Note on the Measurement of e/m by the Hoag Method

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The value of e/m obtained by the Hoag method depends critically on the choice of measuring origin for l, the distance from the oscilloscope deflecting plates to the screen. We have determined the correct origin by solving the equations of motion for the electron trajectories. It is concluded that only a single set of unflared deflecting plates should be present in the oscilloscope tube if one or two percent accuracy is desired.

A POPULAR student experiment in the atomic physics laboratory is the measurement of the specific charge of the electron using a variation of the Busch helical method first suggested by Hoag.[1] An electron beam is accelerated through a potential V in an oscilloscope tube and spread into a horizontal line on the screen by means of a saw tooth or sine wave voltage applied to one set of the deflecting plates. If the tube is now placed coaxially inside of a long current solenoid and the magnetic field slowly increased from zero, the horizontal line will be observed to rotate and shorten until, at a critical field B_1, it is reduced to a small spot on the screen. Further increase of the field causes the spot to spread into a shorter line which rotates until it is once more focused to a spot at a field B_2. If B_n is the field required to produce a focus after a number n of 180° rotations of the line, the formula for computing the electronic specific charge is

$$\frac{e}{m}=\frac{8\pi^2 n^2 V}{B_n{}^2 l^2}\ \frac{\text{coul}}{\text{kg}}, \tag{1}$$

with V given in volts, B_n in webers/m^2, and l the distance in meters between the deflecting plates and the screen.[2]

An x-ray shadowgraph of the tube is provided the student for the accurate measurement of l. In the type of commercial oscilloscope tube generally used for this experiment the distance from the deflector plates to the screen is approximately four to five times the length of the plates. Since l enters Eq. (1) as a square, the correct choice of an origin for its measurement is particularly important. Hoag and Korff state that l should be "measured to the edge and not the center of the condenser plates, since electrons start their deflection from the axis, and their rotation, as they enter the plates." However, when we measured l in this way our calculated values of e/m were consistently too small by about 10%. It therefore seemed worthwhile to check this point by solving the equations of motion for the electron trajectories.

Referring to Fig. 1, the equations of motion may be written down and integrated, first for the region between the deflector plates and second for the zero electric field region beginning at the exit edge of the deflector plates and ending at the screen. If we define

$$\omega=\frac{eB}{m},\quad a=\frac{eE}{m},\quad \theta_1=\frac{\omega l_1}{v_0},$$

$$\theta_2=\frac{\omega l_2}{v_0}\quad\text{and}\quad p=\frac{l_2}{l_1}=\frac{\theta_2}{\theta_1},$$

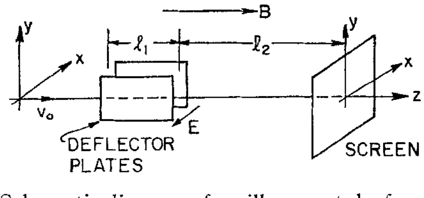

FIG. 1. Schematic diagram of oscilloscope tube for reference in determining electron trajectories.

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[1] J. B. Hoag and S. A. Korff, *Electron and Nuclear Physics* (D. Van Nostrand Company, Inc., Princeton, 1948), third edition, p. 32 and p. 38. See also: L. F. Connell, Am. J. Phys. **17**, 222 (1949); G. Bradley, Am. J. Phys. **24**, 410 (1956).

[2] The solenoid axis should be placed parallel to the earth's magnetic field and measurements made with the solenoid field alternately in each direction. The average of these measurements will eliminate the effect of the earth's field. All elements of the oscilloscope tube should be of nonferromagnetic material.

where l_1 is the length of the deflector plates, l_2 is the distance from the exit edge of these plates to the screen, v_0 is the electron velocity after acceleration through the potential V, and e is understood to be the positive magnitude of the electronic charge, then the coordinates of an electron when it reaches the screen are found to be

$$x=\frac{a}{\omega^2}[\sin\theta_1 \sin\theta_2+(1-\cos\theta_1)\cos\theta_2], \tag{2a}$$

$$y=\frac{a}{\omega^2}[\theta_1+(1-\cos\theta_1)\sin\theta_2-\sin\theta_1 \cos\theta_2]. \tag{2b}$$

If now E is given a constant value and θ_2 replaced by $p\theta_1$, the evaluation of Eqs. (2) for increasing values of θ_1 (increasing B) yields Fig. 2 (for the case $p=5$) which represents the path of a spot on the screen as the magnetic field is increased from zero. In the actual experiment E is a sinusoidally varying field. The spiral line of Fig. 2 would then represent the locus of the right end point of the rotating, shortening line. It is seen that the best focus will not be a point but rather a short vertical line (in practice $\approx\frac{1}{8}$ in. high) through the origin.[3] The size of the focused spot will be determined both by the size of the undeflected spot ($E=B=0$) and the inherent astigmatism.

If we define $l=fl_1+l_2=l_1(f+p)$, Eq. (1) may be converted to

$$\theta_1(n\text{th focus})=\frac{2\pi n}{f+p}. \tag{3}$$

If f is set equal to 0.5 and an arbitrary value given to p, this expression for θ_1 substituted into Eq. (2a) yields $x=0$. Further, if Eq. (2a) is differentiated with respect to p, it is found that $dx/dp=0$ if, and only if, $f=0.5$. Therefore Eq. (1) may be used in calculating e/m provided that l is defined as the distance from the screen to the center of the parallel deflecting plates.

