Escola Politécnica
Universidade de São Paulo

PSI3213
Circuitos Elétricos II
Bloco 8

Potência em RPS: Potências ativa, reativa e complexa
Fator de potência

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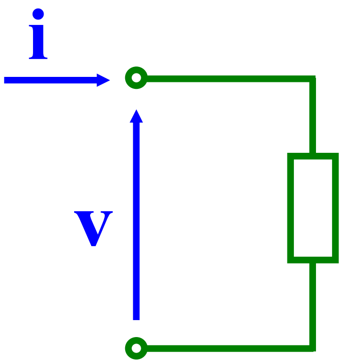
POTÊNCIA



$$p(t) = v(t)\,i(t) \qquad (\,W\,)$$

$$w(t) = \int_{t_1}^{t_2} v(t)\,i(t)\,dt \qquad (\,J\,)$$

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

RPS $\longrightarrow$ $v(t) = V_m \cos \omega t$

$i(t) = I_m \cos(\,\omega t - \varphi\,)$

$\varphi > 0$ indutivo

$\varphi < 0$ capacitivo

$$p(t) = \underbrace{\frac{1}{2} V_m I_m \cos\varphi}_{\text{potência média } P} + \underbrace{\frac{1}{2} V_m I_m \cos(2\omega t - \varphi)}_{\text{potência flutuante}}$$

Valores eficazes: V, I

$P = V\,I \cos\varphi \qquad (\,W,\ kW\,)$

Valor Eficaz

$$P = R\,I^2 = R\,I_{ef}^2$$

Potência média $\rightarrow$

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T i^2(t)\,R\,dt$$

$$= \frac{R}{T}\int_0^T i^2(t)\,dt$$

$$\boxed{I_{ef}^2 = \frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$\boxed{I_{ef} = \left[\frac{1}{T}\int_0^T i^2(t)\,dt\right]^{1/2}}$$

Para senoide :

$$I_{ef} = \left[\frac{1}{T}\int_0^T \left(I_m \cos(\omega t + \theta)\right)^2 dt\right]^{1/2}$$

$$I_{ef} = I_m / \sqrt{2}$$

VALOR EFICAZ E FASORES

O *valor eficaz* de uma *f(t)* periódica, com período *T*, é

$$F_{ef} = \left[\frac{1}{T} \int_T f^2(t)dt \right]^{1/2}$$

Se a *f (t)* for senoidal,

$$F_{ef} = F_{max} / \sqrt{2}$$

Fasores da função

$$\mathbf{v(t) = V_m \cos(\omega\, t + \Phi\ ):}$$

Podem ser *fasores*

de valor máximo: $\hat{V}_m = V_m\, e^{j\,\Phi}$

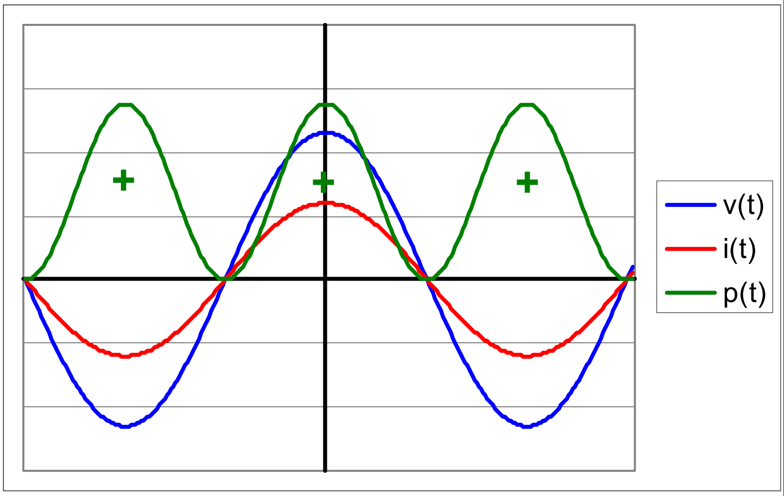
de valor eficaz: $\hat{V}_{ef} = (V_m / \sqrt{2})\, e^{j\Phi}$

R ideal

$\varphi = 0 \rightarrow$
$\cos\varphi = 1$

P=V.I

p(t)= V.I + V.I. cos 2ω.t

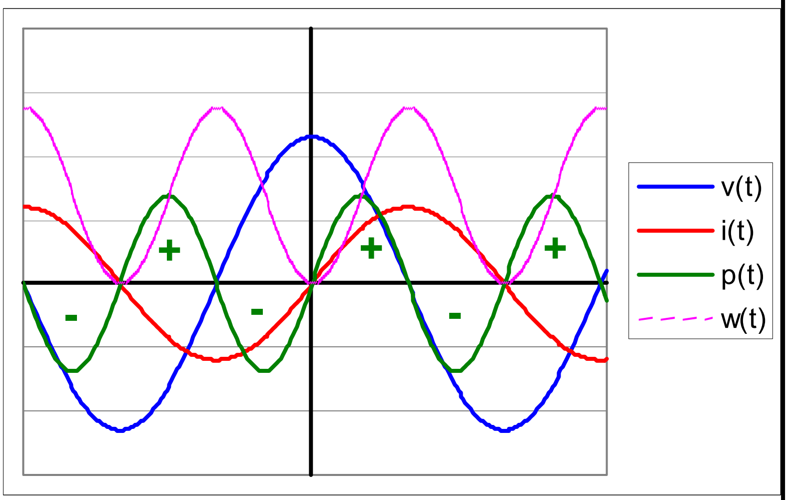


L ideal

$\varphi = \pi/2 \rightarrow$
$\cos\varphi = 0$

P= 0

p(t)= V.I. cos (2ω.t-π/2)

