Light Propagation in Fibres and Related Optical Effects

FIBRE OPTIC TECHNOLOGY COURSE

Universidade do Porto
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2. Light Propagation in Fibres and Related Optical Effects

2. Light Propagation in Fibres and Related Optical Effects

2.1 Wave Nature of the Light

DUAL NATURE

- Waves
  - Electromagnetic radiation consisting of propagating electric and magnetic fields (interference & diffraction)
- Photons
  - Quanta of energy (photoelectric effect)

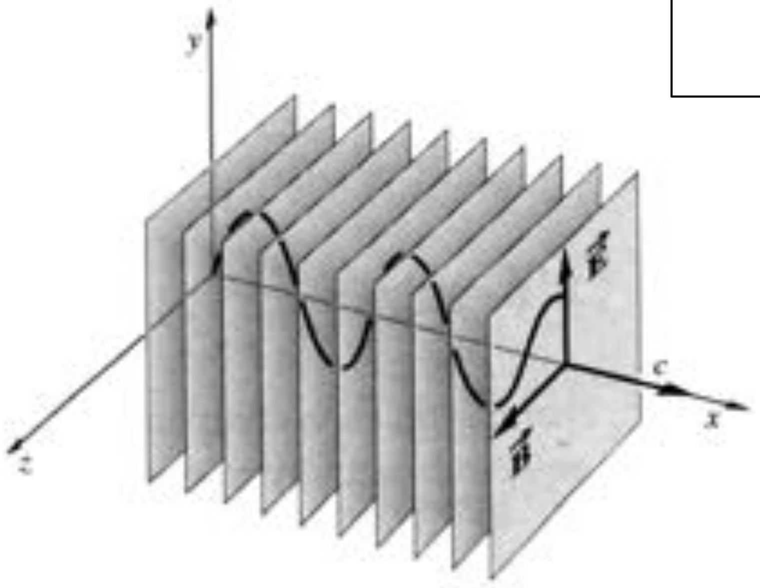
The two views are related: the energy in a photon is proportional to the frequency of the wave.

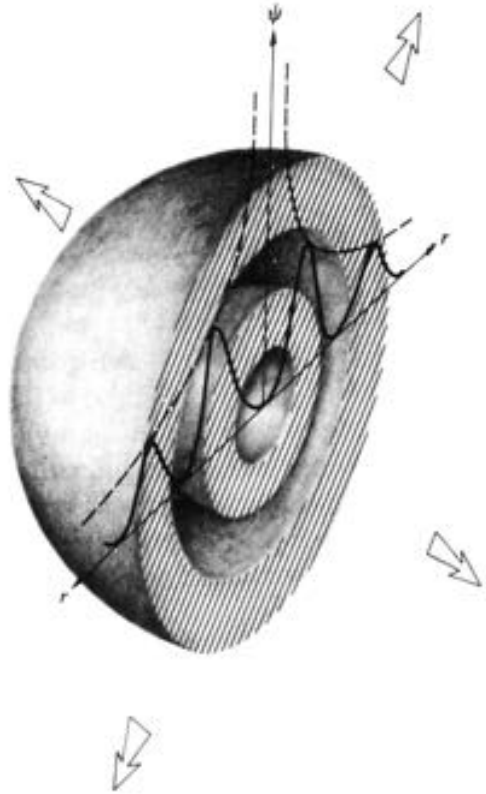
2. Light Propagation in Fibres and Related Optical Effects

2.1 Wave Nature of the Light

Waves

$$\boxed{\nabla^2\psi = \frac{1}{v}\cdot\frac{\partial^2\psi}{\partial t^2}}$$





$$\psi(x,t) = A\cdot\cos\left[kx \pm \omega t)\right]$$

$$\psi(r,t) = \left(\frac{A}{r}\right)\cdot\cos\left[k(r \pm vt)\right]$$

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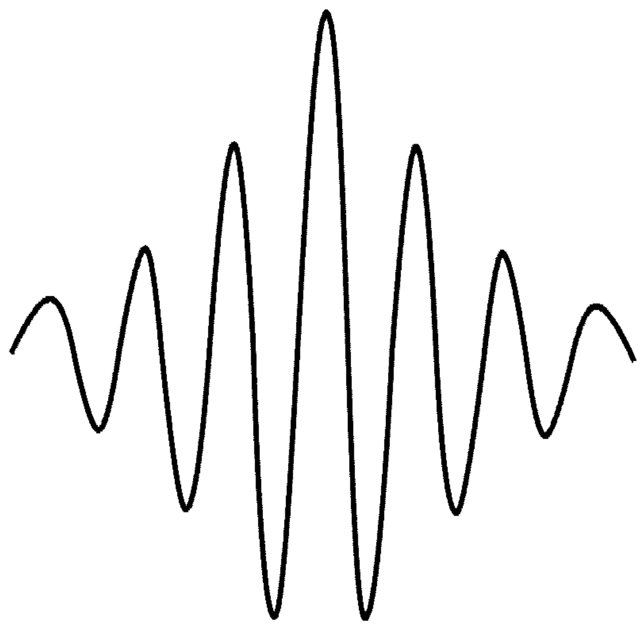
2.1 Wave Nature of the Light

Photon

- Quanta of Energy: Photon (photoelectric effect)

$$\boxed{E = h\nu}$$

$E \rightarrow$ Energy of 1 photon in Joules (J)
$h \rightarrow$ Planck's constant: 6.626×10-34 J-s
$v \rightarrow$ frequency in Hz



A Single Photon
("short" packet wave)

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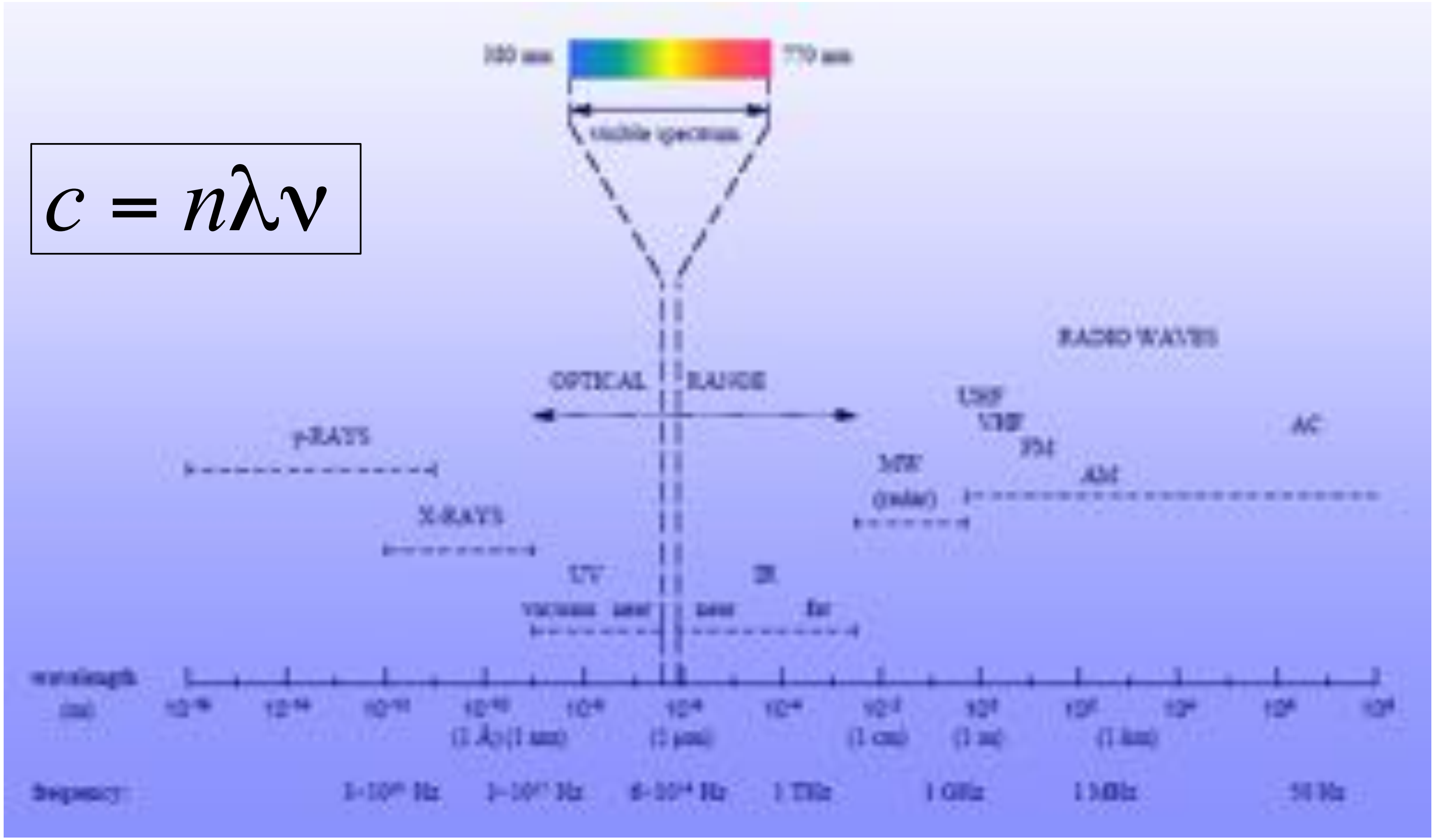
2.1 Wave Nature of the Light

$$\boxed{c = n\lambda\nu}$$



c → Speed of light = 2.9979×10^8 m/s; λ → Wavelength in meters; ν → Frequency in Hz

n → Refractive index (vaccum=1.0000; standard air= 1.0003; silica fibre: 1.44 to 1.48)

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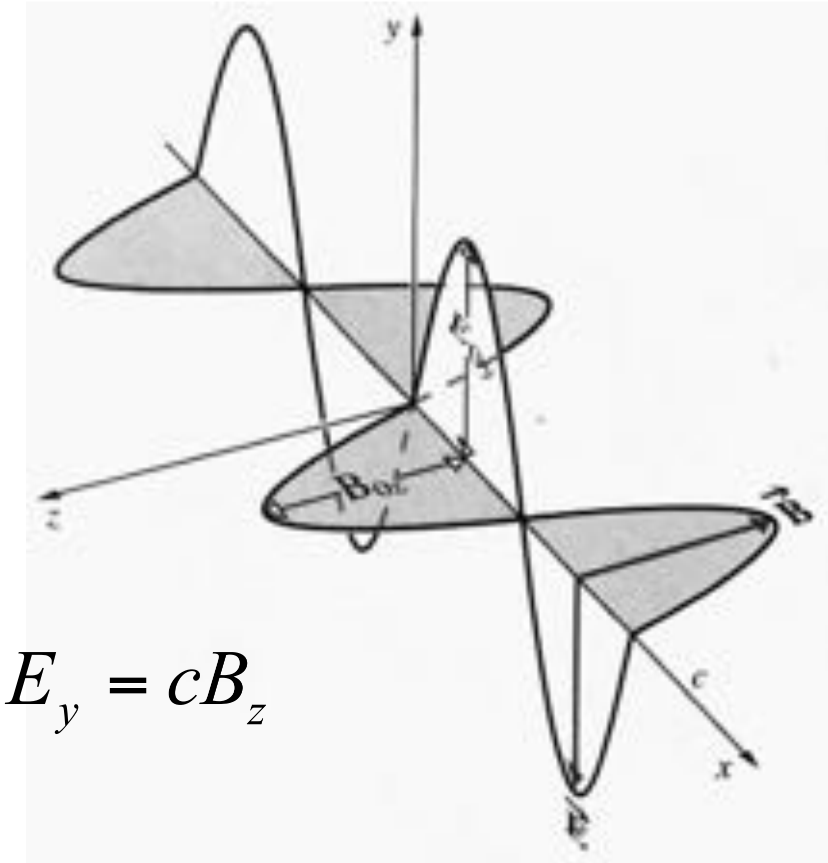
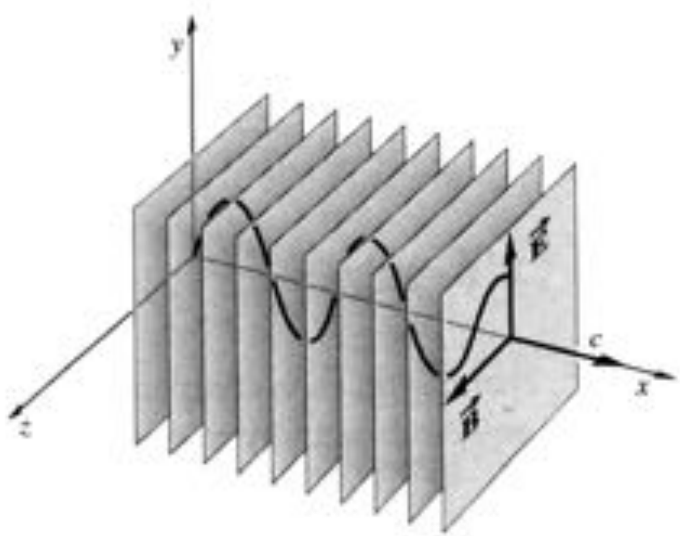
2.1 Wave Nature of the Light



$$E_y = cB_z$$



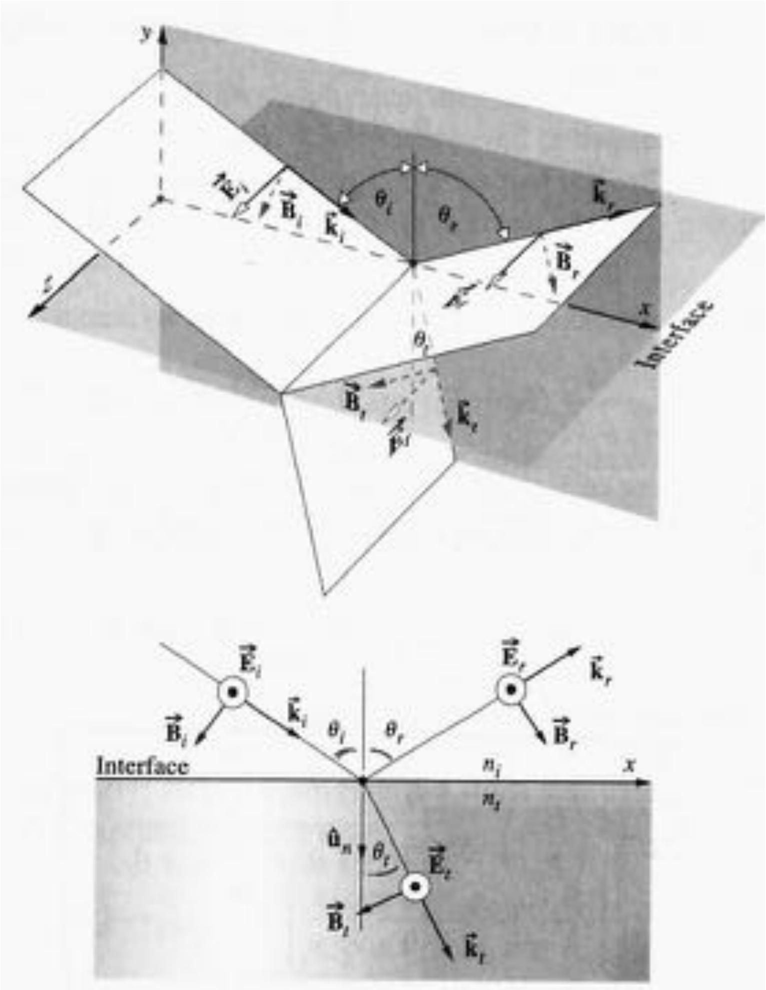
Poynting Vector

$$\boxed{\vec{S} = \frac{1}{\mu_o}\vec{E}\times\vec{B} = c^2\varepsilon_o\vec{E}\times\vec{B}}$$

2. Light Propagation in Fibres and Related Optical Effects

2.2 Rays and Modes representation.

About Reflection (Fresnel Equations): **E** ⊥Plane-of-Incidence



$$\mathrm{v}\vec{B} = \vec{k} \times \vec{E}$$

$$\vec{k} \bullet \vec{E} = 0$$

$$\vec{E}_i = \vec{E}_{oi} \cos(\vec{k}_r \bullet \vec{r} - \omega_r t)$$

$$\vec{E}_t = \vec{E}_{ot} \cos(\vec{k}_t \bullet \vec{r} - \omega_t t)$$

$$\vec{E}_{ot} = \vec{E}_{oi} + \vec{E}_{or}$$

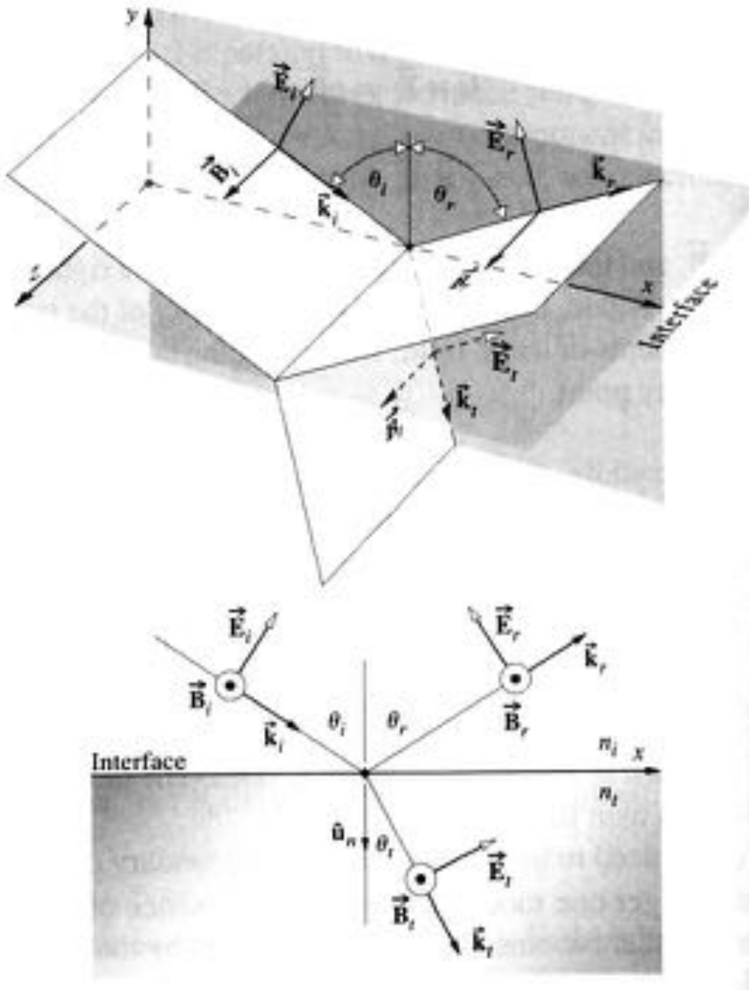
$$r_\perp = \left(\frac{E_{or}}{E_{oi}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{ot}}{E_{oi}}\right)_\perp = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

2. Light Propagation in Fibres and Related Optical Effects

2.2 Rays and Modes representation.

About Reflection (Fresnel Equations): **E** || Plane-of-Incidence



$$r_{\parallel} = \left(\frac{E_{or}}{E_{oi}}\right)_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$t_{\parallel} = \left(\frac{E_{ot}}{E_{oi}}\right)_{\parallel} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

2. Light Propagation in Fibres and Related Optical Effects

2.2 Rays and Modes representation.

Total Internal Reflection (case: $n_i > n_t$): $\theta_i \geq \theta_c$

Critical Angle:

$$\boxed{\sin\theta_c = \frac{n_t}{n_i}}$$

$$r_\perp = \frac{n_i\cos\theta_i - i\sqrt{n_i^2\sin^2\theta_i - n_t^2}}{n_i\cos\theta_i + i\sqrt{n_i^2\sin^2\theta_i - n_t^2}}$$

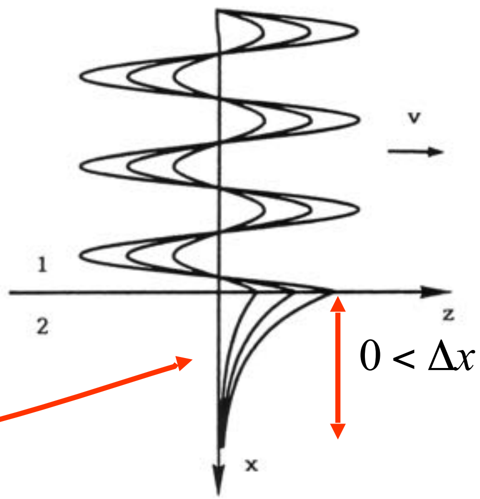
$$r_\parallel = \frac{n_t^2\cos\theta_i - in_i\sqrt{n_i^2\sin^2\theta_i - n_t^2}}{n_t^2\cos\theta_i + in_i\sqrt{n_i^2\sin^2\theta_i - n_t^2}}$$

$$\left.\begin{matrix} r_\perp r_\perp^* = r_\parallel r_\parallel^* = 1 \\ R = 1 \end{matrix}\right\} \Rightarrow I_r = I_i \text{ e } I_t = 0$$

Goos-Hänchen depth

Evanescent wave:

$$\vec{E}_t = \vec{E}_{ot} e^{\mp\beta\gamma} e^{i\left[k_t x n_t \sin\theta_i / n_i - \omega t\right]}$$



$$0 < \Delta x < \frac{\lambda}{4n_1\sqrt{2\Delta}}$$

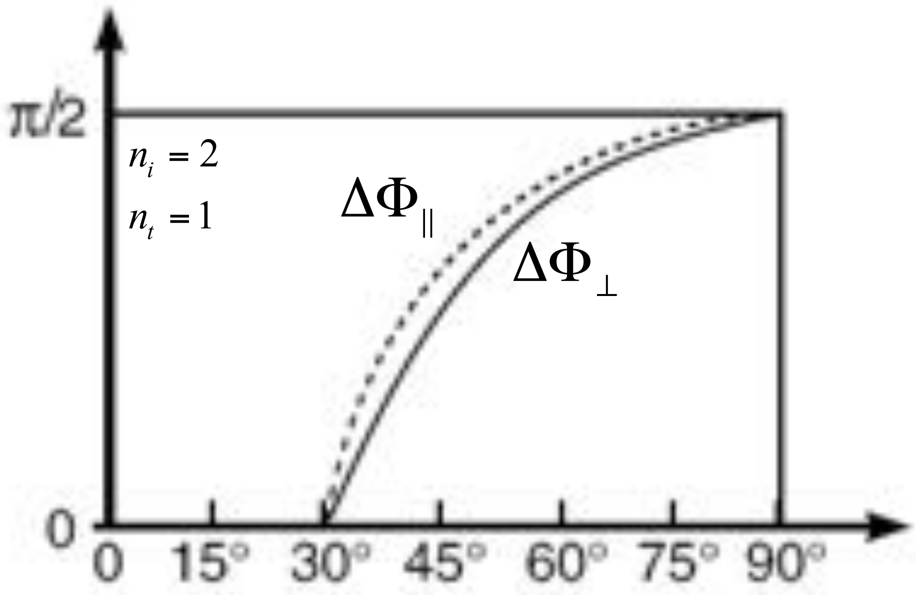
2. Light Propagation in Fibres and Related Optical Effects

2.2 Rays and Modes representation.

Total Internal Reflection (case: $n_i > n_t$): $\theta_i \geq \theta_c$

$$\Delta\Phi_{\perp} = \tan^{-1}\left[\frac{\sqrt{n_i^2 \sin^2\theta_i - n_t^2}}{n_i \cos\theta_i}\right]$$

$$\Delta\Phi_{\parallel} = \tan^{-1}\left[\frac{n_i\sqrt{n_i^2 \sin^2\theta_i - n_t^2}}{n_t^2 \cos\theta_i}\right]$$



$$|r| = rr^* = 1$$

$$r = |r|e^{-i2\Delta\Phi} = e^{-i2\Delta\Phi}$$

Polarization TE $\rightarrow \Delta\Phi_{\perp}$

Polarization TM $\rightarrow \Delta\Phi_{\parallel}$

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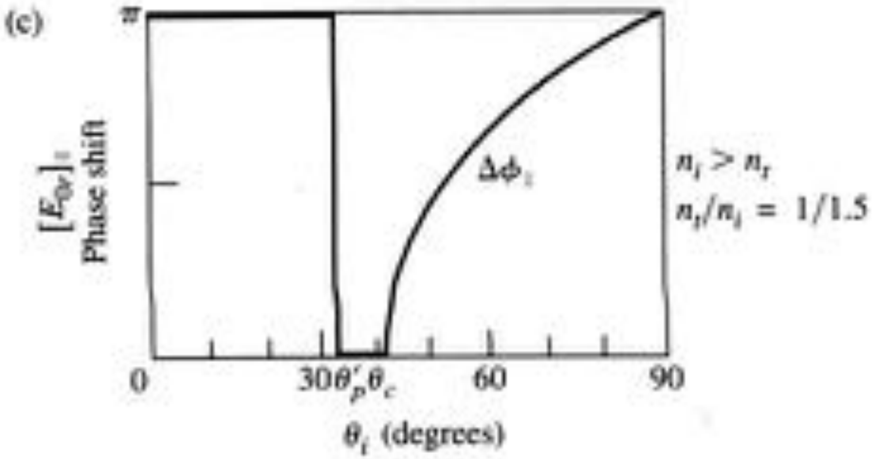
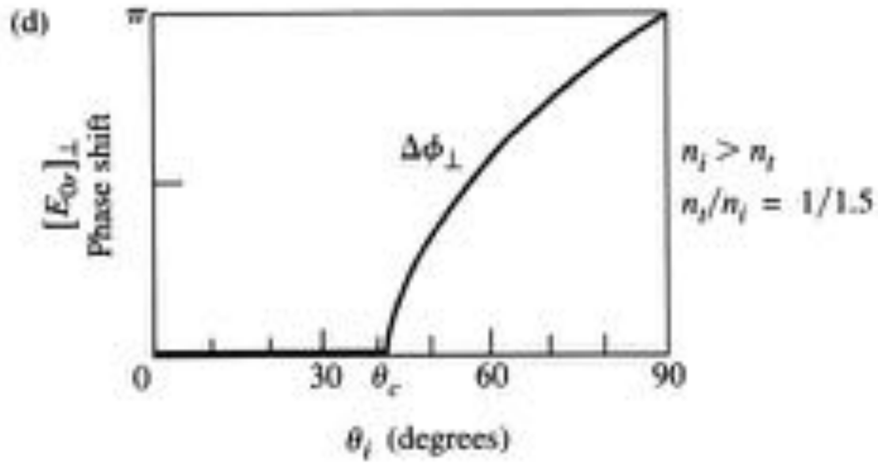
2.2 Rays and Modes representation.

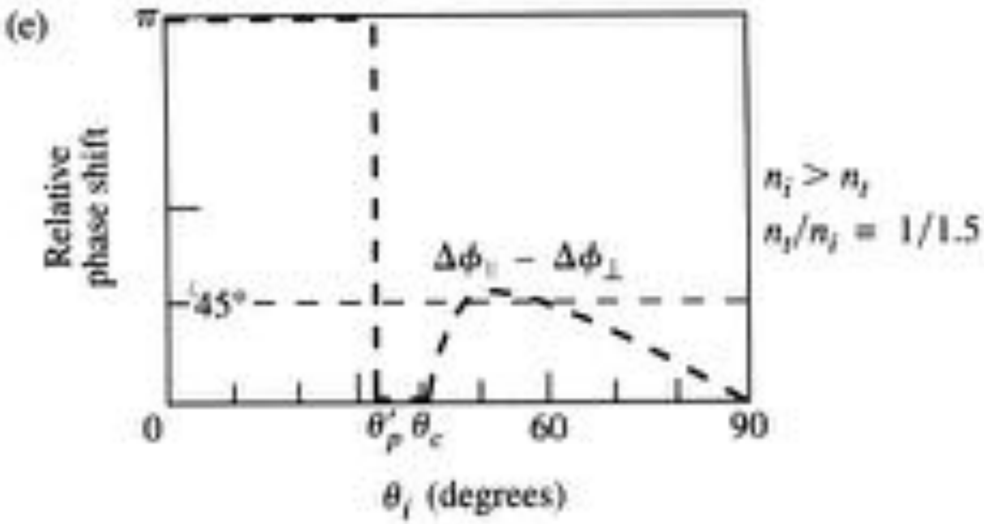
Brewster Angle:

$$\boxed{\tan\theta_p = \frac{n_t}{n_i}}$$

About Reflection (case: $n_i > n_t$): **Phase-shifts**







*The component of electric field normal to plane-of-incidence undergoes a **phase shift of π radians** upon reflection when the incident medium has a lower index than the transmitting medium.*

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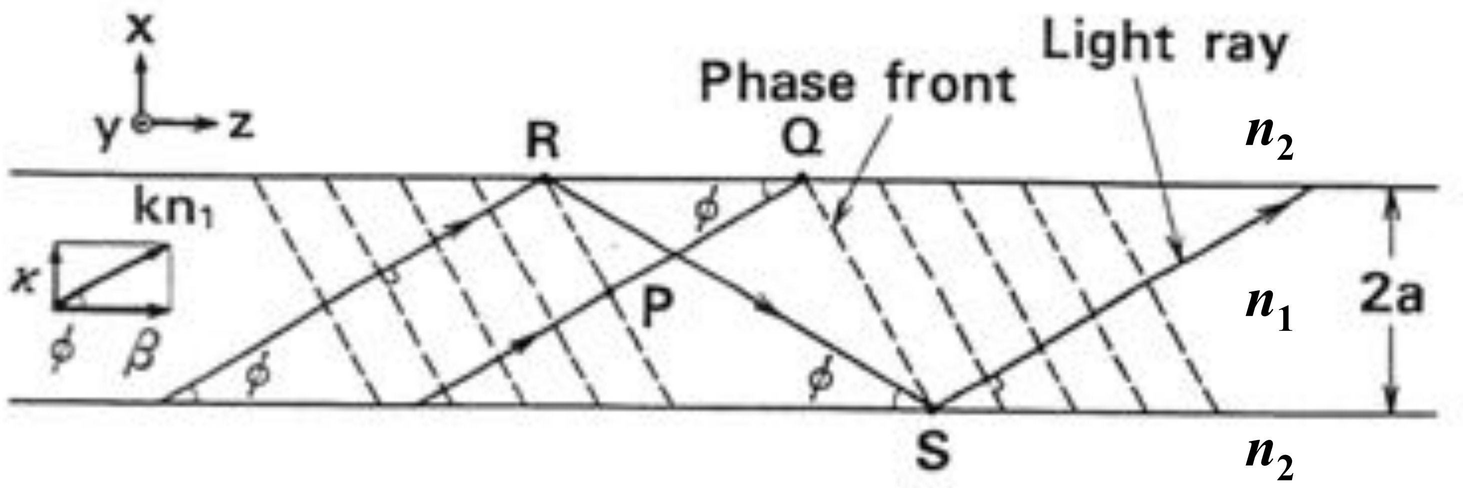
2.2 Rays and Modes representation.



