

Prova da Propriedade ii). (Se $a > b$ e $c > 0$, então $ac > bc$).

(def.)

Se $a > b \Rightarrow (a - b) > 0$.

Usando 1.2.1 (iii) temos $(a - b) \cdot c > 0$ ou $(ac - bc) > 0$ e finalmente, pela definição, $ac > bc$.

1.3 VALOR ABSOLUTO

1.3.1 Definição. O valor absoluto de a , denotado por $|a|$, é definido como

$$|a| = a, \text{ se } a \geq 0$$

$$|a| = -a, \text{ se } a < 0.$$

1.3.2 Interpretação Geométrica. Geometricamente o valor absoluto de a , também chamado módulo de a , representa a distância entre a e 0. Escreve-se então $|a| = \sqrt{a^2}$.

1.3.3 Propriedades.

$$(i) \quad |x| < a \Leftrightarrow -a < x < a, \text{ onde } a > 0.$$

$$(ii) \quad |x| > a \Leftrightarrow x > a \text{ ou } x < -a, \text{ onde } a > 0.$$

$$(iii) \quad \text{Se } a, b \in \mathbb{R}, \text{ então } |a \cdot b| = |a| \cdot |b|.$$

$$(iv) \quad \text{Se } a, b \in \mathbb{R} \text{ e } b \neq 0, \text{ então } \left| \frac{a}{b} \right| = \frac{|a|}{|b|}.$$

$$(v) \quad (\text{Desigualdade triangular})$$

$$\text{Se } a, b \in \mathbb{R}, \text{ então } |a + b| \leq |a| + |b|.$$

$$(vi) \quad \text{Se } a, b \in \mathbb{R}, \text{ então } |a - b| \leq |a| + |b|.$$

$$(vii) \quad \text{Se } a, b \in \mathbb{R}, \text{ então } |a| - |b| \leq |a - b|.$$

Temos,

$$\begin{aligned} g'(x) &= \frac{x \cdot 0 - 1 \cdot 1}{x^2} \\ &= \frac{-1}{x^2}. \end{aligned}$$

4.10.12 Proposição. Se $f(x) = x^{-n}$ onde n é um inteiro positivo e $x \neq 0$, então $f'(x) = -n \cdot x^{-n-1}$.

Prova. Podemos escrever $f(x) = \frac{1}{x^n}$.

Aplicando a proposição 4.10.10, vem

$$\begin{aligned} f'(x) &= \frac{x^n \cdot 0 - 1 \cdot nx^{n-1}}{(x^n)^2} \\ &= \frac{-nx^{n-1}}{x^{2n}} \\ &= -n \cdot x^{n-1} \cdot x^{-2n} \\ &= -n x^{-n-1}. \end{aligned}$$

4.11 EXERCÍCIOS

Nos exercícios de 1 a 22, encontrar a derivada das funções dadas.

1. $f(r) = \pi r^2$

2. $f(x) = 3x^2 + 6x - 10$

3. $f(w) = aw^2 + b$

4. $f(x) = 14 - \frac{1}{2}x^{-3}$

5. $f(x) = (2x + 1)(3x^2 + 6)$

6. $f(x) = (7x - 1)(x + 4)$

7. $f(x) = (3x^5 - 1)(2 - x^4)$

8. $f(x) = \frac{2}{3}(5x - 3)^{-1}(5x + 3)$

9. $f(x) = (x - 1)(x + 1)$

10. $f(s) = (s^2 - 1)(3s - 1)(5s^3 + 2s)$

11. $f(x) = 7(ax^2 + bx + c)$

12. $f(u) = (4u^2 - a)(a - 2u)$

13. $f(x) = \frac{2x + 4}{3x - 1}$

14. $f(t) = \frac{t - 1}{t + 1}$

15. $f(t) = \frac{3t^2 + 5t - 1}{t - 1}$

16. $f(t) = \frac{2 - t^2}{t - 2}$

17. $f(x) = \frac{4 - x}{5 - x^2}$

18. $f(x) = \frac{5x + 7}{2x - 2}$

19. $f(x) = \frac{x + 1}{x + 2}(3x^2 + 6x)$

20. $f(t) = \frac{(t - a)^2}{t - b}$

21. $f(x) = \frac{3}{x^4} + \frac{5}{x^5}$

22. $f(x) = \frac{1}{2}x^4 + \frac{2}{x^6}$

23. Seja $p(x) = (x - a)(x - b)$, a e b constantes. Mostrar que se $a \neq b$ então $p(a) = p(b) = 0$, mas $p'(a) \neq 0$ e $p'(b) \neq 0$.