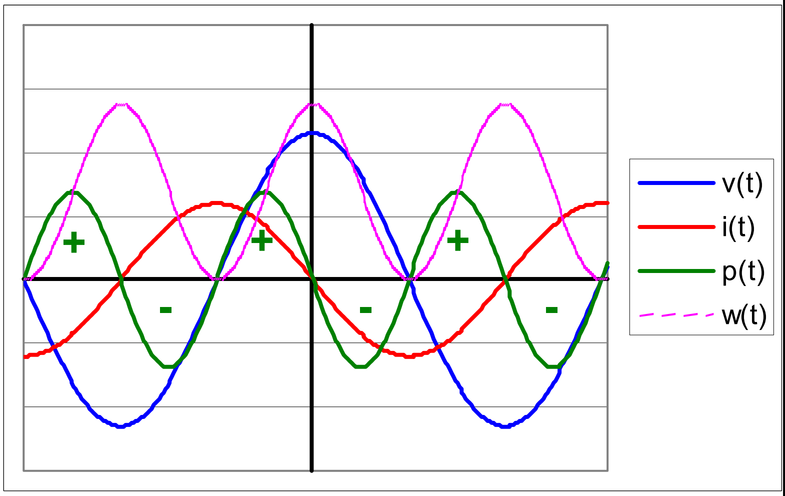


C ideal

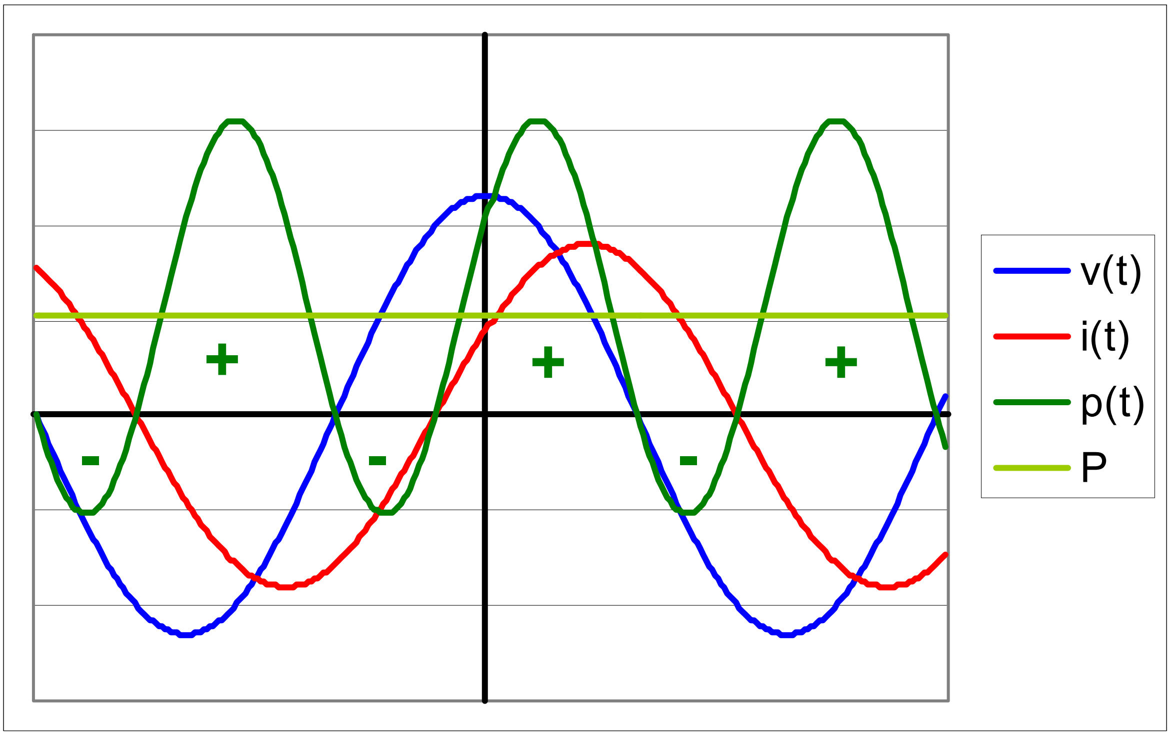
$\varphi = -\pi/2 \rightarrow$
$\cos\varphi = 0$

P= 0

p(t)= V.I. cos (2ω.t+π/2)



Potência em RPS - I



Carga Genérica

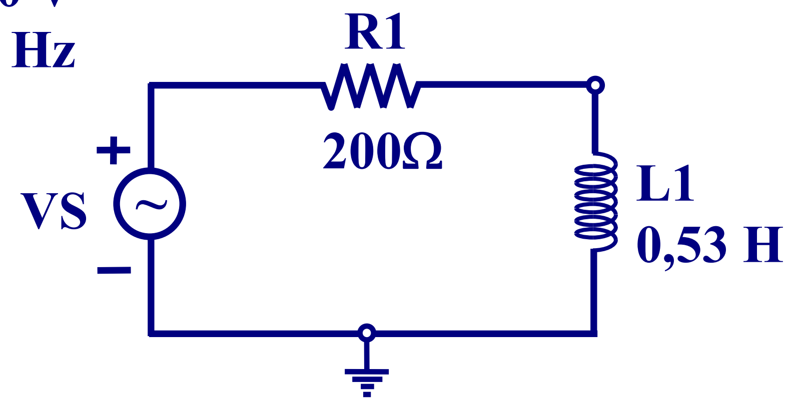
$$p(t) = VI\cos\varphi + VI\cos(2\omega t - \varphi)$$

$$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$$

$$P = VI\cos\varphi > 0 \quad \begin{cases} \text{potência ativa ou} \\ \text{potência real} \end{cases}$$

