PSI3213 – CIRCUITOS ELÉTRICOS II

Solução da Lista 5: Potência e energia em Regime Permanente Senoidal

1 –

Cargas	P(kW)	Q(kVAr)	$\lvert P_{ap} \rvert$ (kVA)	cos ψ
C1	8	6	10	0,8 atr.
C2	12	-16	20	0,6 ad.
C3	5	10	11,18	0,45 atr.
	25	0		

Carga 1: $P = \lvert P_{ap} \rvert \cos\psi \rightarrow \lvert P_{ap} \rvert = 10$ kVA
$Q = \lvert P_{ap} \rvert \operatorname{sen}\psi \rightarrow Q = 6$ kVAr (> 0 ind.)

Carga 2: $P = \lvert P_{ap} \rvert \cos\psi = 12$ kW
$Q = \lvert P_{ap} \rvert \operatorname{sen}\psi = -16$ kVAr

Carga 3: $Y = \dfrac{1}{Z} = \dfrac{1}{2{,}5 + j5} = 0{,}08 - j0{,}16$

$$P_{ap} = G\lvert V \rvert^2 - jB\lvert V \rvert^2 = 5 + j10$$

$$P_{ap}\ \text{total} = (25 + j0)\ \text{kVA}$$

$$\lvert P_{ap} \rvert = \lvert V \rvert\ \lvert I_L \rvert \rightarrow \lvert I_L \rvert = 25000/250 = 100\ \text{Aef}$$

Como $Q_t = 0 \rightarrow \hat{I}_L = 100 \angle 0°$

$$\hat{V}_p = 100(0{,}05 + j0{,}5) = 5 + j50$$

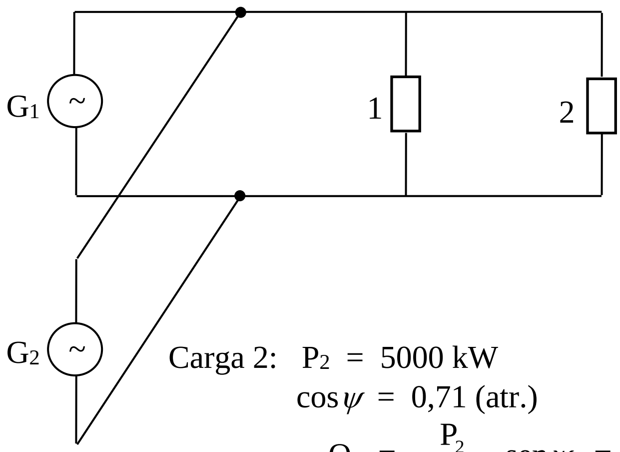
$$\hat{V}_s = \hat{V}_p + 250\angle 0° = 255 + j50 = 259{,}86\angle 11{,}09°$$

$$v_s(t) = 367{,}49\cos(377t + 11{,}09°)\ \ (\text{V,s})$$

Potência real nas cargas: P1 = 8 kW
P2 = 12 kW
P3 = 5 kW

Potência real na linha: $0{,}05 \cdot \lvert I_L \rvert^2 = 0{,}5$ kW

2 –



Carga 1: $P_1 = 3000$ kW
$\cos\psi = 1$
$Q_1 = 0$

Carga 2: $P_2 = 5000$ kW
$\cos\psi = 0{,}71$ (atr.)

$$Q_2 = \frac{P_2}{\cos\psi}\,\text{sen}\,\psi = 4959{,}2\ \text{kVAr}$$

Gerador $G_1 \rightarrow P_{G1} = 5000$ kW
$\cos\psi = 0{,}8$ (atr.)
(convenção de gerador)

$$Q_{G1} = \frac{P_{G1}}{0{,}8}\,\text{sen}\psi = 3750\ \text{kVAr}$$

Teorema da conservação das potências:

$$P_1 + P_2 = P_{G1} + P_{G2} \rightarrow P_{G2} = 3000\ \text{kW}$$
$$\underbrace{Q_1 + Q_2}_{\substack{\text{conv.}\\ \text{receptor}}} = \underbrace{Q_{G1} + Q_{G2}}_{\substack{\text{conv.}\\ \text{gerador}}} \rightarrow Q_{G2} = 1209{,}2\ \text{kVAr}$$

$$\text{tg}\ \psi_2 = \frac{1209{,}2}{3000} = \frac{Q_{G2}}{P_{G2}} = 0{,}40$$

$\cos\psi_2 = 0{,}93$ (atr.) $\leftarrow$ conv. gerador !

