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Respostas da Lista de Exercícios 1

1. $\vec{F} = \frac{qQ}{2\pi^2\epsilon_0 a^2}\hat{y}$

2. $\vec{F} = \frac{q\lambda}{2\pi\epsilon_0\rho}\hat{y}$

3. $\vec{F} = -\frac{Q\sigma_0}{6\epsilon_0}\hat{z}$

4. $\vec{E} = \frac{2\lambda D\ell}{\pi\epsilon_0(D^2+\ell^2)(D^2+2\ell^2)^{1/2}}\hat{z}$

5. $\vec{E} = \frac{\lambda}{4\pi\epsilon_0 y}\left[(\cos\theta_2 - \cos\theta_1)\hat{x} + (\,\mathrm{sen}\,\theta_2 - \,\mathrm{sen}\,\theta_1)\hat{y}\right]$

6. $\vec{E} = -\frac{\lambda_0}{4\epsilon_0 a}\hat{x}$

7. (a) $\alpha = \frac{8}{5}\frac{Q}{\pi R^3}$

   (b)

$$\vec{E} = \begin{cases} \frac{8}{15}\frac{Q}{\pi\epsilon_0 R^3}\vec{r}, & \text{para } r \leq R/2 \\ \frac{Q}{60\pi\epsilon_0 r^2}\left[-1 + 64\left(\frac{r}{R}\right)^3 - 48\left(\frac{r}{R}\right)^4\right]\hat{r}\,, & \text{para } R/2 \leq r \leq R \\ \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}\,, & \text{para } r \geq R \end{cases}$$

   (c) $\frac{Q_1}{Q} = \frac{4}{15}$

   (d) $T = 2\pi\sqrt{\frac{15m\pi\epsilon_0 R^3}{8eQ}}$

   (e) Se $A > R/2$, a força não é mais linear em r $\Rightarrow$ não temos um MHS.

8. (a) $\rho_0 = \frac{e}{\pi a^3}$

(b) $\vec{E} = \frac{e}{4\pi\epsilon_0 r^2} e^{-2r/a} \left(1 + \frac{2r}{a} + \frac{2r^2}{a^2}\right) \hat{r}$

9. $a = \frac{R}{2}$

10. (a)

$$\vec{E} = \begin{cases} \frac{A}{2\epsilon_0} \hat{r}\,, & \text{para } r < R \\ \frac{A}{2\epsilon_0} \frac{R^2}{r^2} \hat{r}\,, & \text{para } r > R \end{cases}$$

(b)

$$V = \begin{cases} \frac{A}{\epsilon_0} \left(R - \frac{r}{2}\right)\,, & \text{para } r < R \\ \frac{AR^2}{2\epsilon_0 r}\,, & \text{para } r > R \end{cases}$$

11. (a) $\vec{\nabla} \cdot \mathcal{C} = 0$

(b) $\oint \vec{\mathcal{C}} \cdot d\vec{a} = 4\pi$

(c) $\int \vec{\nabla} \cdot \mathcal{C}\, d\mathcal{V} = 4\pi \neq 0$. A contribuição à divergência vem do ponto $r = 0$.

(d) $\vec{\nabla} \cdot \mathcal{C} = 4\pi\delta^3(\vec{r})$

12. $\rho = \frac{q}{4\pi r^a}(3 - a)$, $V = \frac{q}{4\pi\epsilon_0 (a-2) r^{a-2}}$

13. $\vec{E} = A(1 + \lambda r) e^{-\lambda r} \frac{\hat{r}}{r^2}\,, \quad \rho = \epsilon_0 A \left[-\frac{\lambda^2}{r} e^{-\lambda r} + 4\pi\delta^3(r)\right]\,, \quad Q = 0$

14. $V = \frac{Q}{4\pi\epsilon_0 h} \ln \left[\frac{\sqrt{R^2+(d+h)^2}+d+h}{\sqrt{R^2+d^2}+d}\right]$

15. (a) $V = \frac{Q}{4\pi\epsilon_0 L} \ln \left(\frac{x+L/2}{x-L/2}\right)$

(b) Demonstração

16. (a) $V = \frac{\sigma}{2\epsilon_0} \left(\sqrt{b^2 + x^2} - \sqrt{a^2 + x^2}\right)$

(b) Demonstração

17. (a) $\vec{E} = -\frac{\sigma}{4\epsilon_0} \hat{z}$

(b) $V_P - V_C = \frac{\sigma R}{2\epsilon_0}(\sqrt{2} - 1)$

18. $V = \frac{\sigma_0 R}{4\epsilon_0} \ln \left(\frac{\sqrt{L^2+R^2}+L}{\sqrt{L^2+R^2}-L}\right)$

19. (a) $V = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{L^2+(x+L)^2}} + \frac{1}{\sqrt{L^2+(x-L)^2}}\right]$

(b) Demonstração

(c) Demonstração

(d) $\omega = \sqrt{\frac{\sqrt{2}\, Qq}{16\pi\epsilon_0 L^3 m}}$

20. $C = \frac{4\pi\epsilon_0}{\frac{1}{\kappa_1}\left(\frac{1}{a}-\frac{1}{c}\right)+\frac{1}{\kappa_2}\left(\frac{1}{c}-\frac{1}{b}\right)}$

21. (a) $C = 2\pi\epsilon_0(K+1)\frac{r_a r_b}{r_b - r_a}$

(b) $E_1 = E_2 = \frac{Q}{2\pi\epsilon_0(K+1)r^2}$

22. $C = \frac{\epsilon_0 ab}{y_0}\ln 2$

23. Demonstração

24. $R = \frac{\rho\pi}{2t\ \ln(b/a)}$

25. $R = \frac{1}{\sigma\pi a}\ln\left(\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}\right)$