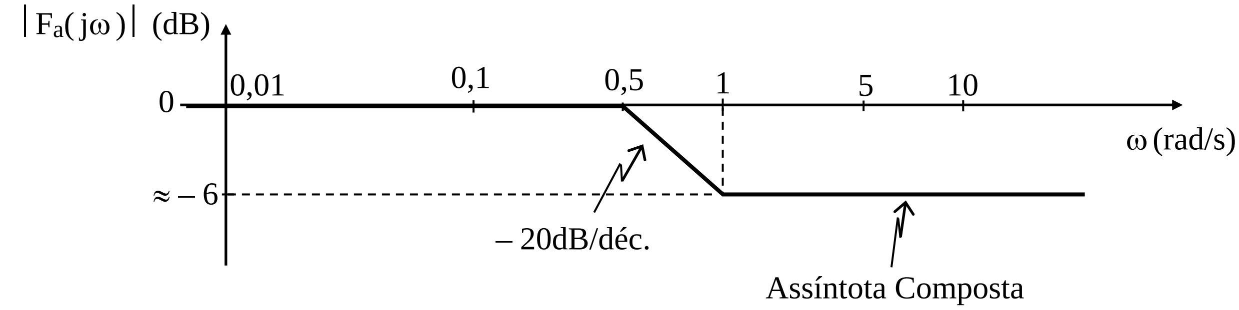
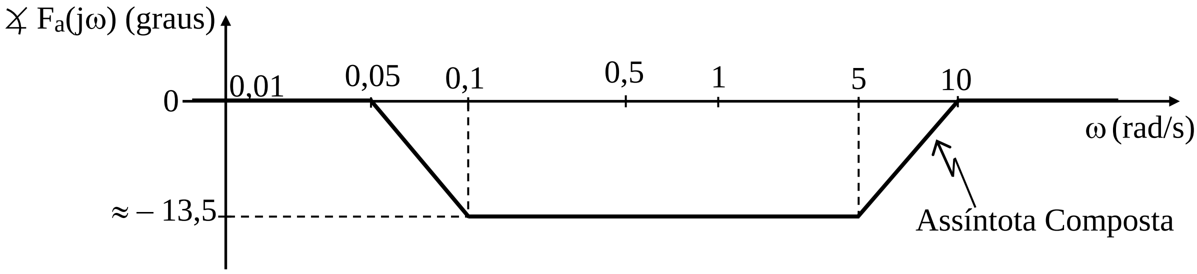
PSI3213 – CIRCUITOS ELÉTRICOS II

Solução da Lista 6: Diagramas de Bode

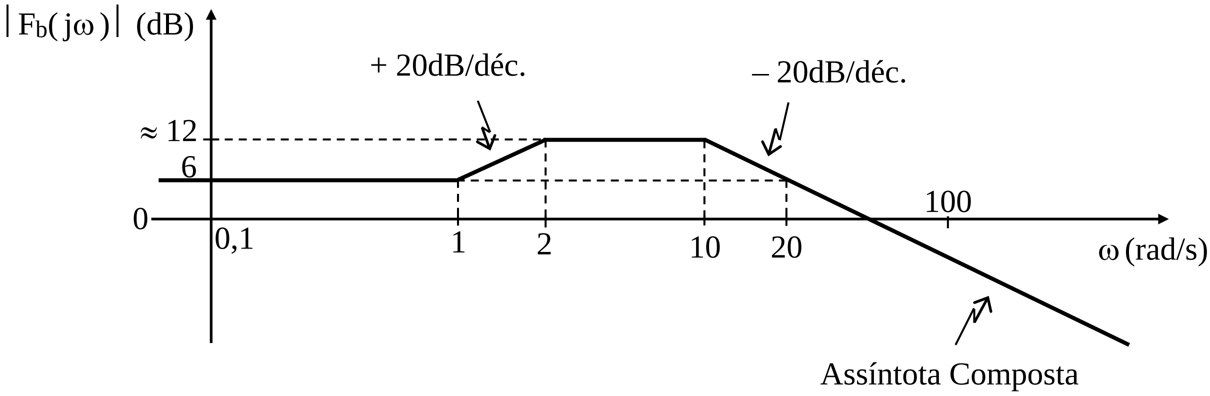
1 – a) $F_a(s) = \dfrac{s+1}{2s+1} \Rightarrow \begin{cases} \text{zero: } z_1 = -1 \\ \text{polo: } p_1 = -\dfrac{1}{2} \end{cases}$