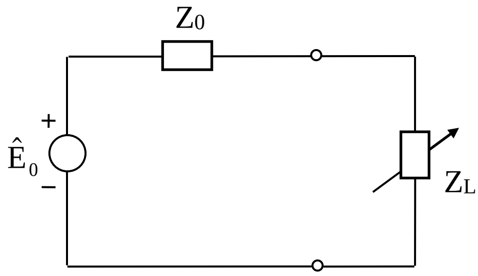
3 – a) $P_{ap\ total} = 300 + j100 = P_{ap1} + P_{ap2}$

$$\left|P_{ap\ total}\right| = \sqrt{(300)^2 + (100)^2} = 316{,}23 = |V|\,|I| \rightarrow |I| = 1{,}58\ \text{Aef}$$

$$P_{total} = R(\omega)|I|^2 \rightarrow R(\omega) = 300/(1{,}58)^2 = 120\ \Omega$$
$$Q_{total} = X(\omega)|I|^2 \rightarrow X(\omega) = 100/(1{,}58)^2 = 40\ \Omega \rightarrow Z = 120 + j40\ \Omega$$

b) $P_1 = R_1|I|^2 \rightarrow R_1 = 40\ \Omega$
$Q_1 = X_1|I|^2 \rightarrow X_1 = 80\ \Omega$
$Z_1 = 40 + j80\ \Omega$

$P_2 = R_2|I|^2 \rightarrow R_2 = 80\ \Omega$
$Q_2 = X_2|I|^2 \rightarrow X_2 = -40\ \Omega$
$Z_2 = 80 - j\,40\ \Omega$



4 – Gerador de Thévenin equivalente:

Tensão em aberto: Divisor de tensão :

$$\hat{E}_0 = \frac{100\angle 0^o \; . \; j3}{25 + j10 + j3} = \frac{300\angle 90^o}{28{,}18 \;\; 27{,}5^o}$$

$$\hat{E}_0 = 10{,}65\angle 62{,}5^o$$

$$\text{Impedância : } Z_0 = \left(25 + j10\right) // \; j3 = \frac{\left(25 + j10\right) j3}{25 + j13}$$

$$Z_0 = 2{,}87\angle 84{,}3 = 0{,}28 + j2{,}85 \; \Omega$$

Condição de máxima transferência de potência ativa à carga Z_L :

a) $Z_L = Z_0^* = 0{,}28 - j\,2{,}85 \;\; \Omega$

b) $P_{máx} = \dfrac{|E_0|^2}{4R} = \dfrac{\left(10{,}65\right)^2}{4 \,.\, 0{,}28} = 101{,}3 \text{ W}$

5 – a)

	P (kW)	Q(kVAr)	$\cos\psi$	$\lvert P_{ap} \rvert$/(kVA)
A	5	0	1	5
B	5	6,67	0,6 atr.	8,33
C	3,72	5,58	0,55 atr.	6,71
	13,72	12,25		

$$\text{Carga C:} \quad Y = \frac{1}{4 + j6} = 0{,}0769 - j0{,}1154 \;\; S$$

$P_{ap} = 0{,}0769\,|V|^2 + 0{,}1154\,|V|^2 = 3{,}72 + j5{,}58$ kVA
$P_{ap\ total} = 13{,}72 + j12{,}25$ kVA
$P_{total} = 13{,}72$ kW $\qquad Q_{total} = 12{,}25$ kVAr

b) $\hat{I}_A = \dfrac{5000}{110}\angle 0° = 45{,}45\angle 0°$ Aef

$\hat{I}_B = \dfrac{8333}{110}\angle\text{-}53{,}13° = 75{,}75\angle\text{-}53{,}13°$ Aef