Goos-Hänchen shift ($\Delta\Phi_{\perp}$)

Propagation Condition: $$kn_1 a \sin\phi - \tan^{-1}\left[\sqrt{\frac{2\Delta}{\sin^2\phi} - 1}\right] = m\frac{\pi}{2} \quad ; m \text{ is an integer}$$

The optical field distribution that satisfies this phase matching condition is called ***MODE***

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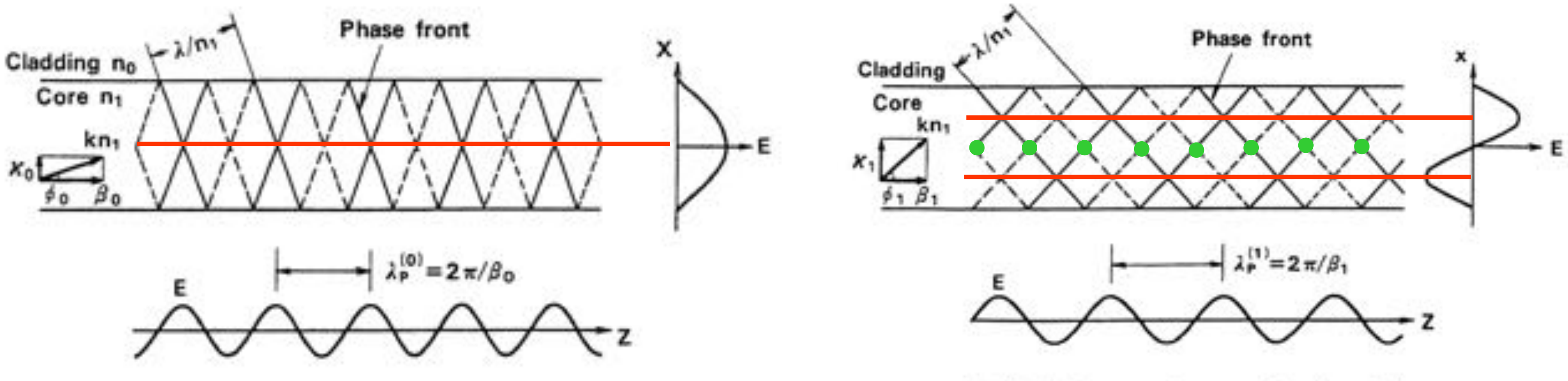
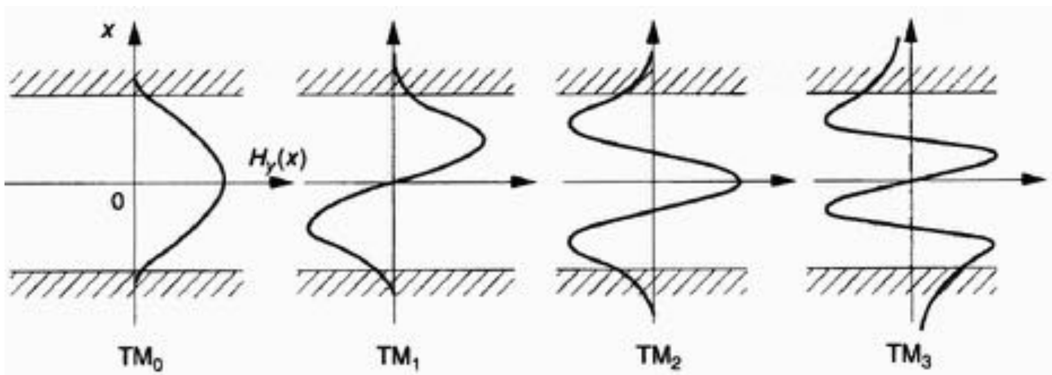
2.2 Rays and Modes representation.

Formation of modes



(a) Fundamental mode (m=0)

(b) Higher-order mode (m=1)



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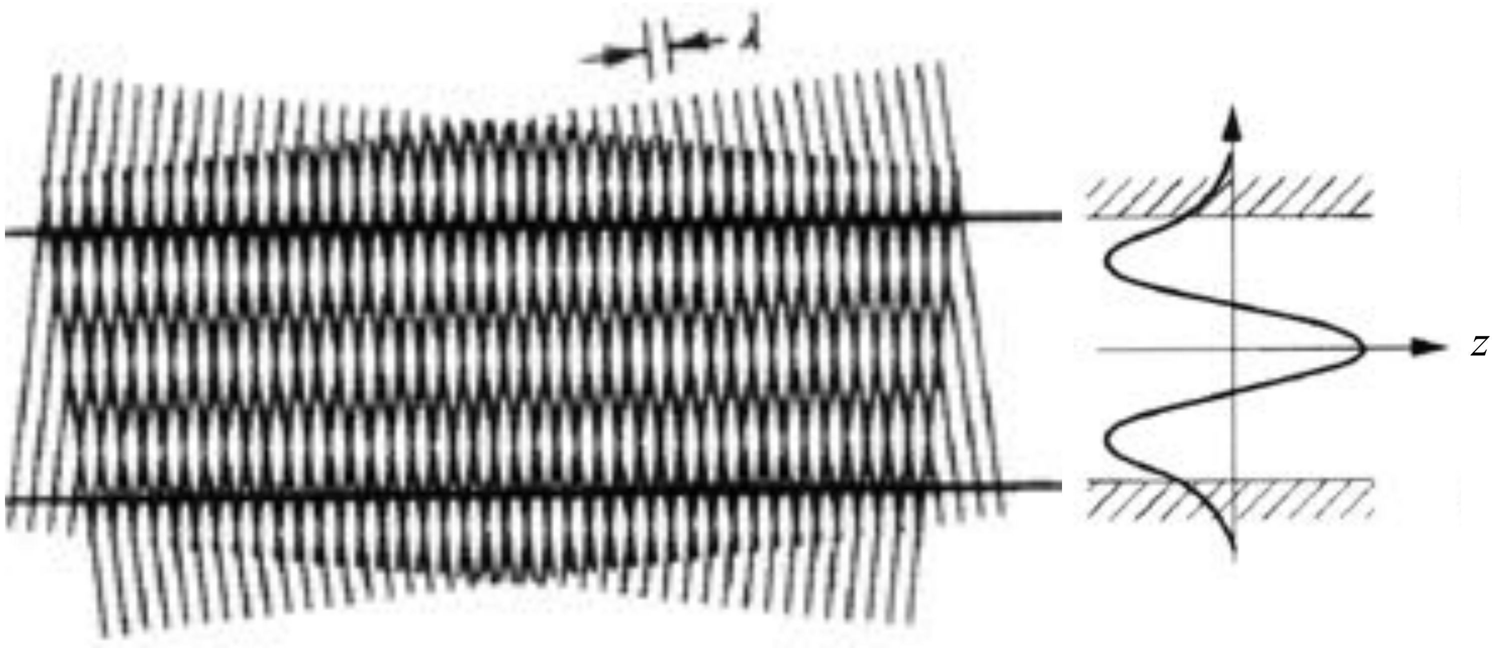
2.2 Rays and Modes representation.



At angles for which the ***condition of self-consistently*** *(i.e., as a wave reflects twice it duplicates itself) is satisfied, the two waves interfere and create a pattern that does not change with z direction.*

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2.2 Rays and Modes representation.

Normalized frequency: $V = kn_1 a\sqrt{2\Delta}$

Propagation constant: $\xi = \dfrac{\sin\phi}{\sqrt{2\Delta}}$

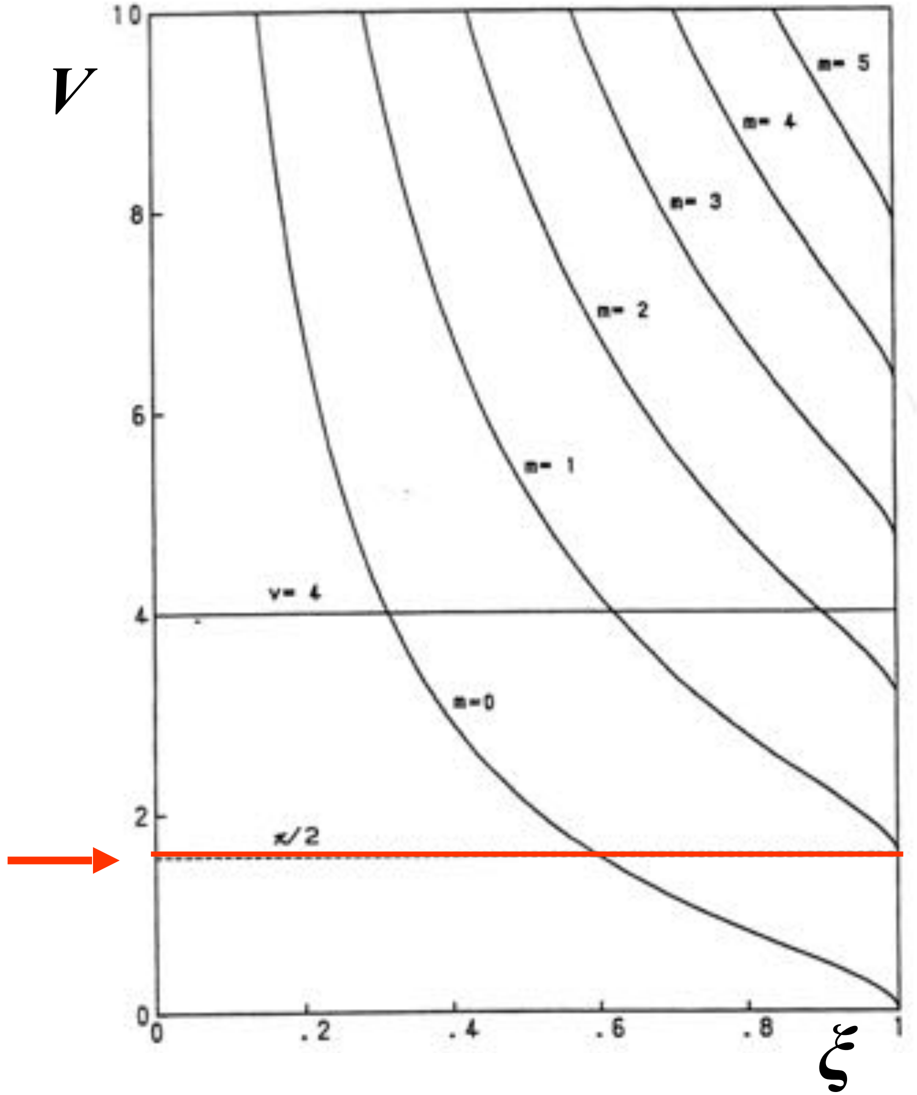
Dispersion Equation

$$V = \frac{\cos^{-1}\xi + m\dfrac{\pi}{2}}{\xi}$$

Single-mode condition

$$V < v_c = \frac{\pi}{2} = 1.571$$

(Slab Waveguide)

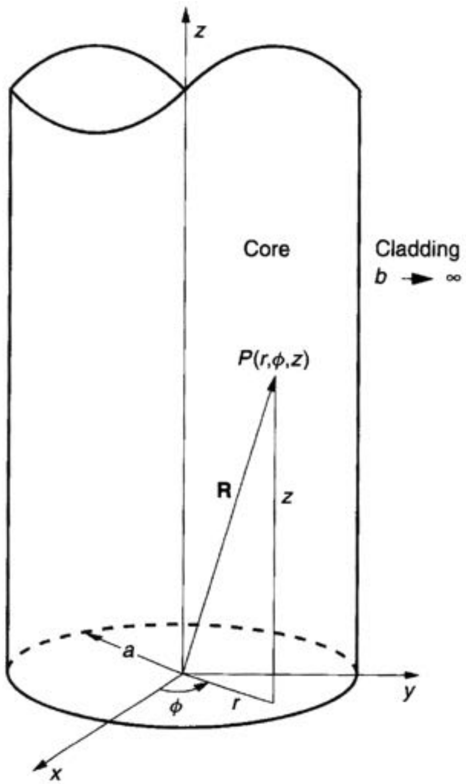


2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

Cylindrical Coordinates (r,ϕ,z)

Coordinate System



$$\nabla f = \frac{\partial f}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial f}{\partial \phi}\mathbf{e}_\phi + \frac{\partial f}{\partial z}\mathbf{e}_z$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{e}_r & \mathbf{e}_\phi & \frac{1}{r}\mathbf{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Notação

$\text{grad} f = \nabla f$

$\text{div}\, \vec{A} = \nabla \bullet \vec{A}$

$\text{rot}\, \vec{A} = \nabla \times \vec{A}$

$\text{lap} f = \nabla^2 f$

2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

Maxwell´s Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t} + \mu\sigma \vec{E}$$

$$\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \bullet \vec{B} = 0$$

$$\Leftrightarrow$$

$$\oint \vec{E} \cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\oint \vec{B} \cdot d\vec{l} = \iint \left(\mu\varepsilon \frac{\partial \vec{E}}{\partial t} + \mu\sigma \vec{E} \right) \cdot d\vec{S}$$

$$\oiint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon} \iiint \rho \cdot dV$$

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

Maxwell´s Equations (2)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \vec{B} = 0$$

Constitutive Equations

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_o \vec{E} + \vec{P}$$

$$\vec{B} = \mu \vec{H} = \mu_o \vec{H} + \vec{M}$$

$$\vec{J} = \sigma \vec{E} \quad \text{(for conductors)}$$

Boundary Conditions

$$\vec{E}_{2t} - \vec{E}_{1t} = 0$$

$$\vec{H}_{2t} - \vec{H}_{1t} = 0$$

$$D_{2n} - D_{1n} = \sigma$$

$$B_{2n} - B_{1n} = 0$$

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2.3 Mode Theory for fibres.

Wave Equations (1)

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{\partial t} = 0$$

$$\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} - \mu\sigma \frac{\partial \vec{H}}{\partial t} = 0$$

In dielectric, non-conducting media, $\sigma = 0$

$$\Rightarrow$$

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

Wave Equations (2)

Waves in photonics are often monochromatic, with a frequency that stays the same across the material boundaries, that is: $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})e^{j\omega t}$

$$\nabla^2 \vec{E}(\vec{r}) + n^2 k^2 \vec{E}(\vec{r}) = 0$$

$$\nabla^2 \vec{H}(\vec{r}) + n^2 k^2 \vec{H}(\vec{r}) = 0$$

$$n^2 k^2 = \mu\varepsilon\omega^2 = \frac{\omega^2}{c^2}$$

Helmholtz Equations

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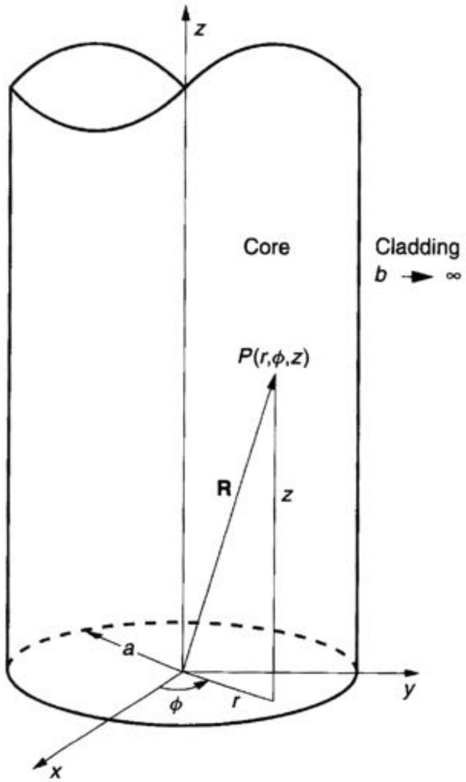
2.3 Mode Theory for fibres.

Cylindrical Coordinates (r,ϕ,z)

Coordinate System



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$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Notação

$\text{grad}\, f = \nabla f$

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$\text{rot}\, \vec{A} = \nabla \times \vec{A}$

$\text{lap}\, f = \nabla^2 f$

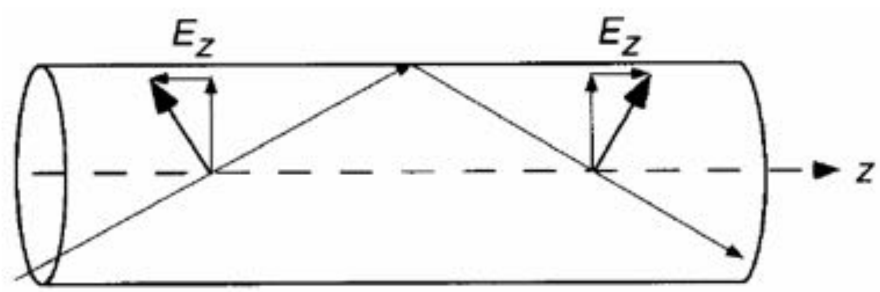
2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

$$\vec{E}(\vec{r},t) = \vec{E}(r,\phi)e^{j(\omega t-\beta z)}$$
$$\vec{H}(\vec{r},t) = \vec{H}(r,\phi)e^{j(\omega t-\beta z)}$$

$$\nabla^2\vec{E}(\vec{r}) + n^2k^2\vec{E}(\vec{r}) = 0$$
$$\nabla^2\vec{H}(\vec{r}) + n^2k^2\vec{H}(\vec{r}) = 0$$

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + n(r,\phi)^2k^2E_z = 0$$
$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial H_z}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z^2} + n(r,\phi)^2k^2H_z = 0$$



The longitudinal component of the electric field does not change though either propagation or reflection at the cylindrical surface

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2.3 Mode Theory for fibres.

Since the equations for E_r and E_θ are coupled, we first solve for E_z (H_z is a solution of the same Helmholtz equation and its solutions have the same form). We find all other field components form E_z and H_z using Mawell's equations.

We look for solution of the form:

$$E_z(r,\phi,z) = R(r)\cdot\Phi(\phi)\cdot Z(z)$$

$$\boxed{r^2\frac{\partial^2 R}{\partial r^2} + r\frac{\partial R}{\partial r} + \frac{\partial^2 \Phi}{\partial \phi^2} + r^2\frac{\partial^2 Z}{\partial z^2} + (knr)^2 = 0}$$

In the core we find:

$$Z(z) = ae^{j\beta z} + be^{-j\beta z}$$

$$\Phi(\phi) = ce^{j\nu\phi} + de^{-j\nu\phi}$$

$$R(r) = gJ_\nu(\kappa r) + hN_\nu(\kappa r)$$

$\kappa^2 = (nk_o)^2 - \beta^2$ and $\nu = 0, 1, 2, \ldots$

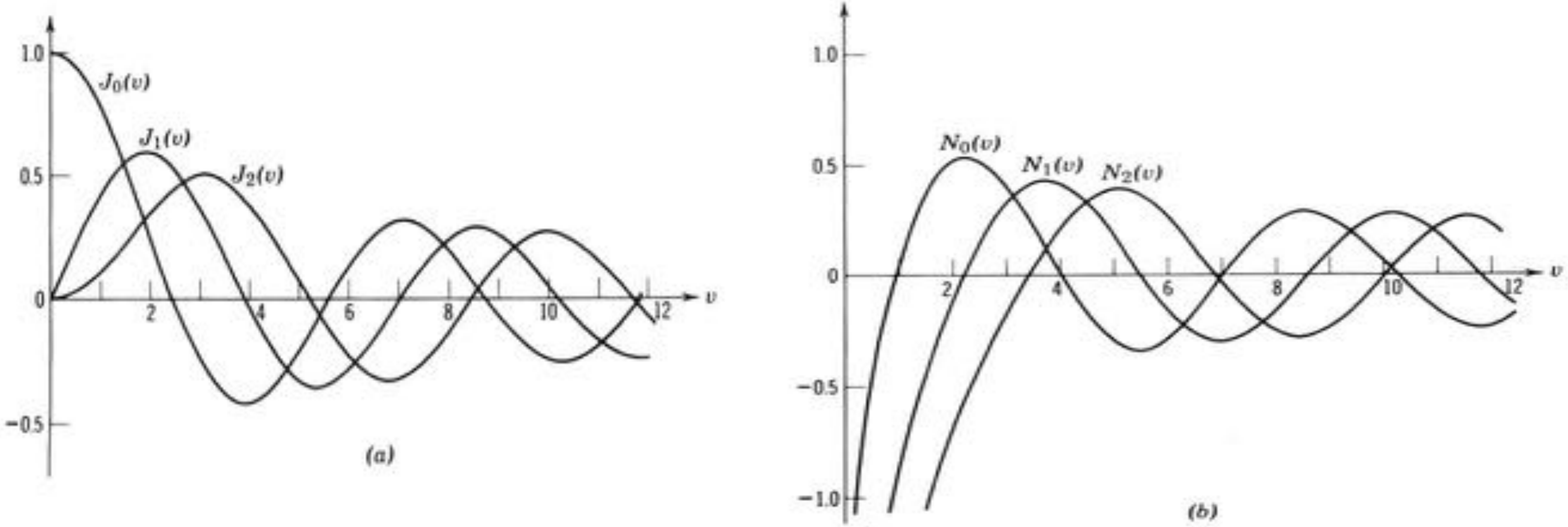
2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

We can simplify these solutions noting that:

- Often we have only forward going waves, thus $a=0$.
- The Neumann function $N_v(\kappa r)$ goes to minus infinity at $r=0$, so it is unphysical ($h=0$).

Therefore $J_v(\kappa r)$ is the proper solution in the core.



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2.3 Mode Theory for fibres.

• Due to predominant propagation of the field along the *z* axis an oscillatory characteristic is assumed for the *z* dependence

$$Z(z) = be^{-j\beta z} \Rightarrow \frac{\partial^2 Z}{\partial z^2} = -\beta$$

• We need both the clockwise and counter-clockwise circulating exponentials that describe the ϕ dependence of the eigenmodes

$$\Phi(\phi) = ce^{j\nu\phi} + de^{-j\nu\phi} \Rightarrow \frac{\partial^2 \Phi}{\partial \phi^2} = -\nu^2\phi$$

Bessel Equation

$$r^2 \frac{\partial^2 R}{\partial r^2} + r\frac{\partial R}{\partial r} + r^2\left(\kappa^2 - \frac{\nu^2}{r^2}\right)R = 0$$

$\kappa^2 = (nk_o)^2 - \beta^2$ and $\nu = 0,1,2,\ldots$

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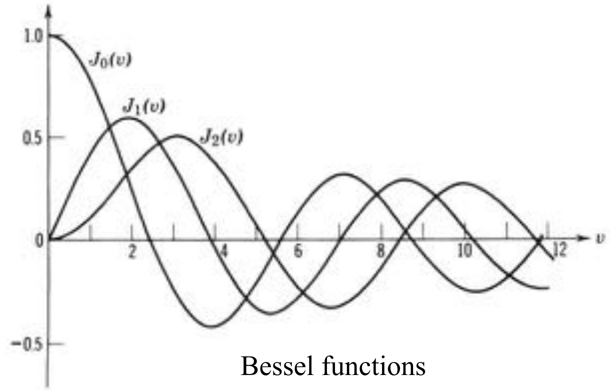
2.3 Mode Theory for fibres.



Bessel functions

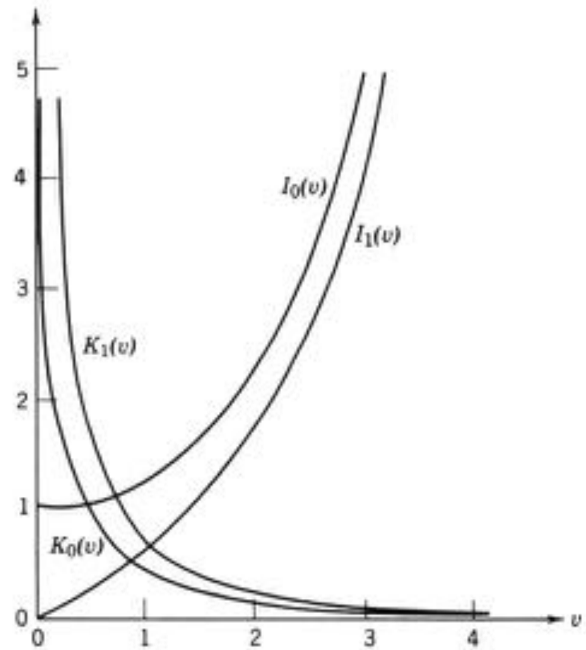
CASE 1: CORE

$$\left(\kappa^2 - \frac{\nu^2}{r^2}\right) > 0 \quad \Rightarrow \quad \boxed{\begin{aligned} E_z &= A J_\nu(\kappa r) e^{j\nu\phi - \beta z} \\ H_z &= B J_\nu(\kappa r) e^{j\nu\phi - \beta z} \end{aligned}}$$

CASE 2: CLADDING

$$\left(\kappa^2 - \frac{\nu^2}{r^2}\right) < 0 \quad \Rightarrow \quad \boxed{\begin{aligned} E_z &= C K_\nu(\gamma r) e^{j\nu\phi - \beta z} \\ H_z &= D K_\nu(\gamma r) e^{j\nu\phi - \beta z} \end{aligned}}$$

$$\gamma^2 = -\kappa^2$$



Modified Bessel functions

A,B,C and *D* are constants determined by the boundary conditions

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2.3 Mode Theory for fibres.

After some maths….
We get this "small" **CHARACTERISTIC EQUATION** for an optical fibre

$$\left(\frac{1}{\kappa a}\cdot\frac{J_\nu'(\kappa a)}{J_\nu(\kappa a)}+\frac{1}{\gamma a}\cdot\frac{K_\nu'(\gamma a)}{K_\nu(\gamma a)}\right)\cdot\left(\frac{1}{\kappa a}\cdot\frac{J_\nu'(\kappa a)}{J_\nu(\kappa a)}+\left(\frac{n_2}{n_1}\right)^2\cdot\frac{1}{\gamma a}\cdot\frac{K_\nu'(\gamma a)}{K_\nu(\gamma a)}\right)=$$

$$=\left[\frac{\beta\nu}{n_1 k_o}\cdot\left(\frac{1}{(\kappa a)^2}+\frac{1}{(\gamma a)^2}\right)\right]^2$$

The characteristic equation is used with:

$$\kappa^2=(n_1k_o)^2-\beta^2$$

$$\gamma^2=\beta^2-(n_2k_o)^2$$

$$\boxed{V^2=(\kappa a)^2+(\gamma a)^2}\quad \text{where}\quad V=k_o a\sqrt{n_1^2-n_2^2}$$

to find values for κ, γ, β and n_{eff}

2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

About the Effective Index (n_{eff})

A plane wave propagates with a phase term e^{jnkz}, where $k=2\pi/\lambda_o$ is the free-space wave-vector.

We can define an effective index for a guided wave that has a phase factor $e^{j\beta z}$ with:

$$\boxed{\beta \equiv \frac{2\pi\, n_{eff}}{\lambda_o}}$$

Then;

$$\frac{2\pi\, n_2}{\lambda_o} < \beta < \frac{2\pi\, n_1}{\lambda_o} \quad \Rightarrow \quad \boxed{n_2 < n_{eff} < n_1}$$

The effective index is an “average” index seen by the guided mode.

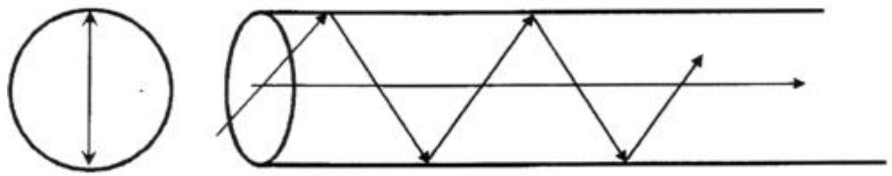
2. Light Propagation in Fibres and Related Optical Effects

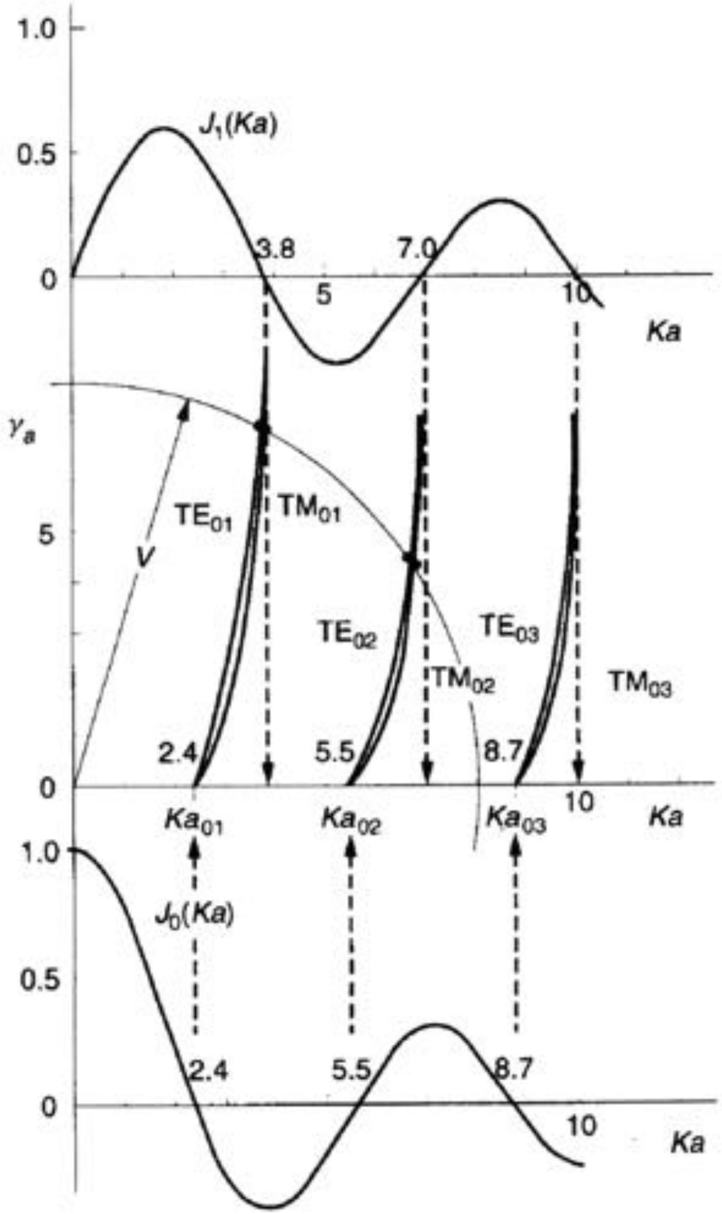
2.3 Mode Theory for fibres.

Meridional Modes (ν = 0)

For modes that correspond to bouncing meridional rays, there is no ϕ dependence. The characteristic equation simplifies greatly. Modes are of two types – $TE_{0\mu}$ (E_z=0) and $TM_{0\mu}$ (H_z=0) with μ=1,2, …. The values κ, γ can be found graphically

Meridional Rays - TE and TM modes





Curves of the characteristic equation of the $TE_{0\mu}$ and $TM_{0\mu}$ modes

2. Light Propagation in Fibres and Related Optical Effects

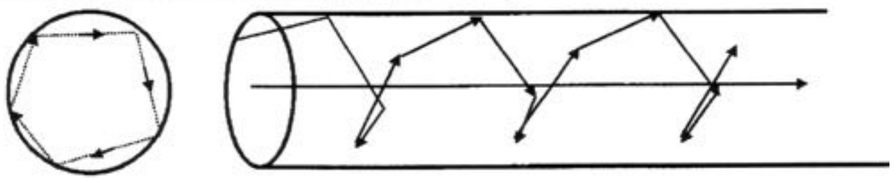
2.3 Mode Theory for fibres.

