13.3 SOLUÇÕES

Revisão técnica: Ricardo Miranda Martins – IMECC – Unicamp

1. $\mathbf{r}'(t) = \langle 2, 3\cos t, -3\,\text{sen}\,t\rangle$,
$|\mathbf{r}'(t)| = \sqrt{4 + 9\cos^2 t + 9\,\text{sen}^2\,t} = \sqrt{13}$,
$L = \int_a^b \sqrt{13}\,dt = \sqrt{13}\,(b - a)$

2. $\mathbf{r}'(t) = \left\langle e^t, e^t(\text{sen}\,t + \cos t), e^t(\cos t - \text{sen}\,t)\right\rangle$,
$|\mathbf{r}'(t)| = e^t\sqrt{1 + 2\,\text{sen}^2\,t + 2\cos^2 t} = \sqrt{3}e^t$,
$L = \int_0^{2\pi} \sqrt{3}e^t\,dt = \sqrt{3}\left(e^{2\pi} - 1\right)$

3. $\mathbf{r}'(t) = \left\langle 6, 6\sqrt{2}t, 6t^2\right\rangle$,
$|\mathbf{r}'(t)| = 6\sqrt{1 + 2t^2 + t^4} = 6\left(1 + t^2\right)$,
$L = \int_0^1 6\left(1 + t^2\right)dt = \left[\frac{1}{3}\cdot 6\left(t + t^3\right)\right]_0^1 = \frac{24}{3} = 8$

4. $\mathbf{r}'(t) = \langle 2, 2t, 2t\rangle$, $|\mathbf{r}'(t)| = 2\sqrt{1 + 2t^2}$
$L = \int_0^1 \sqrt{1 + 2t^2}\,dt = \int_a^b \sqrt{2}\sec^3\theta\,d\theta$
$= \frac{\sqrt{2}}{2}\left[\ln|\sec\theta + \text{tg}\,\theta| + \text{tg}\,\theta\sec\theta\right]_a^b$
$= \frac{\sqrt{2}}{2}\left[\ln\left|\sqrt{1 + 2t^2} + \sqrt{2}t\right| + \sqrt{2}t\sqrt{1 + 2t^2}\right]_0^1$
(ou use a Fórmula 21)
$= \frac{\sqrt{2}}{2}\left[\ln\left|\sqrt{3} + \sqrt{2}\right| + \sqrt{2}\sqrt{3}\right]$
$= \sqrt{3} + \frac{\sqrt{2}}{2}\ln\left(\sqrt{2} + \sqrt{3}\right)$

5. $\mathbf{r}'(t) = \langle \cos t - t\,\text{sen}\,t, \text{sen}\,t + t\cos t, 1\rangle$,
$|\mathbf{r}'(t)| = \sqrt{\cos^2 t + t^2\,\text{sen}^2\,t + \text{sen}^2\,t + t\cos^2 t + 1}$
$= \sqrt{t^2 + 2}$
$L = \int_0^{\pi/2} \sqrt{t^2 + 2}\,dt = \int_a^b \sec^3\theta\,d\theta$
$= \left[\ln\left|\sqrt{t^2 + 2} + t\right| + \frac{1}{2}t\sqrt{t^2 + 2}\right]_0^{\pi/2}$
(ou use a Fórmula 21)
$= \ln\left(\sqrt{\frac{\pi^2}{4} + 2} + \frac{\pi}{2}\right) + \frac{\pi}{4}\sqrt{\frac{\pi^2}{4} + 2} - \ln\sqrt{2}$

