Aula 4 - Controle Adaptativo

SEM5875 - Controle de Sistemas Robóticos

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Teoria de Lyapunov

- Sugestão de Leitura
- Cap. 5 - Estabilidade de Lyapunov
- Disciplina PTC 2417 - Controle Não Linear
- EP - USP - Prof. Paulo Sérgio Pereira da Silva

Parametrização Linear

- Propriedade de robôs manipuladores

$$\tau = M(\mathbf{q})\ddot{\mathbf{q}} + V(\mathbf{q}, \dot{\mathbf{q}}) + G(\mathbf{q}) = Y(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\theta$$

sendo θ o vetor de parâmetros do manipulador e $Y(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ a matriz de regressão.

- Exemplo: manipulador planar dois elos
- Elementos de $M(\mathbf{q})$

$$\begin{aligned}
M_{11}(\mathbf{q}) &= m_1 a_{c1}^2 + m_2(a_1^2 + a_{c2}^2 + 2a_1 a_{c2} cos(q_2)) + I_1 + I_2 \\
M_{12}(\mathbf{q}) &= M_{21}(\mathbf{q}) = m_2(a_{c2}^2 + a_1 a_{c2} cos(q_2)) + I_2 \\
M_{22}(\mathbf{q}) &= m_2 a_{c2}^2 + I_2
\end{aligned}$$

Parametrização Linear

- Vetor dos torques de Coriolis e centrípetos

$$V(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} -2m_2 a_1 a_{c2} sen(q_2)\dot{q}_1\dot{q}_2 + m_2 a_1 a_{c2} sen(q_2)\dot{q}_2^2 \\ m_2 a_1 a_{c2} sen(q_2)\dot{q}_1^2 \end{bmatrix}$$

- Vetor dos torques não inerciais

$$G(\mathbf{q}) = \begin{bmatrix} (m_1 g a_{c1} + m_2 g a_1) cos(q_1) + m_2 g a_{c2} cos(q_1 + q_2) \\ m_2 g a_{c2} cos(q_1 + q_2) \end{bmatrix}$$

- Definindo: $\theta = [m_1 \;\; m_2 \;\; I_1 \;\; I_2]^T$
- Matriz de regressão

$$Y(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \begin{bmatrix} y_{11} & y_{12} & \ddot{q}_1 & \ddot{q}_1 + \ddot{q}_2 \\ 0 & y_{22} & 0 & \ddot{q}_1 + \ddot{q}_2 \end{bmatrix}$$

$$y_{11} = a_{c1}^2 \ddot{q}_1 + g a_{c1} cos(q_1)$$

$$\begin{aligned} y_{12} = {} & (a_1^2 + a_{c2}^2 + 2a_1 a_{c2} cos(q_2))\ddot{q}_1 + (a_{c2}^2 + a_1 a_{c2} cos(q_2))\ddot{q}_2 \\ & - a_1 a_{c2} sen(q_2)(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2) + g a_1 cos(q_1) + g a_{c2} cos(q_1 + q_2)) \end{aligned}$$

$$y_{22} = (a_{c2}^2 + a_1 a_{c2} cos(q_2))\ddot{q}_1 + a_{c2}^2 \ddot{q}_2 + a_1 a_{c2} sen(q_2)\dot{q}_1^2 + g a_{c2} cos(q_1 + q_2)$$

Parametrização Linear

- Forma mais geral

$$M(\mathbf{q})\ddot{\mathbf{r}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{r}} + G(\mathbf{q}) = Y(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})\theta$$

Controle Adaptativo baseado no Torque Calculado

- Torque Calculado + PD

$$\tau = \hat{M}(\mathbf{q})(\ddot{\mathbf{q}}^d + K_v\dot{\mathbf{e}} + K_p\mathbf{e}) + \hat{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \hat{F}(\dot{\mathbf{q}}) + \hat{G}(\mathbf{q})$$