24. Dadas as funções $f(x) = x^2 + Ax$ e $g(x) = Bx$, determinar A e B de tal forma que

$$\begin{cases} f'(x) + g'(x) = 1 + 2x \\ f(x) - g(x) = x^2 \end{cases}$$

25. Dada a função $f(t) = 3t^3 - 4t + 1$, encontrar $f(0) - tf'(0)$.

26. Encontrar a equação da reta tangente à curva $y = \frac{2x + 1}{3x - 4}$ no ponto de abscissa $x = -1$.

27. Encontrar a equação da reta normal à curva $y = (3x^2 - 4x)^2$ no ponto de abscissa $x = 2$.

$$> (33) \quad y = \arg \operatorname{tgh} u \Rightarrow y' = \frac{u'}{1 - u^2}, \quad |u| < 1$$

$$> (34) \quad y = \arg \operatorname{cotgh} u \Rightarrow y' = \frac{u'}{1 - u^2}, \quad |u| > 1$$

$$> (35) \quad y = \arg \operatorname{sech} u \Rightarrow y' = \frac{-u'}{u \sqrt{1 - u^2}}, \quad 0 < u < 1$$

$$> (36) \quad y = \arg \operatorname{cosech} u \Rightarrow y' = \frac{-u'}{|u| \sqrt{1 + u^2}}, \quad u \neq 0.$$

4.15 EXERCÍCIOS

Nos exercícios de 1 a 75 calcular a derivada.