The effect of the deflector plate fringing fields on the electron trajectories must next be considered. Figure 3 is a graph of this field along the center plane at the edge of a semi-infinite paralleled plate condenser[4] with unit distance plate separation. In order to integrate the equations of motion this fringing field was replaced at the entering and exit edges of the deflector plates by an "equivalent" uniform field $E_0/4$ extending a distance from the edges equal to twice the plate separation. Such a field contains approximately the same total electric flux (defined here as $\int Eds$ taken from a point well inside the plates where $E=E_0$ outward to the end of the fringing field) as does the actual field. Assuming a deflector plate length l_1 equal to four times the plate separation, the electron trajectories were calculated. The result is again that Eq. (1) is exact provided that the origin for measuring l is taken at the center of the plates. Since the actual fringing fields can be

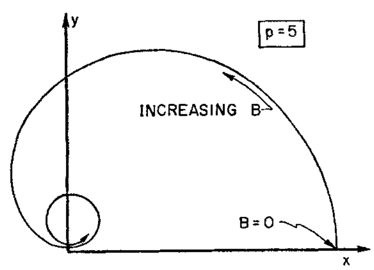

FIG. 2. The locus of a spot on the oscilloscope screen for a constant deflecting field E as the magnetic field B is increased from zero. If E is a sinusoidally varying field the curve would represent the locus of the right end point of the rotating shortening line. The locus of the left end point would be obtained by reflecting the above curve through the origin.

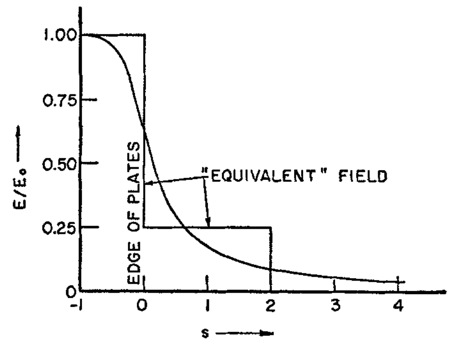

FIG. 3. Fringing field along center plane for semi-infinite parallel plate condenser with unit distance plate separation.

[3] If the magnetic field B is assumed to be zero while the electrons are within the deflecting plates, the equations of motion are simpler to solve and the result is still a rotating straight line. Also in this case, however, no point focus results.

[4] J. H. Jeans, *The Mathematical Theory of Electricity and Magnetism* (Cambridge University Press, New York, 1951), fifth edition, p. 274.

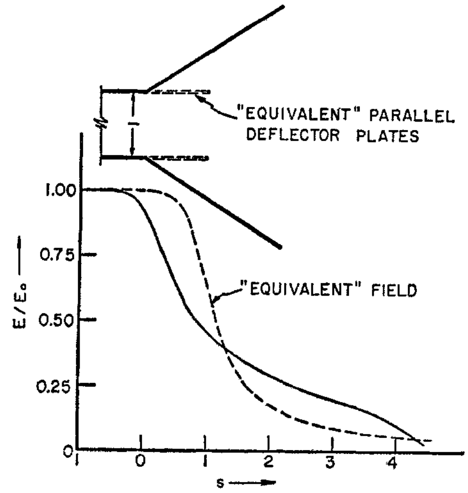

FIG. 4. Flared shape of deflector plates in oscilloscope tube used in this experiment. Below is a plot of the electric field as evaluated along the center plane. Dotted line represents electric field of "equivalent" parallel deflector plates.

thought of as built up by the superposition of a large number of uniform fields of varying magnitude and extent, it would appear that symmetrical fringing fields do not affect the validity of our above conclusion.

The exit edge of the deflecting plates in the Philips GM5655 tube which we used have the shape shown at the top of Fig. 4. The electric field along the center plane was evaluated by using the relaxation method[5] and is plotted in Fig. 4. Also included in Fig. 4 is a plot of the field for an "equivalent" pair of parallel plates extending out one unit of distance from the beginning of the flared section. It is seen that the field of the "equivalent" plates contains approximately the same total electric flux as does the field of the flared plates.

The distance l was measured from the center of this "equivalent" parallel plate condenser. When Eq. (1) was then used to calculate e/m, the average value obtained from several independent experimental determinations was 6% larger than the accepted value. Systematic meter errors were estimated to contribute no more than 2% to the total error. Hence the value of l determined by this procedure would appear to be too small by about 2 or 3%. The flared edges of the deflector plates taper to a blunt point when viewed from above whereas the relaxation calculation assumes two-dimensional geometry. The actual field hence would be expected to fall off more rapidly than does the calculated field, resulting in a larger l value. Another source of error, although probably less important, is the disturbing effect on the entrance fringing field caused by the orthogonal pair of plates lying directly behind the plates which were used.

In view of these complications it is recommended that an oscilloscope tube with parallel deflecting plates (preferably only one pair) be used for this experiment if an accuracy of 1 or 2% is desired. Since the experiment is simple to perform and the reduction of data takes only a short time, it might be considered a worthwhile exercise for the student to include in his written report the solution of the electronic equations of motion and subsequent determination of the point of origin for measuring l.

[5] R. V. Southwell, *Relaxation Methods in Theoretical Physics* (Oxford University Press, New York, 1946).

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The summer meeting of the Association will be held June 25–27 at the University of Colorado, Boulder, Colorado. The summer meeting offers a particularly excellent opportunity to combine a professional meeting with a pleasant vacation. Many members of the Association take their families with them.