1) Calcule a integral

$$\int_0^{\pi/4} \left[t\cos(2t)\vec{i} + \mathrm{sen}(3t)\vec{j} + \frac{1}{t}\vec{k} \right] dt .$$

Sol.: $\int \vec{r}(t)\,dt = \left\langle \int x(t)\,dt, \int y(t)\,dt, \int z(t)\,dt \right\rangle$

$x(t) = t\cos(2t)$

$$\int_0^{\pi/4} t\cos(2t)\,dt = \frac{1}{2} t\,\mathrm{sen}(2t)\Big|_0^{\pi/4} - \frac{1}{2}\int_0^{\pi/4} \mathrm{sen}(2t)\,dt$$

Por partes

$\int u\,dv = uv - \int v\,du$

$u = t$ $\quad du = dt$

$dv = \cos(2t)\,dt$ $\quad v = \frac{1}{2}\mathrm{sen}(2t)$

$$\int_0^{\pi/4} t\cos(2t)\,dt = \frac{1}{2}\frac{\pi}{4}\cdot \underbrace{\mathrm{sen}(\pi/2)}_{1} + \frac{1}{4}\cos(2t)\Big|_0^{\pi/4}$$

$$'' = \frac{\pi}{8} + \frac{1}{4}\Big[\underbrace{\cos(\pi/2)}_{0} - \underbrace{\cos(0)}_{1}\Big]$$

$$'' = \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}$$

$$\boxed{\int_0^{\pi/4} x(t)\,dt = \frac{\pi-2}{8}}$$

$y(t) = \mathrm{sen}(3t)$

$$\int_0^{\pi/4} \mathrm{sen}(3t)\,dt = -\frac{1}{3}\cos(3t)\Big|_0^{\pi/4} = -\frac{1}{3}\Big[\cos(3\pi/4) - \cos(0)\Big]$$

$$= -\frac{1}{3}\Big[-\frac{\sqrt{2}}{2} - 1\Big]$$

$$= \frac{\sqrt{2}+2}{6}$$

(2)

$$\boxed{\int_0^{\pi/4} y(t)\,dt = \frac{\sqrt{2}+2}{6}}$$

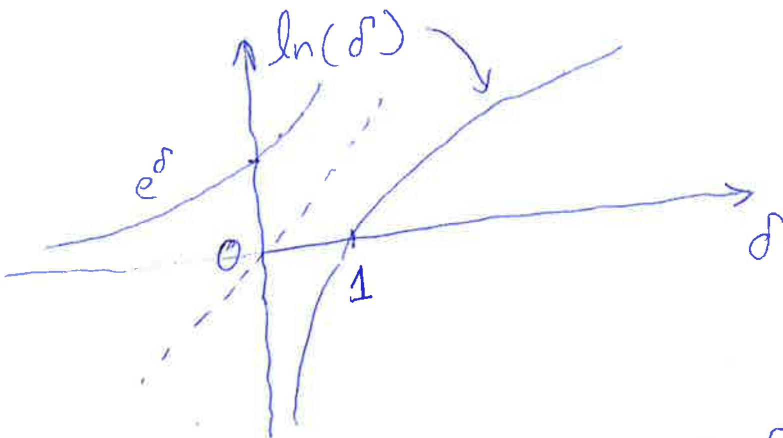
$$z(t) = \frac{1}{t}$$

$$\underbrace{\int_0^{\pi/4} \left(\frac{1}{t}\right) dt}_{\text{É imprópria de 2do Tipo:}} = \lim_{\delta\to 0^+}\left[\int_\delta^{\pi/4}\left(\frac{1}{t}\right)dt\right] = \lim_{\delta\to 0}\left[\ln(t)\Big|_\delta^{\pi/4}\right]$$

$$\lim_{t\to 0}\left(\frac{1}{t}\right) = \infty$$

$$\int_0^{\pi/4}\left(\frac{1}{t}\right)dt = \lim_{\delta\to 0}\left[\ln(\pi/4) - \underbrace{\ln(\delta)}_{-\infty}\right]$$ = Não existe o limite

A integral é divergente



$\ln(\delta) \to -\infty$ quando $\delta \to 0^+$

Logo, não existe a integral $\int_0^{\pi/4} \vec{r}(t)\,dt$.
(é divergente)

2) Encontre a curvatura da função vetorial

$\vec{r}(t) = 3t^2\vec{i} + (1-t)\vec{j} + (1+t)\vec{k}$.

