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OVERVIEW NO. 80

ON THE ENGINEERING PROPERTIES OF MATERIALS

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Abstract—The basic mechanical and thermal properties of engineering materials are surveyed and inter-related. The survey reveals the range of each property, and the sub-range associated with each material class. These and the relationships between properties are displayed on Material Property Charts, of which 10 are presented here. The diagrams have numerous applications which range from the identification of fundamental relationship between material properties to the selection of materials for engineering design.

Résumé—Nous passons en revue et nous relions entre elles les propriétés mécaniques et thermiques fondamentales des matériaux technologiques. Cette revue met en évidence le domaine des valeurs possibles de chaque propriété, et le sous-domaine associé à chaque classe de matériaux. Nous représentons ces domaines et les relations entre propriétés sur des cartes de propriétés des matériaux (dix d'entre elles sont présentées dans cet article). Ces diagrammes ont de nombreuses applications qui vont de l'identification d'une relation fondamentale entre les propriétés du matériau au choix des matériaux à vocation technologique.

Zusammenfassung—Die grundlegenden mechanischen und thermischen Eigenschaften von Konstruktionswerkstoffen werden zusammengestellt und miteinander verknüpft. Diese Zusammenstellung beschreibt die Spanne einer jeden Eigenschaft und die für jede Materialklasse typische (kleinere) Spanne. Diese Spannen und die Zusammenhänge zwischen den Eigenschaften werden in. "Karten der Materialeigenschaften" dargestellt; 10 davon werden hier vorgelegt. Die Diagramme finden vielfältige Anwendung von der Identifizierung fundamentaler Beziehungen zwischen Materialeigenschaften bis zu der Auswahl von Werkstoffen für Konstruktionen.

SYMBOLS, DEFINITIONS AND UNITS

a	thermal diffusivity (m^2/s)
a_c	crack half-length (m)
A, B, C	dimensionless constants (—)
$\bar{c}$	velocity (m/s)
C_e	electronic specific heat (J/kg K)
C_p	specific heat at constant pressure (J/kg K)
C_v	specific heat at constant volume (J/kg K)
E	Youngs modulus (GPa)
E_A	adiabatic modulus (GPA)
E_T	isothermal modulus (GPa)
F	force (N)
G	shear modulus (GPa)
G_{Ic}	toughness or apparent fracture surface energy (J/m^2)
h	heat transfer coefficient (J/m^2 K)
k	Boltzmann's constant (J/K)
K	bulk modulus (GPa)
K_{Ic}	fracture toughness (MPa $m^{1/2}$)
l	mean free path (m)
r_0	atom size (m)
R	radius of pressure vessel (m)
S	bond stiffness (N/m)
t	thickness of section (m)
T	temperature (K)
T_m	melting point (K)
v	elastic wave velocity (m/s)
α	linear expansion coefficient (K^{-1})
η	damping coefficient (—)
γ	surface energy (J/m^2)
γ_G	Gruneisen's constant (—)
λ	thermal conductivity (W/mK)
ρ	density (Mg/m^3)
σ	stress (MPa)
σ_y	yield/crushing/tear strength (MPa)
σ_t	tensile yield/fracture strength (MPa)
ν	Poisson's ratio (—)

1. INTRODUCTION: MATERIAL PROPERTY CHARTS

Each property of an engineering material has a characteristic range of values. The range is enormous: of the ten properties considered here—properties such as modulus, toughness, thermal conductivity—all but one ranges through roughly 5 decades, reflecting the diversity in the atomic mechanisms which determine the value of the property.

It is conventional to classify the solids themselves into the six broad classes shown in Fig. 1—metals, polymers, elastomers, ceramics, glasses and composites. Within a class the range of properties is narrower, and the underlying mechanisms fewer. But classifications of this sort have their dangers, notably those of narrowing vision and of obscuring relationships. Here we aim at a broad review of engineering materials, examining the relationships between the properties of all six classes.

One way of doing this is by constructing *Material Property Charts.* The idea is illustrated by Fig. 2. One property (the modulus, E, in this case) is plotted

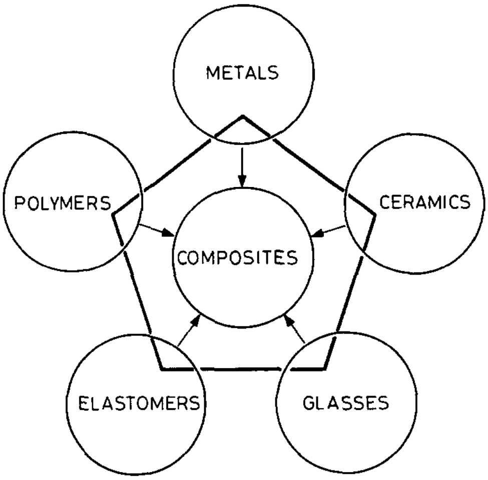


Fig. 1. The menu of engineering materials. Each class has properties which occupy a particular part (or "field") of the Materials Property Charts, which are the central feature of this paper.

against another (the density, ρ) on logarithmic scales. The range of the axes is chosen to include all materials, from the lightest, flimsiest foams to the stiffest, heaviest metals. It is found that data for a given class of materials (polymers, for example) cluster together on the chart; the *subrange* associated with one material class is, in all cases, much smaller than the *full* range of that property. Data for one class can be enclosed in a property-envelope, as shown in Fig. 2. The envelope is constructed to enclose all members of the class.

All this is simple enough—no more than a way of displaying properties in a helpful way. But by choosing the axes and scales appropriately, more can be added. The speed of sound in a solid depends on the modulus, E, and the density, ρ; the longitudinal wave speed v, for instance, is

$$v = \left[\frac{E}{\rho}\right]^{1/2} \tag{1a}$$

or (taking logs)

$$\log E = \log \rho + 2 \log v. \tag{1b}$$

For a fixed value of v, this equation plots as a straight line of slope 1 on Fig. 2. This allows us to add contours of *constant sound velocity* to the chart: they are the family of parallel diagonal lines, linking materials in which sound travels with the same velocity. All the charts allow additional fundamental relationships of this sort to be displayed.

At the more applied end of the spectrum, the charts help in materials selection in engineering design. The performance, in an engineering sense, of load-bearing components is seldom limited by a single property but by one or more combinations of them. The lightest tie rod which will carry a given tensile load without exceeding a given deflection is that with the greatest value of E/ρ. The lightest column which will support a given compressive load without buckling is that with the greatest value of $E^{1/2}/\rho$. The lightest panel which will support a given pressure with minimum deflection is that with the greatest value of $E^{1/3}/\rho$. Figure 3 shows how the chart can be used to select materials which maximise any one of these combinations (1). The condition

$$\frac{E}{\rho} = C \tag{2a}$$

or, taking logs

$$\log E = \log \rho + \log C \tag{2b}$$

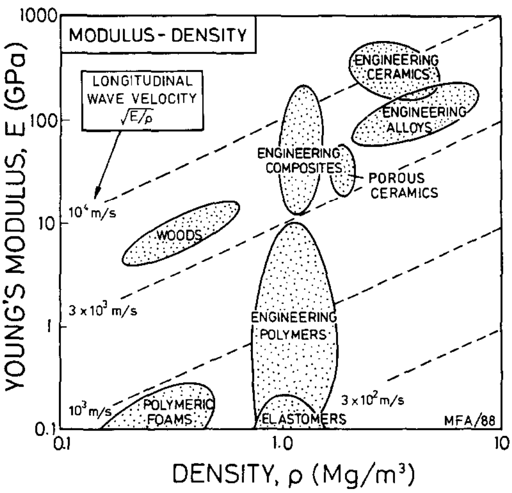


Fig. 2. The idea of a Materials Property Chart: Young's modulus, E, is plotted against the density, ρ, on log scales. Each class of material occupies a characteristic part of the chart. The log scales allow the longitudinal elastic wave velocity $v = (E/\rho)^{1/2}$ to be plotted as a set of parallel contours.

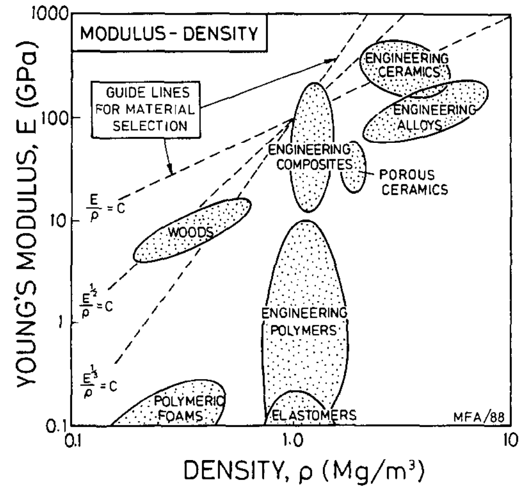


Fig. 3. The same diagram as Fig. 2 but showing guide lines for selecting materials for minimum weight design. Because of the log scales the lines are straight even though they describe non-linear relationships between the properties.

is a family of straight lines of slope 1, one line for each value of the constant C. The condition

$$E^{1/2}/\rho = C \tag{3}$$

gives a family with slope 2; and

$$\frac{E^{1/3}}{\rho} = C \tag{4}$$

gives another set with slope 3. One member of each family is shown on Fig. 3, labelled "Guide Lines for Material Selection". The others are found by translating the appropriate guide line sideways.

It is now easy to read off the subset of materials which are optimal for each loading geometry. If a straight-edge is laid parallel to the $E^{1/2}/\rho = C$ line, all the materials which lie on the line will perform equally well as a light column loaded in compression; those above the line are better (they can withstand greater loads), those below, worse. If the straight-edge is translated towards the top left corner of the diagram while retaining the same slope, the choice narrows. At any given position of the edge, two materials which lie on it are equally good, and only those which remain above are better. The same procedure, applied to the tie (E/ρ) or plate in bending ($E^{1/3}/\rho$), lead to different equivalences and optimal subsets of materials. There are numerous such *criteria for optimal materials selection*, some of which are summarised in Fig. 4. All of these appear on one or another of the charts described below.

Among the mechanical and thermal properties, there are 10 which are of primary importance, both in characterising the material, and in engineering design. They are listed in Table 1: they include density, modulus, strength, toughness, thermal conductivity, diffusivity and expansion. The charts display data for these properties for the 9 classes of materials listed in Table 2. The class-list is expanded from the original 6 by distinguishing *engineering composites* from *foams* and from *woods* though all, in the most general sense, are composites; and by distinguishing the high-strength *engineering ceramics* (like silicon carbide) from the low strength, *porous ceramics* (like brick). Within each class, data are plotted for a representative set of materials, chosen both to span the full range of behaviour for the class, and to include the most common and most widely used members of it. In this way the envelope for a class encloses data not only for the materials listed in Table 2, but for virtually all other members of the class as well.