EXEMPLO

Amplitude = 200 V
Frequência = 60 Hz



$$V_{ef} = 200 / \sqrt{2}$$

$$I_{ef} = 0,5$$

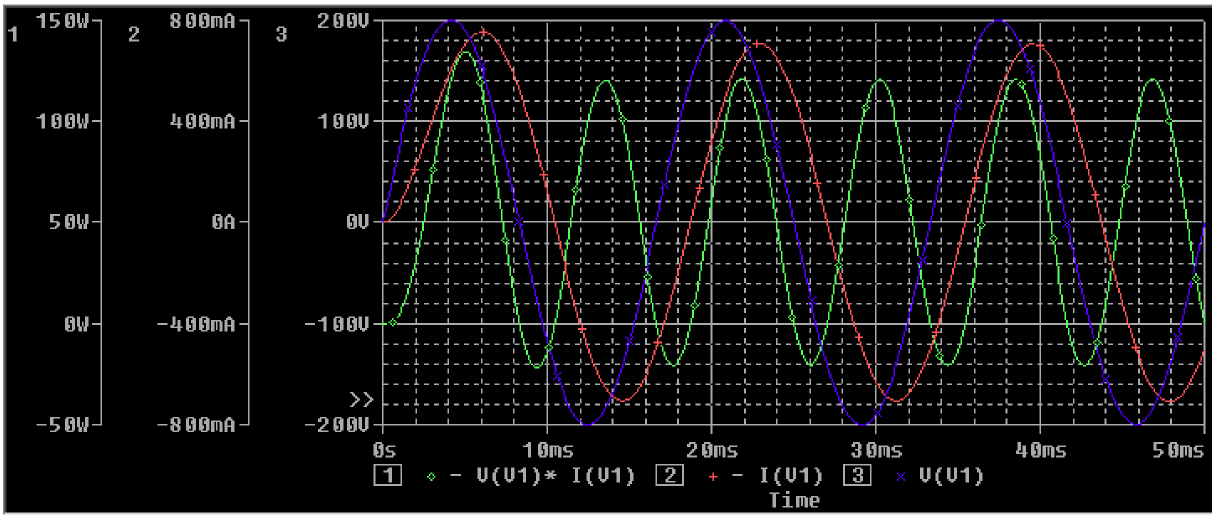
$$Z = 200 + j\,200\ \Omega$$

$$P = VI \cos\varphi = 50\ W$$

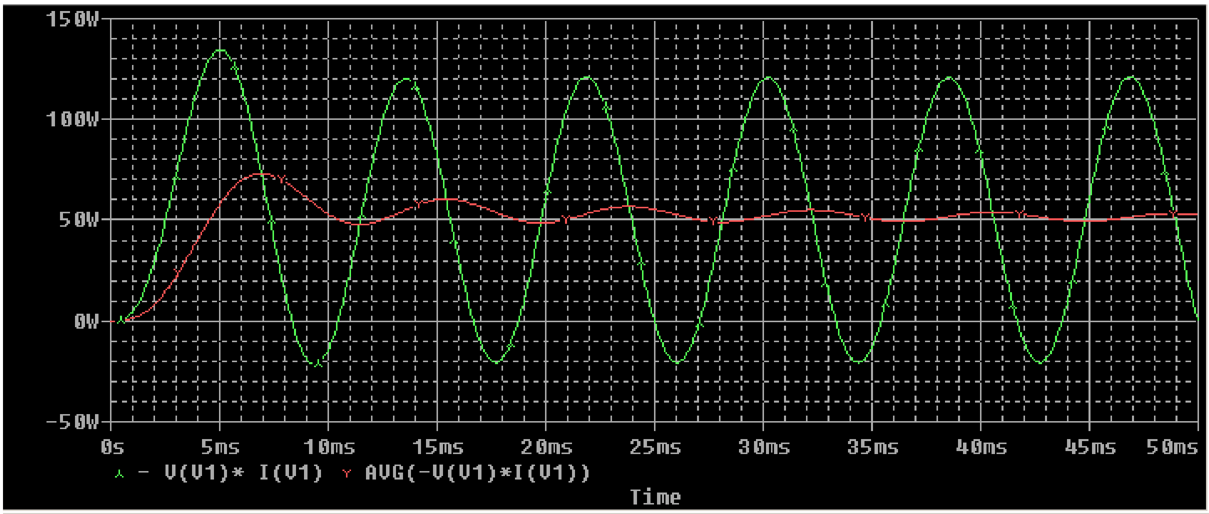
$$VI \cos\varphi - VI < p(t) < VI \cos\varphi + VI$$

$$-20,71\ W < p(t) < 120,71\ W$$

EXEMPLO



x v(t) + i(t) ◇ p(t)



$$- 20{,}71\ \text{W} < p(t) < 120{,}71\ \text{W}$$

$$P = \frac{1}{t}\int_0^t p(\tau)d\tau = \text{AVG}\,(\text{v(t)} * \text{i(t)}) = 50\ \text{W}$$

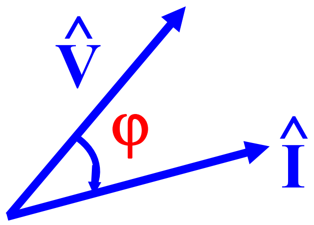
Potência aparente

$$|P_{ap}| = VI \qquad (\text{VA}, \ \text{kVA})$$

Fator de Potência :

– Caso geral : $\text{f. p.} = \dfrac{P}{VI} = \dfrac{P}{|P_{ap}|}$

– RPS : $\text{f. p.} = \cos\varphi = \dfrac{P}{VI}$



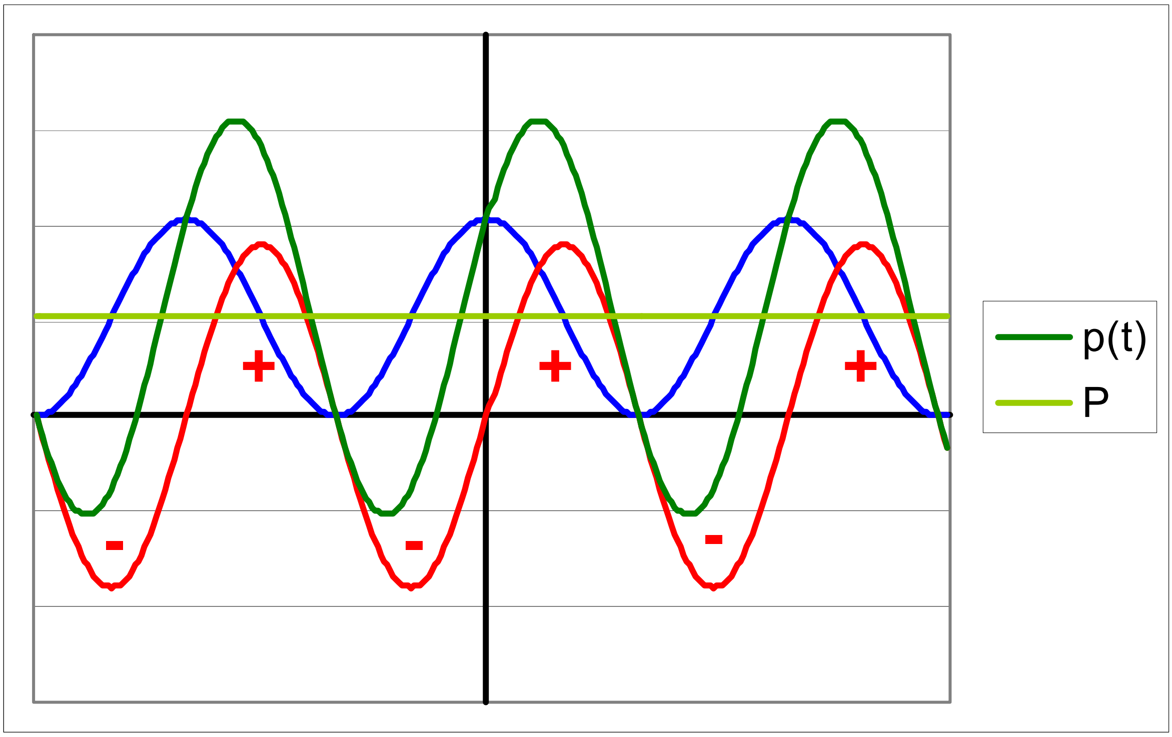
$$\begin{cases} \text{atrasado : } & 0 < \varphi < \pi/2 \\ \text{adiantado: } & -\dfrac{\pi}{2} < \varphi < 0 \end{cases}$$

