

Lista de exercícios 1 – Integrais I

A) Exercícios extraídos do **capítulo 7** de MORETTIN, P.A; S. HAZZAN; W.O. BUSSAB. **Cálculo. Funções de uma e várias variáveis.** Editora Saraiva, São Paulo, 2003, 413p.

1. Obtenha as integrais indefinidas a seguir:

- | | |
|---|--|
| a) $\int 2x^3 dx$ | j) $\int (x^{-3} + x^2 - 5x) dx$ |
| b) $\int (x^2 + 3x) dx$ | k) $\int \sqrt{x} dx$ |
| c) $\int (x^2 - 3x) dx$ | l) $\int 5\sqrt[3]{x} dx$ |
| d) $\int (5 - x) dx$ | m) $\int (\sqrt{x} + \sqrt[3]{x}) dx$ |
| e) $\int 5 dx$ | n) $\int \left(\frac{x^2 - 3x + 5}{x^2} \right) dx$ |
| f) $\int (3x^3 - 2x^2 + 8x - 6) dx$ | o) $\int \left(\frac{1}{1+x^2} + x^2 \right) dx$ |
| g) $\int \frac{5}{x} dx$ | p) $\int 2e^x dx$ |
| h) $\int \left(x^2 + \frac{6}{x} \right) dx$ | q) $\int (3e^x + x^3) dx$ |
| i) $\int (\sin x + \cos x) dx$ | r) $\int (\sin x - 5e^x) dx$ |

2. Mostre que $\int 2^x dx = \frac{2^x}{\ln 2} + c$.

3. Mostre que $\int \frac{2x}{x^2 + 3} dx = \ln(x^2 + 3) + c$.

4. Mostre que $\int e^{3x} dx = \frac{1}{3} e^{3x} + c$.

5. Sabendo-se que o custo marginal é $C_{mg}(x) = 0,08x + 3$ e que o custo fixo é \$ 100,00, obtenha a função custo.

Respostas

1. a) $\frac{x^4}{2} + c$
- | | |
|---|--|
| b) $\frac{x^3}{3} + \frac{3x^2}{2} + c$ | k) $\frac{2x^{3/2}}{3} + c$ |
| c) $\frac{x^3}{3} - \frac{3x^2}{2} + c$ | l) $\frac{15x^{4/3}}{4} + c$ |
| d) $5x - \frac{x^2}{2} + c$ | m) $\frac{2x^{3/2}}{3} + \frac{3x^{4/3}}{4} + c$ |
| e) $5x + c$ | n) $x - 3 \ln x - \frac{5}{x} + c$ |
| f) $\frac{3x^4}{4} - \frac{2x^3}{3} + 4x^2 - 6x + c$ | o) $\arctg x + \frac{x^3}{3} + c$ |
| g) $5 \ln x + c$ | p) $2e^x + c$ |
| h) $\frac{x^3}{3} + 6 \ln x + c$ | q) $3e^x + \frac{x^4}{4} + c$ |
| i) $-\cos x + \sin x + c$ | r) $-\cos x - 5e^x + c$ |
| j) $-\frac{1}{2x^2} + \frac{x^3}{3} - \frac{5x^2}{2} + c$ | |

Nos exercícios 1 a 10, calcular a integral e em seguida, derivar as respostas para conferir os resultados.

1. $\int \frac{dx}{x^3}$
2. $\int \left(9t^2 + \frac{1}{\sqrt{t^3}} \right) dt$
3. $\int (ax^4 + bx^3 + 3c) dx$
4. $\int \left(\frac{1}{\sqrt{x}} + \frac{x\sqrt{x}}{3} \right) dx$
5. $\int (2x^2 - 3)^2 dx$
6. $\int \frac{dx}{\sin^2 x}$
7. $\int \left(\sqrt{2y} - \frac{1}{\sqrt{2y}} \right) dy$
8. $\int \frac{\sqrt{2} dt}{3t^2 + 3}$
9. $\int x^3 \sqrt{x} dx$
10. $\int \frac{x^5 + 2x^2 - 1}{x^4} dx$

Nos exercícios de 11 a 30, calcular as integrais indefinidas.

11. $\int \frac{x^2}{x^2 + 1} dx$
12. $\int \frac{x^2 + 1}{x^2} dx$
13. $\int \frac{\sec x}{\cos^2 x} dx$
14. $\int \sqrt{\frac{9}{1 - x^2}} dx$
15. $\int \sqrt{\frac{4}{x^4 - x^2}} dx$
16. $\int \frac{8x^4 - 9x^3 + 6x^2 - 2x + 1}{x^2} dx$
17. $\int \left(\frac{e^t}{2} + \sqrt{t} + \frac{1}{t} \right) dt$
18. $\int \cos \theta \cdot \operatorname{tg} \theta d\theta$
20. $\int (t + \sqrt{t} + \sqrt[3]{t} + \sqrt[4]{t} + \sqrt[5]{t}) dt$
21. $\int \frac{x^{-1/3} - 5}{x} dx$
23. $\int \sec^2 x (\cos^3 x + 1) dx$
24. $\int \frac{dx}{(ax)^2 + a^2}, a \neq 0, \text{ constant}$
25. $\int \frac{x^2 - 1}{x^2 + 1} dx$
26. $\int \sqrt[3]{8(t - 2)^6 \left(t + \frac{1}{2}\right)^3} dt$
27. $\int (e^t - \sqrt[4]{16t} + \frac{3}{t^3}) dt$
28. $\int \frac{\ln x}{x \ln x^2} dx$
29. $\int \operatorname{tg}^2 x \operatorname{cosec}^2 x dx$
30. $\int (x - 1)^2 (x + 1)^2 dx$

Nota: Utilize, quando necessário, para estes exercícios, a tabela de integrais básicas.