Sol.: $\vec{r}(t) = \langle 3t^2, 1-t, 1+t \rangle$

$$\boxed{K(t) = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}}$$

Fórmula para a Curvatura

$\vec{r}\,'(t) = \langle 6t, -1, 1 \rangle \longrightarrow |\vec{r}\,'| = \sqrt{36t^2+2}$

$\downarrow$

$\vec{r}\,''(t) = \langle 6, 0, 0 \rangle$

$$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6t & -1 & 1 \\ 6 & 0 & 0 \end{vmatrix} = \langle 0, 6, 6 \rangle = 6\langle 0, 1, 1 \rangle$$

$|\vec{r}\,'(t) \times \vec{r}\,''(t)| = 6\sqrt{2}$

$$K(t) = \frac{6\sqrt{2}}{\left[2(18t^2+1)\right]^{3/2}} = \frac{\overset{3}{\cancel{6}}\cancel{\sqrt{2}}}{\cancel{2}\cancel{\sqrt{2}}\,(18t^2+1)^{3/2}}$$

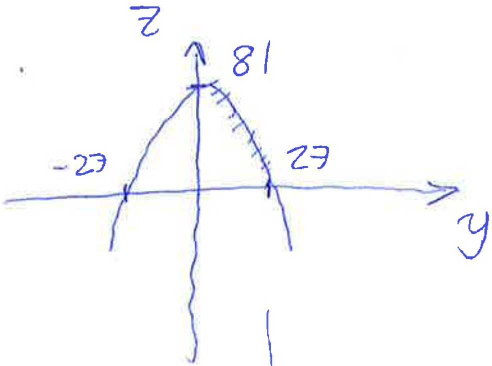
$$\boxed{K(t) = \frac{3}{(18t^2+1)^{3/2}}}$$

3) Esboce o gráfico da função

$$f(x,y) = 81 - x^2 - \frac{y^2}{9}.$$

Sol.: $z = 81 - x^2 - \dfrac{y^2}{9}$

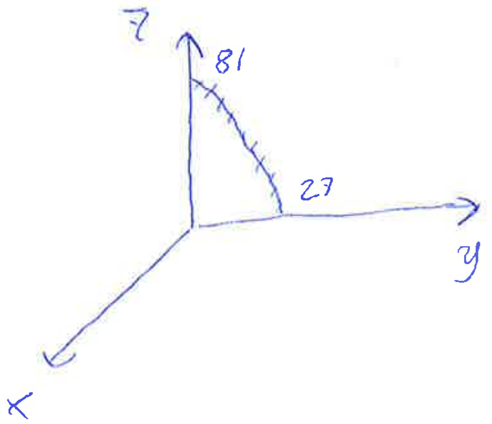
- Se $x = 0 \Rightarrow z = 81 - \dfrac{y^2}{9}$ (Parábola)



$(y,z) = (0,81)$

$z = 0 \rightarrow y^2 = 81 \cdot 9 \Rightarrow y = \pm 9 \cdot 3 = \pm 27$

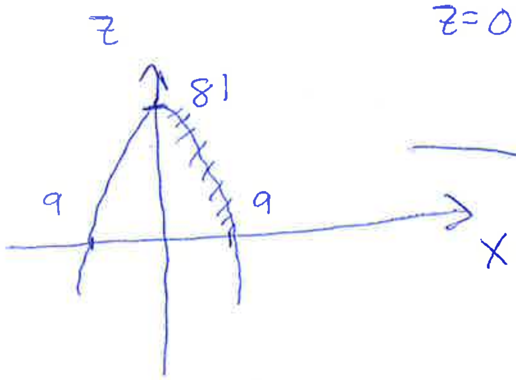
$(\underset{y}{27},\underset{z}{0})$ e $(\underset{y}{-27},\underset{z}{0})$

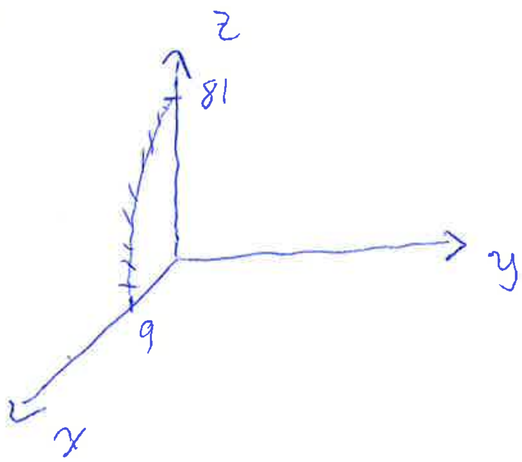


- Se $y = 0 \Rightarrow z = 81 - x^2$ (Parábola)