Carga resistiva → $\text{f.p.} = 1$

Carga puramente reativa → $\text{f.p.} = 0$

Bipolo receptor → $0 \le \text{f.p.} \le 1$

Potência em RPS - II



p(t)

$VI\cos\varphi\ (1 + \cos 2\omega t)$

$VI\operatorname{sen}\varphi\ \operatorname{sen} 2\omega t$

$$p(t) = VI\cos\varphi\ (1 + \cos 2\omega t) + VI\operatorname{sen}\varphi\ \operatorname{sen} 2\omega t$$

Potência Reativa

$Q \triangleq VI \operatorname{sen} \varphi$ (VAr , kVAr)

$Q > 0 \rightarrow$ indutivo
$Q < 0 \rightarrow$ capacitivo

$$|P_{ap}| = VI = \sqrt{P^2 + Q^2}$$

Convenção do receptor

	X	φ	Q	I
Bip. Indutivo	> 0	> 0	> 0	atras.
Bip. Capacitivo	< 0	< 0	< 0	adiant.

Teorema da Conservação das Potências

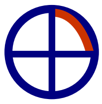
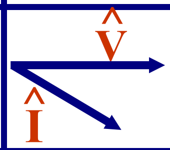
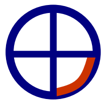
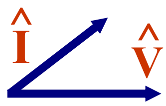
– Rede passiva linear
– Geradores sincronizados
– RPS
– Relações $\hat{V}_k$ e $\hat{J}_k$ → convenção do receptor

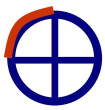
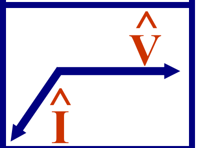
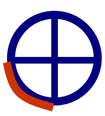
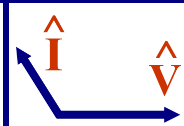
• Soma complexa P_{ap} nos bipolos = soma complexa P_{ap} fornecidas pelos geradores

• Soma P recebidas pelos bipolos = soma P fornecidas pelos geradores

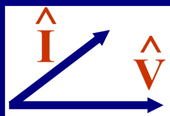
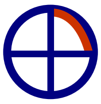
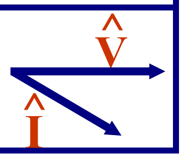
• Soma algébrica Q nos bipolos = soma algébrica Q fornecidas pelos geradores

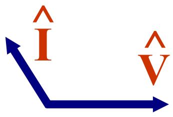
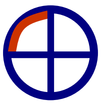
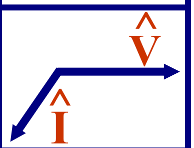
Bipolo em Convenção de Receptor

Recebe Potência	φ	cosφ	P	Q	I	
Indutivo		> 0	> 0	> 0	atras.	
Capacitivo		> 0	> 0	< 0	adiant.	

Fornece Potência	φ	cosφ	P	Q	I	
Indutivo		< 0	< 0	> 0	atras.	
Capacitivo		< 0	< 0	< 0	adiant.	

Bipolo em Convenção de Gerador

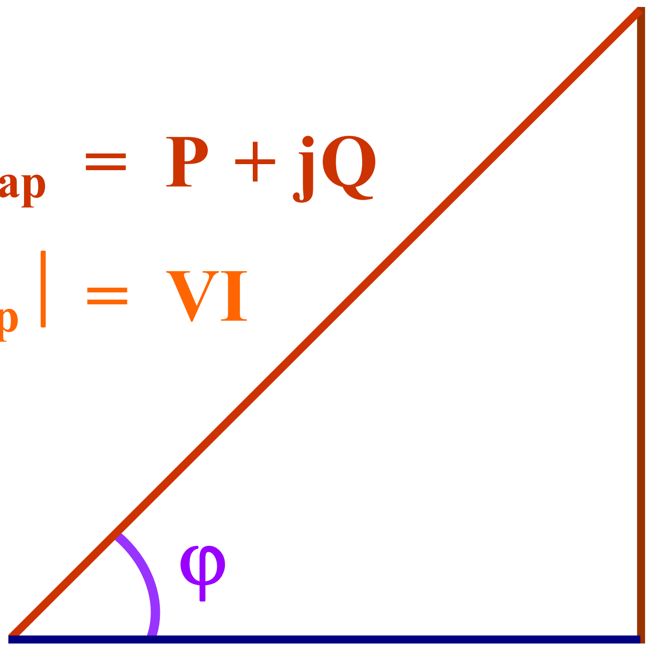
Fornece Potência	φ	cosφ	P	Q	I	
Indutivo		> 0	> 0	< 0	adiant.	
Capacitivo		> 0	> 0	> 0	atras.	

Recebe Potência	φ	cosφ	P	Q	I	
Indutivo		< 0	< 0	< 0	adiant.	
Capacitivo		< 0	< 0	> 0	atras.	

