

Part I

Many doors are now open (they open according to where we give our attention). Once through, looking back, no wall or doors are seen. Why was anyone for so long closed in? *Sounds one hears are music.* (1967b)*

Relations between theory and practice in Western music have always been somewhat strained, but by the early years of this century they had reached a breaking point. Unable to keep up with the radical changes that were occurring in compositional practice, harmonic theory had become little more than an exercise in “historical musicology,” and had ceased to be of immediate relevance to contemporary music. This had not always been so. Most of the important theorists of the past — from Guido and Franco through Tinctoris and Zarlino to Rameau (and even Riemann) — had not only been practicing composers, but their theoretical writings had dealt with questions arising in their own music and that of their contemporaries. Arnold Schoenberg (one of the last of the great composer-theorists) was acutely aware of the disparities between what could be said about harmony (ca. 1911) and then-current developments in compositional practice. Near the end of his *Harmonielehre* he expresses

* A list of Cage’s writings referred to in this text may be found in chronological order at the end. Quotations are identified by date within the text, in order to clarify the evolutionary development of his ideas. Any emphases (italics) are my own. Other sources are referenced in footnotes, indicated by superscripts.

the belief that “continued evolution of the theory of harmony is not to be expected at present.”¹ I choose to interpret this statement of Schoenberg’s as announcing a *postponement* of that evolution, however — not the end of it.

One of the reasons for the current disparity between harmonic theory and compositional practice is not hard to identify: the very *meaning* of the word “harmony” has come to be so narrowly defined that it can only be thought of as applying to the materials and procedures of the diatonic/triadic tonal system of the last two or three centuries. The word has a very long and interesting history, however, which suggests that it need not be so narrowly defined, and that the “continued evolution of the theory of harmony” might depend on — among other things — a broadening of our definition of “harmony.”

... and perhaps, of “theory” as well. By “theory” I mean essentially what any good dictionary tells us it means — e.g.:

... the analysis of a set of facts in relation to one another ...
the general or abstract principles of a body of fact, a science,
or an art ... a plausible or scientifically acceptable general
principle or body of principles offered to explain phenomena
...²

... which is to say, something that current textbook versions of “the theory of harmony” are decidedly *not* — any more than a book of etiquette, for example, can be construed as a “theory of human behavior,” or a cookbook a “theory of chemistry.”

It seems to me that what a true theory of harmony would have to be now is a theory of *harmonic perception* (one component in a more general theory of musical perception) — consistent with the most recent data available from the fields of acoustics and psychoacoustics, but also taking into account the greatly extended range of musical experiences available to us today. I would suggest, in addition, that such a theory ought to satisfy the following conditions:

First, it should be *descriptive* — not pre- (or pro-) scriptive — and thus, *aesthetically neutral*. That is, it would not presume to tell a composer what should or should not be done, but rather what the results might be if a given thing *is* done.

Second, it should be culturally/stylistically *general* — as relevant to music of the twentieth (or twenty-first!) century as it is to that of the eighteenth (or thirteenth) centuries, and as pertinent to the music of India or Africa or the Brazilian rainforest as it is to that of Western Europe or North America.

Finally — in order that such a theory might qualify as a “theory” at all, in the most pervasive sense in which that word is currently used (outside of music, at least) — it should be (whenever and to the maximum extent possible) *quantitative*. Unless the propositions, deductions, and predictions of the theory are formulated quantitatively, there is no way to verify the theory, and thus no basis for comparison with other theoretical systems.

Is such a theory really needed? Perhaps not — music seems to have done very well without one for a long time now. On the other hand, one might answer this question the way Ghandi is said to have done when asked what he thought of Western civilization: “It would be nice.” (1968)

Is such a theory *feasible* now? I think it is, or at least that the time has come for us to make some beginnings in that direction — no matter how tentative. Furthermore, I believe that the work of John Cage, while posing the greatest conceivable *challenge* to any such effort, yet contains many fertile seeds for theoretical development — some of them not only useful, but *essential*.

Such an assertion may come as a surprise to many — no doubt including Cage himself, since he has never shown any inclination to call himself a theorist, nor any interest in what he calls “harmony.” The bulk of his writings — taken together — sometimes seem more like that “thick presence all at once of a naked self-obscuring body of history” (to quote his description of a painting by Jasper Johns; 1964) than a “body of principles” constituting a theory. But these writings include some of the most cogent examples of pure but practical theory to be found anywhere in the literature on twentieth-century music. His work encourages us to re-examine all of our old habits of thought, our assumptions, and our definitions (of “theory,” of “harmony” — of “music” itself) — even where (as with “harmony”) he has not done so himself. His own precise definitions of “material,” “method,” “structure,” “form,” etc. — even

where needing some revision or extension to be maximally useful today — can serve as suggestive points of departure for our own efforts.

I propose to examine some of Cage's theoretical ideas a little more closely, and then to consider their possible implications for a new theory of harmony. Before proceeding, however, I want to clarify one point. Some of Cage's critics (even friendly ones) seem to think that he is primarily a philosopher, rather than a composer — and my own focusing on his contributions as theorist might be misunderstood to imply a similar notion on my own part. This would be a mistake. I believe, in fact, that it is primarily *because of his music* — his very substantial credibility as a *composer* — that we are drawn into a consideration of his philosophical and theoretical ideas. To imagine otherwise is to “put the cart before the horse.” In a letter defending the music of Erik Satie, Cage once wrote:

More and more it seems to me that relegating Satie to the position of having been very influential but in his own work finally unimportant is refusing to accept the challenge he so bravely gave us . . . (1951)

The same thing can truly be said of John Cage himself.

Definitions . . . Structure in music is its divisibility into successive parts from phrases to long sections. Form is content, the continuity. Method is the means of controlling the continuity from note to note. *The material of music is sound and silence.* Integrating these is composing. (1949)

Cage's earliest concerns — and his most notorious later innovations — had to do with *method* — “the means of controlling the continuity from note to note.” His music includes an astonishing variety of different methods, from one “dealing with the problem of keeping repetitions of individual tones as far apart as possible” (1933–34) and “unorthodox twelve-tone” procedures (1938) through the “considered improvisation” of the *Sonatas and Interludes* and other works of the '40s, to “moves on . . . charts analogous to those used in constructing a magic square” (1951), chance operations based on the *I Ching* (from 1951 to the present), the use of transparent “templates made or found” (1952–), the “observation of imperfections in the paper” on which a score was written (1952–), etc. (1958, 1961). Surely no other composer in the history of music has so thoroughly explored this aspect of composition — but not merely because of some fascination with “method” for its own sake. On the contrary, Cage's frequent changes of method have always resulted from a new and more penetrating analysis of the *material* of music, and of the nature of musical activity in general.