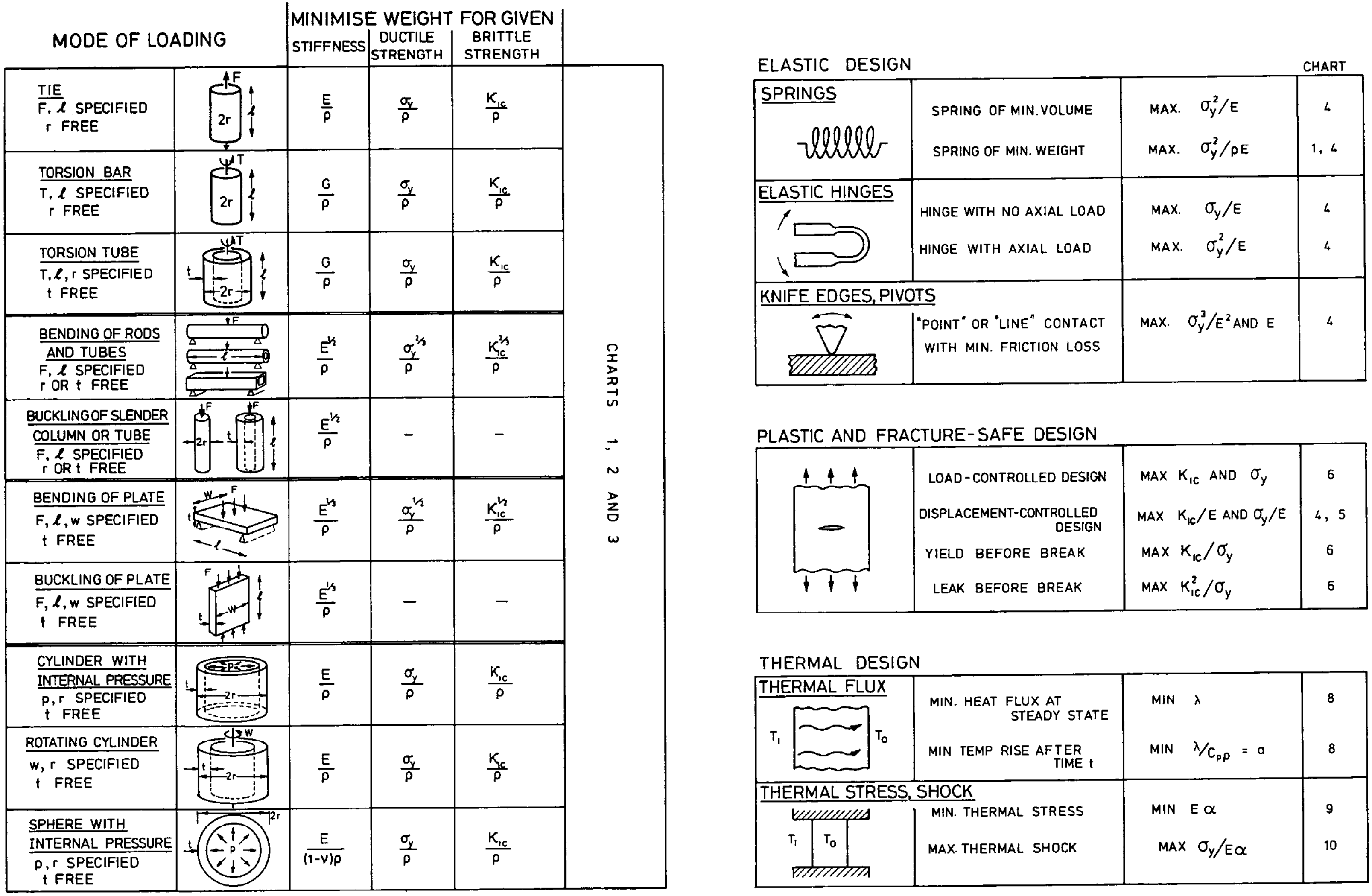
2. DATA AND DATA SOURCES

The data plotted on the charts shown below have been assembled over several years from a wide variety of sources. As far as possible, the data have been *validated*: cross-checked by comparing values from more than one source, and they have been examined for consistency with physical rules. The charts show a *range* of values for each property of each material. Sometimes the range is narrow: the modulus of a metal, for instance, varies by only a few percent about its mean value. Sometimes it is wide: the strength of a ceramic can vary by a factor of 100 or more. The reasons for the range of values vary: heat treatment and mechanical working have a profound effect on yield strength, damping and toughness of metals. Crystallinity and degree of cross-linking greatly influence the modulus of polymers. Grain size and porosity change considerably the fracture strength of ceramics. And so on. These *structure-sensitive* properties appear as elongated balloons within the envelopes on the charts. A balloon encloses a typical range for the value of the property for a single material. Envelopes (heavier lines) enclose the balloons for a class.

The framework of this study is one of maximum breadth at relatively low precision. In this context, a number of approximations are possible: that the shear modulus is roughly 3/8 E; that the hardness is roughly $3\sigma_y$ (where σ_y is the yield strength); that the two specific heats C_p and C_v are (for solids) equal; and so forth. This allows us to deal with one modulus (Young's), one wave velocity (the longitudinal), and so on; the others are proportional to the one discussed here, and relationships, ranges and physical origins are the same.

Now to the data sources themselves. First, there are the standard handbooks: *The American Institute of Physics Handbook* [2]; the *Handbook of Physics and Chemistry* [3]; the *Landolt–Bornstein Tables* [4]; the *Materials Engineering "Materials Selector"* [5] and the *Fulmer Materials Optimiser* [6]. More specialised data can be found in the compilations by Simmons and Wang [7] for moduli, Lazan [8] for damping, Frost and Ashby [9] and the *Atlas of Creep and Stress Rupture Curves* [10] for strength at temperature, and in the major reference texts on specific materials such as the *ASM Metals Handbooks* [11] and Smithells [12] for metals, the *Handbook of Glass Properties* by Bansal and Doremus [13] for glasses, the *Handbook of Plastics and Elastomers* [14], the *International Plastics Selector* [15], the *Plastics Technology Handbook* of Chanda and Roy [16], and the *Handbook of Elastomers* of Bhowmick and Stephens, [17] for polymers, the *Handbook of Physical Constants* [18] for rocks and minerals, Creyke *et al.* [19] and Morrell [20] for ceramics, *Composites* [21] and the *Engineering Guide to Composite Materials* [22] for composites, Dinwoodie [23] for wood and wood products and Gibson and Ashby [24] for foams. Much of the data (particularly those for moduli, strength, toughness and thermal conductivity) are derived from scientific journals and conference proceedings, and from manufacturers data sheets for their products.

We now introduce charts which display the 10 properties and allow useful relationships between them to be explored.



MODE OF LOADING		MINIMISE WEIGHT FOR GIVEN STIFFNESS	DUCTILE STRENGTH	BRITTLE STRENGTH	
TIE F, ℓ SPECIFIED r FREE		$\frac{E}{\rho}$	$\frac{\sigma_y}{\rho}$	$\frac{K_{IC}}{\rho}$	CHARTS 1, 2 AND 3
TORSION BAR T, ℓ SPECIFIED r FREE		$\frac{G}{\rho}$	$\frac{\sigma_y}{\rho}$	$\frac{K_{IC}}{\rho}$	
TORSION TUBE T, ℓ, r SPECIFIED t FREE		$\frac{G}{\rho}$	$\frac{\sigma_y}{\rho}$	$\frac{K_{IC}}{\rho}$	
BENDING OF RODS AND TUBES F, ℓ SPECIFIED r OR t FREE		$\frac{E^{1/2}}{\rho}$	$\frac{\sigma_y^{2/3}}{\rho}$	$\frac{K_{IC}^{2/3}}{\rho}$	
BUCKLING OF SLENDER COLUMN OR TUBE F, ℓ SPECIFIED r OR t FREE		$\frac{E^{1/2}}{\rho}$	–	–	
BENDING OF PLATE F, ℓ, w SPECIFIED t FREE		$\frac{E^{1/3}}{\rho}$	$\frac{\sigma_y^{1/2}}{\rho}$	$\frac{K_{IC}^{1/2}}{\rho}$	
BUCKLING OF PLATE F, ℓ, w SPECIFIED t FREE		$\frac{E^{1/3}}{\rho}$	–	–	
CYLINDER WITH INTERNAL PRESSURE p, r SPECIFIED t FREE		$\frac{E}{\rho}$	$\frac{\sigma_y}{\rho}$	$\frac{K_{IC}}{\rho}$	
ROTATING CYLINDER w, r SPECIFIED t FREE		$\frac{E}{\rho}$	$\frac{\sigma_y}{\rho}$	$\frac{K_{IC}}{\rho}$	
SPHERE WITH INTERNAL PRESSURE p, r SPECIFIED t FREE		$\frac{E}{(1-\nu)\rho}$	$\frac{\sigma_y}{\rho}$	$\frac{K_{IC}}{\rho}$	

ELASTIC DESIGN

			CHART
SPRINGS	SPRING OF MIN. VOLUME	MAX. σ_y^2/E	4
	SPRING OF MIN. WEIGHT	MAX. $\sigma_y^2/\rho E$	1, 4
ELASTIC HINGES	HINGE WITH NO AXIAL LOAD	MAX. σ_y/E	4
	HINGE WITH AXIAL LOAD	MAX. σ_y^2/E	4
KNIFE EDGES, PIVOTS	"POINT" OR "LINE" CONTACT WITH MIN. FRICTION LOSS	MAX. σ_y^3/E^2 AND E	4

PLASTIC AND FRACTURE-SAFE DESIGN

	LOAD-CONTROLLED DESIGN	MAX K_{IC} AND σ_y	6
	DISPLACEMENT-CONTROLLED DESIGN	MAX K_{IC}/E AND σ_y/E	4, 5
	YIELD BEFORE BREAK	MAX K_{IC}/σ_y	6
	LEAK BEFORE BREAK	MAX K_{IC}^2/σ_y	6

THERMAL DESIGN

THERMAL FLUX	MIN. HEAT FLUX AT STEADY STATE	MIN λ	8
	MIN TEMP RISE AFTER TIME t	MIN $\lambda/C_p\rho = a$	8
THERMAL STRESS, SHOCK	MIN. THERMAL STRESS	MIN $E\alpha$	9
	MAX. THERMAL SHOCK	MAX $\sigma_y/E\alpha$	10

Fig. 4. Some of the property-combinations which determine performance in design.

Table 1. Basic subset of material properties

Property	Units
Density, ρ	(Mg/m^3)
Youngs modulus, E	(GPa)
Strength, σ_y	(MPa)
Fracture toughness, K_{Ic}	(MPa m$^{1/2}$)
Toughness, G_{Ic}	(J/m^2)
Damping coefficient, η	(—)
Thermal conductivity, λ	(W/m K)
Thermal diffusivity, a	(m^2/s)
Volume specific heat, $C_p \rho$	(J/m^3 K)
Thermal expansion coefficient, α	(1/K)

3. THE MATERIAL-PROPERTY DIAGRAMS

3.1. The modulus–density chart (Chart 1, Fig. 5)

Modulus and density are among the most self-evident of material properties. Steel is used for stiff beams; rubber for compliant cushions. The density of lead makes it good for sinkers; that of cork makes it good for floats. Figure 5 shows the full range of Young's modulus, E, and density, ρ, for engineering materials.

Table 2. Material classes and members of each class

Engineering alloys (The metals and alloys of engineering)	Aluminium alloys	Al alloys
	Lead alloys	Lead alloys
	Magnesium alloys	Mg alloys
	Nickel alloys	Ni alloys
	Steels	Steels
	Tin alloys	Tin alloys
	Titanium alloys	Ti alloys
	Zinc alloys	Zn alloys
Engineering polymers (The thermoplastics and thermosets of engineering)	Epoxies	EP
	Melamines	MEL
	Polycarbonate	PC
	Polyesters	PEST
	Polyethylene, high density	HDPE
	Polyethylene, low density	LDPE
	Polyformaldehyde	PF
	Polymethylmethacrylate	PMMA
	Polypropylene	PP
	Polytetrafluorethylene	PTFE
	Polyvinylchloride	PVC
Engineering ceramics (Fine ceramics capable of load-bearing application)	Alumina	Al_2O_3
	Diamond	C
	Sialons	Sialons
	Silicon carbide	SiC
	Silicon nitride	Si_3N_4
	Zirconia	ZrO_2
Engineering composites (The composites of engineering practice. A distinction is drawn between the properties of a ply—"Uniply"—and of a laminate—"Laminates")	Carbon fibre reinforced polymer	CFRP
	Glass fibre reinforced polymer	GFRP
	Kevlar fibre reinforced polymer	KFRP
Porous ceramics (Traditional ceramics, cements, rocks and minerals)	Brick	Brick
	Cement	Cement
	Common rocks	Rocks
	Concrete	Concrete
	Porcelain	Pcln
	Pottery	Pot
Glasses (Ordinary silicate glass)	Borosilicate glass	B-glass
	Soda glass	Na-glass
	Silica	SiO_2
Woods (Separate envelopes describe properties parallel to the grain and normal to it, and wood products)	Ash	Ash
	Balsa	Balsa
	Fir	Fir
	Oak	Oak
	Pine	Pine
	Wood products (ply, etc)	Wood products
Elastomers (Natural and artificial rubbers)	Natural rubber	Rubber
	Hard butyl rubber	Hard butyl
	Polyurethanes	PU
	Silicone rubber	Silicone
	Soft butyl rubber	Soft butyl
Polymer foams (Foamed polymers of engineering)	These include:	
	Cork	Cork
	Polyester	PEST
	Polystyrene	PS
	Polyurethane	PU

