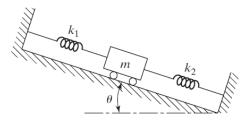
## DEPARTAMENTO DE ENGENHARIA NAVAL E OCEÂNICA - EPUSP



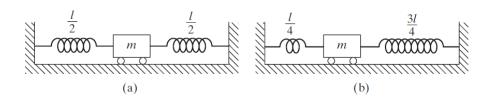
## PNV-3314 Dinâmica de Sistemas I

## Lista de Exercícios Nº 1

- 2.1 An industrial press is mounted on a rubber pad to isolate it from its foundation. If the rubber pad is compressed 5 mm by the self weight of the press, find the natural frequency of the system.
- 2.2 A spring-mass system has a natural period of 0.21 sec. What will be the new period if the spring constant is (a) increased by 50 percent and (b) decreased by 50 percent?
- 2.3 A spring-mass system has a natural frequency of 10 Hz. When the spring constant is reduced by 800 N/m, the frequency is altered by 45 percent. Find the mass and spring constant of the original system.
- 2.4 A helical spring, when fixed at one end and loaded at the other, requires a force of 100 N to produce an elongation of 10 mm. The ends of the spring are now rigidly fixed, one end vertically above the other, and a mass of 10 kg is attached at the middle point of its length. Determine the time taken to complete one vibration cycle when the mass is set vibrating in the vertical direction.
- 2.6 The maximum velocity attained by the mass of a simple harmonic oscillator is 10 cm/s, and the period of oscillation is 2 s. If the mass is released with an initial displacement of 2 cm, find (a) the amplitude, (b) the initial velocity, (c) the maximum acceleration, and (d) the phase angle.
- 2.8 An automobile having a mass of 2,000 kg deflects its suspension springs 0.02 m under static conditions. Determine the natural frequency of the automobile in the vertical direction by assuming damping to be negligible.
- **2.9** Find the natural frequency of vibration of a spring-mass system arranged on an inclined plane, as shown in Fig. 2.52.



**2.40** A helical spring of stiffness *k* is cut into two halves and a mass *m* is connected to the two halves as shown in Fig. 2.81(a). The natural time period of this system is found to be 0.5 s. If an identical spring is cut so that one part is one-fourth and the other part three-fourths of the original length, and the mass *m* is connected to the two parts as shown in Fig. 2.81(b), what would be the natural period of the system?

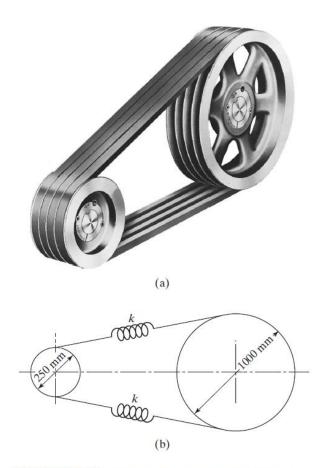


- **2.51** Determine the displacement, velocity, and acceleration of the mass of a spring-mass system with k = 500 N/m, m = 2 kg,  $x_0 = 0.1$  m, and  $\dot{x_0} = 5$  m/s.
- **2.52** Determine the displacement (x), velocity  $(\dot{x})$ , and acceleration  $(\ddot{x})$  of a spring-mass system with  $\omega_n = 10$  rad/s for the initial conditions  $x_0 = 0.05$  m and  $\dot{x}_0 = 1$  m/s. Plot x(t),  $\dot{x}(t)$ , and  $\ddot{x}(t)$  from t = 0 to 5 s.
- **2.53** The free-vibration response of a spring-mass system is observed to have a frequency of 2 rad/s, an amplitude of 10 mm, and a phase shift of 1 rad from t = 0. Determine the initial conditions that caused the free vibration. Assume the damping ratio of the system as 0.1.
- 2.54 An automobile is found to have a natural frequency of 20 rad/s without passengers and 17.32 rad/s with passengers of mass 500 kg. Find the mass and stiffness of the automobile by treating it as a single-degree-of-freedom system.
- **2.55** A spring-mass system with mass 2 kg and stiffness 3,200 N/m has an initial displacement of  $x_0 = 0$ . What is the maximum initial velocity that can be given to the mass without the amplitude of free vibration exceeding a value of 0.1 m?
- **2.59** An oil drum of diameter 1 m and a mass of 500 kg floats in a bath of salt water of density  $\rho_w = 1050 \text{ kg/m}^3$ . Considering small displacements of the drum in the vertical direction (x), determine the natural frequency of vibration of the system.
- **2.65** A pulley 250 mm in diameter drives a second pulley 1,000 mm in diameter by means of a belt (see Fig. 2.90). The moment of inertia of the driven pulley is 0.2 kg-m<sup>2</sup>. The belt connecting these pulleys is represented by two springs, each of stiffness *k*. For what value of *k* will the natural frequency be 6 Hz?
- 2.98 The ratio of successive amplitudes of a viscously damped single-degree-of-freedom system is found to be 18:1. Determine the ratio of successive amplitudes if the amount of damping is (a) doubled, and (b) halved.
- **2.99** Assuming that the phase angle is zero, show that the response x(t) of an underdamped single-degree-of-freedom system reaches a maximum value when