Before 1951, Cage's methods (or rather, his “composing means”) were designed to achieve two things traditionally assumed to be indispensable to the making of art: on the one hand, spontaneity and freedom of expression (at the level of “content” or “form”), and on the other, a measure of structural control over the musical material. What was *unique* about his compositional procedures stemmed from his efforts to define these things (“form”, “structure”, etc.) in a way which would be consistent with the essential nature of the musical material, and with the nature of auditory perception. These concerns have continued undiminished

through his later work as well, but in addition he has shown an ever-increasing concern with the larger *context* in which musical activity takes place:

The novelty of our work derives . . . from our having moved away from simply private human concerns toward the world of nature and society of which all of us are a part. Our intention is to affirm this life, not to bring order out of chaos nor to suggest improvements in creation, but simply to wake up to the very life we're living, which is so excellent once one gets one's mind and one's desires out of the way and lets it act of its own accord. (1956a)

In this spirit, he had begun, as early as 1951, a series of *renunciations* of those very things his earlier methods had been designed to ensure — first, *expressivity*, and soon after that, *structural controls*. The method he chose to effect these renunciations (after some preliminary work with “moves on charts . . .”) involved the use of chance operations, and in writing about the *Music of Changes* (1951) he said:

It is thus possible to make a musical composition the continuity of which is free of individual taste and memory (psychology) and also of the literature and “traditions” of the art . . . Value judgments are not in the nature of this work as regards either composition, performance, or listening. The idea of relation (the idea: 2) being absent, anything (the idea: 1) may happen. A “mistake” is beside the point, for once anything happens it authentically is. (1952)

This statement generated a shock-wave which is still reverberating throughout the Western cultural community, because it was interpreted

as a negation of many long-cherished assumptions about the creative process in art. But there is an important difference between a “negation” and a “renunciation” which has generally been overlooked: to renounce something is not to deny others their right to have it — though it does throw into question the notion that such a thing is universally *necessary*. On the other hand, such things as taste, tradition, value judgments, etc., not only can be but often (and habitually) *are* used in ways which are profoundly negative. Cage’s “renunciations” since 1951 should therefore not be seen as “negations” at all, but rather as efforts to *give up the old habits of negation* — the old *exclusions* of things from the realm of aesthetic validity, the old *limitations* imposed on musical imagination, the old *boundaries* circumscribing the “art of music.” And the result? As he has said:

. . . nothing was lost when everything was given away. In fact, everything was gained. In musical terms, *any sounds may occur in any combination and in any continuity*. (1957)

The fact that his own renunciations need not be taken as negations should have been clearly understood when he said, for example:

The activity of movement, sound, and light, we believe, is expressive, but what it expresses is determined by each one of you . . . (1956a)

or again:

. . . the coming into being of something new does not by that fact deprive what was of its proper place. Each thing has its

own place . . . and the more things there are, as is said, the merrier. (1957)

but here, it seems, his critics were not listening.

It should go without saying (though I know it won't) that **we don't need those old "habits of negation" anymore** — neither in life (where they are so often used in ways that are very destructive), nor in art. Still less do we need them in a theory of harmony — and this is one of the reasons I find Cage's work and thought to be essential to new theoretical efforts. His "renunciations" have created an intellectual climate in which **it is finally possible to envision a theory of harmony which is both "general" and "aesthetically neutral" — a climate in which a truly *scientific* theory of musical perception might begin to be developed.**

Composing's one thing, performing's another, listening's a third. What can they have to do with one another? (1955)

While the question of method is naturally of interest to a composer — and has been, in Cage's case, the subject of greatest concern to his critics — **what is actually *perceived* in a piece of music is not method as such, but *material, form, and structure*.** Cage's most radical earlier innovations had involved extensions of material, and these may one day turn out to have more profound implications for theory than his investigations of method. **The pieces for percussion ensemble, for prepared piano, and for electrical devices** — composed during the late '30s and '40s — **greatly extended the**

range of musical materials, first to include *noises* as well as tones, and then *silence* as well as sound.

These extensions were not without precedent, of course. As Cage has said, it was “Edgard Varèse who fathered forth noise into twentieth-century music” (1959b) and who

. . . more clearly and actively than anyone else of his generation . . . established the present nature of music . . . [which] . . . arises from an acceptance of *all audible phenomena as material proper to music*. (1959b)

But Cage was the first to deal with the *theoretical* consequences of this acceptance. Since “harmony” and other kinds of pitch-organization did not seem applicable to noise,

The present methods of writing music . . . will be inadequate for the composer, who will be faced with *the entire field of sound*. (1937)

More specifically,

In writing for these [electrically produced] sounds, as in writing for percussion instruments alone, the composer is dealing with material that does not fit into the orthodox scales and harmonies. It is therefore necessary to find some other organizing means than those in use for symphonic instruments . . . A method analogous to the twelve-tone system may prove useful, but . . . *because of the nature of the materials involved*, and because their duration characteristics can be easily controlled and related, *it is more than likely that the unifying means will be rhythmic*. (1942)

This statement, which reads like a prediction, was actually a description of the state of affairs that had already prevailed in Cage's work since the *First Construction (In Metal)* of 1939, but it was not until 1948 that the idea took the form of a general principle — even a rather dogmatic one:

In the field of structure, the field of the definition of parts and their relation to a whole, there has been only one new idea since Beethoven. And that new idea can be perceived in the work of Anton Webern and Erik Satie. With Beethoven the parts of a composition were defined by means of harmony. With Satie and Webern they are defined by means of time lengths . . . There can be no right making of music that does not structure itself from the very roots of sound and silence — lengths of time . . . (1948)

A year later this principle is repeated, but with a slightly different emphasis:

Sound has four characteristics: pitch, timbre, loudness, and duration. The opposite and necessary coexistent of sound is silence. Of the four characteristics of sound, only duration involves both sound and silence. Therefore, a structure based on durations . . . is correct (corresponds with the nature of the material), whereas harmonic structure is incorrect (derived from pitch, which has no being in silence). (1949)

Cage was right, of course, in emphasizing the fundamental importance of time and time-structure in music, but — as compelling and persuasive as this argument is — there is a serious flaw in it. On the one hand, *all* music manifests some sort of temporal structure (including harmonically

organized music; Beethoven), and on the other hand, neither Webern nor Satie nor Cage himself had ever managed to “define” the successive parts of a composition purely “by means of time lengths.” Such time lengths — in order to be perceived as “parts” — must be *articulated* by some other means, and these means may or may not include the specifically “harmonic” devices of cadence, modulation, etc. In the works of Cage intentionally organized according to this concept of time-structure (as in the music of Satie and Webern), the successive parts in the structure are in fact articulated by various kinds of *contrast* — changes of dynamic level, texture, tempo, pitch-register, thematic material, etc. — and such contrast-devices have *always* been used (with or without the benefit of “harmony”) to articulate temporal structure.