$x = 0 \rightarrow z = 81$

$z = 0 \rightarrow x = \pm 9$

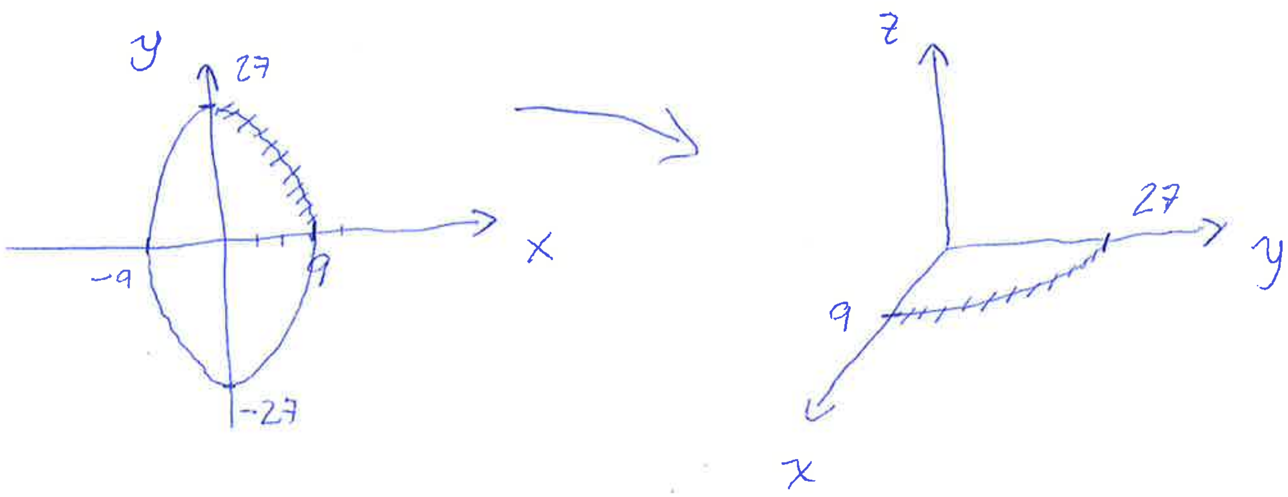




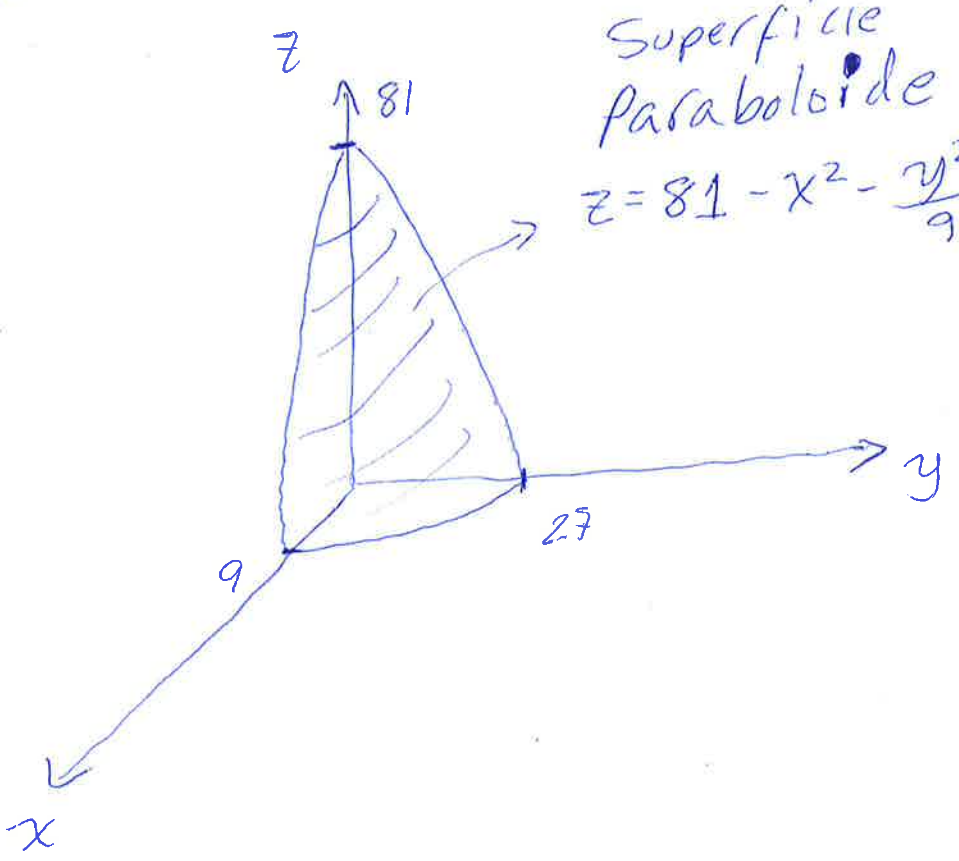
- Se $z=0 \Rightarrow x^2 + \frac{y^2}{9} = 81$

$$\frac{x^2}{81} + \frac{y^2}{81 \cdot 9} = 1$$

$$\left(\frac{x}{9}\right)^2 + \left(\frac{y}{27}\right)^2 = 1 \quad \text{Elipse}$$



