spectrum e.g., **Fe I** or **Na;Mg;**

Lower Wavelength: or Upper Wavenumber (in cm^{-1})

Upper Wavelength: or Lower Wavenumber (in cm^{-1})

Units:

Ritz Wavelength Air (Å)	A_{ki} (s^{-1})	$\log(g_i f_{ik})$	E_i (eV)		E_k (eV)	Lower Level Conf., Term, J	Upper Level Conf., Term, J
3 933.663	1.47e+08	0.135	0.00000	-	3.150984	$3p^6 4s \ ^2S \ 1/2$	$3p^6 4p \ ^2P^\circ \ 3/2$
3 968.469	1.4e+08	-0.18	0.00000	-	3.123349	$3p^6 4s \ ^2S \ 1/2$	$3p^6 4p \ ^2P^\circ \ 1/2$

Lower level: $^2S_{1/2}$, $J=1/2$, $g = 2J + 1 = 2$

Upper level: $^2P_{3/2}$, $J=3/2$, $g = 2J + 1 = 4$

População do nível n (Boltzmann Equation)

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$

$$\frac{N_n}{N_m} = \frac{g_n}{g_m} e^{-\Delta\chi/kT}$$

Boltzmann population of a level n :

$$\frac{N_n}{N} = \frac{g_n e^{-\chi_n/kT}}{g_1 + g_2 e^{-\chi_2/kT} + g_3 e^{-\chi_3/kT} + \dots}$$

$$= \frac{g_n}{u(T)} e^{-\chi_n/kT}$$

$$u(T) = \sum g_i e^{-\chi_i/kT}$$

$u(T)$ é a função de partição

População do nível n (Boltzmann Equation)

$$\frac{N_n}{N} = \frac{g_n}{u(T)} e^{-\chi_n/kT}$$

$$\frac{N_n}{N} = \frac{g_n}{u(T)} 10^{-\theta \chi_n}$$

Energia de excitação χ_n em eV

$$\theta = 5040/T$$

$$5040 = (\log e) / k ; k = 8.61733 \times 10^{-5} \text{ eV K}^{-1}$$

BONUS MATERIAL: forma geral da equação de Boltzmann

Ionização: Notação

- Hidrogênio neutro: H , H^0 ou $H\text{ I}$
- Hidrogênio ionizado: H^+ ou $H\text{ II}$
- Ferro neutro: Fe , Fe^0 ou $Fe\text{ I}$
- Ferro ionizado: Fe^+ ou $Fe\text{ II}$
- Ferro três vezes ionizado: Fe^{3+} , $Fe\text{ IV}$

Populações:

- N_n : elétron no nível de excitação n
- N_{I} : neutro; N_{II} : ionizado, N_{III} : 2 vezes ionizado
- N_n^{III} : nível de excitação n , 2 vezes ionizado

Energias de ionização

Table D.1. *Atomic weights and ionization potentials.*

No.	Element	Symbol	Weight	I_1	I_2	I_3
1	Hydrogen	H	1.008	13.598	–	–
2	Helium	He	4.003	24.587	54.418	–
3	Lithium	Li	6.941	5.392	75.640	122.454
4	Beryllium	Be	9.012	9.323	18.211	153.897
5	Boron	B	10.811	8.298	25.155	37.931
6	Carbon	C	12.011	11.260	24.383	47.888
7	Nitrogen	N	14.007	14.543	29.601	47.449
8	Oxygen	O	15.994	13.618	35.117	54.936
9	Fluorine	F	18.998	17.423	34.971	62.708
10	Neon	Ne	20.179	21.565	40.963	63.45
11	Sodium	Na	22.990	5.139	47.286	71.620
12	Magnesium	Mg	24.305	7.646	15.035	80.144
13	Aluminum	Al	26.982	5.986	18.829	28.448
14	Silicon	Si	28.086	8.152	16.346	33.493
15	Phosphorus	P	30.974	10.487	19.769	30.203
16	Sulfur	S	32.06	10.360	23.338	34.79
17	Chlorine	Cl	35.45	12.968	23.814	39.61
18	Argon	Ar	39.95	15.760	27.63	40.74
19	Potassium	K	39.10	4.341	31.63	45.806
20	Calcium	Ca	40.08	6.113	11.872	50.913

Estágios de Ionização

A equação de Saha

$$\frac{N_{II}}{N_I} P_e = \frac{(2\pi m_e)^{2/3} (kT)^{5/2}}{h^3} \frac{2u_1(T)}{u_0(T)} e^{-I/kT}$$

N_{II}/N_I : ratio of ions to neutrals

P_e : electron pressure

I : ionization potential

T : temperature

u_1/u_0 : ratio of ionic to neutral partition functions

k : Boltzmann constant, h : Planck constant, m_e = mass e-

The amount of ionization depends inversely on P_e

A equação de Saha

$$\frac{N_{II}}{N_I} P_e = \frac{(2\pi m_e)^{2/3} (kT)^{5/2}}{h^3} \frac{2u_1(T)}{u_0(T)} e^{-I/kT}$$

