Universidade de São Paulo
Escola Superior de Agricultura “Luiz de Queiroz"
Departamento de Ciências Exatas
LCE 0220 - Cálculo II
Funções Gama e Beta

1) Mostrar que $\Gamma(1/2) = \sqrt{\pi}$

$$\beta\left(\frac{1}{2},\frac{1}{2}\right) = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(1)} = \left(\Gamma\left(\frac{1}{2}\right)\right)^2$$

Mas, $\beta\left(\frac{1}{2},\frac{1}{2}\right) = 2\int_0^{\frac{\pi}{2}} [senx]^{2*\frac{1}{2}-1}\ [cosx]^{2*\frac{1}{2}-1}\ dx$

$$= 2\int_0^{\frac{\pi}{2}} [senx]^0\ [cosx]^0\ dx$$

$$= 2\int_0^{\frac{\pi}{2}} dx$$

$$= 2\ \left(\frac{\pi}{2} - 0\right) = \pi$$

Logo, $\left(\Gamma\left(\frac{1}{2}\right)\right)^2 = \pi\ e\ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

2) Demonstração da reparametrização

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

Resolvendo por substituição temos:

Se $x = (sen(u))^2 \quad u = arcsen(\sqrt{x})$

Assim, quando $x \to 0$, $u \to 0$ e quando $x \to 1$, $u \to \frac{\pi}{2}$

Ainda, se $x = (sen(u))^2$ $\mathrm{d}x = 2sen(u)\cos(u)\,du$

Logo,

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx = \int_0^{\frac{\pi}{2}} (sen^2(u))^{m-1}(1-sen^2(u))^{n-1}\,2sen(u)\cos(u)\,du$$

$= 2\int_0^{\frac{\pi}{2}} (sen(u))^{2m-1}(cos(u))^{2n-1}$