Aula 11

Energia potencial magnética

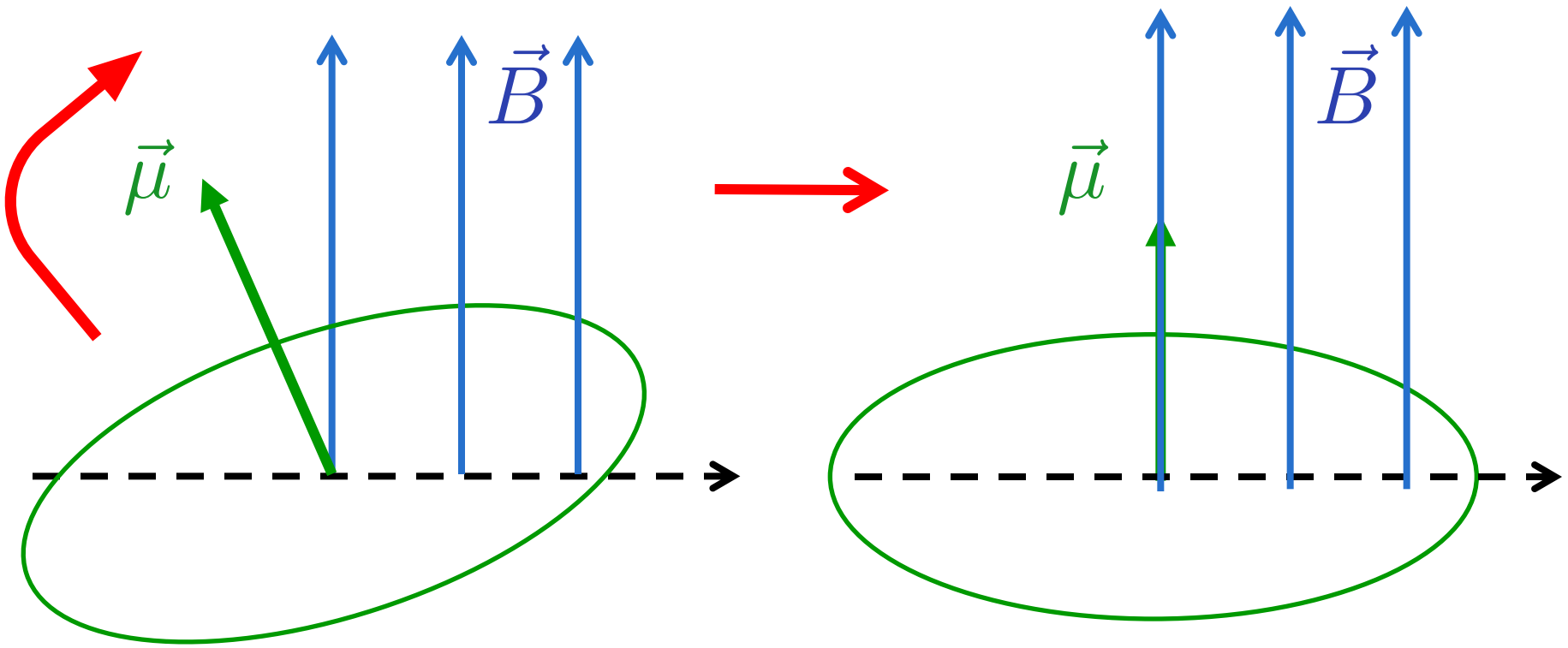
Interação hiperfina no hidrogênio

A linha de 21 cm

A rotação das galáxias

A formação das primeiras estrelas

Espira tenta se alinhar como uma bússola !



mais energia !

menos energia !

Energia potencial de orientação

$$U = -\vec{\mu} \,.\, \vec{B}$$

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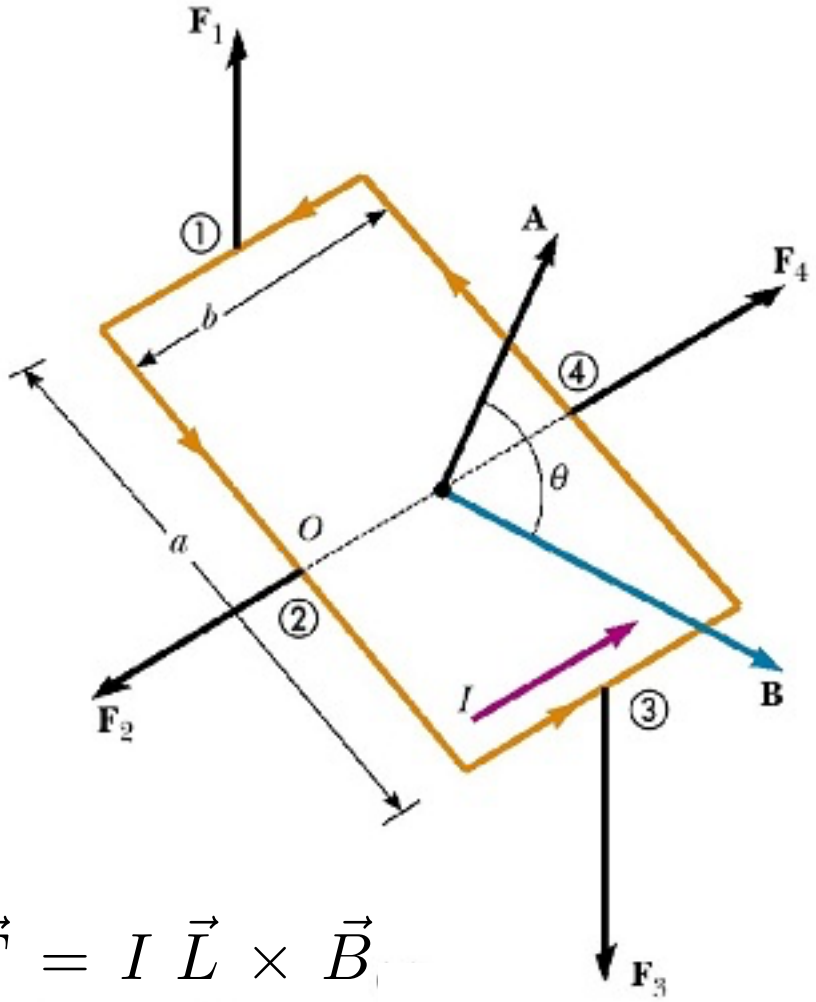
Espira sofre um torque !



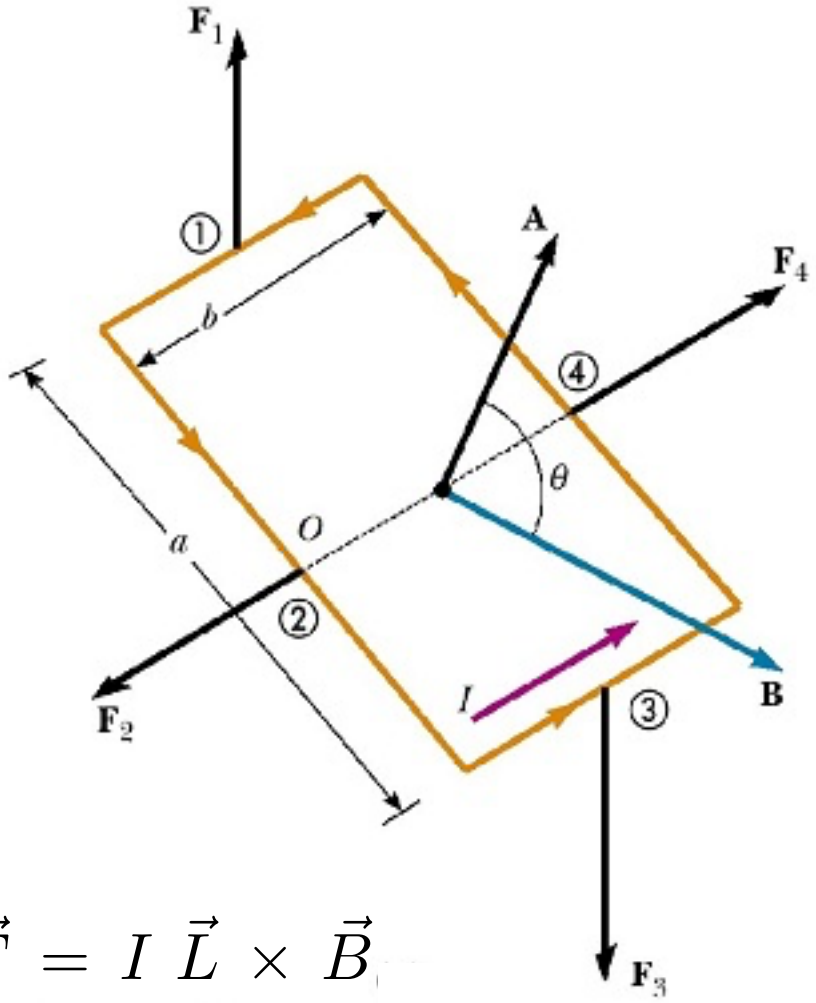
$$\vec{F} = I\,\vec{L} \times \vec{B}$$

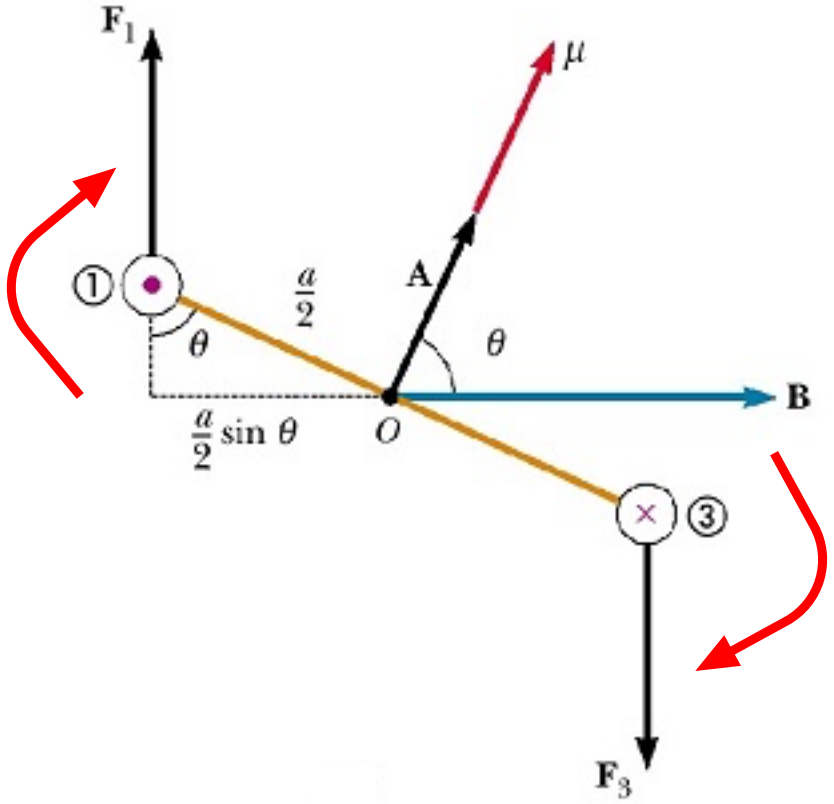
$$F = I\,b\,B$$

Espira sofre um torque !

$$\vec{\mu} = I\,\vec{A}$$

$$\mu = I\,a\,b$$





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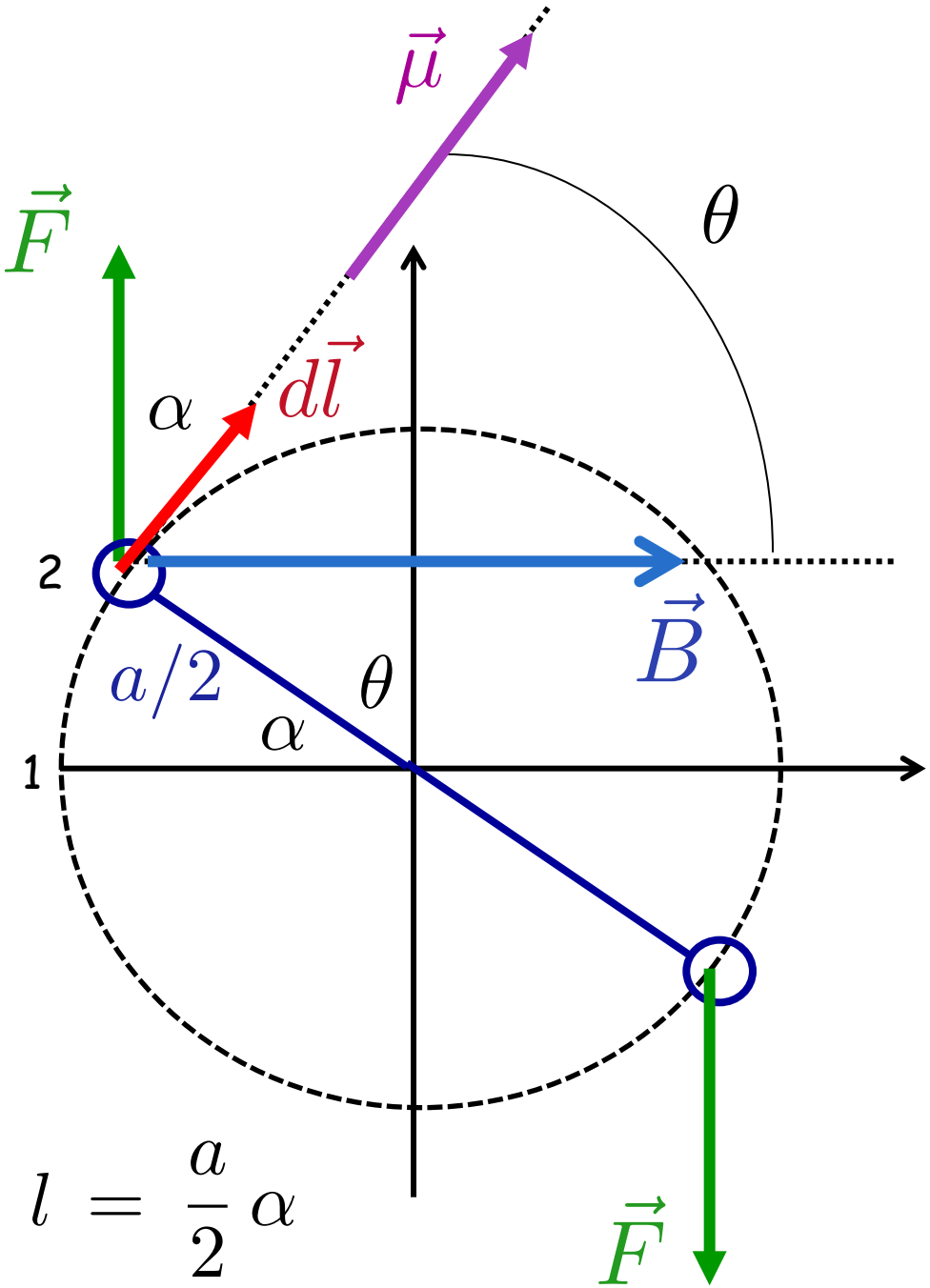
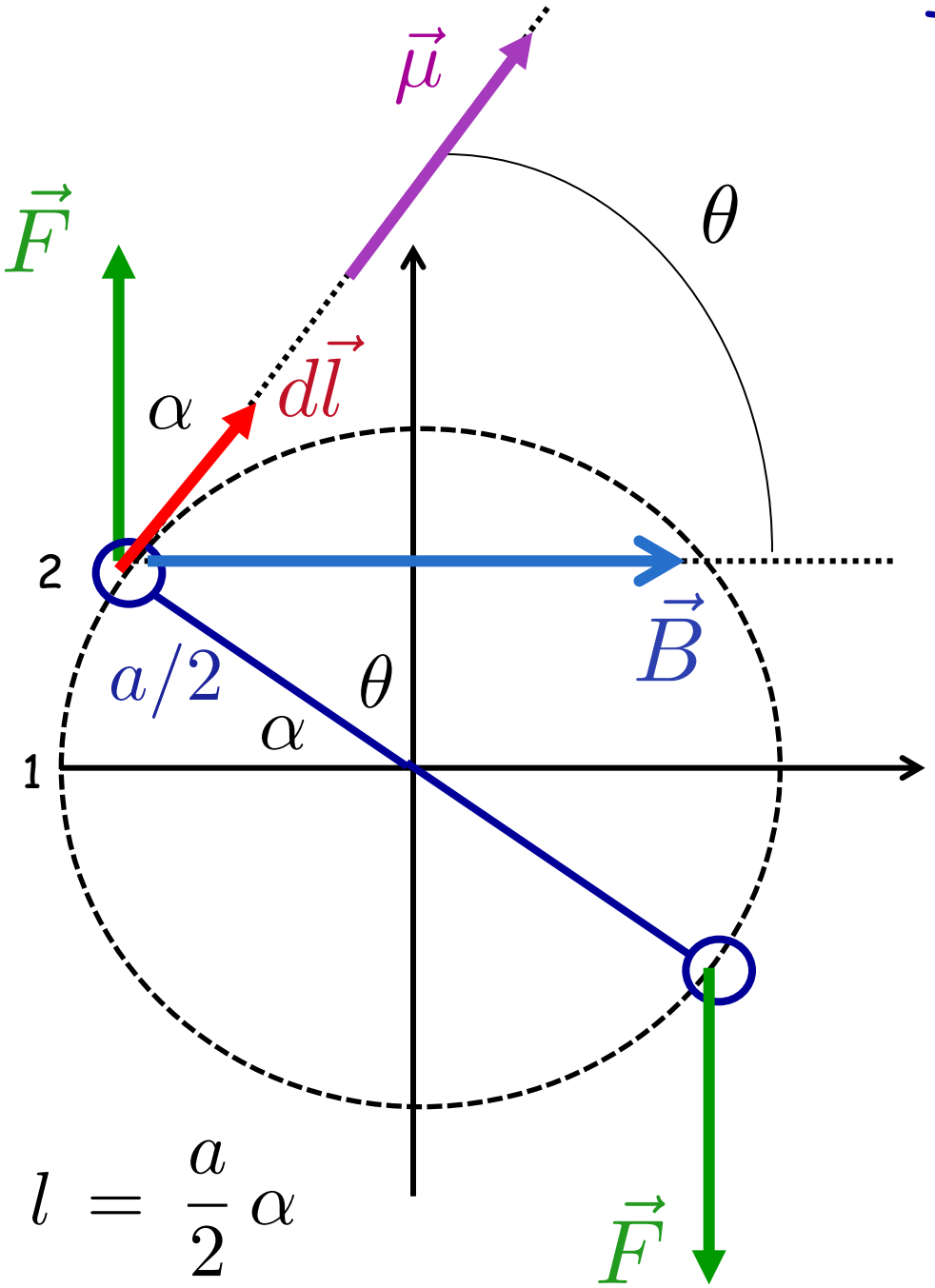


$$l = \frac{a}{2}\alpha$$

Trabalho da força F quando a espira vai de 1 a 2. :