Practical form:

$$\log \frac{N_{II}}{N_I} P_e = \frac{-5040}{T} I + 2.5 \log T + \log \frac{u_1}{u_0} - 0.1762$$

Or:

$$\frac{N_{II}}{N_I} = \frac{\Phi(T)}{P_e}$$

$$\Phi(T) = 1.2020 \times 10^9 \frac{u_1}{u_0} \theta^{-5/2} 10^{-\theta I}$$

BONUS MATERIAL: outras formas da equação de Saha

Fração neutro / (neutro + ionizado)

Saha:

$$\frac{N_{II}}{N_I} = \frac{\Phi(T)}{P_e}$$

$$\Phi(T) = 1.2020 \times 10^9 \frac{u_1}{u_0} \theta^{-5/2} 10^{-\theta I}$$

Neutral fraction (relative to neutrals + ionized):

$$\frac{N_I}{(N_I + N_{II})} = \frac{1}{1 + (N_{II} / N_I)}$$

Ionized fraction:

$$\frac{N_{II}}{(N_I + N_{II})} = \frac{1}{1 + (N_I / N_{II})}$$

Fração f_i

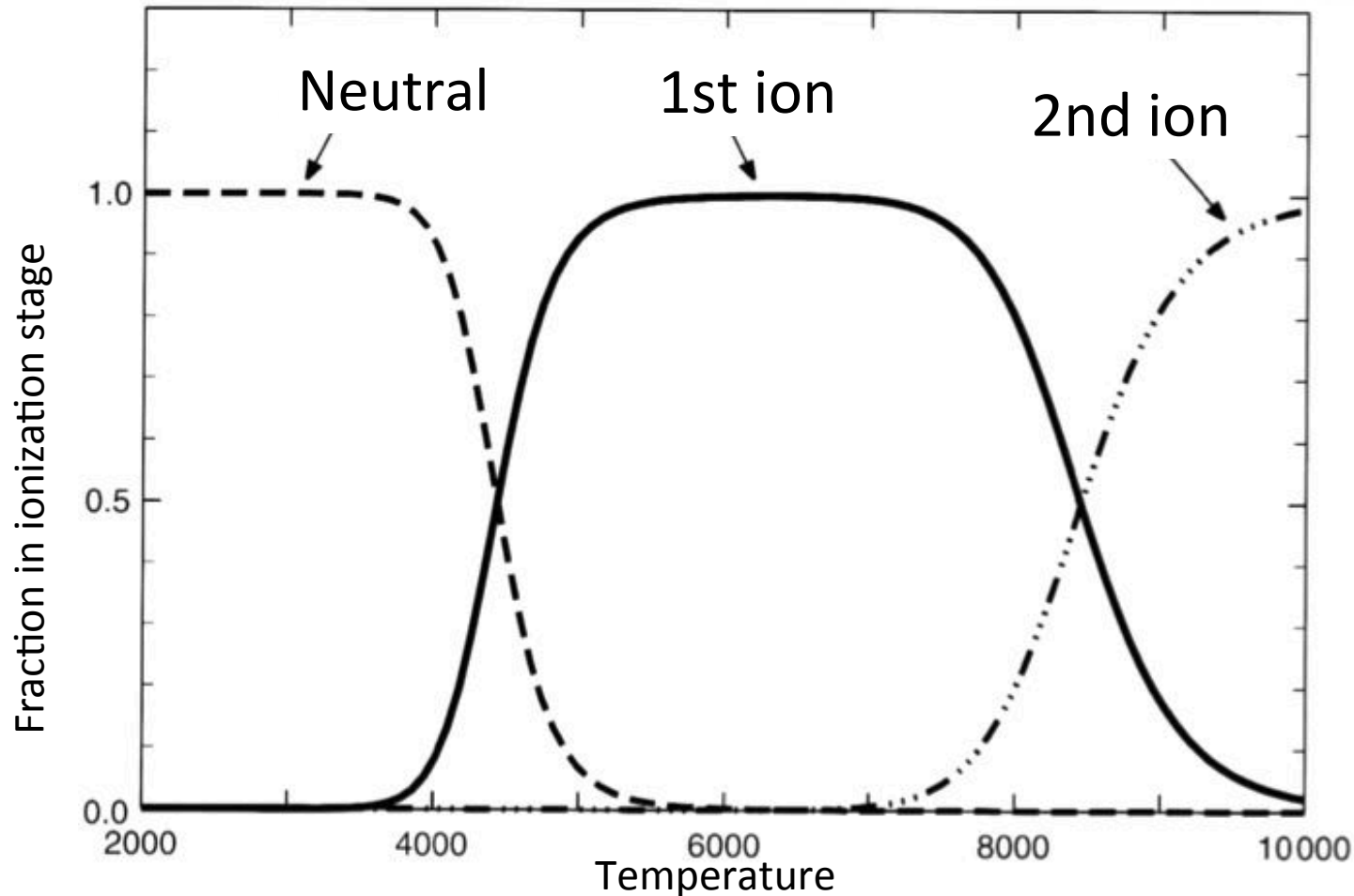
ionização $n_i /$ (todos os estágios n)