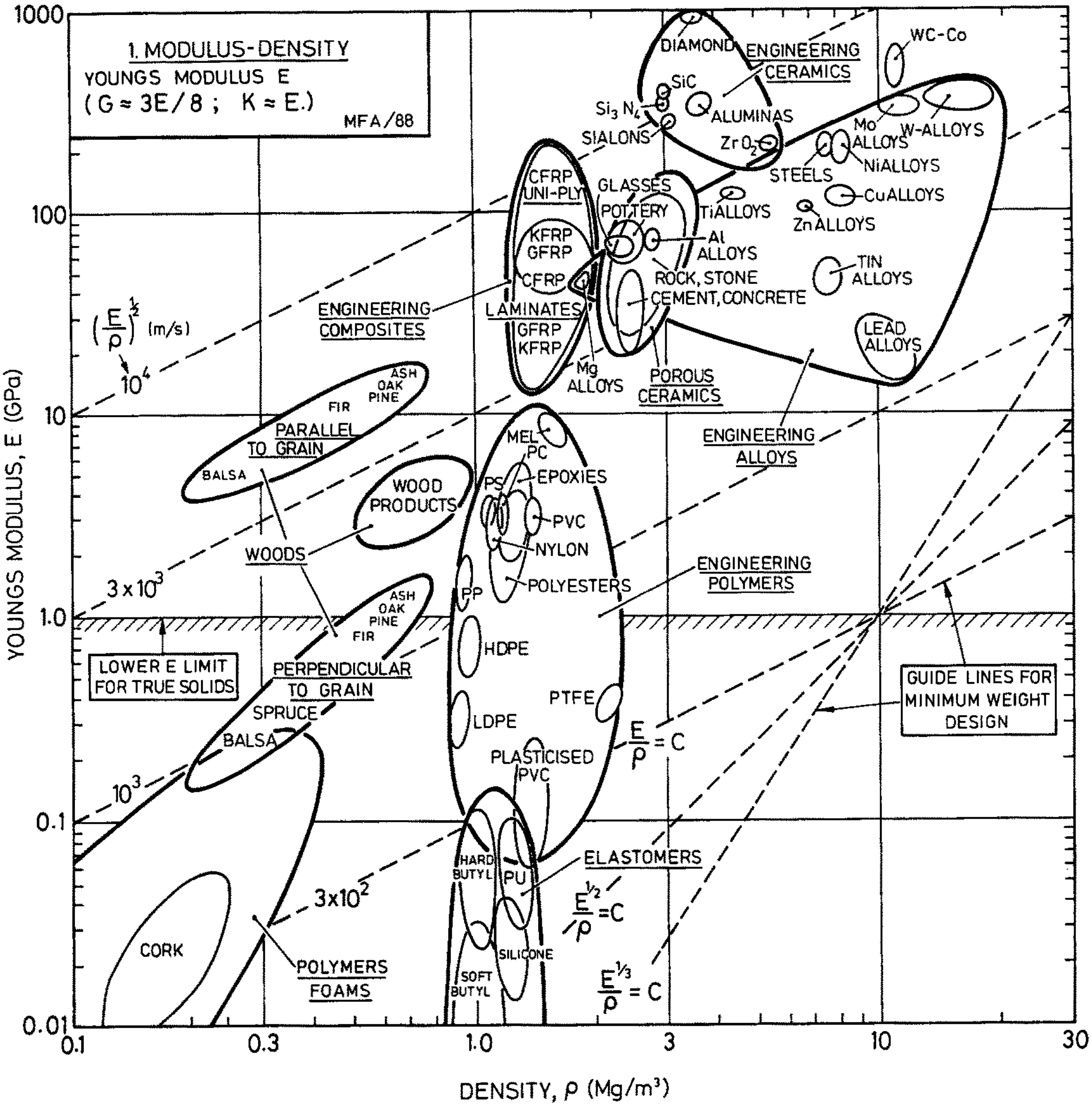


Fig. 5. Chart 1: Youngs modulus, E, plotted against density, ρ. The heavy envelopes enclose data for a given class of material. The diagonal contours show the longitudinal wave velocity. The guide lines of constant E/ρ, $E^{1/2}/\rho$ and $E^{1/3}/\rho$ allow selection of materials for minimum weight, deflection-limited, design.

Data for members of a particular class of material cluster together and can be enclosed by an envelope (heavy line). The same class-envelopes appear on all the diagrams: they correspond to the main headings in Table 2. The members of a class were chosen to span the full property-range of that class, so the class-envelopes enclose data not only for the members listed in Table 2, but for other unlisted members also.

What determines the density of a solid? At its simplest, it depends on three factors: the (mean) atomic weight of its atoms or ions, their (mean) size, and the way they are packed. The size of atoms does not vary much: most have a volume within a factor of two of $2 \times 10^{-29}\,\mathrm{m}^3$. Packing fractions do not vary much either—a factor of two, more or less: close-packing gives a packing fraction of 0.74; open networks, typified by the diamond–cubic structure, give about 0.34. The spread of density comes from that of atomic weight, from 1 for hydrogen to 207 for lead. Metals are dense because they are made of heavy atoms, packed more or less closely; polymers have low densities because they are made of light atoms in a linear, 2 or 3-dimensional network. Ceramics, for the most part, have lower densities than metals because they contain light O, N or C atoms. Even the lightest atoms, packed in the most open way, give solids with a density of around $1\,\mathrm{Mg/m^3}$. Materials with lower densities than this are foams, containing substantial pore space.

The *moduli* of most materials depend on two factors: bond stiffness, and the density of bonds per unit area. A bond is like a spring: it can be characterised by a spring constant, S (units: N/m). Young's modulus, E, is roughly

$$E = \frac{S}{r_0} \tag{1}$$

where r_0 is the atom size [r_0^3 is the (mean) atomic or ionic volume]. The wide range of moduli is largely caused by the range of value of S. The covalent bond is stiff ($S = 20$–200 N/m); the metallic and the ionic a little less so ($S = 15$–100 N/m). Diamond has a very high modulus because the carbon atom is small (giving a high bond density) and its atoms are linked by very strong bonds ($S = 200$ N/m). Metals have high moduli because close-packing gives a high bond density and the bonds are strong. Polymers contain both strong covalent bonds and weak hydrogen or Van-der-Waals bonds ($S = 0.5$–2 N/m); it is the weak bonds which stretch when the polymer is deformed, giving low moduli.

But even large atoms ($r_0 = 3 \times 10^{-10}$ m) bonded with weak bonds ($S = 0.5$ N/m) have a modulus of roughly

$$E = \frac{0.5}{3 \times 10^{-10}} \approx 1\ \text{GPa}. \tag{2}$$

This is the *lower limit* for true solids. The chart shows that many materials have moduli that are lower than this: they are either elastomers, or foams—materials made up of cells with a large fraction of pore space. Elastomers have a low E because the weak secondary bonds have melted (their glass temperature is below room temperature) leaving only the very weak "entropic" restoring force associated with tangled, long-chain molecules; and foams have low moduli because the cell walls bend (allowing large displacements) when the material is loaded.

The chart shows that the modulus of engineering materials spans 5 decades,† from 0.01 GPa (low density foams) to 1000 GPa (diamond); the density spans a factor of 2000, from less than 0.1 to 20 Mg/m^3. At the level of approximation of interest here (that required to reveal the relationship between the properties of materials classes) we may approximate the shear modulus, G, by $3E/8$ and the bulk modulus, K, by E, for all materials except elastomers (for which $G = E/3$ and $K \gg E$).

The log-scales allow more information to be displayed. The velocity of elastic waves in a material, and the natural vibration frequencies of a component made of it, are proportional to $(E/\rho)^{1/2}$; the quantity $(E/\rho)^{1/2}$ itself is the velocity of longitudinal waves in a thin rod of the material. Contours of constant $(E/\rho)^{1/2}$ are plotted on the Chart, labelled with the longitudinal wave speed: it varies from less than 50 m/s (soft elastomers) to a little more than 10^4 m/s (fine ceramics). We note that aluminium and glass, because of their low densities, transmit waves quickly despite their low moduli. One might have expected the sound velocity in foams to be low because of the low modulus; but the low density almost compensates. That in wood, across the grain, is low; but along the grain, it is high—roughly the same as steel—a fact made use of in the design of musical instruments.

The diagram helps in the common problem of material selection for applications in which weight must be minimised. Guide lines corresponding to three common geometries of loading are drawn on the diagram. They are used in the way described in Section 1 to select materials for elastic design at minimum weight.

3.2. The strength–density chart (Chart 2, Fig. 6)

The modulus of a solid is a well-defined quantity with a sharp value. The strength is not.

The word "strength" needs definition. For metals and polymers it is the *yield strength*, but since the range of materials includes those which have been worked, the range spans initial yield to ultimate strength; for most practical purposes it is the same in tension and compression. For brittle ceramics, it is the *crushing strength in compression*, not that in tension which is about 15 times smaller; the envelopes for brittle materials are shown as broken lines as a reminder of this. For elastomers, strength means the *tear-strength*. For composites, it is the *tensile failure strength* (the compressive strength can be less, because of fibre buckling).

Figure 6 shows these strengths, for which we will use the symbol σ_y (despite the different failure mechanisms involved), plotted against density, ρ. The considerable vertical extension of the strength-balloon for an individual material reflects its wide range, caused by degree of alloying, work hardening, grain size, porosity and so forth. As before, members of a class group together and can be enclosed in an envelope (heavy line). Each occupies a characteristic area of the chart, and, broadly speaking, encompasses not only the materials listed in Table 2, but most other members of the class also.

The range of strength for engineering materials, like that of the modulus, spans about 5 decades: from less than 0.1 MPa (foams, used in packaging and energy-absorbing systems) to 10^4 MPa (the strength of diamond, exploited in the diamond-anvil press). The single most important concept in understanding this wide range is that of the *lattice resistance* or *Peierls stress*: the intrinsic resistance of the structure to plastic shear. Metals are soft and ceramics hard because the non-localised metallic bond does little to prevent dislocation motion, whereas the more localised covalent and ionic bonds of the ceramic (which must be broken and reformed when the structure is sheared) lock the dislocations in place. In

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†Very low density foams and gels (which can be thought of as molecular-scale fluid filled foams) can have moduli far lower than this. As an example, gelatin (as in Jello) has a modulus of about 5×10^{-5} GPa. Their strengths and fracture toughness too, can be below the lower limit of the charts.

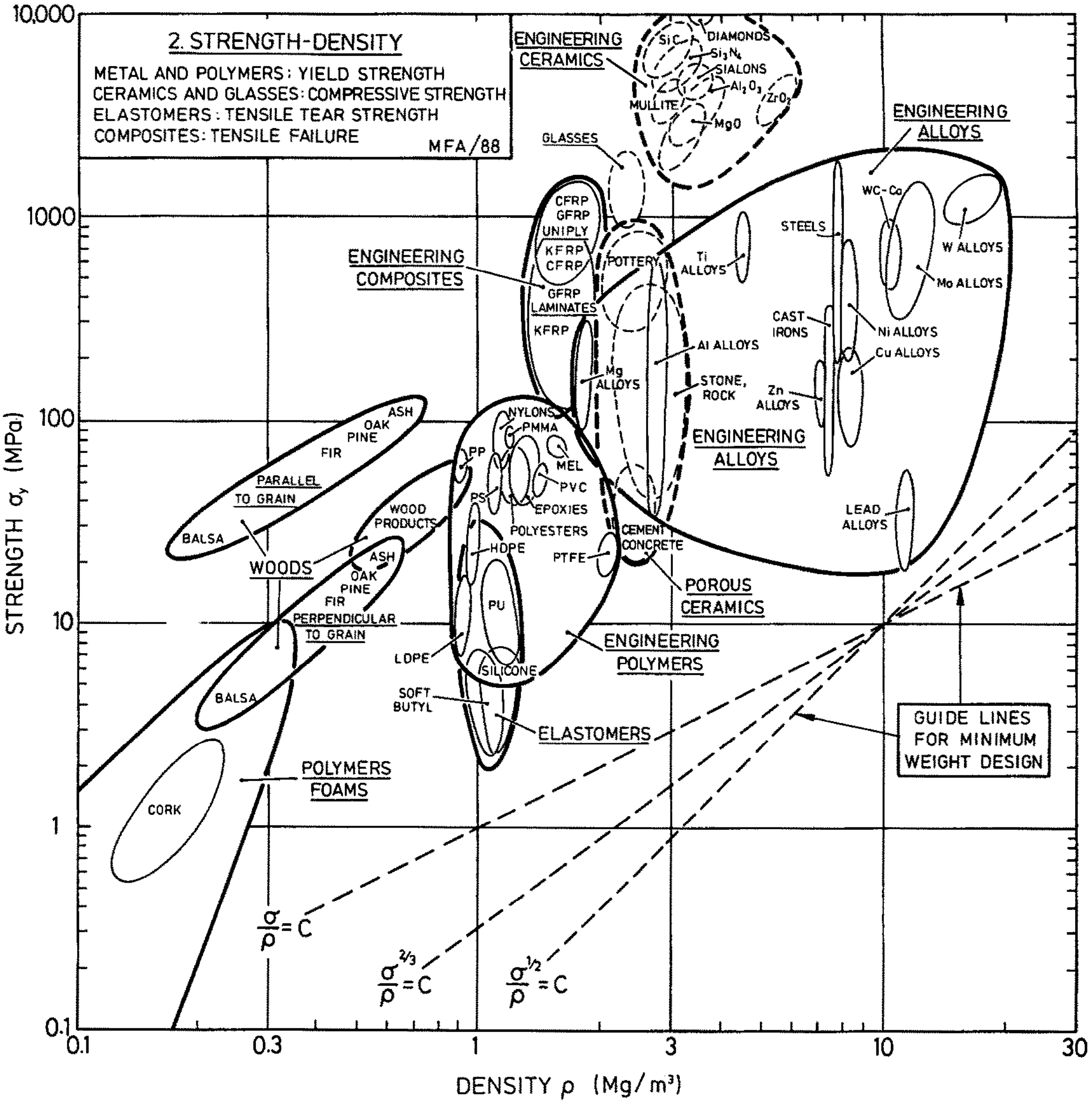

Fig. 6. Chart 2: Strength, σ_y, plotted against density, ρ (yield strength for metals and polymers, compressive strength for ceramics, tear strength for elastomers and tensile strength for composites). The guide lines of constant σ_y/ρ, $\sigma_y^{2/3}/\rho$ and $\sigma_y^{1/2}/\rho$ are used in minimum weight, yield-limited, design.

non-crystalline solids we think instead of the energy associated with the unit step of the flow process: the relative slippage of two segments of a polymer chain, or the shear of a small molecular cluster in a glass network. Their strength has the same origin as that underlying the lattice resistance: if the unit step involves breaking strong bonds (as in an inorganic glass), the material will be strong; if it only involves the rupture of weak bonds (the Van-der-Waals bonds in polymers for example), it will be weak.