$$dW = \vec{F} \,.\, d\vec{l}$$



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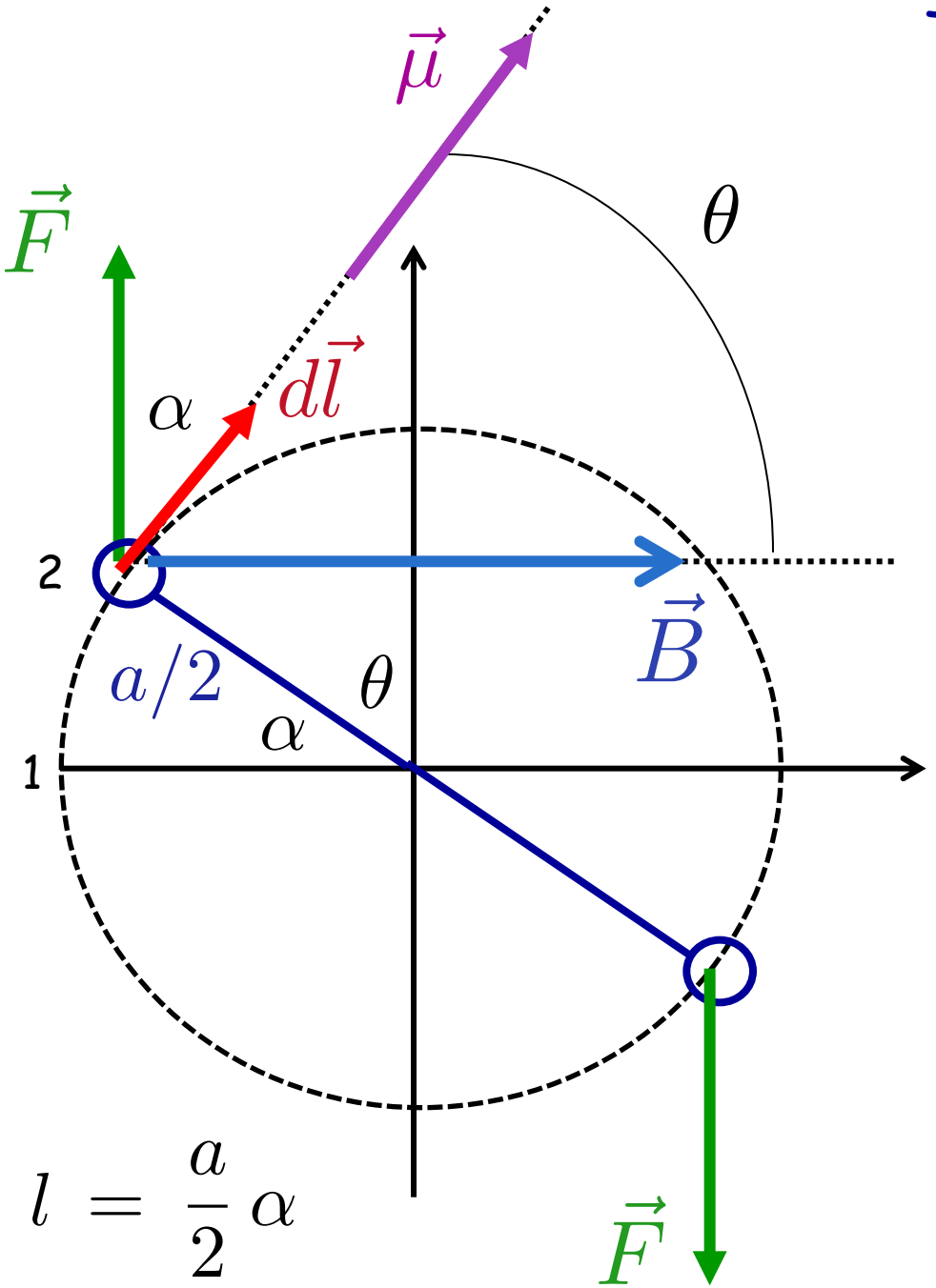
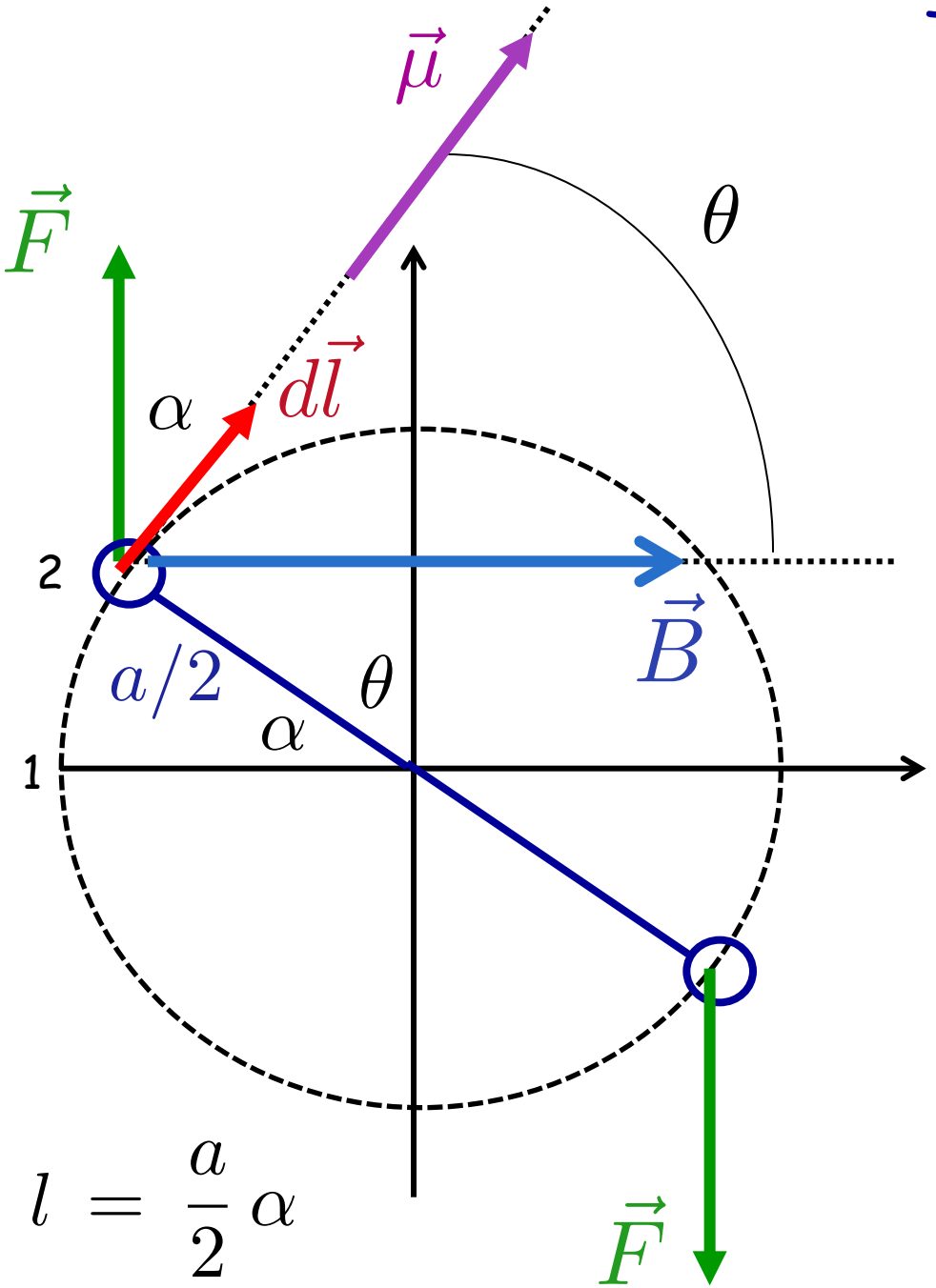


Trabalho da força F quando a espira vai de 1 a 2. :

$$dW = \vec{F} \,.\, d\vec{l}$$

$$= F\, dl\, cos\alpha$$



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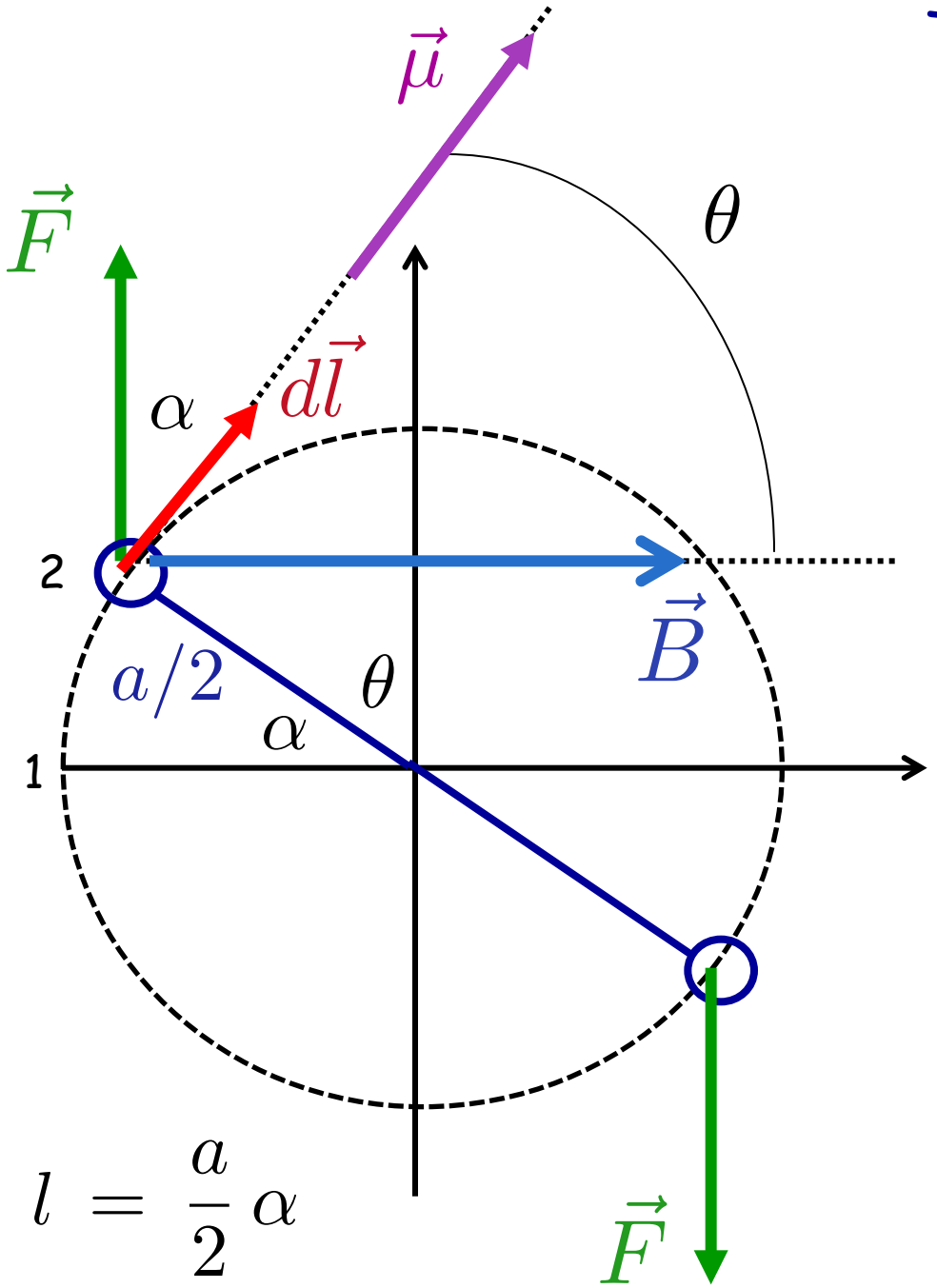

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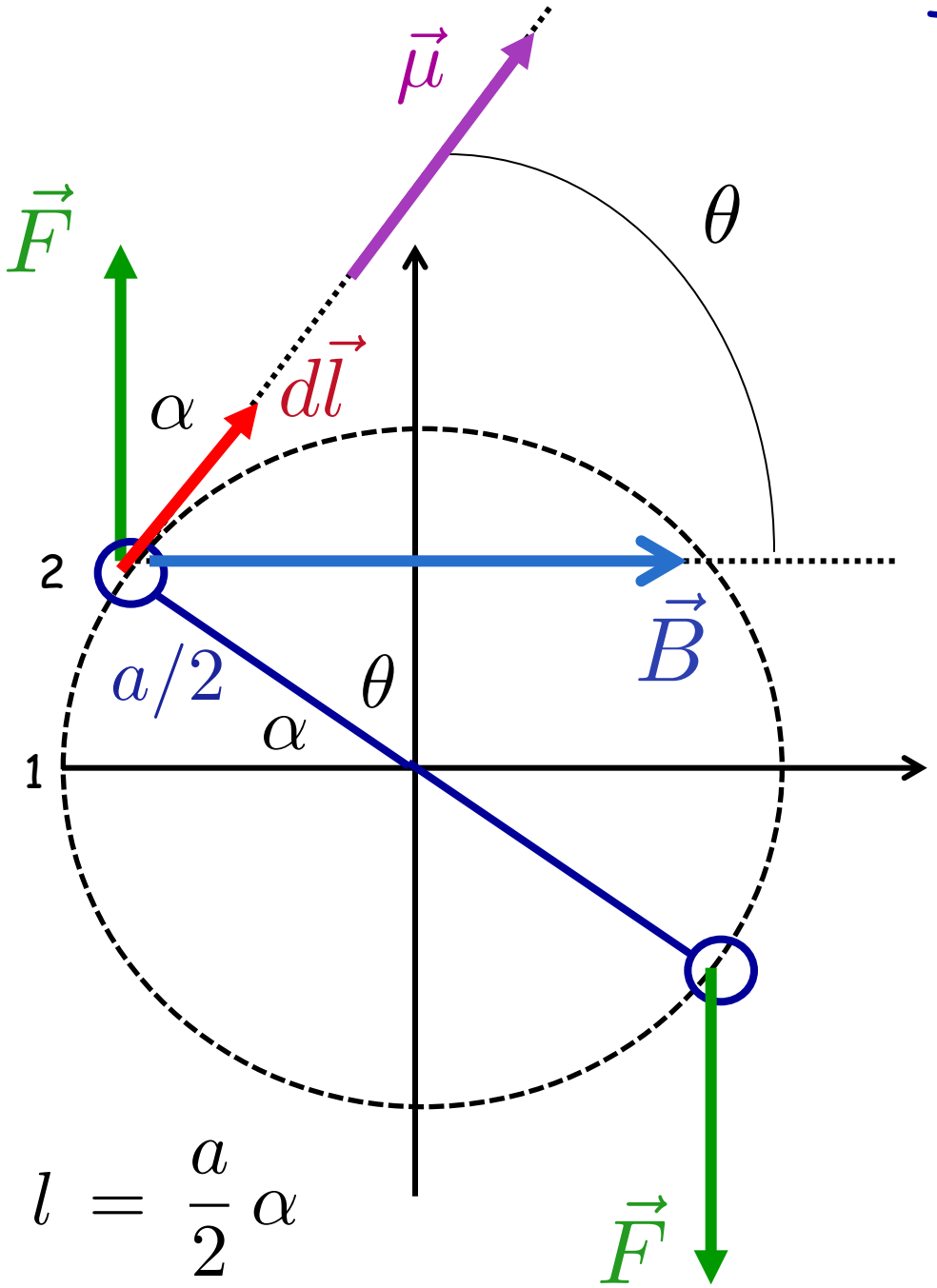
$$W = \int_0^{\alpha} F\, \frac{a}{2}\, cos\,\alpha\; d\alpha$$



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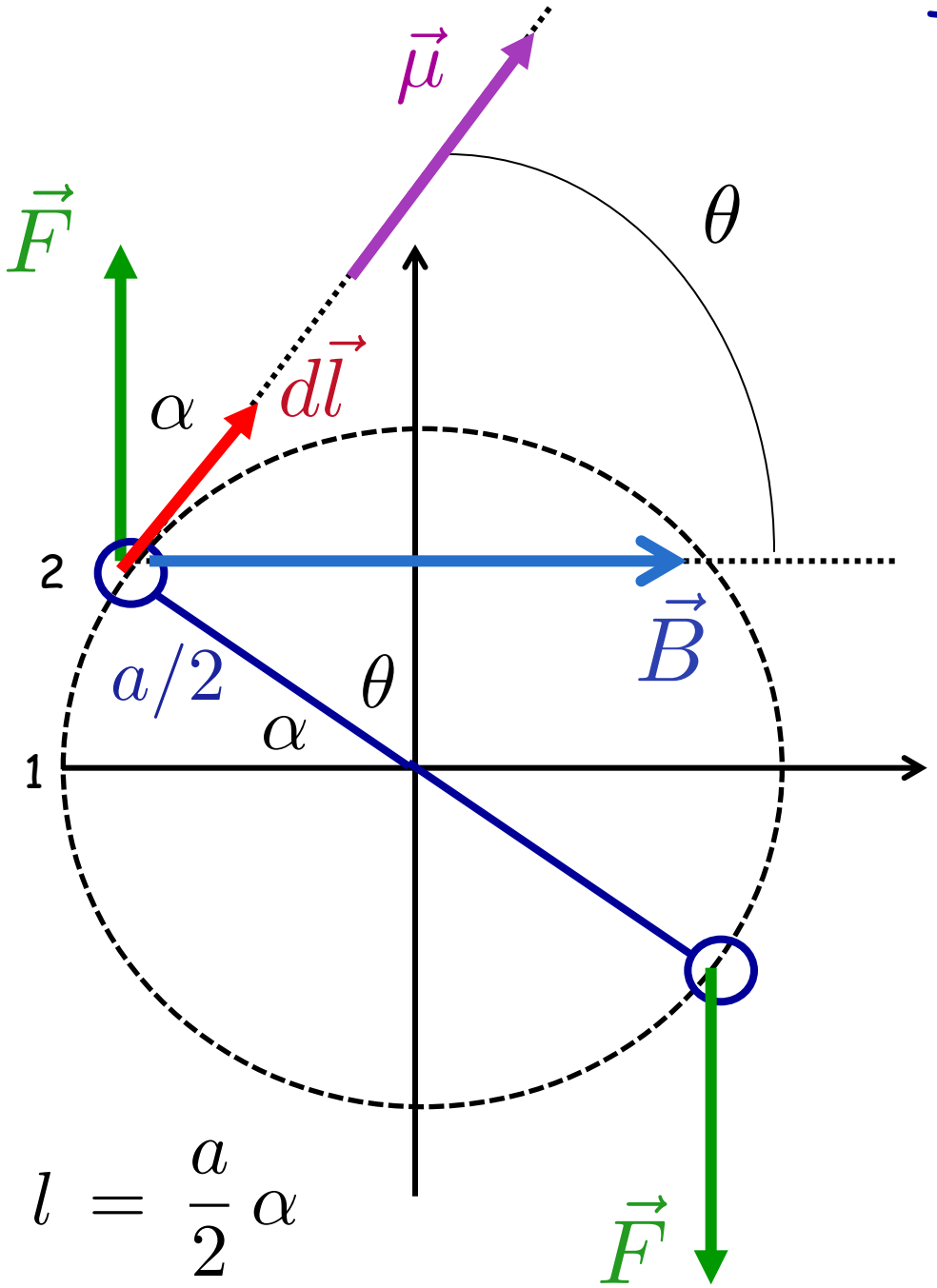

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$$W = F\, \frac{a}{2}\, cos\,\theta$$

$$W = F \frac{a}{2} cos\,\theta$$

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Somando a contribuição da barra inferior (3) :

$$W = F\, a\, cos\,\theta$$

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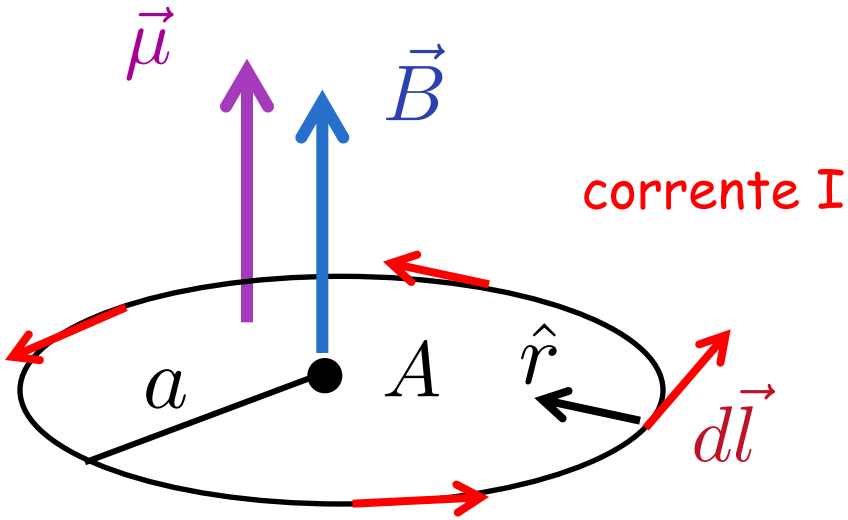
$$\boxed{U = -\,\vec{\mu}\,.\,\vec{B}}$$