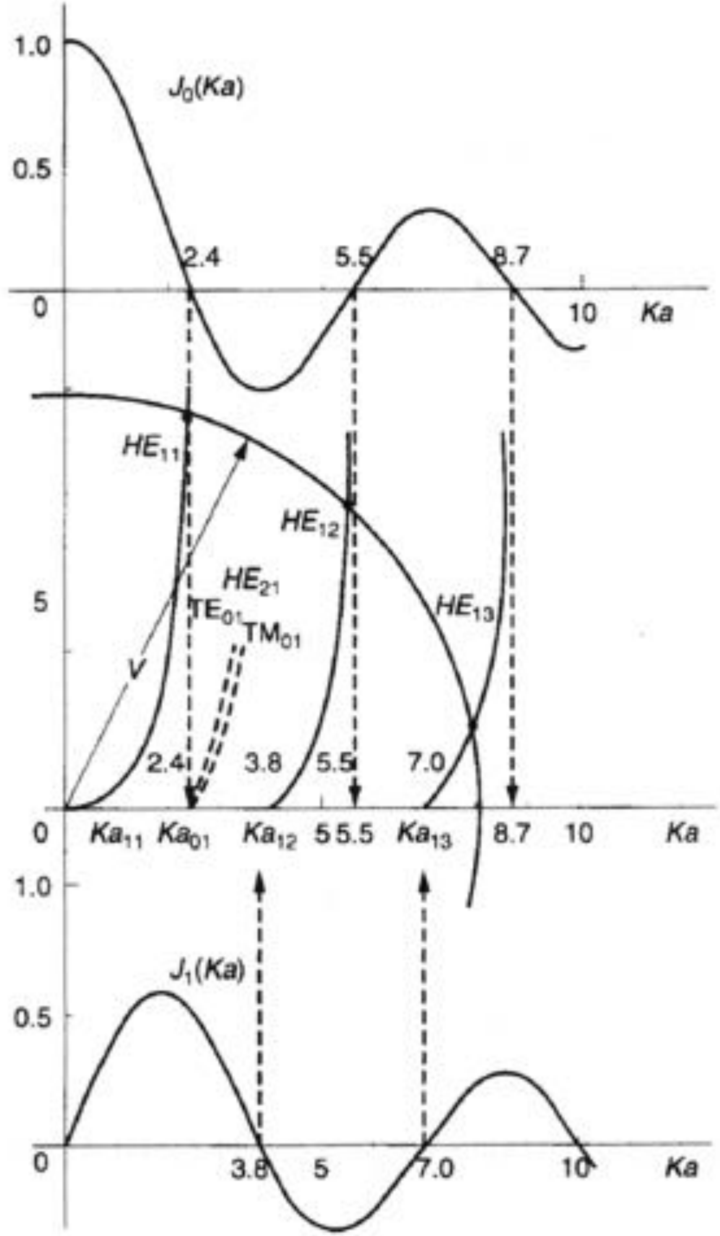
Skew Modes ($\nu \neq 0$)

Modes have both $E_z \neq 0$ and $H_z \neq 0$ and thus are called "**hybrid**" modes. The hybrid modes are labeled $EH_{\nu\mu}$ and $HE_{\nu\mu}$ depending on whether E_z or H_z is dominant.

The values κ, γ can be found graphically

Skew Ray Trajectory- EH and HE mode paths





Curves of the characteristic equation of the $HE_{1\mu}$ modes

2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

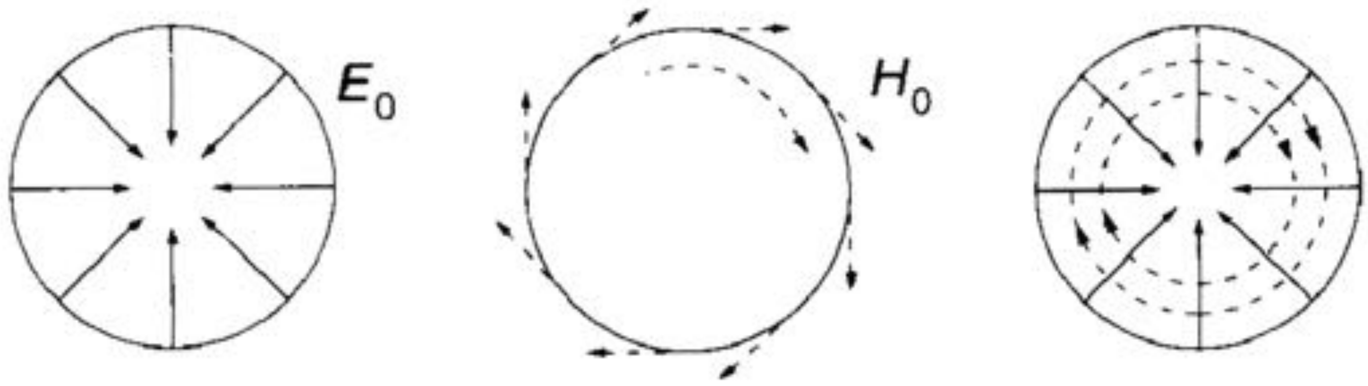
Field Distributions in Optical Fibers (1)

TE Modes

$$E_r \sim 0$$
$$E_\phi \sim J_1(\kappa_{0\mu} r)$$
$$H_r \sim -J_1(\kappa_{0\mu} r)$$
$$H_\phi \sim 0$$

$$E_r \sim -J_1(\kappa_{0\mu} r)$$
$$E_\phi \sim 0$$
$$H_r \sim 0$$
$$H_\phi \sim J_1(\kappa_{0\mu} r)$$

TM Modes



E_0 field H_0 field TM_{01} mode

This Bessel function (J_1) has a zero at the origin and one maximum in the core

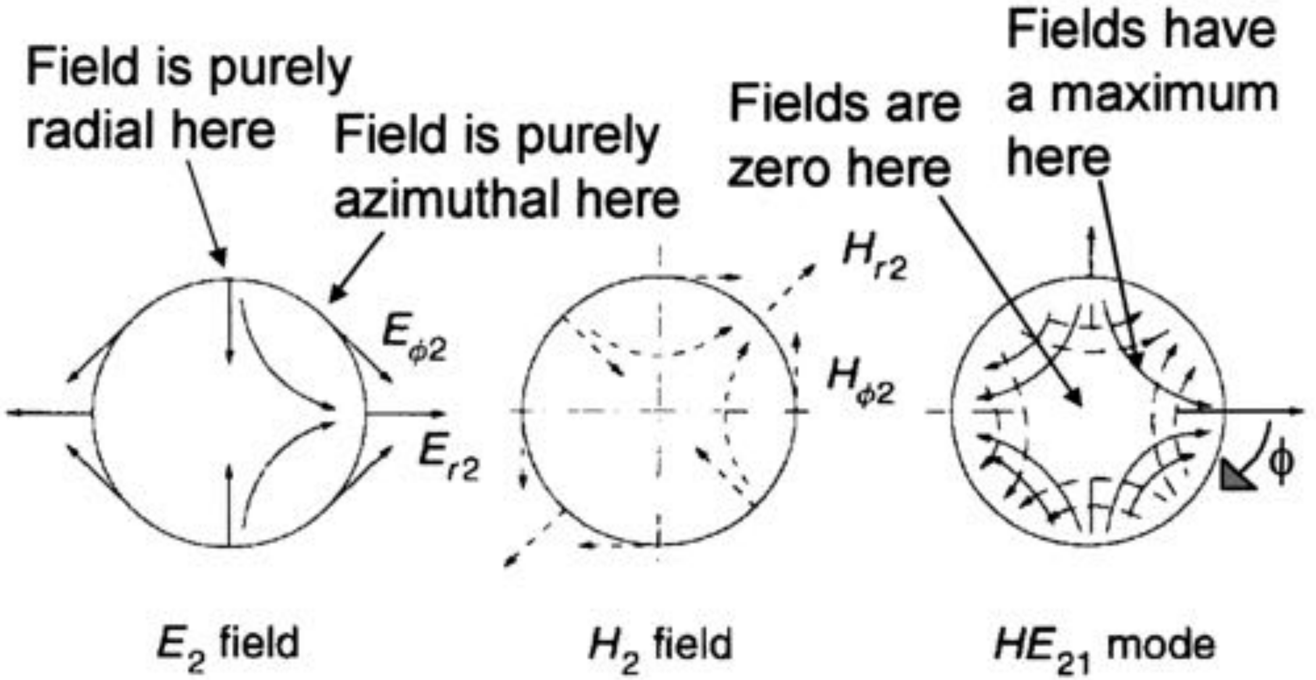
2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

Field Distributions in Optical Fibers (2)

HE_{21} Mode

$$E_r \sim J_1(\kappa_{21} r)\cos(2\phi) \qquad H_r \sim -J_1(\kappa_{21} r)\sin(2\phi)$$
$$E_\phi \sim -J_1(\kappa_{21} r)\sin(2\phi) \qquad H_\phi \sim -J_1(\kappa_{21} r)\cos(2\phi)$$



This Bessel function (J_1) has a zero at the origin and one maximum in the core

2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

Linearly Polarized (LP) Modes

When the refractive index of the core $n_1 \approx n_2$, the characteristic equation (CE) can be simplified. This is called the "**weakly guiding approximation**". The CE can be written in the unified form as:

$$\frac{J_m(\kappa a)}{\kappa J_{m-1}(\kappa a)} = \frac{K_m(\gamma a)}{\gamma K_{m-1}(\gamma a)}$$

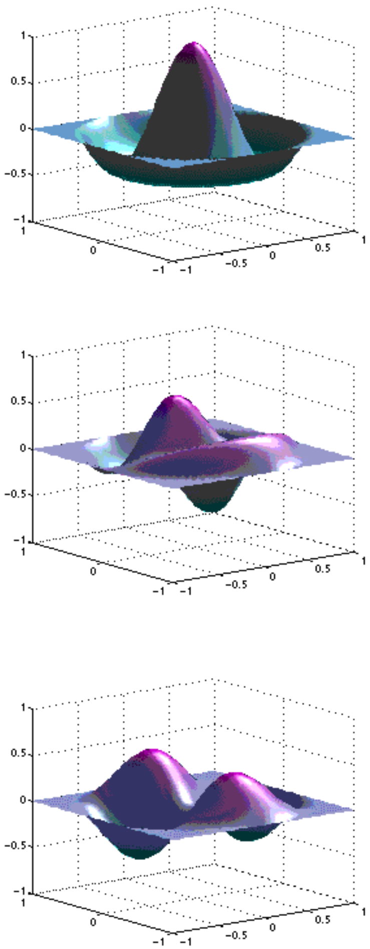
$$m = \begin{cases} 1 \rightarrow \text{ for TM and TE modes} \\ \nu + 1 \rightarrow \text{ for EH modes} \\ \nu - 1 \rightarrow \text{ for HE modes} \end{cases}$$

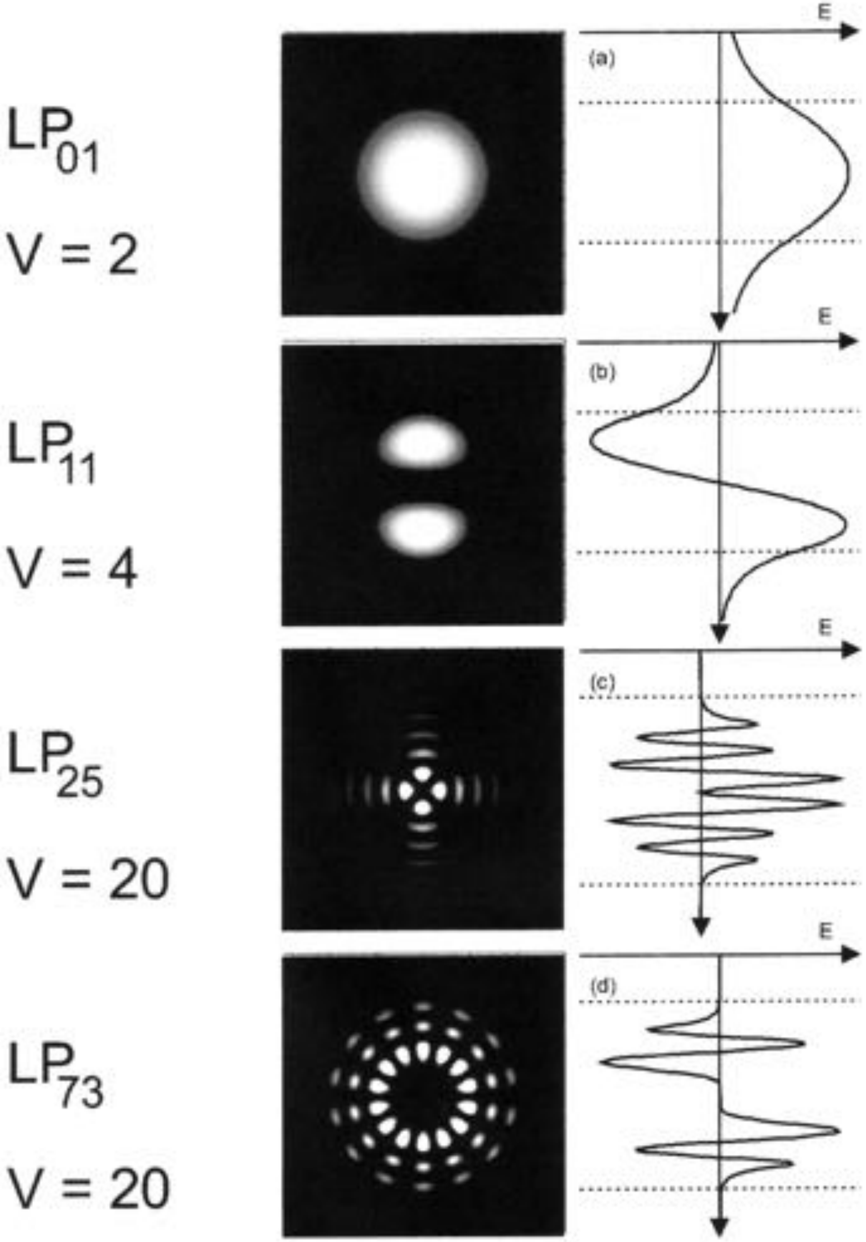
LP modes can be constructed from sums of EH and HE modes that have the same propagation constant.

2. Light Propagation in Fibres and Related Optical Effects

2.3 Mode Theory for fibres.

Linearly Polarized (LP) Modes





2. Light Propagation in Fibres and Related Optical Effects

2.4 The Cut-off wavelength and V-number.

$$u = \kappa a = a\sqrt{k_o^2 n_1^2 - \beta^2}$$

$$V^2 = (\kappa a)^2 + (\gamma a)^2$$

$$w = \gamma a = a\sqrt{\beta^2 - k_o^2 n_2^2}$$

$$\boxed{V = k_o a\sqrt{n_1^2 - n_2^2} = k_o a n_1^2 \sqrt{2\Delta}}$$

This is a dimensionless number that determines how many modes a fibre can support

$$\boxed{b = \frac{w^2}{V^2} = \frac{(\beta / k_o)^2 - n_2^2}{n_1^2 - n_2^2}}$$

Normalized Propagation Constant*

$$\boxed{\lambda_{cutoff} = \frac{2\pi a}{V}\sqrt{n_1^2 - n_2^2}}$$

Cut-off Wavelength

* D. Gloge, "*Weakly guiding fibers*", Applied Optics **10**, 2252-2258 (1971)

2. Light Propagation in Fibres and Related Optical Effects

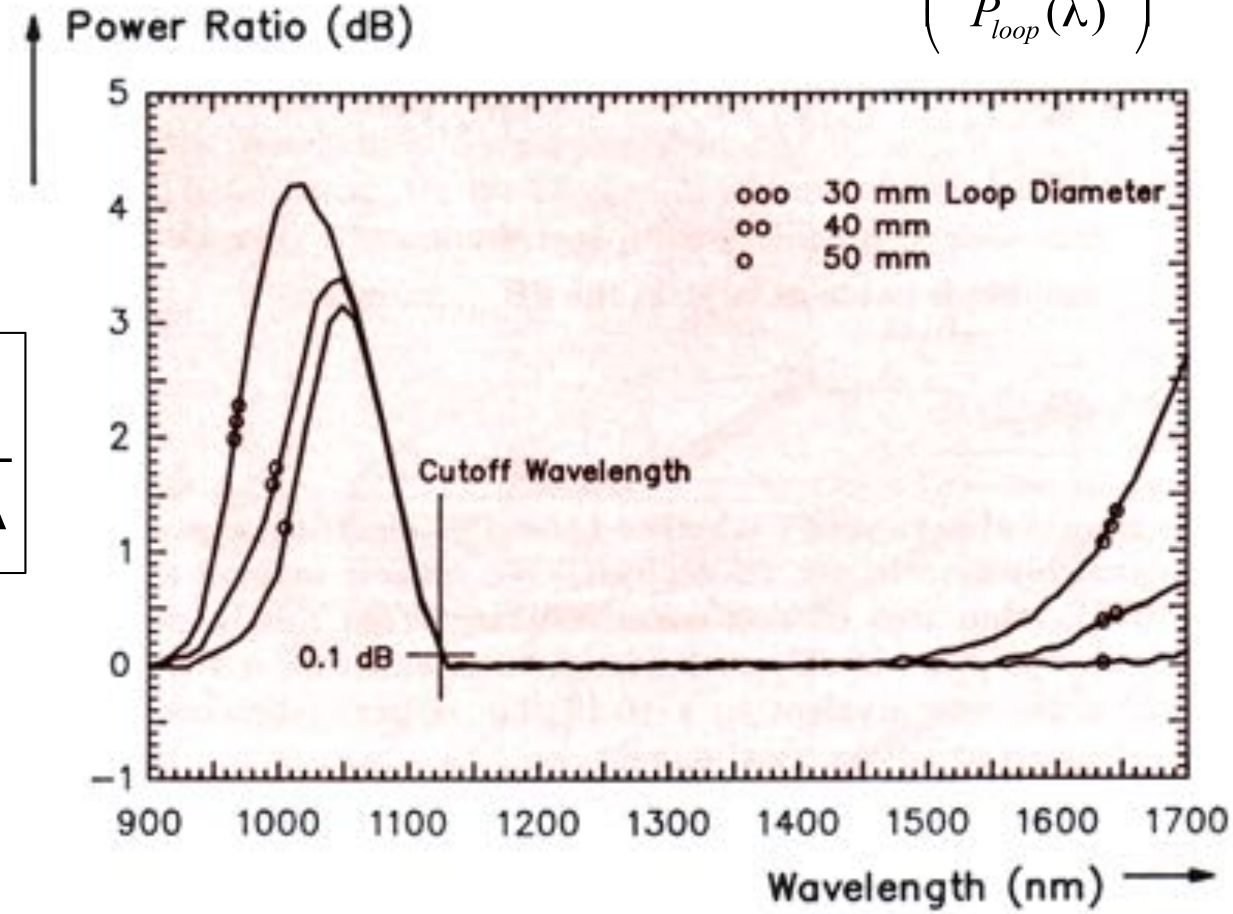
2.4 The Cut-off wavelength and V-number.

$$R(\lambda) = 10\log\left(\frac{P_{straight}(\lambda)}{P_{loop}(\lambda)}\right)$$

Singlemode Condition

$$V < V_{\min} = 2.405$$

$$\lambda > \lambda_{cutoff} = 3.7 a n_1 \sqrt{\Delta}$$



See recommendation ITU-T G.650

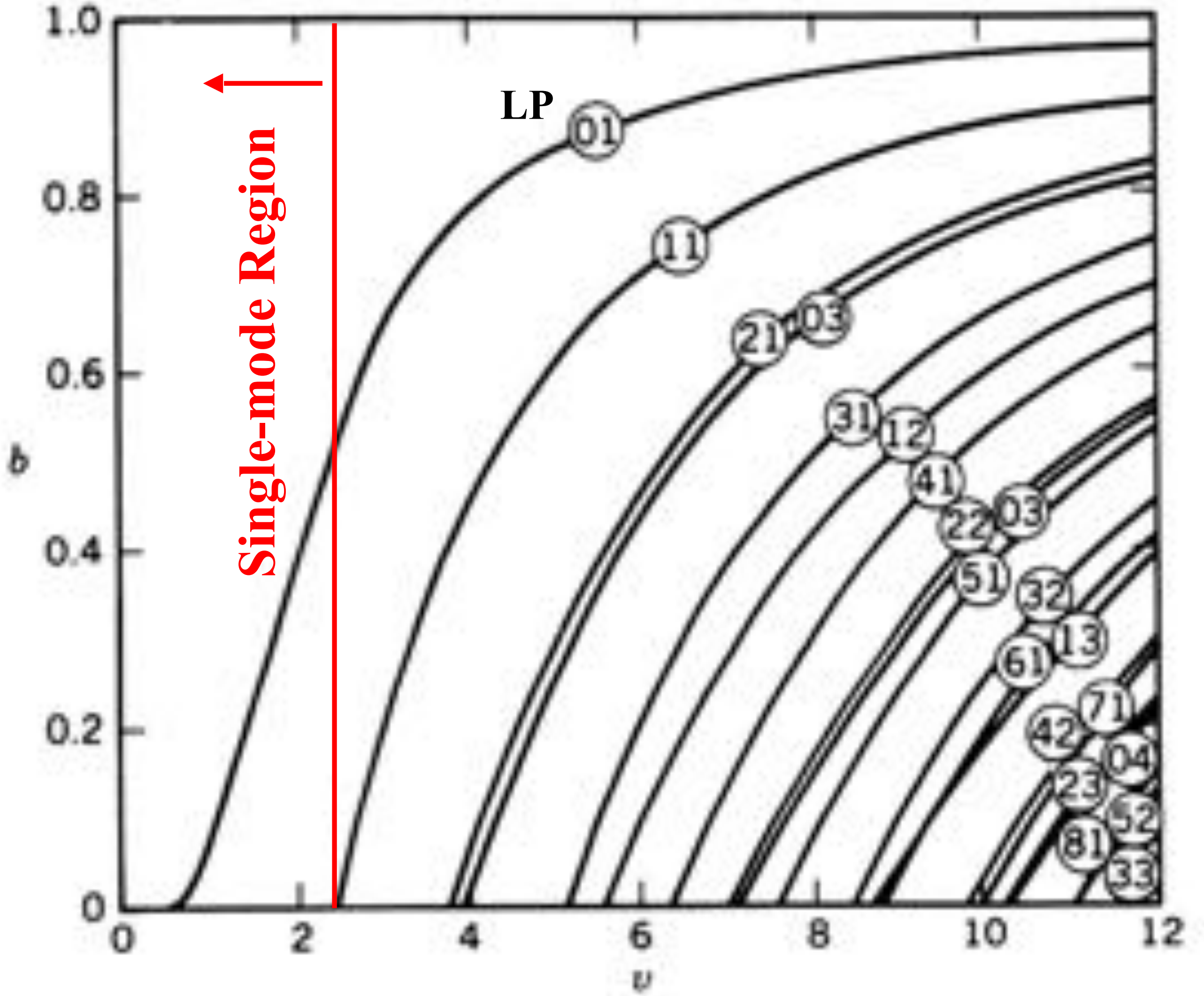
2. Light Propagation in Fibres and Related Optical Effects

2.5 Singlemode and multimode fibre propagation.

Single-mode Condition

$$V \leq 2.405$$

Step-index singlemode fibre



2. Light Propagation in Fibres and Related Optical Effects

2.5 Singlemode and multimode fibre propagation.

Numerical Aperture

$$NA = \sqrt{n_1^2 - n_2^2}$$

Solid Angle

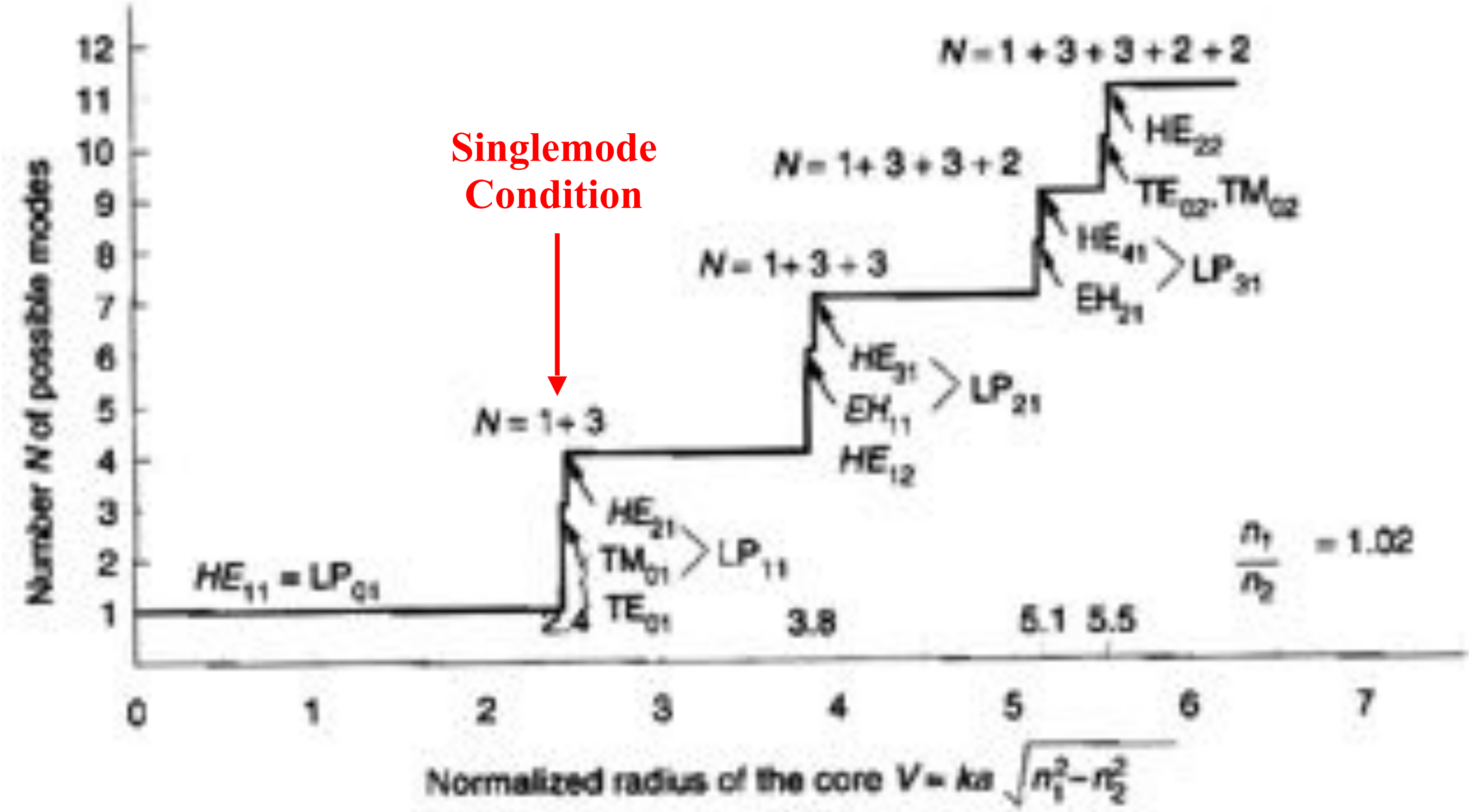
$$\Omega = \pi \sin^2 \theta \approx \pi \theta^2 = \pi \sqrt{n_1^2 - n_2^2}$$

$$\boxed{M = \frac{V^2}{2}}$$

Total number of Modes in the fibre
(Step-index Fibre)

2. Light Propagation in Fibres and Related Optical Effects

2.5 Singlemode and multimode fibre propagation.



In the weakly guiding approximation, the big steps in this figure become perfectly vertival (eg. TE_{01}, TM_{01} and HE_{21} have the same V at cutoff. Groups of modes with the same cutoff also have the same propagation constant.

2. Light Propagation in Fibres and Related Optical Effects

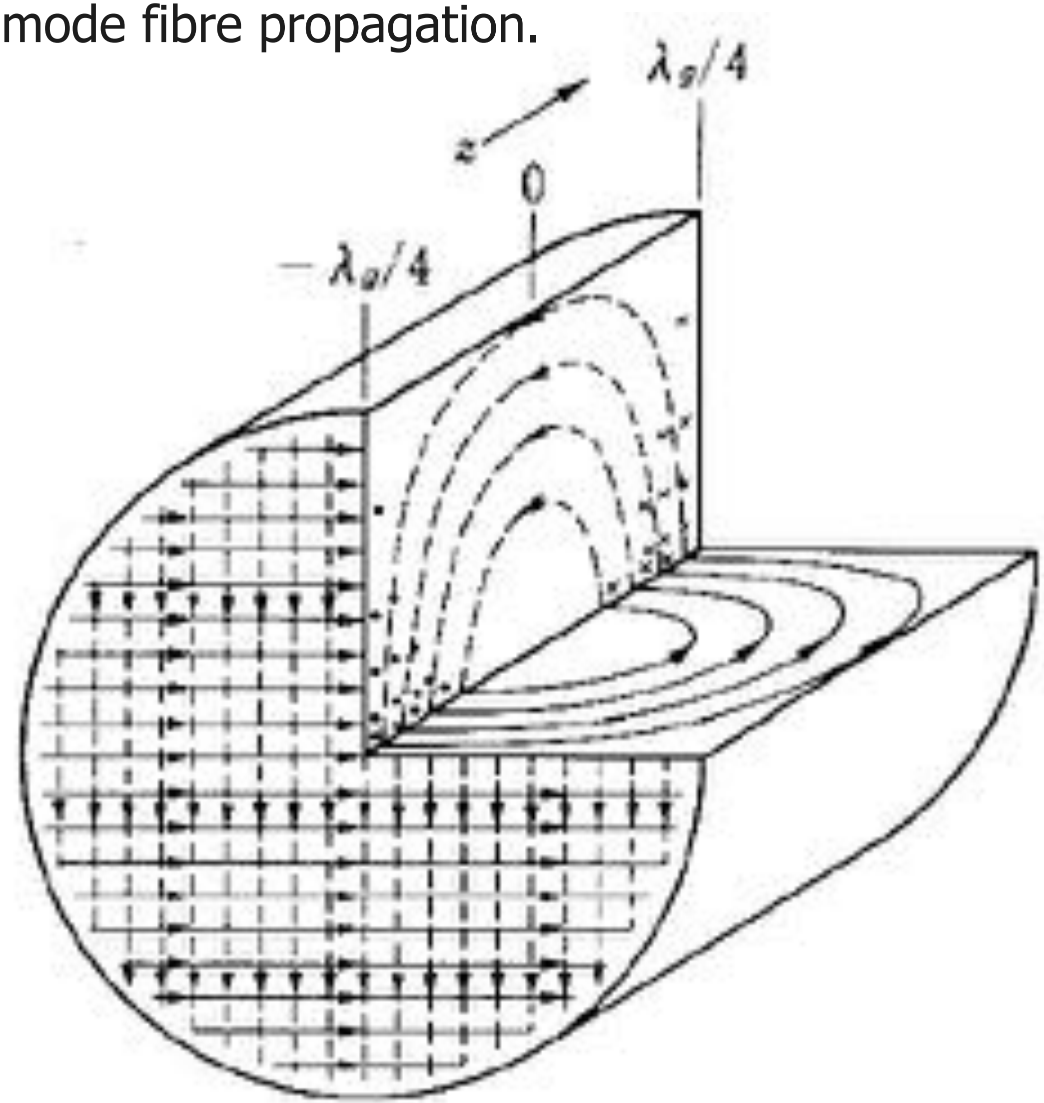
2.5 Singlemode and multimode fibre propagation.