When the lattice resistance is low, the material can be strengthened by introducing obstacles to slip: in metals, by adding alloying elements, particles, grain boundaries and even other dislocations ("work hardening") and in polymers by cross-linking or orientation of chains so that strong covalent as well as weak Van-der-Waals bonds are broken. When the lattice resistance is high, further hardening is superfluous—the problem becomes that of suppressing fracture (next section).

An important use of the chart is in materials selection in plastic design. Figure 4 lists the combinations (such as σ_y/ρ, $\sigma_y^{2/3}/\rho$ and $\sigma_y^{1/2}/\rho$) which enter the equations for minimum-weight design of ties, columns, beams and plates, and for yield-limited design of moving components in which inertial forces are important (for details and examples see Refs [1], [6] and [24]). Guide lines with slopes of 1, $\frac{2}{3}$ and $\frac{1}{2}$, corresponding to these combinations are shown in Fig. 6. They are used to identify an optimal subset of materials as described in Section 1.

3.3. The fracture toughness–density chart (Chart 3, Fig. 7)

Increasing the plastic strength of a material is useful only as long as it remains plastic and does not

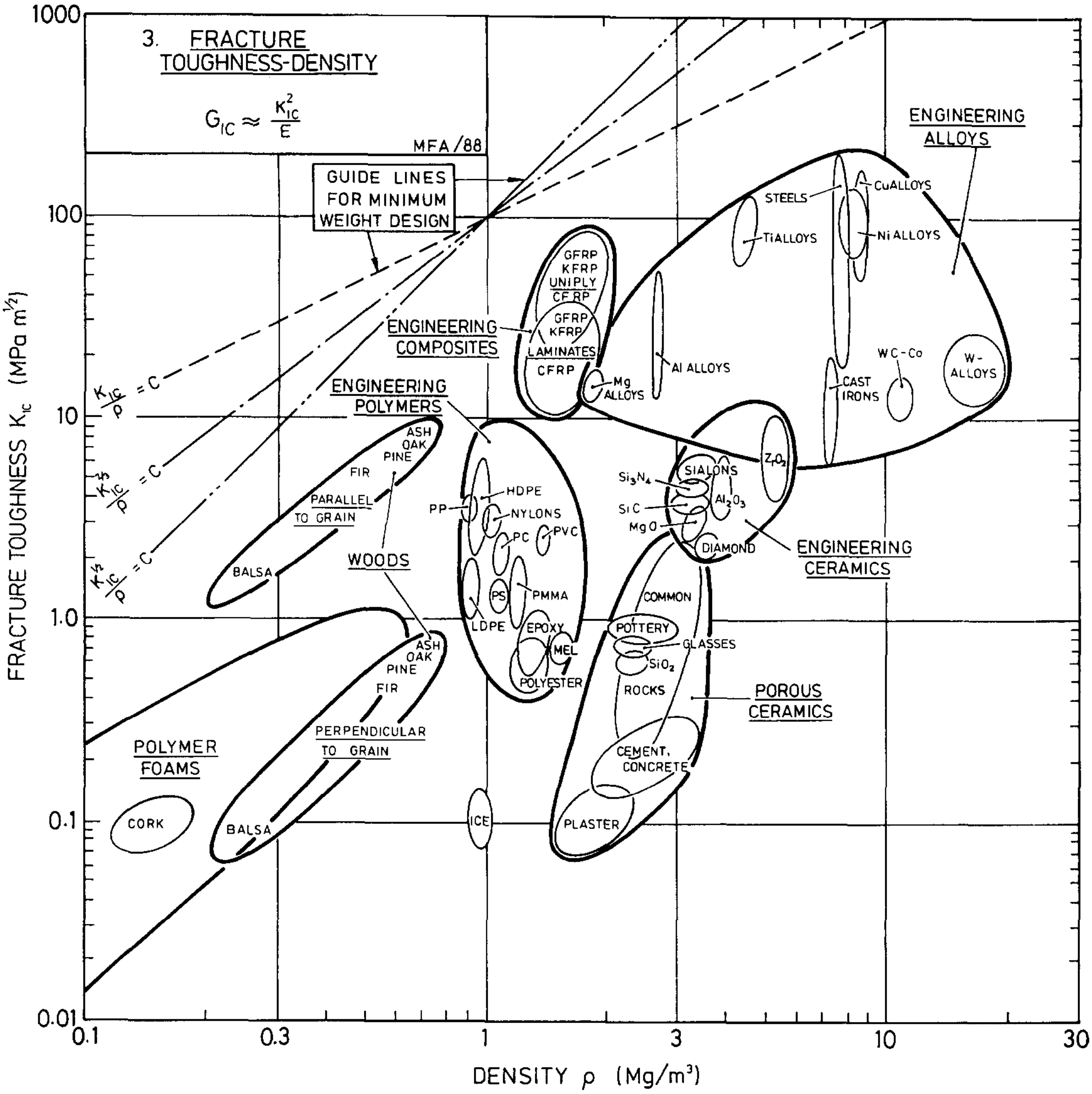

Fig. 7. Chart 3: Fracture toughness, K_{Ic}, plotted against density, ρ. The guide lines of constant K_{Ic}/ρ, $K_{Ic}^{2/3}/\rho$ and $K_{Ic}^{1/2}/\rho$ help in minimum weight, fracture-limited, design.

fail by fast fracture. The resistance to the propagation of a crack is measured by the *fracture toughness*, K_{Ic}. It is plotted against density in Fig. 7. The range is large: from 0.01 to over 100 MPa m$^{1/2}$. At the lower end of this range are brittle materials which, when loaded, remain elastic until they fracture. For these, linear-elastic fracture mechanics works well, and the fracture toughness itself is a well-defined property. At the upper end lie the super-tough materials, most of which show substantial plasticity before they break. For these the values of K_{Ic} are approximate, derived from critical J-integral (J_c) and critical crack-opening displacement (δ_c) measurements [by writing $K_{Ic} = (EJ_c)^{1/2}$, for instance]. They are helpful in providing a ranking of materials. The figure shows one reason for the dominance of metals in engineering: they almost all have values of K_{Ic} above 20 MPa m$^{1/2}$, a value often quoted as a minimum for conventional design.

There are a number of fundamental points to be made about the fracture toughness, but they are best demonstrated with Charts 5 and 6, coming later. Here we simply note that minimum-weight design, when the design criterion is that of preventing brittle fracture from a flaw of given size, requires that K_{Ic}/ρ, $K_{Ic}^{2/3}/\rho$ or $K_{Ic}^{1/2}/\rho$ (depending on loading geometry) be maximised (see Fig. 4). Guide-lines corresponding to constant values of these parameters are plotted on the diagram. They are used as described in Section 1.

3.4. The modulus-strength chart (Chart 4, Fig. 8)

High-tensile steel makes good springs. But so does rubber. How is it that two such different materials are both suited for the same task? This and other questions are answered by Fig. 8, the most useful of all the charts.

It shows Young's modulus, E, plotted against strength, σ_y. The qualifications on "strength" are

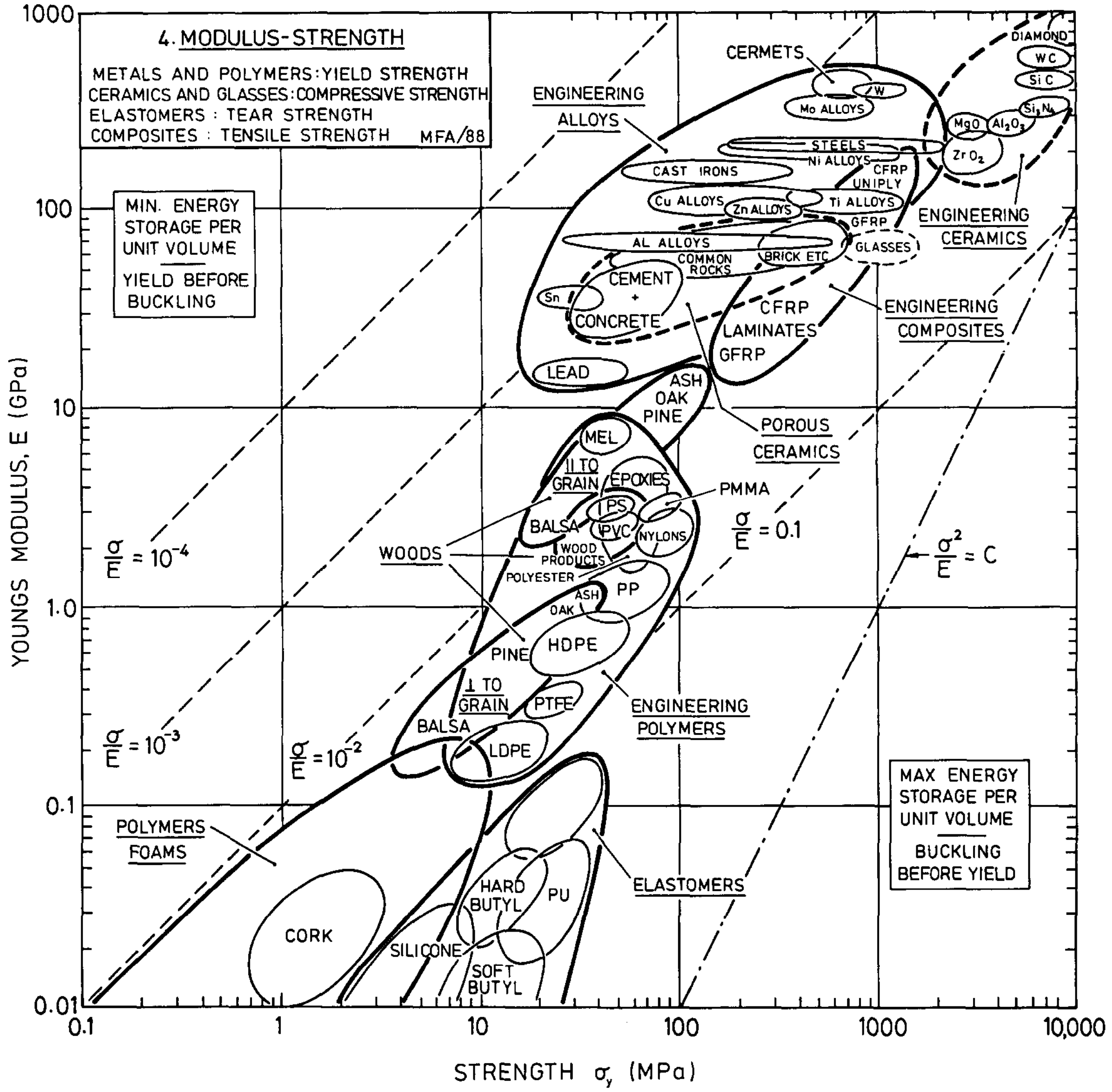

Fig. 8. Chart 4: Youngs modulus, E, plotted against strength σ_y. The guide line of constant σ_y^2/E helps with the selection of materials for springs, pivots and knife-edges; those of constant σ_y/E with choosing materials for elastic hinges.

the same as before: it means yield strength for metals and polymers, compressive crushing strength for ceramics, tear strength for elastomers and tensile strength for composites and woods; the symbol σ_y is used for them all. The ranges of the variables, too, are the same. Contours of *normalised strength*, σ_y/E, appear as a family of straight parallel lines.

Examine these first. Engineering polymers have normalised strengths between 0.01 and 0.1. In this sense they are remarkably strong: the value for metals are at least a factor of 10 smaller. Even ceramics, in compression, are not as strong, and in tension they are weaker (by a further factor of 15 or so). Composites and woods lie on the 0.01 contour, as good as the best metals. Elastomers, because of their exceptionally low moduli, have values of σ_y/E larger than any other class of material: 0.1 to 10.

The *ideal strength* of a solid is set by the range of interatomic forces. It is small—a bond is broken if it is stretched by more than 10% or so. So the force needed to break a bond is roughly

$$F = \frac{Sr_0}{10}$$

where S is the bond stiffness (Section 3.1). If shear *breaks* bonds, the strength of a solid should be roughly

$$\sigma_y \approx \frac{F}{r_0^2} = \frac{S}{10r_0} = \frac{E}{10}. \qquad (4)$$

The chart shows that, for some polymers, it is. Most solids are weaker, for two reasons.