$$f_i = \frac{n_i}{n_1 + n_2 + n_3 + n_4 + \dots}$$

$$f_i = \frac{\left(\frac{n_i}{n_1}\right)}{1 + \left(\frac{n_2}{n_1}\right) + \left(\frac{n_3}{n_1}\right) + \left(\frac{n_4}{n_1}\right) + \dots} = \frac{\left(\frac{n_i}{n_{i-1}}\right)\left(\frac{n_{i-1}}{n_{i-2}}\right)\dots\left(\frac{n_2}{n_1}\right)}{1 + \left(\frac{n_2}{n_1}\right) + \left(\frac{n_3}{n_2}\right)\left(\frac{n_2}{n_1}\right) + \left(\frac{n_4}{n_3}\right)\left(\frac{n_3}{n_2}\right)\left(\frac{n_2}{n_1}\right) + \dots}$$

Almost all contributions to a given element is accounted for by 3 ionization stages

$$\frac{N_n}{N_{\text{total}}} = \frac{N_n}{N_0 + N_1 + N_2 + \dots}$$
$$= \frac{N_n/N_1}{N_0/N_1 + 1.0 + N_2/N_1 + \dots}$$



Fração neutro / (neutro + ionizado)

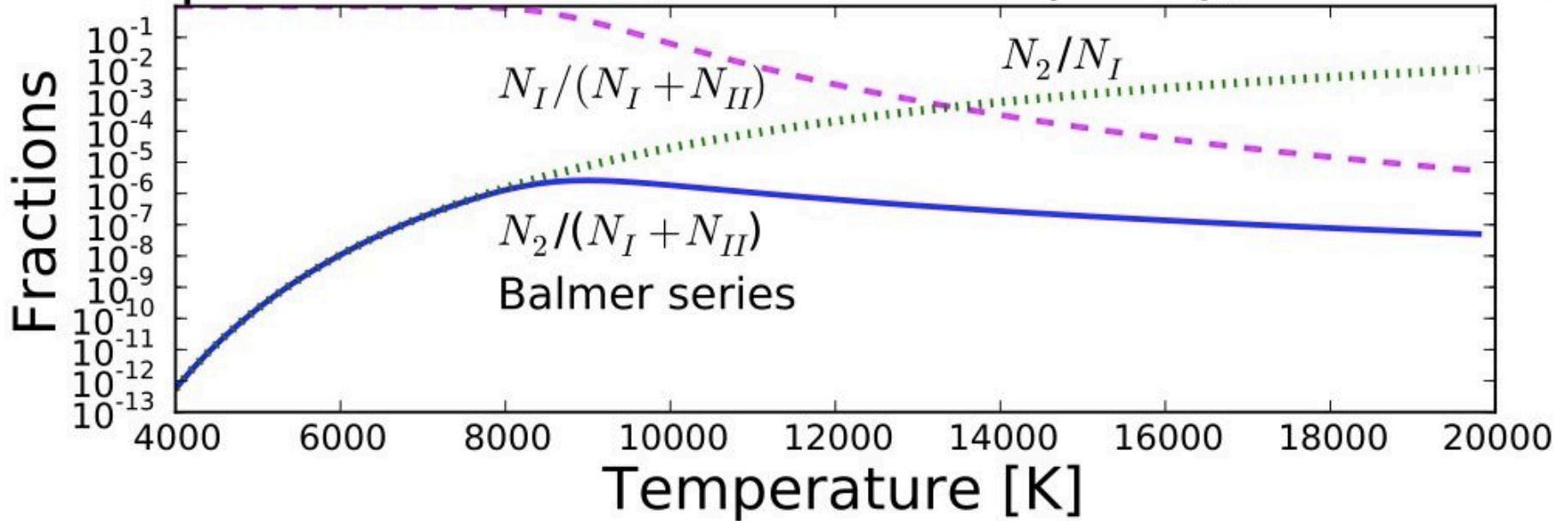
$$\frac{N_n}{N_I} = \frac{g_n}{u(T)} 10^{-\theta \chi_n}$$

$$\frac{N_{II}}{N_I} = \frac{\Phi(T)}{P_e}$$

$$\frac{N_I}{(N_I + N_{II})} = \frac{1}{1 + (N_{II} / N_I)}$$

$$\Phi(T) = 1.2020 \times 10^9 \frac{u_1}{u_0} \theta^{-5/2} 10^{-\theta I}$$

Populations Saha, Boltzmann(n=2), combined



Linhas H e K em espectros estelares

Ritz Wavelength Air (Å)	A_{ki} (s ⁻¹)	$\log(g_i f_{ik})$	E_i (eV)		E_k (eV)	Lower Level Conf., Term, J	Upper Level Conf., Term, J
3 933.663	1.47e+08	0.135	0.00000	-	3.150984	3p ⁶ 4s ² S 1/2	3p ⁶ 4p ² P° 3/2
3 968.469	1.4e+08	-0.18	0.00000	-	3.123349	3p ⁶ 4s ² S 1/2	3p ⁶ 4p ² P° 1/2

Lower level: $^2S_{1/2}$, $J=1/2$, $g = 2$, $\chi = 0.0$ eV

H and K (Ca II) lines. Saha, Boltzmann, combined