Interação hiperfina no átomo de hidrogênio

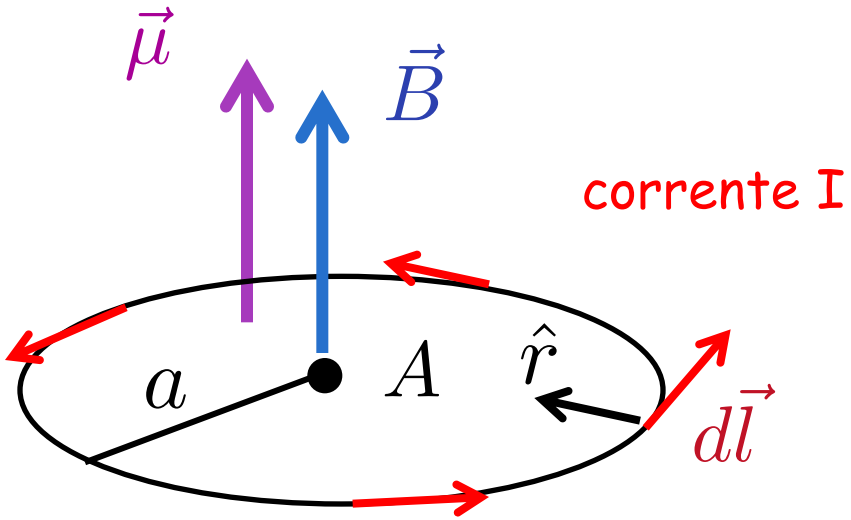
Modelo:
interação magnética entre duas espiras

Campo magnético de uma espira



$$d\vec{l} \times \hat{r} = |d\vec{l} \times \hat{r}|\, \hat{k} = dl\, \hat{k}$$

Campo magnético de uma espira



Biot-Savart :

$$d\vec{B} = \frac{\mu_0 \, I}{4\,\pi} \, \frac{d\vec{l} \times \hat{r}}{r^2}$$

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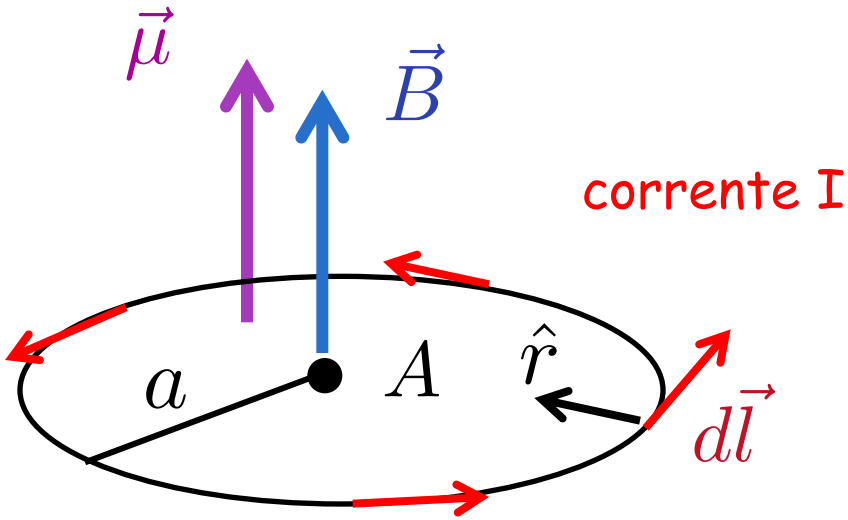

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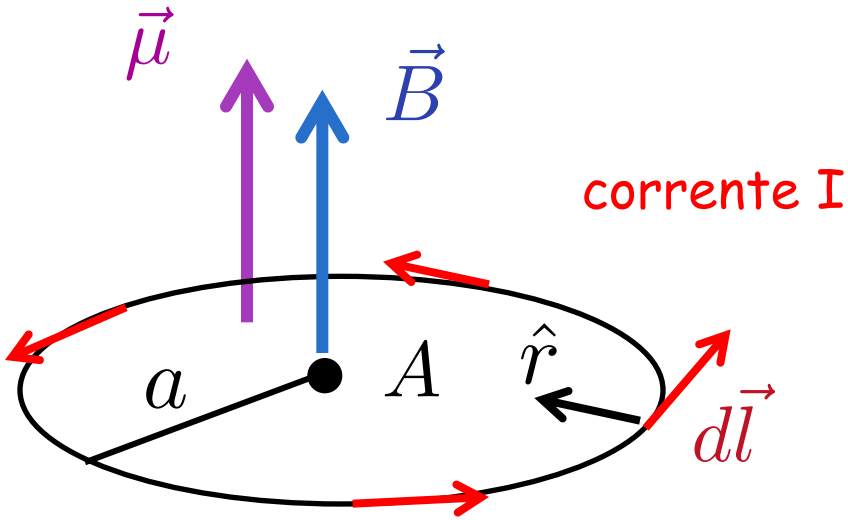
Campo magnético de uma espira



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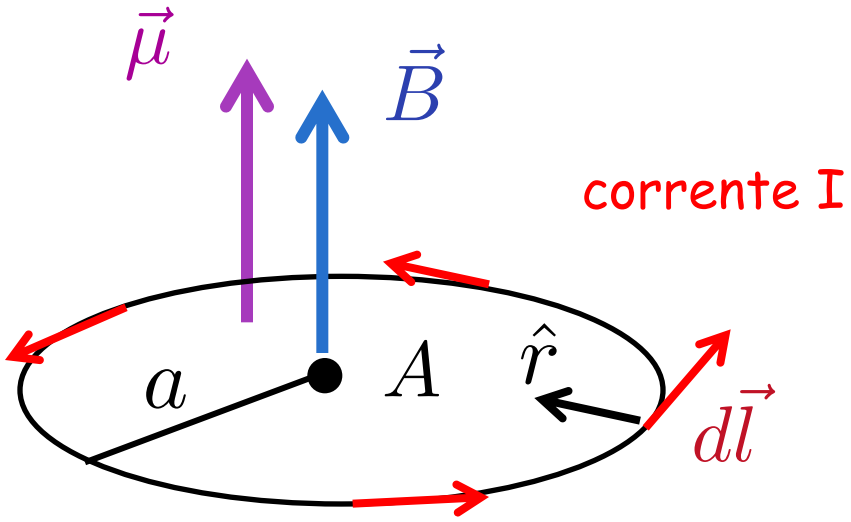
$$\vec{\mu} = I\, A\, \hat{k} = I\, \pi\, a^2\, \hat{k}$$

Campo magnético de uma espira



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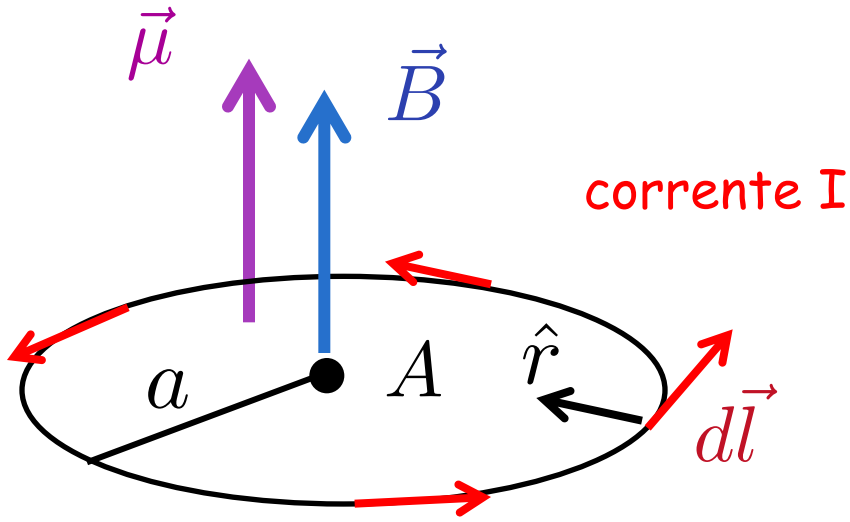
$$\vec{\mu} = I\, A\, \hat{k} = I\, \pi\, a^2\, \hat{k}$$

$$\boxed{\vec{B} = \frac{\mu_0}{2\,\pi\, a^3}\, \vec{\mu}}$$

Campo magnético de uma espira



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$$d\vec{l}\times\hat{r} = |d\vec{l}\times\hat{r}|\,\hat{k} = dl\,\hat{k}$$

$$\vec{\mu} = I\,A\,\hat{k} = I\,\pi\,a^2\,\hat{k}$$

momento magnético gera campo magnético !

$$\vec{B} = \frac{\mu_0}{2\,\pi\,a^3}\,\vec{\mu}$$

Spin gera momento magnético : $\vec{\mu} = \gamma \vec{S}$

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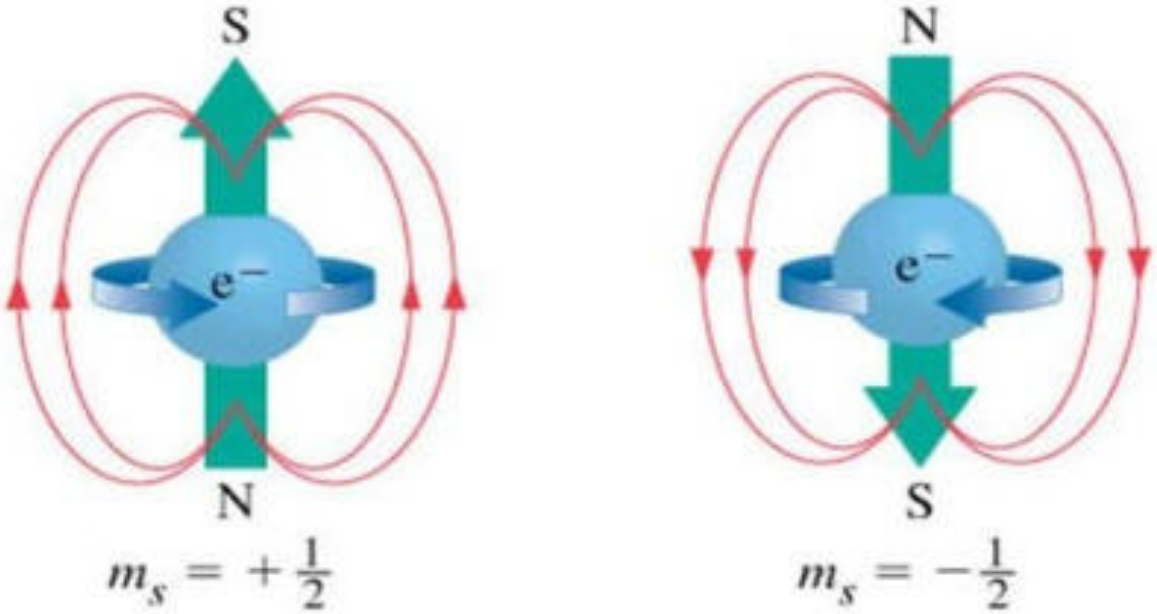
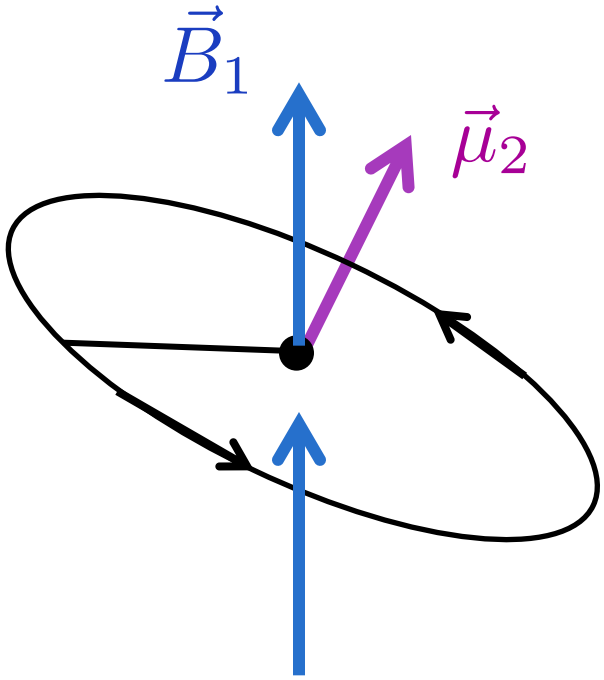
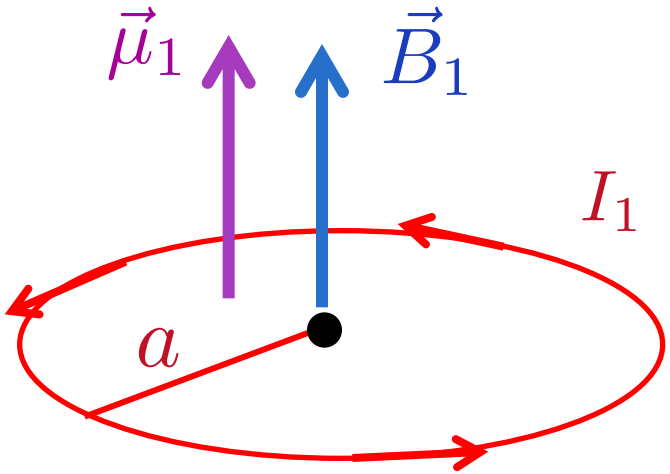
Spin gera
campo magnético !



Interação entre duas espiras





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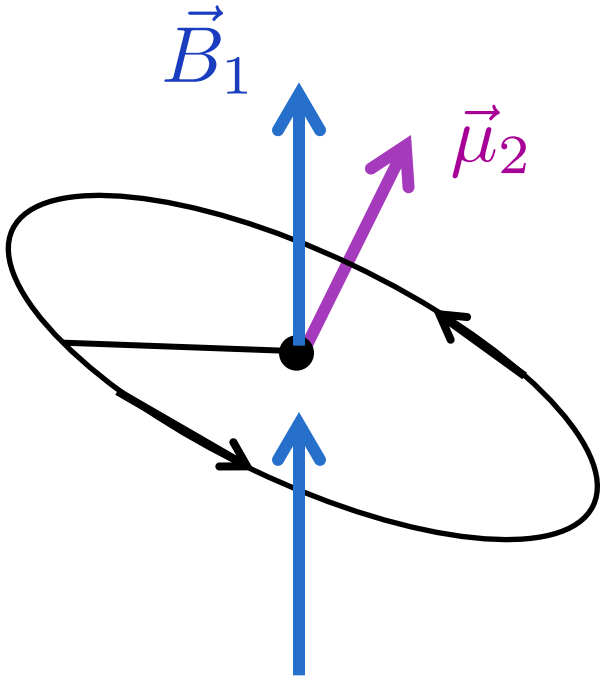
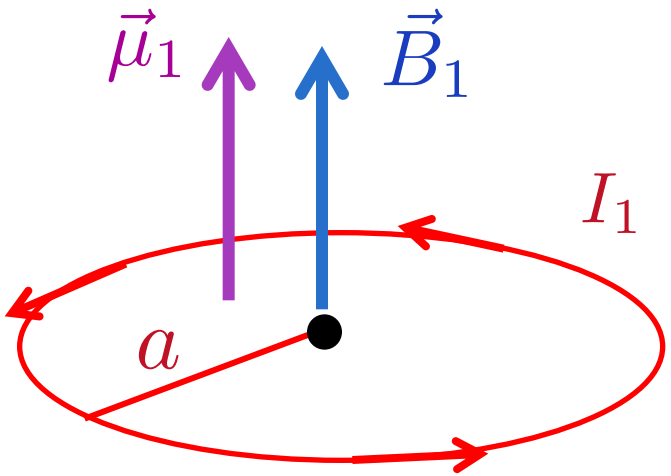


$$\vec{B}_1 = \frac{\mu_0}{2\,\pi\,a^3}\,\vec{\mu}_1$$



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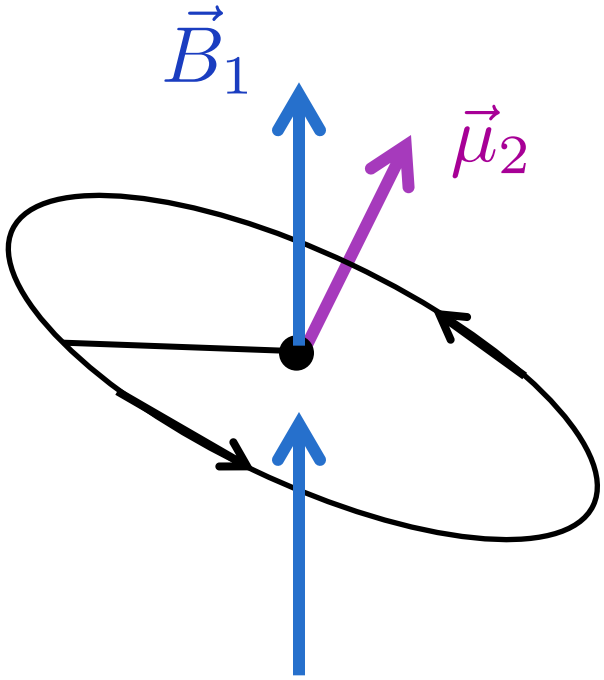
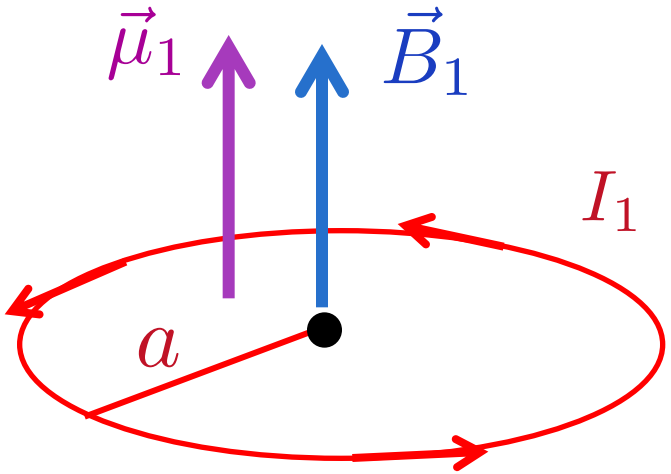


$$\vec{B}_1 = \frac{\mu_0}{2\,\pi\,a^3}\,\vec{\mu}_1$$

$$U = -\,\vec{\mu}_2\,.\,\vec{B}_1$$



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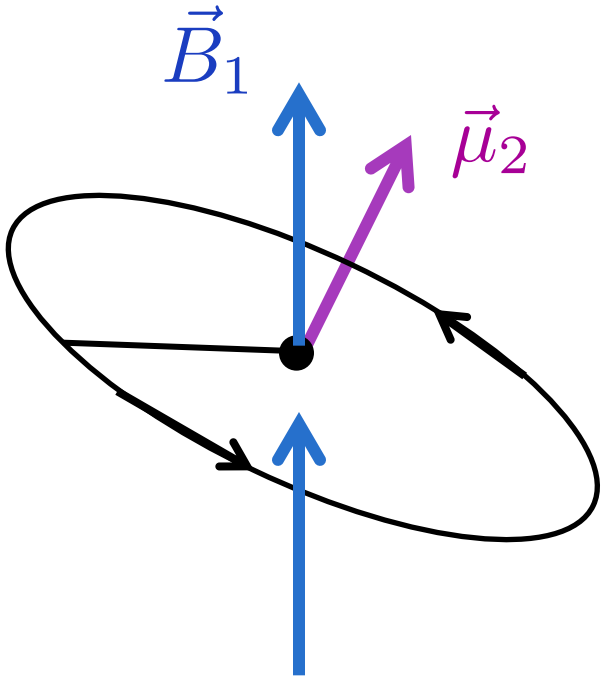