First, non-localised bonds (those in which the cohesive energy derives from the interaction of one atom with large number of others, not just with its nearest neighbours) are not broken when the structure is sheared. The metallic bond, and the ionic bond for certain directions of shear, are like this; very pure metals, for example, yield at stresses as low as $E/10{,}000$, and strengthening mechanisms (Section 3.2) are needed to raise the strength. The covalent bond *is* localised; and covalent solids do, for this reason, have yield strength which, at low temperatures, are as high as $E/10$. It is hard to measure them (though it can sometimes be done by indentation) because of the second reason for weakness: they generally contain defects—concentrators of stress—from which shear or fracture can propagate, often at stresses well below the "ideal" $E/10$. Elastomers are anomalous (they have strengths of about E) because the modulus does not derive from bond-stretching, but from the change in entropy of the tangled molecular chains when the material is deformed.

In the design of columns and beams, the ratio σ_y/E often appears. Structures which have a high value of σ_y/E will deflect or buckle before they yield; those with low σ_y/E do the opposite. The best materials for an elastic hinge (a thin web or ligament that bends elastically, forming the hinge of a box or container, for example) are those with the maximum value of σ_y/E: the diagram immediately identifies them as elastomers and certain polymers (the polyethylenes, for example).

Finally, to return to springs. The best material for a spring is that with the greatest value of σ_y^2/E (because it stores the most elastic energy per unit volume, $\frac{1}{2}\sigma^2/E$, before it yields). A guide-line corresponding to the condition

$$\frac{\sigma_y^2}{E} = C$$

is plotted on the diagram; it, or any line parallel to it, links materials that are equally good by this criterion. If such a line is drawn through the middle of the elastomers, it just touches spring steeel. Ceramics must be rejected because they are weaker, by the factor of 15, in tension, but glass, which can be made defect-free, makes good springs. Slightly further to the right lie CFRP and GFRP. All are good for springs.

3.5. The fracture toughness-modulus chart (*Chart 5, Fig. 9*)

The fracture toughnesses of most polymers are less than those of most ceramics. Yet polymers are widely used in engineering structures; ceramics, because they are "brittle", are treated with much more caution. Figure 9 helps resolve this apparent anomaly. It shows the fracture toughness, K_{Ic}, plotted against Young's modulus, E. The restrictions described in Section 3.3 apply to the values of K_{Ic}: when small, they are well defined; when large, they are useful only as a ranking for material selection.

Consider first the question of the *necessary condition for fracture*. It is that sufficient external work be done, or elastic energy released, to supply the true surface energy (2γ per unit area) of the two new surfaces which are created. We write this as

$$G \geqslant 2\gamma \tag{5}$$

where G is the energy release-rate. Using the standard relation $K \approx (EG)^{1/2}$ between G and stress intensity K, we find

$$K \geqslant (2E\gamma)^{1/2}. \tag{6}$$

Now the surface energies, γ, of solid materials scale as their moduli; to an adequate approximation $\gamma = Er_0/20$, where r_0 is the atom size, giving

$$K \geqslant E\left[\frac{r_0}{20}\right]^{1/2}. \tag{7}$$

We identify the right-hand side of this equation with a lower-limiting value of K_{Ic}, when, taking r_0 as 2×10^{-10} m,

$$\frac{(K_{IC})}{E}\ \text{min} = \left[\frac{r_0}{20}\right]^{1/2} \approx 3 \times 10^{-6}\ \text{m}^{1/2}.$$

This criterion is plotted on the chart as a shaded, diagonal band near the lower right corner (the width of the band reflects a realistic range of r_0 and of the constant C in $\gamma = Er_0/C$). It defines a *lower limit* on values of K_{Ic}: it cannot be less then this unless some other source of energy (such as a chemical reaction, or the release of elastic energy stored in the special dislocation structures caused by fatigue loading) is available, when it is given a new symbol such as $(K_{Ic})_{scc}$. We note that the most brittle ceramics lie close to the threshold: when they fracture, the energy absorbed is only slightly more than the surface energy. With metals and polymers the energy absorbed by fracture is vastly greater, almost always because of *plasticity* associated with crack propagation. We come to this in a moment, with the next chart.

Plotted on Fig. 9 are contours of *toughness*, G_{Ic}, a measure of the apparent fracture surface-energy ($G_{Ic} \approx K_{Ic}^2/E$). The true surface energies, γ, of solids lie in the range 10^{-4} to 10^{-3} kJ/m^2. The diagram shows that the values of the toughness start at 10^{-3} kJ/m^2 and range through almost six decades to 10^3 kJ/m^2. On this scale, ceramics are low (10^{-3}–10^{-1} kJ/m^2), much lower than polymers (10^{-1}–10 kJ/m^2)—and this is part of the reason that design with polymers is easier than with ceramics. This is not to say that engineering design relies purely on G_{Ic}: it is more complicated than that. When the modulus is high, deflections are small. Then designers are concerned about the *loads* the structure can support. In load-limited design, the fracture toughness, K_{Ic}, is what matters: it determines, for a given crack length, the stress the structure can support. Experience shows that a value of K_{Ic} above about

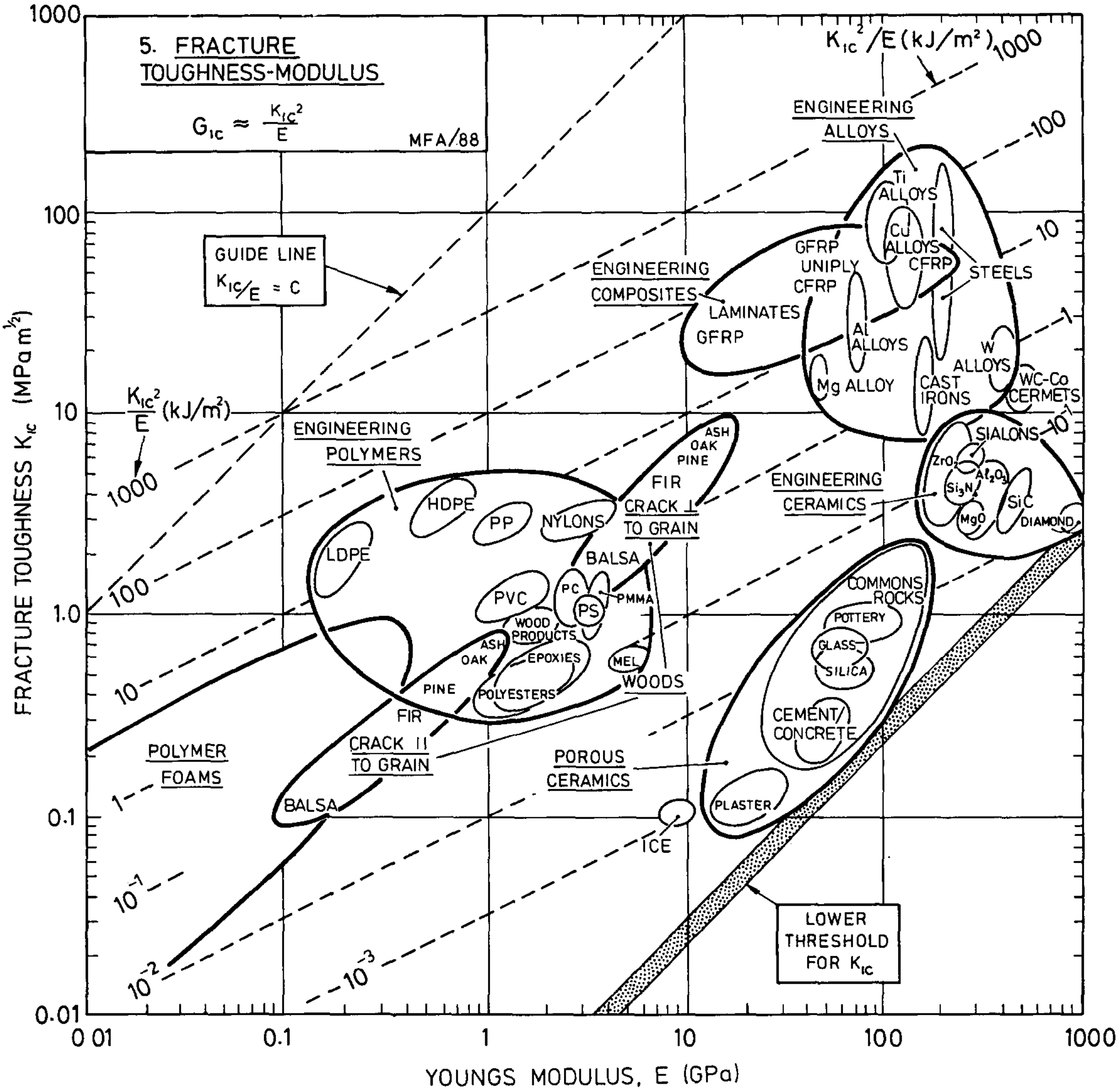

Fig. 9. Chart 5: Fracture toughness, K_{Ic}, plotted against Youngs modulus, E. The family of lines are of constant K_{Ic}^2/E (roughly, of G_{Ic}, the fracture energy). These, and the guide-line of constant K_{Ic}/E, help in design against fracture. The shaded band shows the "necessary condition" for fracture. Fracture can, in fact, occur below this limit under conditions of corrosion, or cyclic loading.

20 MPa$^{1/2}$ is necessary for conventional load-limited design methods to be viable. Only metals and composites meet this requirement.

Polymers, woods and foams have low moduli. Design with low-modulus materials focusses on limiting the *displacement*, requiring a high value of K_{Ic}/E, (or, equivalently, $(G_{Ic}/E)^{1/2}$). Polymers, woods and foams meet these requirements better than metals, as the guide-line of $K_{Ic}/E = C$ on the chart shows. The problem with ceramics is that they are poor by either criterion. The solution—since ceramics have other properties too good to ignore—lies in further progress in toughening them, and in new design methods which allow for their brittleness in tension.

That, of course, is still too simple. The next section adds further refinements.

3.6. The fracture toughness–strength chart (Chart 6, Fig. 10)

The stress concentration at the tip of a crack generates a *process-zone*: a plastic zone in ductile solids, a zone of microcracking in ceramics, a zone of delamination, debonding and fibre pull-out in composites. Within the process zone, work is done against plastic and frictional forces; it is this which accounts for the difference between the measured fracture energy G_{Ic} and the true surface energy 2γ. The amount of energy dissipated must scale roughly with the size of the zone d_y, given (by equating the stress field of the crack at $r = d_y$ to the strength σ_y of the material) by

$$d_y = \frac{K_{Ic}^2}{\pi\sigma_y^2} \qquad (9)$$

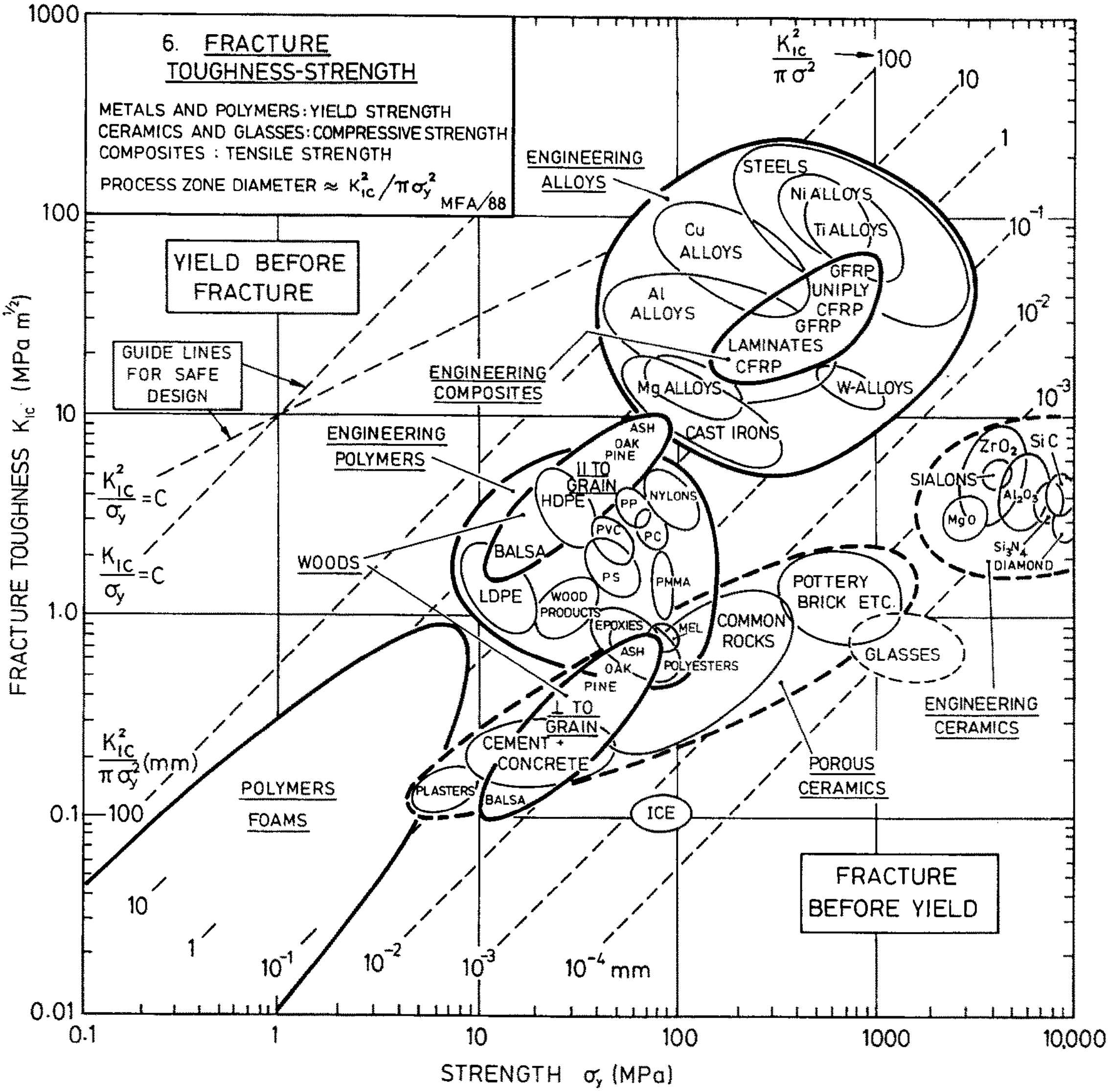

Fig. 10. Chart 6: Fracture-toughness, K_{Ic}, plotted against strength, σ_y. The contours show the value of $K_{Ic}^2/\pi\sigma_y^2$—roughly, the diameter of the process-zone at a crack tip (units: mm). The guide lines of constant K_{Ic}/σ_y and K_{Ic}^2/σ_y are used in yield-before-break and leak-before-break design.