LP-mode designation	Traditional-mode designation and number of modes	Number of degenerate modes
LP_{01}	$HE_{11} \times 2$	2
LP_{11}	$TE_{01}, TM_{01}, HE_{21} \times 2$	4
LP_{21}	$EH_{11} \times 2, HE_{31} \times 2$	4
LP_{02}	$HE_{12} \times 2$	2
LP_{31}	$EH_{21} \times 2, HE_{41} \times 2$	4
LP_{12}	$TE_{02}, TM_{02}, HE_{22} \times 2$	4
LP_{41}	$EH_{31} \times 2, HE_{51} \times 2$	4
LP_{22}	$EH_{12} \times 2, HE_{32} \times 2$	4
LP_{03}	$HE_{13} \times 2$	2
LP_{51}	$EH_{41} \times 2, HE_{61} \times 2$	4

2. Light Propagation in Fibres and Related Optical Effects

2.5 Singlemode and multimode fibre propagation.

HE$_{11}$ mode



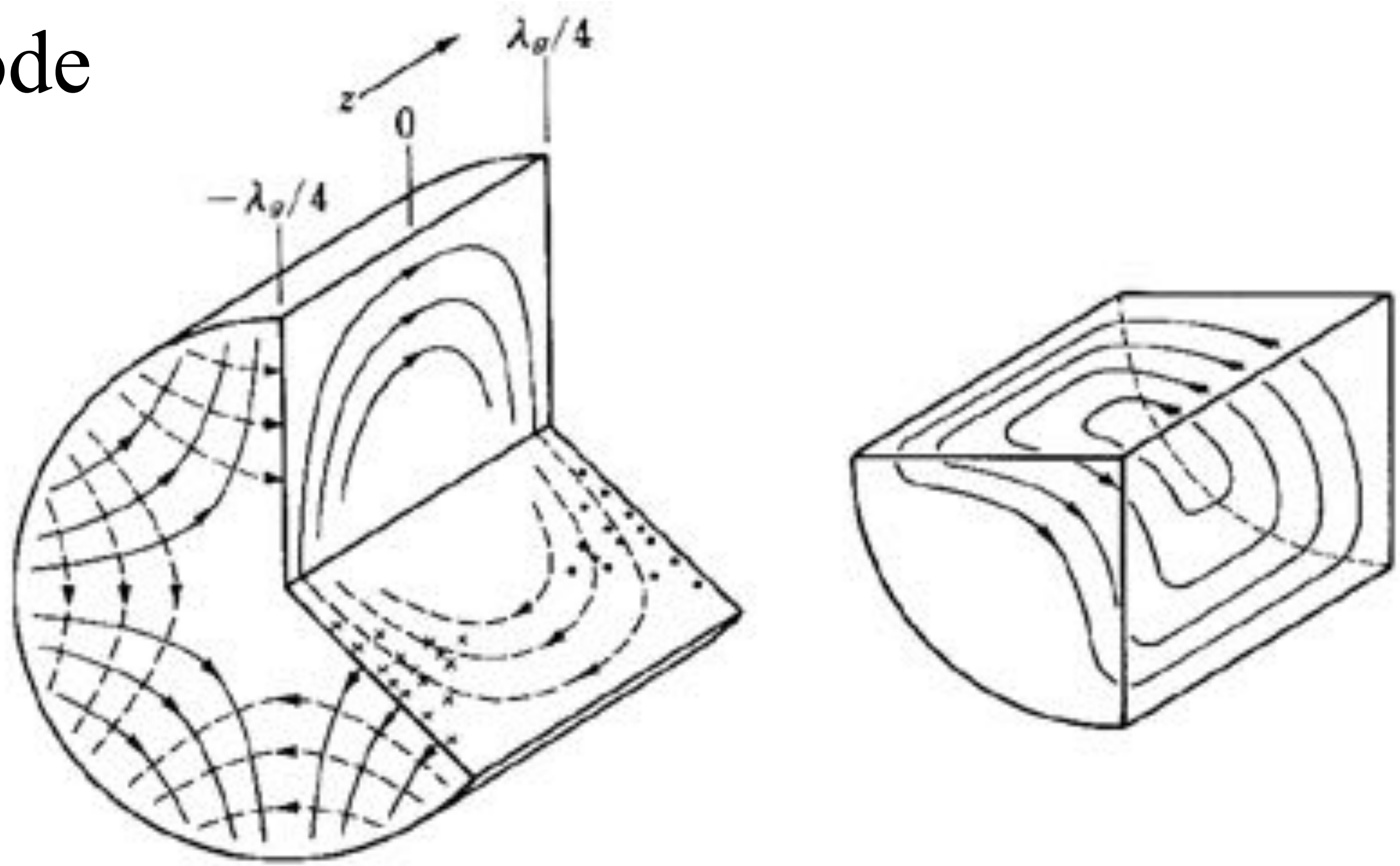
Solid lines – **E** field
Dashed lines – **H** field

With permission of C.D. Cantrell and D.M. Hollenbeck, "Fiberoptic Mode Functions: A Tutorial", Erick Jonsson School of Eng. and Computer Science, Univ. Texas at Dallas, Course EE6314.

2. Light Propagation in Fibres and Related Optical Effects

2.5 Singlemode and multimode fibre propagation.

HE_{21} mode



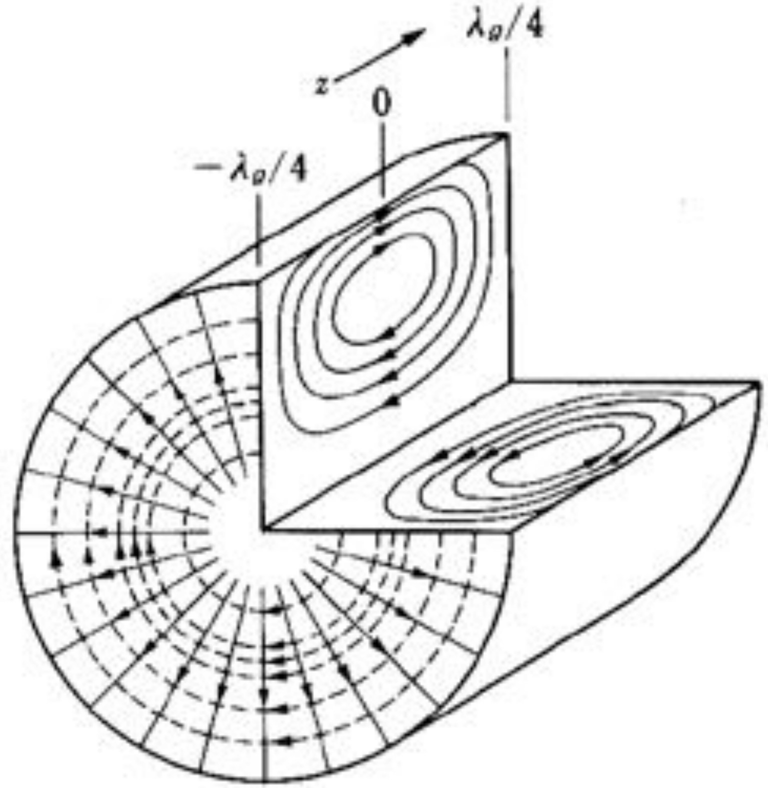
Solid lines – **E** field
Dashed lines – **H** field

With permission of C.D. Cantrell and D.M. Hollenbeck, “Fiberoptic Mode Functions: A Tutorial”, Erick Jonsson School of Eng. and Computer Science, Univ. Texas at Dallas, Course EE6314.

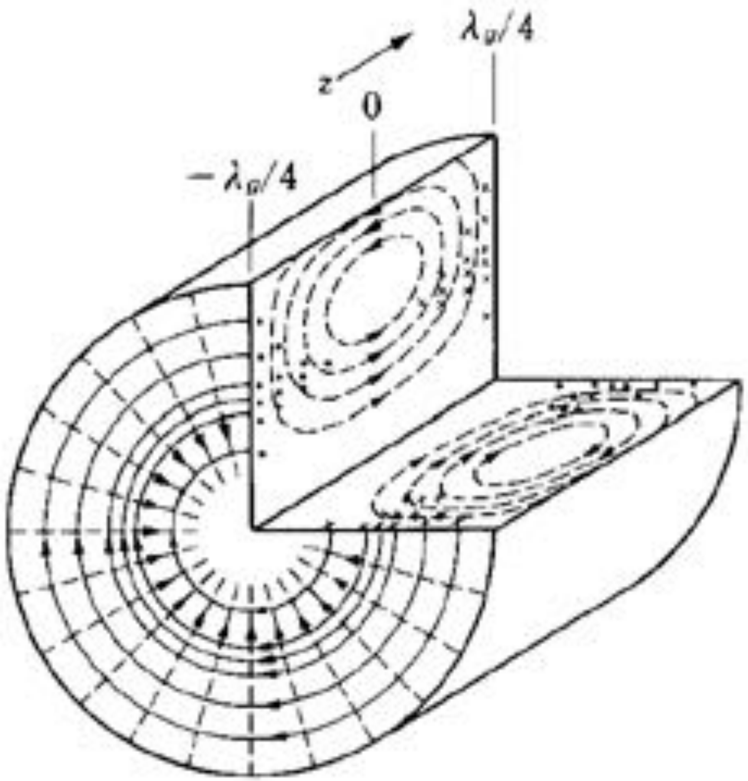
2. Light Propagation in Fibres and Related Optical Effects

2.5 Singlemode and multimode fibre propagation.

TM_{01} mode



TE_{01} mode



Solid lines – **E** field
Dashed lines – **H** field

With permission of C.D. Cantrell and D.M. Hollenbeck, "Fiberoptic Mode Functions: A Tutorial", Erick Jonsson School of Eng. and Computer Science, Univ. Texas at Dallas, Course EE6314.

2. Light Propagation in Fibres and Related Optical Effects

2.5 Singlemode and multimode fibre propagation.

Optical Power in the Mode

$$\boxed{S_z = \frac{1}{2}\left(\vec{E}\times\vec{H}^*\right)\cdot\vec{u}_z = \frac{1}{2}\left(E_r H_\theta^* - E_\theta H_r^*\right)}$$

$$P_{core} = \int_0^{2\pi}\int_0^{a} S_z r\cdot dr\cdot d\phi = \frac{1}{2}\int_0^{2\pi}\int_0^{a} r\left(E_r H_\theta^* - E_\theta H_r^*\right)\cdot dr\cdot d\phi$$

$$P_{cladding} = \int_0^{2\pi}\int_a^{\infty} S_z r\cdot dr\cdot d\phi = \frac{1}{2}\int_0^{2\pi}\int_a^{\infty} r\left(E_r H_\theta^* - E_\theta H_r^*\right)\cdot dr\cdot d\phi$$

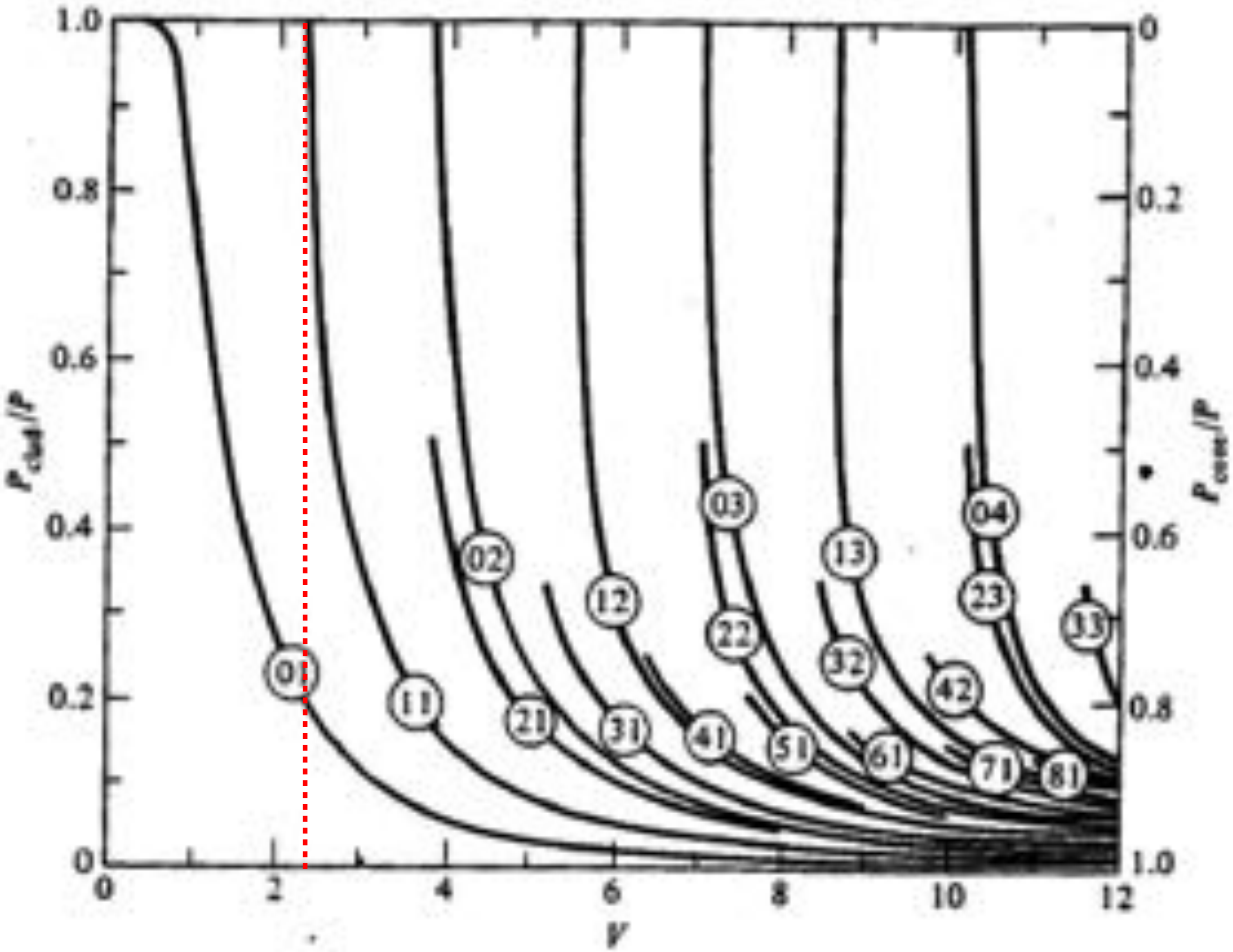
$$\frac{P_{core}}{P_{total}} = 1 - \frac{(\kappa a)^2}{V^2}\left[1 - \frac{K_m^2(\gamma a)}{K_{m-1}^2(\gamma a)K_{m+1}^2(\gamma a)}\right]$$

$$\frac{P_{cladding}}{P_{total}} = 1 - \frac{P_{core}}{P_{total}}$$

2. Light Propagation in Fibres and Related Optical Effects

2.5 Singlemode and multimode fibre propagation.

Optical Power in the Mode LP_{ml}



2. Light Propagation in Fibres and Related Optical Effects

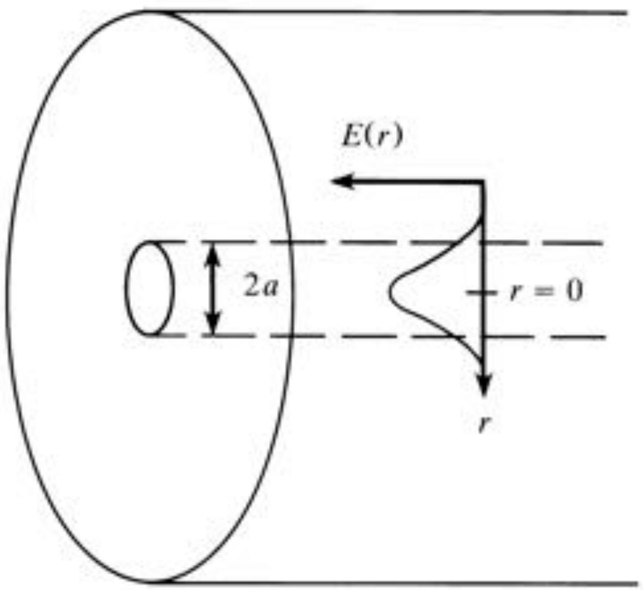
2.6 Mode Field Diameter (MFD).

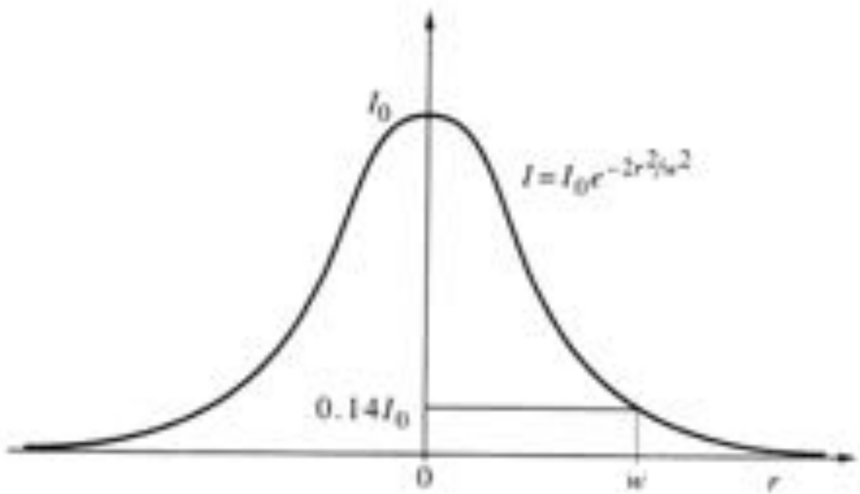
Gaussian Beam (1)

$$E(r) = E_o e^{-\left(\frac{r}{w}\right)^2}$$

$$\Downarrow$$

$$I(r) = I_o e^{-2\left(\frac{r}{w_o}\right)^2}$$





2. Light Propagation in Fibres and Related Optical Effects

2.6 Mode Field Diameter (MFD).

Gaussian Beam (2)

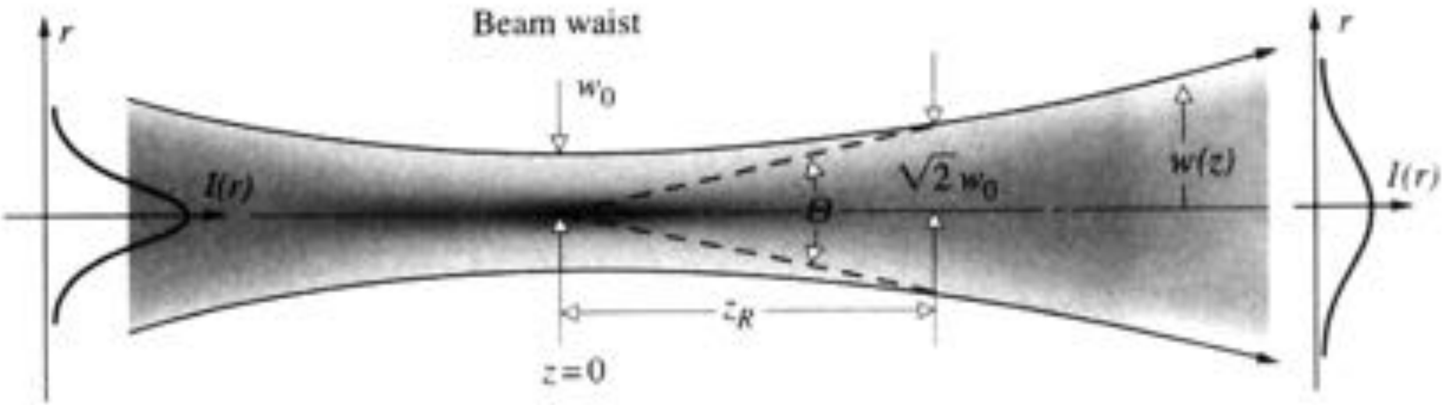
Rayleigh Range: $z_R = \dfrac{\pi w_o^2}{\lambda}$

Divergence Angle: $\Theta = \dfrac{2\lambda}{\pi w_o}$

$$w(z) = w_o \left[1 + \left(\frac{z}{z_R} \right)^2 \right]^{1/2}$$

$$R(z) = z \left[1 + \left(\frac{z_R}{z} \right)^2 \right]$$

for z large $\Rightarrow w(z) \approx \dfrac{\lambda z}{\pi w_o}$



H. Kolgelnik and T. Li, "Laser beams and resonators", Applied Optics **5**, 1550-1566 (1966)

2. Light Propagation in Fibres and Related Optical Effects

2.6 Mode Field Diameter (MFD).

Petermann I Integral (Near-Field):

$$MFD_I = 2\sqrt{2} \cdot \left[\frac{\int_0^\infty E^2(r) r^3 \cdot dr}{\int_0^\infty E^2(r) r \cdot dr} \right]^{1/2}$$

Petermann II Integral* (Far-Field):

$$MFD_{II} = \frac{\lambda}{\pi} \cdot \left[\frac{2\int_{-\theta}^{\theta} I(\theta) \sin\theta \cos\theta \cdot d\theta}{\int_{-\theta}^{\theta} I(\theta) \sin^3\theta \cos\theta \cdot d\theta} \right]^{1/2}$$

*TIA/EIA FOTP-191, ITU-T G.650E

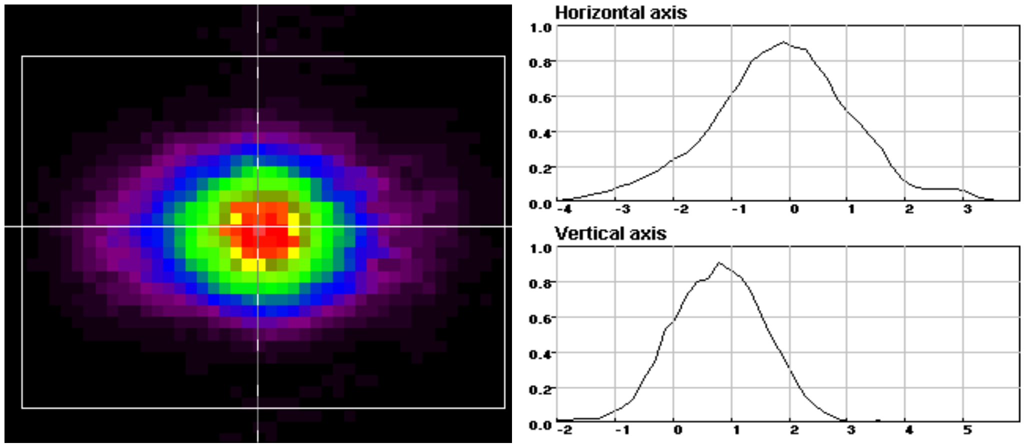
M. Artiglia et al, "Mode Field Diameter Measurements in Single-Mode Optical Fibers", Journal of Lightwave Technology, Vol. 7, No. 8. (1989)

2. Light Propagation in Fibres and Related Optical Effects

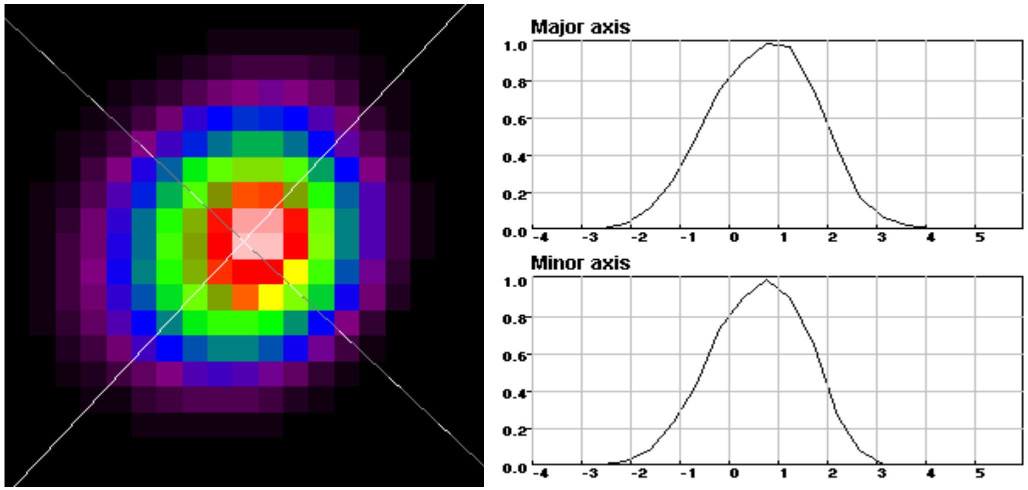
2.6 Mode Field Diameter (MFD).

Near-Field Profiles

100X



40X

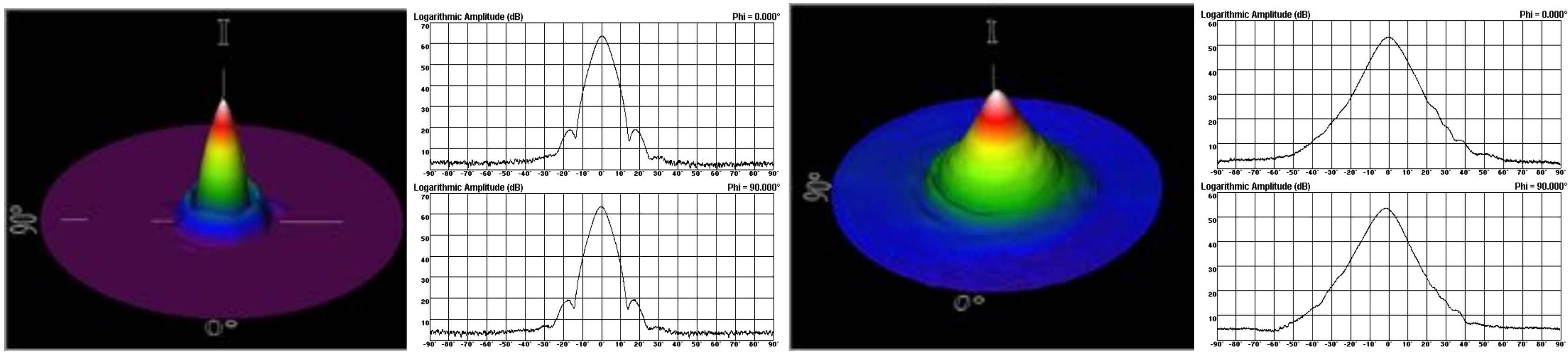


J. L. Guttman, "*Mode-Field Diameter and "Spot Size" Measurements of Lensed and Tapered Specialty Fibers*", NIST Symposium on Optical Fiber Measurements, September 24-26, 2002

2. Light Propagation in Fibres and Related Optical Effects

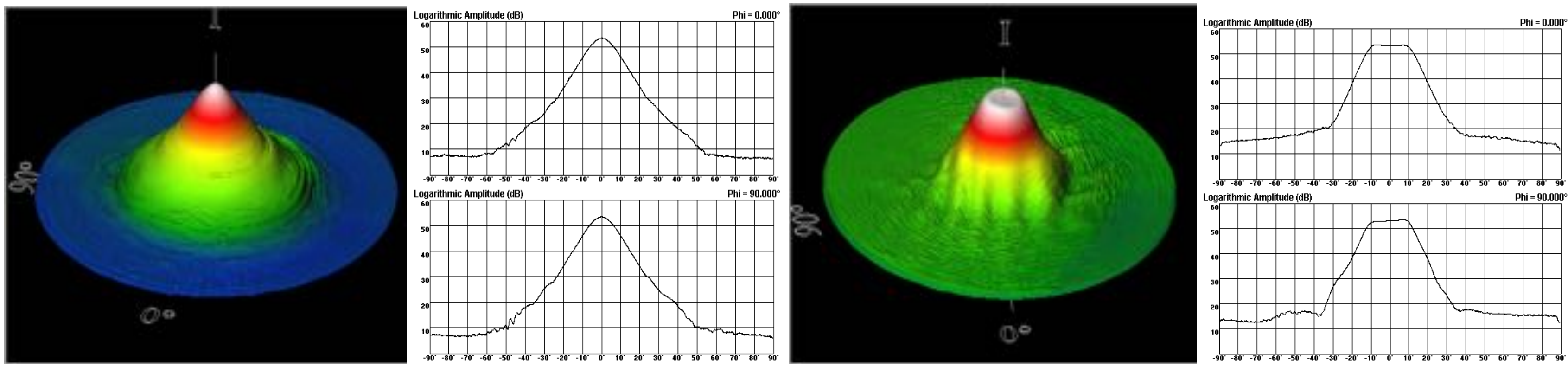
2.6 Mode Field Diameter (MFD).