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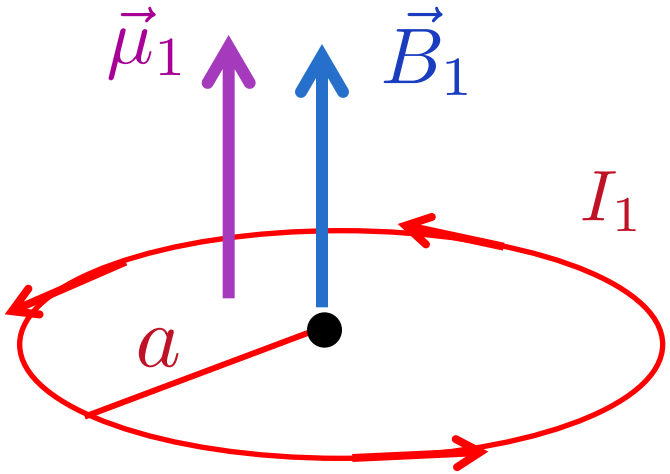
$$H = U = -\,\frac{\mu_0}{2\,\pi\,a^3}\;\vec{\mu}_1\,.\,\vec{\mu}_2$$



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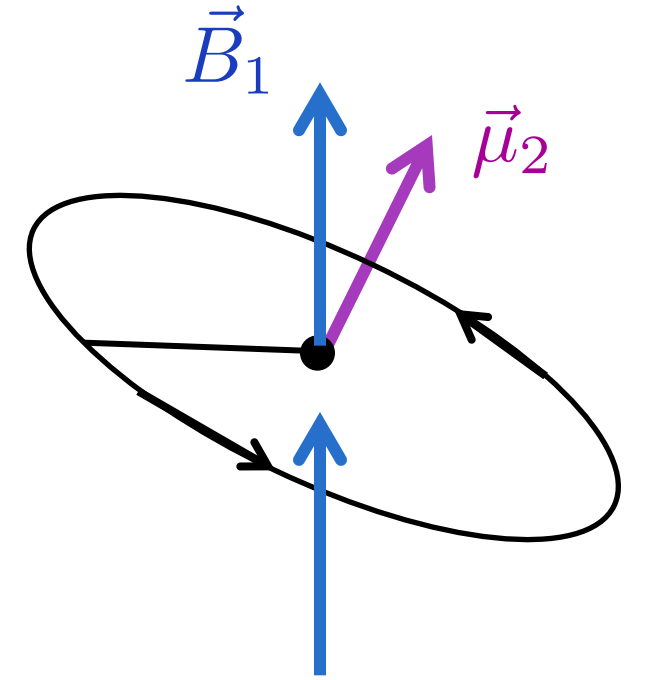

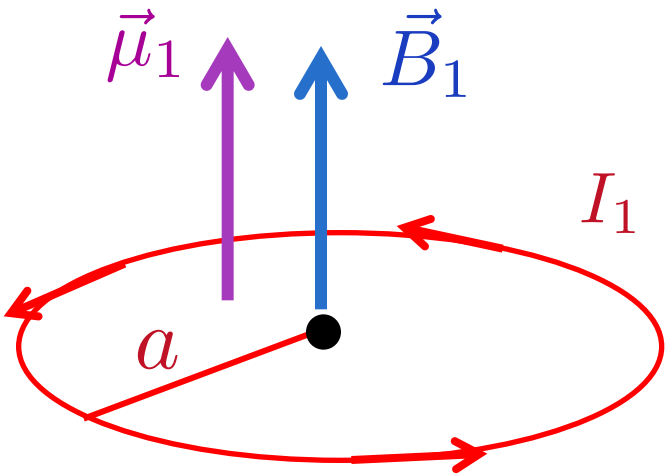

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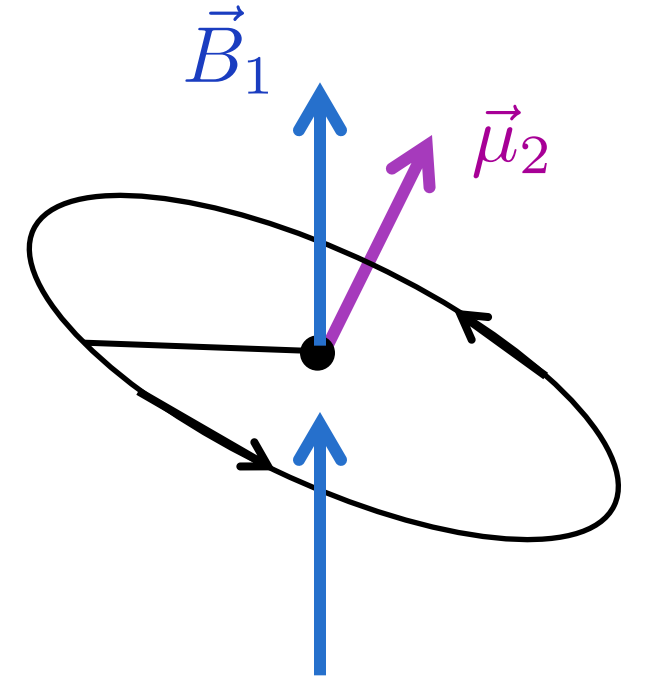
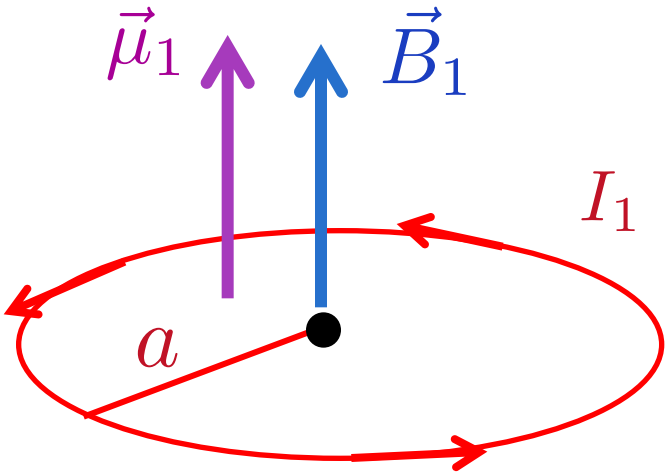
$$U = -\,\vec{\mu}_2\,.\,\vec{B}_1$$

$$H = U = -\,\frac{\mu_0}{2\,\pi\,a^3}\;\vec{\mu}_1\,.\,\vec{\mu}_2$$

$$\begin{cases} \vec{\mu}_1 = \gamma_1\,\vec{S}_1 \\ \vec{\mu}_2 = -\,\gamma_2\,\vec{S}_2 \end{cases}$$

Interação entre duas espiras





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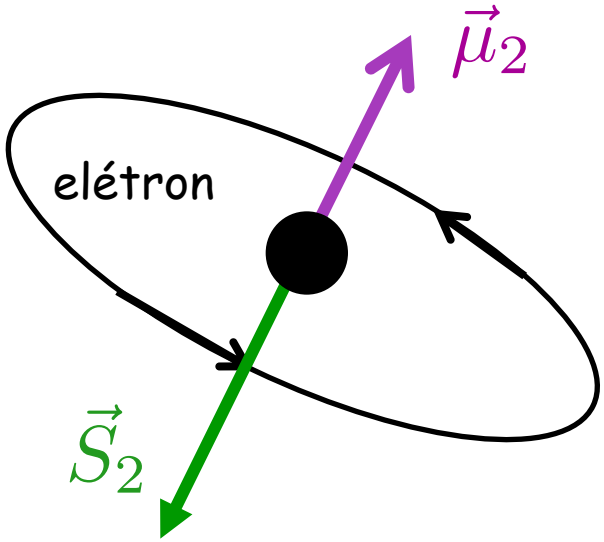
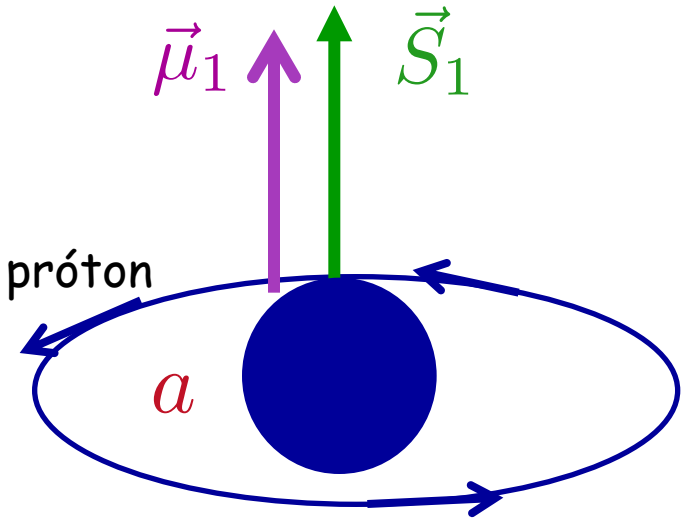
$$H = \frac{\mu_0\,\gamma_1\,\gamma_2}{2\,\pi\,a^3}\;\vec{S}_1\,.\,\vec{S}_2$$

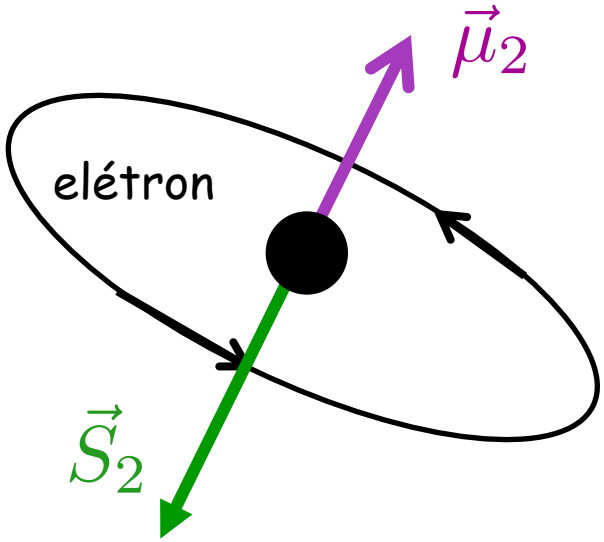
$$H = \frac{\mu_0 \, \gamma_1 \, \gamma_2}{2 \, \pi \, a^3} \, \vec{S}_1 \, . \, \vec{S}_2$$

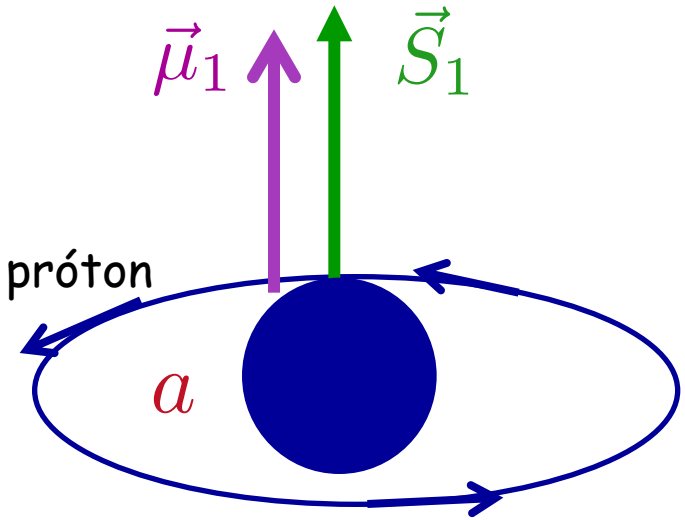


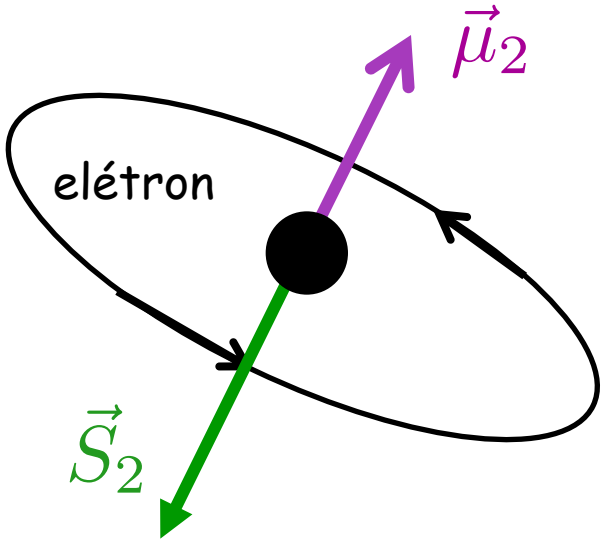


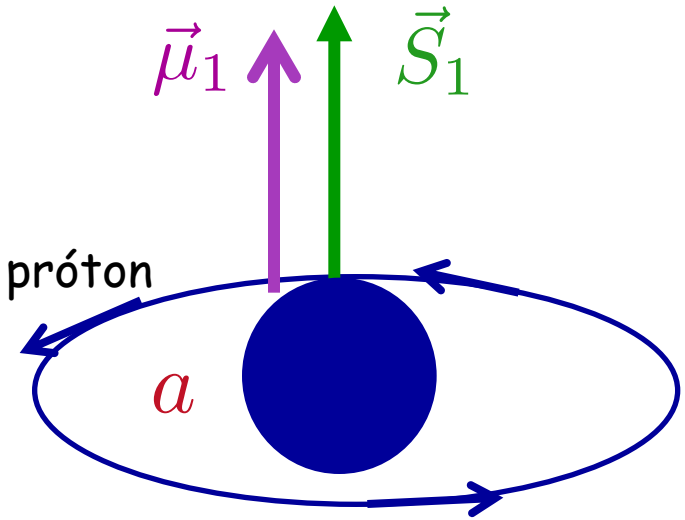
$$H = \frac{\mu_0 \, \gamma_1 \, \gamma_2}{2 \, \pi \, a^3} \, \vec{S}_1 \, . \, \vec{S}_2$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$





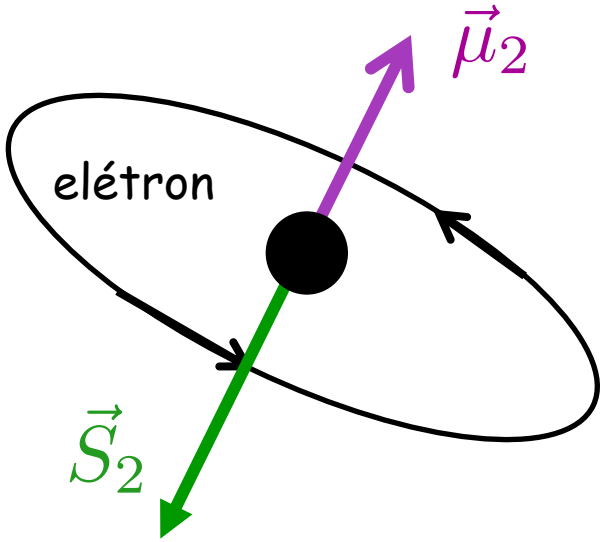


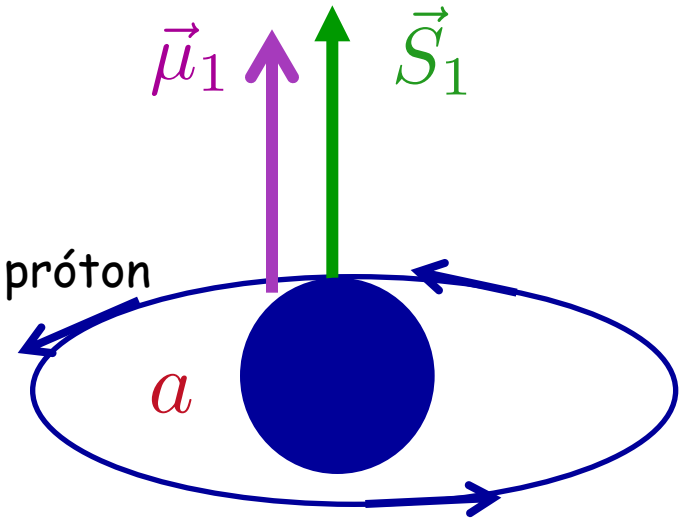


$$H = \frac{\mu_0 \, \gamma_1 \, \gamma_2}{2 \, \pi \, a^3} \, \vec{S}_1 \, . \, \vec{S}_2$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

$$\vec{S}^2 = \vec{S}_1^2 + \vec{S}_2^2 + 2 \, \vec{S}_1 \, . \, \vec{S}_2$$





$$H = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \vec{S}_1 . \vec{S}_2$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

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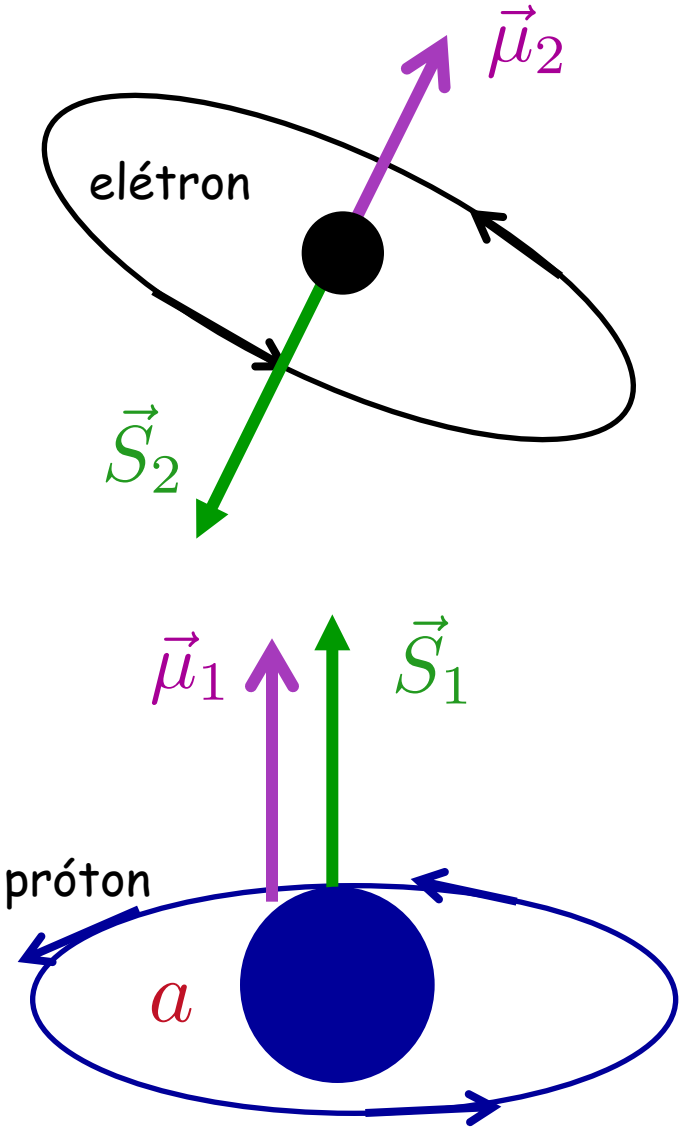

