the bottom of the vessel will have different influences. (3) The amplitudes of vibration decrease rapidly during the timing so that the extent of the motion of the liquid varies considerably. (4) The periods, though determined to 0.01 sec, have to be squared and differenced to get the moments of inertia I's. Sometimes these differences are small; and this leads to errors in the moments of inertia. The graphs are drawn to average the experimental results; there was a fair amount of scatter so that to avoid confusion the individual readings are not shown for water, mercury, and ether.

[1] D. R. Inglis, Am. J. Phys. 26, 82–89 (1958), esp. p. 85.
[2] See reference 1, p. 84.
[3] J. Satterly, Am. J. Phys. 24, 527 (1956). See also 25, 72 (1957).
[4] A. L. Romanoff and A. J. Romanoff, *The Avian Egg* (John Wiley & Sons, Inc., New York, 1949), p. 403.

Phase Velocity and Group Velocity

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It is well known that a group of water waves travels forward at a speed which is only half the speed of the individual waves in the group. The distinction between phase velocity and group velocity is apt to puzzle students at first, and the following apparatus can help.

A sheet of metal about 8 by 18 in. has its two long sides bent over to form flanges about 1 in. deep (see Fig. 1). An oval hole is then cut in the face of the sheet. Two wooden rollers of about 1 in. diam are held between the flanges by screws, one roller near each end, so that the apparatus will roll on a blackboard. A strip of paper $5\frac{1}{2}$ in. wide is passed round both rollers and joined to form an endless belt. On this paper is painted a wavy profile carrying about twelve waves in the total length of the belt. Finally the face of the metal sheet is painted half black and half white to disguise the presence of the hole in the sheet. When this apparatus is rolled across the blackboard the visible waves travel at twice the speed of the apparatus, appearing at the rear and disappearing at the front in the manner of real waves in a group.

One may begin by describing on the blackboard the case where a short group of water waves is set up by a wave maker at one end of a long tank such as is used for testing models of ships. An observer might see at a glance that there are perhaps four waves in the group. Nevertheless, if he counts the waves as they pass him he will count eight, twice as many as can be seen in the group at any one time. Turning to the apparatus, four waves can probably be seen on it, but when it is placed on the drawing of the tank and rolled slowly past some fixed mark, the waves travel faster than the apparatus itself, and eight waves in all pass the fixed mark. The demonstration is quite striking and usually needs to be repeated more slowly so that it can be seen how new waves continually appear at the rear and disappear at the front as they do in a real wave group on water.

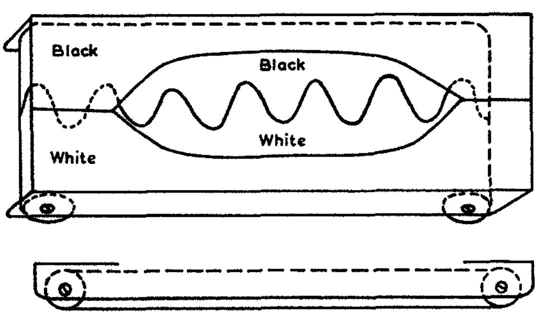

FIG. 1.

e/m by the Hoag Method

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In the "Note on the measurement of e/m by the Hoag method,"[1] the authors point out that the distance l in the expression for e/m should be measured from the center of the deflection plates to the screen. This fact was also reported in my talk, "Modified helical method for determining e/m," delivered at the annual AAPT meeting in New York City on February 1, 1958 but not yet published in detail. The authors of reference 1, also consider the effect of the fringing electric fields of the deflection plates, and using a two-step approximation to the field distribution, find that measuring from the center of the plates is still correct. It is then surmised that this conclusion is valid for any symmetrical (the same at entrance as at exit) fringing fields. This conjecture is correct and its proof is the subject of this letter.

Consider an electron traveling along the Z axis (also the axis of the uniform magnetic field B) and subjected to an X component of electric field $E(t)$, which begins as it passes the origin of coordinates, increases until time, $t_1/2$, when it passes the center of the plates and then decreases symmetrically to zero at time t_1; the electron then continues to the screen under the action of the magnetic field alone. We are concerned with the projection of the motion on the X-Y plane and the time it takes for this projection to return as nearly as possible to the origin (focus condition). The equations for this projected motion are

$$m\dot{v}_y = Bev_x \tag{1}$$

$$m\dot{v}_x = Ee - Bev_y, \tag{2}$$

subject to,

$$\text{at } t=0 \quad v_x = v_y = x = y = 0. \tag{3}$$

Defining

$$\omega = \frac{Be}{m} \quad \text{and} \quad f(t) = \frac{E(t)}{B}, \tag{4}$$

Eqs. (1) and (2) become

$$\dot{v}_y = \omega v_x \tag{5}$$

$$\dot{v}_x = \omega(f(t) - v_y). \tag{6}$$

Combining Eqs. (5) and (6) gives

$$\ddot{v}_y + \omega^2 v_y = \omega^2 f(t). \tag{7}$$

The solution of Eq. (7) is

$$v_y = \omega \int_0^t f(\alpha) \sin\omega(t-\alpha) d\alpha. \tag{8}$$

Integrating Eq. (5) and using Eq. (8) gives

$$x = \frac{v_y}{\omega} = \int_0^t f(\alpha) \sin\omega(t-\alpha) d\alpha. \tag{9}$$