Triângulo de Potência

$$P_{ap} = P + jQ$$

$$|P_{ap}| = VI$$



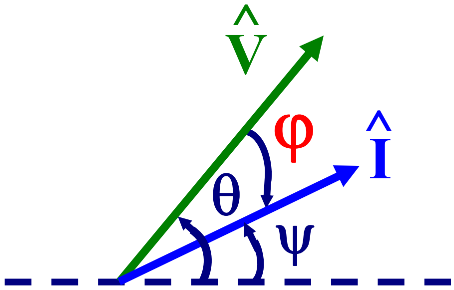
$$Q = VI\,\mathrm{sen}\varphi$$

$$P = VI\cos\varphi$$

$$\boxed{\mathrm{tg}\,\varphi = \frac{Q}{P}}$$

P_{ap} → **Potência Aparente Complexa**

Potência e Fasores



$$\hat{V} = V\,e^{j\theta}$$

$$\hat{I} = I\,e^{j\psi}$$

$$\varphi = \theta - \psi$$

$$\hat{V}\,.\,\hat{I}^{*} = VI\,e^{j(\theta - \psi)}$$

$$= VI\cos\varphi + j\,VI\,\mathrm{sen}\,\varphi$$

ou seja :

$$\boxed{\hat{V}\,.\,\hat{I}^{*}} = P + j\,Q = VI\,e^{j\varphi}$$

Potência aparente complexa $\rightarrow P_{ap}$

P $\rightarrow$ potência ativa

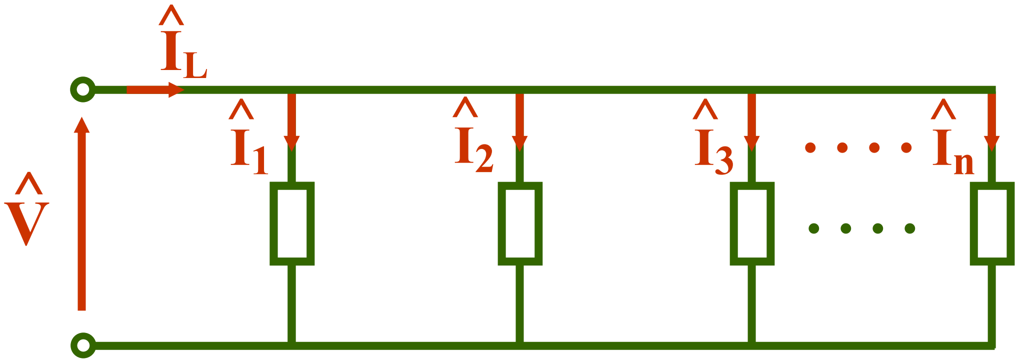
Q $\rightarrow$ potência reativa

$$|P_{ap}| = \sqrt{P^2 + Q^2}$$

$$\mathrm{tg}\,\varphi = \frac{Q}{P}$$

$$\cos\varphi = \frac{P}{|P_{ap}|} = \text{f.p.}$$

Circuito de Distribuição Monofásico

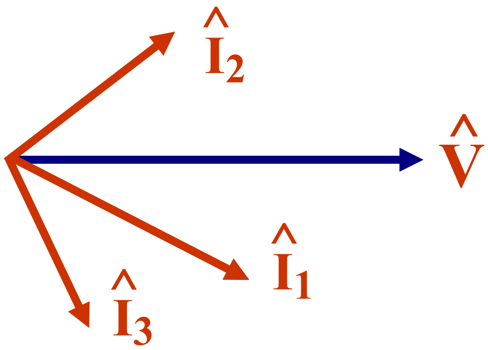


Corrente de linha ⇨

$$\hat{I}_L = \sum_{k=1}^{n} \hat{I}_k$$

Potência aparente :

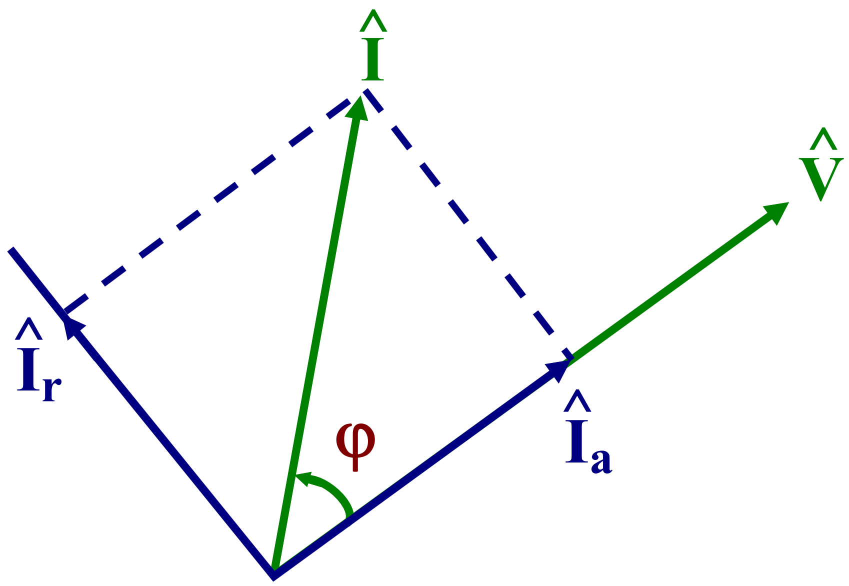
$$P_{ap} = \hat{V}\,.\,\hat{I}_L^* = \sum_{k=1}^{n} \hat{V}\,.\,\hat{I}_k^* = \sum_{k=1}^{n} P_{ap_k}$$



$$P_{ap} = \sum_{k=1}^{n} P_k + j \sum_{k=1}^{n} Q_k$$

$$|\hat{I}_L| = \frac{|P_{ap}|}{|\hat{V}|} \qquad \cos\varphi_L = \frac{\sum P_k}{|P_{ap}|}$$