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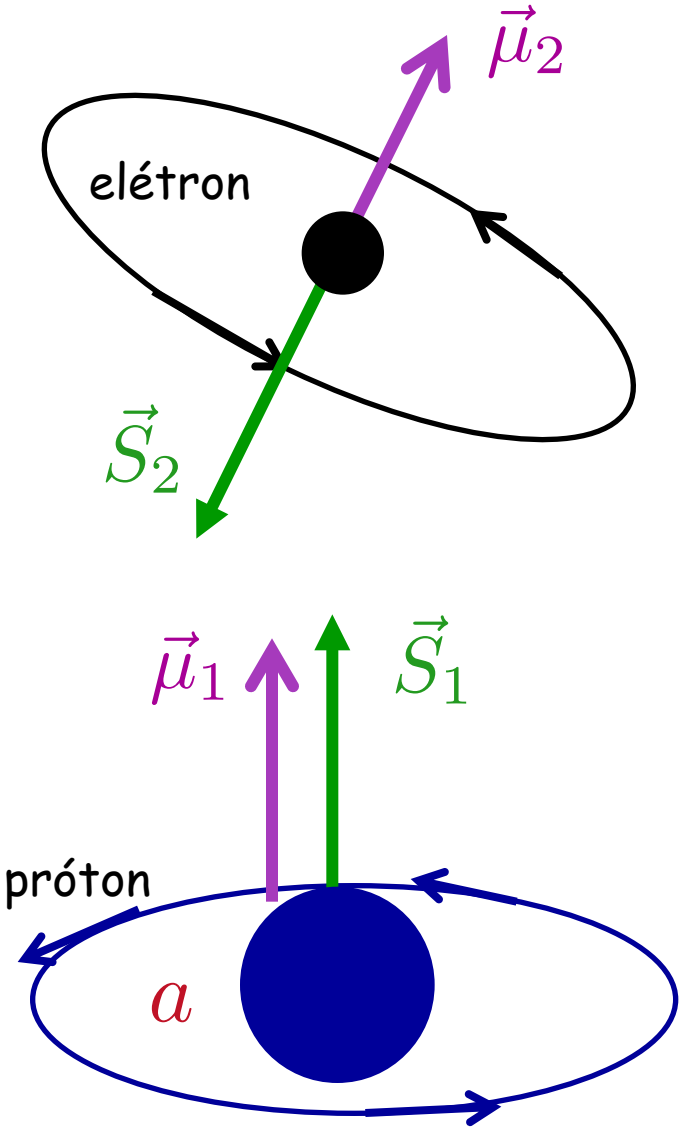

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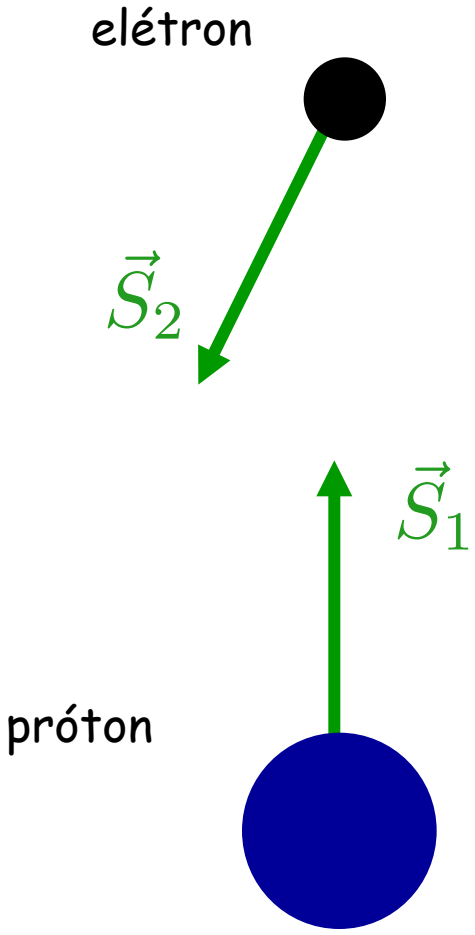
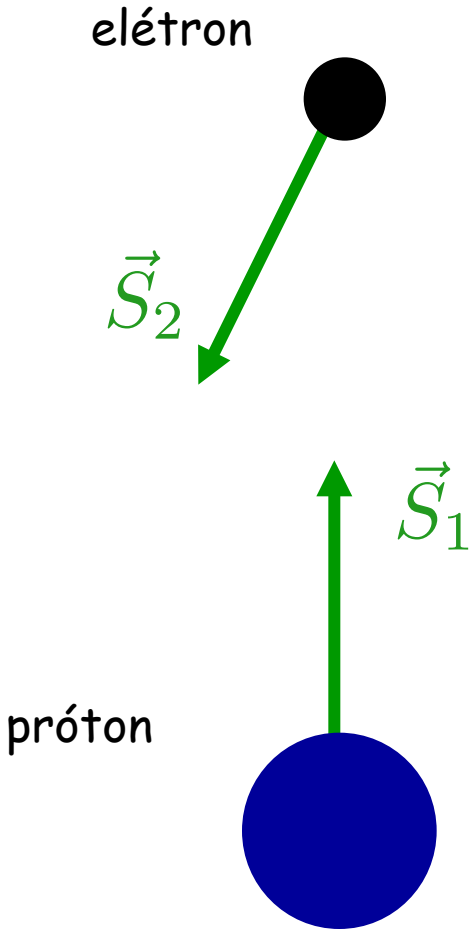
$$H \rightarrow \hat{H} \qquad \vec{S} \rightarrow \hat{S}$$

Energia de interação hiperfina (spin-spin)



Energia de interação hiperfina (spin-spin)

$$\Delta E = \langle \hat{H} \rangle = \langle \chi \,|\, \hat{H} \,|\, \chi \rangle$$



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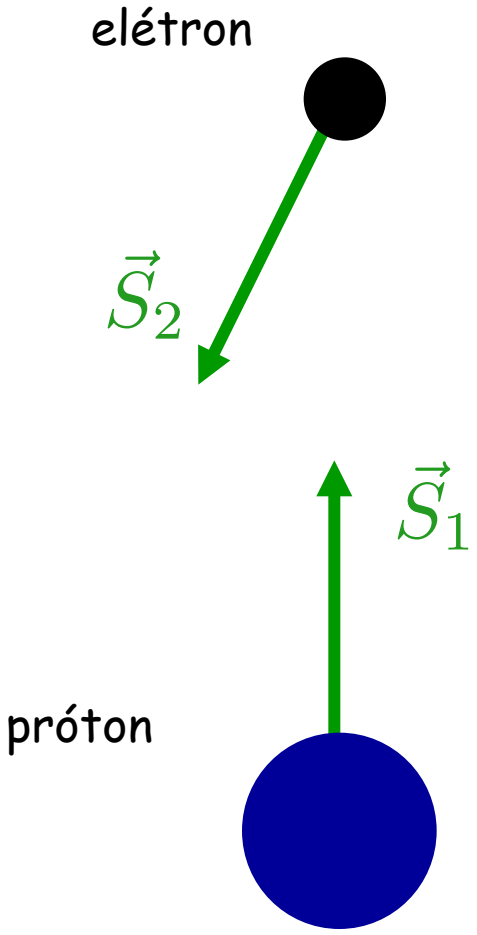
$$|\,\chi\,\rangle = |s, m\rangle$$

Estado de spin de duas partículas com spin total s e projeção m



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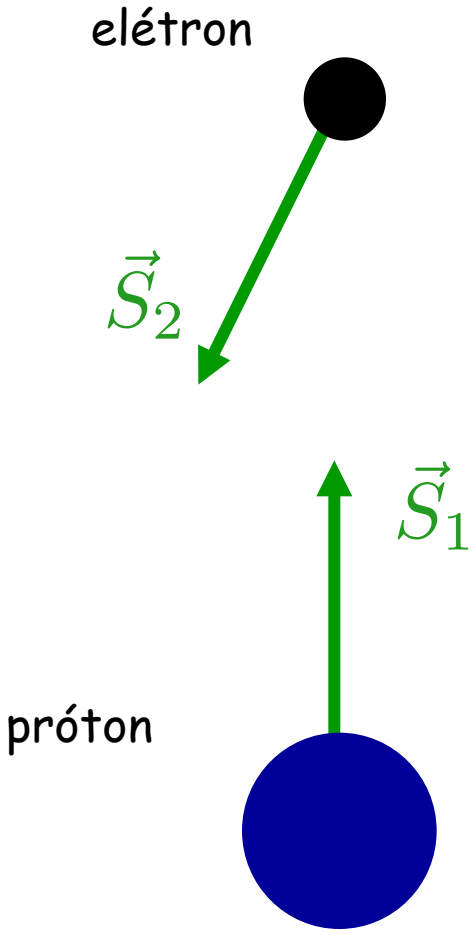

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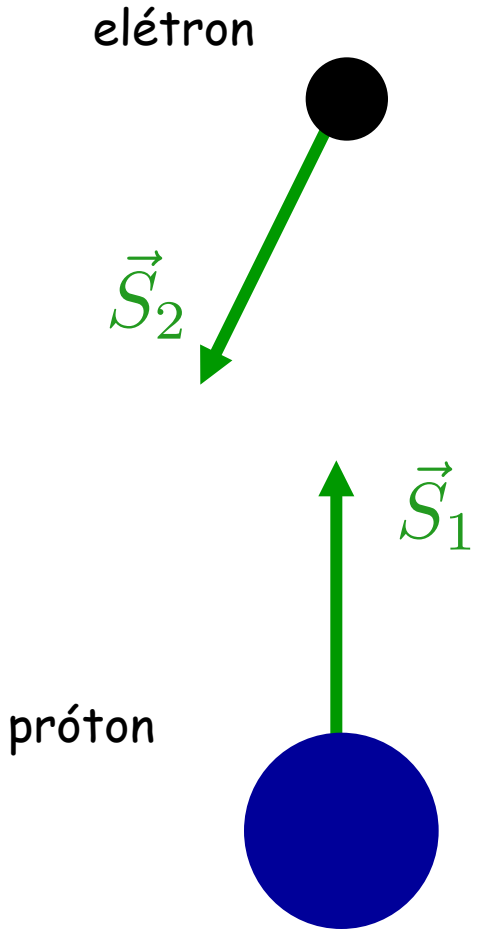

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Singleto : $|0\ 0\rangle$

Tripleto : $\left\{ \begin{array}{l} |1\ 1\rangle \\ |1\ 0\rangle \\ |1\ -1\rangle \end{array} \right.$

$$\Delta E \;=\; \frac{\mu_0\,\gamma_1\,\gamma_2}{2\,\pi\,a^3}\;\left(\;\frac{\langle s\,m|\vec{S}^2\,|s\,m\rangle\;-\;\langle s\,m|\vec{S}_1^2|s\,m\rangle\;-\;\langle s\,m|\vec{S}_2^2|s\,m\rangle}{2}\right)$$

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Singleto :

$$|0\ 0\rangle = \frac{1}{\sqrt{2}} |\frac{1}{2}\ +\frac{1}{2}\rangle |\frac{1}{2}\ -\frac{1}{2}\rangle - \frac{1}{\sqrt{2}} |\frac{1}{2}\ -\frac{1}{2}\rangle |\frac{1}{2}\ +\frac{1}{2}\rangle$$

$$\Delta E = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(\frac{\langle s\, m|\vec{S}^2\, |s\, m\rangle - \langle s\, m|\vec{S}_1^2|s\, m\rangle - \langle s\, m|\vec{S}_2^2|s\, m\rangle}{2} \right)$$

Singleto :

$$|0\ 0\rangle = \frac{1}{\sqrt{2}} |\frac{1}{2} + \frac{1}{2}\rangle |\frac{1}{2} - \frac{1}{2}\rangle - \frac{1}{\sqrt{2}} |\frac{1}{2} - \frac{1}{2}\rangle |\frac{1}{2} + \frac{1}{2}\rangle$$

$$\langle 0\ 0|\, \hat{S}^2\, |0\ 0\rangle = 0$$

$$\Delta E = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(\frac{\langle s\, m | \vec{S}^2 | s\, m \rangle - \langle s\, m | \vec{S}_1^2 | s\, m \rangle - \langle s\, m | \vec{S}_2^2 | s\, m \rangle}{2} \right)$$

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$$\Delta E = \frac{\mu_0\,\gamma_1\,\gamma_2}{2\,\pi\,a^3}\left(\frac{\langle s\,m|\vec{S}^2\,|s\,m\rangle - \langle s\,m|\vec{S}_1^2|s\,m\rangle - \langle s\,m|\vec{S}_2^2|s\,m\rangle}{2}\right)$$

Singleto :

$$|0\ 0\rangle = \frac{1}{\sqrt{2}}\,|\frac{1}{2}\ +\frac{1}{2}\rangle\,|\frac{1}{2}\ -\frac{1}{2}\rangle - \frac{1}{\sqrt{2}}\,|\frac{1}{2}\ -\frac{1}{2}\rangle\,|\frac{1}{2}\ +\frac{1}{2}\rangle$$

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$$\Delta E_{singlet} = \frac{\mu_0\,\gamma_1\,\gamma_2}{2\,\pi\,a^3}\,(-\frac{3}{4}\,\hbar^2)$$

$$\Delta E \;=\; \frac{\mu_0\,\gamma_1\,\gamma_2}{2\,\pi\,a^3}\;\left(\;\frac{\langle s\,m|\vec{S}^2\,|s\,m\rangle\;-\;\langle s\,m|\vec{S}_1^2|s\,m\rangle\;-\;\langle s\,m|\vec{S}_2^2|s\,m\rangle}{2}\right)$$

$$\Delta E = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(\frac{\langle s\, m | \vec{S}^2 | s\, m \rangle - \langle s\, m | \vec{S}_1^2 | s\, m \rangle - \langle s\, m | \vec{S}_2^2 | s\, m \rangle}{2} \right)$$

Tripleto :

$$|1 \; -1\rangle = |\frac{1}{2} \; -\frac{1}{2}\rangle \, |\frac{1}{2} \; -\frac{1}{2}\rangle$$

(por exemplo)

$$\Delta E = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(\frac{\langle s\, m | \vec{S}^2 | s\, m \rangle - \langle s\, m | \vec{S}_1^2 | s\, m \rangle - \langle s\, m | \vec{S}_2^2 | s\, m \rangle}{2} \right)$$

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$$|1 \ -1\rangle = |\frac{1}{2} \ -\frac{1}{2}\rangle \, |\frac{1}{2} \ -\frac{1}{2}\rangle$$

(por exemplo)

$$\langle 1 \ -1 | \, \hat{S}^2 \, | 1 \ -1 \rangle = 2 \hbar^2$$

$$\Delta E = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(\frac{\langle s\, m | \vec{S}^2 | s\, m \rangle - \langle s\, m | \vec{S}_1^2 | s\, m \rangle - \langle s\, m | \vec{S}_2^2 | s\, m \rangle}{2} \right)$$

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(por exemplo)

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$$\Delta E = \frac{\mu_0\,\gamma_1\,\gamma_2}{2\,\pi\,a^3}\left(\frac{\langle s\,m|\vec{S}^2\,|s\,m\rangle - \langle s\,m|\vec{S}_1^2|s\,m\rangle - \langle s\,m|\vec{S}_2^2|s\,m\rangle}{2}\right)$$

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$$\langle 1\ -1\,|\,\hat{S}_2^2\,|1\ -1\rangle = \frac{3}{4}\,\hbar^2$$

$$\Delta E_{triplet} = \frac{\mu_0\,\gamma_1\,\gamma_2}{2\,\pi\,a^3}\,(+\frac{1}{4}\,\hbar^2)$$

$$\Delta E_{hf} \;=\; \Delta E_{triplet} - \Delta E_{singlet} \;=\; \frac{\mu_0\,\gamma_1\,\gamma_2}{2\,\pi\,a^3} \qquad\qquad \gamma \;=\; \frac{g\,e}{2\,m}$$

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$$\Delta E_{hf} \;\simeq\; 5.884 \times 10^{-6}\,\mathrm{eV}$$

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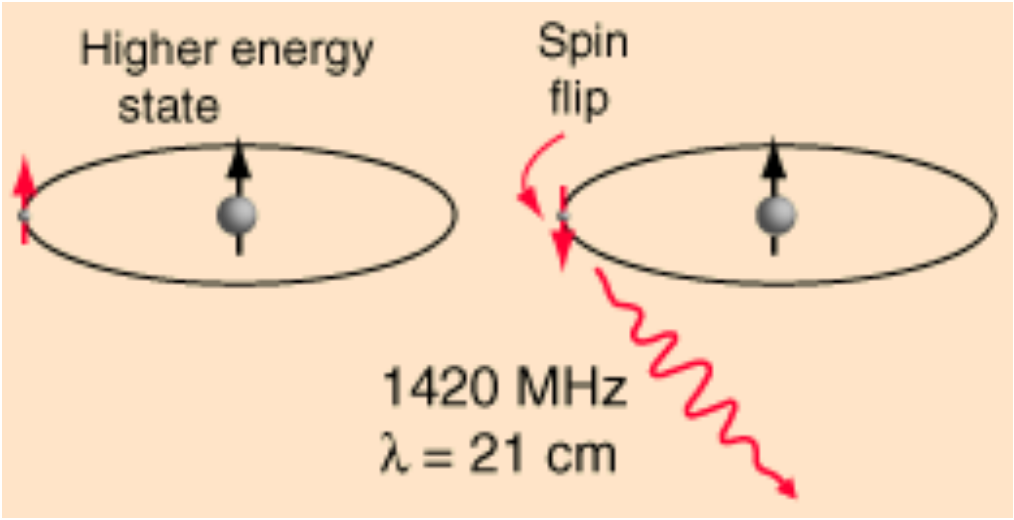
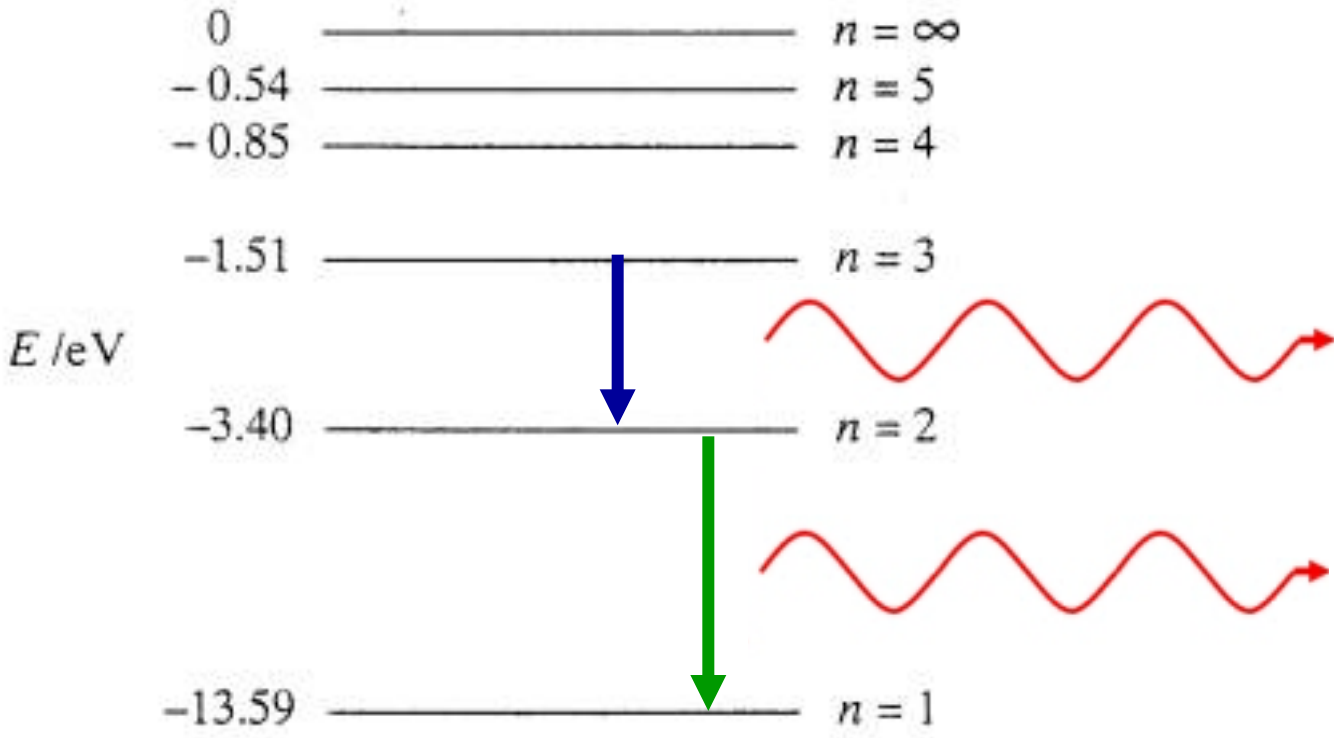
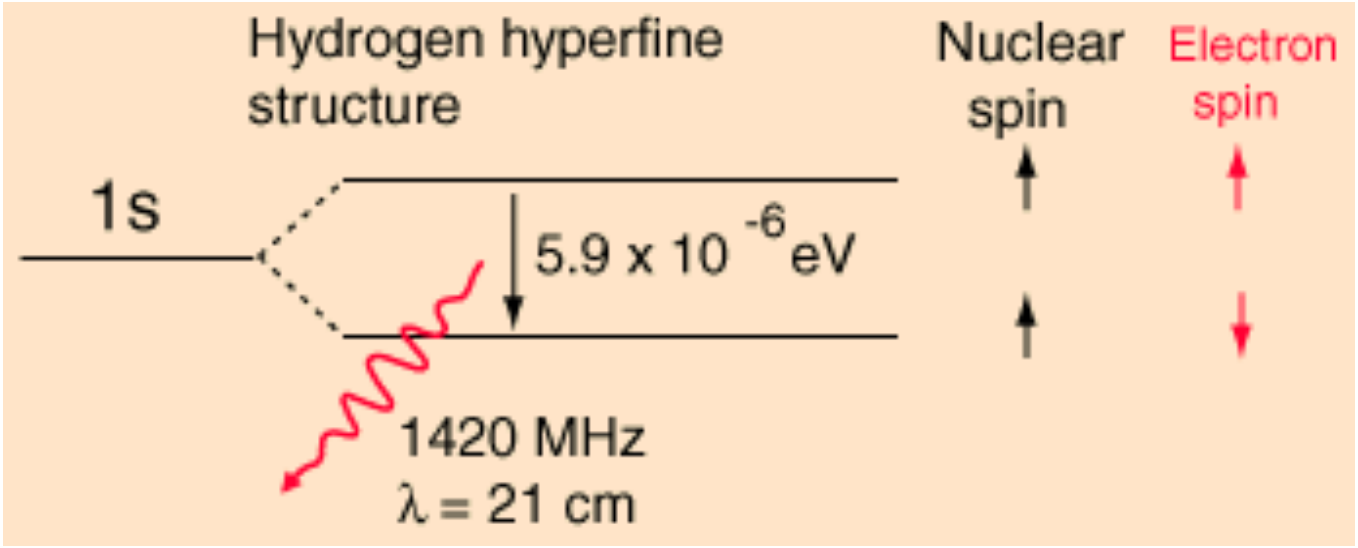
(micro-ondas : 12 cm)