$$\sin \omega_d t = \sqrt{1 - \zeta^2}$$

and a minimum value when

$$\sin \omega_d t = -\sqrt{1 - \zeta^2}$$



 $\begin{array}{ll} FIGURE\ 2.90 & (\mbox{Photo courtesy of Reliance Electric} \\ \mbox{Company.}) \end{array}$ 

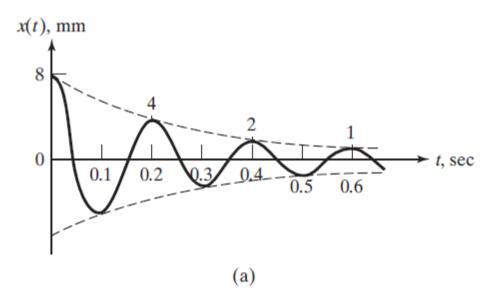
Also show that the equations of the curves passing through the maximum and minimum values of x(t) are given, respectively, by

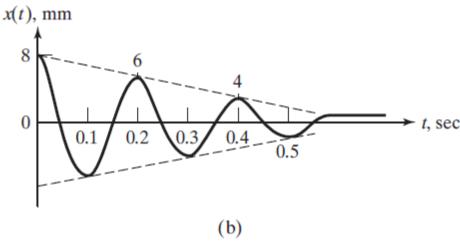
$$x = \sqrt{1 - \zeta^2} X e^{-\zeta \omega_n t}$$

and

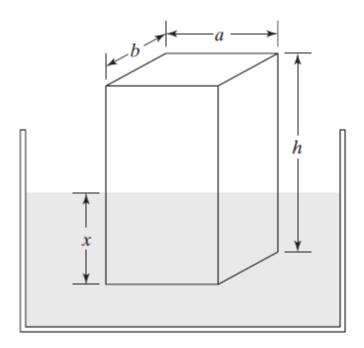
$$x = -\sqrt{1 - \zeta^2} X e^{-\zeta \omega_n t}$$

- **2.100** Derive an expression for the time at which the response of a critically damped system will attain its maximum value. Also find the expression for the maximum response.
- **2.101** A shock absorber is to be designed to limit its overshoot to 15 percent of its initial displacement when released. Find the damping ratio  $\zeta_0$  required. What will be the overshoot if  $\zeta$  is made equal to (a)  $\frac{3}{4}\zeta_0$ , and (b)  $\frac{5}{4}\zeta_0$ ?
- 2.102 The free-vibration responses of an electric motor of weight 500 N mounted on different types of foundations are shown in Figs. 2.107(a) and (b). Identify the following in each case: (i) the nature of damping provided by the foundation, (ii) the spring constant and damping coefficient of the foundation, and (iii) the undamped and damped natural frequencies of the electric motor.





- **2.103** For a spring-mass-damper system, m=50 kg and k=5,000 N/m. Find the following: (a) critical damping constant  $c_c$ , (b) damped natural frequency when  $c=c_c/2$ , and (c) logarithmic decrement.
- 2.108 A body vibrating with viscous damping makes five complete oscillations per second, and in 50 cycles its amplitude diminishes to 10 percent. Determine the logarithmic decrement and the damping ratio. In what proportion will the period of vibration be decreased if damping is removed?
- 2.109 The maximum permissible recoil distance of a gun is specified as 0.5 m. If the initial recoil velocity is to be between 8 m/s and 10 m/s, find the mass of the gun and the spring stiffness of the recoil mechanism. Assume that a critically damped dashpot is used in the recoil mechanism and the mass of the gun has to be at least 500 kg.
- 2.110 A viscously damped system has a stiffness of 5,000 N/m, critical damping constant of 0.2 N-s/mm, and a logarithmic decrement of 2.0. If the system is given an initial velocity of 1 m/s, determine the maximum displacement of the system.
- 2.118 A wooden rectangular prism of cross section 40 cm × 60 cm, height 120 cm, and mass 40 kg floats in a fluid as shown in Fig. 2.105. When disturbed, it is observed to vibrate freely with a natural period of 0.5 s. Determine the density of the fluid.



**FIGURE 2.105**