We needn’t be too concerned, however, with the “dogmatic” aspect of these statements, since it was to be only a few years later that Cage would cease to be concerned with determinate structure at all. What is more important is the way in which he was thinking about *the nature of sound*:

A sound does not view itself as thought, as ought, as needing another sound for its elucidation . . . it is occupied with the performance of its characteristics: before it has died away it must have made perfectly exact its frequency, its loudness, its length, its overtone structure, the precise morphology of these and of itself . . . It does not exist as one of a series of discrete steps, but as transmission in all directions from the field’s center. (1955)

This line of thought gradually crystallized into a conception of what Cage calls “sound-space” — that perceptual “space” in which music (*any* music)

must exist. His clearest and most complete description of this concept is perhaps the following:

The situation made available by these [tape-recording] means is essentially a total sound-space, the limits of which are ear-determined only, the position of a particular sound in this space being the result of five determinants: frequency or pitch, amplitude or loudness, overtone structure or timbre, duration, and morphology (how the sound begins, goes on, and dies away). By the alteration of any one of these determinants, the position of the sound in sound-space changes. Any sound at any point in this total sound-space can move to become a sound at any other point . . . musical action or existence can occur at any point or along any line or curve . . . in total sound-space; . . . we are . . . technically equipped to transform our contemporary awareness of nature's manner of operation into art. (1957)

Note that the list of “four characteristics” given in 1949 has now been increased to “five determinants,” and in a later passage a sixth one is added (“an order of succession”; 1958a). Even so, such a list is by no means exhaustive, and important clues regarding the nature of harmonic perception will emerge from a consideration of the “determinants,” parameters, or what I will call *dimensions* of “sound-space” which are missing from all of these lists.

By his own definitions (pre-1951), *form* is “content, the continuity,” and *method* is “the means of controlling the continuity” — i.e. of controlling *form*. After 1951, of course, Cage's methods were no longer intended to “control” form in this same sense, and yet a certain necessary causal relationship still holds between method and form — no matter what the

intention — and as a result most of Cage’s works since 1951 exemplify an important new formal type which I have elsewhere called “ergodic.”³ I use this term (borrowed from thermodynamics) to mean *statistically homogeneous* at some hierarchical level of formal perception. For example, it can be said about many of Cage’s post-1951 pieces (and something like this often *is* said, though usually with negative implications not intended here) that any 2- or 3-minute segment of the piece is essentially the same as any other segment of corresponding duration, even though the details are quite different in the two cases. I interpret this to mean that certain *statistical properties* are in fact “the same” — or so nearly identical that no distinction can be made in perception.

The relation between the ergodic form and Cage’s later methods involving chance and/or indeterminacy is this: an ergodic form will always and inevitably be the result when a range of possibilities (with respect to the sound-elements in a piece, and their characteristics) is given at the outset of the compositional process, and remains unchanged during the realization of the work. Such a form is quite unlike the dramatic and/or rhetorical forms we are accustomed to in most earlier music, and has been the cause of much of the negative response to Cage’s music of the last thirty years. A different attitude is obviously required of the listener to be able to enjoy an ergodic piece — and it is perhaps ironic that it is an attitude which most people are able to adopt quite easily in situations outside the usual realm of “art” (e.g., the sounds of a forest). In this respect, many of Cage’s pieces represent an “imitation of nature” in more

than just “her manner of operation,” but in her “forms” (or, as I’m sure Cage would prefer to say, her “processes”) as well.

Cage’s inclusion of “all audible phenomena as material proper to music” did not mean that distinctions were no longer to be made. On the contrary, it now became possible to distinguish many more varieties of elementary sounds — some of which Cage called “aggregates.” In writing about his *Sonatas and Interludes* for prepared piano (1946–48) he says:

. . . a static gamut of sounds is presented, no two octaves repeating relations. However, one could hear interesting differences between certain of these sounds. On depressing a key, sometimes a single frequency was heard. In other cases . . . an interval [i.e. a dyad]; in still others *an aggregate of pitches and timbres*. Noticing the nature of this gamut led to selecting a comparable one for the *String Quartet* . . . (1958a)

This concept of the aggregate is, I believe, extremely important for any new theory of harmony, since such a theory must deal with the question: under what conditions will a multiplicity of elementary acoustic signals be perceived as a “single sound?” When this question is asked about a compound tone containing several harmonic partials, its relevance to the problems of harmony becomes immediately evident.

Aside from their possible implications for a theory of *harmony*, as such, Cage’s extensions of the range of musical materials to include “all audible phenomena” have created a whole new set of problems for the theorist, but his efforts to understand the *nature* of those materials have also

indicated ways in which these problems might be solved. One of his statements about composition might also be applied to theory:

Something more far-reaching is necessary: a composing of sounds *within a universe predicated upon the sounds themselves* rather than upon the mind which can envisage their coming into being. (1958a)

. . . when Schoenberg asked me whether I would devote my life to music, I said, “Of course.” After I had been studying with him for two years, Schoenberg said, “In order to write music, you must have a feeling for harmony.” I explained to him that I had no feeling for harmony. He said that I would always encounter an obstacle, that it would be as though I came to a wall through which I could not pass. I said, “In that case I will devote my life to beating my head against that wall.” (1959a)

This metaphor of the wall — and other sorts of boundaries, barriers, or enclosures — is a recurring one in Cage’s writings:

. . . once a circle is drawn my necessity is to get outside of it
. . . No doubt there is a threshold in all matters, but once through the door — no need to stand there as though transfixed — the rules disappear. (1962)

. . . my philosophy in a nutshell. Get out of whatever cage you happen to be in. (1972)

There were many such walls, but “harmony” — in its narrowest sense (the materials and procedures of traditional, tonal, textbook harmony) — was for Cage a particularly obstructive one:

Harmony, so-called, is a forced abstract vertical relation which blots out the spontaneous transmitting nature of each of the sounds forced into it. It is artificial and unrealistic. (1954)

Seeking an interpenetration and non-obstruction of sounds . . . a composer at this moment . . . renounces harmony and its effect of fusing sounds in a fixed relationship. (1963)

Series equals harmony equals mind of man (unchanged, used as obstacle . . .) (1966)

Only once does he suggest the possibility of defining the word differently:

This music is not concerned with harmoniousness as generally understood, where the quality of harmony results from a blending of several elements. Here we are concerned with the coexistence of dissimilars, and the central points where fusion occurs are many: the ears of the listeners wherever they are. *This disharmony*, to paraphrase Bergson’s statement about disorder, *is simply a harmony to which many are unaccustomed*. (1957)

Here, Cage was closer than he may have realized to Schoenberg (in the latter’s writings, at least, if not in his teaching) — as when he had said:

What distinguishes dissonances from consonances is not a greater or lesser degree of beauty, but a greater or lesser

degree of *comprehensibility* . . . The term *emancipation of the dissonance* refers to [this] comprehensibility . . . ⁴

What is it then, in Cage's vision that lies beyond these "walls?" An *open field* — and this is an image that he evokes again and again in his writings:

I have never gratuitously done anything for shock, though what I have found necessary to do I have carried out, occasionally and only after struggles of conscience, even if it involved actions apparently outside the "boundaries of art." For "art" and "music" when anthropocentric (involved in self-expression), seem trivial and lacking in urgency to me. We live in a world where there are things as well as people. Trees, stones, water, everything is expressive. I see this situation in which I impermanently live as a complex interpenetration of centers moving out in all directions without impasse. This is in accord with contemporary awareness of the operations of nature. I attempt to let sounds be themselves in a space of time . . . I am more and more realizing . . . that I have ears and can hear. My work is intended as a demonstration of this; you might call it an affirmation of life. (1956b)

This open field is thus life itself, in all its variety and complexity, and an art activity "imitating nature in her manner of operation" only becomes possible when the limitations imposed by "self-expression," "individual taste and memory," the literature and traditions of an "anthropocentric" art — and of course, "harmony" — have all been questioned so deeply and critically that they no longer circumscribe that activity — no longer define "boundaries." Not that these things will cease to exist, but "looking

back, no wall or doors are seen . . . *Sounds one hears are music.*” No better definition of “music” — for our time — is likely to be found.