- Valores estimados dos parâmetros são mantidos fixos
- Controle adaptativo: valores estimados dos parâmetros variam com o tempo
- Pode-se reescrever

$$\tau = \hat{M}(\mathbf{q})(\ddot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{e}) + \hat{M}(\mathbf{q})\ddot{\mathbf{q}} + \hat{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \hat{F}(\dot{\mathbf{q}}) + \hat{G}(\mathbf{q})$$

$$\tau = \hat{M}(\mathbf{q})(\ddot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{e}) + Y(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\hat{\theta}$$

Controle Adaptativo baseado no Torque Calculado

- Considerando: $\tau = Y(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\theta$
- Equação do erro:

$$\ddot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{e} = \hat{M}^{-1}(\mathbf{q})Y(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\tilde{\theta}$$

- sendo $\tilde{\theta} = \theta - \hat{\theta}$
- Espaço de estados: $\mathbf{x} = [\mathbf{e} \quad \dot{\mathbf{e}}]^T$

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{e}} \\ \ddot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ I \end{bmatrix}\hat{M}^{-1}(\mathbf{q})Y(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\tilde{\theta}$$

Controle Adaptativo baseado no Torque Calculado

- Funçao de Lyapunov

$$V = \mathbf{x}^T P \mathbf{x} + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$

- $P \Rightarrow$ Matriz simétrica definida positiva
- $\Gamma \Rightarrow$ Matriz diagonal definida positiva

$$\dot{V} = \mathbf{x}^T P \dot{\mathbf{x}} + \dot{\mathbf{x}}^T P \mathbf{x} + 2\tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}}$$

$$\begin{aligned} \dot{V} =& \mathbf{x}^T P (A\mathbf{x} + B\hat{M}^{-1}(\mathbf{q}) Y(\cdot)\tilde{\theta}) \\ &+ (A\mathbf{x} + B\hat{M}^{-1}(\mathbf{q}) Y(\cdot)\tilde{\theta})^T P \mathbf{x} + 2\tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \end{aligned}$$

Controle Adaptativo baseado no Torque Calculado

- Manipulando

$$\dot{V} = -\mathbf{x}^T Q\mathbf{x} + 2\tilde{\theta}^T(\Gamma^{-1}\dot{\tilde{\theta}} + Y^T(\cdot)\hat{M}^{-1}(\mathbf{q})B^T P\mathbf{x})$$

- $Q \Rightarrow$ Matriz simétrica definida positiva que satisfaz a equação de Lyapunov:

$$A^T P + PA = -Q$$

- Definindo:

$$\dot{\tilde{\theta}} = -\Gamma Y^T(\cdot)\hat{M}^{-1}(\mathbf{q})B^T P\mathbf{x}$$

- Então

$$\dot{V} = -\mathbf{x}^T Q\mathbf{x}$$

Controle Adaptativo baseado no Torque Calculado

- Parâmetro θ é constante
- Lembrando: $\tilde{\theta} = \theta - \hat{\theta}$
- Lei de adaptação:

$$\dot{\hat{\theta}} = \Gamma Y^T(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \hat{M}^{-1}(\mathbf{q}) B^T P \mathbf{x}$$

- $\mathbf{x}$ é assintoticamente estável
- $\tilde{\theta}$ limitado se $\hat{M}^{-1}(\mathbf{q})$ existe

Controle Adaptativo baseado no Torque Calculado

- Resumo
- Lei de controle:

$$\tau = \hat{M}(\mathbf{q})(\ddot{\mathbf{q}}^d + K_v\dot{\mathbf{e}} + K_p\mathbf{e}) + \hat{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \hat{F}(\dot{\mathbf{q}}) + \hat{G}(\mathbf{q})$$

- Lei de adaptação:

$$\dot{\hat{\theta}} = \Gamma Y^T(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\hat{M}^{-1}(\mathbf{q})B^T P\mathbf{x}$$