E = 1.89 eV

E = 10.19 eV



O fóton hiperfino tem energia MUITO MENOR !

É um só !

Astronomia de 21 cm !

A "linha de 21 cm"

1944 Jan Oort : o Universo pode estar cheio de hidrogênio !

A "linha de 21 cm"

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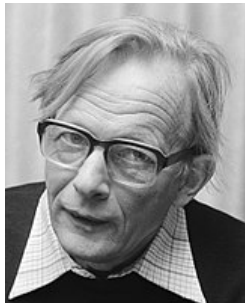
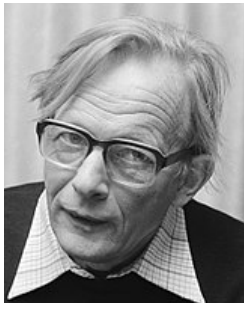


1945 H. Van de Hulst : H emite os fotons de 21 cm

Eles podem atravessar a poeira interestelar

A "linha de 21 cm"

1944 Jan Oort : o Universo pode estar cheio de hidrogênio !



1945 H. Van de Hulst : H emite os fotons de 21 cm
Eles podem atravessar a poeira interestelar

1951 H. Ewen e E. Purcell: medida
da linha de 21 centímetros

O Universo está cheiro de hidrogênio !

(1 átomo por cm^3)

Estes átomos emitem fotons, emitem a "linha 21 cm" !

Podemos ajustar o telescópio e tirar "fotos" da "luz" 21 cm :

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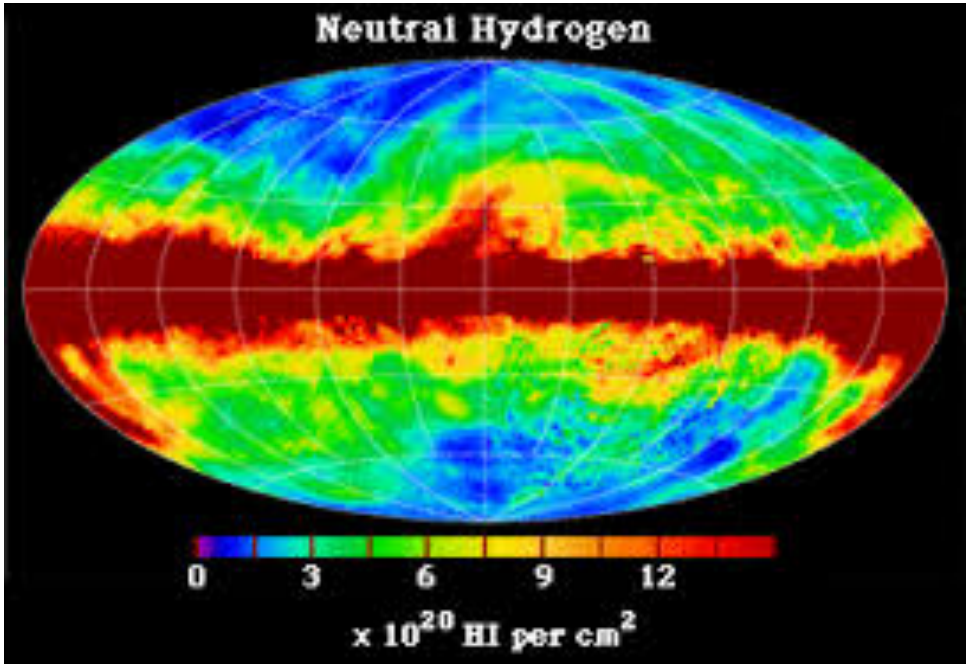
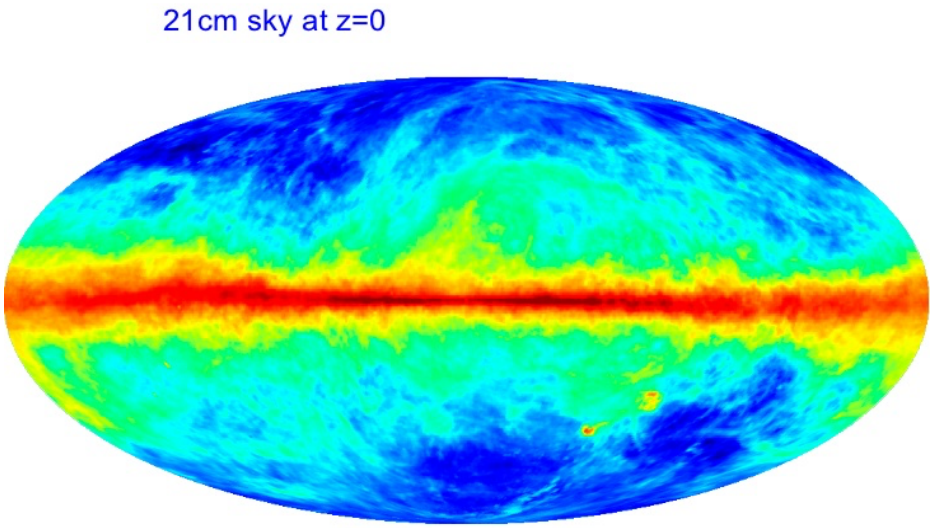
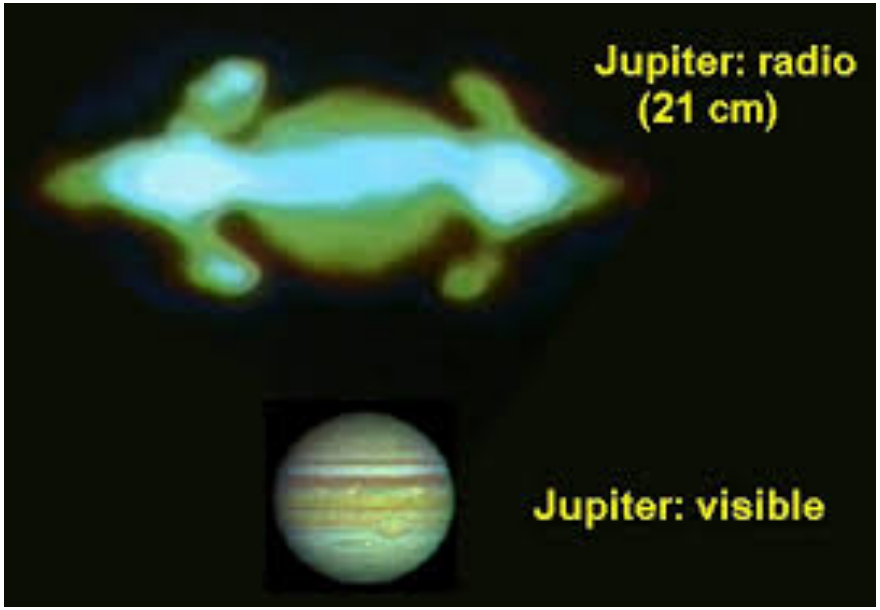
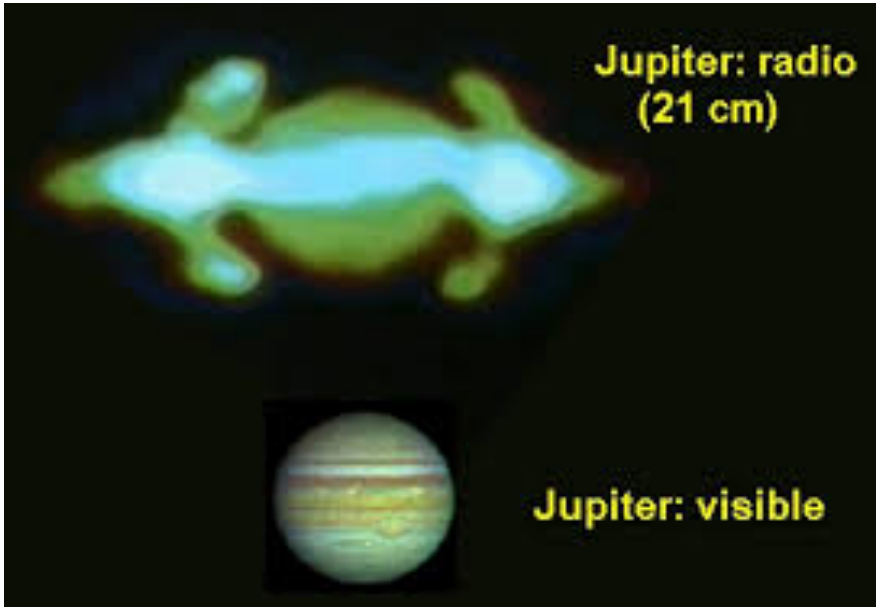




Kalberla et al 2005, from WMAP LAMBDA


Jupiter: visible


Jupiter: radio
(21 cm)
Jupiter: visible


Jupiter: radio
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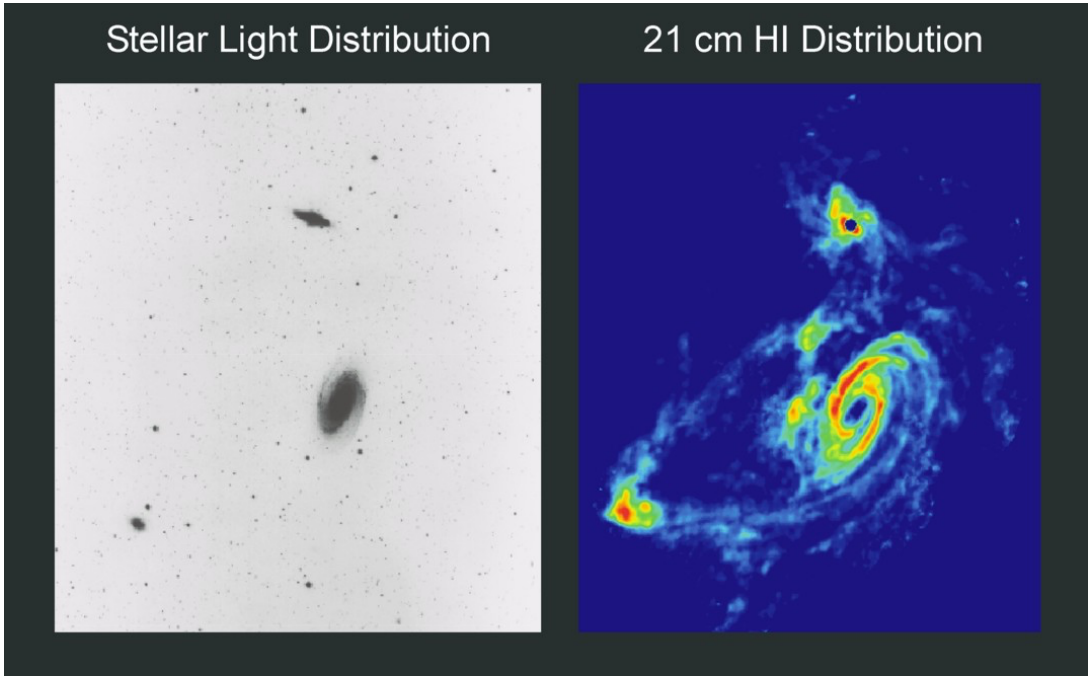
Jupiter: visible


Stellar Light Distribution
21 cm HI Distribution

Os fotons de 21 cm são muito penetrantes !

Atravessam o meio intergalático ! (IGM)

Trazem informações de partes remotas da galáxia !

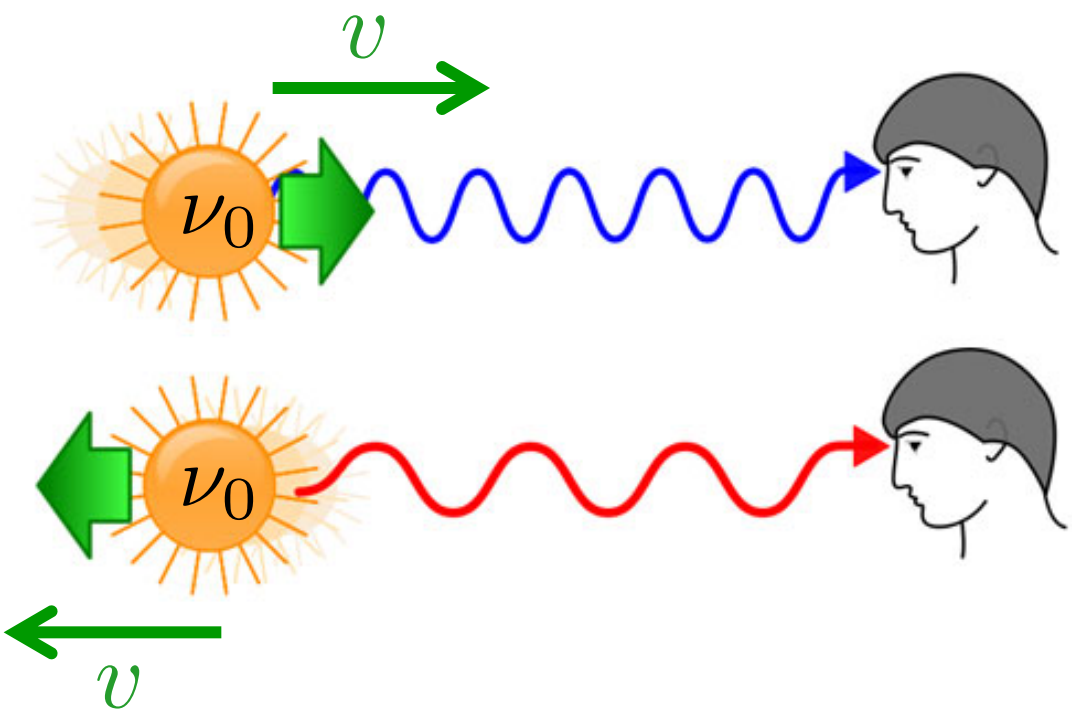
Os átomos de H se movem junto com as estrelas

A luz emitida sofre efeito Doppler !