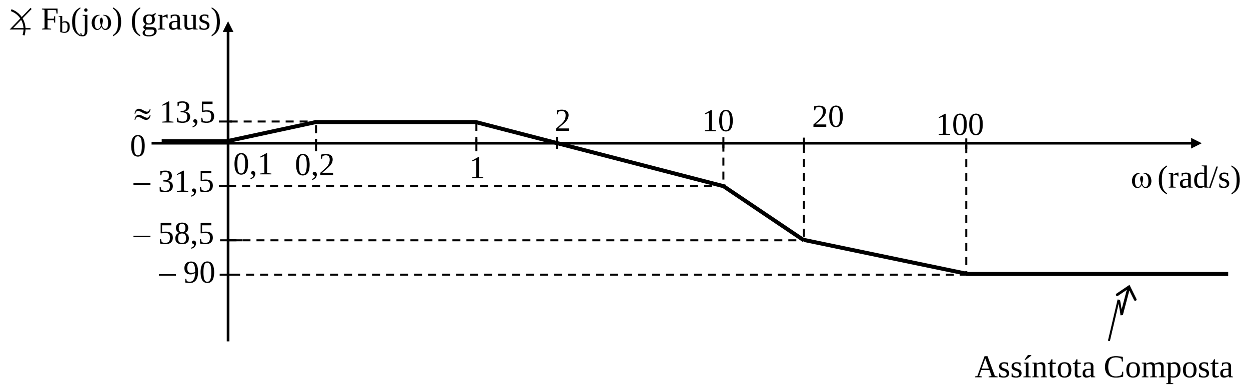




b) $F_b(s) = \dfrac{40(s+1)}{(s+2)(s+10)} = \underbrace{2}_{=K} \dfrac{(s+1)}{\left(\dfrac{s}{2}+1\right)\left(\dfrac{s}{10}+1\right)} \Rightarrow \begin{cases} \text{zero: } z_1 = -1 \\ \text{polos: } \begin{cases} p_1 = -2 \\ p_2 = -10 \end{cases} \end{cases}$