Far-Field Profiles



a.) Fiber #1

b.) Fiber #2



J. L. Guttman, "*Mode-Field Diameter and "Spot Size" Measurements of Lensed and Tapered Specialty Fibers*", NIST Symposium on Optical Fiber Measurements, September 24-26, 2002

2. Light Propagation in Fibres and Related Optical Effects

2.6 Mode Field Diameter (MFD).

Petermann II Integral* (Far-Field):

(*TIA/EIA FOTP-191, ITU-T G.650E)

$$MFD_{II} = \frac{\lambda}{\pi} \cdot \left[\frac{2\int_{-\theta}^{\theta} I(\theta)\sin\theta\cos\theta \cdot d\theta}{\int_{-\theta}^{\theta} I(\theta)\sin^{3}\theta\cos\theta \cdot d\theta} \right]^{1/2}$$

- Errors [1,2]
    - Obliquity Factor and Aperture Field
- Elliptical Fiber [3]
    - Radial Symmetry for Hankel Transform
- Field Within Fiber vs Field at Focus

[1] M. Young, “Mode-field Diameter of single-mode optical fiber by far-field scanning”, Applied Optics, Vol. 37, No. 24, August 1998
[2] R. C. Wittmann and M. Young, “Are the Formulas for Mode-Field Diameter Correct?”, NIST SOFM 1998
[3] M. Artiglia et al, “Mode Field Diameter Measurements in Single-Mode Optical Fibers”, Journal of Lightwave Technology, Vol. 7, No. 8. August 1989

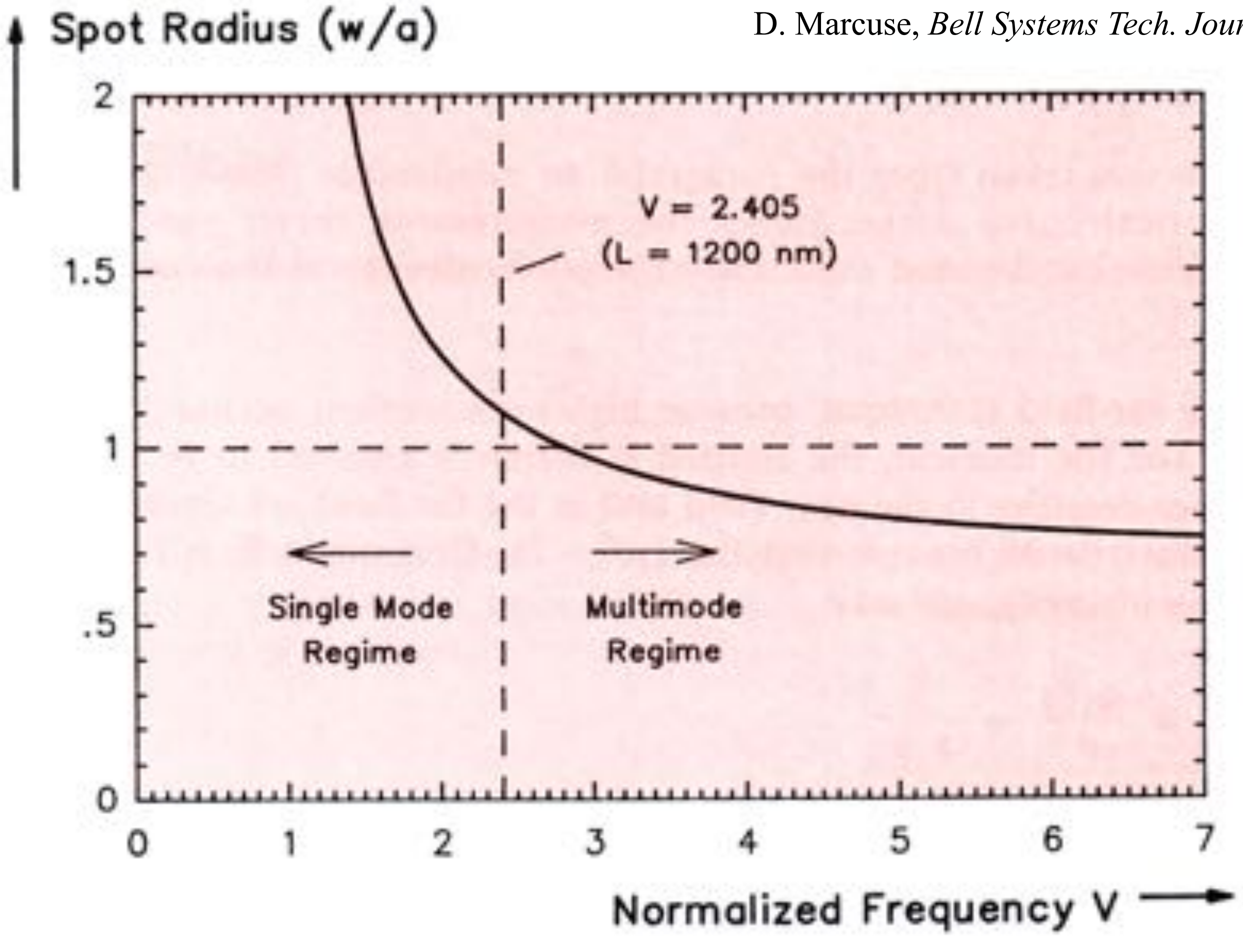
2. Light Propagation in Fibres and Related Optical Effects

2.6 Mode Field Diameter (MFD).

Marcuse Model:

$$w = a\left[0.65 + \frac{1.619}{V^{1.5}} + \frac{2.879}{V^{6}}\right]$$

D. Marcuse, *Bell Systems Tech. Journal* **56**, 703-718 (1977)



2. Light Propagation in Fibres and Related Optical Effects

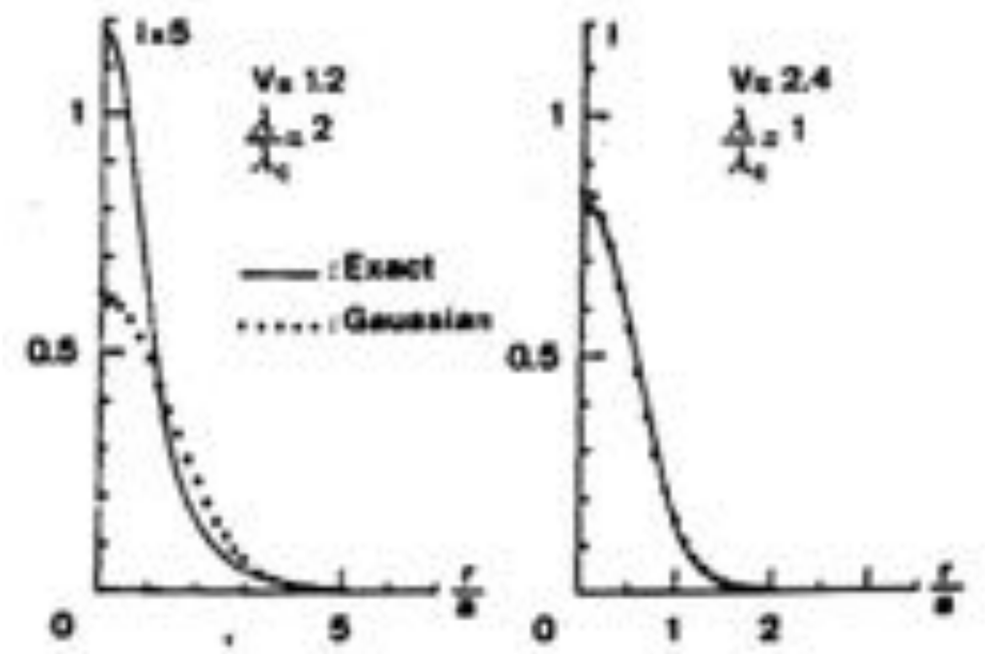
2.6 Mode Field Diameter (MFD).

For the best fit between a Gaussian function and the Bessel function in the core we can use the Marcuse Model:

$$w = a\left[0.65 + \frac{1.619}{V^{1.5}} + \frac{2.879}{V^{6}}\right]$$

Satisfying this condition gives about 96% overlap between the Gaussian and the Bessel function mode profiles. At the cut-off condition ($V \approx 2.405$) we obtain:

$$w \approx 1.1a$$



2. Light Propagation in Fibres and Related Optical Effects

2.6 Mode Field Diameter (MFD).

Other MFD models

Snyder and Sammut Model $$w = a\frac{1}{(\ln V)^{1.5}} \quad \text{for } V>1$$

Myslinski Model $$w = a\left[0.761 + \frac{1.237}{V^{1.5}} + \frac{1.429}{V^{6}}\right]$$

Desurvire Model $$w = a\left[0.759 + \frac{1.289}{V^{1.5}} + \frac{1.041}{V^{6}}\right]$$

Whitley Model $$w = a\left[0.616 + \frac{1.66}{V^{1.5}} + \frac{0.987}{V^{6}}\right]$$

Several models have been developed to obtain better agreement with experimentally observed data (particularly gain factors $\rightarrow$ Erbium doped fibres)

2. Light Propagation in Fibres and Related Optical Effects

2.7 Absorption and Scattering losses.

SCATTERING

- LINEAR ⇒
  - Rayleigh (diameter physical anomalies < $\lambda/10$)
  - Mie (diameter physical anomalies > $\lambda/10$)
- NON-LINEAR ⇒
  - Brillouin (acoustic phonon → SBS)
  - Raman (optical phonon → SRS)

Rayleigh Scattering Coefficient ⇒ Transmission Loss factor

$$\gamma_R = \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta_T k_B T_F \quad \Longrightarrow \quad F_{Rayleigh} = e^{-\gamma_R L}$$

n – refractive index of the material, β_T – isothermal compressibility, p – photoelastic coefficient, T_F – *Solidification temperature,* k_B – Boltzman constant, L - the fibre length.

2. Light Propagation in Fibres and Related Optical Effects

2.7 Absorption and Scattering losses.

ABSORPTION

- INTRINSIC ⇒ Interaction of free electrons and the λ within the fibre material.
- EXTRINSIC ⇒ Impurities atoms in glass material (metal ions)
- INDUCED ⇒ Curvature (micro and macrobending)

Urbach's rule (empirical relationship)

$$\alpha_{UV}[dB/km] = Ce^{\frac{\lambda_{UV}}{\lambda}} = 1.108\times10^{-3}e^{\frac{4.582}{\lambda[\mu m]}}$$

$$\alpha_{IR}[dB/km] = 4\times10^{-11}e^{\frac{48.48}{\lambda[\mu m]}}$$

Macrobending (critical radius)

$$R_{MMF} = \frac{3\lambda n_1^2}{4\pi(n_1^2 - n_2^2)}$$

$$R_{SMF} = \frac{20\lambda}{\sqrt{n_1^2 - n_2^2}}\left(2.748 - \frac{0.996\lambda}{\lambda_{cutoff}}\right)^{-3}$$

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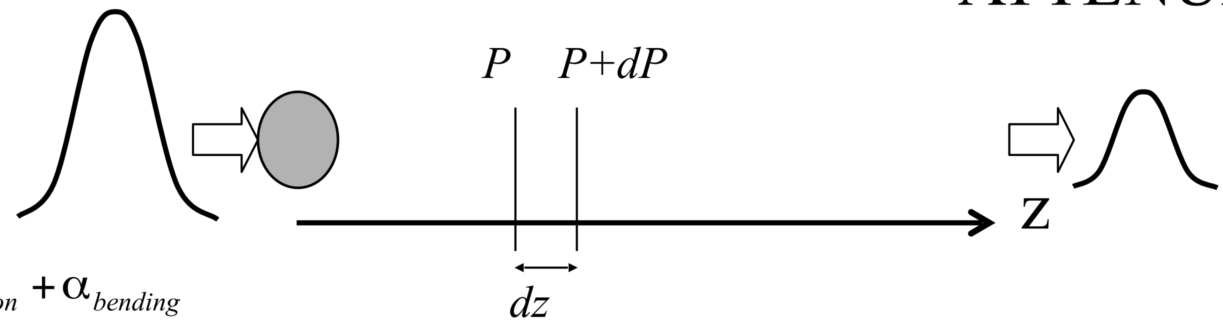
2. Light Propagation in Fibres and Related Optical Effects

2.7 Absorption and Scattering losses.

ATTENUATION



$$\alpha = \alpha_{scattering} + \alpha_{absorption} + \alpha_{bending}$$

$$\frac{dP}{dz} = -\alpha L \Rightarrow P(L) = P(0)e^{-\alpha L} \Rightarrow \alpha = \frac{1}{L}\ln\left[\frac{P(0)}{P(L)}\right] \quad [1/\text{km or } 1/\text{m}]$$

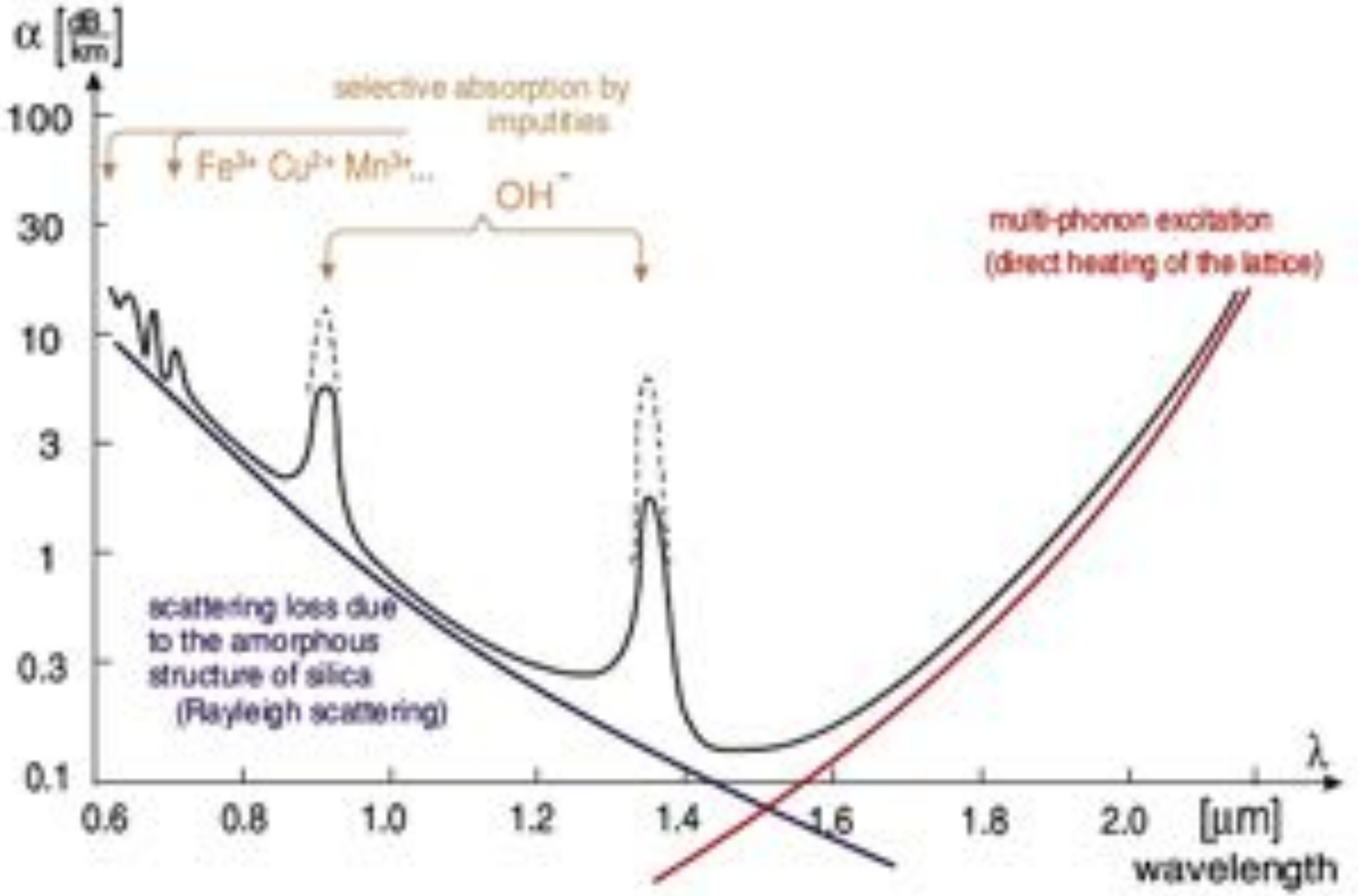
$$\frac{P(0)}{P(L)} = e^{\alpha L} \Leftrightarrow 10\log\left[\frac{P(0)}{P(L)}\right] = 10\log\left[e^{\alpha L}\right] = 10\alpha L \log e = 4.343\alpha$$

$$\alpha[dB/km] = \frac{10}{L}\log\left[\frac{P(0)}{P(L)}\right]$$

$$\alpha[dB/km] = 4.343\alpha[1/km]$$

2. Light Propagation in Fibres and Related Optical Effects

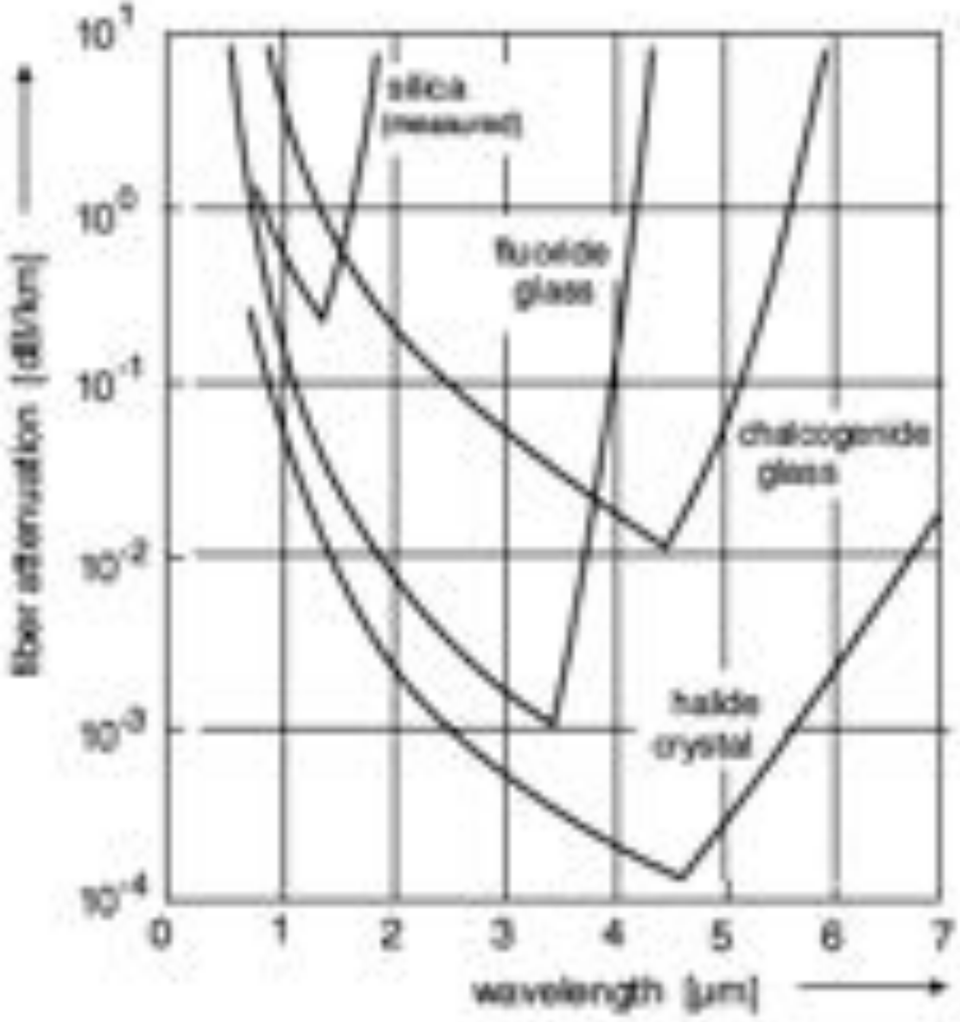
2.7 Absorption and Scattering losses.



2. Light Propagation in Fibres and Related Optical Effects

2.7 Absorption and Scattering losses.

Theoretically expected minimum attenuation



Governed by:
1. Rayleigh scattering at short λ
2. Multi-phonon absorption at long λ

2. Light Propagation in Fibres and Related Optical Effects

2.7 Absorption and Scattering losses.

Typical minimum attenuation values for several fibres

Fibre	***λ (nm)***	***α (dB/km)***
MMF-GI	850	2.5
SMF	1300	0.5
SMF	1550	≤0.25
F-POF (Fluorinated)	800 to 1340	60
ZBLAN	1550	0.02
SMFF (Fluoride glass)	2300	0.005

2. Light Propagation in Fibres and Related Optical Effects

2.8 Phase velocity and group velocity.

Group Velocity

$$v_g = \frac{d\omega}{d\beta}$$

$$\Downarrow$$

$$v_g = \frac{c}{\left(n_1 - \lambda \frac{dn_1}{d\lambda} \right)}$$

Guide Group Index (n_g)

Phase Velocity

$$v_{ph} = \frac{\omega}{\beta}$$

$$\Downarrow$$

$$v_{ph} = \frac{c}{n_1}$$

2. Light Propagation in Fibres and Related Optical Effects

2.9 Dispersion: Group delay and Material dispersion.

Group Delay

$$\tau = \frac{L}{v_g} = L\frac{d\beta}{d\omega} = \frac{L}{c}\cdot\frac{d\beta}{dk} = L\beta_1$$

Dispersion Parameter

$$D = \frac{1}{L}\cdot\frac{d\tau}{d\lambda_o}$$

Origin of the Dispersion: Frequency dependence of the mode index $n(\omega)$

$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \beta_0 + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2 + \ldots$$

$$\beta_1 = \frac{1}{c}\left(n + \omega\frac{dn}{d\omega}\right) = \frac{n_g}{c} = \frac{1}{v_g}$$

$$\beta_2 = \frac{1}{c}\left(2\frac{dn}{d\omega} + \omega\frac{d^2n}{d\omega^2}\right) \simeq \frac{\omega}{c}\cdot\frac{d^2n}{d\omega^2} \simeq \frac{\lambda^3}{2\pi c^2}\cdot\frac{d^2n}{d\lambda^2}$$

2. Light Propagation in Fibres and Related Optical Effects

2.9 Dispersion: Group delay and Material dispersion.

Dispersion Parameter:

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2}\beta_2 \simeq -\frac{\lambda}{c}\cdot\frac{d^2 n}{d\lambda^2}$$

Group Velocity Dispersion (GVD):

(Contains the information about the variation of the group velocity with wavelength)

$$\beta_2 = \frac{d\beta_1}{d\omega} = \frac{d}{d\omega}\left(\frac{1}{v_g}\right) = -\frac{1}{v_g^2}\cdot\frac{dv_g}{d\omega}$$

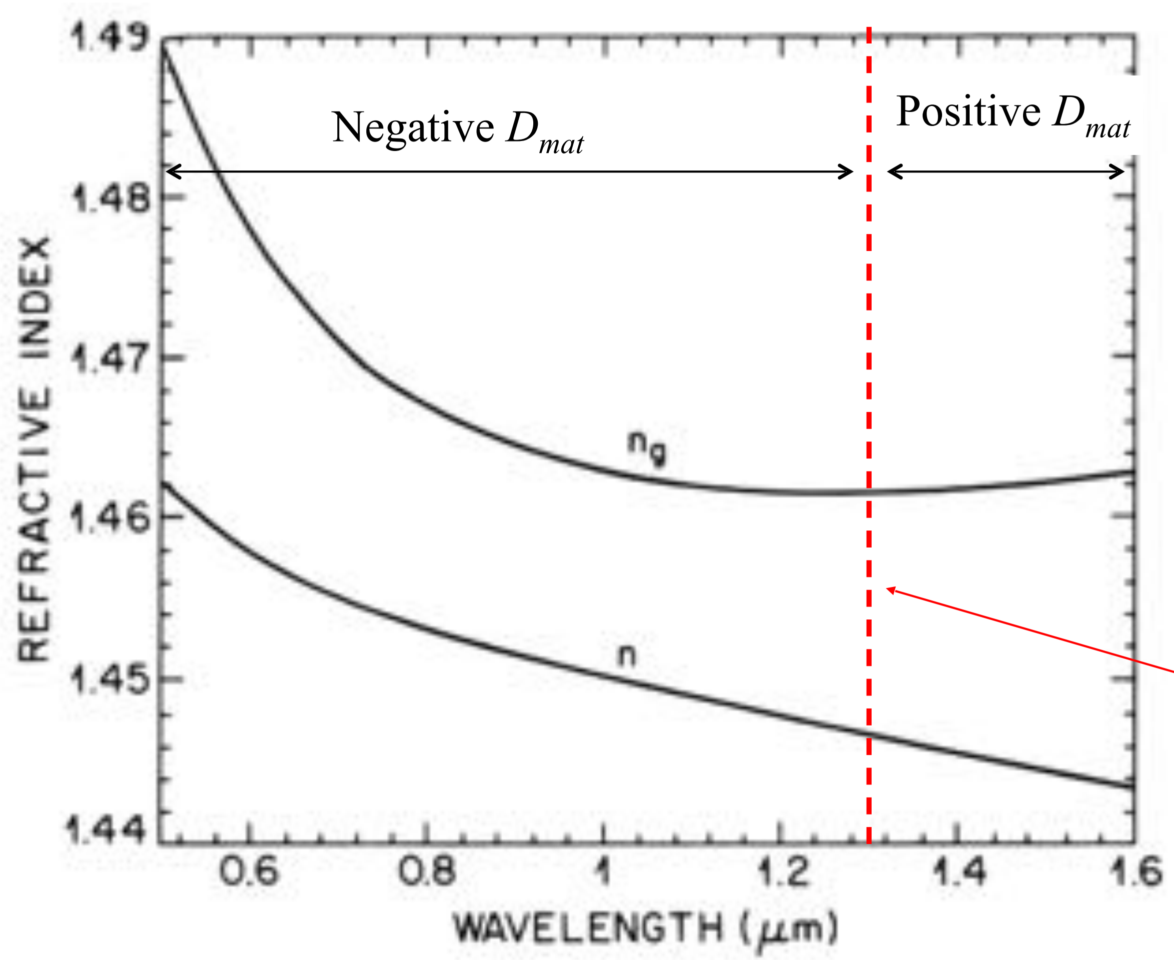
If a pulse with spectral width ($\Delta\delta$) input to a fibre with length L, the output pulse broadening is:

$$\delta t = \frac{d}{d\omega}\left(\frac{L}{v_g}\right)\delta\omega = L\beta_2\cdot\delta\omega = \frac{d}{d\lambda}\left(\frac{L}{v_g}\right)\delta\lambda = LD\cdot\delta\lambda$$

Limitation on the bit rate: $\delta t < \dfrac{1}{B}$ or $B\delta t = BL\beta_2\cdot\delta\omega BLD\cdot\delta\lambda < 1$

2. Light Propagation in Fibres and Related Optical Effects

2.9 Dispersion: Group delay and Material dispersion.



Material Dispersion

$$D_{mat} = \frac{1}{c} \cdot \frac{dn_g}{d\lambda} = -\frac{\lambda}{c} \cdot \frac{d^2 n_1}{d\lambda^2}$$

$$\boxed{\frac{dn_g}{d\lambda} = 0}$$

Zero-dispersion wavelength = 1276 nm

2. Light Propagation in Fibres and Related Optical Effects

2.10 Chromatic Dispersion (CD)

Normalized propagation constant*: $b = \frac{(\beta / k_o)^2 - n_2^2}{n_1^2 - n_2^2}$ $\xrightarrow{\text{for small } \Delta}$ $b \cong \frac{(\beta / k_o) - n_2}{n_1 - n_2}$

$$\Longrightarrow \quad \beta = k_o n_2 (1 + b\Delta) = k_o n_{eff}$$

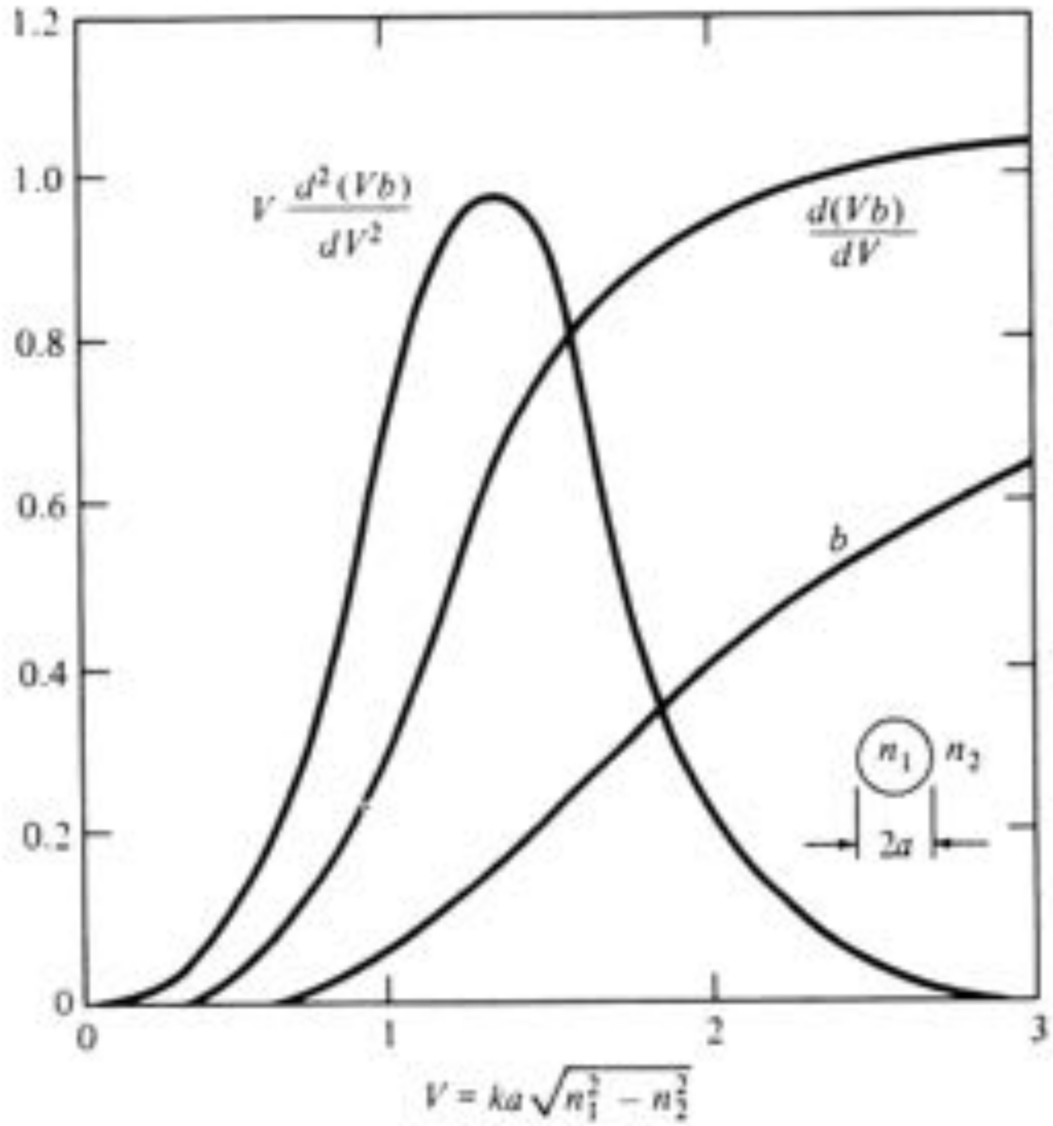
$$D = D_{mat} + D_{wg} \quad \left\{ \begin{array}{l} \boxed{D_{mat} = \frac{1}{c} \cdot \frac{dn_g}{d\lambda} = -\frac{\lambda}{c} \cdot \frac{d^2 n_1}{d\lambda^2}} \\ \\ \boxed{D_{wg} = -\frac{n_2 \Delta}{c\lambda} V \frac{d^2 (Vb)}{dV^2}} \end{array} \right.$$

* See D. Gloge, "*Weakly guiding fibers*", Applied Optics **10**, 2252-2258 (1971)