$$\nu = \nu_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$$

frequência aumenta

a fonte se aproxima

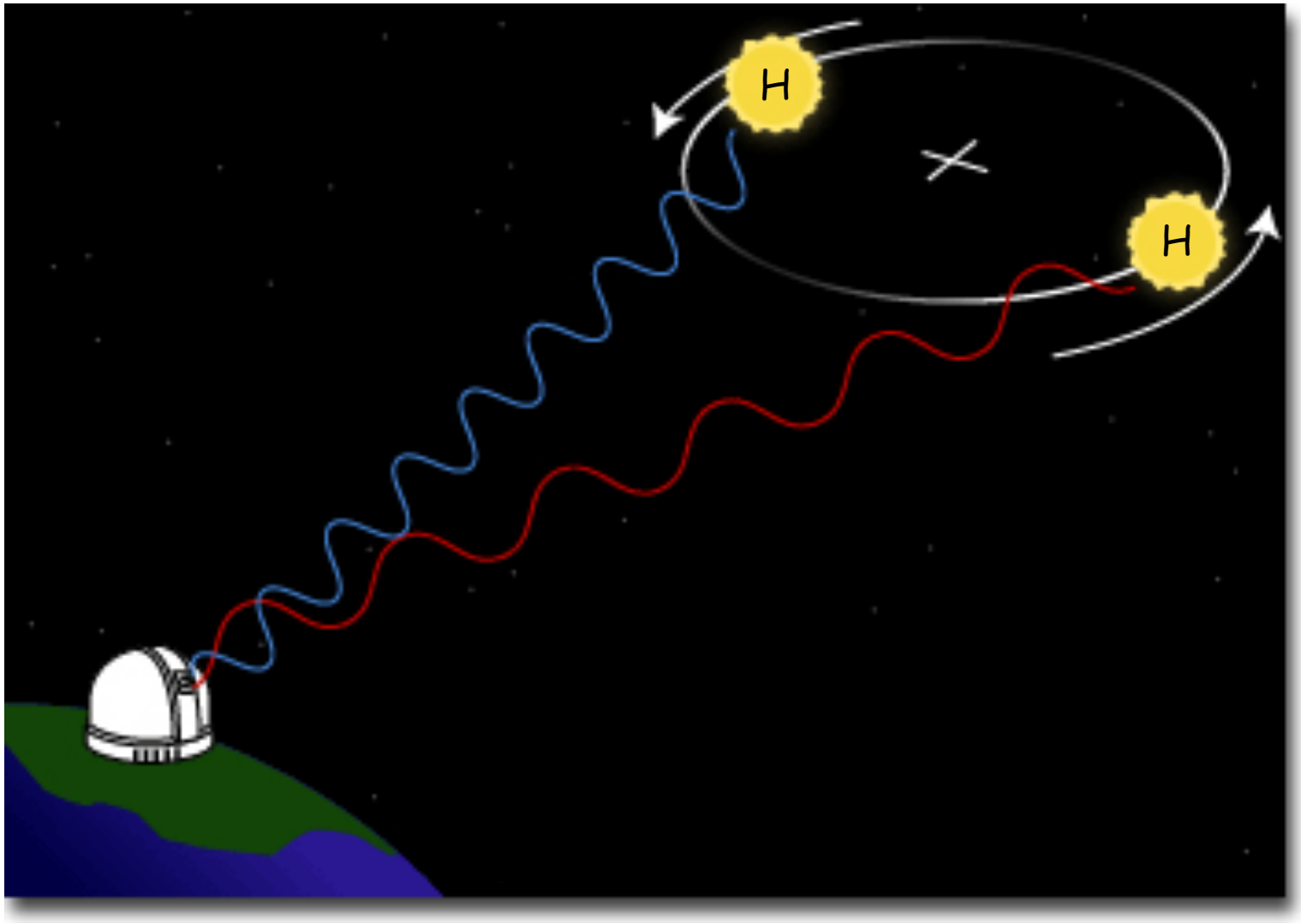


observador parado

a fonte se afasta

$$\nu = \nu_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$$

frequência diminui

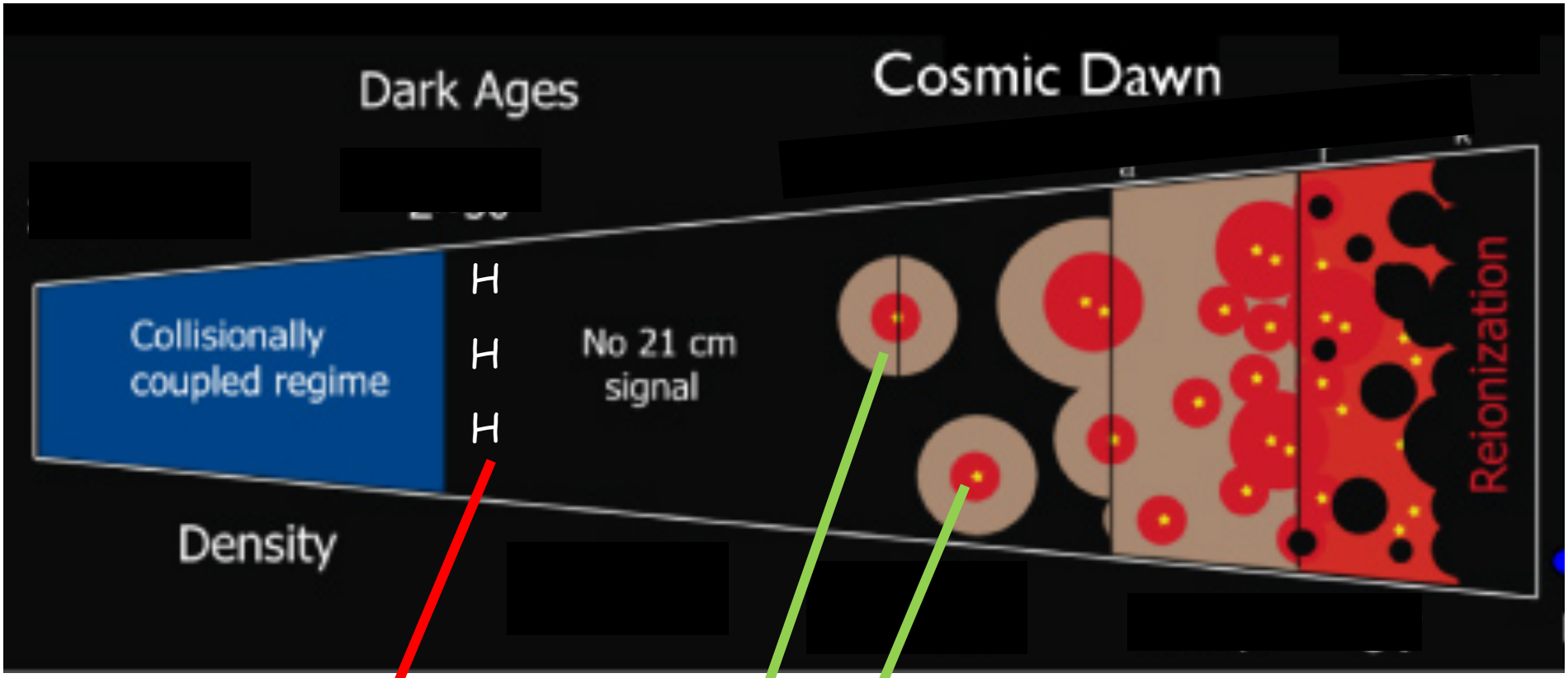


Medida precisa da rotação das galáxias !

Cosmologia de 21 cm !

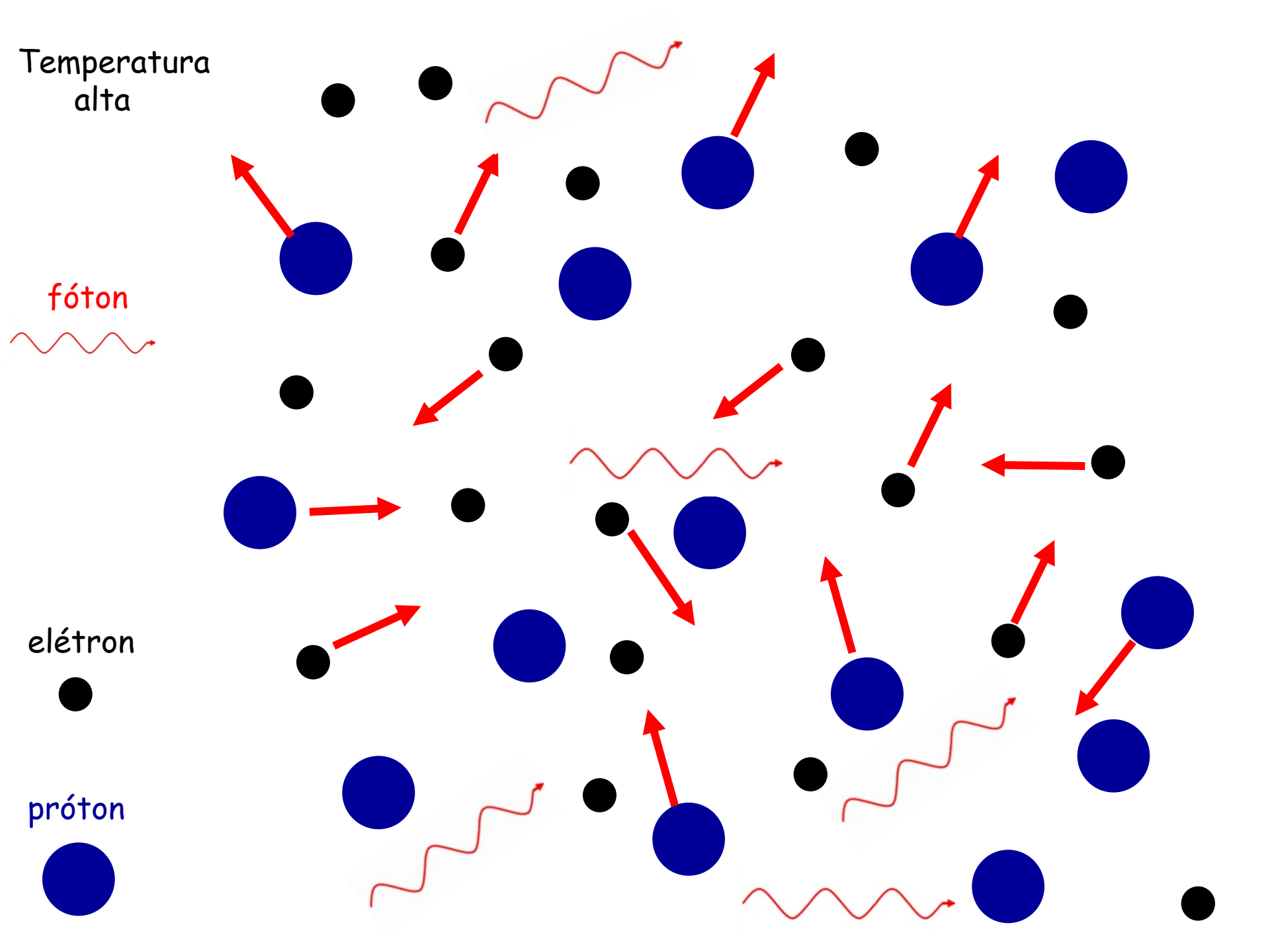
Cosmologia de 21 cm !

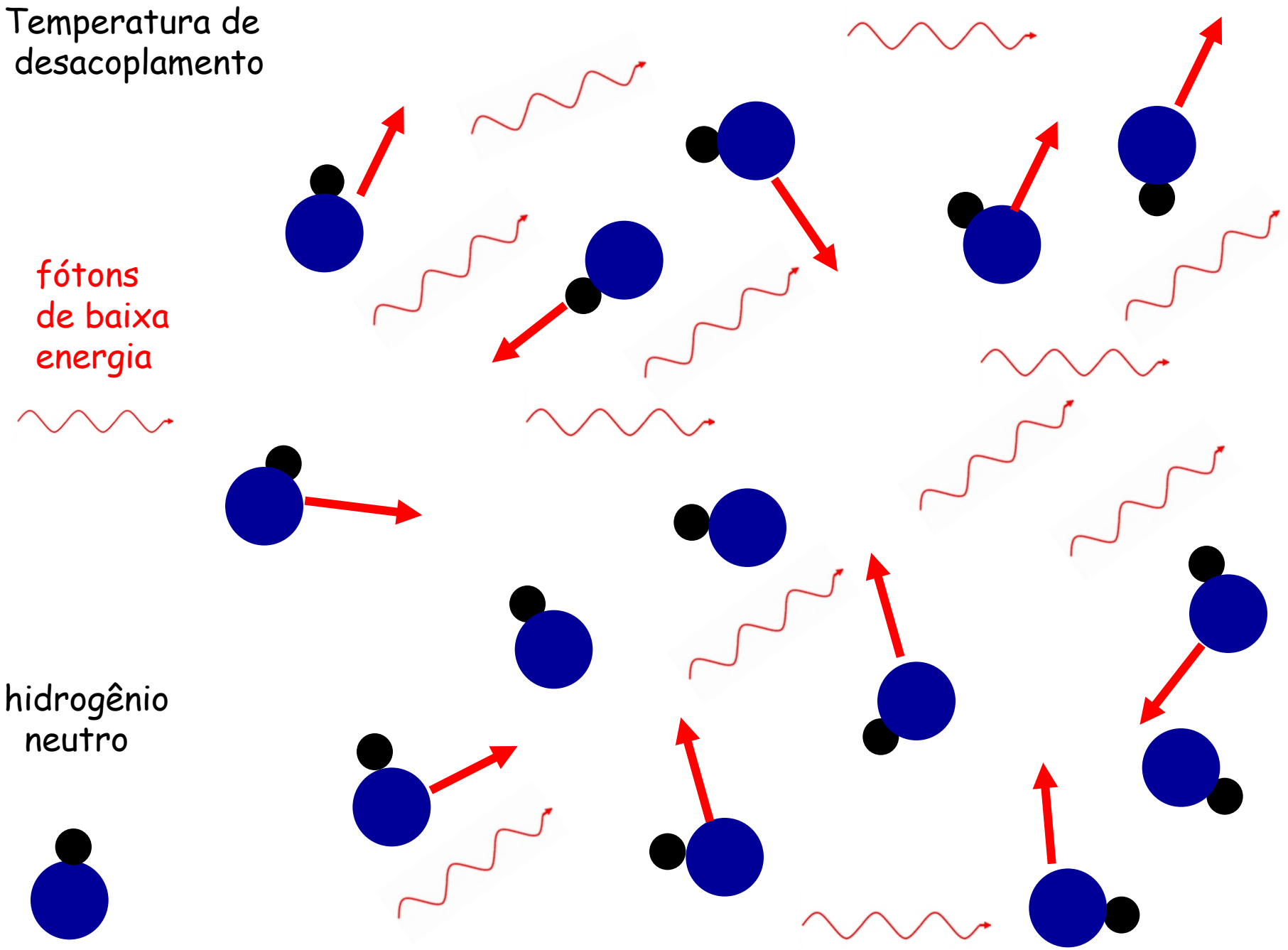
H foi produzido no Universo primordial



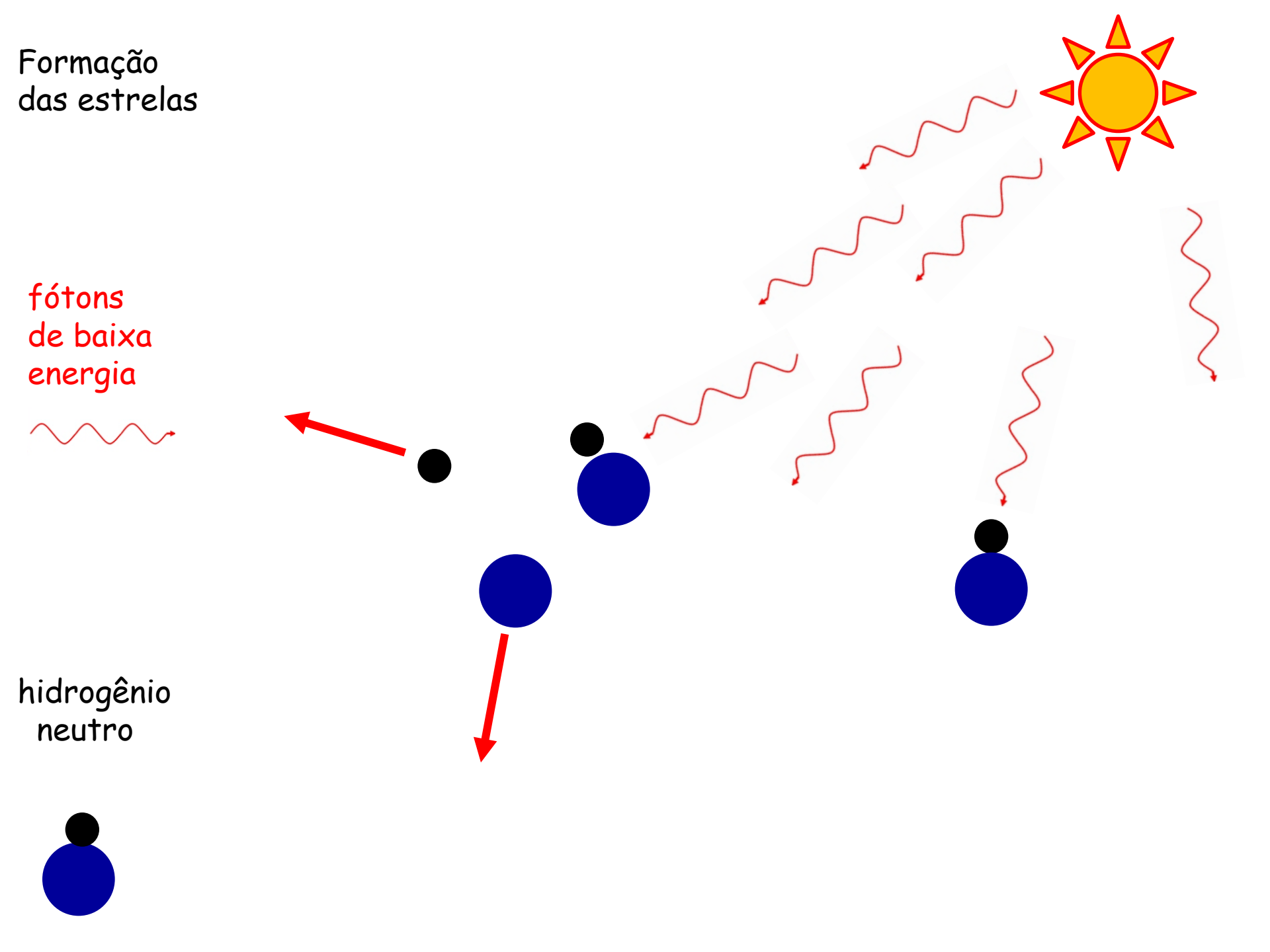
Formação de H

Formação das primeiras estrelas !