The field — thus understood as life or nature — is much more than just music, but the “sound-space” of musical perception is one part of that total field, and Cage would have us approach it in a similar way. Its limits are “*ear-determined only*,” the position of a sound within this field is a function of *all aspects of sound*, and

. . . each aspect of sound . . . is to be seen as a *continuum*, not as a series of discrete steps favored by conventions . . .
(1959b)

This “total sound-space” has turned out to be more complex than Cage could have known, and within it a place will be found for specifically *harmonic* relations — and thus, for “harmony” — but not until this word has been redefined to free *it* from the walls that have been built around it.

Originally, the word “harmony” simply meant *a fitting together* of things in the most mundane sense — as might be applied to pieces of something put together by a craftsman. It was later adapted by the Pythagoreans to serve a much broader philosophical/religious purpose, describing the order of the cosmos. Its specifically musical uses must have been derived from the earlier sense of it, but for the Pythagoreans, the way the tones of a stretched string “fit together” was seen as an instance — in microcosm — of that cosmic order. Even so, it did not refer to simultaneous sounds, but simply to certain *relations between pitches*.

Similarly for Aristoxenus: the discipline of “harmonics” was the science of melody, considered with respect to pitch (and thus to be distinguished from “rhythmics” — the science of melody with respect to time). These senses of the word “harmony” are carried through in the writings of the mediaeval theorists. Only after the beginnings of polyphony in about the ninth century did the word begin to carry a different connotation, and since that time its meaning has become more and more restricted. Apel defines it as “the vertical aspect of music”⁵ — i.e. chord structure, and (to a limited extent) relationships between successive chords. But in fact the word has come to imply only a certain limited set of such relationships — a certain *type* of vertical structure. Thus, even in the case of some kinds of music in which tones *are* heard simultaneously (e.g. Indonesian gamelan music) it has been said that “harmony” is not involved. But it is absurd to imagine that the Indonesian musician is not concerned with the “vertical” aspect of his music. The word “harmony” obviously needs to be freed from its implied restriction to triadic/tonal music — but this is not enough. Even in a purely “horizontal” or monophonic/melodic situation, the realities of musical perception cannot be described without reference to *harmonic relations* between tones. Clearly, *a new theory of harmony will require a new definition of “harmony,”* of “harmonic relations,” etc., and I believe that such definitions will emerge from a more careful analysis of the “total sound-space” of musical perception.

Part II

This project will seem fearsome to many, but on examination it gives no cause for alarm. Hearing sounds which are just sounds immediately sets the theorizing mind to theorizing, and the emotions of human beings are continually aroused by encounters with nature. (1957)

Minimum ethic: Do what you said you'd do. Impossible? (1965)

[More stringent ethic:] . . . make affirmative actions, and not . . . negative . . . critical or polemical actions . . . (1961)

Cage has always emphasized the *multidimensional* character of sound-space, with pitch as just one of its dimensions. This is perfectly consistent with current acoustical definitions of pitch, in which — like its physical correlate, frequency — it is conceived as a *one-dimensional continuum* running from low to high. But our perception of relations between pitches is more complicated than this. The phenomenon of “octave-equivalence,” for example, cannot be represented on such a one-dimensional continuum, and octave-equivalence is just one of several specifically *harmonic relations* between pitches — i.e. relations other than merely “higher” or “lower.” This suggests that the single acoustical variable, frequency, must give rise to *more than one dimension* in sound-space — that the “space” of pitch perception is itself multidimensional. This multidimensional space of pitch-perception will be called *harmonic space*.

The metrical and topological properties of harmonic space have only begun to be investigated, but a provisional model of such a space which seems consistent with what we already know about harmonic perception will be outlined here, and may eventually help to clarify aspects of harmonic perception which are not yet very well understood. In this model, pitches are represented by points in a multidimensional space, and each is labeled according to its frequency ratio with respect to some reference pitch (1/1). Thus, the pitch one octave above the reference pitch is labeled 2/1, that a perfect fifth below 1/1 is labeled 2/3, etc. But since our perception of pitch intervals involves some degree of approximation, these frequency ratios must be understood to represent pitches within a certain *tolerance range* — i.e., a range of relative frequencies within which some slight mistuning is possible without altering the harmonic identity of an interval. The actual magnitude of this tolerance range would depend on several factors, and it is not yet possible to specify it precisely, but it seems likely that it would vary inversely with the ratio-complexity of the interval. That is, the smaller the integers needed to designate the frequency ratio for a given interval, the larger its tolerance range would be. What Harry Partch called “the language of ratios”⁶ is thus assumed to be the appropriate language for the analysis and description of harmonic relations — but only if it is understood to be qualified and limited by the concept of interval tolerance.

For a given set of pitches, the number of dimensions of the implied harmonic space would correspond to the number of *prime factors* required to specify their frequency ratios with respect to the reference pitch. Thus, the harmonic space implied by a “Pythagorean” scale, based

exclusively on fifths ($3/2$), fourths ($4/3$), and octaves ($2/1$), is two-dimensional, since the frequency ratios defining its constituent intervals involve only powers of 2 and 3 (see Figure 1). The harmonic space implied by a “just” scale, which includes natural thirds ($5/4$, $6/5$) and sixths ($5/3$, $8/5$), is three-dimensional, since its frequency ratios include powers of 5, as well as 2 and 3. A scale incorporating the natural minor seventh ($7/4$) and other “septimal” intervals would imply a harmonic space of four dimensions, and Partch’s “11-limit” scale would imply a harmonic space of five dimensions (corresponding to the prime factors 2, 3, 5, 7, and 11) — if (and only if) we assume that all of its constituent intervals are distinguishable. Whether all such intervals among a given set of pitches are in fact distinguishable depends, of course, on the tolerance range, and it is this which prevents an unlimited proliferation of “dimensions” in harmonic space. That is, at some level of scale-complexity, intervals whose frequency ratios involve a higher-order prime factor will be indistinguishable from similar intervals characterized by simpler frequency ratios, and the prime factors in these simpler ratios will define the dimensionality of harmonic space in the most general sense.