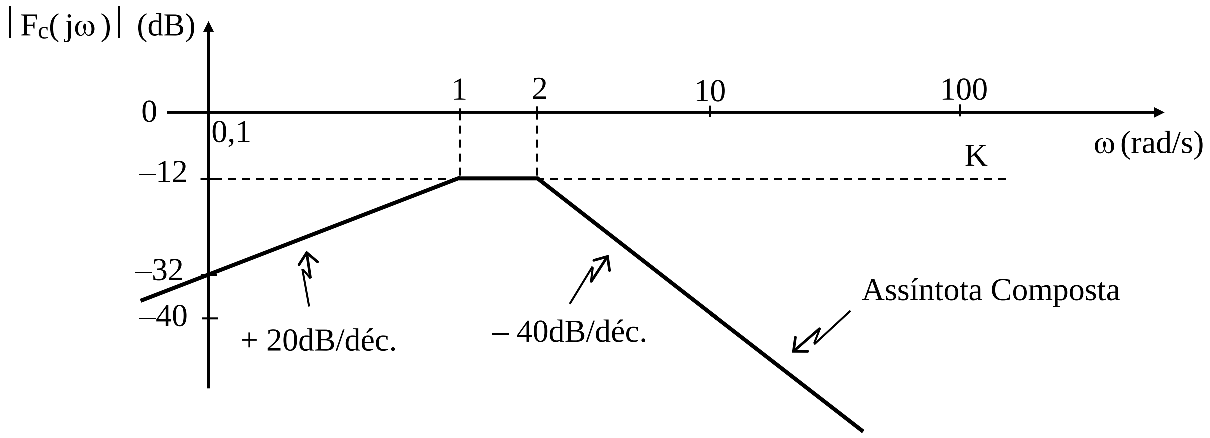
$20 \log|K| \approx 6 \text{ dB}$

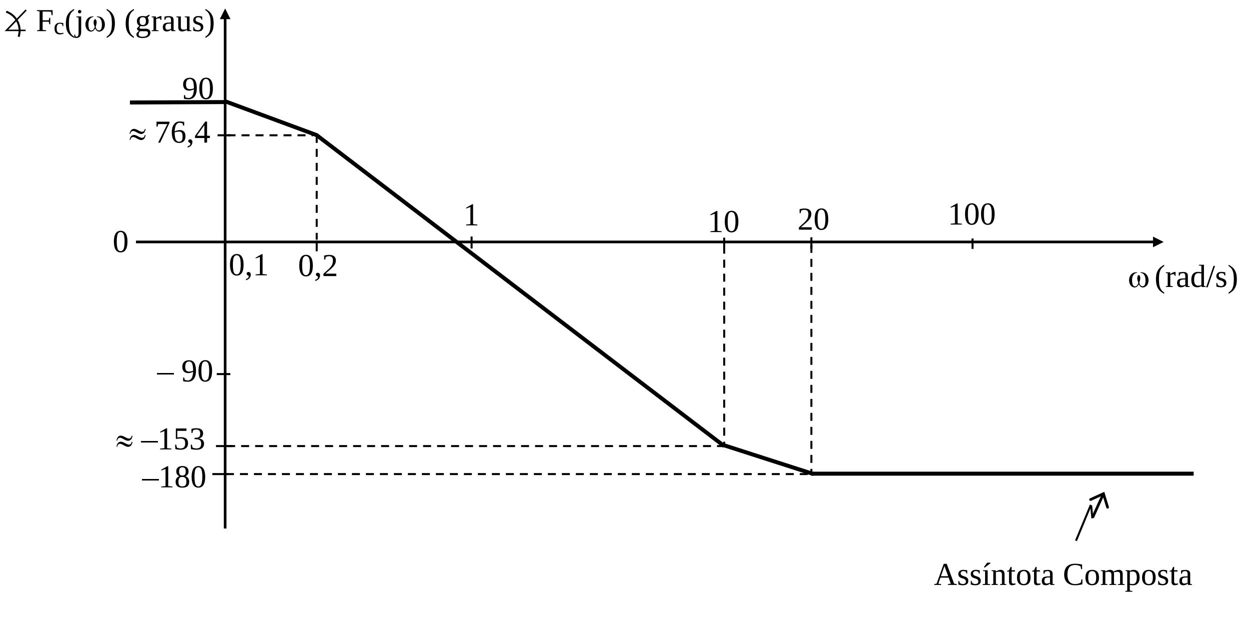




c) $$F_c(s) = \frac{s}{(s+2)^2(s+1)} = \underbrace{\frac{1}{4}}_{=K} \frac{s}{\left(\frac{s}{2}+1\right)^2(s+1)} \Rightarrow \begin{cases} \text{zero}: z_1 = 0 \\ \text{polos}: \begin{cases} p_1 = p_2 = -2 \\ p_3 = -1 \end{cases} \end{cases}$$

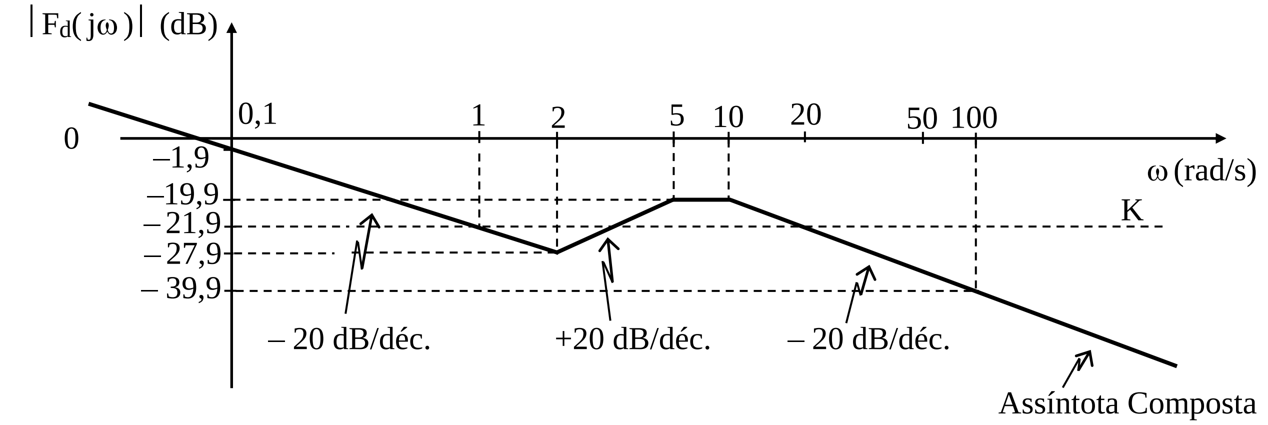
$$20 \log |K| \approx 20 \log\left(\frac{1}{4}\right) \approx -12 \text{ dB}$$

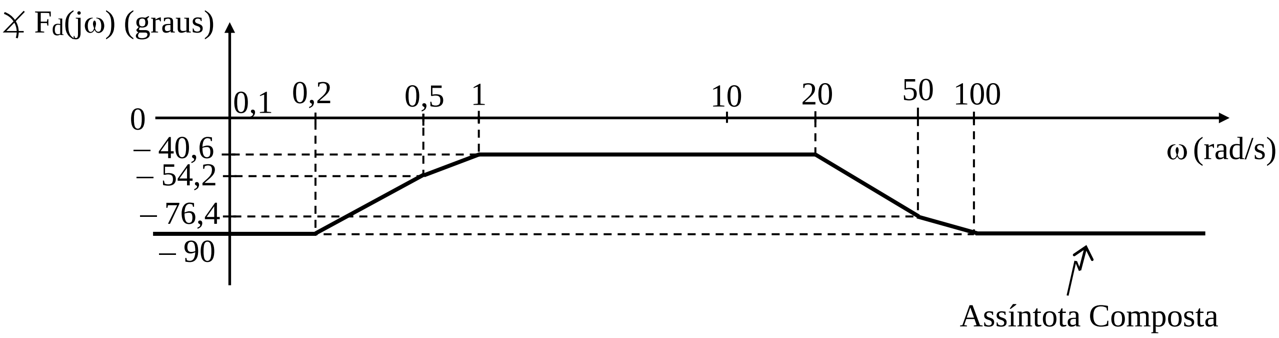




$$\text{d)}\quad F_d(s) = \frac{(s+2)^2}{s(s+5)(s+10)} = \underbrace{\frac{2}{25}}_{=K}\cdot\frac{\left(\frac{s}{2}+1\right)^2}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{10}+1\right)} \Rightarrow \begin{cases} \text{zeros}: & z_1 = z_2 = -2 \\ \text{polos}: & \begin{cases} p_1 = 0 \\ p_2 = -5 \\ p_3 = -10 \end{cases} \end{cases}$$