Juntando tudo



Superfície Parabolóide Elíptico

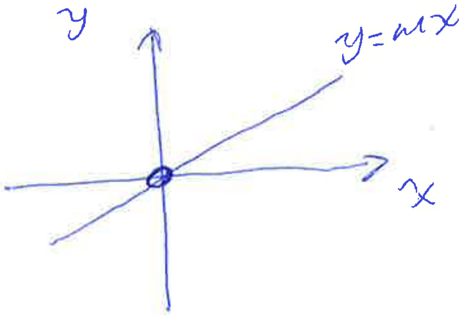
$z = 81 - x^2 - \frac{y^2}{9}$

4) Determine o limite $\lim_{(x,y)\to(0,0)} \left[\frac{7xy}{x^2+5y^2}\right]$, ou mostre que não existe.

Sol.: $\lim_{(x,y)\to(0,0)} \left[\frac{7xy}{x^2+5y^2}\right] = \text{"}\frac{0}{0}\text{"}$

Vamos tentar provar que não existe. Considere retas da forma $y = mx$ $(m \in \mathbb{R})$ que passam pela origem.

m é um parâmetro



$$\underset{\substack{(x,y)\to(0,0) \\ \boxed{\text{seguindo } y=mx}}}{\lim} \left[\frac{7xy}{x^2+5y^2}\right] = \underset{\substack{(x,mx)\to(0,0)\\ x\to 0}}{\lim} \left[\frac{7x\,mx}{x^2+5m^2x^2}\right]$$

$$= \lim_{x\to 0} \left[\frac{7m\cancel{x^2}}{\cancel{x^2}[1+5m^2]}\right]$$

$$= \lim_{x\to 0} \left[\frac{7m}{1+5m^2}\right]$$

$$= \frac{7m}{1+5m^2}$$

Como o resultado depende de $\underline{m}$ o limite não é o mesmo por todas as retas, logo não existe.

5) Sejam $z = e^{xy}$, $x = 2u + v$ e $y = \frac{u}{v}$

encontre $\frac{\partial z}{\partial u}$ e $\frac{\partial z}{\partial v}$ usando a regra da cadeia.

Sol.:

$$z(x,y) = e^{xy}$$

$$x(u,v) = 2u + v$$

$$y(u,v) = \frac{u}{v}$$

$$\begin{tikzcd}
 & & z \arrow[dl, "\frac{\partial z}{\partial x}"'] \arrow[dr, "\frac{\partial z}{\partial y}"] & & \\
 & x \arrow[dl, "\frac{\partial x}{\partial u}"'] \arrow[d, "\frac{\partial x}{\partial v}"] & & y \arrow[d, "\frac{\partial y}{\partial u}"'] \arrow[dr, "\frac{\partial y}{\partial v}"] & \\
u & v & & u & v
\end{tikzcd}$$

$$\frac{\partial z}{\partial u} = \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial x}{\partial u}\right) + \left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial y}{\partial u}\right)$$

$$\frac{\partial z}{\partial u} = y e^{xy} \cdot 2 + x e^{xy} \cdot \frac{1}{v}$$

$$\frac{\partial z}{\partial u} = 2\frac{u}{v} e^{(2u+v)\frac{u}{v}} + \frac{2u+v}{v} e^{(2u+v)\frac{u}{v}}$$

$$= \frac{e^{(2u+v)\frac{u}{v}}}{v}\left[2u + 2u + v\right]$$

$$\boxed{\frac{\partial z}{\partial u} = \frac{4u+v}{v} e^{(2u+v)\frac{u}{v}}}$$

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$$\frac{\partial z}{\partial v} = \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial x}{\partial v}\right) + \left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial y}{\partial v}\right)$$

$$'' = y e^{xy} \cdot 1 + x e^{xy}\left(\frac{-u}{v^2}\right)$$

$$\frac{\partial z}{\partial v} = \frac{u}{v} e^{(2u+v)\frac{u}{v}} + (2u+v) e^{(2u+v)\frac{u}{v}} \left(\frac{-u}{v^2}\right)$$

$$= \frac{e^{(2u+v)\frac{u}{v}}}{v}\left[u - \frac{(2u+v)u}{v}\right]$$

$$\frac{\partial z}{\partial v} = \frac{u}{v} e^{(2u+v)\frac{u}{v}}\left[1 - \frac{2u+v}{v}\right]$$

$$\boxed{\frac{\partial z}{\partial v} = -\frac{2u^2}{v^2} e^{(2u+v)\frac{u}{v}}}$$