D. Gloge, "*Dispersion in weakly guiding fibers*", Applied Optics **10**, 2442-2445 (1971).

2. Light Propagation in Fibres and Related Optical Effects

2.10 Chromatic Dispersion (CD)

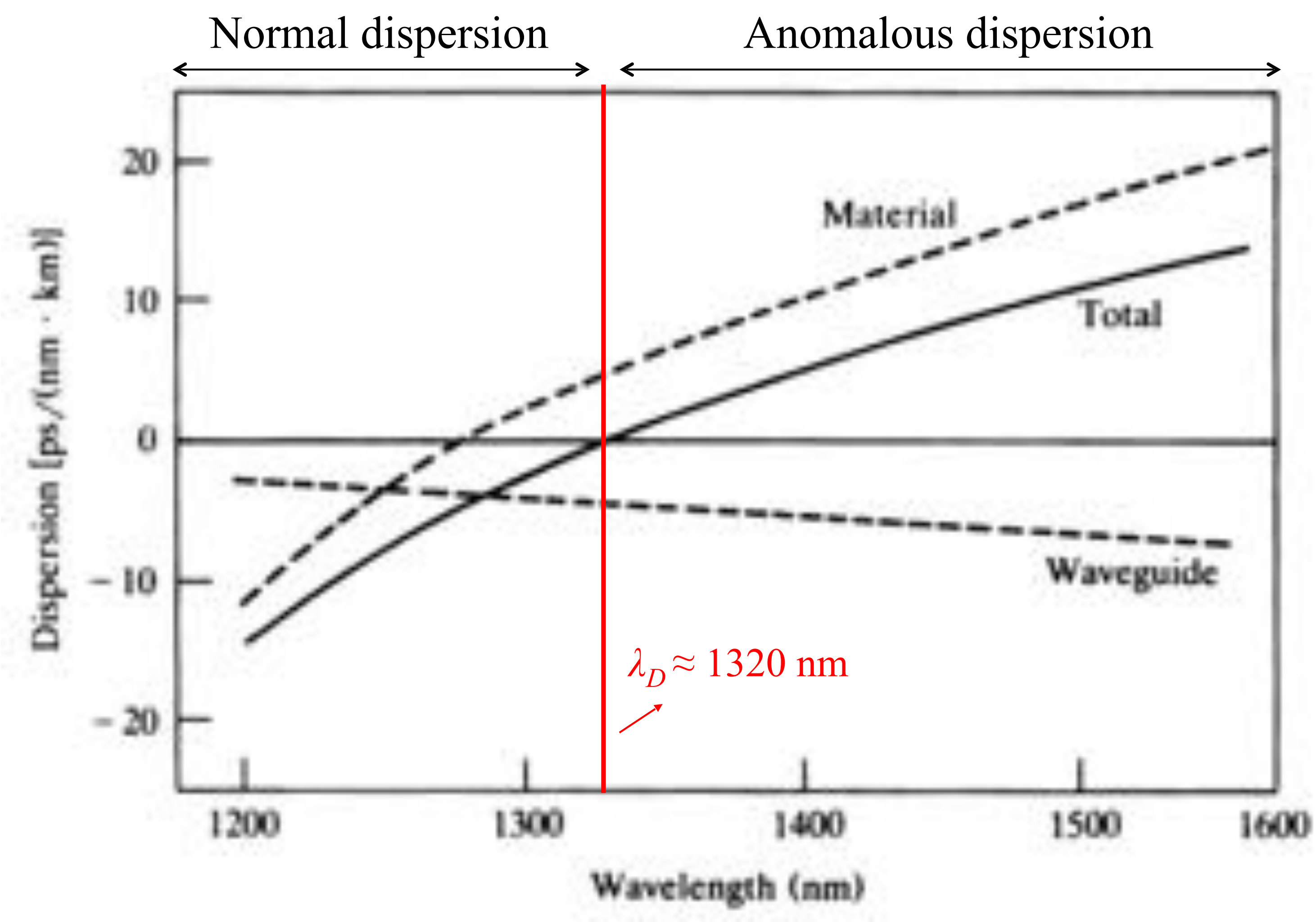


Waveguide Dispersion

$$D_{wg} = -\frac{n_2\Delta}{c\lambda} V \frac{d^2(Vb)}{dV^2}$$

2. Light Propagation in Fibres and Related Optical Effects

2.10 Chromatic Dispersion (CD)

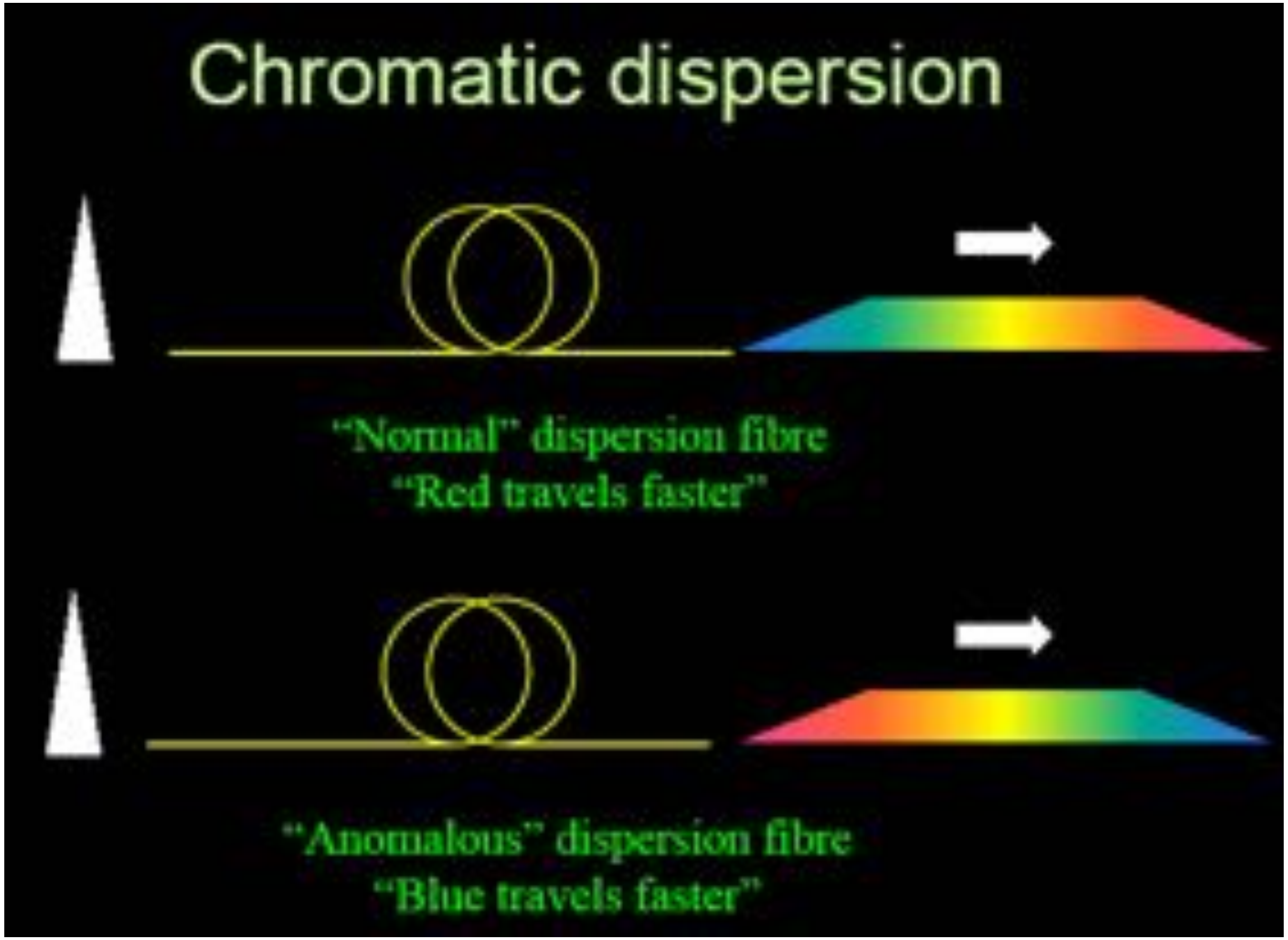


2. Light Propagation in Fibres and Related Optical Effects

2.10 Chromatic Dispersion (CD)

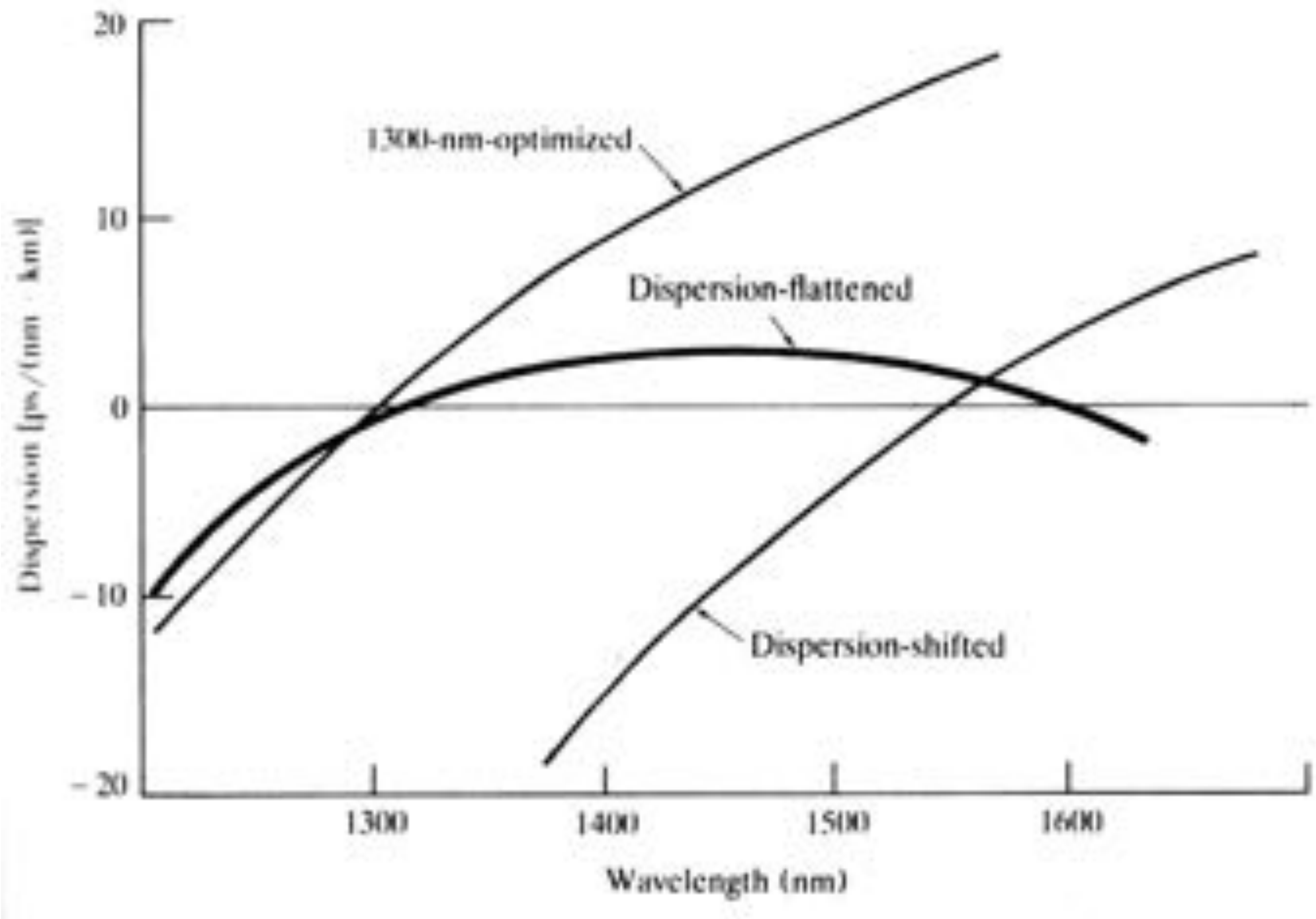
Normal Dispersion
($D < 0$)

Anomalous Dispersion
($D > 0$)



2. Light Propagation in Fibres and Related Optical Effects

2.10 Chromatic Dispersion (CD)

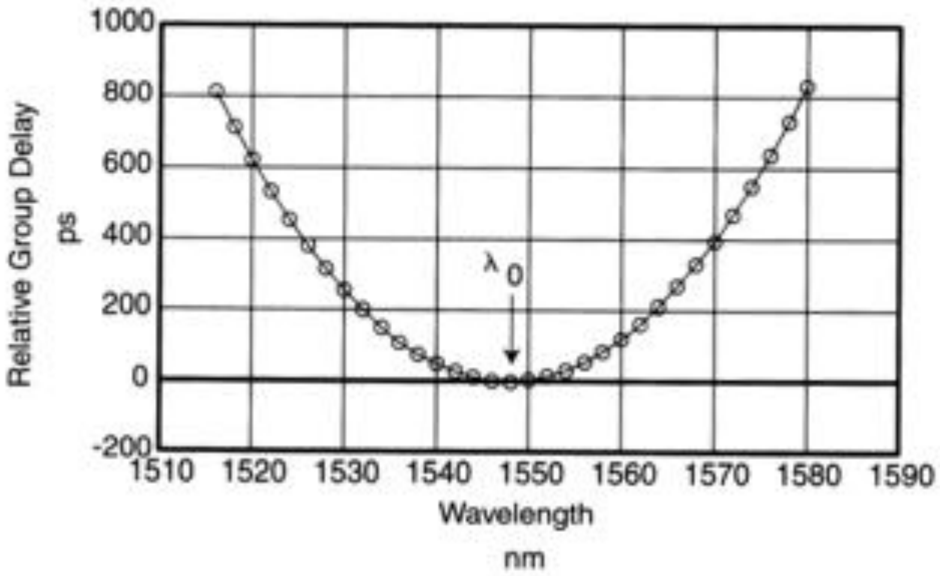


P.K. Bachman *et.al.*, “Dispersion-flattened single-mode fibres prepared with PCVD: Performance, limitations, design and optimization”, J. Lightwave Technology **4**, 858-863 (1986)

2. Light Propagation in Fibres and Related Optical Effects

2.10 Chromatic Dispersion (CD)

$$D(\lambda) = \frac{1}{L} \cdot \frac{d\tau_g}{d\lambda}$$

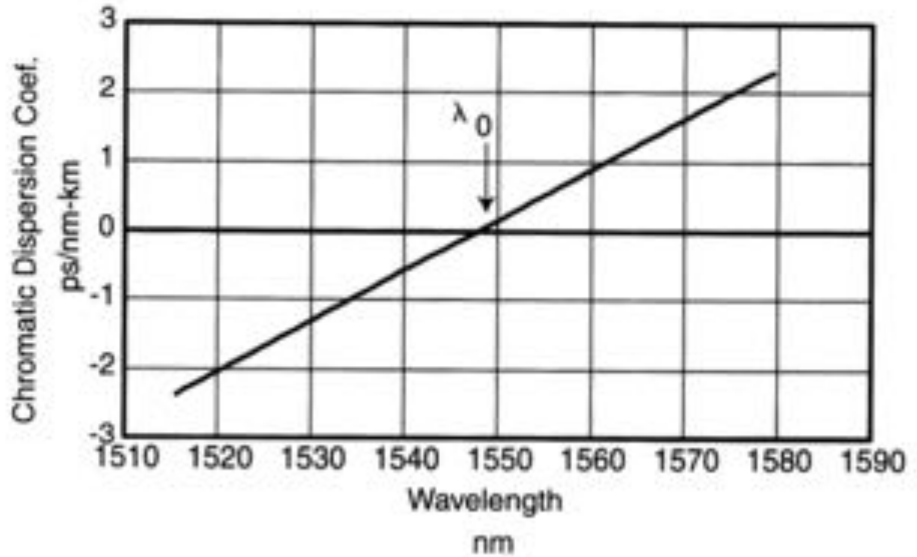


Slope (ps/km·nm²)

$$D(\lambda) = S_o(\lambda - \lambda_0)$$

$$D(\lambda) = \frac{\lambda S_o}{4}\left[1 - \left(\frac{\lambda_0}{\lambda}\right)^4\right]$$

(See recommendation ITU-T G.652)



Typical values for S_0 are 0.092 ps/(km·nm²) for SMF, and between 0.06 and 0.08 ps/(km·nm2) for DSF

2. Light Propagation in Fibres and Related Optical Effects

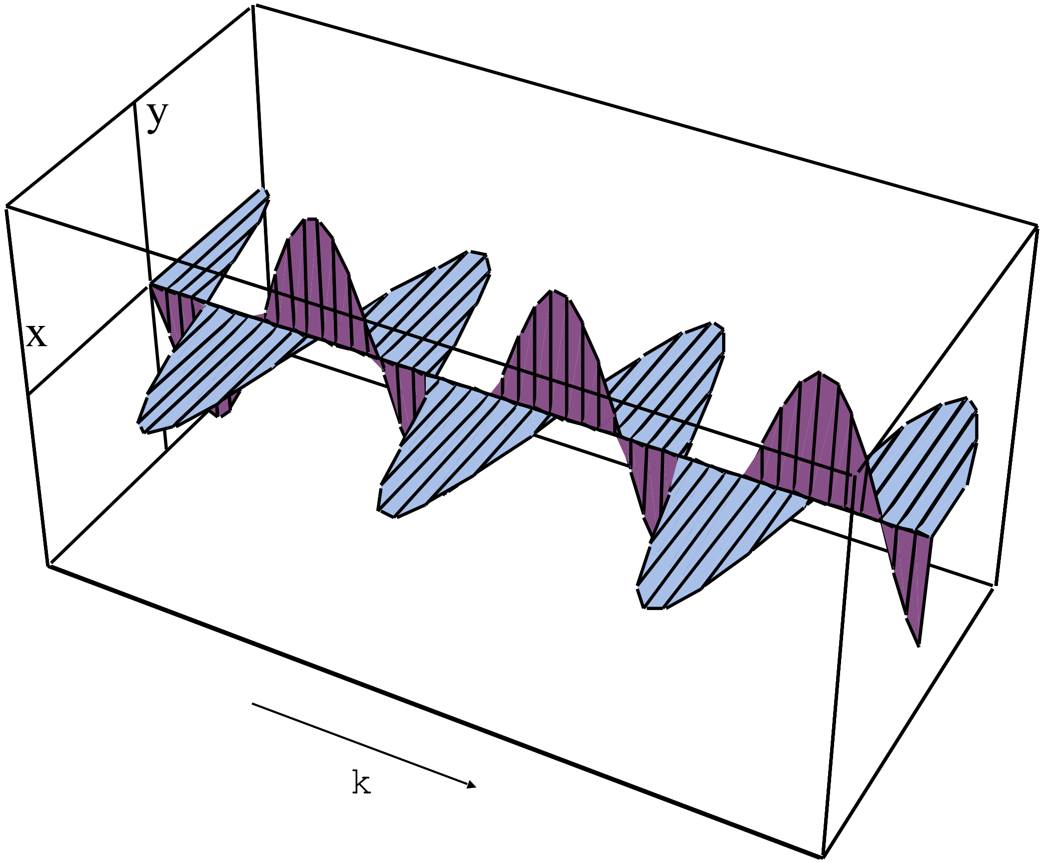
2.10 Chromatic Dispersion (CD)

Fiber Type and Trade Name	A_{eff} (μm^2)	λ_{ZD} (nm)	D (C band) ps/(km-nm)	Slope S ps/(km-nm^2)
Corning SMF-28	80	1302–1322	16 to 19	0.090
Lucent AllWave	80	1300–1322	17 to 20	0.088
Alcatel ColorLock	80	1300–1320	16 to 19	0.090
Corning Vascade	101	1300–1310	18 to 20	0.060
TrueWave-RS	50	1470–1490	2.6 to 6	0.050
Corning LEAF	72	1490–1500	2 to 6	0.060
TrueWave-XL	72	1570–1580	−1.4 to −4.6	0.112
Alcatel TeraLight	65	1440–1450	5.5 to 10	0.058

2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

POLARIZATION



$$\vec{E}_x = \vec{e}_x E_{0x} \cos(\omega t - \beta z + \phi_x)$$

$$\vec{E}_y = \vec{e}_y E_{0y} \cos(\omega t - \beta z + \phi_y)$$

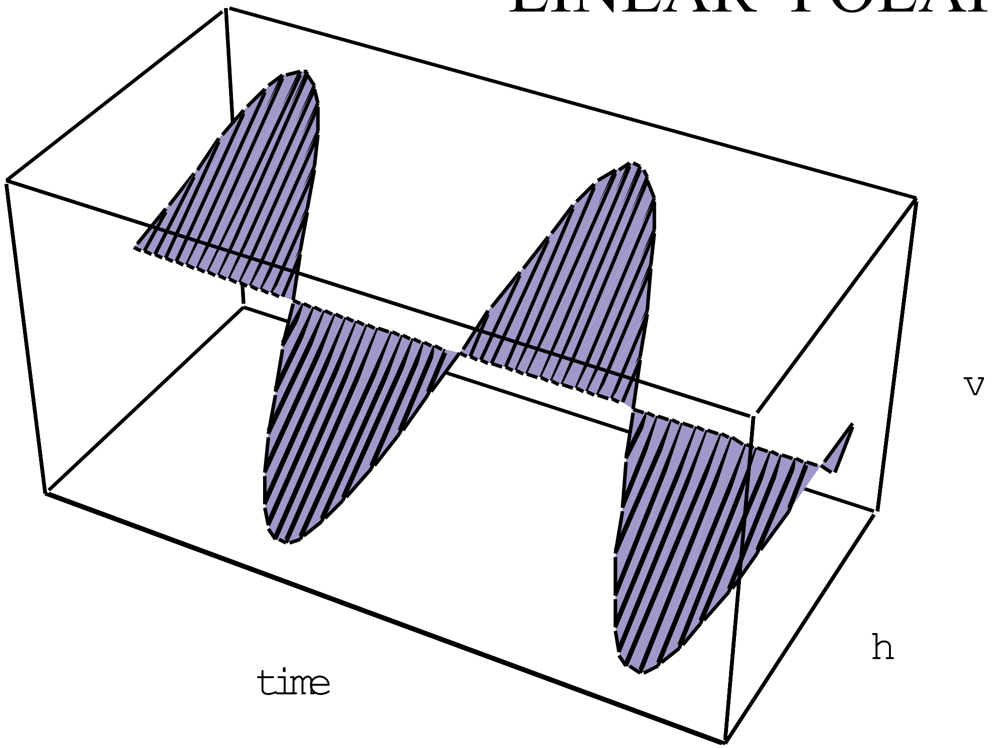
$$\beta = kn = \frac{2\pi}{\lambda} n$$

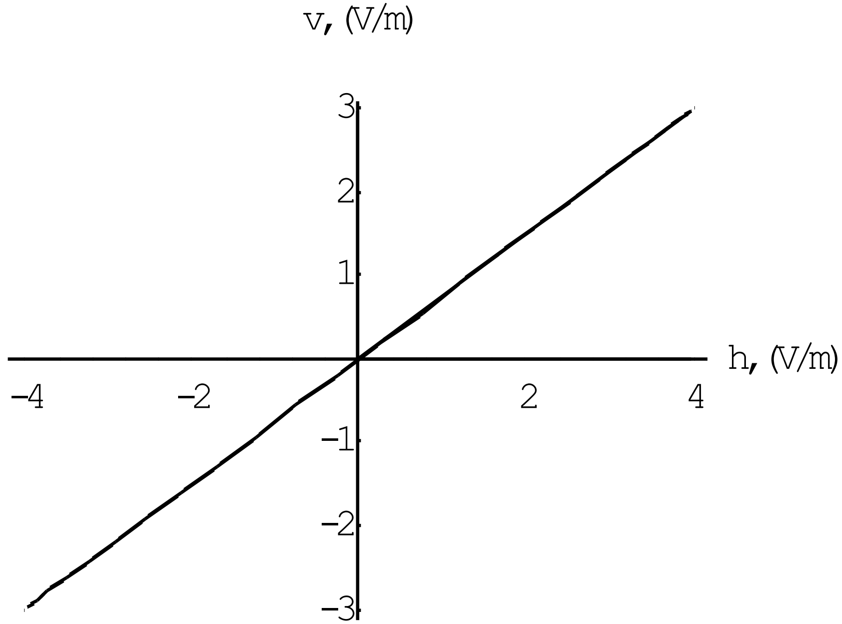
$$\vec{E}_{TOTAL} = \vec{E}_x + \vec{E}_y$$

2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

LINEAR POLARIZATION



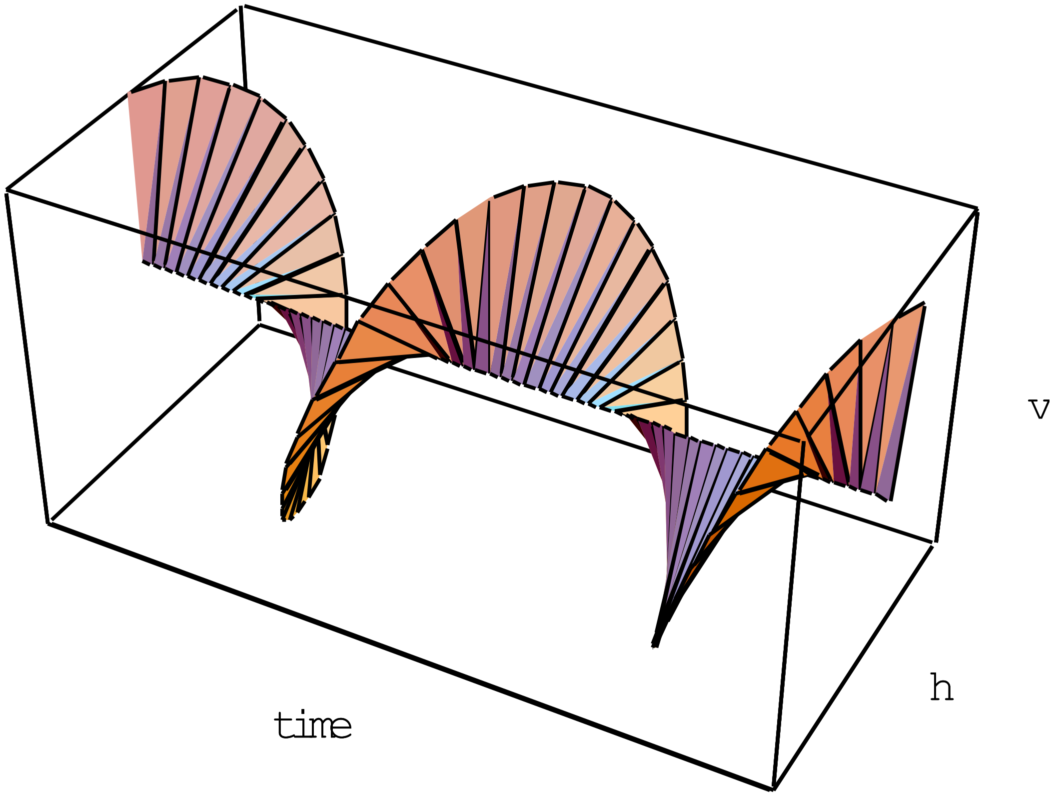


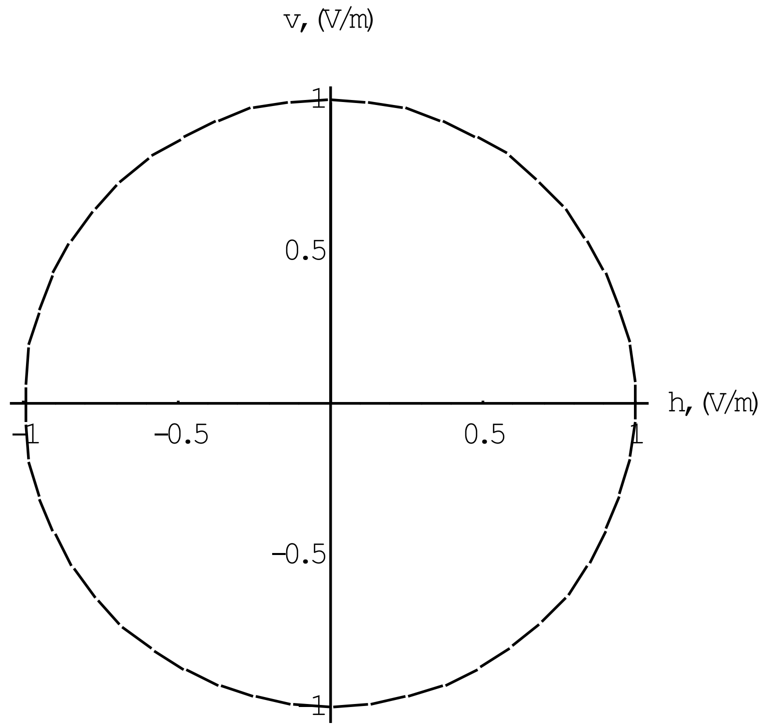
$$\vec{E}_{TOTAL} = \vec{e}_x E_{0x} \cos(\omega t - \beta z) + \vec{e}_y E_{0y} \cos(\omega t - \beta z + \pi)$$

2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

CIRCULAR POLARIZATION



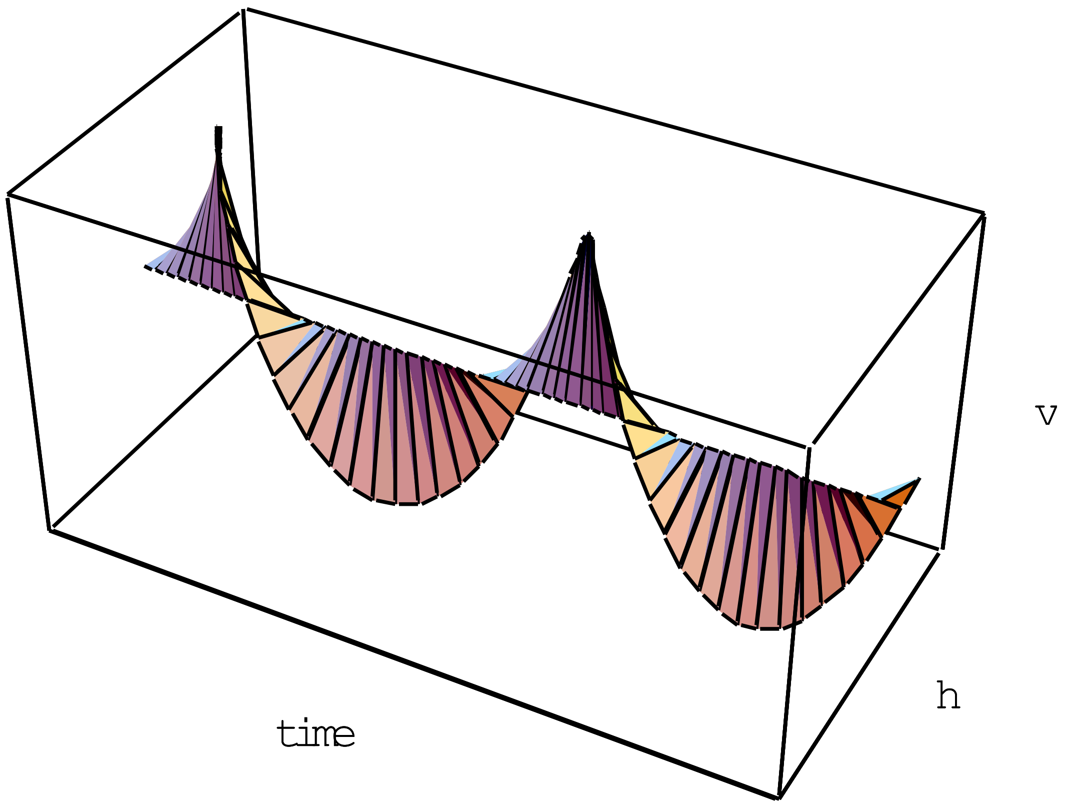


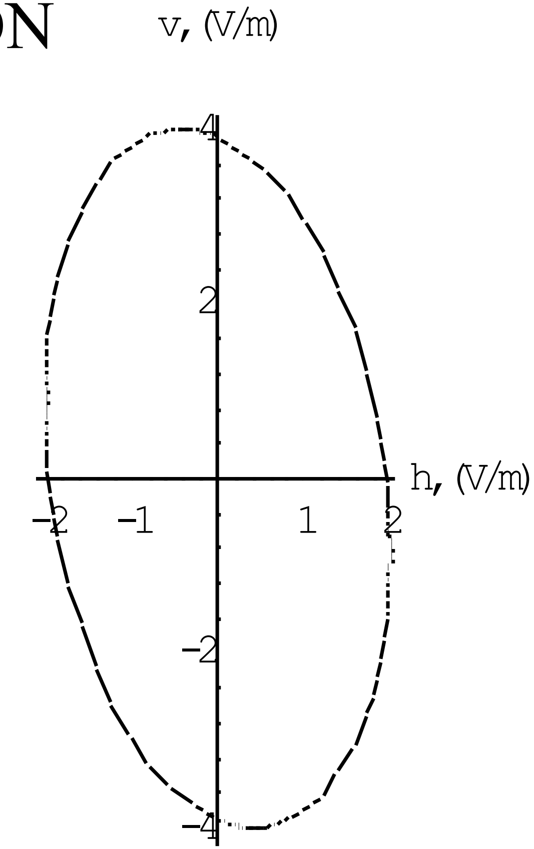
$$\vec{E}_{TOTAL} = E_{0x}\left[\vec{e}_x \sin(\omega t - \beta z) + \vec{e}_y \cos(\omega t - \beta z)\right]$$

2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

ELLIPTICAL POLARIZATION





$$\vec{E}_{TOTAL} = \vec{e}_x E_{0x} \sin(\omega t - \beta z + \phi_1) + \vec{e}_y E_{0y} \cos(\omega t - \beta z + \phi_2)$$

2. Light Propagation in Fibres and Related Optical Effects

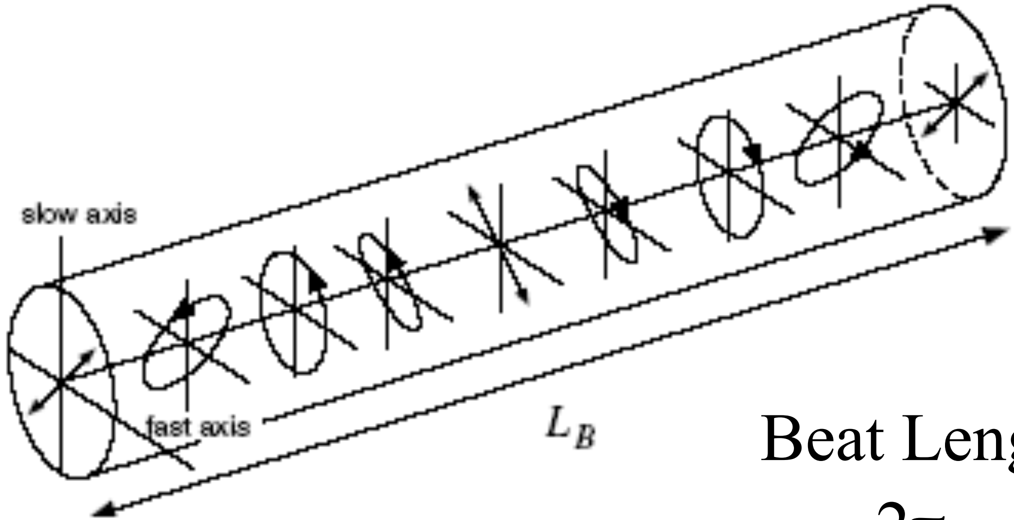
2.11 Polarization effects and Birefringence.

