DEPARTAMENTO DE ENGENHARIA NAVAL E OCEÂNICA – EPUSP

PNV-3314 Dinâmica de Sistemas I

Lista de Exercícios Nº 3

5.1 Derive the equations of motion of the system shown in Fig. 5.20.

5.2 Derive the equations of motion of the system shown in Fig. 5.21.

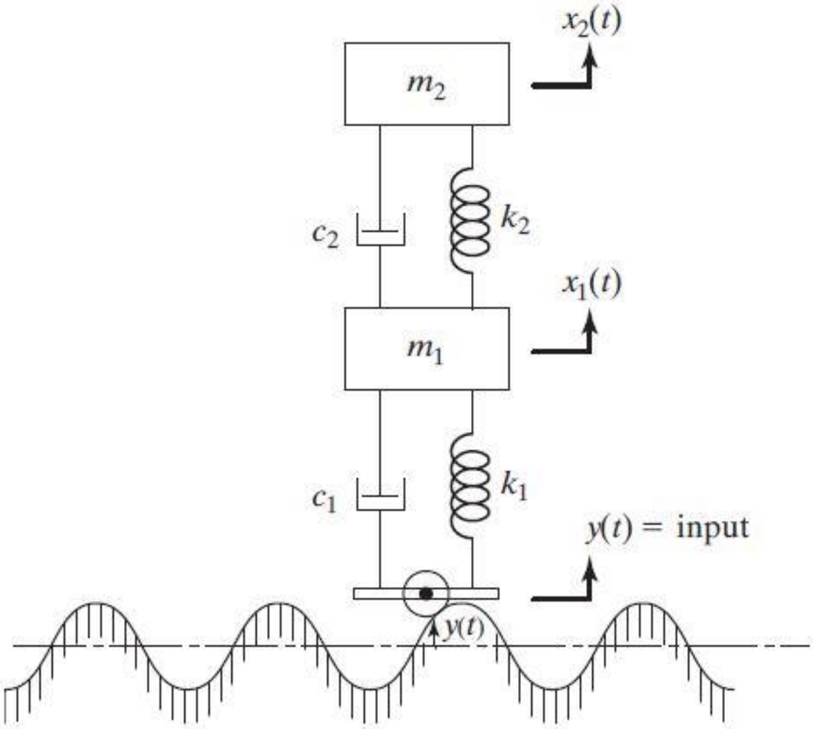

FIGURE 5.20

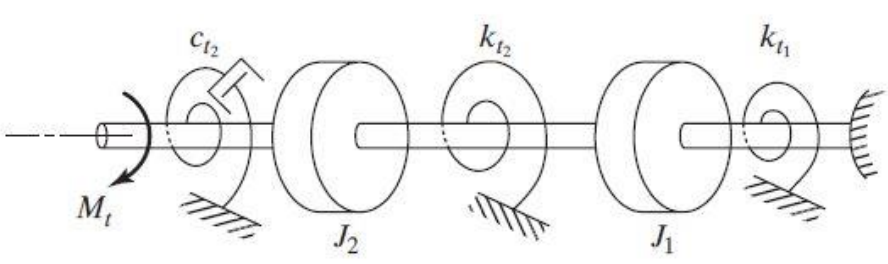

FIGURE 5.21

5.4 A two-mass system consists of a piston of mass m_1, connected by two elastic springs, that moves inside a tube as shown in Fig. 5.23. A pendulum of length l and end mass m_2 is connected to the piston as shown in Fig. 5.23. (a) Derive the equations of motion of the system in terms of $x_1(t)$ and $\theta(t)$. (b) Derive the equations of motion of the system in terms of the $x_1(t)$ and $x_2(t)$. (c) Find the natural frequencies of vibration of the system.