For $t > t_1$, since $f(t) = 0$,

$$x = \int_0^{t_1} f(\alpha) \sin\omega(t-\alpha) d\alpha \tag{10}$$

or

$$x(t>t_1) = \sin\omega t \int_0^{t_1} f(\alpha) \cos\omega\alpha d\alpha - \cos\omega t \int_0^{t_1} f(\alpha) \sin\omega\alpha d\alpha.$$

Finally

$$x(t>t_1) = A \sin\omega t + B \cos\omega t \tag{11}$$

$$= C \cos(\omega t + \phi), \tag{12}$$

in which A, B, and therefore C are constants. Since we know that for $t > t_1$, the total motion (projected) must be circular, Eq. (12) establishes that the center of the circle must lie on the Y axis. This means that the closest approach of the electron to the origin, (which corresponds to the focus condition from which the e/m formula is derived) must occur when the Y coordinate is a minimum, the condition for which is $v_y = 0$. Substituting in Eq. (8), we wish to solve for t, for times greater than t_1, the equation

$$\int_0^t f(\alpha) \sin\omega(t-\alpha) d\alpha = 0. \tag{13}$$

Since we need solutions for $t > t_1$, and $f(t) = 0$ for $t > t_1$, the upper limit may be replaced by t_1, so

$$\int_0^{t_1} f(\alpha) \sin\omega(t-\alpha) d\alpha = 0, \tag{14}$$

and is solved by

$$t = \frac{t_1}{2} + \frac{n\pi}{\omega}, \tag{15}$$

as may be seen by substituting Eq. (15) in Eq. (14), and noting that with respect to $\frac{1}{2}t_1$ as a reference, the integral is that of an even function multiplied by an odd function over a symmetrical range and is consequently zero.

Now if the magnetic field has been adjusted for the best focus of electrons on the screen, then the electrons move from the origin to the screen in one of the times given by Eq. (15); for the first focus $n = 2$. Multiplying Eq. (15) by v_z, the axial speed of the electrons to the screen gives

$$v_z t = v_z \frac{t_1}{2} + \frac{2\pi}{\omega} v_z. \tag{16}$$

Since $v_z t =$ the distance from the origin to the screen, and $\frac{1}{2} v_z t_1 =$ the distance from the origin to the center of the deflection plates, we may write Eq. (16) as

$$l_c = \frac{2\pi}{\omega} v_z, \tag{17}$$

in which l_c is the distance from the center of the plates to the screen. Combining Eq. (17) with the relation

$$\tfrac{1}{2} m v_z^2 = Ve \tag{18}$$

yields the desired formula

$$\frac{e}{m} = \frac{8\pi^2 V}{B^2 l_c^2} \quad \text{(for first focus)}.$$

As a by-product of the early discussion above, it may perhaps be worthwhile to explicitly mention the following "theorem" about charged particle motion. In crossed electric and magnetic fields (right angles), with B_z constant and E_x any time function (but spatially uniform) which acts for a finite interval, a charged particle initially at rest at the origin of coordinates will end up going around a circle whose center lies on the Y axis; as a corollary, if the particle is to end up at rest, it will have to repose on the Y axis. This is still true if initially the electron has an X component of velocity; if it also has an initial Y component of velocity, the X coordinate of the center of the final circle will be the same as if no electric field had acted.

[1] Soemitro, Prawirowardojo, and Dickinson, Am. J. Phys. **26**, 316 (1958).

AMERICAN JOURNAL OF PHYSICS VOLUME 27, NUMBER 2 FEBRUARY, 1959

LETTERS TO THE EDITOR

Leaning Tower of *The Physical Reviews*

THE note by Paul B. Johnson, "Leaning Tower of Lire,"[1] reminded me of an incident that occurred some years ago, when I first solved a problem similar to the one he proposed. My line of argument, I feel, was simpler to see physically than the mathematical manipulation used by Johnson to solve his Eqs. (3_r). By choosing the center of gravity of a leaning array of r coins as my origin, and placing the ($r+1$ st) coin with its rim directly under this origin so that its center has unit abscissa, it was evident that the center of gravity of the ($r+1$) coins would be displaced by a distance $1/(r+1)$ from the origin. Since the rim of each added coin is placed under the center of gravity of the stack above it, the sum $\Sigma\, 1/(r+1)$ also gives the displacement between the top and bottom coins, and hence may be made arbitrarily large. To prove this result "physically," a fellow graduate student and I stacked bound volumes of *The Physical Review* one evening, until an astonishingly large offset was obtained, and left them to be discovered the next morning by a startled physics librarian. Because of the books' compressibility, a safety