$$20\log|K| = 20\log\left(\frac{2}{25}\right) \approx -21{,}9\ \text{dB}$$





2 – Analisando cada frequência individualmente e considerando desprezível o erro de uma assíntota em cinco ou dez vezes sua frequência característica, temos:

- $\omega = 1$ rad/s : erro de $\approx +3$ dB devido ao zero em $\omega = 1$ rad/s + erro de ≈ -1 dB devido ao polo em $\omega = 2$ rad/s $\Rightarrow$ ERRO $\approx +2$ dB

- $\omega = 2$ rad/s : erro de ≈ -3 dB devido ao polo em $\omega = 2$ rad/s + erro de $\approx +1$ dB devido ao zero em $\omega = 1$ rad/s $\Rightarrow$ ERRO ≈ -2 dB

- $\omega = 10$ rad/s : erro de ≈ -3 dB devido ao polo em $\omega = 10$ rad/s $\Rightarrow$ ERRO ≈ -3 dB

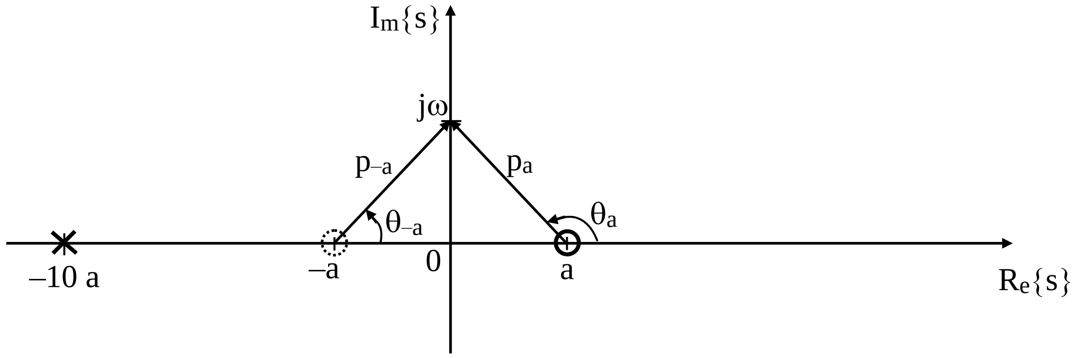
- $\omega = 20$ rad/s : erro de ≈ -1 dB devido ao polo em $\omega = 10$ rad/s $\Rightarrow$ ERRO ≈ -1 dB

3 – $G(s) = \dfrac{s-a}{s+10a}$, $a > 0$

$$\Rightarrow \quad G(s) = \left(\frac{\cancel{a}}{10\cancel{a}}\right) \frac{\left(\frac{s}{a} - 1\right)}{\left(\frac{s}{10a} + 1\right)} = \underbrace{\frac{1}{10}}_{=K} \frac{\left(\frac{s}{a} - 1\right)}{\left(\frac{s}{10a} + 1\right)} \quad \Rightarrow \begin{cases} \text{zero: } z_1 = a \\ \text{polo: } p_1 = -10a \end{cases}$$

$$20 \log |K| = 20 \log\left(\frac{1}{10}\right) = -20 \text{ dB}$$

Como há um zero no semiplano direito, vamos verificar como ele difere de um zero no semiplano esquerdo a partir do seguinte diagrama de polos e zeros.



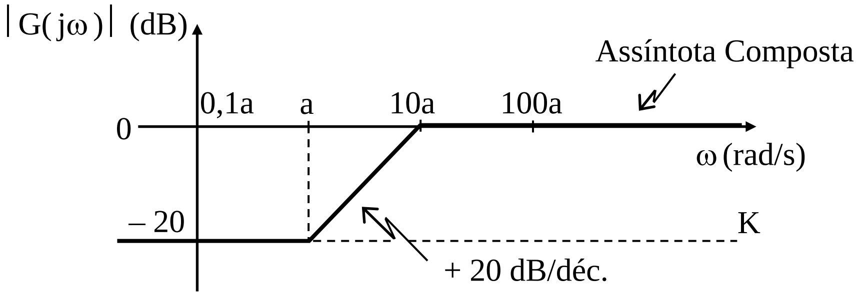
Notamos que os módulos da resposta em frequência para os zeros em s = a e s = – a são iguais, ou seja,