Respostas

11. $x - \operatorname{arc} \operatorname{tg} x + c$
12. $x - \frac{1}{x} + c$
13. $\sec x + c$
14. $3 \operatorname{arc} \operatorname{sen} x + c$
15. $2 \operatorname{arc} \sec x + c$
16. $\frac{8x^3}{3} - \frac{9x^2}{2} + 6x - 2 \ln |x| - \frac{1}{x} + c$
17. $\frac{1}{2} e^t + \frac{2}{3} t^{3/2} + \ln |t| + c$
18. $-\cos \theta + c$
20. $\frac{t^2}{2} + \frac{2}{3} t^{3/2} + \frac{3}{4} t^{4/3} + \frac{4}{5} t^{5/4} + \frac{5}{6} t^{6/5} + c$
21. $\frac{-3}{3\sqrt{x}} - 5 \ln |x| + c$
23. $\operatorname{sen} x + \operatorname{tg} x + c$
24. $\frac{1}{a^2} \operatorname{arc} \operatorname{tg} x + c$
25. $x - 2 \operatorname{arc} \operatorname{tg} x + c$
26. $\frac{t^4}{2} - \frac{7t^3}{3} + 2t^2 + 4t + c$
27. $e^t - \frac{8}{5} t^{5/4} - \frac{3}{2t^2} + c$
28. $\frac{1}{2} \ln |x| + c$
29. $\operatorname{tg} x + c$
30. $\frac{x^5}{5} - \frac{2}{3} x^3 + x + c$

Tabela de derivadas

$f(x) = x^a \Rightarrow f'(x) = ax^{a-1}$	
$f(x) = a^x \Rightarrow f'(x) = a^x \ln a$	$f(x) = e^x \Rightarrow f'(x) = e^x$
$f(x) = u(x)^{v(x)} \Rightarrow f'(x) = u(x)^{v(x)} \left[v'(x) \ln u(x) + v(x) \frac{u'(x)}{u(x)} \right]$	
$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a} = \frac{1}{x} \log_a e$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \sin x \Rightarrow f'(x) = \cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
$f(x) = \operatorname{tg} x \Rightarrow f'(x) = \sec^2 x$	$f(x) = \operatorname{cotg} x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$
$f(x) = \sec x \Rightarrow f'(x) = \sec x \operatorname{tg} x$	$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \operatorname{cotg} x$
$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$	$f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$
$f(x) = \arctg x \Rightarrow f'(x) = \frac{1}{1+x^2}$	$f(x) = \operatorname{arccotg} x \Rightarrow f'(x) = -\frac{1}{1+x^2}$
$f(x) = \operatorname{arcsec} x \Rightarrow f'(x) = \frac{1}{ x \sqrt{x^2-1}}$	$f(x) = \operatorname{arccosec} x \Rightarrow f'(x) = -\frac{1}{ x \sqrt{x^2-1}}$

Tabela de integrais básicas

1. $\int dx = x + C$	2. $\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$
3. $\int x^{-1} dx = \ln x + C$	4. $\int a^x dx = \frac{a^x}{\ln a} + C$
5. $\int e^x dx = e^x + C$	6. $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctg \frac{x}{a} + C$
7. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C \quad (a \neq 0)$	8. $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left \frac{a-x}{a+x} \right + C \quad (a \neq 0)$
9. $\int \frac{1}{\sqrt{x^2+a}} dx = \ln \left x + \sqrt{x^2+a} \right + C \quad (a \neq 0)$	10. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a} + C \quad (a > 0)$
11. $\int \sin x dx = -\cos x + C$	12. $\int \cos x dx = \sin x + C$
13. $\int \operatorname{tg} x dx = -\ln \cos x + C$	14. $\int \operatorname{cotg} x dx = \ln \sin x + C$
15. $\int \sec x dx = \ln \operatorname{tg} x + \sec x + C$	16. $\int \operatorname{cosec} x dx = \ln \operatorname{cosec} x - \operatorname{cotg} x + C$
17. $\int \sec^2 x dx = \operatorname{tg} x + C$	18. $\int \operatorname{cosec}^2 x dx = -\operatorname{cotg} x + C$
19. $\int \sec x \operatorname{tg} x dx = \sec x + C$	20. $\int \operatorname{cosec} x \operatorname{cotg} x dx = -\operatorname{cosec} x + C$
21. $\int \sin^2 x dx = \frac{1}{2}(x - \sin x \cos x) + C$	22. $\int \cos^2 x dx = \frac{1}{2}(x + \sin x \cos x) + C$