The one-dimensional continuum of pitch-height (i.e. “pitch” as ordinarily defined) can be conceived as a central *axis of projection* within this harmonic space. The position of a “point” along this pitch-height axis may be specified, as usual, by the logarithm of the fundamental frequency of the corresponding tone, and the distance (or *pitch-distance*) between two such points by the difference between their log-frequency values. That is,

$PD(f_a, f_b) \propto \log(a) - \log(b) = \log(a/b)$, where f_a and f_b are the fundamental frequencies of the two tones, $a = f_a/\gcd(f_a, f_b)$, $b = f_b/\gcd(f_a, f_b)$, and $a \geq b$

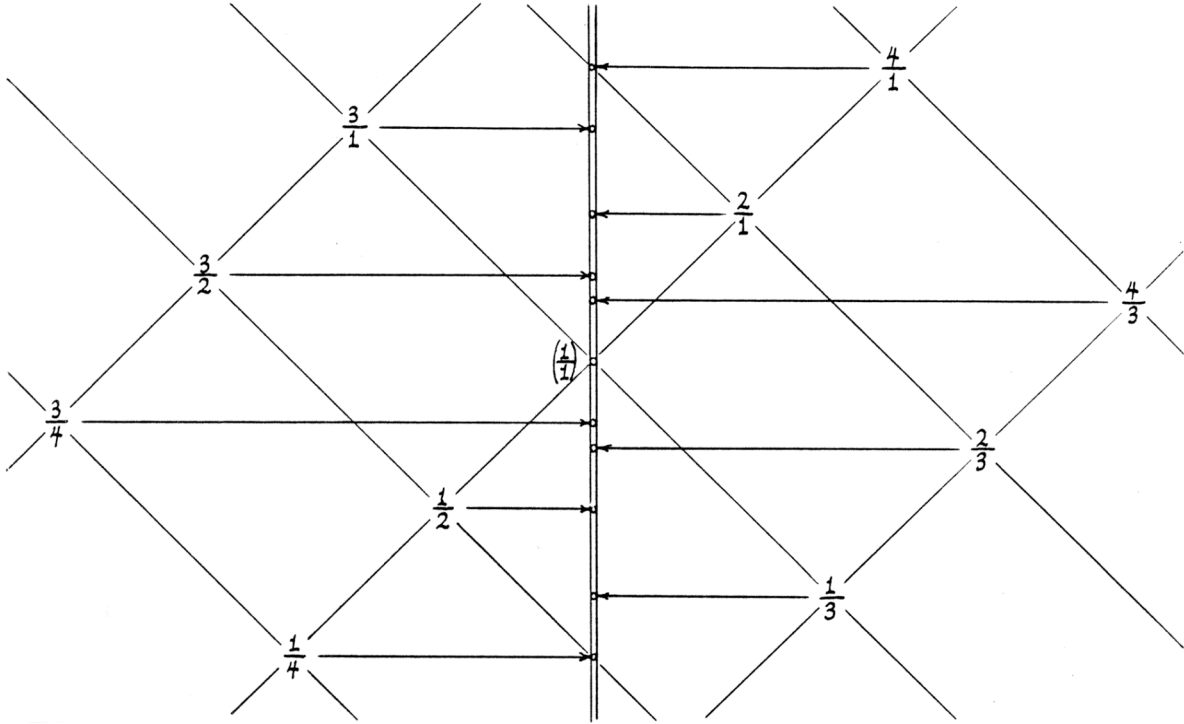


Figure 1. The 2,3 plane of harmonic space, showing the pitch-height projection axis

Although the pitch-height axis is effectively continuous, harmonic space itself is not. Instead, it consists of a discontinuous network or *lattice* of points. A distance measure which I call *harmonic distance* can be defined between any two points in this space as proportional to the sum of the distances traversed on a shortest path connecting them (i.e. along the line segments shown in the figures). (The “metric” on harmonic space is thus not a Euclidian one, but rather a “city-block” metric.) This measure of harmonic distance can be expressed algebraically as follows:

$$HD(f_a, f_b) \propto \log(a) + \log(b) = \log(ab)$$

Here again, the tolerance condition must be kept in mind, and it is useful in this connection to formulate it as follows: an interval is represented by *the simplest ratio within the tolerance range* around its actual relative frequencies, and any measure on the interval is the measure on that simplest ratio.

In this model of harmonic space, octave-equivalence is represented by another sort of projection — of points in a direction parallel to the “2-vectors” (the right-ascending diagonals in Figures 1 and 2; vertical lines in Figure 3). Alternatively, it can be conceived as a “collapsing” of the harmonic space in this same direction yielding a reduced *pitch-class projection space* with one-fewer dimensions. In a 2-dimensional harmonic space, this will be another projection *axis*, as shown in Figure 2. In a 3-dimensional (2,3,5) harmonic space, the pitch-class projection space will be a 2-dimensional (3,5) *plane*, as in Figure 3. This pitch-class projection plane can be used to display the primary (“5-limit”) harmonic relations of triadic/tonal music. For example, the diatonic major and minor scales appear as shown in Figure 4 (using Partch’s labeling convention, whereby a given pitch-class is identified by the ratio it has in the first octave above 1/1). With the addition of two scale degrees not included in Figure 4 (the minor 2nd and the augmented 4th), these two scales can be combined into a composite structure (similar to what Alexander Ellis called the “harmonic duodene”⁷) which shows many of the primary harmonic relations available within the 12-tone chromatic scale (see Figure 5).