$$1. \quad f(x) = 10(3x^2 + 7x - 3)^{10}$$

$$2. \quad f(x) = \frac{1}{3}(2x^5 + 6x^{-3})^5$$

$$3. \quad f(x) = \frac{1}{a}(bx^2 + ax)^3$$

$$4. \quad f(x) = (3x^2 + 6x)^{10} - \frac{1}{x^2}$$

$$5. \quad f(t) = (7t^2 + 6t)^7 (3t - 1)^4$$

$$6. \quad f(x) = (5x - 2)^6 (3x - 1)^3$$

$$7. \quad f(t) = \left(\frac{7t + 1}{2t^2 + 3} \right)^3$$

$$8. \quad f(x) = (2x - 5)^4 + \frac{1}{x + 1} - \sqrt{x}$$

$$9. \quad f(x) = \sqrt[3]{(3x^2 + 6x - 2)^2}$$

$$10. \quad f(t) = (4t^2 - 5t + 2)^{-1/3}$$

$$11. \quad f(x) = \frac{2x}{\sqrt{3x - 1}}$$

$$12. \quad f(x) = \frac{7x^2}{2 \sqrt[5]{3x + 1}} + \sqrt{3x + 1}$$

$$13. \quad f(t) = \sqrt{\frac{2t + 1}{t - 1}}$$

$$14. \quad f(x) = 2e^{3x^2 + 6x + 7}$$

15. $f(x) = \frac{1}{3} e^{3-x}$

16. $f(x) = e^{\sqrt{x}}$

17. $f(x) = 2^{3x^2 + 6x}$

18. $f(x) = \left(\frac{1}{2}\right)^{\ln 2x}$

19. $f(s) = (7s^2 + 6s - 1)^3 + 2e^{-3s}$

20. $f(t) = \frac{e^{-t^2} + 1}{t}$

21. $f(t) = e^{t/2} (t^2 + 5t)$

22. $f(t) = \frac{\sqrt{e^t - 1}}{\sqrt{e^t + 1}}$

23. $f(x) = \log_2(2x + 4)$

24. $f(x) = \frac{1}{a} (bx^2 + c) - \ln x$

25. $f(s) = \log_3 \sqrt{s+1}$

26. $f(x) = \frac{1}{2} \ln(7x^2 - 4)$

27. $f(x) = \ln \left(\frac{1}{x} + \frac{1}{x^2} \right)$

28. $f(x) = \ln \left(\frac{1+x}{1-x} \right)$

29. $f(x) = \frac{a^{3x}}{b^{3x^2 - 6x}}$

30. $f(t) = \left(\frac{a}{b} \right)^{\sqrt{t}}$

31. $f(t) = (2t + 1)^{t^2 - 1}$

32. $f(x) = (e^{x^2} + 4)^{\sqrt{x}}$

33. $f(s) = \frac{1}{2} (a + bs)^{\ln(a + bs)}$

34. $f(x) = \operatorname{sen}(2x + 4)$

35. $f(u) = \cos(\pi/2 - u)$

36. $f(\theta) = 2 \cos(2\theta^2 - 3\theta + 1)$

37. $f(\theta) = 2 \cos \theta^2 \cdot \operatorname{sen} 2\theta$

38. $f(\alpha) = \frac{1 + \cos 2\alpha}{2}$

39. $f(x) = \operatorname{sen}^3(3x^2 + 6x)$

40. $f(\theta) = \operatorname{sen}^2 \theta + \cos^2 \theta$

41. $f(x) = 3 \operatorname{tg}(2x + 1) + \sqrt{x}$

42. $f(s) = \operatorname{cotg}^4(2s - 3)^2$

43. $f(x) = \frac{3 \sec^2 x}{x}$

44. $f(x) = \left(\frac{1}{\operatorname{sen} x} \right)^2$

45. $f(x) = e^{2x} \cos 3x$

46. $f(x) = \frac{\operatorname{sen}(x + 1)}{e^x}$

47. $f(\theta) = -\operatorname{cosec}^2 \theta^3$

48. $f(x) = \operatorname{sen}^2(x/2) \cos^2(x/2)$

49. $f(x) = a \sqrt{\cos bx}$

50. $f(t) = \ln \cos^2 t$

51. $f(u) = (u \operatorname{tg} u)^2$

52. $f(x) = \log_2(3x - \cos 2x)$

53. $f(\theta) = a \operatorname{cotg} \theta, a > 0$

54. $f(t) = e^{2 \cos 2t}$

55. $f(x) = (\operatorname{arc sen} x)^2$

56. $f(x) = \operatorname{arc cos} \frac{2x}{3}$

57. $f(t) = t \operatorname{arc cos} 3t$

58. $f(s) = \frac{\operatorname{arc sen} s/2}{s + 1}$

59. $f(t) = \operatorname{arc cos}(\operatorname{sen} t)$

60. $f(x) = \operatorname{arc tg} \frac{1}{1 - x^2}$

61. $f(x) = \operatorname{arc sec} \sqrt{x}$

62. $f(x) = \operatorname{senh}(2x - 1)$

63. $f(t) = t^2 \operatorname{arc cosec}(2t + 3)$

64. $f(t) = \ln [\cosh(t^2 - 1)]$

65. $f(x) = \frac{\ln(\operatorname{sen} hx)}{x}$

66. $f(t) = \operatorname{tgh}(4t^2 - 3)^2$

67. $f(t) = [\operatorname{cotgh}(t + 1)^2]^{1/2}$

68. $f(x) = \operatorname{sech}[\ln x]$

69. $f(x) = \left[\operatorname{cosech} \frac{(3x+1)}{x} \right]^3$

70. $f(x) = (\arg \operatorname{senh} x)^2$

71. $f(x) = x \arg \cosh x - \sqrt{x^2 - 1}$

72. $f(x) = \arg \operatorname{tgh} \frac{1}{2} x^2$

73. $f(x) = x \arg \operatorname{cotgh} x^2$

74. $f(x) = (x+1) \arg \operatorname{sech} 2x$

75. $f(x) = \frac{1}{2} [\arg \cosh x^2]^2$

76. Encontrar $f'(x)$.

a) $f(x) = \begin{cases} 1-x & , x \leq 0 \\ e^{-x} & , x > 0 \end{cases}$

b) $f(x) = \ln |3-4x|$

c) $f(x) = e^{|2x-1|}$

77. Calcular $f'(0)$, se $f(x) = e^{-x} \cos 3x$.

78. Calcular $f'(1)$, se $f(x) = \ln(1+x) + \operatorname{arc sen} x/2$.

79. Dada $f(x) = e^{-x}$, calcular $f(0) + xf'(0)$.

80. Dada $f(x) = 1 + \cos x$, mostrar que $f(x)$ é par e $f'(x)$ é ímpar.

81. Dada a $f(x) = \operatorname{sen} 2x \cos 3x$, mostrar que $f(x)$ é ímpar e $f'(x)$ é par.

82. Dada a função $f(x) = \frac{1}{2} \operatorname{sen} 2x$, calcular $f'(x)$ e verificar que f e f' são periódicas de mesmo período.