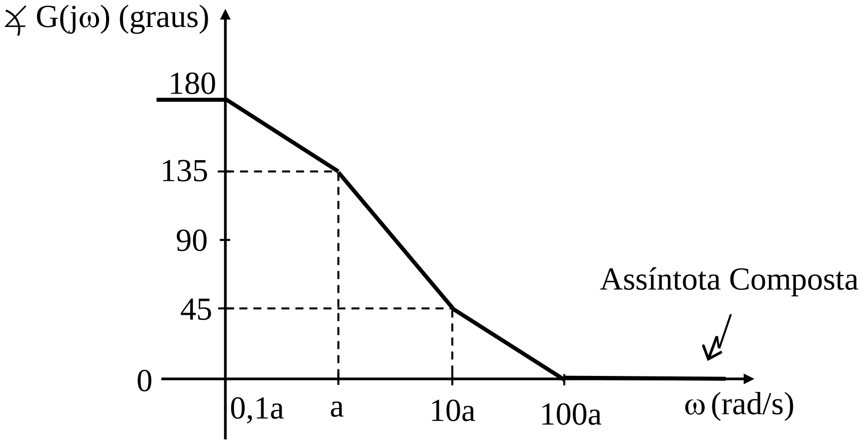
$$p_{-a} = p_a$$

Notamos também que a soma das fases da resposta em frequência para os zeros em s = a e s = –a é de 180º. Logo,

$$\theta_a = 180^o - \theta_{-a}$$

Assim, para construir o diagrama de Bode, vamos supor inicialmente o zero em s = –a e depois corrigir sua contribuição na fase da resposta em frequência de acordo com a expressão acima.





Possível aplicação: Circuito passa-altas para remoção de nível DC em sinal senoidal de frequência $\omega > 10a$.

4 – Entrada : $e_S(t) = \delta(t) \longleftrightarrow E_S(s) = 1$

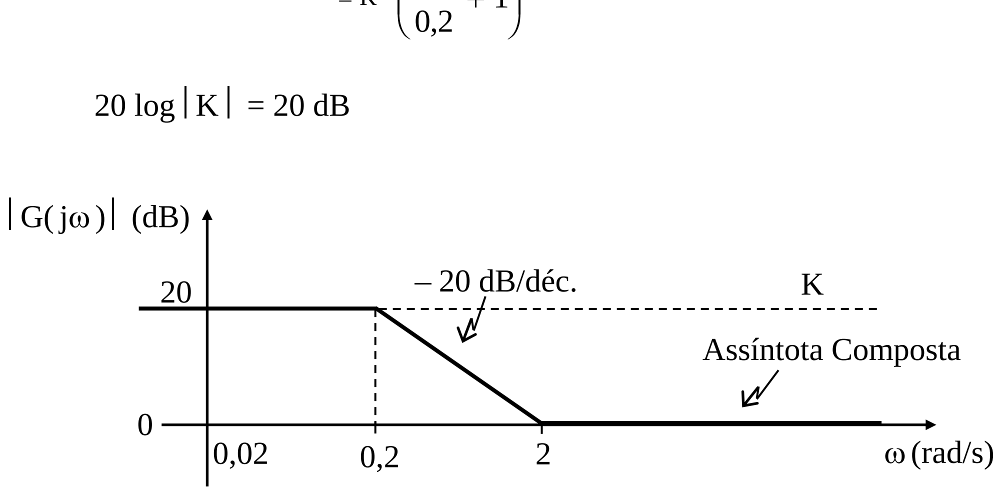
Saída: $h(t) = \delta(t) + 1{,}8\, e^{-0{,}2\,t}\, H(t) \longleftrightarrow H(s) = 1 + \dfrac{1{,}8}{s+0{,}2}$

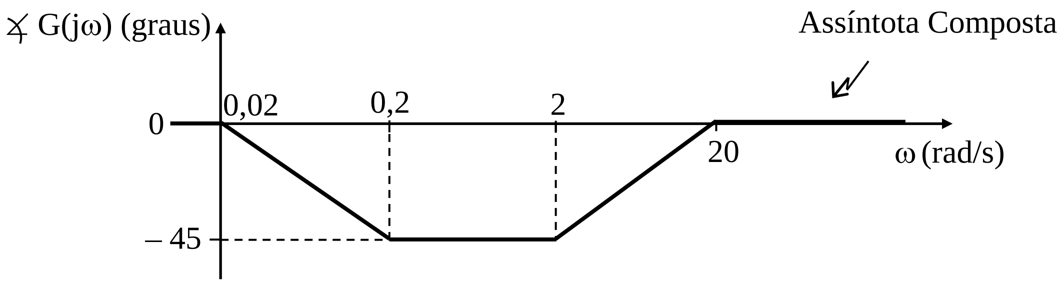
a) Função de rede:

$$G(s) = \frac{H(s)}{\underbrace{\cancel{E_S(s)}}_{1}} = 1 + \frac{1{,}8}{s+0{,}2} \Rightarrow G(s) = \frac{s+2}{s+0{,}2} = \underbrace{10}_{=K}\,\frac{\left(\dfrac{s}{2}+1\right)}{\left(\dfrac{s}{0{,}2}+1\right)}$$

$$G(s) = \frac{s+2}{s+0{,}2} = \underbrace{10}_{=K}\,\frac{\left(\dfrac{s}{2}+1\right)}{\left(\dfrac{s}{0{,}2}+1\right)} \rightarrow \begin{cases} \text{zero: } z_1 = -2 \\ \text{polo: } p_1 = -0{,}2 \end{cases}$$

$20 \log |K| = 20$ dB





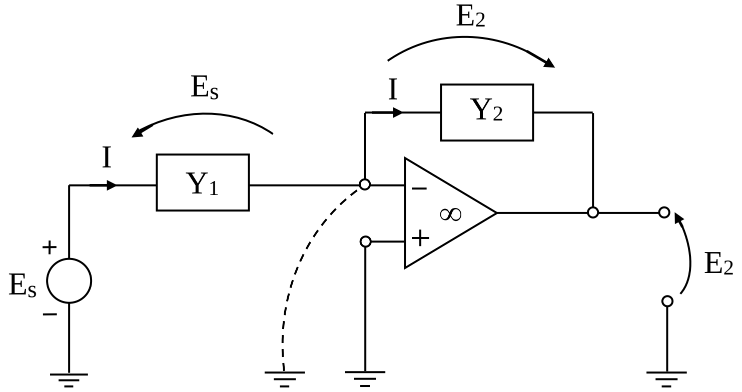
b) $e_s(t) = \cos(0{,}1t) + 10\cos(4t)$

$$\omega = 0{,}1\,\frac{\text{rad}}{\text{s}} :\ G(j0{,}1) \approx 10e^{-j\,31{,}4^o}$$

$$\omega = 4\,\frac{\text{rad}}{\text{s}} :\ G(j4) \approx 1e^{-j\,31{,}4^o}$$

$$\Rightarrow\ e_0(t) = 10\cos(0{,}1\,t - 31{,}4^o) + 10\cos(4t - 31{,}4^o)$$

5 – a)



$$I = E_sY_1 = -E_2Y_2 \ \Rightarrow\ G_v(s) = \frac{E_2(s)}{E_s(s)} = -\frac{Y_1(s)}{Y_2(s)}$$

b) $Y_1(s) = s0{,}25.10^{-6} + \dfrac{1}{100.10^3} \ \Rightarrow$

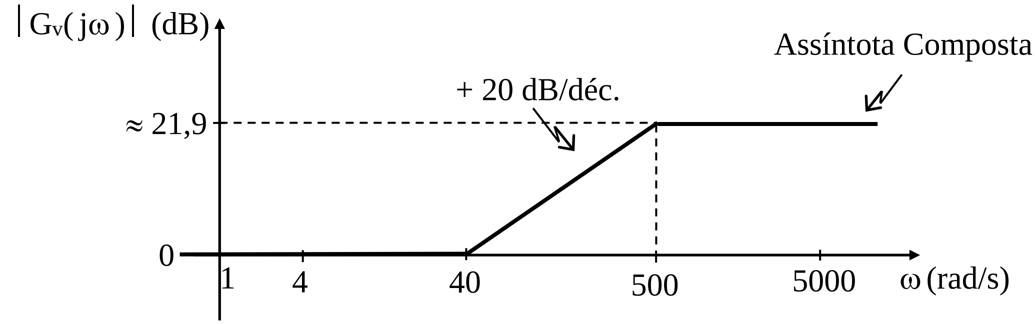
$$Y_1(s) = s0{,}25.10^{-6} + 10^{-5} = 10^{-6}(s0{,}25 + 10)s$$