and with the strength σ_y of the material within it. Figure 10—fracture toughness against strength—shows that the size of the zone, d_y (broken lines), varies enormously, from atomic dimensions for very brittle ceramics and glasses to almost 1 metre for the most ductile of metals. At a constant zone size, fracture toughness tends to increase with strength (as expected): it is this that causes the data plotted in Fig. 10 to be clustered around the diagonal of the chart.

The diagram has application in selecting materials for the safe design of load bearing structures. First some obvious points. Fast fracture occurs when

$$\sigma > C\frac{K_{Ic}}{\sqrt{\pi a_c}} \qquad (10)$$

where $2a_c$ is the length of the longest crack in the structure, and C is a constant near unity (we assume, below, that $C = 1$). The crack which will just propagate when the stress equals the yield strength has a length

$$a_c = \frac{K_{Ic}^2}{\pi\sigma_y^2} \qquad (11)$$

that is, the critical crack length is the same as the process zone size: the contours on the diagram. A valid fracture toughness test (one that gives a reliable value of the plane-strain fracture toughness K_{Ic}) requires a specimen with all dimensions larger than 10 times d_y; the contours, when multiplied by 10, give a quick idea of this.

There are two criteria for materials selection involving K_{Ic} and σ_y. First, safe design at a given *load* requires that the structure will *yield before it breaks*. If the minimum detectable crack size is $2a_c$, then this condition can be expressed as

$$\frac{K_{Ic}}{\sigma_y} \geqslant \sqrt{\pi a_c}. \qquad (12)$$

The safest material is the one with the greatest value of K_{Ic}/σ_y: it will tolerate the longest crack. But, though safe, it may not be efficient. The section required to carry the load decreases as σ_y increases. We want high K_{Ic}/σ_y *and* high σ_y. The reader may wish to plot two lines onto the figure, isolating the material which best satisfies both criteria at once: it is steel. It is this which gives steel its pre-eminence as the material for highly stressed structures when weight is not important.

One such structure is the pressure vessel. Here safe design requires that the vessel *leaks before it breaks*: leakage is not catastrophic, fast fracture is. To ensure this, the vessel must tolerate a crack of length, $2a_c$, equal to the wall thickness t, and this leads to a different criterion for materials selection. From the last equation, the leak-before-break criterion is

$$\frac{K_{Ic}^2}{\sigma_y^2} \geqslant \pi t.$$

But the pressure, p, that the vessel can support is limited by yield, so that, for a thin walled cylindrical vessel of radius R,

$$\frac{pR}{t} \leqslant \sigma_y.$$

Substituting for t gives

$$p \leqslant \frac{1}{\pi R}\left[\frac{K_{Ic}^2}{\sigma_y}\right].$$

The greatest pressure is carried by the vessel with the largest value of K_{Ic}^2/σ_y. A guide line of K_{Ic}^2/σ_y is shown on the chart. It, and the yield-before-break line, are used in the way described in Section 1. Again, steel and copper are optimal.

3.7. *The loss coefficient—modulus chart (Chart 7, Fig. 11)*

Bells, traditionally, are made of bronze. They can be (and sometimes are) made of glass; and they could (if you could afford it) be made of silicon carbide. Metals, glasses and ceramics all, under the right circumstances, have low intrinsic damping, or "internal friction", an important material property when structures vibrate. We measure intrinsic damping by the *loss coefficient*, η, which is plotted in Fig. 11. Other measures include the *specific damping capacity* D/U (the energy D dissipated per cycle of vibrational energy U), the *log decrement*, Δ (the log of the ratio of successive amplitudes), the *phase lag*, δ, between stress and strain and the *resonance factor*, Q. When the damping is small ($\eta < 0.01$) these measures are related by

$$\eta = \frac{D}{2\pi U} = \frac{\Delta}{\pi} = \tan\delta = \frac{1}{Q} \qquad (13)$$

but when the damping is large, the definitions are no longer equivalent. Large η's are best measured by recording a symmetric load cycle and dividing the area of the stress–strain loop by 2π times the peak energy stored.

There are many mechanisms of intrinsic damping and hysteresis. Some (the "damping" mechanisms) are associated with a process that has a specific time constant; then the energy loss is centred about a characteristic frequency. Others (the "hysteresis" mechanisms) are associated with time-independent mechanisms, and absorb energy at all frequencies.

One damping mechanism, common to all materials, is a thermoelastic effect. A suddenly-applied tensile stress causes a true solid to cool slightly as it expands (elastomers are not true solids, and show the opposite effect). As it warms back to its initial temperature it expands further, giving additional strain that lags behind the stress. The anisotropy of moduli means that a polycrystal, even when uniformly loaded, shows a thermoelastic damping because neighbouring grains distort—and thus cool—by differing amounts. The damping is proportional to the difference between the adiabatic modulus, E_A and that measured at constant temperature, E_T. A thermodynamic analysis (e.g. [24]) shows that

$$\eta = C\frac{E_A - E_T}{E_T} = \frac{CT\alpha^2 E_T}{\rho C_v} \qquad (14)$$

where α is the coefficient of linear thermal expansion, C_v the specific heat, T the temperature and C a constant. This leads to the shaded line on the Chart marked "thermal damping". Single crystals and glasses lie below the line, because, when loaded uniformly, no temperature gradients exist.

The loss coefficient of most materials is far higher than this. In metals a large part of the loss is hysteretic, caused by dislocation movement: it is high in soft metals like lead and aluminium, but heavily alloyed metals like bronze, and high-carbon steels have low loss because the solute pins the dislocations. Exceptionally high loss is found in the Mu–Cu alloys, because of a strain-induced martensite transformation, and in magnesium, perhaps because of reversible twinning. The elongated balloons for metals span the large range accessible by alloying and working. Engineering ceramics have low damping because the enormous lattice resistance (Section 3.2) pins dislocations in place at room temperature. Porous ceramics, on the other hand, are filled with cracks, the surfaces of which rub, dissipating energy, when the material is loaded; the high damping of some cast irons has a similar origin. In polymers, chain segments slide against each other when loaded; the relative motion lowers the compliance and dissipates energy. The ease with which they slide depends on the ratio of the temperature (in this case, room

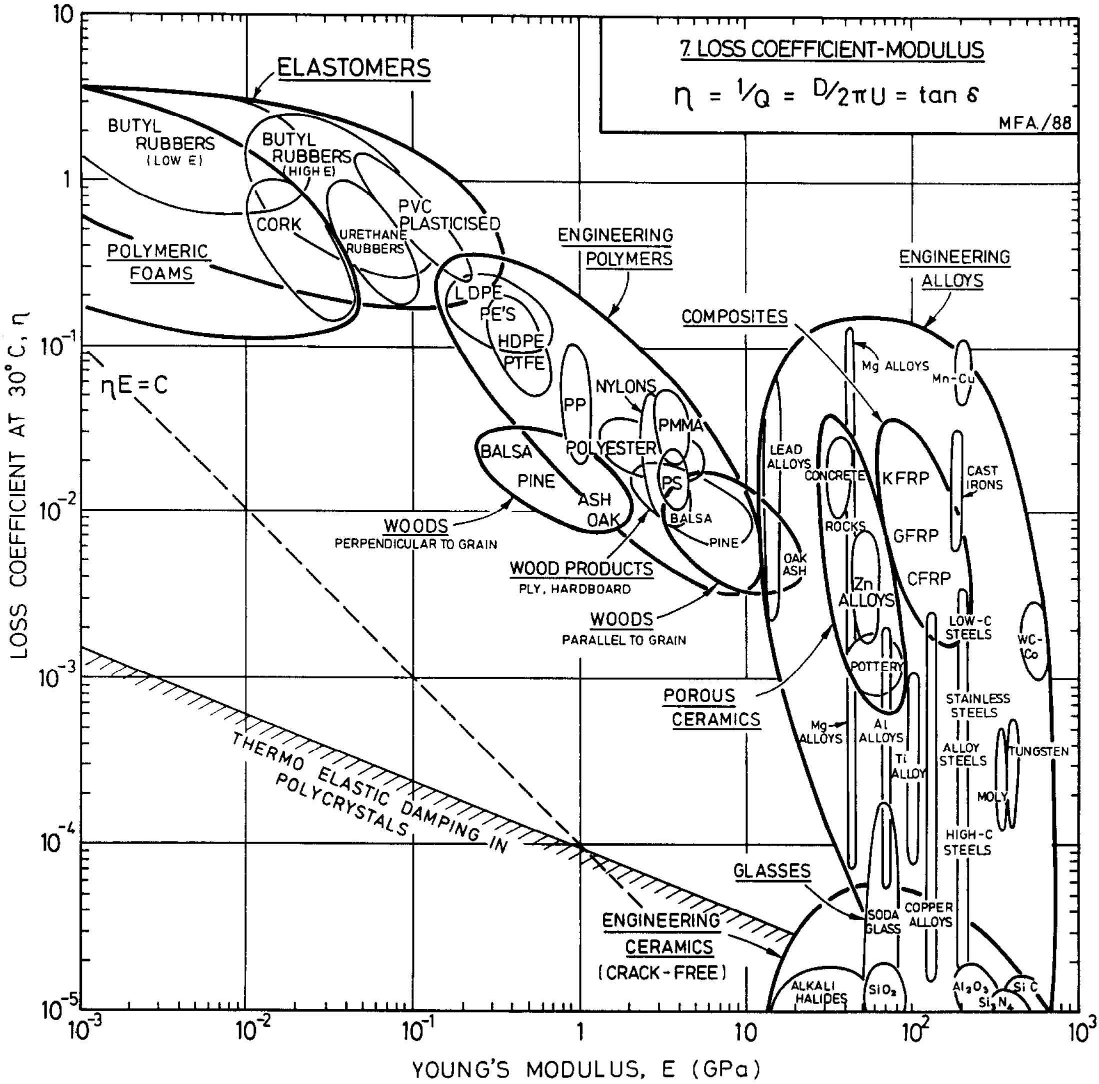

Fig. 11. Chart 7: The loss coefficient, η, plotted against Youngs modulus, E. the guide-line corresponds to the condition $\eta = C/E$.

temperature) to the glass temperature, T_g, of the polymer. When $T/T_g < 1$, the secondary bonds are "frozen", the modulus is high and the damping is relatively low. When $T/T_g > 1$, the secondary bonds have melted, allowing easy chain slippage: the modulus is low and the damping is high. This accounts for the obvious inverse dependence of η on E for polymers in Fig. 11; indeed, to a first approximation

$$\eta = \frac{4 \times 10^{-2}}{E} \qquad (15)$$

with E in *GPa*.

3.8. *The thermal conductivity–thermal diffusivity chart (Chart 8, Fig. 12)*

The material property governing the flow of heat through a material at steady state is the *thermal conductivity*, λ (units: J/mK); that governing transient heat flow is the *thermal diffusivity*, a (units: m^2/s). They are related by

$$a = \frac{\lambda}{\rho C_p} \qquad (16)$$

where ρ is the density and C_p the specific heat, measured in J/kg·K; the quantity ρC_p is the *volumetric specific heat*. Figure 12 relates conductivity, diffusivity and volumetric specific heat, at room temperature.