83. Seja $f(x)$ derivável e periódica de período T . Mostrar que f' também é periódica de período T .

$$e) \frac{4x^3 - 8x^2 + 4x - 1}{(x - 1)^2} \quad f) \frac{-1 - 8x(x - 1)^2}{(x - 1)^2} \quad g) \frac{-4x}{x - 1}$$

11. a) $(3/4, +\infty)$ b) $(-\infty, 3/4)$

SEÇÃO 4.9

1. $f'(3^+) = 2 ; f'(3^-) = -2$
2. $f'(1^+) = 2 ; f'(1^-) = 1$
3. $f'(-2^+) = 2 ; f'(-2^-) = -2$
4. $f'(-1^+) = 0 ; f'(-1^-) = 2 ; f'(1^+) = -2 ; f'(1^-) = 0$
5. $f'(-2^+) = 0 ; f'(-2^-) = 4 ; f'(2^+) = 2 ; f'(2^-) = 0$

6. b) é contínua c) 2 ; -2 ; 2 ; -2 d) $f'(x) = \begin{cases} 2x, & \text{se } |x| < 1, \\ -2x, & \text{se } |x| > 1, \end{cases} D = \mathbf{R} - \{-1, 1\}$

SEÇÃO 4.11

1. $2\pi r$
2. $6x + 6$
3. $2aw$
4. $\frac{3}{2x^4}$
5. $18x^2 + 6x + 12$
6. $14x + 27$
7. $-27x^8 + 30x^4 + 4x^3$
8. $\frac{-20}{(5x - 3)^2}$
9. $2x$

10. $(s^2 - 1)(3s - 1)(15s^2 + 2) + 3(s^2 - 1)(5s^3 + 2s) + 2s(3s - 1)(5s^3 + 2s)$

11. $7(2ax + b)$

12. $-24u^2 + 8au + 2a$

13. $\frac{-14}{(3x - 1)^2}$

14. $\frac{2}{(t + 1)^2}$

15. $\frac{3t^2 - 6t - 4}{(t - 1)^2}$

16. $\frac{-t^2 + 4t - 2}{t^2 - 4t + 4}$

17. $\frac{-x^2 + 8x - 5}{(5 - x^2)^2}$

18. $\frac{-24}{(2x - 2)^2}$

19. $\frac{6x^3 + 27x^2 + 36x + 12}{(x + 2)^2}$

20. $\frac{t^2 - 2bt - a^2 + 2ab}{(t - b)^2}$

21. $\frac{-12}{x^5} - \frac{25}{x^6}$

22. $2x^3 - \frac{12}{x^7}$

24. $A = B = 1/2$

25. $4t + 1$

26. $11x + 49y + 4 = 0$

27. $x + 64y - 1026 = 0$

28. $x - y - 2\sqrt{2} + 2 = 0 ; x - y + 2 + 2\sqrt{2} = 0$

29. $(2, 2/3) ; (1, 5/6)$

30. $a = 3 ; b = 2$

SEÇÃO 4.15

1. $100 (3x^2 + 7x - 3)^9 (6x + 7)$

2. $\frac{10}{3} (2x^5 + 6x^{-3})^4 (5x^4 - 9x^{-4})$

3. $\frac{3}{a} (bx^2 + ax)^2 (2bx + a)$

4. $60(3x^2 + 6x)^9(x + 1) + \frac{2}{x^3}$

5. $(7t^2 + 6t)^6 (3t - 1)^3 [12(7t^2 + 6t) + 7(3t - 1)(14t + 6)]$

6. $(5x - 2)^5 (3x - 1)^2 (135x - 48)$

7. $\frac{3(7t + 1)^2 (-14t^2 - 4t + 21)}{(2t^2 + 3)^4}$

8. $8(2x - 5)^3 - \frac{1}{(x + 1)^2} - \frac{1}{2\sqrt{x}}$

9. $\frac{4(x + 1)}{\sqrt[3]{3x^2 + 6x - 2}}$

10. $-\frac{1}{3} (4t^2 - 5t + 2)^{-4/3} (8t - 5)$

11. $\frac{3x - 2}{(3x - 1)\sqrt{3x - 1}}$

12. $-\frac{21}{10} x^2 (3x + 1)^{-6/5} + 7x (3x + 1)^{-1/5} + \frac{3}{2} (3x + 1)^{-1/2}$