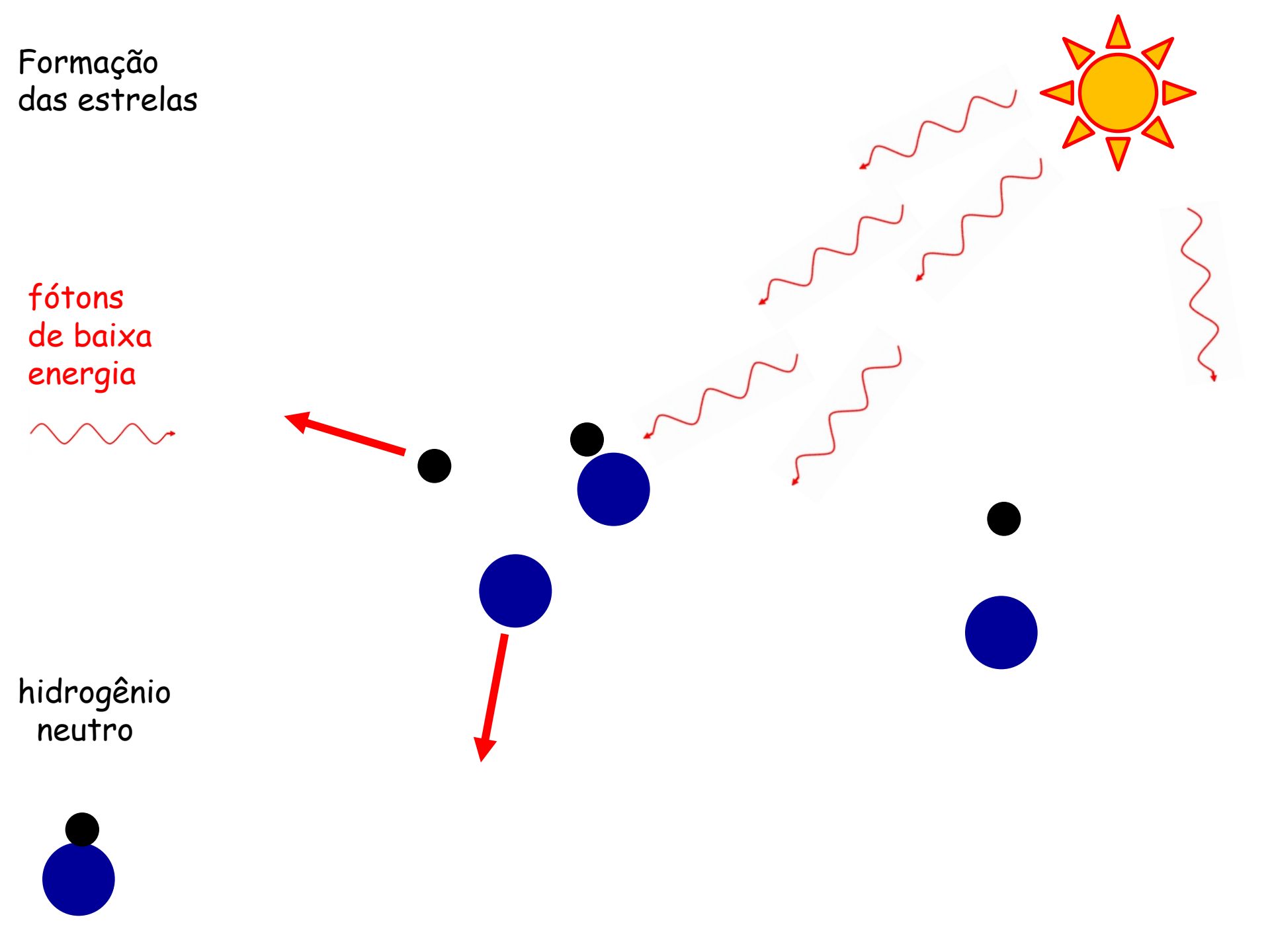

Temperatura alta
fóton
elétron
próton


Temperatura de desacoplamento
fótons de baixa energia
hidrogênio neutro

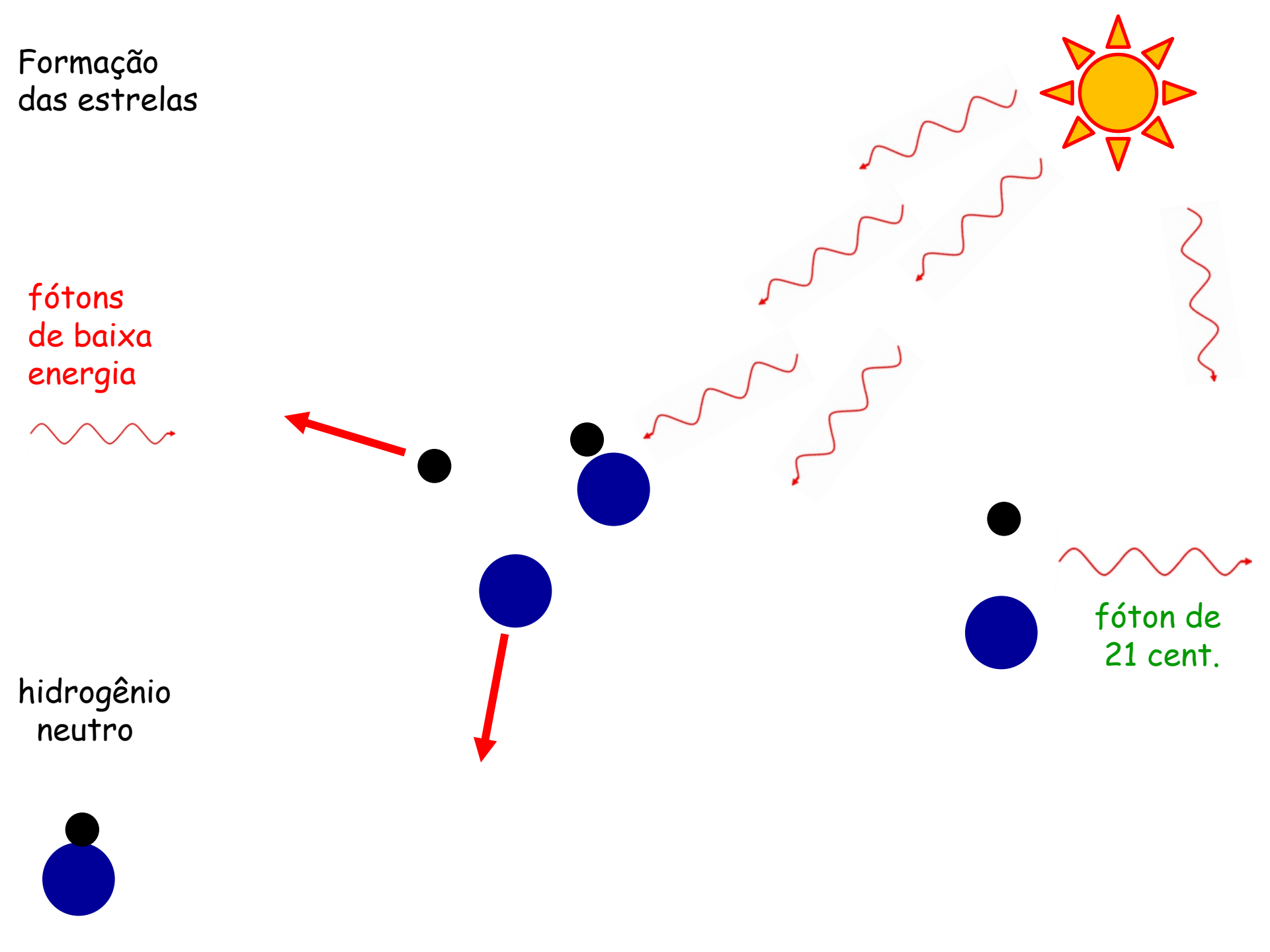
Formação
das estrelas



Formação
das estrelas



Formação
das estrelas



Formação
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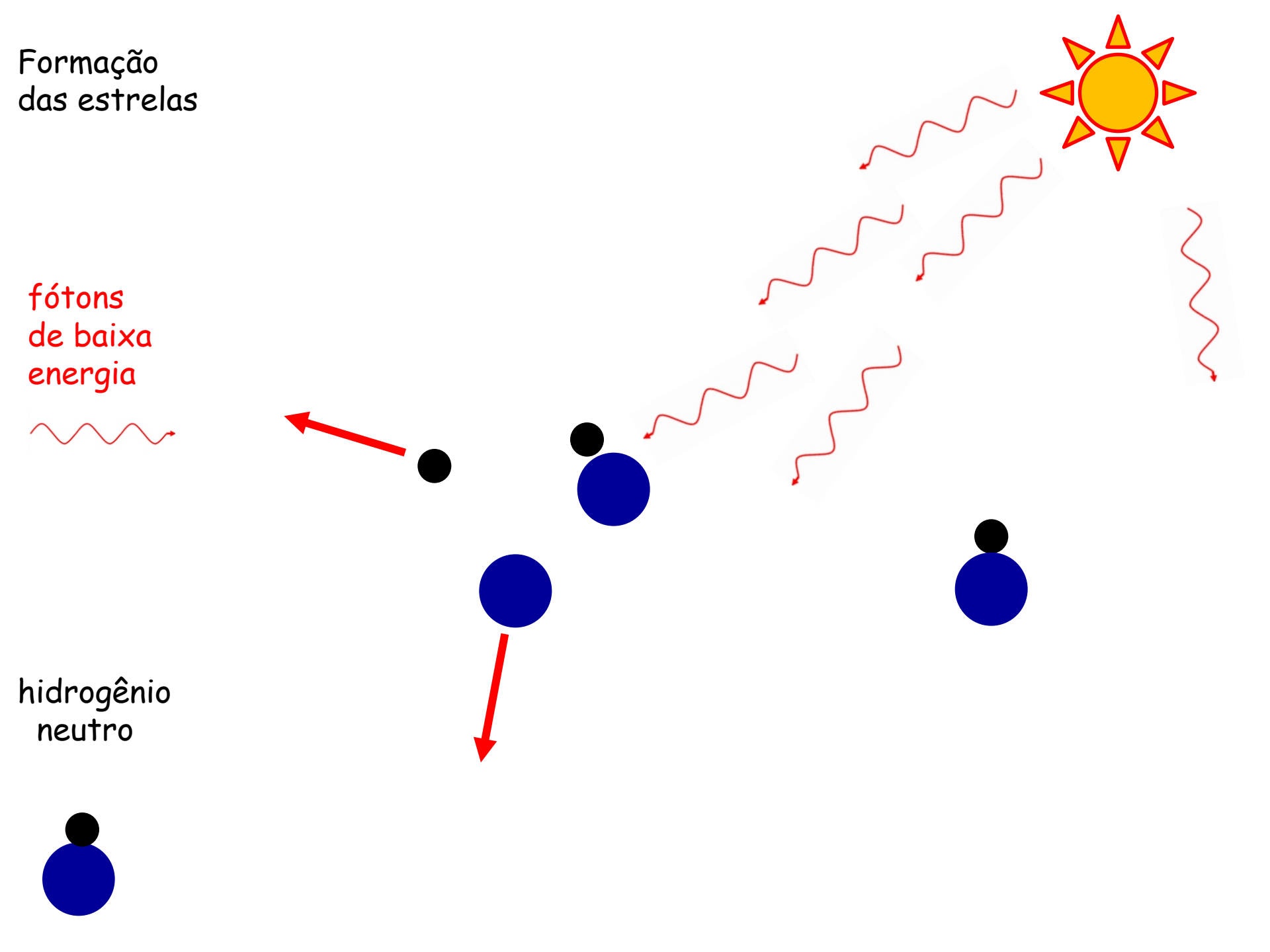
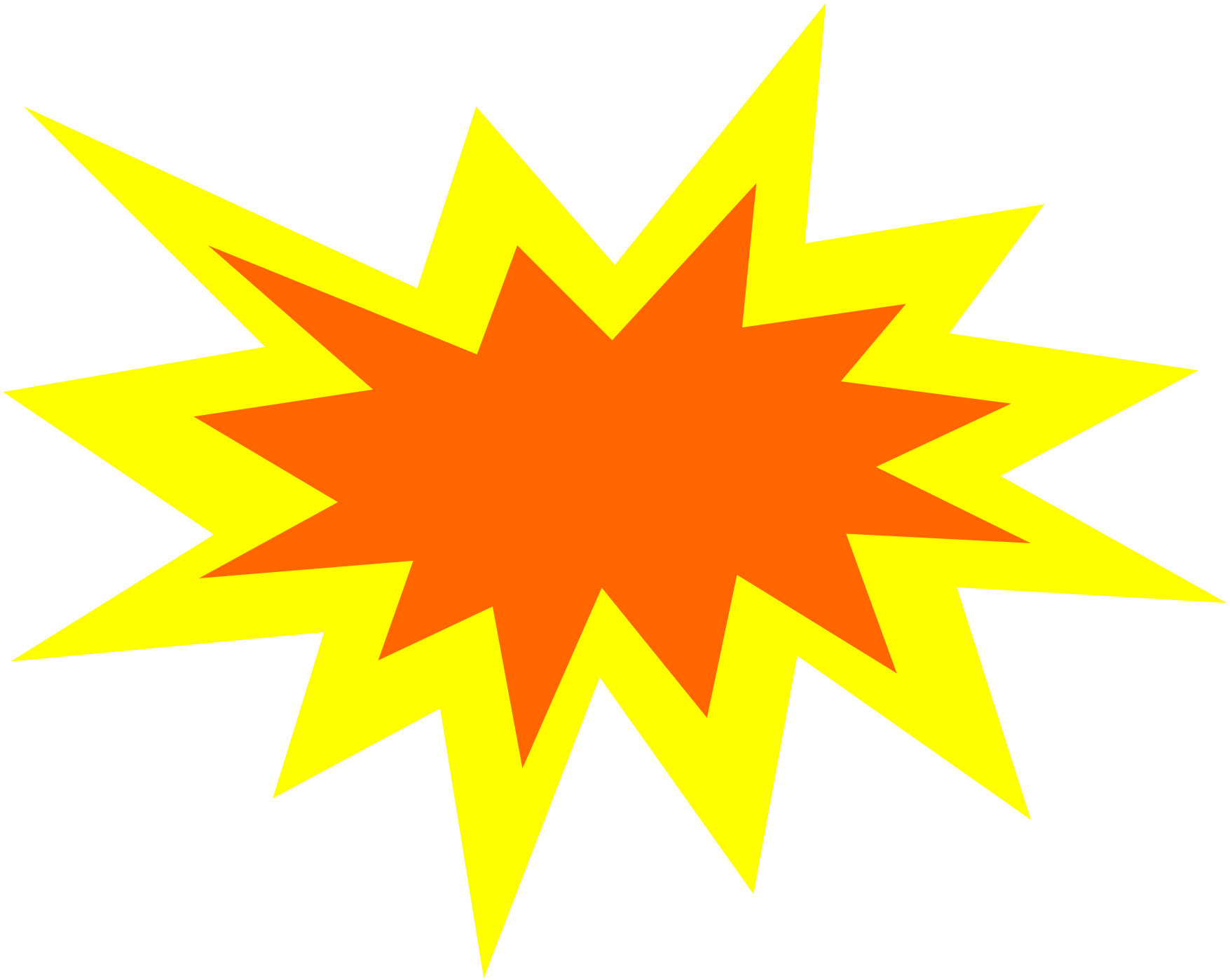


Conclusão

Spin útil em astronomia : rotação das galáxias

Spin útil em cosmologia : nascimento das estrelas

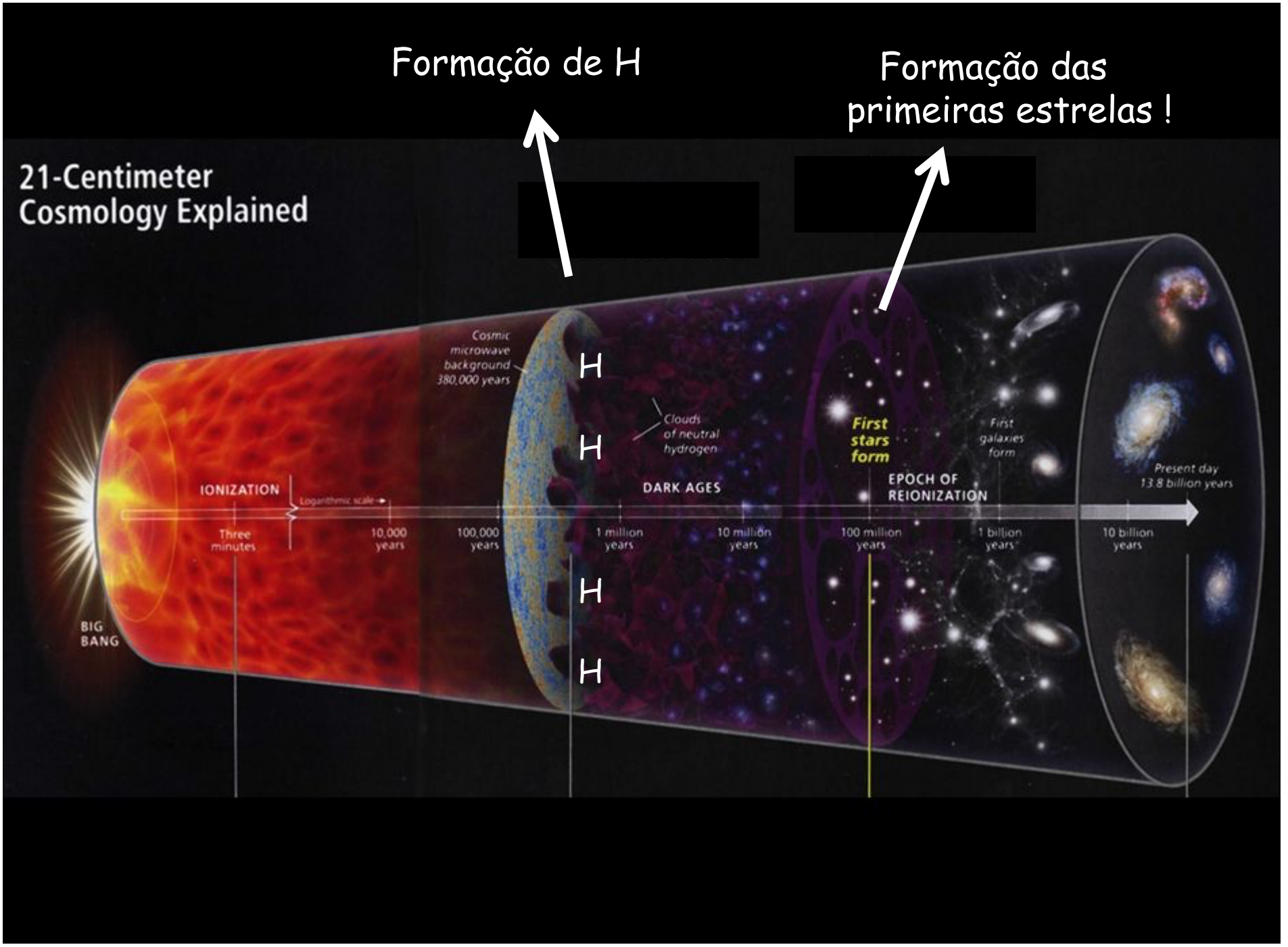


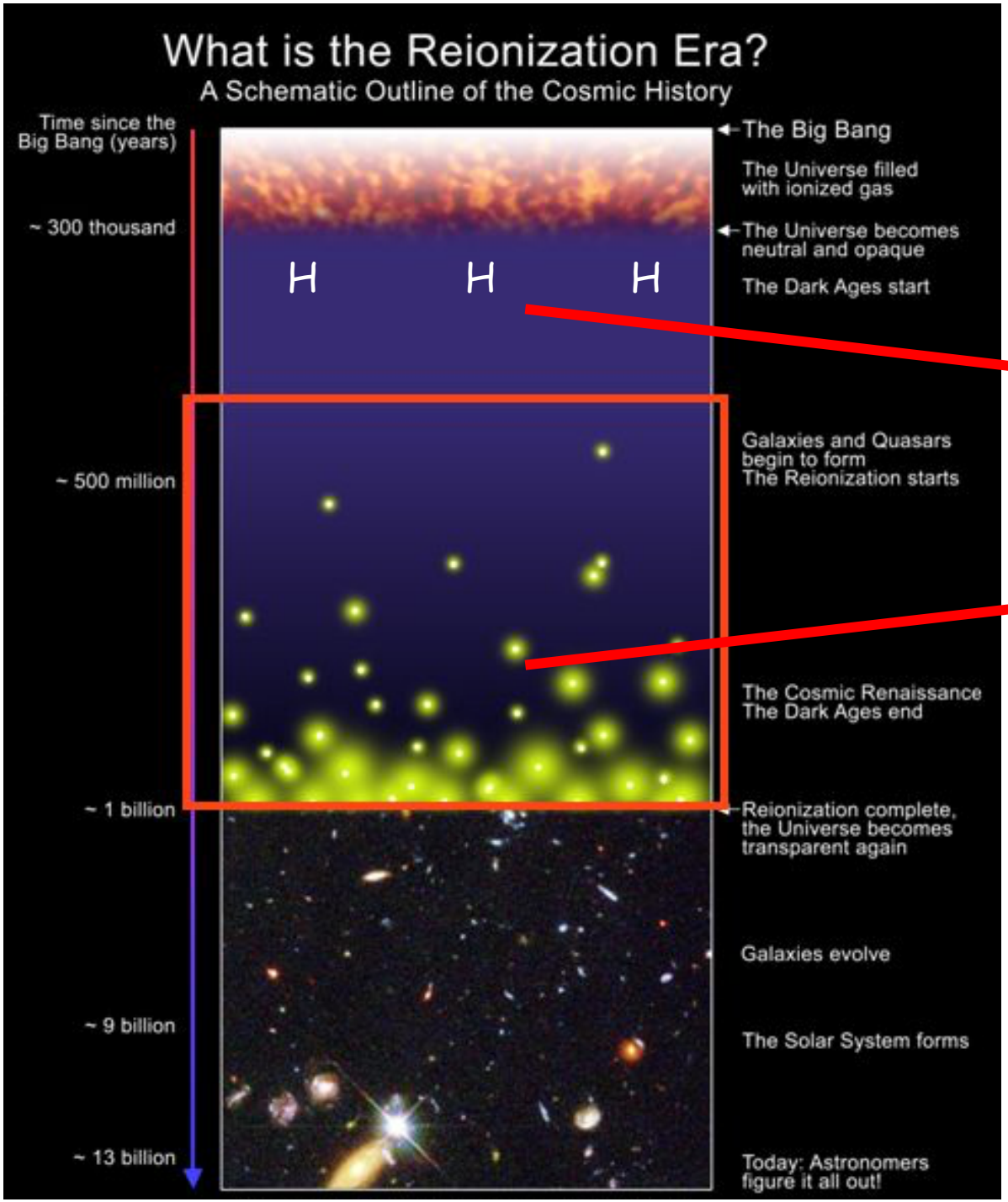
Beginnings

- In the '50s & '60s, Radio telescopes first discovered the "signature" line of Hydrogen at 21 cm wavelength ***between*** stars.
  - The hydrogen gas was found (in abundance) in the "empty" space between stars.
  - Note: Domestic microwave ovens use a signal of ~ 12 cm wavelength to cook food
- This gas was "dark" in optical wavelengths but "bright" in radio wavelengths.

Formação de H

Formação das primeiras estrelas !





Formação de H

Formação das primeiras estrelas !