The data span almost 5 decades in λ and a. Solid materials are strung out along the line

$$\rho C_p \approx 3 \times 10^6\ \mathrm{J/m^3\,K} \qquad (17)$$

This can be understood by noting that a solid containing N atoms has $3N$ vibrational modes. Each (in the classical approximation) absorbs thermal energy kT at the absolute temperature T, and the vibrational specific heat is $C_p \approx C_v = 3Nk$ (J/K) where k is Bolzmann's constant. The volume per atom, Ω, for almost all solids lies within a factor of two of 2×10^{-29} m^3, so the volume of N atoms is

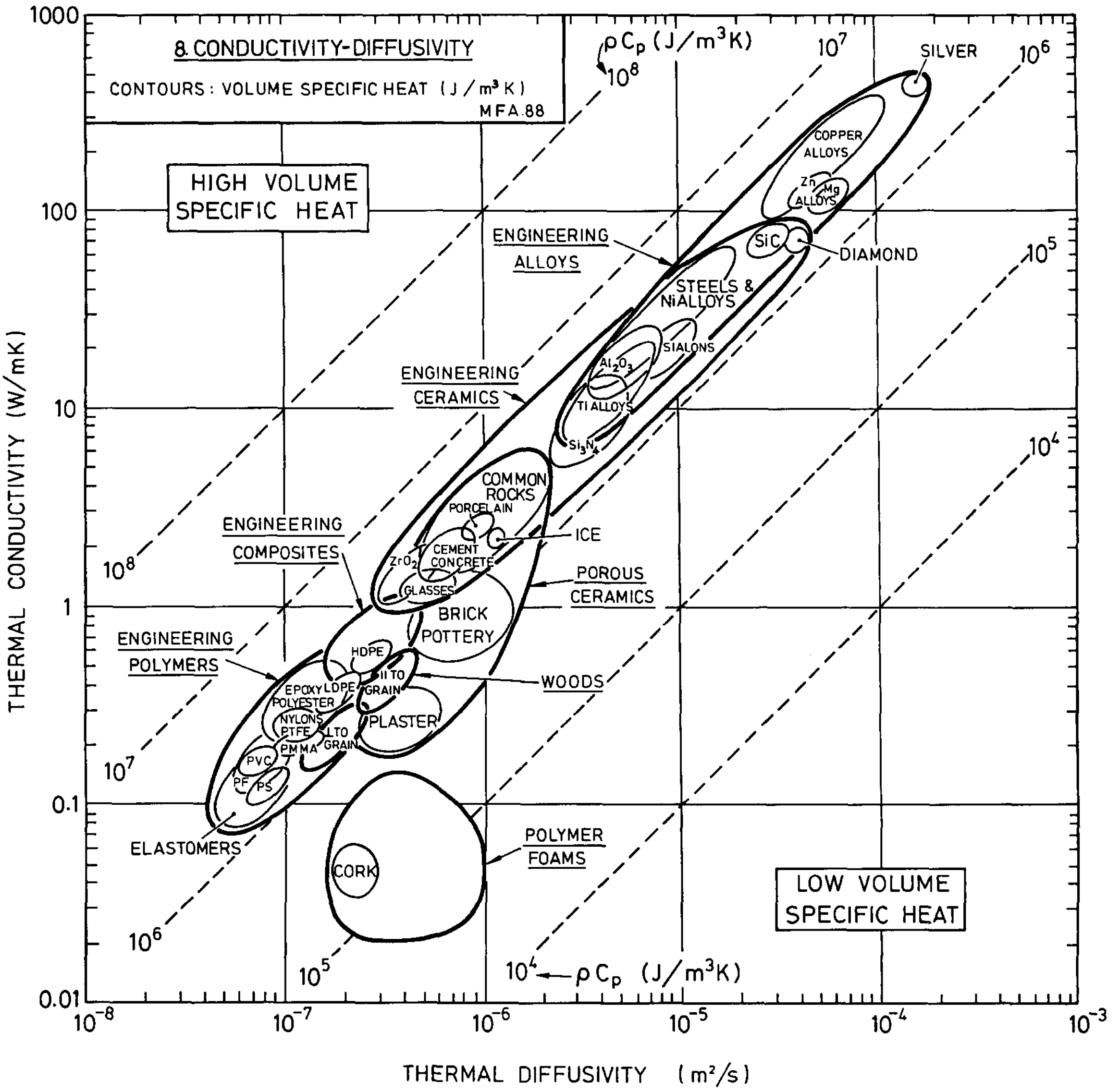

Fig. 12. Chart 8: Thermal conductivity, λ, plotted against thermal diffusivity, a. the contours show the volume specific heat, ρC_p. All three properties vary with temperature; the data here are for room temperature.

2×10^{-29} N. The volume specific heat is then (as the Chart shows)

$$\rho C_v = 3Nk/N\Omega = 3k/\Omega = 3 \times 10^6 \text{ J/m}^3\text{ K}. \quad (18)$$

For solids, C_p and C_v differ very little; at the level of approximation of this paper we assume them to be equal. As a general rule, then

$$\lambda = 3 \times 10^6 a$$

(λ in J/mK and a in m²/s). Some materials deviate from this rule: they have lower-than-average volumetric specific heat. A few, like diamond, are low because their Debye temperatures lie well above room temperature; then heat absorption is not classical, some modes do not absorb kT and the specific heat is less than $3Nk$. The largest deviations are shown by porous solids: foams, low density firebrick, woods and so on. Their low density means that they contain fewer atoms per unit volume and, averaged over the volume of the structure, ρC_v is low. The result is that, although foams have low *conductivities* (and are widely used for insulation because of this) their thermal *diffusivities* are not low: they may not transmit much heat, but they reach a steady state quickly.

The range of λ and of a reflect the mechanisms of heat transfer in each class of solid. Electrons conduct the heat in pure metals such as copper, silver and aluminium (top right of chart). The conductivity is described by

$$\lambda = \tfrac{1}{3} C_e \bar{c} l \quad (19)$$

where C_e is the electron specific heat per unit volume, $\bar{c}$ is the electron velocity (2×10^5 m/s) and l the electron mean free path, typically 10^{-7} m in pure metals. In solid solution (steels, nickel-based and titanium alloys) the foreign atoms scatter electrons,

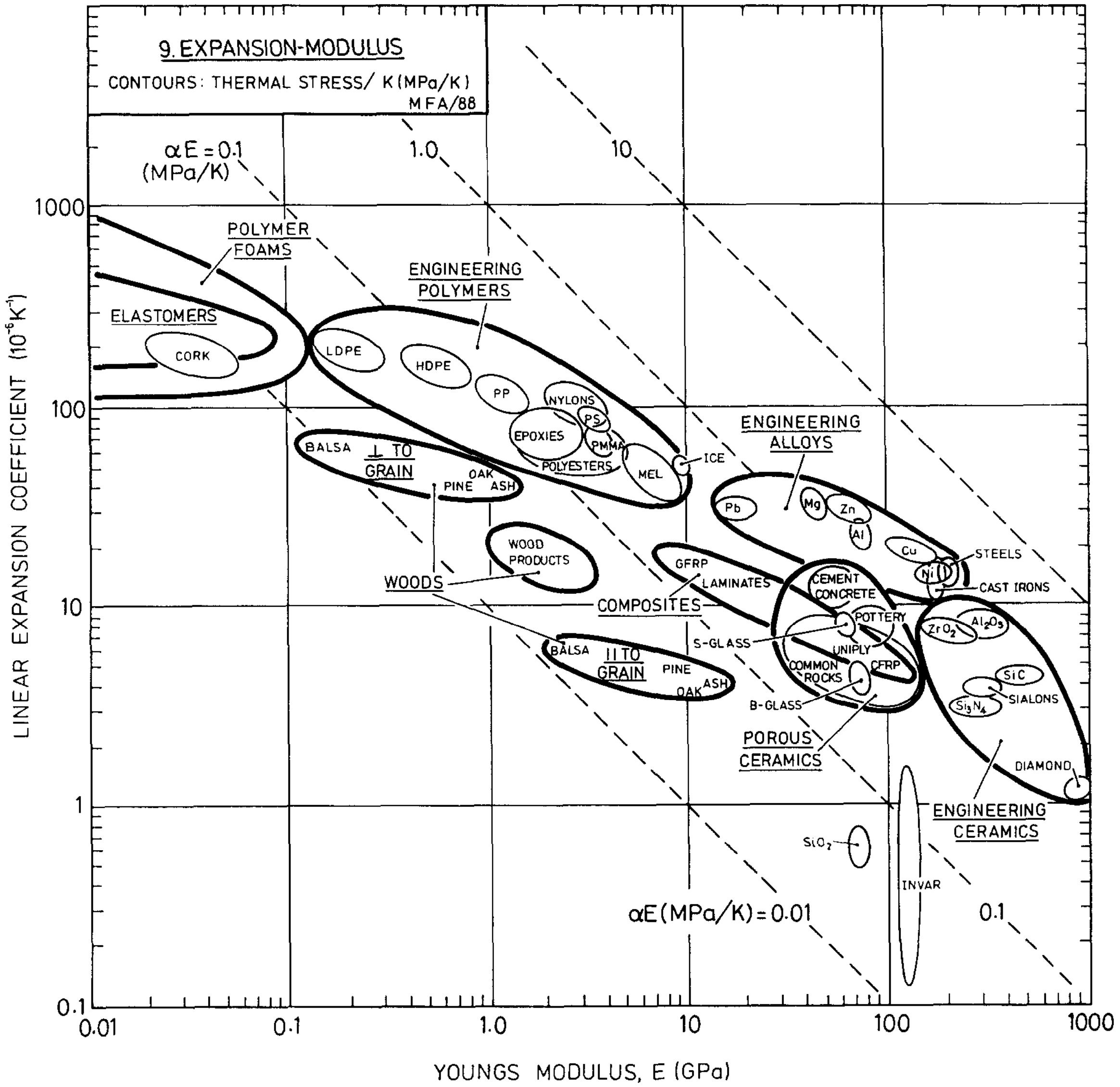

Fig. 13. Chart 9: The linear expansion coefficient, α, plotted against Young's modulus, E. The contours show the thermal stress created by a temperature change of 1°C if the sample is axially constrained. A correction factor C is applied for biaxial or triaxial constraint (see text).

reducing the mean free path to atomic dimensions ($\approx 10^{-10}$ m), much reducing λ and a.

Electrons do not contribute to conduction in ceramics and polymers. Heat is carried by phonons—lattice vibrations of short wavelength. They are scattered by each other (through an anharmonic interaction) and by impurities, lattice defects and surfaces; it is these which determine the phonon mean free path, l. The conductivity is still given by equation (19) which we write as

$$\lambda = \tfrac{1}{3}\rho C_p \bar{c} l \qquad (20)$$

but now $\bar{c}$ is the elastic wave speed (around 10^3 m/s—see Chart 1) and ρC_p is the volumetric specific heat. If the crystal is particularly perfect, and the temperature is well below the Debye temperature, as in diamond at room temperature, the phonon conductivity is high: it is for this reason that single crystal diamond, silicon carbide, and even alumina have conductivities almost as high as copper. The low conductivity of glass is caused by its irregular amorphous structure: the characteristic length of the molecular linkages (about 10^{-9} m) determines the mean free path. Polymers have low conductivities because the elastic wave speed $\bar{c}$ is low (Chart 1), and the mean free path in the disordered structure is small.

The best insulators are highly porous materials like firebrick, cork and foams. Their conductivity is limited by that of the gas in their cells, and (in very low density polymer foams) by heat transfer by radiation though the transparent cell walls.

3.9. The thermal expansion–modulus chart (Chart 9, Fig. 13)

Almost all solids expand on heating. The bond between a pair of atoms behaves like a linear-elastic spring when the relative displacement of the atoms is small; but when it is large, the spring is non-linear. Most bonds become stiffer when the atoms are pushed together, and less stiff when they are pulled apart. The thermal vibration of atoms, even at room temperature, involves large displacements; as the temperature is raised, the non-linear spring constant of the bond pushes the atoms apart, increasing their mean spacing. The effect is measured by the *linear expansion coefficient*

$$\alpha = \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}T} \tag{21}$$

where l is a linear dimension of the body. A quantitative development of this theory leads to the relation

$$\alpha = \frac{\gamma_G \rho C_v}{3E} \tag{22}$$

where γ_G is Gruneisen's constant; its value ranges between about 0.4 and 4, but for most solids it is near 1. Since ρC_v is almost constant [equation (18)], the equation tells us that α is proportional to $1/E$. Figure 13 shows that this is so. Diamond, with the highest modulus, has one of the lowest coefficients of expansion; elastomers with the lowest moduli expand the most. Some materials with a low coordination number (silica, and some diamond-cubic or zinc-blende structured materials) can absorb energy preferentially in transverse modes, leading to a very small (even a negative) value of γ_G and a low expansion coefficient—that is why SiO_2 is exceptional. Others, like Invar, contract as they lose their ferromagnetism when heated through the Curie temperature and, over a narrow range of temperature, show near-zero expansion, useful in precision equipment and in glass–metal seals.