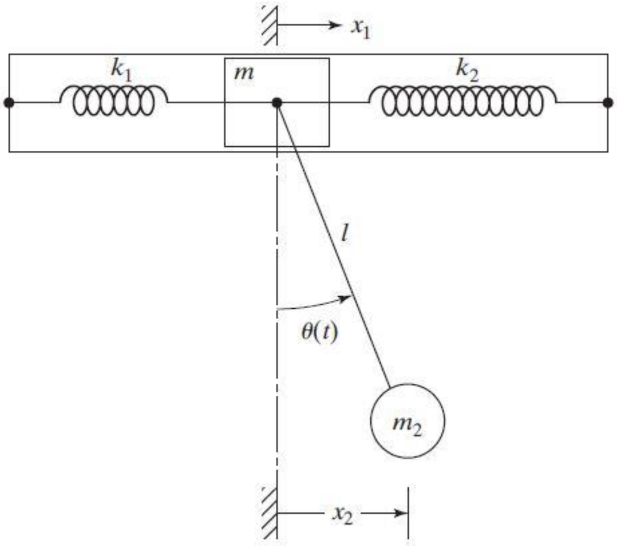

FIGURE 5.23

5.5 Find the natural frequencies of the system shown in Fig. 5.24, with $m_1 = m$, $m_2 = 2m$, $k_1 = k$, and $k_2 = 2k$. Determine the response of the system when $k = 1000$ N/m, $m = 20$ kg, and the initial values of the displacements of the masses m_1 and m_2 are 1 and -1, respectively.

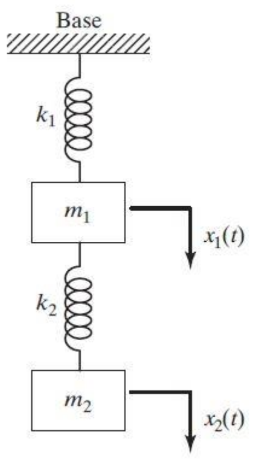

FIGURE 5.24

5.8 A machine tool, having a mass of $m = 1000$ kg and a mass moment of inertia of $J_0 = 300$ kg-m^2, is supported on elastic supports, as shown in Fig. 5.27. If the stiffnesses of the supports are given by $k_1 = 3000$ N/mm and $k_2 = 2000$ N/mm, and the supports are located at $l_1 = 0.5$ m and $l_2 = 0.8$ m, find the natural frequencies and mode shapes of the machine tool.

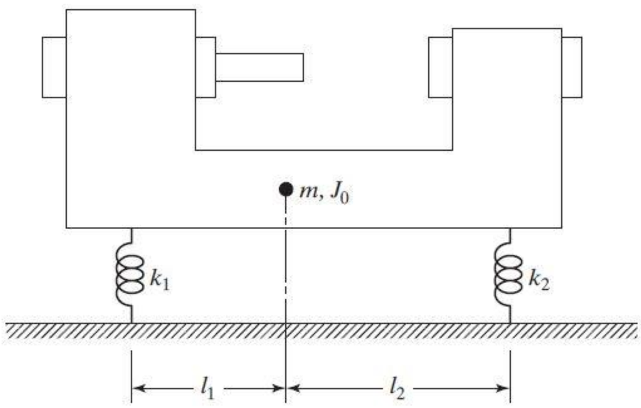

FIGURE 5.27

5.12 One of the wheels and leaf springs of an automobile, traveling over a rough road, is shown in Fig. 5.30. For simplicity, all the wheels can be assumed to be identical and the system can be idealized as shown in Fig. 5.31. The automobile has a mass of $m_1 = 1000$ kg and the leaf springs have a total stiffness of $k_1 = 400$ kN/m. The wheels and axles have a mass of $m_2 = 300$ kg and the tires have a stiffness of $k_2 = 500$ kN/m. If the road surface varies sinusoidally with an amplitude of $Y = 0.1$ m and a period of $l = 6$ m, find the critical velocities of the automobile.

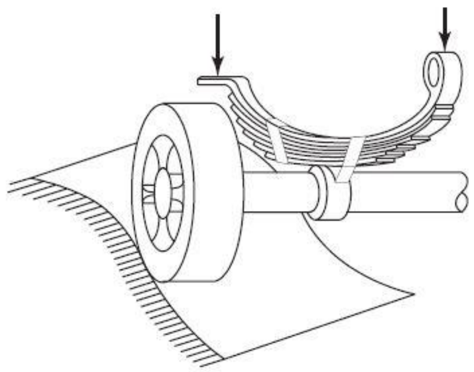

FIGURE 5.30

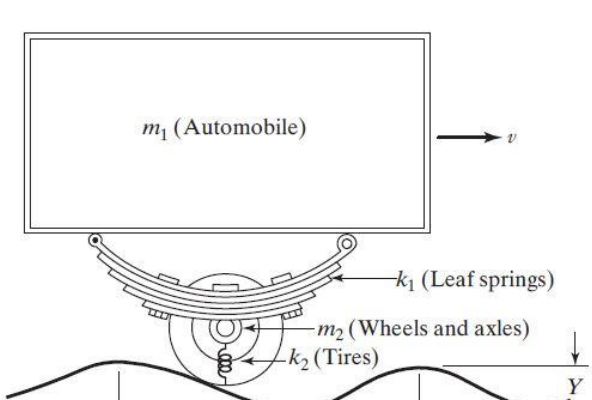

FIGURE 5.31

5.14 Find the natural frequencies and mode shapes of the system shown in Fig. 5.24 for $m_1 = m_2 = m$ and $k_1 = k_2 = k$.