Corrente Complexa



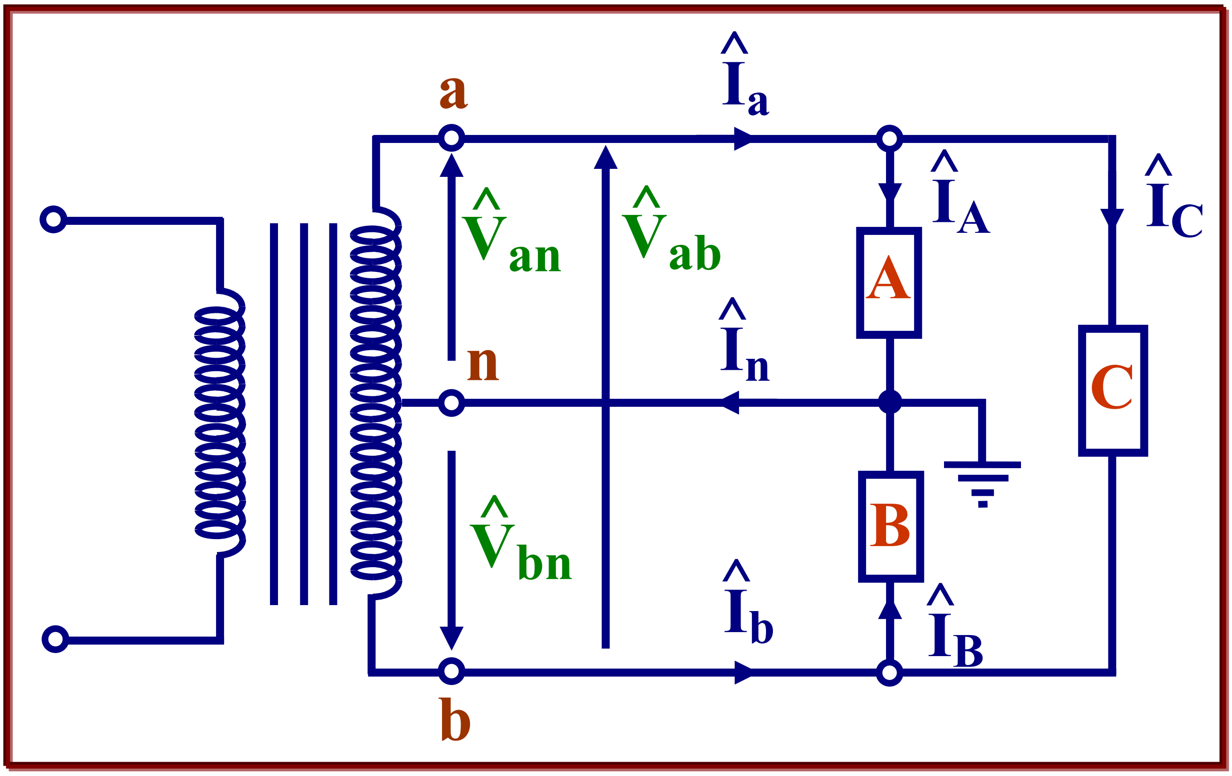
$$P_{ap} = \underbrace{VI\cos\varphi}_{VI_a} + j\,\underbrace{VI\,\mathrm{sen}\varphi}_{VI_r}$$

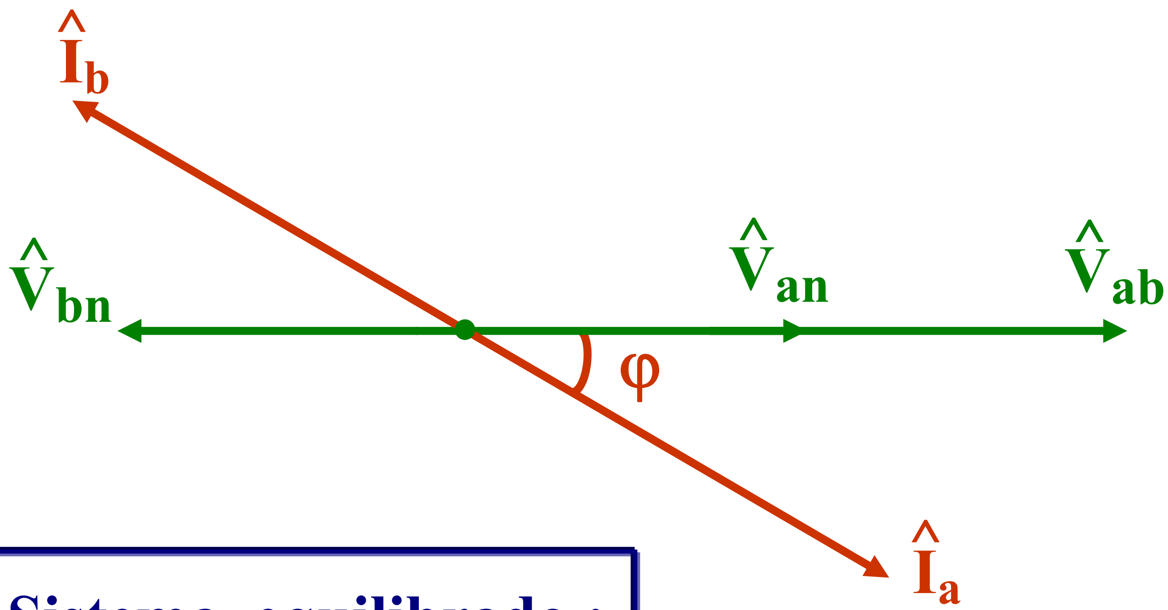
$VI_a \rightarrow$ **P** $\Rightarrow$ **Potência " real "**

$VI_r \rightarrow$ **Q** $\Rightarrow$ **Potência em quadratura**

Monofásico a 3 fios

(110 / 220 V)





Sistema equilibrado :

$\hat{I}_n = 0$

Cálculo da Potência Ativa no caso não senoidal

$$v(t) = V_0 + \sum_{k=1}^{N} \sqrt{2}\, V_k \cos\left(k\,\omega_0\,t + \theta_k\right)$$

$$i(t) = I_0 + \sum_{k=1}^{N} \sqrt{2}\, I_k \cos\left(k\,\omega_0\,t + \psi_k\right)$$