6. $\mathbf{r}'(t) = e^t(\cos t + \text{sen}\,t)\,\mathbf{i} + e^t(\cos t - \text{sen}\,t)\,\mathbf{j}$,
$ds/dt = |\mathbf{r}'(t)| = e^t\sqrt{(\cos t + \text{sen}\,t)^2 + (\cos t - \text{sen}\,t)^2}$
$= e^t\sqrt{2\cos^2 t + 2\,\text{sen}^2\,t} = \sqrt{2}e^t$
$s(t) = \int_0^t |\mathbf{r}'(u)|\,du = \int_0^t \sqrt{2}e^u\,du = \sqrt{2}\left(e^t - 1\right) \Rightarrow$
$\frac{1}{\sqrt{2}}s + 1 = e^t \Rightarrow t(s) = \ln\left(\frac{1}{\sqrt{2}}s + 1\right)$. Portanto,
$\mathbf{r}(t(s)) = \left(\frac{1}{\sqrt{2}}s + 1\right)\left[\text{sen}\left(\ln\left(\frac{1}{\sqrt{2}}s + 1\right)\right)\mathbf{i} + \cos\left(\ln\left(\frac{1}{\sqrt{2}}s + 1\right)\right)\mathbf{j}\right]$

7. $\mathbf{r}'(t) = 2\,\mathbf{i} + \mathbf{j} - 5\,\mathbf{k}$,
$ds/dt = |\mathbf{r}'(t)| = \sqrt{4 + 1 + 25} = \sqrt{30}$ e
$s(t) = \int_0^t |\mathbf{r}'(u)|\,du = \int_0^t \sqrt{30}\,du = \sqrt{30}\,t$
$\Rightarrow t(s) = \frac{1}{\sqrt{30}}s$. Portanto,
$\mathbf{r}(t(s)) = \left(1 + \frac{2}{\sqrt{30}}s\right)\mathbf{i} + \left(3 + \frac{1}{\sqrt{30}}s\right)\mathbf{j} - \frac{5}{\sqrt{30}}s\,\mathbf{k}$.

8. $|\mathbf{r}'(t)|$
$= \sqrt{\left(-3\cos^2 t\,\text{sen}\,t\right)^2 + \left(3\,\text{sen}^2\,t\cos t\right)^2 + \left(-2\,\text{sen}\,2t\right)^2}$
$= \sqrt{9\,\text{sen}^2\,t\cos^2 t\left(\cos^2 t + \text{sen}^2\,t\right) + 4\,\text{sen}^2\,2t}$
$= \sqrt{\frac{9}{4}\left(2\,\text{sen}\,t\cos t\right)^2 + 4\,\text{sen}^2\,2t} = \sqrt{\left(\frac{9}{4} + 4\right)\text{sen}^2\,2t}$
$= \frac{5}{2}\,\text{sen}\,2t$
Então para $0 \le t \le \frac{\pi}{2}$,
$s(t) = \int_0^t |\mathbf{r}'(u)|\,du = \frac{5}{2}\int_0^t \text{sen}\,2u\,du$
$= -\frac{5}{4}\left[\cos 2u\right]_0^t = \frac{5}{4}\left(1 - \cos 2t\right)$
Portanto, $t(s) = \frac{1}{2}\cos^{-1}\left(1 - \frac{4}{5}s\right)$ e
$\mathbf{r}(t(s)) = \cos^3\left[\frac{1}{2}\cos^{-1}\left(1 - \frac{4}{5}s\right)\right]\mathbf{i} + \text{sen}^3\left[\frac{1}{2}\cos^{-1}\left(1 - \frac{4}{5}s\right)\right]\mathbf{j} + \cos\left[2\cdot\frac{1}{2}\cos^{-1}\left(1 - \frac{4}{5}s\right)\right]\mathbf{k}$
$= \cos^3\left[\frac{1}{2}\cos^{-1}\left(1 - \frac{4}{5}s\right)\right]\mathbf{i} + \text{sen}^3\left[\frac{1}{2}\cos^{-1}\left(1 - \frac{4}{5}s\right)\right]\mathbf{j} + \left[1 - \frac{4}{5}s\right]\mathbf{k}$