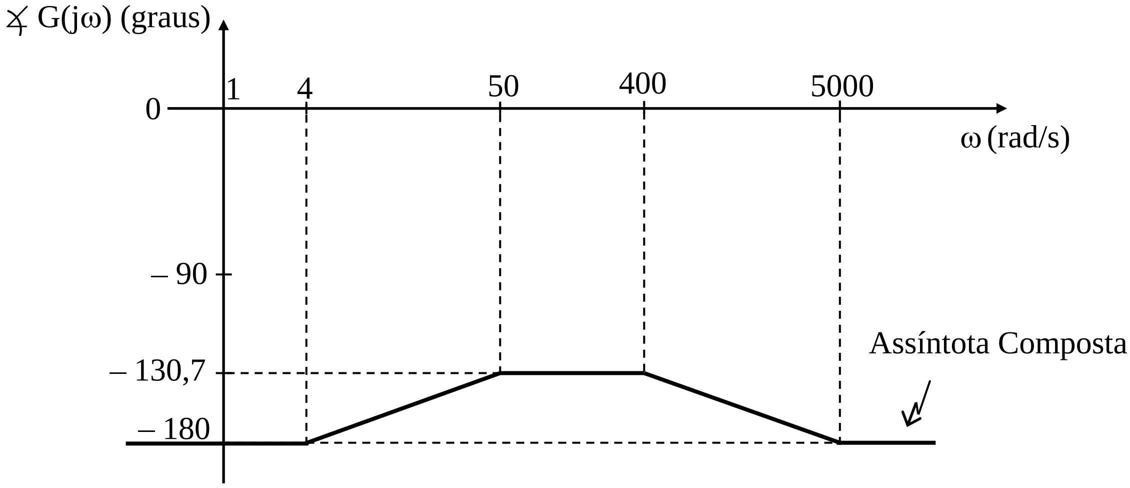
$$Y_2(s) = s0{,}02.10^{-6} + \frac{1}{100.10^3} = 10^{-6}(s0{,}02 + 10)s$$

$$\Rightarrow\ G_v(s) = -\frac{Y_1(s)}{Y_2(s)} = -\frac{s0{,}25+10}{s0{,}02 + 10}$$

$$\Rightarrow \quad G_v(s) = \underbrace{-1}_{=K} \frac{\left(\frac{s}{40}+1\right)}{\left(\frac{s}{500}+1\right)} \quad \Rightarrow \quad \begin{cases} \text{zero}: z_1 = -40 \\ \text{polo}: p_1 = -500 \end{cases}$$

$$\begin{cases} 20 \log |K| = 20 \log 1 = 0 \text{ dB} \\ \measuredangle K = \measuredangle(-1) = \pm 180^o \end{cases}$$



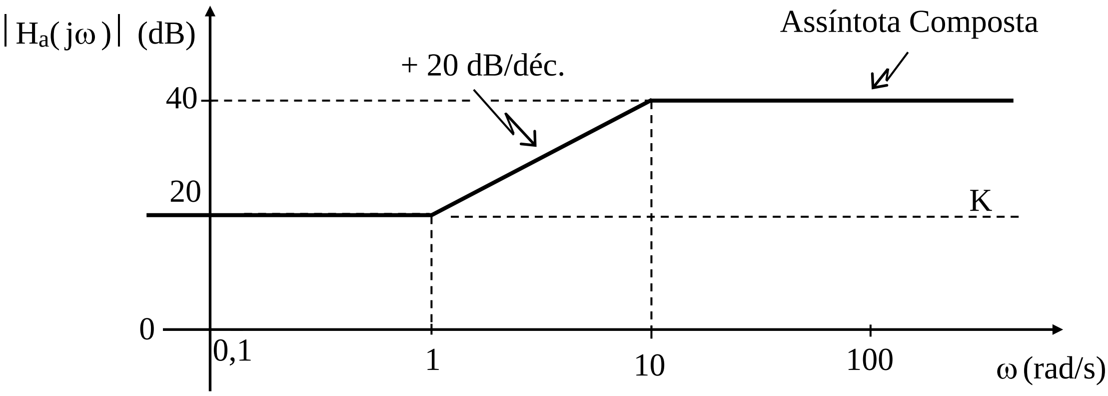


$$6 - a) \qquad H_a(s) = \frac{100\left(s^2+s+1\right)}{10(s+1)\left(\frac{s}{10}+1\right)} = 10 \, \frac{s^2+s+1}{(s+1)\left(\frac{s}{10}+1\right)}$$

$$\Rightarrow \begin{cases} \text{zeros:} \begin{cases} z_1 = -\dfrac{1}{2} + j\dfrac{\sqrt{3}}{2} \\ z_2 = -\dfrac{1}{2} - j\dfrac{\sqrt{3}}{2} \end{cases} \\ \text{polos:} \begin{cases} p_1 = -1 \\ p_2 = -10 \end{cases} \end{cases}$$

Par conjugado: fator quadrático $\left[\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1\right] \Rightarrow \begin{cases}\omega_n = 1 \text{ rad/s} \\ \zeta = 0,5\end{cases}$

