Answer to Question #53. Measuring Planck's constant by means of an LED

The experiment which attempts to use light-emitting diodes (LEDs) to obtain Planck's constant, h, has been misrepresented since at least 1974[1] with distressing regularity.[2] These experimental descriptions indicate that there is either a ''turn on'' or a diffusion potential V_D, nearly equal to the band gap energy of the semiconductor divided by the electronic charge, that can be found from a tangent to an I vs V graph of the diode's characteristics. These descriptions also assume direct recombination of charges within the semiconductor to produce the light, so that $eV_D = h\nu$, where e is the electronic charge and ν is the frequency of the light. Then, effectively, $h = eV_D\lambda/c$, they say.

Herrmann and Schätzle[3] questioned the possibility of obtaining V_D by plotting the I vs V characteristics for an LED, because the rectifier equation describes the diode current as a function of the voltage across the diode junction V. This equation is

$$I(V) = I_0\left[\exp\left(\frac{V}{V_t}\right) - 1\right],$$

which appears to contain no V_D and has no other V intercept except when I and V equal zero.

Their objection is well founded because that is the correct equation. However, their question misinterprets the application of this equation to the experiment in three places.

1. The constant V_t should have been $\eta kT/e$, where η is about 1.62 for typical LEDs. If all charges diffuse directly across the diode junction, $\eta = 1$; if the electrons and holes recombine in the diode junction area to produce the current, $\eta = 2$. This constant characterizes the diode current for a particular type of semiconductor and cannot arbitrarily be set to unity.

2. For LEDs, the inverse saturation current I_0 is not the appropriate constant to consider because it obscures the physics of the diode. Most texts derive I_0 in terms of minority charge carrier density.[4] Unfortunately, this density and, thus, I_0 depend exponentially on the temperature and the band gap energy. The quantities that remain constant in the semiconductor are the impurity atomic density, the charge diffusion properties, and the diode area. The expressions get bulky, but after collecting all the constants, $I_0 \propto \exp(-V_g/V_t)$, where V_g is the band gap potential and V_t is defined above. Since I_0 ranges from 1.5×10^{-21} A for a typical green LED to 2.2×10^{-11} A for a 940-nm IR LED, most student labs cannot measure I_0 directly to obtain h from the V_g's. However, if $I > 2$ nA so that the one in the rectifier equation is negligible, then $I \propto \exp[(V - V_g)/V_t]$. That expression does contain V_g and could yield h.

3. The junction potential difference V is not directly measurable due to ohmic resistance within the LED package. The V can be replaced in the rectifier equation by $V = V_m - R$, where V_m is the voltmeter reading in the external diode circuit. The internal resistance of the diode, R, is itself a function of V_g. Typically, R varies from 9 Ω for a green LED to 0.7 Ω for a 940-nm IR LED. At $I = 10$ mA in a green LED, $V = V_m - 0.09$ V, about half the observed voltage difference at that current for a green and orange LED pair, $V_{m_g} - V_{m_0} = 0.20$ V. This resistance term is only negligible in LEDs at currents below 1 mA.

As the question noted, the erroneous experimental procedure described at the beginning of this answer does *seem* to give the correct result for h. This is because students usually draw the tangents on their I vs V graphs at nearly the same currents for each diode of a set. The V intercepts of these tangents, except for an additive constant, are then identical to measuring all diode voltages at a single, constant current. This is the key to the apparent successful determination of h from the slope of the V intercept vs λ^{-1} graph. That slope eliminates the additive constant. The result from the intercept data is identical to voltage measurements taken at a single current, and the bulk of the data collected for the original I vs V graph is irrelevant.

In a proper measurement all the diode voltages are observed at a single current, e.g., a set of diodes are connected in series. Then the quantity $(V - V_g)$ is a constant because $I \propto \exp[(V - V_g)/V_t]$. If the value of I has been selected to avoid unwanted complications from the one in the rectifier equation and from R, i.e., $0.1\ \text{mA} \geqslant I \geqslant 2\ \text{nA}$, then $V \propto V_g$. The slope of the graph of V vs λ^{-1} for a set of direct recombination LEDs is hc/e. It is interesting to note that most red LEDs emit light after first trapping the electrons and holes in shallow traps so $eV_g > h\nu$. These red LEDs cannot be used to obtain h. Their electrical characteristics are nearly identical to those of green LEDs.

A better way to measure h, because it gives the student more insight into the operation of LEDs, is to measure the V_m of an LED of each color for $0.1\ \mu\text{A} \leqslant I \leqslant 300\ \mu\text{A}$. A graph of $\ln(I)$ vs V_m for each LED allows the student to see the exponential form of the rectifier equation. (If the measurements are extended to 30 mA, the effects of R can also be observed.) The slope of the linear portion of these graphs is $\eta kT/e$ so that η may be obtained, and the intercept of these graphs is $\ln(I_0)$. Then, for the whole set of LEDs, plot $\eta \ln(I_0)$ vs λ^{-1}. The slope of this graph is hc/kT. This procedure yields a value about 8% larger than the accepted value of h. To do better one needs to look more closely at λ, but that is a different question.

[1] P. J. O'Connor and L. R. O'Connor, ''Measuring Planck's constant using a light emitting diode,'' Phys. Teach. **12**, 423–425 (1974).

[2] L. Nieves *et al.*, ''Measuring the Planck Constant with LED's,'' Phys. Teach. **35**, 108–109 (1997).

[3] F. Herrmann and D. Schätzle, ''Question #53. Measuring Planck's constant by means of an LED,'' Am. J. Phys. **64** (12), 1448 (1996).

[4] C. L. Hemenway *et al.*, *Physical Electronics* (Wiley, New York, 1967), p. 280; J. C. Sprott, *Introduction to Modern Electronics* (Wiley, New York, 1981), p. 123.

Roger Morehouse
Department of Physics
California State Polytechnic University, Pomona
Pomona, California 91768
rlmorehouse@csupomona.edu