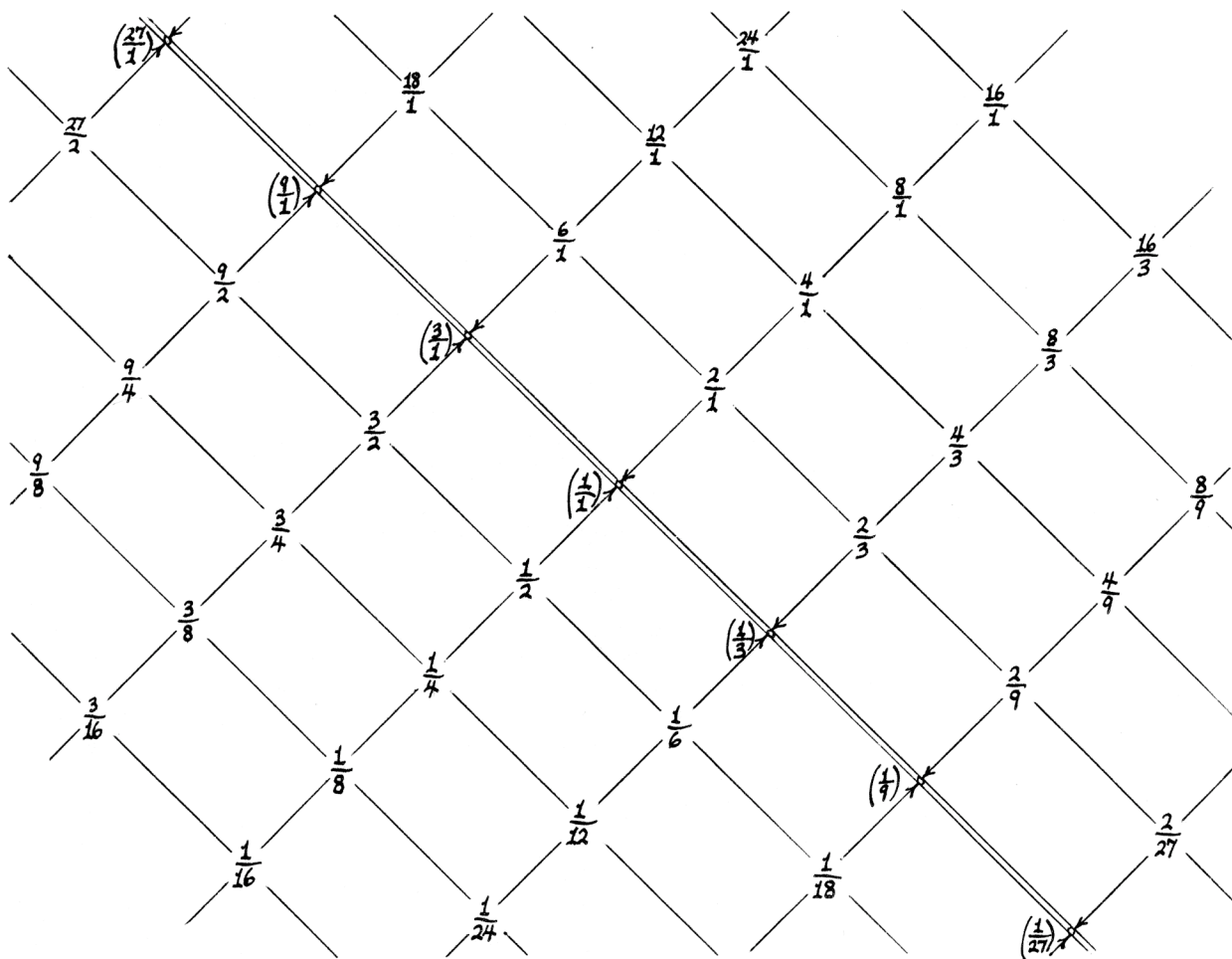


Figure 2. The 2,3 plane of harmonic space, showing the pitch-class projection axis.

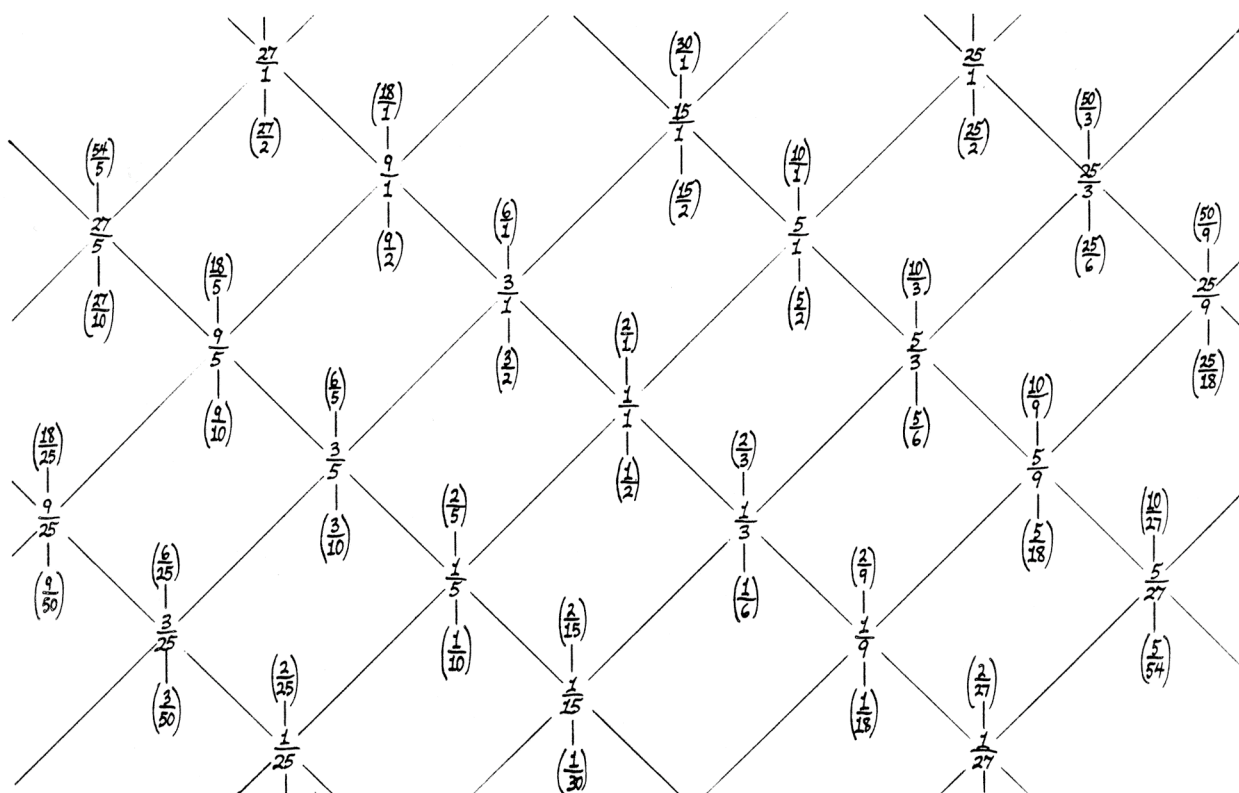


Figure 3. The 3,5 plane of harmonic space as a pitch-class projection plane within 2,3,5 space.

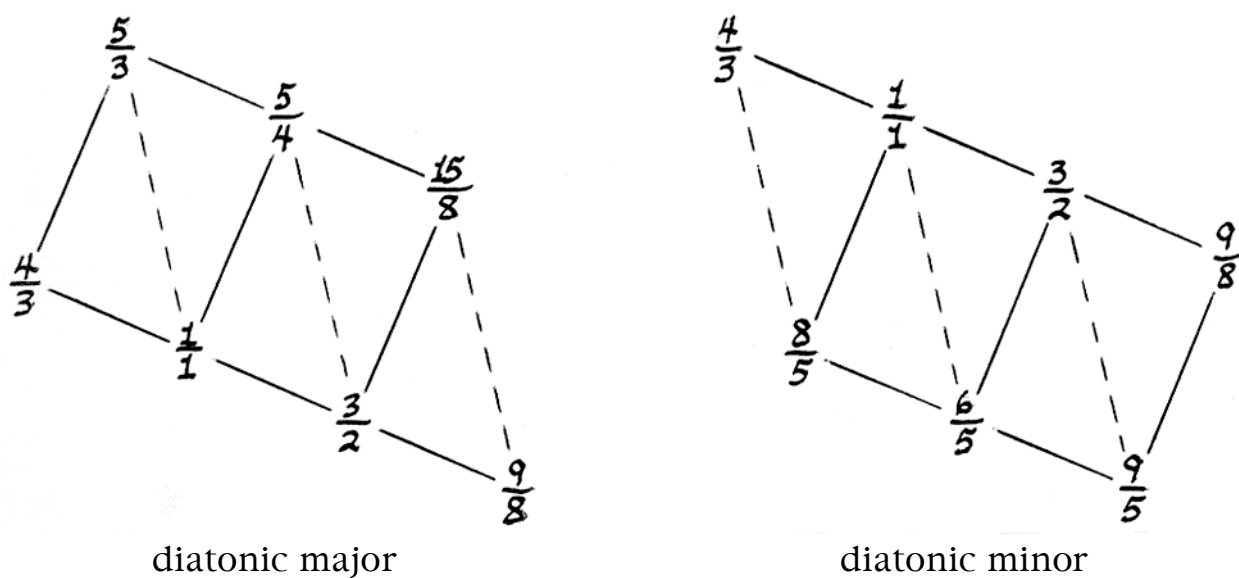


Figure 4. Primary harmonic relations within the diatonic scales.