9. (a) $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \dfrac{1}{\sqrt{16 + 9}}\langle 4\cos 4t, 3, -4\,\text{sen}\,4t\rangle$
$= \frac{1}{5}\langle 4\cos 4t, 3, -4\,\text{sen}\,4t\rangle$
$\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \dfrac{5}{16\cdot 5}\langle -16\,\text{sen}\,4t, 0, -16\cos 4t\rangle$
$= \langle -\text{sen}\,4t, 0, -\cos 4t\rangle$
(b) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{16}{5\cdot 5} = \dfrac{16}{25}$

10. (a) $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \dfrac{1}{6(1 + t^2)}\left\langle 6, 6\sqrt{2}t, 6t^2\right\rangle$
$= \dfrac{1}{1 + t^2}\left\langle 1, \sqrt{2}t, t^2\right\rangle$
$\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$
$= \dfrac{1}{|\mathbf{T}'(t)|}\left[-\dfrac{2t}{(1 + t^2)^2}\left\langle 1, \sqrt{2}t, t^2\right\rangle + \dfrac{1}{1 + t^2}\left\langle 0, \sqrt{2}, 2t\right\rangle\right]$
$= \dfrac{1 + t^2}{\sqrt{2}}\left[\dfrac{1}{(1 + t^2)^2}\left\langle -2t, \sqrt{2}\left(1 - t^2\right), 2t\right\rangle\right]$
$= \dfrac{1}{\sqrt{2}(1 + t^2)}\left\langle -2t, \sqrt{2}\left(1 - t^2\right), 2t\right\rangle$
(b) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{\sqrt{2}}{1 + t^2}\cdot\dfrac{1}{6(1 + t^2)}$
$= \dfrac{1}{3\sqrt{2}(1 + t^2)^2}$

11. (a) $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$

$$= \frac{1}{\sqrt{2\,\text{sen}^2\, t + 2\cos^2 t}} \left\langle -\sqrt{2}\,\text{sen}\, t,\ \cos t,\ \cos t \right\rangle$$

$$= \tfrac{1}{\sqrt{2}} \left\langle -\sqrt{2}\,\text{sen}\, t,\ \cos t,\ \cos t \right\rangle$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$$

$$= \frac{1}{\sqrt{2\cos^2 t + 2\,\text{sen}^2\, t}} \left\langle -\sqrt{2}\cos t,\ -\text{sen}\, t,\ -\text{sen}\, t \right\rangle$$

$$= \tfrac{1}{\sqrt{2}} \left\langle -\sqrt{2}\cos t,\ -\text{sen}\, t,\ -\text{sen}\, t \right\rangle$$

(b) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{1}{\sqrt{2}}$

12. (a) $\mathbf{r}'(t) = \left\langle t^2, 2t, 2 \right\rangle \quad \Rightarrow$

$|\mathbf{r}'(t)| = \sqrt{t^4 + 4t^2 + 4} = \sqrt{(t^2+2)^2} = t^2 + 2$. Assim

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{t^2+2}\left\langle t^2, 2t, 2 \right\rangle.$$

$$\mathbf{T}'(t) = \frac{-2t}{(t^2+2)^2}\left\langle t^2, 2t, 2 \right\rangle + \frac{1}{t^2+2}\langle 2t, 2, 0\rangle$$

(pelo Teorema 13.2.3 #3)

$$= \frac{1}{(t^2+2)^2}\left\langle -2t^3, -4t^2, -4t \right\rangle + \frac{1}{(t^2+2)^2}\left\langle 2t^3 + 4t, 2t^2 + 4, 0 \right\rangle$$

$$= \frac{1}{(t^2+2)^2}\left\langle 4t, 4 - 2t^2, -4t \right\rangle$$

$$|\mathbf{T}'(t)| = \frac{1}{(t^2+2)^2}\sqrt{16t^2 + (16 - 16t^2 + 4t^4) + 16t^2}$$

$$= \frac{1}{(t^2+2)^2}\sqrt{4t^4 + 16t^2 + 16}$$

$$= \frac{1}{(t^2+2)^2}\sqrt{4(t^2+2)^2} = \frac{2\left(t^2+2\right)}{(t^2+2)^2}$$

$$= \frac{2}{t^2+2}$$

Portanto,

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{1/\left(t^2+2\right)^2}{2/(t^2+2)}\left\langle 4t, 4 - 2t^2, -4t \right\rangle$$

$$= \frac{1}{t^2+2}\left\langle 2t, 2 - t^2, -2t \right\rangle$$

(b) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{2/\left(t^2+2\right)}{t^2+2} = \dfrac{2}{(t^2+2)^2}$