13. $\frac{-3}{2(t - 1)^{3/2} (2t + 1)^{1/2}}$

14. $12e^{3x^2 + 6x + 7} (x + 1)$

15. $-\frac{1}{3} e^{3-x}$

16. $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

17. $2^{3x^2 + 6x} 6(x + 1) \ln 2$

18. $\frac{2^{\ln 2x} \ln 2}{x}$

19. $6 [(7s^2 + 6s - 1)^2 (7s + 3) - e^{-3s}]$

20. $\frac{-2t^2 e^{-t^2} - e^{-t^2} - 1}{t^2}$

21. $e^{t/2} (1/2t^2 + 9/2t + 5)$

22. $\sqrt{\frac{e^t + 1}{e^t - 1}} \cdot \frac{e^t}{(e^t + 1)^2}$

23. $\frac{2}{2x + 4} \log_2 e$

24. $\frac{2bx^2 - a}{ax}$

25. $\frac{\log_3 e}{2(s + 1)}$

26. $\frac{7x}{7x^2 - 4}$

27. $\frac{-x - 2}{x(x + 1)}$

28. $\frac{2}{1 - x^2}$

29. $\frac{3(\ln a) a^{3x} - a^{3x} (6x - 6) \ln b}{b^{3x^2 - 6x}}$

30. $\left(\frac{a}{b}\right)^{\sqrt{t}} \ln\left(\frac{a}{b}\right) \cdot \frac{1}{2\sqrt{t}}$

31. $2t(2t + 1)^{t^2 - 1} \ln(2t + 1) + 2(2t + 1)^{t^2 - 2} (t^2 - 1)$

32. $(e^{x^2} + 4)^{\sqrt{x}} \ln(e^{x^2} + 4) \frac{1}{2\sqrt{x}} + 2x\sqrt{x}(e^{x^2} + 4)^{\sqrt{x} - 1} e^{x^2}$

33. $\frac{b(a + bs)^{\ln(a + bs)} \ln(a + bs)}{a + bs}$

34. $2 \cos(2x + 4)$

35. $\sin\left(\frac{\pi}{2} - u\right)$

36. $-2 \sin(2\theta^2 - 3\theta + 1)(4\theta - 3)$

37. $4 \cos\theta^2 \cos 2\theta - 4\theta \sin 2\theta \sin\theta^2$

38. $-\sin 2\alpha$

39. $3 \sin^2(3x^2 + 6x) \cos(3x^2 + 6x)(6x + 6)$

40. 0

41. $6 \sec^2(2x + 1) + \frac{1}{2\sqrt{x}}$

42. $-16(2s - 3) \cotg^3(2s - 3)^2 \operatorname{cosec}^2(2s - 3)^2$

43. $\frac{6x \sec^2 x \operatorname{tg} x - 3 \sec^2 x}{x^2}$

44. $\frac{-2 \cos x}{\operatorname{sen}^3 x}$

45. $e^{2x}(2 \cos 3x - 3 \sin 3x)$

46. $\frac{\cos(x + 1) - \operatorname{sen}(x + 1)}{e^x}$

47. $6 \theta^2 \operatorname{cosec}^2 \theta^3 \cdot \cot \theta^3$

48. $-\sin^3 \frac{x}{2} \cos \frac{x}{2} + \cos^3 \frac{x}{2} \sin \frac{x}{2}$

49. $\frac{-ab \sin bx}{2\sqrt{\cos bx}}$

50. $-2 \tan t$

51. $2u^2 \sec^2 u \tan u + 2u \tan^2 u$

52. $\frac{3 + 2 \sin 2x}{3x - \cos 2x} \log_2 e$

53. $-a^{\cot \theta} \ln a \operatorname{cosec}^2 \theta$

54. $-4 \sin 2t e^{2\cos 2t}$

55. $\frac{2 \operatorname{arc sen} x}{\sqrt{1-x^2}}$

56. $\frac{-2}{\sqrt{9-4x^2}}$

57. $\frac{-3t}{\sqrt{1-9t^2}} + \operatorname{arc cos} 3t$

58. $\frac{1}{(s+1)^2} \left(\frac{s+1}{\sqrt{4-s^2}} - \operatorname{arc sen} \frac{s}{2} \right)$