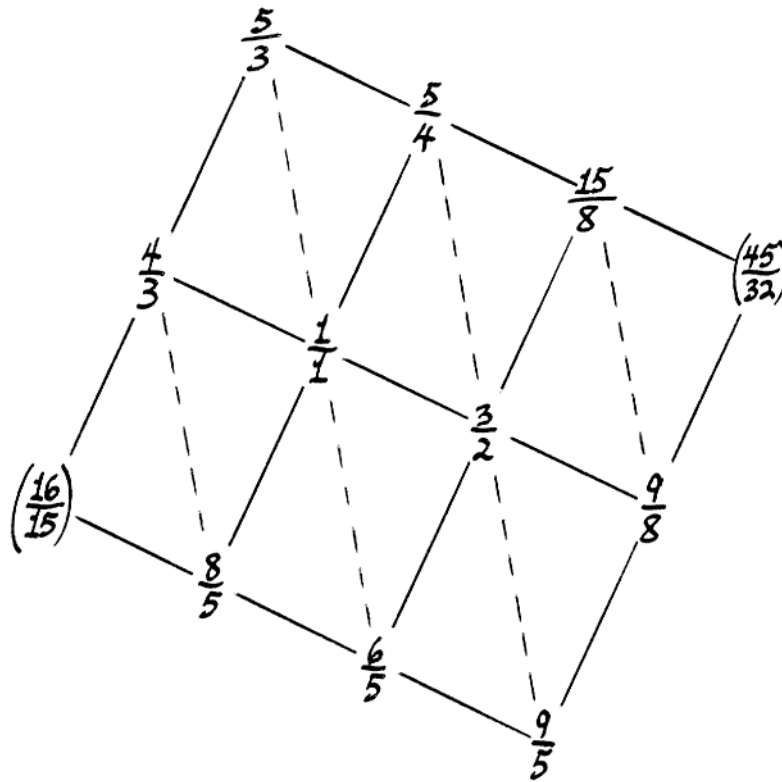


Figure 5. Primary harmonic relations within the chromatic scale.

In representing what has become an equally tempered version of this chromatic scale with low-integer ratios in harmonic space we implicitly assume a fairly large tolerance range (on the order of 15 cents or more), but this is precisely what is implied by the use of our tempered scale for triadic/tonal music. Thus it is no wonder that the evolution of harmony as a clearly functional force in Western music reached a *cul de sac* around 1910. New compositional approaches to harmony will almost certainly involve new “microtonal” scales and tuning systems, and this model of harmonic space provides a useful tool for the design of such systems, as well as for the analysis of old ones. For example, Ben Johnston has for several years now been using what he calls “ratio lattices” — identical in

every respect to those described here — for this very purpose of designing new scales and tuning systems. Although he does not use the term “harmonic space” explicitly, he does refer to “harmonic neighborhoods” demonstrated by the lattice structures, and he distinguishes between what he calls the harmonic and the melodic “modes of perception” in a way which is entirely consistent with the concept of harmonic space presented here.⁸

The physiological correlate of the pitch-height projection axis is surely the *basilar membrane* of the inner ear, while that of the surrounding harmonic space (and of the pitch-class projection space) is assumed to be a set of *pitch-processing centers* in the central nervous system (including some form of short-term memory). The functional characteristics of harmonic space will naturally depend on those of its physiological correlate, and a theory of harmonic perception based on this concept requires the elaboration of a viable model of the auditory system. No such model has yet been developed, but preliminary work in that direction suggests the following:

- 1) Before a point in harmonic space can become activated, the corresponding point on the pitch-height axis must be clearly defined. That is, there must be both *pitch-saliency* and relative *stability* of pitch — and this requires *time*. During the first few hundredths of a second after the onset of a tone, its “image” on the pitch-height axis will not be a well-defined point, but will be spread over some considerable portion of the pitch-height axis, above and below the point representing its nominal pitch. With time, the spread of this image will gradually be reduced to an

effective point (i.e. a region confined to the tolerance range), and the corresponding point in harmonic space will then be activated.

2) Once activated, a point in harmonic space will remain active for some considerable amount of time after the tonal stimulus has stopped sounding. That is, points in harmonic space are characterized by a certain *persistence* (due to a sort of neural “resonance” in short-term memory). The extent of this persistence depends primarily on the number and nature of the sounds which follow the first one.

Note that both of these functional characteristics of harmonic space would involve *time* — and they provide some clues to the question that was asked earlier, in regard to Cage’s concept of the *aggregate*: “Under what conditions will a multiplicity of elementary acoustic signals be perceived as a ‘single sound?’” From a purely physical standpoint, nearly every sound we hear is some sort of “aggregate,” made up of a large number of components. But during the first few tens of milliseconds after the onset of a sound it is impossible to distinguish those individual components. As the sound continues, of course, it may gradually become possible to make such distinctions, and these will depend on the separability of these components’ “images” — either in harmonic space or on the pitch-height axis alone. There are, however, two common acoustical situations in which a multiplicity of components resists this kind of aural “analysis” almost indefinitely: (1) noise bands, and (2) compound tones with harmonic partials.

In the first case — though there may originally have been a large number of individual frequency components (as in a “tone cluster”) — their mutual interferences are such that no one of them remains stable long enough to elicit a tonal percept (i.e. long enough for its image to become a well-defined point on the pitch-height axis). Thus, points in harmonic space will not be activated by a noise band, but its image will appear as a cluster of contiguous points (or regions) along the pitch-height axis.

In the second case, the points in harmonic space activated by the several harmonic partials (assuming them to be stable) also form a “cluster of contiguous points” — but now projected outward (and upward, in the shape of an inverted cone) from the pitch-height axis into the surrounding regions of harmonic space.

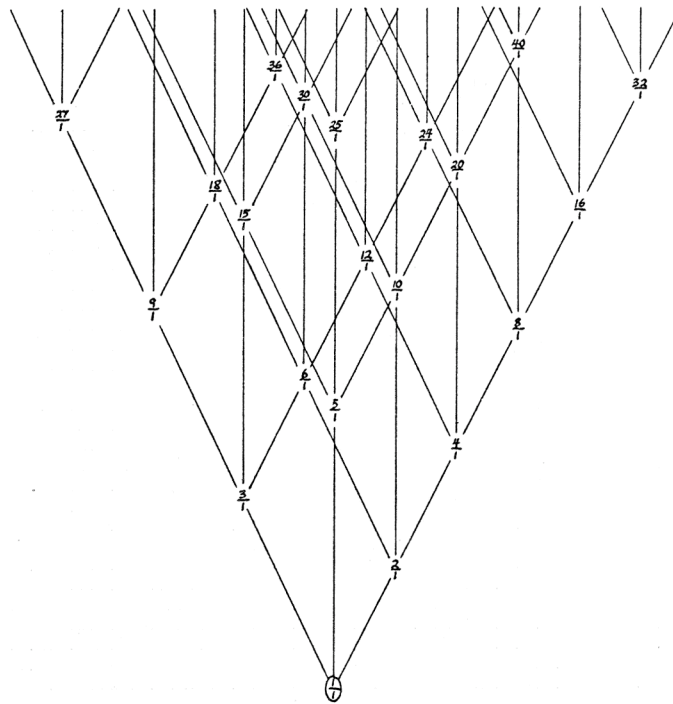


Figure 6. The harmonic containment “cone” in 2,3,5 space.