$20 \log|K| = 20 \log 10 = 20 \text{ dB}$

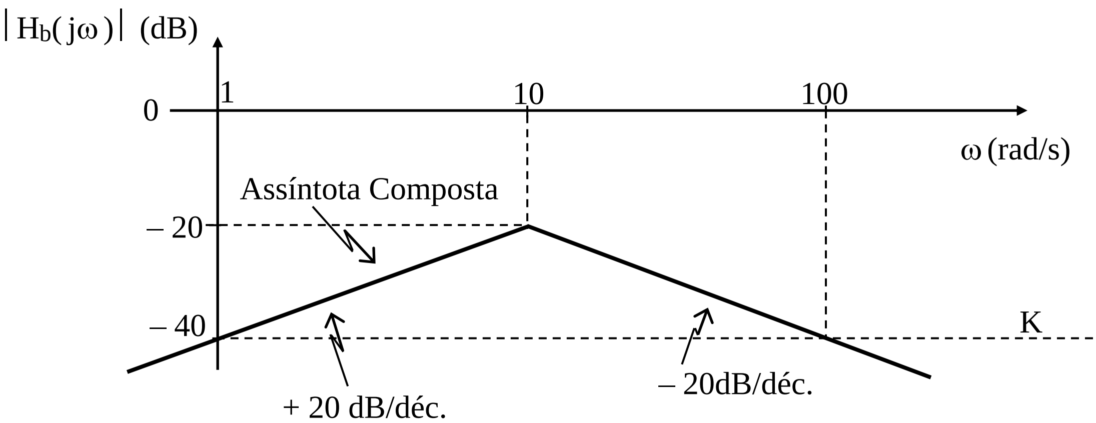


b) $$H_b(s) = \frac{s}{s^2+10s+100} = \underbrace{\frac{1}{100}}_{=K} \cdot \frac{s}{\left[\left(\frac{s}{10}\right)^2 + \frac{s}{10} + 1\right]}$$

$$\Rightarrow \begin{cases} \text{zero: } \{ z_1 = 0 \\ \text{polos: } \begin{cases} p_1 = -5 + j5\sqrt{3} \\ p_2 = -5 - j5\sqrt{3} \end{cases} \end{cases}$$

Par conjugado: fator quadrático $\left[\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1\right] \Rightarrow \begin{cases}\omega_n = 10 \text{ rad/s} \\ \zeta = 0,5\end{cases}$

$20 \log|K| = 20 \log\left(\frac{1}{100}\right) = -40 \text{ dB}$



7 – a) $|K| = 1$

$$\left.\begin{array}{l} \text{zeros:}\begin{cases} z_1 = 0 \\ z_2 = -30 \end{cases} \\ \\ \text{polos:}\begin{cases} p_1 = -1 \\ p_2 = p_3 = -8 \\ p_4 = -100 \end{cases} \end{array}\right\} \Rightarrow \quad F(s) = \pm 1 \frac{s.\left(\frac{s}{30} + 1\right)}{(s+1)\underbrace{\left(\frac{s}{8} + 1\right)^2}_{\zeta = 1}\left(\frac{s}{100} + 1\right)}$$

$$\Rightarrow \quad F(s) = \pm \frac{1}{3\cancel{0}} .8^2.10\cancel{0} \frac{s(s+30)}{(s+1)(s+8)^2(s+100)}$$

$$\Rightarrow \quad F(s) = \pm \frac{640}{3} \frac{s(s+30)}{(s+1)(s+8)^2(s+100)}$$

b) $\text{zeros:}\left\{ z_1 = z_2 = -100 \right.$

$$\text{polos:}\begin{cases} p_1 = 0 \\ \text{fator quadrático}\begin{cases} \omega_n = 20\frac{\text{rad}}{\text{s}} \\ \zeta = 0{,}1 \end{cases} \\ p_4 = -900 \end{cases}$$

$$\begin{cases} 0{,}8\,\text{rad/s} \rightarrow 20\,\text{dB} \\ 1\,\text{rad/s} \rightarrow \;? \end{cases} \Rightarrow \quad -20\log\left(\frac{1}{0{,}8}\right) = -20\log(1{,}25)$$

$$\Rightarrow \quad \omega = 1\,\text{rad/s} \rightarrow (20 - 20\log(1{,}25))\,\text{dB}$$

$$\Rightarrow \quad |K| = 10^{(1-\log(1{,}25))} = \frac{10}{1{,}25} = 8$$

$$\Rightarrow \quad H(s) = \frac{(\pm 8)\left(\frac{s}{100} + 1\right)^2}{s\left[\left(\frac{s}{20}\right)^2 + \frac{2.0{,}1s}{20} + 1\right]\left(\frac{s}{900} + 1\right)} =$$

$$= \pm 8.\frac{1}{100^2}.20^2.\,900.\,\frac{(s+100)^2}{s\left(s^2 + 4s + 400\right)(s+900)} = \pm\, 288 \frac{(s+100)^2}{s\left(s^2 + 4s + 400\right)(s+900)}$$

c) $K \in \mathbb{R}_+$

$$\begin{cases} \text{zero}: & z_1 = -10 \\ \text{polos}: & \begin{cases} p_1 = 0 \\ p_2 = -100 \end{cases} \end{cases}$$

$$G(s) = K \cdot \frac{\left(\frac{s}{10} + 1\right)}{s\left(\frac{s}{100} + 1\right)} = K' \cdot \frac{s+10}{s(s+100)} \quad , \quad K,\ K' \in \mathbb{R}_+$$