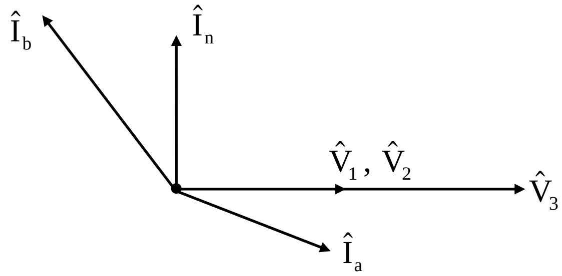
$\hat{I}_C = \dfrac{220\angle 0°}{4 + j6} = 30{,}51\angle\text{-}56{,}31°$ Aef

$\hat{I}_a = \hat{I}_A + \hat{I}_C = 67{,}34\angle\text{-}22{,}15°$ Aef

$$\hat{I}_b = -\left(\hat{I}_B + \hat{I}_C\right) = 106{,}23 \angle 125{,}96° \quad \text{Aef}$$

$$\hat{I}_n = \hat{I}_A - \hat{I}_B = 60{,}6 \angle 90° \quad \text{Aef}$$

c)



6 –

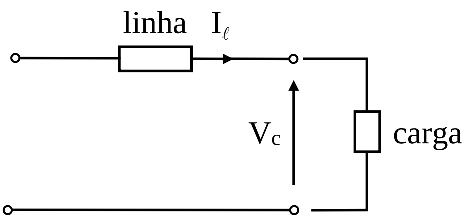
Cargas	P (kW)	Q (kVAr)	cos φ	$\lvert P_{ap} \rvert$ (kVA)
i	250	0	1	250
ii	1500	726,48	0,9 at.	1666,67
iii	1000	750	0,8 at.	1250
iv	700	-339,02	0,9 ad.	777,78

$\Sigma P = 3450$ kW $\qquad \Sigma Q = 1137{,}45$ kVAr

a) $P_{ap\,t} = 3450 + j\,1137{,}45$ kVA

$$\cos\varphi t = \frac{P_t}{\lvert P_{ap\,t} \rvert} = \frac{3450}{3632{,}67} = 0{,}95 \text{ atrasado}$$

b)



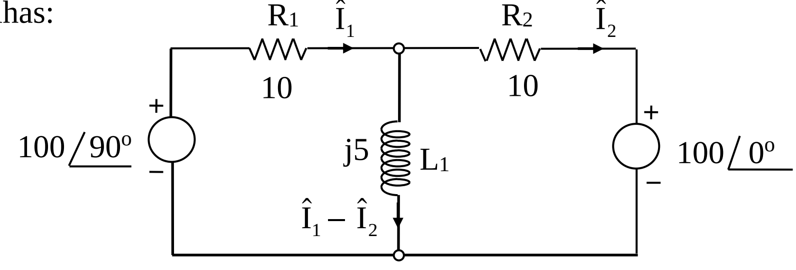
Mesmo aquecimento $\longrightarrow$ mesma $\lvert I_\ell \rvert$

$P_t = V_c\, I_\ell \cos \varphi_t = 3450$ kW

Para $\cos\varphi_t' = 1$, mantendo fixos V_c e I_ℓ $\rightarrow$

$$\longrightarrow P_t' = V_c\, I_\ell = \frac{3450}{0{,}95} = 3632{,}67 \text{ kW}$$

7 – Por análise de malhas:



2ª LK :

$$10\,\hat{I}_1 + j5\left(\hat{I}_1 - \hat{I}_2\right) = 100 \angle 90^{\mathrm{o}}$$

$$j5\left(\hat{I}_1 - \hat{I}_2\right) - 10\,\hat{I}_2 = 100\underline{/0^o} \quad \text{que fornece o sistema :}$$

$$\begin{bmatrix} 10 + j5 & -j5 \\ j5 & -10 - j5 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 100\underline{/90^\circ} \\ 100 \end{bmatrix}$$