5.16 Find the natural frequencies of the system shown in Fig. 5.6 for $k_1 = 300$ N/m, $k_2 = 500$ N/m, $k_3 = 200$ N/m, $m_1 = 2$ kg, and $m_2 = 1$ kg.

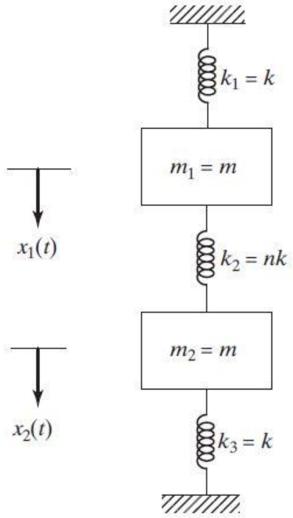

FIGURE 5.6 Two-degree-of-freedom system.

5.17 Find the natural frequencies and mode shapes of the system shown in Fig. 5.24 for $m_1 = m_2 = 1$ kg, $k_1 = 2000$ N/m, and $k_2 = 6000$ N/m.

5.19 For the system shown in Fig. 5.6, $m_1 = 1$ kg, $m_2 = 2$ kg, $k_1 = 2000$ N/m, $k_2 = 1000$ N/m, $k_3 = 3000$ N/m, and an initial velocity of 20 m/s is imparted to mass m_1. Find the resulting motion of the two masses.

5.20 For Problem 5.17, calculate $x_1(t)$ and $x_2(t)$ for the following initial conditions:

a. $x_1(0) = 0.2, \dot{x}_1(0) = x_2(0) = \dot{x}_2(0) = 0.$
b. $x_1(0) = 0.2, \dot{x}_1(0) = x_2(0) = 0, \dot{x}_2(0) = 5.0.$

5.31 Two identical pendulums, each with mass m and length l, are connected by a spring of stiffness k at a distance d from the fixed end, as shown in Fig. 5.35.

a. Derive the equations of motion of the two masses.

b. Find the natural frequencies and mode shapes of the system.

c. Find the free-vibration response of the system for the initial conditions $\theta_1(0) = a$, $\theta_2(0) = 0$, $\dot{\theta}_1(0) = 0$, and $\dot{\theta}_2(0) = 0$.

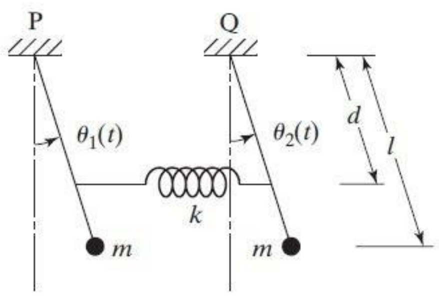

FIGURE 5.35

5.33 An airplane standing on a runway is shown in Fig. 5.37. The airplane has a mass $m = 20{,}000$ kg and a mass moment of inertia $J_0 = 50 \times 10^6$ kg-m^2. If the values of stiffness and damping constant are $k_1 = 10$ kN/m and $c_1 = 2$ kN-s/m for the main landing gear and $k_2 = 5$ kN/m and $c_2 = 5$kN-s/m for the nose landing gear, (a) derive the equations of motion of the airplane, and (b) find the undamped natural frequencies of the system.