One more useful fact: the moduli of materials scale approximately with their melting point, T_m (see, for example, Ref. [9])

$$E \approx \frac{100\,kT_m}{\Omega} \tag{23}$$

where k is Boltzmann's constant and Ω the volume-per-atom in the structure. Substituting this and equation (18) for ρC_v into equation (22) for α gives

$$\alpha = \frac{\gamma_G}{100\,T_m} \tag{24}$$

—the expansion coefficient varies inversely with the melting point, or (equivalently stated), for all solids the thermal strain, just before they melt, is the same. The result is useful for estimating and checking expansion coefficients.

Whenever the thermal expansion or contraction of a body is prevented, thermal stresses appear; if large enough, they cause yielding, fracture, or elastic collapse (buckling). It is common to distinguish between thermal stress caused by external constraint (a rod, rigidly clamped at both ends, for example) and that which appears without external constraint because of temperature gradients in the body. All scale as the quantity αE, shown as a set of diagonal contours on Fig. 13. More precisely: the stress $\Delta\sigma$ produced by a temperature change of 1°C in a constrained system, or the stress per °C caused by a sudden change of surface temperature in one which is not constrained, is given by

$$C\Delta\sigma = \alpha E \tag{25}$$

where $C = 1$ for axial constraint $(1 - \nu)$, for biaxial constraint or normal quenching, and $(1 - 2\nu)$ for triaxial constraint, where ν is Poisson's ratio. These stresses are large: typically 1 MPa/K; they can cause a material to yield, or crack, or spall, or buckle, when it is suddenly heated or cooled. The resistance of materials to such damage is the subject of the next section.

3.10. The normalised strength—thermal expansion chart (Chart 10, Fig. 14)

When a cold ice-cube is dropped into a glass of gin, it cracks audibly. The ice is failing by thermal shock. The ability of a material to withstand such stresses is measured by its *thermal shock resistance*. It depends on its thermal expansion coefficient, α, and its normalised strength, σ_t/E. They are the axes of Fig. 14, on which contours of constant $\sigma_t/\alpha E$ are plotted. The tensile strength, σ_t, requires definition, just as σ_y did. For brittle solids, it is the tensile fracture strength (roughly equal to the modulus of rupture, or MOR). For ductile metals and polymers, it is the tensile yield strength; and for composites it is the stress which first causes permanent damage in the form of delamination, matrix cracking of fibre debonding.

To use the chart, we note that a temperature change of ΔT, applied to a constrained body—or a sudden change ΔT of the surface temperature of a body which is unconstrained—induces a stress

$$\sigma = \frac{E\alpha\Delta T}{C} \tag{26}$$

where C was defined in the last section. If this stress exceeds the local strength σ_t of the material, yielding or cracking results. Even if it does not cause the component to fail, it weakens it. Then a measure of the thermal shock resistance is given by

$$\frac{\Delta T}{C} = \frac{\sigma_t}{\alpha E}. \tag{27}$$

This is not quite the whole story. When the constraint is internal, the thermal conductivity of the material becomes important. Instant cooling requires an infinite heat transfer coefficient, h, when the body is quenched. Water quenching gives a high h, and then the values of ΔT calculated from equation (27) give an approximate ranking of thermal shock resistance. But when heat transfer at the surface is poor and the thermal conductivity of the solid is high (thereby reducing thermal gradients) the thermal stress is less than that given by equation (26) by a

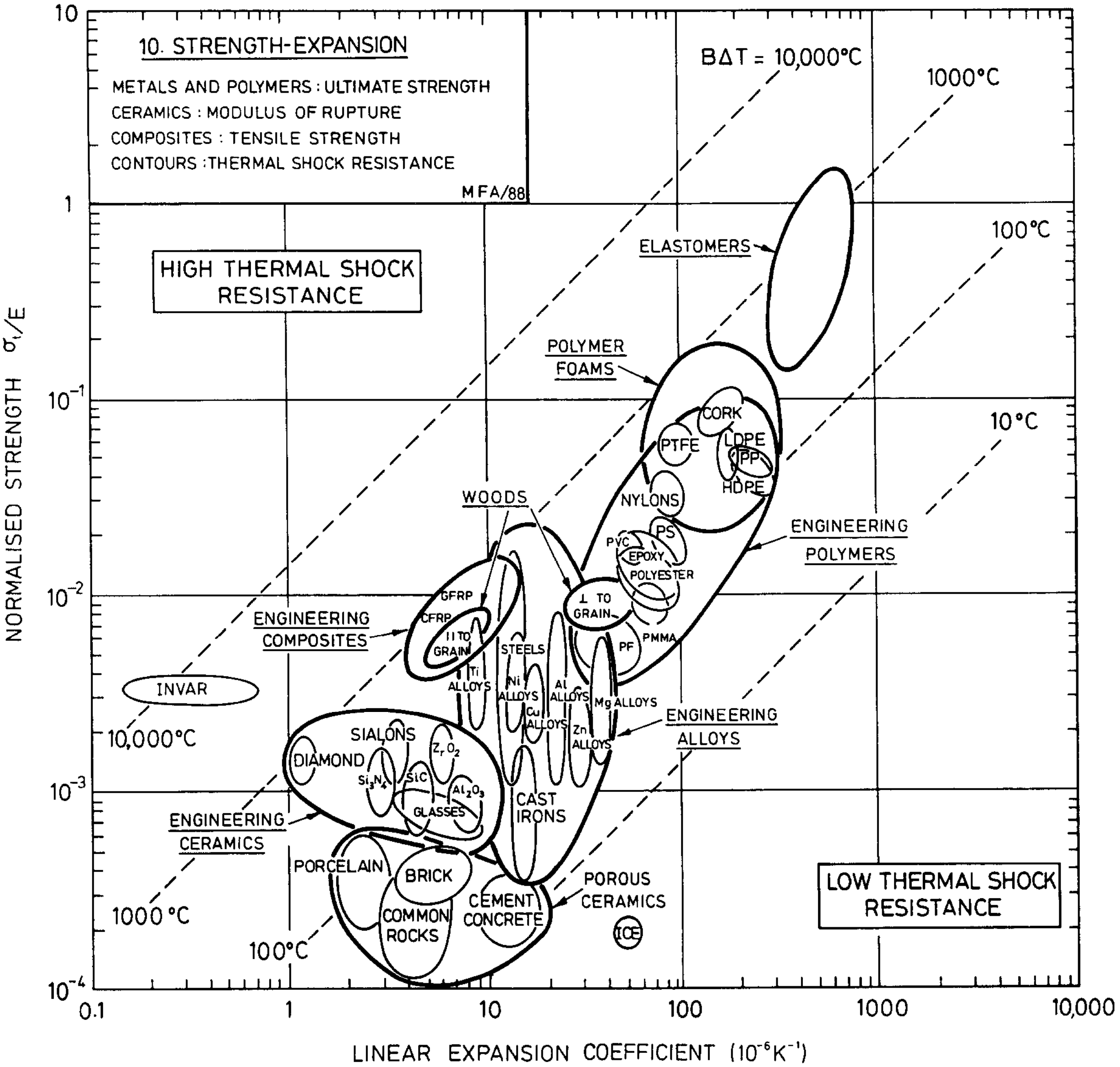

Fig. 14. Chart 10: The normalised tensile strength, σ_t/E, plotted against linear coefficient of expansion, α. The contours show a measure of the thermal shock resistance, ΔT. Corrections must be applied for constraint, and to allow for thermal conduction during quenching.

factor A which, to an adequate approximation, is given by

$$A = \frac{th/\lambda}{1 + th/\lambda} \tag{28}$$

where t is a typical dimension of the sample in the direction of heat flow; the quantity th/λ is usually called the Biot modulus. Table 3 gives typical values of A, for each class, using a section size of 10 mm.

The equation defining the thermal shock resistance, ΔT, now becomes

$$B\Delta T = \frac{\sigma_t}{\alpha E} \tag{29}$$

Table 3. Values of the factor A (Section t = 10 mm)

Conditions	Foams	Polymers	Ceramics	Metals
Air flow, slow (h = 10 W/m^2 K)	0.75	0.5	3×10^{-2}	3×10^{-3}
Black body radiation 500 to 0°C (h = 40 W/m^2 K)	0.93	0.6	0.12	1.3×10^{-2}
Air flow, fast (h = 10^2 W/m^2 K)	1	0.75	0.25	3×10^{-2}
Water quench, slow (h = 10^3 W/m^2 K)	1	1	0.75	0.23
Water quench, fast (h = 10^4 W/m^2 K)	1	1	1	0.1–0.9

where $B = C/A$. The contours on the diagram are of $B\ \Delta T$. The table shows that, for rapid quenching, A is unity for all materials except the high conductivity metals: then the thermal shock resistance is simply read from the contours, with appropriate correction for the constraint (the factor C). For slower quenches, ΔT is larger by the factor $1/A$, read from the table.

4. CONCLUSIONS AND APPLICATIONS

Most research on materials concerns itself, quite properly, with precision and detail. But it is occasionally helpful to stand back and view, as it were, the general lie of the land; to seek a framework into which the parts can be fitted; and when they do not fit, to examine the interesting exceptions. These charts are an attempt to do this for the mechanical and thermal properties of materials. There is nothing new in them except the mode of presentation, which summarises information in a compact, accessible way. The logarithmic scales are wide—wide enough to include materials as diverse as polymer foams and high performance ceramics—allowing a comparison between the properties of all classes of solid. And by choosing the axes in a sensible way, more information can be displayed: a chart of modulus E against density ρ reveals the longitudinal wave velocity $(E/\rho)^{1/2}$; a plot of fracture toughness K_{Ic} against modulus E shows the fracture surface energy G_{Ic}: a diagram of thermal conductivity λ against diffusivity, a, also gives the volume specific heat ρC_{v}; expansion, α, against normalised strength, σ_{t}/E, gives thermal shock resistance ΔT.

The most striking feature of the charts is the way in which members of a material class cluster together. Despite the wide range of modulus and density associated with metals (as an example), they occupy a field which is distinct from that of polymers, or that of ceramics, or that of composites. The same is true of strength, toughness, thermal conductivity and the rest: the fields sometimes overlap, but they always have a characteristic place within the whole picture. The position of the fields and their relationship can be understood in simple physical terms: the nature of the bonding, the packing density, the lattice resistance and the vibrational modes of the structure (themselves a function of bonding and packing), and so forth. It may seem odd that so little mention has been made of microstructure in determining properties. But the charts clearly show that the first-order difference between the properties of materials has its origins in the mass of the atoms, the nature of the interatomic forces and the geometry of packing. Alloying, heat treatment and mechanical working all influence microstructure, and through this, properties, giving the elongated balloons shown on many of the charts; but the magnitude of their effect is less, by factors of 10, than that of bonding and structure.

The charts have other applications. One is the checking of data. Computers consume data; their output is no better than their input; garbage in (it is often said), garbage out. That creates a need for *data valition*; ways of checking that the value assigned to a material property is reasonable, that it lies within an expected field of values. The charts define the limits of the fields. A value, if it lies within the field, is reasonable; if it lies outside it may not be wrong, but it is exceptional and should, perhaps, be questioned.

Another concerns the nature of data. The charts (this is not meant to sound profound) are a section through a multi-dimensional property-space. Each material occupies a small volume in this space; classes of material occupy a somewhat larger volume. If data for a material (a new polymer, for instance) lie outside its characteristic volume, then the material is, in some sense, novel. The physical basis of the property deserves investigation and explanation.

There is another facet to this: that of "the new material looking for an application". Established materials have applications; they are known. The first-order approach to identifying applications for a new material is to plot its position on charts and examine its environment: is it lighter, or stiffer, or stronger than its neighbours? Does it have a better value of a design-limiting combination like σ_{y}^2/E than they? Then it may compete in the applications they currently enjoy.

Finally, the charts help in problems of materials selection. In the early stages of the design of a component or structure, *all* materials should be considered; failure to do so may mean a missed opportunity for innovation or improvement. The number of materials available to the engineer is enormous (estimates range from 50,000 to 80,000). But any design is limited by certain material properties—by stiffness E or strength σ_{y} for instance, or by combinations such as $E^{1/2}/\rho$ or $K_{\text{Ic}}^2/\sigma_{\text{y}}$. These *design-limiting properties* are precisely those used as the axes of the charts, or the "guide lines" plotted on them. By following the procedures of Section 1, a subset of materials is isolated which best satisfies the primary demands made by the design; secondary constraints then narrow the choice to one or a few possibilities (examples in Refs [1] and [24]).

A final word. Every effort has been made to include in the charts a truly representative range of materials; to find reliable data for them; and to draw the envelopes to enclose all reasonably common members of a class to which they belong (not just those specifically listed). I am aware that the charts must still be imperfect, and hope that anyone with better information will extend them.

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