- Equação de Lyapunov:

$$A^T P + PA = -Q$$

Controle Adaptativo relacionado à Matriz de Inércia

- Define-se

$$r = \Lambda \mathbf{e} + \dot{\mathbf{e}}$$

- Equação dinâmica:

$$\tau = -M(\mathbf{q})\dot{\mathbf{r}} - C(\mathbf{q}, \dot{\mathbf{q}})\mathbf{r} + Y(\cdot)\theta$$

- sendo

$$Y(\cdot)\theta = M(\mathbf{q})(\ddot{\mathbf{q}}^d + \Lambda\dot{\mathbf{e}}) + C(\mathbf{q}, \dot{\mathbf{q}})(\dot{\mathbf{q}}^d + \Lambda\mathbf{e}) + F(\dot{\mathbf{q}}) + G(\mathbf{q})$$

- Função de Lyapunov

$$V = \frac{1}{2}\mathbf{r}^T M(\mathbf{q})\mathbf{r} + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta}$$

- Derivando

$$\dot{V} = \mathbf{r}^T M(\mathbf{q})\dot{\mathbf{r}} + \frac{1}{2}\mathbf{r}^T \dot{M}(\mathbf{q})\mathbf{r} + \tilde{\theta}^T \Gamma^{-1}\dot{\tilde{\theta}}$$

- Substituindo $M(\mathbf{q})\dot{\mathbf{r}}$

$$\dot{V} = \mathbf{r}^T (Y(\cdot)\theta - \tau) + \mathbf{r}^T \left(\frac{1}{2}\dot{M}(\mathbf{q}) - C(\mathbf{q}, \dot{\mathbf{q}})\right)\mathbf{r} + \tilde{\theta}^T \Gamma^{-1}\dot{\tilde{\theta}}$$

$$\dot{V} = \mathbf{r}^T (Y(\cdot)\theta - \tau) + \tilde{\theta}^T \Gamma^{-1}\dot{\tilde{\theta}}$$

- Selecionando

$$\tau = Y(\cdot)\hat{\theta} + K_v \mathbf{r}$$

- Temos

$$\dot{V} = -\mathbf{r}^T K_v \mathbf{r} + \tilde{\theta}^T(\Gamma^{-1}\dot{\tilde{\theta}} + Y^T(\cdot)\mathbf{r})$$

- Selecionando

$$\dot{\hat{\theta}} = -\dot{\tilde{\theta}} = \Gamma Y^T(\cdot)\mathbf{r}$$

- Temos

$$\dot{V} = -\mathbf{r}^T K_v \mathbf{r}$$

Controle Adaptativo relacionado à Matriz de Inércia

- Resumo:
- Lei de controle:

$$\tau = Y(\cdot)\hat{\theta} + K_v \Lambda \mathbf{e} + K_v \dot{\mathbf{e}}$$

- Lei de adaptação:

$$\dot{\hat{\theta}} = \Gamma Y^T(\cdot)(\Lambda \mathbf{e} + \dot{\mathbf{e}})$$

- sendo

$$Y(\cdot)\hat{\theta} = \hat{M}(\mathbf{q})(\ddot{\mathbf{q}}^d + \Lambda \dot{\mathbf{e}}) + \hat{C}(\mathbf{q}, \dot{\mathbf{q}})(\dot{\mathbf{q}}^d + \Lambda \mathbf{e}) + \hat{F}(\dot{\mathbf{q}}) + \hat{G}(\mathbf{q})$$

- UARM II

$$\theta = \begin{bmatrix} m_1 l_{c1}^2 \\ m_2 l_1^2 \\ m_2 l_1 l_{c2} \\ m_3 l_1^2 \\ m_3 l_{c3}^2 \\ m_3 l_1 l_{c3} \\ I_1 \\ I_3 \end{bmatrix}$$

$$Y(\mathbf{q}, \dot{\mathbf{q}}, y_1, y_2)$$

$$y_1 = \ddot{\mathbf{q}}^d + \Lambda \dot{\mathbf{e}}$$

$$y_2 = \dot{\mathbf{q}}^d + \Lambda \mathbf{e}$$

Tarefa - Entrega na Próxima Aula

- Implementar o primeiro método no simulador do UARM II