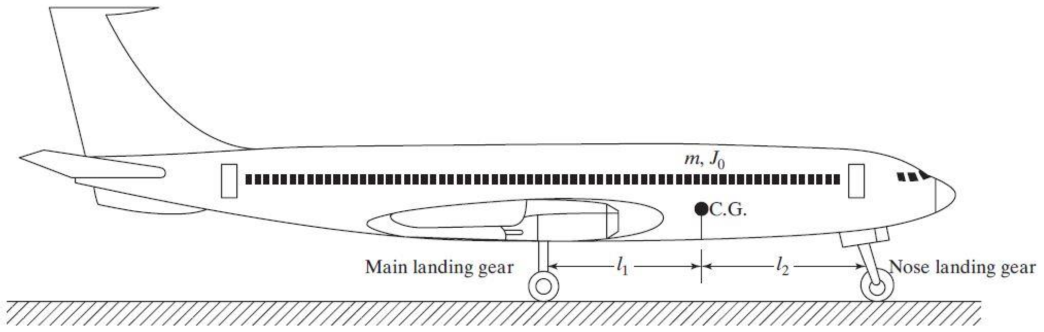

FIGURE 5.37

5.34 The mass and stiffness matrices and the mode shapes of a two-degree-of-freedom system are given by

$$[m] = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \quad [k] = \begin{bmatrix} 12 & -k_{12} \\ -k_{12} & k_{22} \end{bmatrix}, \quad \vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 9.1109 \end{Bmatrix}, \quad \vec{X}^{(2)} = \begin{Bmatrix} -9.1109 \\ 1 \end{Bmatrix}$$

If the first natural frequency is given by $\omega_1 = 1.7000$, determine the stiffness coefficients k_{12} and k_{22} and the second natural frequency of vibration, ω_2.

5.35 The mass and stiffness matrices and the mode shapes of a two-degree-of-freedom system are given by

$$[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad [k] = \begin{bmatrix} 27 & -3 \\ -3 & 3 \end{bmatrix}, \quad \overline{X}^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}, \quad \overline{X}^{(2)} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

If the first natural frequency is given by $\omega_1 = 1.4142$, determine the masses m_1 and m_2 and the second natural frequency of the system.

Section 5.4 Torsional System

5.36 Determine the natural frequencies and normal modes of the torsional system shown in Fig. 5.38 for $k_{t2} = 2k_{t1}$ and $J_2 = 2J_1$.

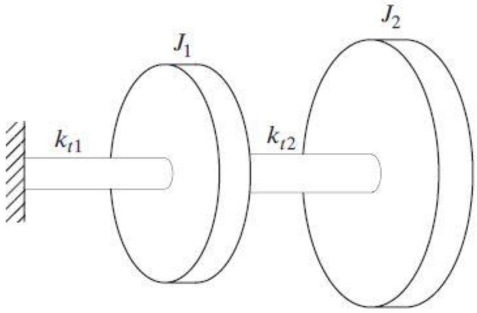

FIGURE 5.38

5.46 An automobile is modeled with a capability of pitch and bounce motions, as shown in Fig. 5.45. It travels on a rough road whose surface varies sinusoidally with an amplitude of 0.05 m and a wavelength of 10 m. Derive the equations of motion of the automobile for the following data: mass = 1,000 kg, radius of gyration = 0.9 m, $l_1 = 1.0$ m, $l_2 = 1.5$ m, $k_f = 18$ kN/m, $k_r = 22$ kN/m, velocity = 50 km/hr.

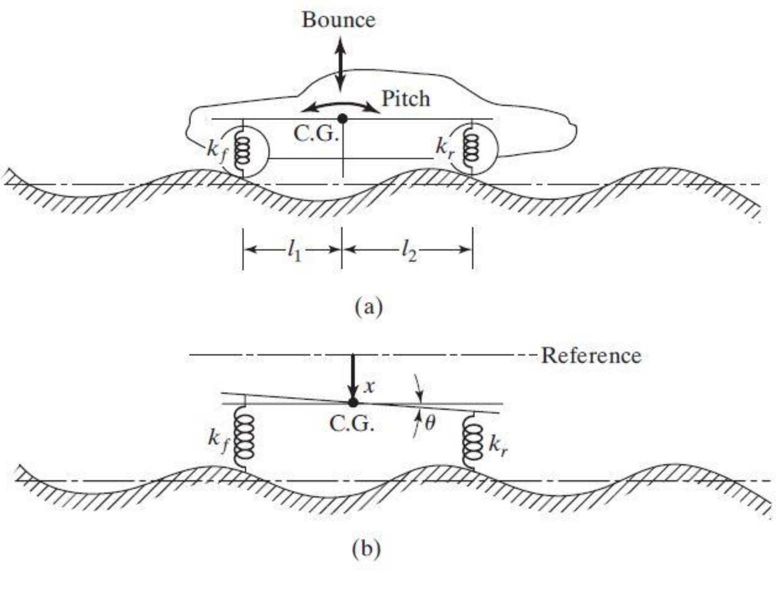

FIGURE 5.45

Fonte: Rao, S.S. *Mechanical Vibrations*, 5th Ed., Prentice Hall, 2011