59. -1

60. $\frac{2x}{x^4 - 2x^2 + 2}$

61. $\frac{1}{2x \sqrt{x-1}}$

62. $2 \cosh (2x-1)$

63. $\frac{-2t^2}{|2t+3| \sqrt{(2t+3)^2-1}} + 2t \operatorname{arc cosec} (2t+3)$

64. $2t \operatorname{tgh} (t^2-1)$

65. $\frac{x \operatorname{cotgh} x - \ln (\operatorname{senh} x)}{x^2}$

66. $16t (4t^2-3) \operatorname{sech}^2 (4t^2-3)^2$

67. $\frac{-(t+1) \operatorname{cosech}^2 (t+1)^2}{\sqrt{\operatorname{cotgh} (t+1)^2}}$

68. $\frac{-\operatorname{sech}(\ln x) \operatorname{tgh}(\ln x)}{x}$

69. $\frac{3}{x^2} \left(\operatorname{cosech} \frac{3x+1}{x} \right)^3 \operatorname{cotgh} \left(\frac{3x+1}{x} \right)$

70. $\frac{2 \operatorname{arg} \operatorname{senh} x}{\sqrt{x^2+1}}$

71. $\operatorname{arg} \cosh x$

72. $\frac{4x}{4-x^4}$

73. $\frac{2x^2}{1-x^4} + \arg \operatorname{cotgh} x^2$

74. $\frac{-(x+1)}{x\sqrt{1-4x^2}} + \arg \operatorname{sech} 2x$

75. $\frac{2x \arg \cosh x^2}{\sqrt{x^4-1}}$

76. a) $f'(x) = \begin{cases} -1 & , x \leq 0 \\ -e^{-x} & , x > 0 \end{cases}$

b) $\frac{4}{4x-3}$

c) $f'(x) = \begin{cases} 2e^{2x-1} & , x > 1/2 \\ -2e^{1-2x} & , x < 1/2 \end{cases}$

77. -1

78. $\frac{3+2\sqrt{3}}{6}$

79. $1-x$

90. a) $\frac{\pi(2k+1)}{4}, k \in \mathbb{Z}$

b) $k\pi, k \in \mathbb{Z}$

SEÇÃO 4.20

1. $y^v = 0$

2. $y''' = 6a$

3. $y^{(10)} = 0$

4. $y'' = \frac{-3}{(3-x^2)\sqrt{3-x^2}}$

5. $y^{iv} = \frac{24}{(x-1)^5}$

6. $y''' = 8e^{2x+1}$

7. $y^{iv} = \frac{1}{e^x}$

8. $y'' = \frac{-1}{x^2}$

9. $y^{vii} = -a^7 \cos ax$

10. $y^v = \frac{1}{16} \operatorname{sen} \frac{x}{2}$

11. $y''' = 2 \sec^4 x + 4 \sec^2 x \cdot \operatorname{tg}^2 x$

12. $y'' = \frac{-2x}{(1+x^2)^2}$

13. a) $\operatorname{sen} x$

b) $\cos x$

18. a) $\frac{-x^2}{y^2}$

b) $\frac{-3x^2 - 2xy}{x^2 + 2y}$

c) $-\sqrt{\frac{y}{x}}$

d) $\frac{1-y^3}{3xy^2 + 4y^3 + 1}$

e) -1

f) $\frac{y}{\sec^2 y - x}$

g) $\frac{1}{e^y - 1}$

19. retas tangentes: $x - \sqrt{3}y + 2 = 0$ e $x + \sqrt{3}y + 2 = 0$

retas normais: $\sqrt{3}x + y - 2\sqrt{3} = 0$ e $\sqrt{3}x - y - 2\sqrt{3} = 0$