What is actually perceived in this case, of course, is a single tone with a pitch corresponding to that of the vertex of the “cone” — whether or not a component of that frequency is actually present in the sound — and a timbre determined by the relative amplitudes of the partials.

On the basis of these examples, the initial question might be answered as follows: a multiplicity of elementary acoustic signals will be perceived as a “single sound” — even long after the initial onset — when their images form *a cluster of contiguous points* either in harmonic space or on the pitch-height projection axis alone.

The two most important problems in earlier harmonic theory — regarding the nature of consonance and dissonance, and the tonic phenomenon (including the whole question of chord roots) — have not yet been mentioned here. I suspect that harmonic theorists in the future will be far less concerned with these problems than earlier theorists were, but I think the concept of harmonic space may shed some light on them, for what it’s worth. The problem of consonance and dissonance has been considerably confused by the fact that these terms have been used to mean distinctly different things in different historical periods.⁹ And yet there is one simple generalization that can be applied to nearly all of these different conceptions of consonance and dissonance, which is that tones represented by proximate points in harmonic space tend to be heard as being in a consonant relation to each other, while tones represented by more widely separated points are heard as mutually dissonant. Now this statement serves neither to clarify the distinctions between different

senses of consonance and dissonance mentioned above nor to “explain” any one of them. It does, however, indicate an important correlation between consonance and dissonance and what I am calling harmonic space.

Regarding the “tonic phenomenon,” our model does not, in itself, suggest either an explanation or a measure of it, but we can incorporate into the model the simple observation that there is a kind of directed “field of force” in harmonic space, such that a tone represented by a given point will tend to “become tonic” with respect to tones/points to the “right” of it (in most of my diagrams — i.e. in the 3/2 or “dominant” direction). Such a tone seems capable of absorbing those other tones into what might be called its “tonic field,” and to be absorbed, in its turn, into the tonic field of another tone to the “left” of it (i.e. in the 2/3 or “subdominant” direction), or “below” it. This is analogous to the way in which the harmonic partials in a compound tone seem to be absorbed into the fundamental, but this analogy must not be carried too far, or taken too literally. The harmonic (or “overtone”) series has too often been invoked to explain both consonance and dissonance (e.g. Helmholtz⁷) and the tonic/chord-root phenomenon (e.g. Rameau¹⁰). But the harmonic series cannot truly explain either of these things (any more than this concept of harmonic space can explain them). Although there is one sense of consonance and dissonance which does depend on the harmonic series (and in respect to this one sense of the terms I believe Helmholtz was essentially correct), there are other senses which remain applicable to tones even in the absence of harmonic partials. And it is not — as Rameau postulated — the *son fondamentale* which “generates” the triad, but the

other way around: when there is a sense that a particular pitch is the root of a chord it is surely the chord itself which creates that sense.

To understand the real relation between the harmonic series and musical perception we must ask the following question: why is it that a compound tone consisting of many harmonic partials is normally and immediately perceived as a *single tone*, rather than as a “chord?” The science of psychoacoustics does not yet provide a satisfactory answer to this question, but I predict that — when it does — it will be seen that it is the nature of harmonic perception in the auditory system which “explains” the unique perceptual character of the harmonic series, not (again) the other way around. The harmonic series is not so much a causal factor in harmonic perception as it is a physical manifestation of a *principle* which is also manifested (though somewhat differently) in harmonic perception. That principle involves the mutual compatibility — as elements in a unitary gestalt or “system” (whether physical-acoustical or psychoacoustical) — of frequencies exhibiting certain rational relations to each other.

We can now define *harmony* as *that aspect of musical perception which depends on harmonic relations between pitches — i.e. relations other than “higher” or “lower”*. Thus defined, “harmony” will still include all of those things it now includes — the “vertical aspect of music,” chord-structure, etc. — but it is no longer limited to these, and it is certainly not limited to the “materials and procedures of the diatonic/triadic tonal system . . .” It would, for example, also include pitch-relations manifested in a purely melodic or monophonic situation, and — by this definition —

nearly all music will be found to involve harmony in some way (not just Western “part-music”). In addition, the model of harmonic space outlined here suggests an important “first principle” for a new theory of harmony — that *there is some (set of) specifically harmonic relation(s) between any two salient and relatively stable pitches*.

Yet, by definition, “harmony” does still have some limits in its application, and these are important to recognize. In the case of any music in which no salient and stable pitches occur at all (and there is a great deal of such music in the contemporary literature), harmony — even by this broader definition — would not be relevant. A theory of harmony, therefore, can only be one component in a more general theory of musical perception, and that more general theory must *begin* — as the work of John Cage repeatedly demonstrates — with the primary dimension common to all music: *time*.

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A Chronological Bibliography of Writings by John Cage

The titles of books in which these articles are currently to be found (not necessarily where they were first printed) are abbreviated as follows (the page numbers given with these abbreviations are those on which each article begins):

- S: *Silence* (Middletown, Ct.: Wesleyan University Press, 1961)
CPC: *Cage/Peters Catalogue*, ed. Robert Dunn
(New York: C.F. Peters Corp., 1962)
AYM: *A Year From Monday*
(Middletown, Ct.: Wesleyan University Press, 1967)
JC: *John Cage*, ed. Richard Kostelanetz (New York: Praeger, 1971)
M: *M* (Middletown, Ct.: Wesleyan University Press, 1973)
EW: *Empty Words*
(Middletown, Ct.: Wesleyan University Press, 1979)
FB: *For the Birds*. In conversation with Daniel Charles
(Salem, N.H.: Marion Boyars, 1981)

- 1937 The Future of Music (S 3)
1942 For More New Sounds (JC 64)
1948 Defense of Satie (JC 77)
1949 Forerunners of Modern Music (S 62)
1951 Satie Controversy (letters; JC 89)
1952 To Describe the Process of Composition Used in
Music of Changes and *Imaginary Landscape No. 4* (S 57)
1954 45' for a Speaker (S 146)
1955 Experimental Music: Doctrine (S 13)
1956a In This Day (S 94)
1956b letter to Paul Henry Lang (JC 116)
1957 Experimental Music (S 7)
1958a Composition as Process I. Changes (S 18)

- 1958b Edgard Varèse (S 83)
- 1959a Indeterminacy (S 260)
- 1959b History of Experimental Music in the United States (S 67)
- 1961 Interview with Roger Reynolds (D 45)
- 1962 Rhythm Etc. (AYM 120)
- 1963 Happy New Ears! (AYM 30)
- 1964 Jasper Johns: Stories and Ideas (AYM 73)
- 1965 Diary: How to Improve the World
(You Will Only Make Matters Worse) 1965 (AYM 3)
- 1966 Seriously Comma (AYM 26)
- 1967a Diary: How to Improve the World . . . Continued 1967
(AYM 145).
- 1967b Afterword (to AYM 163)
- 1968 Diary: How to Improve the World . . . Continued 1968
(Revised) (M 3)
- 1972 Diary: How to Improve the World . . . Continued 1971–72
(M 195)