Resolvendo :

$\hat{I}_1 = 5\underline{/90^o}$ Aef $\qquad \hat{I}_2 = 11{,}18\underline{/-206{,}56^o}$ Aef

$P_{ap\,g1} = 100\underline{/90^o} \,.\, \hat{I}_1^* = 500 + j\,0$ VA fornecida

$P_{ap\,g2} = 100\underline{/0^o} \,.\, \left(-\hat{I}_2^*\right) = 1000 + j\,500$ VA fornecida

$P_{R1} = R_1 \,.\, \left|\hat{I}_1\right|^2 = 10\,.\,25 = 250$ W

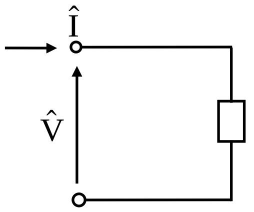
$P_{R2} = R_2 \,.\, \left|\hat{I}_2\right|^2 = 10\,.\,(11{,}18)^2 = 1250$ W

$Q_{L1} = 5\,.\left(\left|\hat{I}_1 - \hat{I}_2\right|\right)^2 = 500$ Var

Portanto: $P_{g1} + P_{g2} = P_{R1} + P_{R2} = 1500$ W

$Q_{g1} + Q_{g2} = Q_{L1} = 500$ Var

8 –



a) Sabe-se que: $P_{ap} = \hat{V}.\hat{I}^* = P + j\,Q \longrightarrow$

$$\longrightarrow \hat{I}^* = \frac{P + jQ}{\hat{V}}$$

$$\longrightarrow \hat{I} = \frac{P - jQ}{\hat{V}^*} = \frac{P}{\hat{V}^*} - j\,\frac{Q}{\hat{V}^*}$$

(aplicando-se propriedades dos números complexos)

b) $\hat{I}_a = \hat{I}_1 + \hat{I}_2$ $\qquad$ Carga 1: P = 12 kW $\qquad \hat{V} = \hat{E}_1 = 127\underline{/\,0^o}$

$$Q = \sqrt{P_{ap}^2 - P^2} = 21.931{,}71 \text{ Var}$$

$$\longrightarrow \hat{I}_1 = \frac{12.000}{127} - \frac{j\,21.931{,}71}{127} \quad \text{(item a)}$$

$$\hat{I}_1 = 196{,}85\underline{/-61{,}31^o} \text{ Aef}$$

Carga 2: $Q = -12.000$ kVAr $\qquad P = \sqrt{P_{ap}^2 - Q^2} = 48.538{,}64$ W

$$\hat{V} = \hat{E}_1 + \hat{E}_2 = 127\underline{/\,60^o} \text{ Vef}$$

$$\hat{I}_2 = \frac{48.538{,}64}{127\underline{/-60^\circ}} + \frac{j\,12.000}{127\underline{/-60^\circ}} = 393{,}69\underline{/\,73{,}89^o} \text{ Aef}$$

$$\hat{I}_a = \hat{I}_1 + \hat{I}_2 = 289{,}41\underline{/45{,}25^o} \text{ Aef}$$

c) $P_{ap\ g1}$ = 25,88 - j 26,10 kVA

Carga 1 $\longrightarrow$ P_1 = 12 kW Q_1 = 21,93 kVAr

Carga 1' $\longrightarrow$ P_1' = 12 kW Q_1' = 21,93 kVAr

Carga 2 $\longrightarrow$ P_2 = 48,54 kW Q_2 = –12 kVAr

Pela conservação das potências:

$P_{g2} = P_1 + P_1' + P_2 - 25{,}88 = 46{,}66$ kW

$Q_{g2} = Q_1 + Q_1' + Q_2 + 26{,}10 = 57{,}96$ kVAr

Portanto:

$P_{ap\ g2}$ = 46,66 + j 57,96 kVA fornecida.