13. (a) $\mathbf{T}(t) = \dfrac{1}{\sqrt{2 + e^{2t} + e^{-2t}}}\left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle$

$$= \frac{1}{e^t + e^t}\left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle$$

$$= \frac{1}{e^{2t}+1}\left\langle \sqrt{2}e^{2t}, e^{2t}, -1 \right\rangle$$

$$\mathbf{T}'(t) = \frac{-2e^{2t}}{(e^{2t}+1)^2}\left\langle \sqrt{2}e^t, e^{2t}, -1 \right\rangle + \frac{1}{e^{2t}+1}\left\langle \sqrt{2}e^t, 2e^{2t}, 0 \right\rangle$$

$$= \frac{1}{(e^{2t}+1)^2}\left\langle -2\sqrt{2}e^{3t} + \sqrt{2}e^{3t} + \sqrt{2}e^t,\ -2e^{4t} + 2e^{4t} + 2e^{2t}, 2e^{2t} \right\rangle$$

$$= \frac{1}{(e^{2t}+1)^2}\left\langle \sqrt{2}\left(e^t - e^{3t}\right), 2e^{2t}, 2e^{2t} \right\rangle$$

$$|\mathbf{T}'(t)| = \frac{1}{(e^{2t}+1)^2}\sqrt{2\left(e^{2t} + e^{6t} - 2e^{4t}\right) + 8e^{4t}}$$

$$= \frac{\sqrt{2}e^t}{(e^{2t}+1)^2}\left(e^{2t}+1\right) = \frac{\sqrt{2}e^t}{e^{2t}+1}$$

$$\mathbf{N}(t) = \frac{1}{(e^{2t}+1)^2}\,\frac{e^{2t}+1}{\sqrt{2}e^t}\left\langle \sqrt{2}\left(e^t - e^{3t}\right), 2e^{2t}, 2e^{2t} \right\rangle$$

$$= \frac{1}{\sqrt{2}\left(e^{2t}+1\right)e^t}\left\langle \sqrt{2}\left(e^t - e^{3t}\right), 2e^{2t}, 2e^{2t} \right\rangle$$

$$= \frac{1}{e^{2t}+1}\left\langle 1 - e^{2t}, \sqrt{2}e^t, \sqrt{2}e^t \right\rangle$$

(b) $\kappa(t) = \dfrac{\sqrt{2}e^t}{(e^{2t}+1)} \cdot \dfrac{1}{e^t + e^{-t}} = \dfrac{\sqrt{2}e^{2t}}{(e^{2t}+1)^2}$

14. (a) $\mathbf{T}(t) = \dfrac{1}{\sqrt{4t^2 + 4t^4 + 1}}\left\langle 2t, 2t^2, 1 \right\rangle$

$$= \frac{1}{2t^2+1}\left\langle 2t, 2t^2, 1 \right\rangle$$

$$\mathbf{T}'(t) = -\left(2t^2+1\right)^{-2}(4t)\left\langle 2t, 2t^2, 1 \right\rangle + \left(2t^2+1\right)^{-1}\langle 2, 4t, 0\rangle$$

$$= \frac{1}{(2t^2+1)^2}\left\langle -8t^2 + 4t^2 + 2,\ -8t^3 + 8t^3 + 4, -4t \right\rangle$$

$$= \frac{1}{(2t^2+1)^2}\left\langle 1 - 2t^2, 2t, -2t \right\rangle$$

$$|\mathbf{T}'(t)| = \frac{1}{(2t^2+1)^2}\sqrt{1 - 4t^2 + 4t^4 + 8t^2} = \frac{2}{2t^2+1}$$

$$\mathbf{N}(t) = \frac{2}{(2t^2+1)^2}\cdot\frac{2t^2+1}{2}\left\langle 1 - 2t^2, 2t, -2t \right\rangle$$

$$= \frac{1}{2t^2+1}\left\langle 1 - 2t^2, 2t, -2t \right\rangle$$

(b) $\kappa(t) = \dfrac{2}{2t^2+1}\cdot\dfrac{1}{2t^2+1} = \dfrac{2}{(2t^2+1)^2}$