Birefringence

$$B = (n_x - n_y)$$

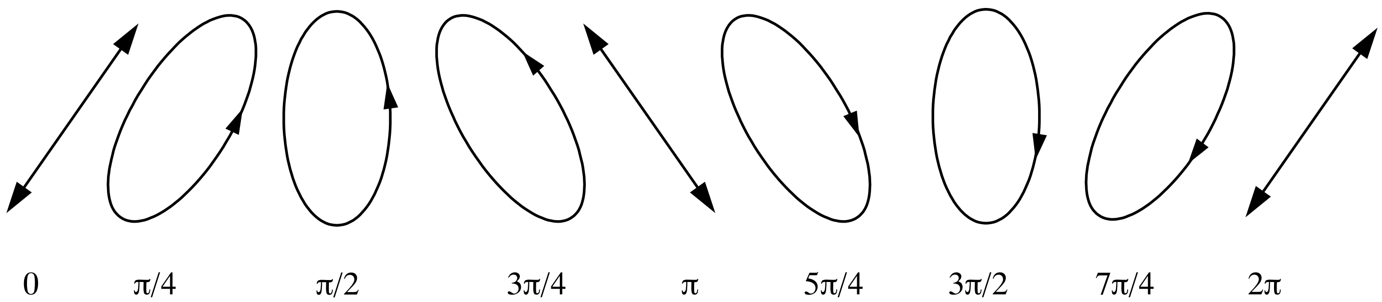
or equivalently,

$$\Delta\beta = \frac{2\pi}{\lambda}(n_x - n_y)$$



Beat Length

$$L_B = \frac{2\pi}{\Delta\beta} = \frac{\lambda}{B}$$



Birefringence effect of polarized light in the fibre

2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

Jones Calculus* - applicable only to polarized waves

(*) E. Hecht, "*Optics*", 4Ed. Addison Wesley, Chapter 8, 2002.

$$\vec{E}_{in} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0y} e^{i\varphi_y} \end{bmatrix} \Rightarrow \boxed{A_1} \Rightarrow \vec{E}_{out} = \begin{bmatrix} E_{xout} \\ E_{yout} \end{bmatrix}$$

$$\begin{bmatrix} E_{xout} \\ E_{yout} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} E_{xin} \\ E_{yin} \end{bmatrix}$$

Dividing both terms by:

$$\vec{E}_{in} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0x} e^{i\varphi_x} \end{bmatrix} = E_{0x} e^{i\varphi_x} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \xRightarrow{\sqrt{2} E_{0x} e^{i\varphi_x}} \vec{E}_{45^\circ} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

Jones Matrix

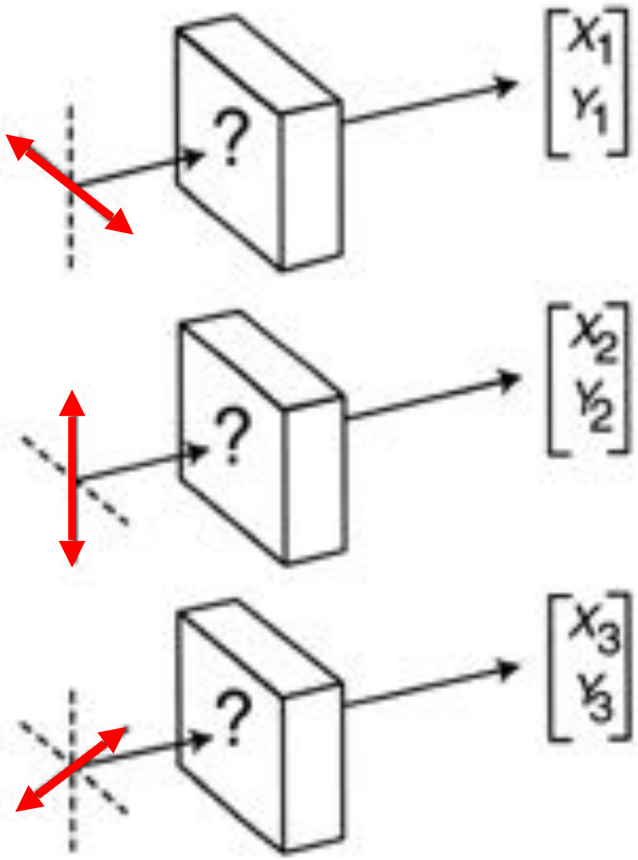
from E. Hecht, "*Optics*", 4Ed. Addison Wesley, Chapter 8, 2002.

Linear optical element		Jones matrix
Horizontal linear polarizer	↔	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Vertical linear polarizer	↕	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Linear polarizer at +45°	↗	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
Linear polarizer at −45°	↘	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
Quarter-wave plate, fast axis vertical		$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
Quarter-wave plate, fast axis horizontal		$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
Homogeneous circular polarizer right	↻	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Homogeneous circular polarizer left	↺	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$

2.11 Polarization effects and Birefringence.

Measurement of the Jones Matrix of an optical element

[from D.Derickson, "*Fiber Optic Test and Measurement*" Prentice Hall, Ch. 6 (1998)]



$$K_1 = \frac{X_1}{Y_1} \qquad K_2 = \frac{X_2}{Y_2}$$

$$K_3 = \frac{X_3}{Y_3} \qquad K_4 = \frac{K_3 - K_2}{K_1 - K_3}$$

Jones matrix $\mathbf{M} = C \begin{bmatrix} K_1 K_4 & K_2 \\ K_4 & 1 \end{bmatrix}$

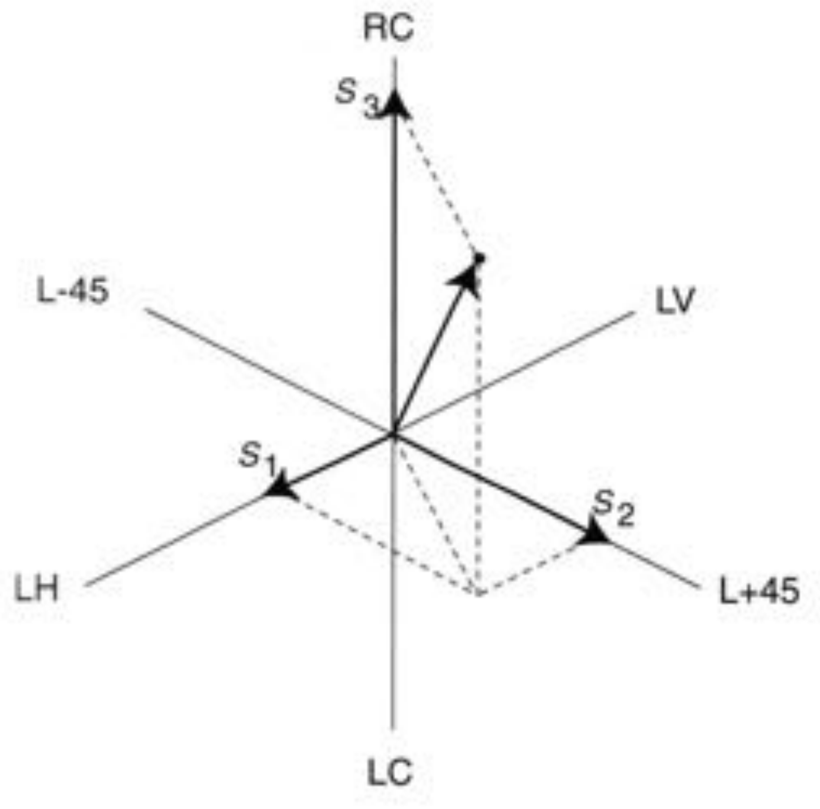
2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

Stokes Parameters – applicable to both totally or partially polarized light.
The elements describe the optical power in particular reference polarization states.

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I_{total} \\ I_{LH} - I_{LV} \\ I_{L+45^o} - I_{L-45^o} \\ I_{RC} - I_{LC} \end{bmatrix}$$

$$DOP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$



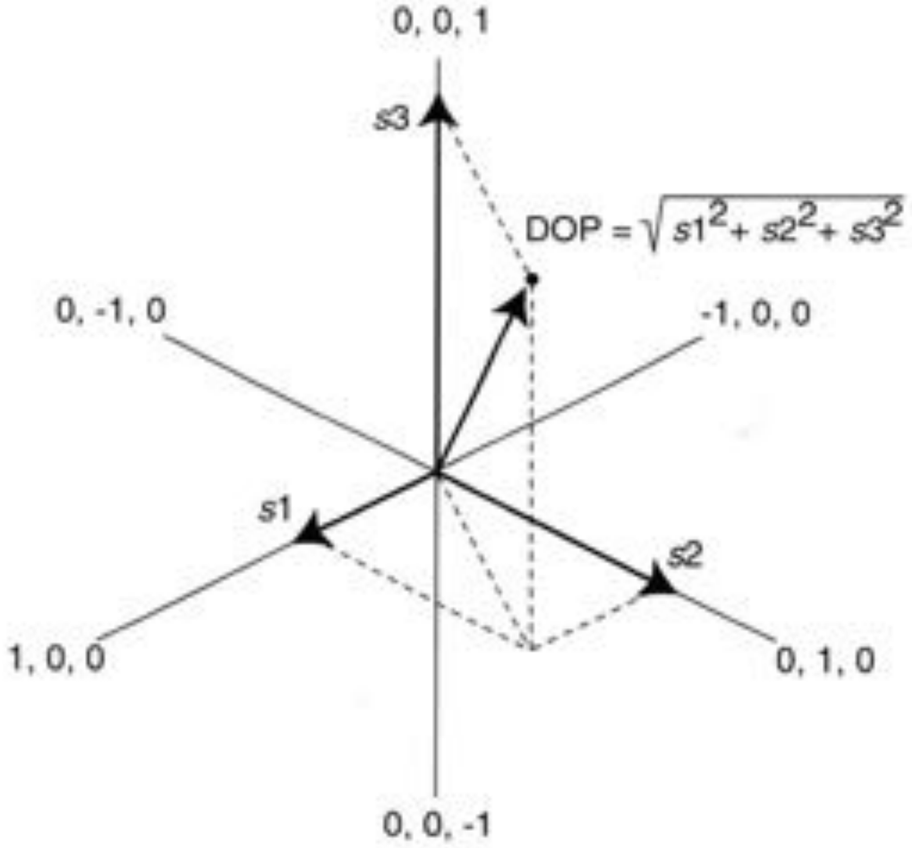
2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

Normalized Stokes Parameters

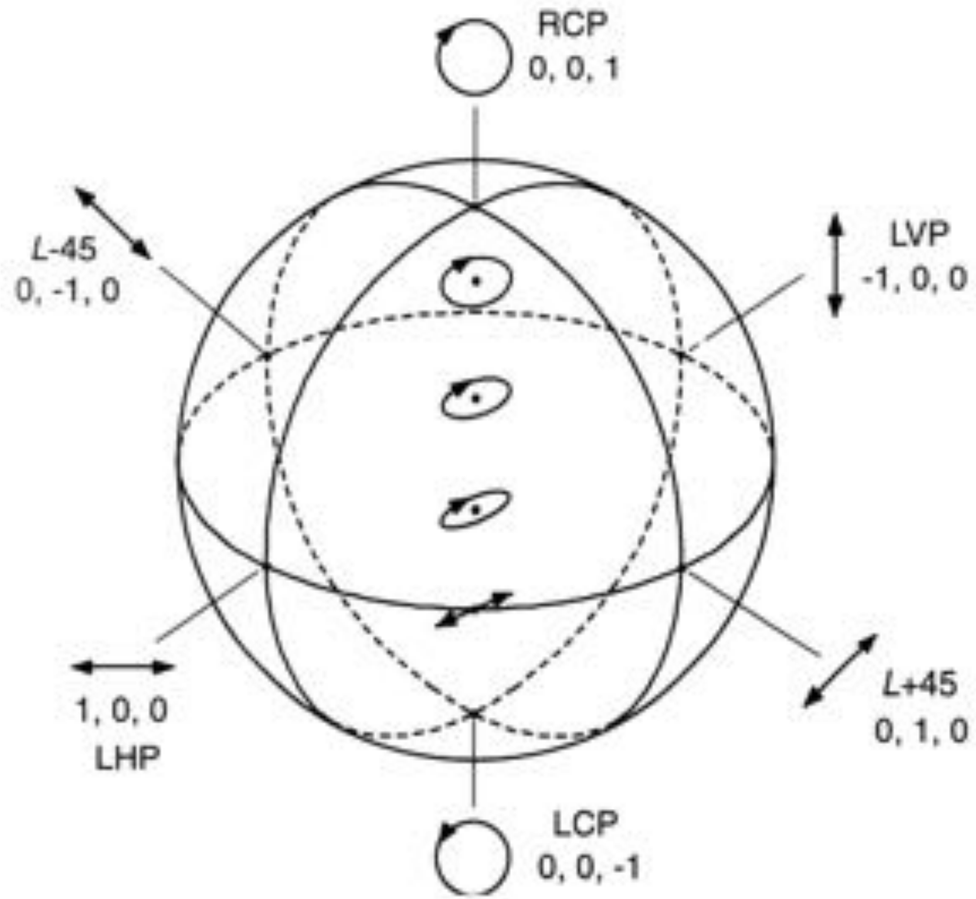
$$\begin{bmatrix} 1 \\ s1 \\ s2 \\ s3 \end{bmatrix} = \frac{1}{S_0} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

$$DOP = \sqrt{s1^2 + s2^2 + s3^2}$$



2.11 Polarization effects and Birefringence.

Poincaré Sphere – graphical tool in real, 3D space that allows convenient description of polarized signals and of polarization transformations caused by propagation through devices.

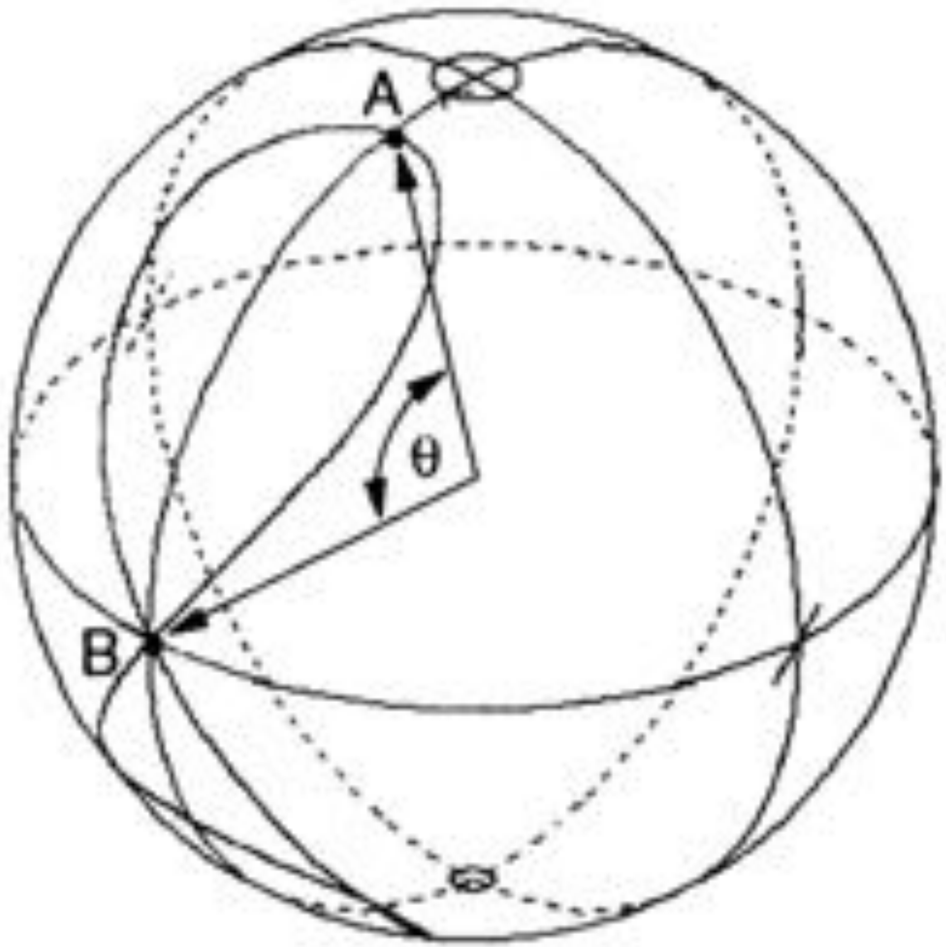


2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

Measurement of retardance θ of a near $\lambda/4$-*wave* retarder

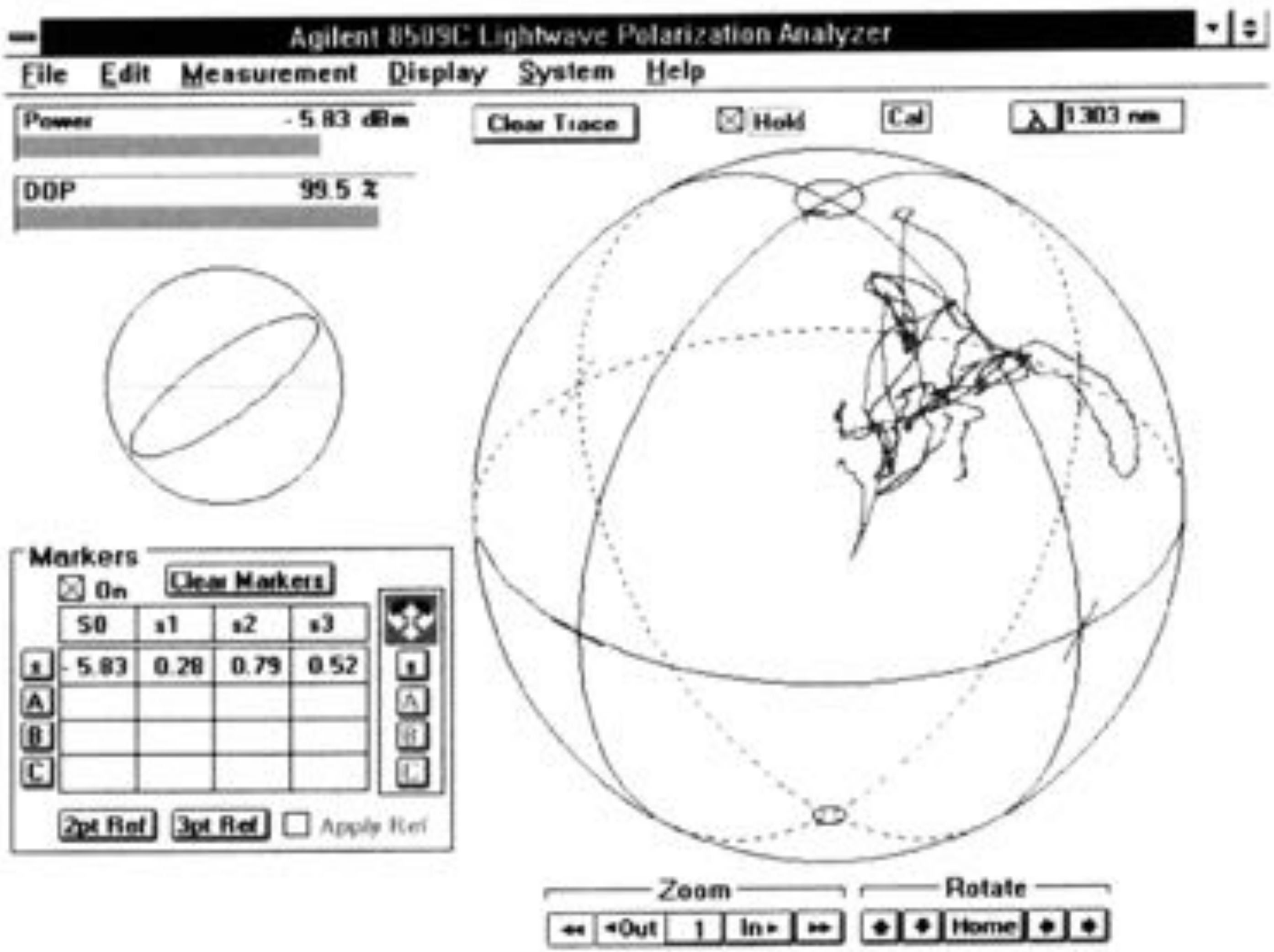
[from D.Derickson, "*Fiber Optic Test and Measurement*" Prentice Hall, Ch. 6 (1998)]



2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

Twisting of a singlemode fibre [from Agilent HP8509C Application Note]



2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

Muller Matrix – applicable to any degree of polarization.

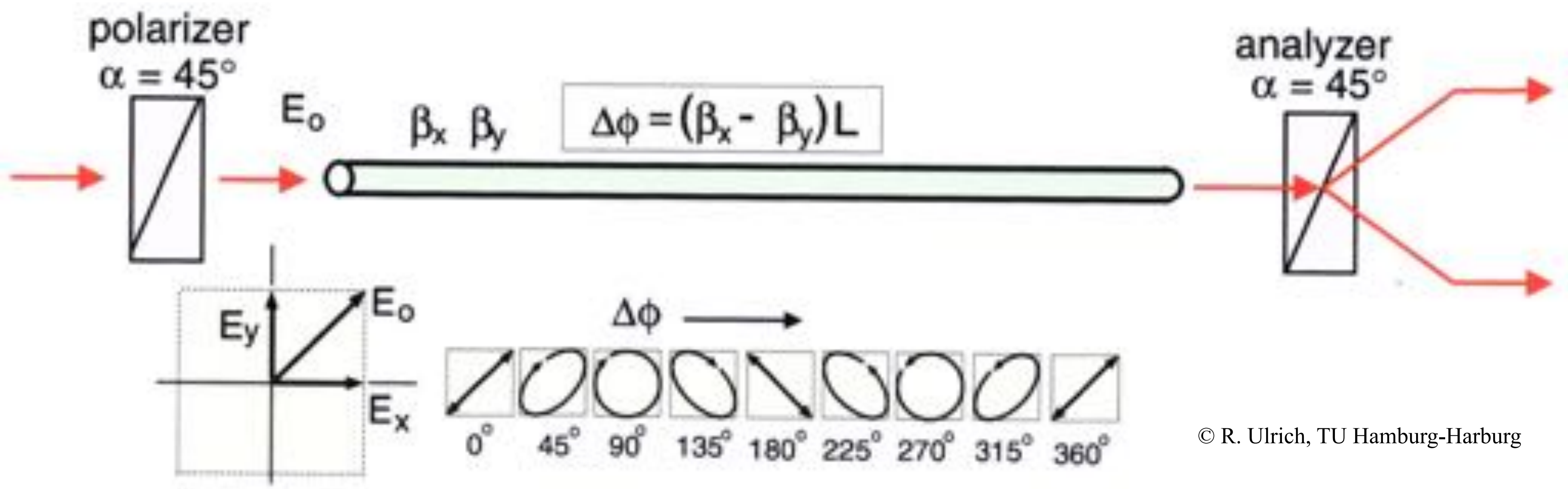
$$\begin{bmatrix} S_0^{out} \\ S_1^{out} \\ S_2^{out} \\ S_3^{out} \end{bmatrix} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{bmatrix} \cdot \begin{bmatrix} S_0^{in} \\ S_1^{in} \\ S_2^{in} \\ S_3^{in} \end{bmatrix}$$

Linear optical element		Mueller matrix
Horizontal linear polarizer	↔	$\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Vertical linear polarizer	↕	$\frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Linear polarizer at +45°	⤢	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

Two polarization modes E_x , E_y of a singlemode fibre



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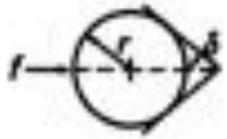
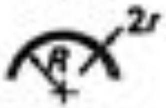
2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

From K.T.V.Grattan & B.T. Meggitt, "*Optical Fiber Sensor Technology*", Chapman & Hall (1995)

Table 2.2 External sources of birefringence in single mode fibers

Source	Configuration	Birefringence
Lateral pressure		$b = \dfrac{4\sigma f}{\pi r E}$
V-groove clamp		$b = 2\sigma\,(1 - \cos 2\delta \sin\delta)\,\dfrac{f}{\pi r E}$
Free bending		$b = \dfrac{\sigma r^2}{4R^2}$
Bending under tension		$b = \dfrac{\sigma r \varepsilon (2 - 3\nu)}{2R\,(1 - \nu)}$ + bend birefringence
Bending over rough surface under tension		Retardation from one kink $= \dfrac{k_0 \sigma h}{R}\left(\dfrac{F}{\pi E}\right)^{1/2}$ + bend and tension effects (when $hRF \gg (E\pi r^4/8)$)
Twist		Polarization rotation $= \sim 0.16\rho\ \mathrm{rad\,m^{-1}}$ for silica fibers

2. Light Propagation in Fibres and Related Optical Effects

2.11 Polarization effects and Birefringence.

From K.T.V.Grattan & B.T. Meggitt, "*Optical Fiber Sensor Technology*", Chapman & Hall (1995)

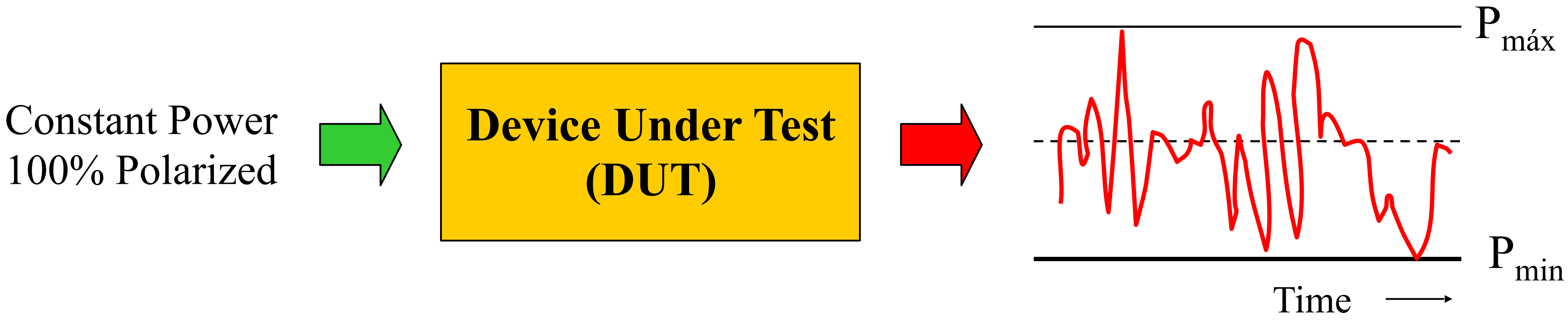
External electric field		$b = \dfrac{2\pi K E_F^2}{k_0}$
Magnetic field		Polarization rotation $= VH$ rad m^{-1}
Optical electric field		$b = n_{2b} I$
Polarized ultraviolet light		Unpredictable

f : force per unit length
r : fiber radius
ε : Fiber strain
E_F : Electric field in fiber
F : Fiber tension
$\sigma = n_e^3 (1 + \nu)(P_{11} - P_{12})$ where P_{ij} are components of the elasto-optic tensor ($P_{11} - P_{12}$ = 0.15 for silica)
E : Young's modulus of core (7.3×10^{10} N m^{-2} for silica)
ν : Poisson's ratio (0.17 for silica)
K : Kerr electro-optic coefficient (9×10^{-15} cm V^{-2} for silica)
n_{2b} : Anisotropic optical Kerr coefficient (4–6×10^{-16} cm^2 W^{-1} for silica)
V : Verdet constant ($\sim 5 \times 10^{-6}$ rad A^{-1} for silica at 633 nm)

2. Light Propagation in Fibres and Related Optical Effects

2.12 Polarization Dependence Loss (PDL) .

PDL measures the peak-to-peak difference in transmission for light with various states of polarization.