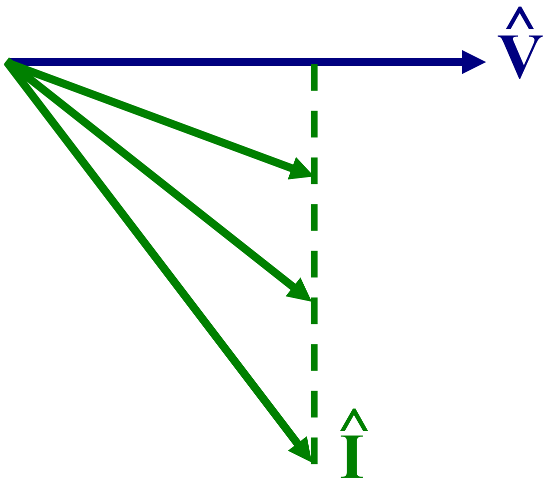
$$P = \frac{1}{T_0} \int_{T_0} v(t)\, i(t)\, dt$$

$$P = P_0 + P_1 + P_2 + \ldots. + P_N$$

$$P = V_0\, I_0 + \sum_{k=1}^{N} V_k\, I_k \cos \varphi_k$$

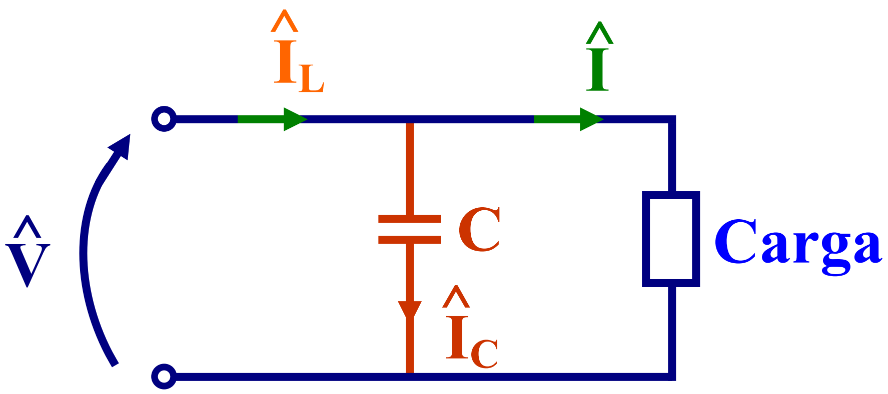
$$\varphi_k = \theta_k - \psi_k$$

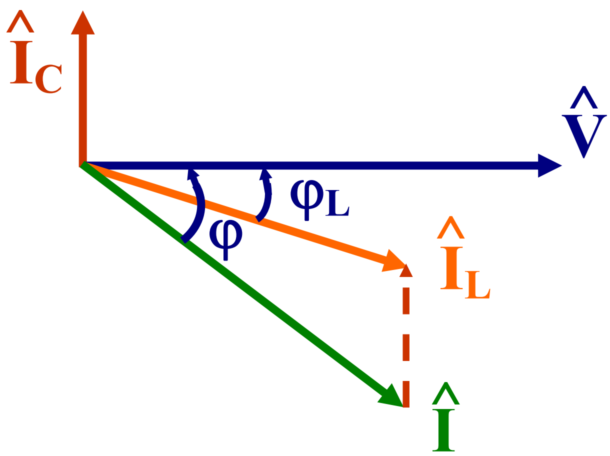
Correção de Fator de Potência



f.p. deve ser
> 0,92 atrasado

Manter P





$$P_{ap} = P + jQ + jQ_C$$

$$C = \frac{P\,(\,tg\,\varphi \; - \; tg\,\varphi_L\,)}{\omega\, V^2}$$

Fator de Potência e Energia

Energia Ativa : $E_a = \int_{\Delta t} P\,dt$

Energia reativa : $E_r = \int_{\Delta t} Q\,dt$

$\Delta t = 1$ **hora, p. ex.**

Fator de Potência Médio em Δt :

$$F.P. = \frac{E_A}{\sqrt{E_A^2 + E_R^2}}$$

Correção do F.P. em Δt :

$$P_{ap}\,cap = \hat{V} . \hat{I}^* = \hat{V}\,(\,\hat{V} . j\omega C\,)^*$$

$$= |\,\hat{V}\,|^2 . (-j\omega C)$$

$$Q\,cap = -\omega C\,V^2$$

Energia Reativa no Capacitor em Δt:

$$E\,cap = -\omega C\,V^2 . \,\Delta t$$

Potência & Impedância / Admitância

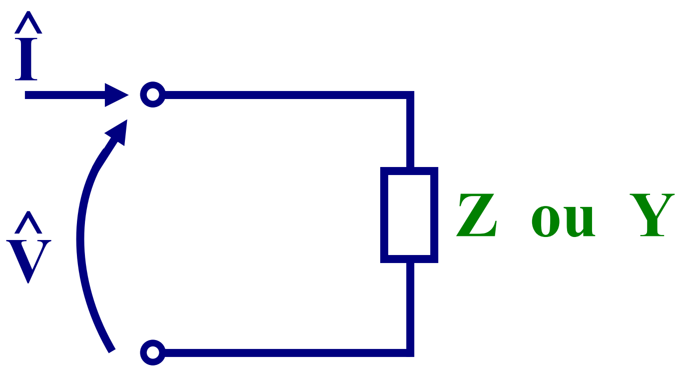
→ Generalização da Lei de Joule

$$Z(j\omega) = R(\omega) + jX(\omega)$$

$$P_{ap} = \underbrace{R(\omega)\,|\hat{I}|^2}_{P} + j\,\underbrace{X(\omega)\,|\hat{I}|^2}_{Q}$$

$$Y(j\omega) = G(\omega) + jB(\omega)$$

$$P_{ap} = \underbrace{G(\omega)\,|\hat{V}|^2}_{P} + j\,\underbrace{(-B(\omega))\,|\hat{V}|^2}_{Q}$$