15. $\mathbf{r}'(t) = \mathbf{j} - 2t\,\mathbf{k}$, $\mathbf{r}''(t) = -2\,\mathbf{k}$, $|\mathbf{r}'(t)|^3 = \left(4t^2+1\right)^{3/2}$,

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = |-2\mathbf{i}| = 2$,

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{2}{(4t^2+1)^{3/2}}$$

16. $\mathbf{r}'(t) = \langle 1, -1, 6t\rangle$, $\mathbf{r}''(t) = \langle 0, 0, 6\rangle$,

$|\mathbf{r}'(t)|^3 = \left(\sqrt{2 + 36t^2}\right)^3 = \left[2\left(1 + 18t^2\right)\right]^{3/2}$,

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = |\langle -6, -6, 0\rangle| = 6\sqrt{2}$,

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{6\sqrt{2}}{[2(1+18t^2)]^{3/2}} = \frac{3}{(1+18t^2)^{3/2}}$$

17. $\mathbf{r}'(t) = \langle 6t^2, -6t, 6\rangle$, $\mathbf{r}''(t) = \langle 12t, -6, 0\rangle$,

$|\mathbf{r}'(t)|^3 = 6^3\left(t^4 + t^2 + 1\right)^{3/2}$,

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \left|36\left\langle 1, 2t, t^2\right\rangle\right| = 36\sqrt{1 + 4t^2 + t^4}$,

$$\boldsymbol{\kappa}(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{36\sqrt{1 + 4t^2 + t^4}}{6^3\left(t^4 + t^2 + 1\right)^{3/2}} = \frac{\sqrt{1 + 4t^2 + t^4}}{6\left(t^4 + t^2 + 1\right)^{3/2}}$$

18. $\mathbf{r}'(t) = \langle 2t, 2t - 4, 2\rangle$, $\mathbf{r}''(t) = \langle 2, 2, 0\rangle$,

$|\mathbf{r}'(t)|^3 = \left(4t^2 + 4t^2 - 16t + 16 + 4\right)^{3/2} = 8\left(2t^2 - 4t + 5\right)^{3/2}$,

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = 4\left|\langle -1, 1, 2\rangle\right| = 4\sqrt{6}$,

$$\boldsymbol{\kappa}(t) = \frac{|\mathbf{r}'(\mathbf{t}) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{4\sqrt{6}}{8\left(2t^2 - 4t + 5\right)^{3/2}} = \frac{\sqrt{6}}{2\left(2t^2 - 4t + 5\right)^{3/2}}$$

19. $\mathbf{r}'(t) = \langle \cos t, -\operatorname{sen} t, \cos t\rangle$,

$\mathbf{r}''(t) = \langle -\operatorname{sen} t, -\cos t, -\operatorname{sen} t\rangle$,

$|\mathbf{r}'(t)|^3 = \left(\sqrt{\cos^2 t + 1}\right)^3$,

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = |\langle 1, 0, -1\rangle| = \sqrt{2}$,

$$\boldsymbol{\kappa}(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{\sqrt{2}}{\left(1 + \cos^2 t\right)^{3/2}}$$

20. $\mathbf{r}'(t) = \left\langle \sqrt{2}, e^t, -e^{-t}\right\rangle$. O ponto $(0, 1, 1)$ corresponde a $t = 0$, e $\mathbf{r}'(0) = \left\langle \sqrt{2}, 1, -1\right\rangle$

$\Rightarrow |\mathbf{r}'(0)| = \sqrt{\left(\sqrt{2}\right)^2 + 1^2 + (-1)^2} = 2$.

$\mathbf{r}''(t) = \left\langle 0, e^t, e^{-t}\right\rangle \quad\Rightarrow\quad \mathbf{r}''(0) = \langle 0, 1, 1\rangle$.

$\mathbf{r}'(0) \times \mathbf{r}''(0) = \langle 2, -\sqrt{2}, \sqrt{2}\rangle$,

$|\mathbf{r}'(0) \times \mathbf{r}''(0)| = \sqrt{2^2 + \left(-\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{8} = 2\sqrt{2}$

Então $\kappa(0) = \dfrac{|\mathbf{r}'(0) \times \mathbf{r}''(0)|}{|\mathbf{r}'(0)|^3} = \dfrac{2\sqrt{2}}{2^3} = \dfrac{\sqrt{2}}{4}$.

21. $y' = \dfrac{1}{2\sqrt{x}}$, $y'' = -\dfrac{1}{4(x)^{3/2}}$,

$$\kappa(x) = \frac{|y''(x)|}{\left[1 + (y'(x))^2\right]^{3/2}} = \frac{1}{4\left|x^{3/2}\right|}\,\frac{1}{[1 + 1/(4x)]^{3/2}} = \frac{2}{(4x + 1)^{3/2}}$$

22. $y' = \cos x$, $y'' = -\operatorname{sen} x$,

$$\kappa(x) = \frac{|y''(x)|}{\left[1 + (y'(x))^2\right]^{3/2}} = \frac{|\operatorname{sen} x|}{\left(1 + \cos^2 x\right)^{3/2}}$$

23. $y' = \dfrac{1}{x}$, $y'' = -\dfrac{1}{x^2}$,

$$\kappa(x) = \frac{|y''(x)|}{\left[1 + (y'(x))^2\right]^{3/2}} = \left|\frac{-1}{x^2}\right| \frac{1}{\left(1 + 1/x^2\right)^{3/2}} = \frac{1}{x^2}\,\frac{\left(x^2\right)^{3/2}}{\left(x^2 + 1\right)^{3/2}} = \frac{|x|}{\left(x^2 + 1\right)^{3/2}}$$

24.
$$\kappa(t) = \frac{|\dot{x}\ddot{y} - \ddot{x}\dot{y}|}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}} = \frac{\left|\left(3t^2\right)(2) - (6t)(2t)\right|}{\left(9t^4 + 4t^2\right)^{3/2}} = \frac{6t^2}{\left(t^2\right)^{3/2}\left(9t^2 + 4\right)^{3/2}} = \frac{6t^2}{|t|^3\left(9t^2 + 4\right)^{3/2}} = \frac{6}{|t|\left(9t^2 + 4\right)^{3/2}}$$

25.
$$\kappa(t) = \frac{|\dot{x}\ddot{y} - \ddot{x}\dot{y}|}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}$$
$$= \frac{\left|(\operatorname{sen} t + t\cos t)(-2\operatorname{sen} t - t\cos t) - (2\cos t - t\operatorname{sen} t)(\cos t - t\operatorname{sen} t)\right|}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}$$
$$= \frac{\left|\left(-2\operatorname{sen}^2 t - 3t\operatorname{sen} t\cos t - t^2\cos^2 t\right) - \left(2\cos^2 t - 3t\cos t\operatorname{sen} t + t^2\operatorname{sen}^2 t\right)\right|}{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}$$
$$= \frac{\left|-\left(\operatorname{sen}^2 t + \cos^2 t\right)\left(2 + t^2\right)\right|}{\left(\operatorname{sen}^2 t + 2t\cos t\operatorname{sen} t + \cos^2 t + t^2\cos^2 t - 2t\operatorname{sen} t\cos t + t^2\operatorname{sen} t\right)^{3/2}}$$
$$= \frac{2 + t^2}{\left(1 + t^2\right)^{3/2}}$$