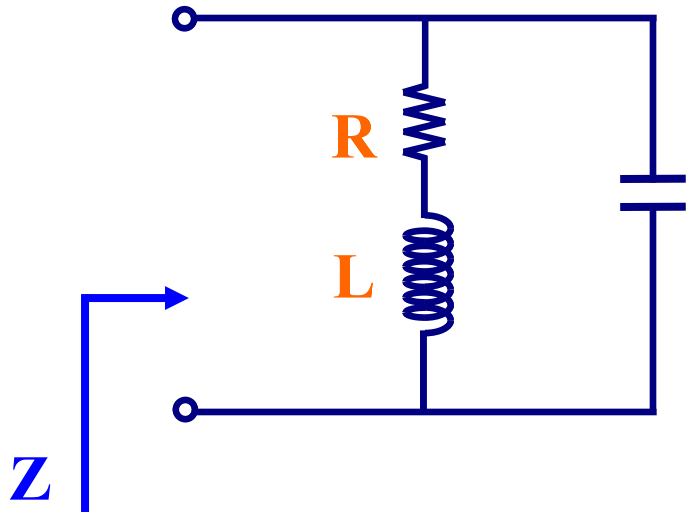


$$P_{ap} = \hat{V}.\hat{I}^*$$

$$\hat{V} = Z\,\hat{I}$$

$$\hat{I} = Y\,\hat{V}$$

Exemplo



$$X_L = \omega L$$

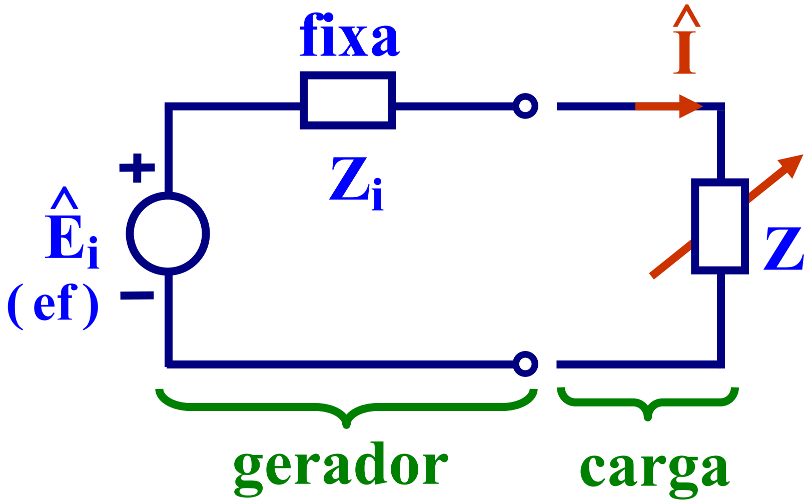
$$X_C = \frac{-1}{\omega C}$$

$$Z = \frac{(R + j X_L)(j X_C)}{R + j(X_L + X_C)}$$

$$Z = \underbrace{\frac{R X_C^2}{R^2 + (X_L + X_C)^2}}_{R(\omega)} + j \underbrace{\frac{X_C\left[R^2 + X_L(X_L + X_C)\right]}{R^2 + (X_L + X_C)^2}}_{X(\omega)}$$

$$Y = \frac{1}{Z} = \underbrace{\frac{R X_C^2}{X_C^2(R^2 + X_L^2)}}_{G(\omega)} - j \underbrace{\frac{X_C\left[R^2 + X_L(X_L + X_C)\right]}{X_C^2(R^2 + X_L^2)}}_{B(\omega)}$$

Transferência de Potência em RPS



$Z_i = R_i + j X_i$

$Z = R + j X$

Potência ativa : $P = R \, |\hat{I}|^2$

$$P = \frac{R}{(R + R_i)^2 + (X + X_i)^2} \, |E_i|^2$$

Condição de máximo :

$$P_{máx} = \frac{|E_i|^2}{4R} \qquad \text{para}$$

$\eta = 50\ \%$ $\boxed{Z = Z_i^*}$

Casamento de Impedâncias !

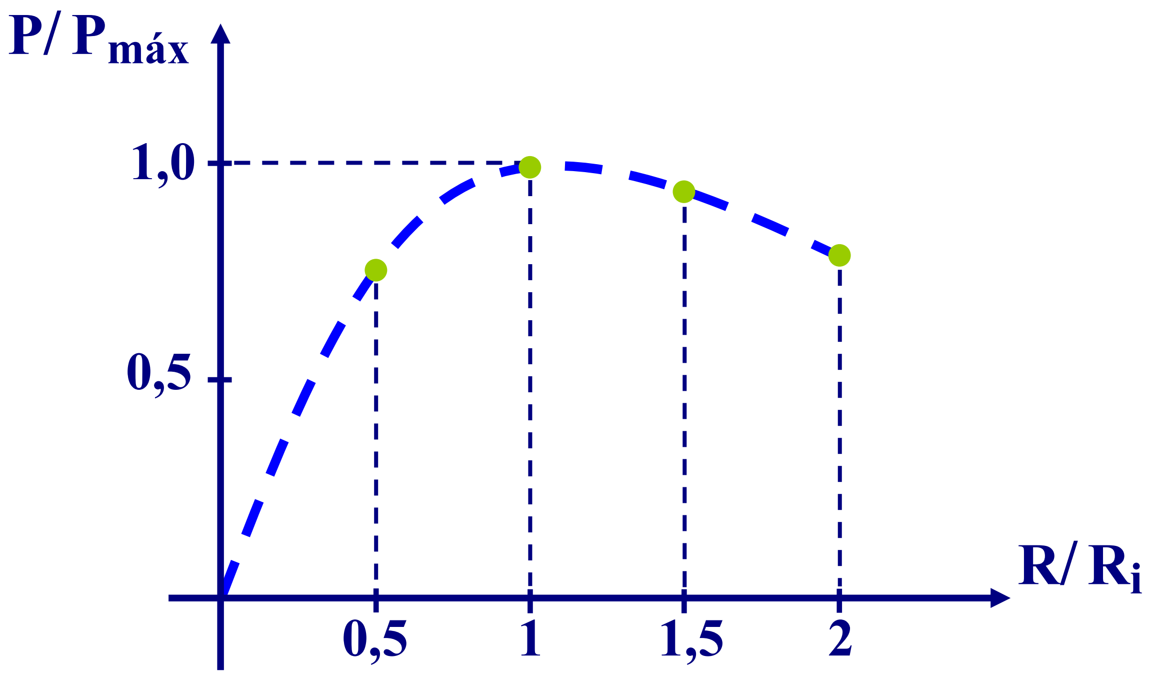
Exemplo : com Transformador Ideal

tal que $\begin{cases} R = R_i \\ X = -X_i \end{cases}$

Transferência de Potência em RPS

$$\frac{P}{P_{máx}} = \frac{4R\,/\,R_i}{(1 + R/R_i)^2 + (X + X_i)^2/R_i^2}$$

$$\frac{P}{P_{máx}} = 4\,\frac{R}{R_i}\,\frac{1}{(1 + R/R_i)^2}$$



Condições:
$$\begin{cases} (X + X_i)^2 / R_i^2 << 1 \\ R \approx R_i \end{cases}$$