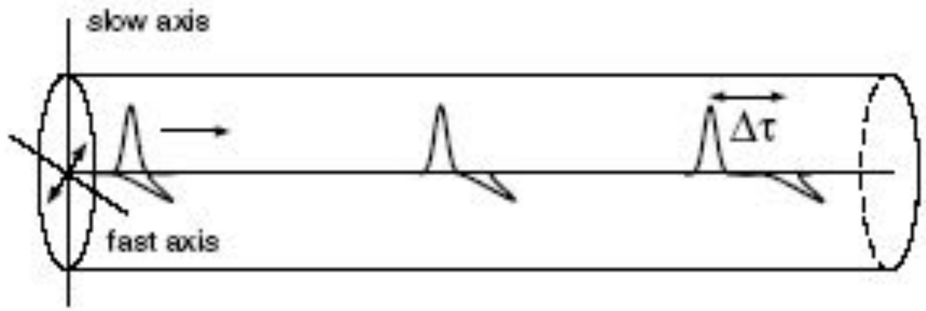
$$PDL_{dB} = 10\log\left(\frac{P_{max}}{P_{min}}\right)$$

2. Light Propagation in Fibres and Related Optical Effects

2.13 Polarization Mode Dispersion (PMD).

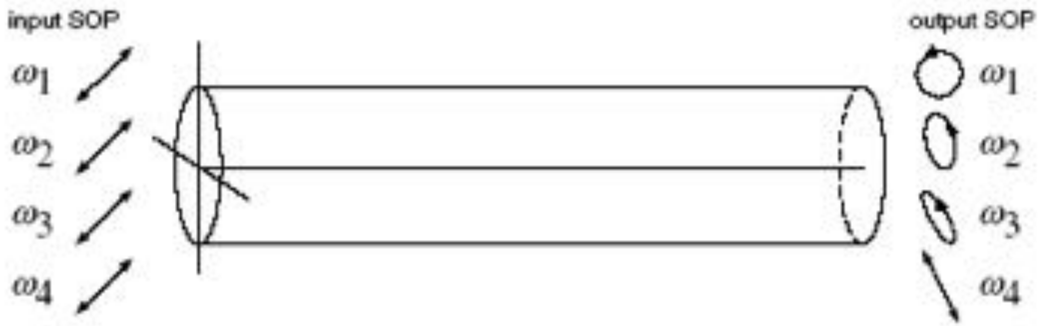
PMD is a fundamental property of singlemode optical fibre and components in which signal energy at a given wavelength is resolved into two orthogonal polarization modes of slightly different propagation velocity. The resulting difference in propagation time between polarization modes is called the differential group delay ($\Delta\tau$).

$$\Delta\tau = \frac{L}{\Delta v_g} = L\frac{d\Delta\beta}{d\omega}$$
$$= L\left(\frac{\Delta n}{c} + \frac{\omega}{c}\cdot\frac{d\Delta n}{d\omega}\right)$$



Also, Frequency-domain manifestation

$$\phi = L\Delta\beta \rightarrow \frac{\Delta\phi}{\Delta\omega} = L\frac{d\beta}{d\omega} = \Delta\tau$$

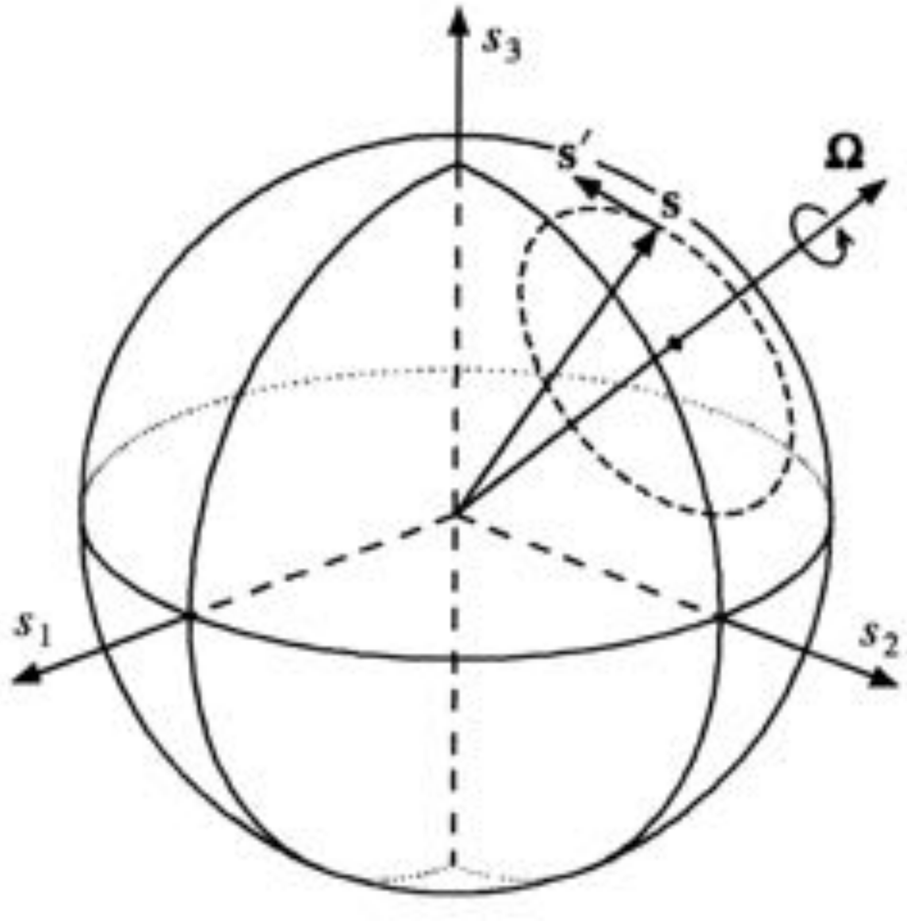


2. Light Propagation in Fibres and Related Optical Effects

2.13 Polarization Mode Dispersion (PMD).

PMD is characterized by the PMD-vector, $\mathbf{\Omega}(\omega)$, in Stokes space, around which an output state of polarization (SOP), **s** , rotates when the carrier frequency is changed.

$$\vec{S}\,' = \frac{d\vec{S}}{d\omega} = \vec{\Omega} \times \vec{S}$$



And the differential group delay (DGD) is:

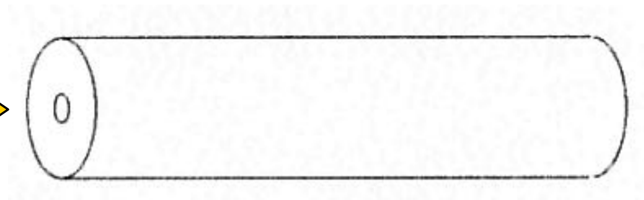
$$\Delta\tau = \left|\vec{\Omega}\right|$$

2. Light Propagation in Fibres and Related Optical Effects

2.13 Polarization Mode Dispersion (PMD).

CASE 1 - Ideal fibre section ($\beta_x = \beta_y$)

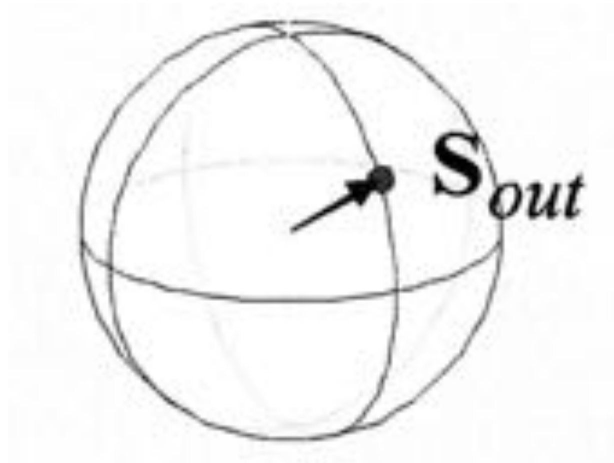
$$\vec{E}_{in}(\omega) = \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}$$



$$\vec{E}_{out}(\omega) = \left[\vec{J}\right] \bullet \left[\vec{E}_{in}\right] = e^{-j\beta L}\vec{E}_{in}$$

$$\vec{J}(\omega) = \begin{bmatrix} e^{-j\beta L} & 0 \\ 0 & e^{-j\beta L} \end{bmatrix}$$

$$\boxed{\Delta\tau = 0}$$

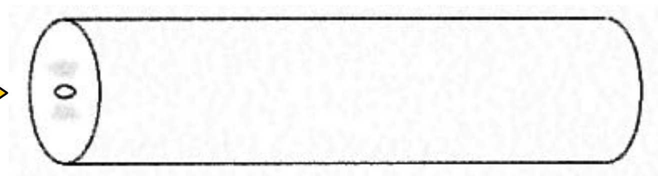


2. Light Propagation in Fibres and Related Optical Effects

2.13 Polarization Mode Dispersion (PMD).

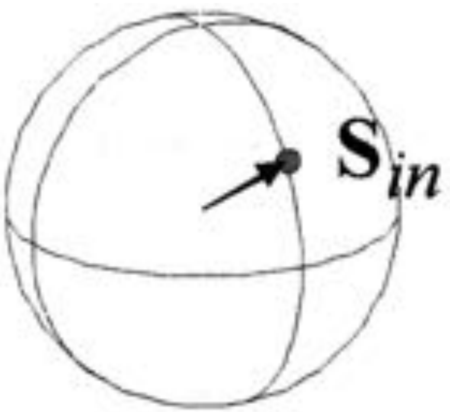
CASE 2 - linearly birefringent fibre section ($\beta_x \neq \beta_y$)

$$\vec{E}_{in}(\omega) = \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}$$

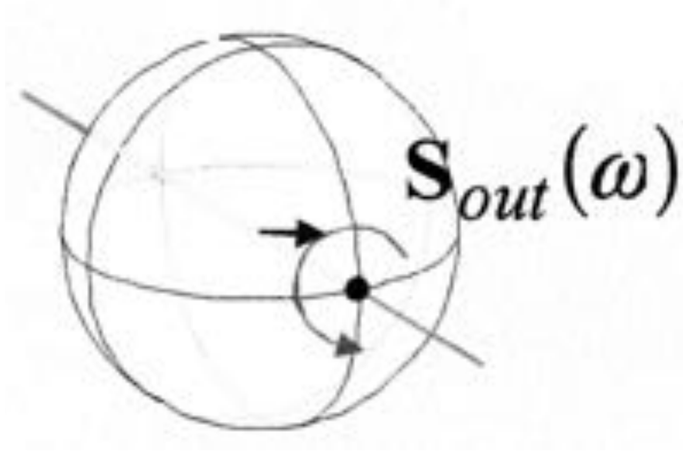


$$\vec{E}_{out}(\omega) = \begin{bmatrix} e^{-j\beta_x L} E_{x,in} \\ e^{-j\beta_y L} E_{y,in} \end{bmatrix}$$

$$\vec{J}(\omega) = \begin{bmatrix} e^{-j\beta_x L} & 0 \\ 0 & e^{-j\beta_y L} \end{bmatrix}$$



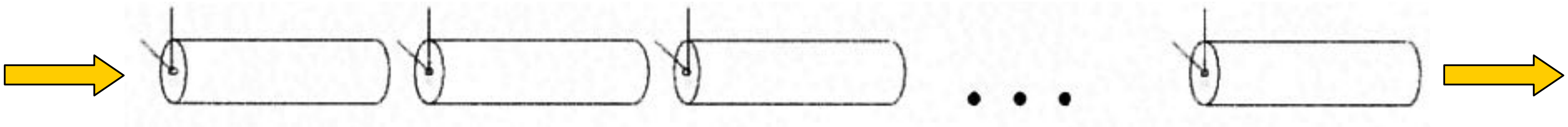
$$\Delta\tau = L \cdot \frac{d}{d\omega}\left(\frac{2\pi}{\lambda}\Delta n\right) \cong \Delta n \cdot \frac{L}{c}$$



2. Light Propagation in Fibres and Related Optical Effects

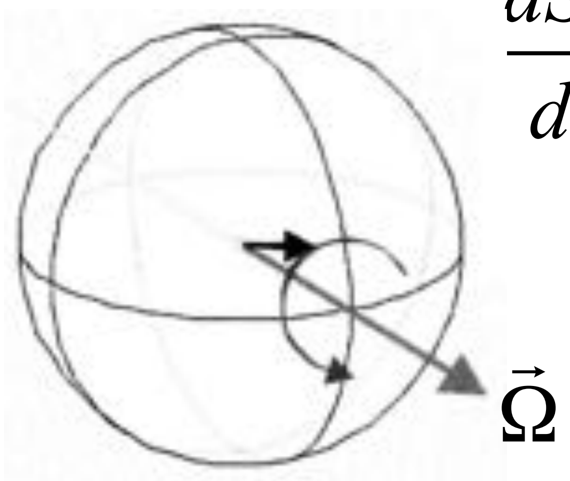
2.13 Polarization Mode Dispersion (PMD).

CASE 3 - Birefringence axes aligned: PM fibre



$$\vec{J}_{total}(\omega) = \begin{bmatrix} e^{-j\beta_x L} & 0 \\ 0 & e^{-j\beta_y L} \end{bmatrix}$$

$$\frac{d\vec{S}_{out}}{d\omega} = \vec{\Omega} \times \vec{S}_{out}$$

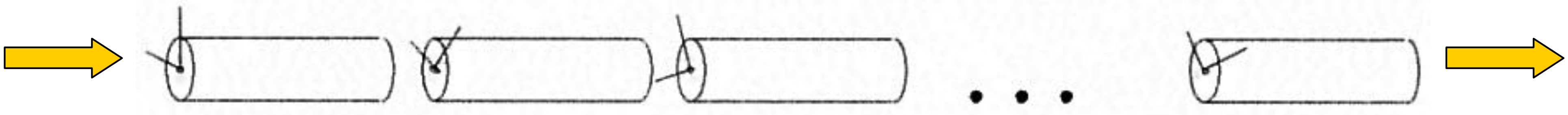


$$\Delta\tau = \left|\vec{\Omega}\right|$$

2. Light Propagation in Fibres and Related Optical Effects

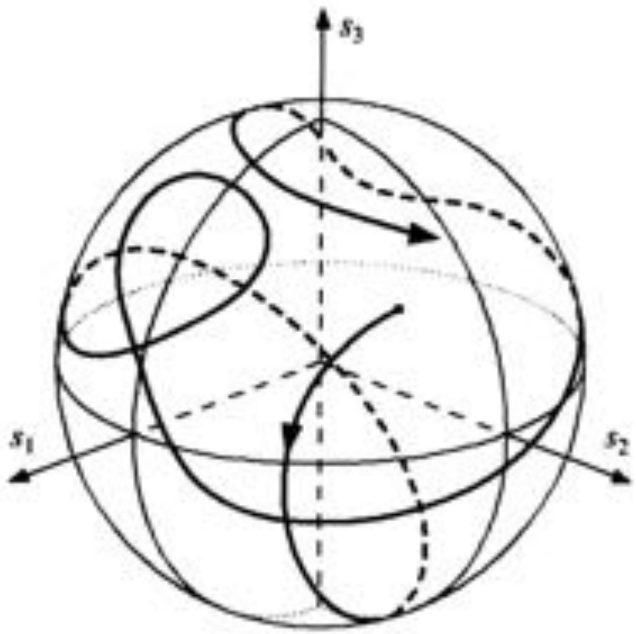
2.13 Polarization Mode Dispersion (PMD).

CASE 4 - Random orientation of birefringence axes aligned: *standard long-length fibre*



$$\vec{J}_{total} = [R_n]\cdot[J_n]\cdots[R_2]\cdot[J_2]\cdot[R_1]\cdot[J_1]$$

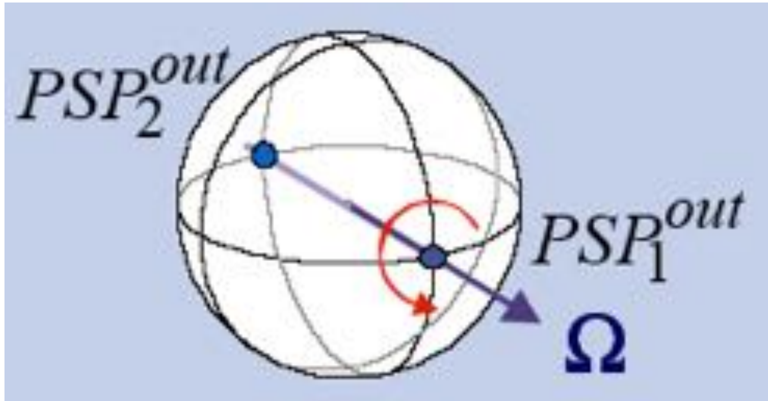
$$\frac{d\vec{S}_{out}}{d\omega} = \vec{\Omega}(\omega)\times\vec{S}_{out}$$



2. Light Propagation in Fibres and Related Optical Effects

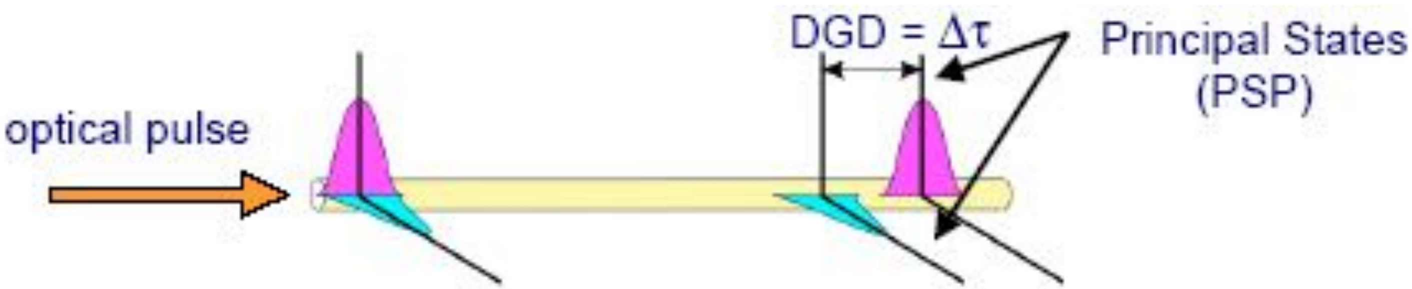
2.13 Polarization Mode Dispersion (PMD).

• **Frequency domain scenario**: PMD vector and principal states of polarization (PSP)



$$\frac{d\vec{S}_{out}}{d\omega} = 0 \quad \text{if} \quad \vec{S}_{out} = PSP_{1,2}^{out}$$

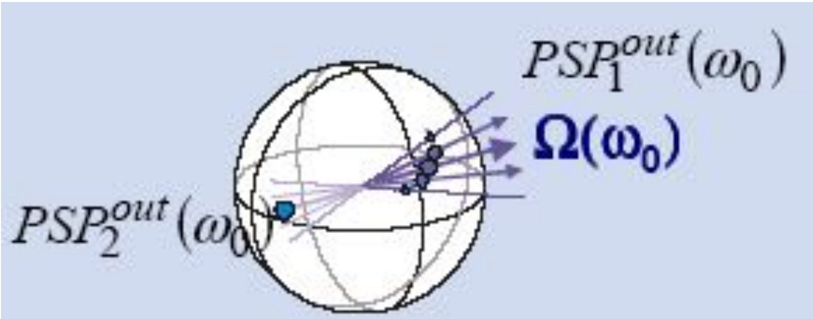
• **Time domain scenario**: Pulse splitting



2. Light Propagation in Fibres and Related Optical Effects

2.13 Polarization Mode Dispersion (PMD).

- **Frequency domain scenario :** frequency dependence of PSP



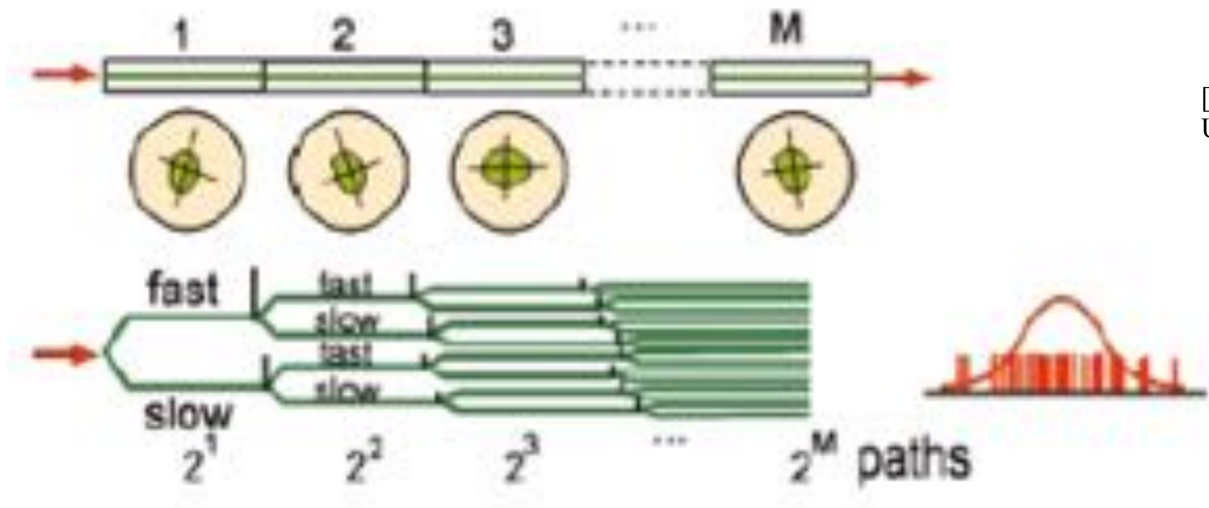
$$\vec{\Omega}(\omega) = \vec{\Omega}(\omega_0) + \vec{\Omega}_\omega \Delta\omega + \ldots.$$

$$\vec{\Omega}_\omega = \left.\frac{d\vec{\Omega}}{d\omega}\right|_{\omega=\omega_0}$$

1st order approximation is valid within "PSP-bandwidth":

$$\Delta\omega_{PSP} \cong \frac{1}{\langle \Delta\tau \rangle}$$

- **Time domain scenario**: multi-path pulse transmission



[See J.L. Santos, Ph.D. Thesis, Chap.7, Univ. Porto, (1992)]

2. Light Propagation in Fibres and Related Optical Effects

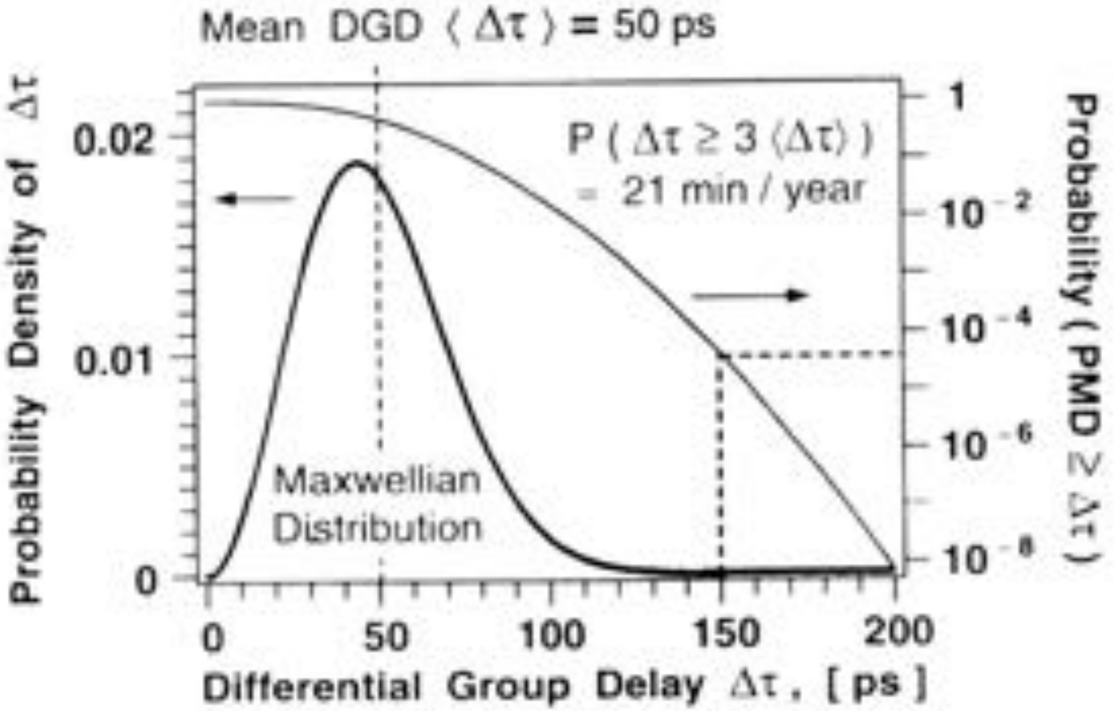
2.13 Polarization Mode Dispersion (PMD).

Example : Probability of $\Delta\tau \geq 3\langle\Delta\tau\rangle$ is $4 \cdot 10^{-5}$

Probability of finding DGD at value Δτ is given by the Maxwellian density distribution

$$\sqrt{\frac{2}{\pi}} \cdot \frac{\Delta\tau^2}{\alpha^3} e^{-\frac{\Delta\tau^2}{2\alpha^2}}$$

with $\langle\Delta\tau\rangle = \alpha\sqrt{8/\pi}$



Fiber Type	Mean DGD Coefficient $\langle\Delta\tau\rangle/\sqrt{L}$	Average DGD for $L = 625$ km
Modern Low PMD	$\leq$ 0.1 ps / √ km	2.5 ps
Older High PMD	~ 2 ps / √ km	50 ps

Receiver Sensitivity Penalty for $\Delta\tau = 50$ ps
at 2.5 Gb/s : < 0.1 dB
at 10 Gb/s : > 4 dB

(c.f. A. Hamel *et al.*, Techn. Digest **NFOEC'97**, p. 7, (1997))

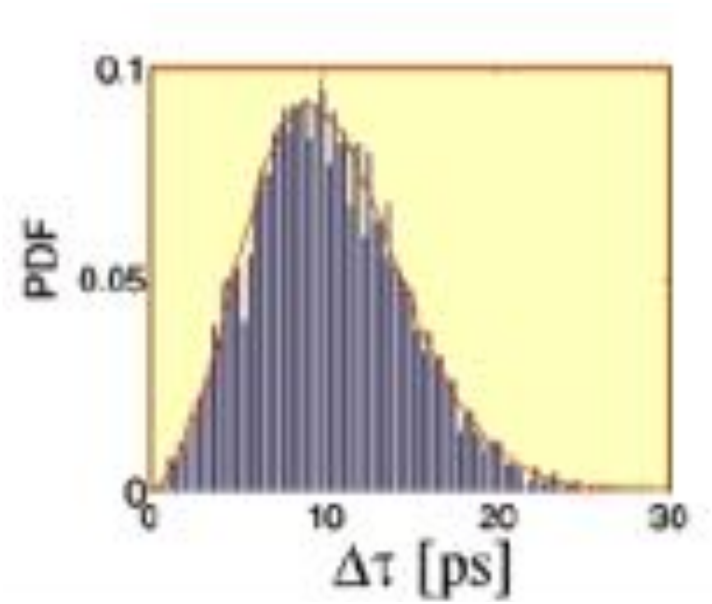
2. Light Propagation in Fibres and Related Optical Effects

2.13 Polarization Mode Dispersion (PMD).

Consequences of PMD

- DGD ($\Delta\tau$) causes:
  - Pulse broadening
  - Reduction of eye openings / increase in BER
  - Additional power penalty
  - Increase of outage probability
- Instantaneous $\Delta\tau$
  - Is a random variable
  - It varies due to environment (temnperature, strain)
  - It can surpass its mean value by far

$$\Delta\tau_{PMD} = \sqrt{\langle \Delta\tau^2 \rangle} \propto \sqrt{L}$$



2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

- Self-Phase Modulation (SPM) – pulses distiort as they propagate.
- Cross-Phase Modulation (XPM) – pulses interfere with one another
- Modulation Instability (MI) – CW beams break into pulses
- Solitons – a nonlinear way of transmitting pulses
- Four-Wave Mixing (FWM)
- Optical Kerr Effect – electric field imposed induces linear birefringence

Parametric Process
(light induced modulation)

- Stimulated Brillouin Scattering (SBS) – inelastic scattering from acoustic phonons
- Stimulated Raman Scattering (SRS) – inelastic scattering from molecular resonances

Non-Parametric Process

- Supercontinuum Generation (SG) – “white light” generation

→ **Observed effect**

2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

Nonlinear optics is a result of anharmonic excitation of the medium. The induced polarization is given by:

$$\vec{P} = \varepsilon_0 \left(\chi_1 \cdot \vec{E} + \chi_2 \cdot \vec{E}\vec{E} + \chi_3 \cdot \vec{E}\vec{E}\vec{E} + \ldots \right)$$

where χ_1, χ_2, χ_3 are 1^{st}, 2^{nd}, 3^{rd} order susceptibilities. χ_2 vanishes in centro-symmetric materials like glass, so the lowest-oder nonlinear term is χ_3.

One manifestation of this is the **nonlinear refractive index**:

$$n = n_0 + n_{2K} \left| E \right|^2$$

where n_{2K} is the nonlinear Kerr coefficient and is directly related to χ_3. In most glasses, n_2 is positive, so the refractive index of the material increases at higher intensities. The value of n_{2K} for SiO_2 is ~3.2×10^{-20} m^2W^{-1}.

2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

DEFINITIONS

- **Nonlinear Coefficient** (γ) – measure of the strength of the nonlinear response of a particular fibre at frequency ω_0 with effective area A_{eff} and n_{2k} known.

$$\gamma = \frac{n_{2K}\omega_0}{cA_{eff}}$$

- **Effective Length** (L_{eff}) – effective nonlinear length of a fibre with physical length L and loss given by α.

$$L_{eff} = \frac{1-e^{-\alpha L}}{\alpha}$$

- **Nonlinear Length** (L_{NL}) – the fibre length required for nonlinear effects to become important, for a given peak pump power. Explicity, the length for development of a phase shift of unity.

$$L_{NL} = \frac{1}{\gamma P_0}$$

- **Dispersion Length** (L_d) – length over which the pulse length τ_0 is significantly dispersed in a fibre with β_2.

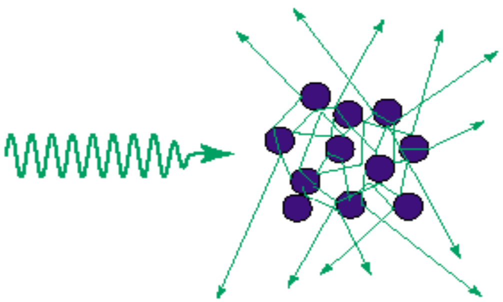
$$L_d = \frac{\tau_0^2}{\beta_2}$$

2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

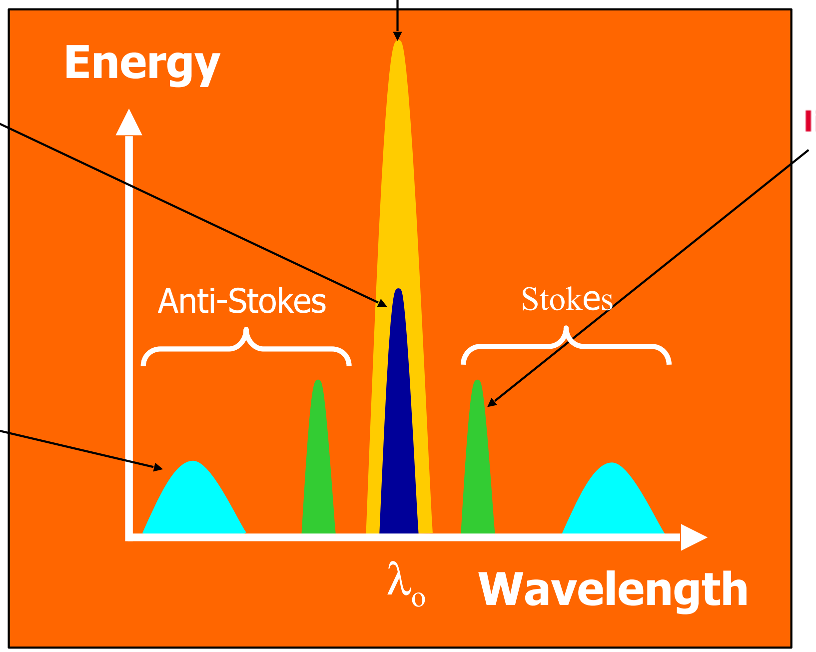
Scattering spectra of an optical fibre

Rayleigh radiation (λ_o)

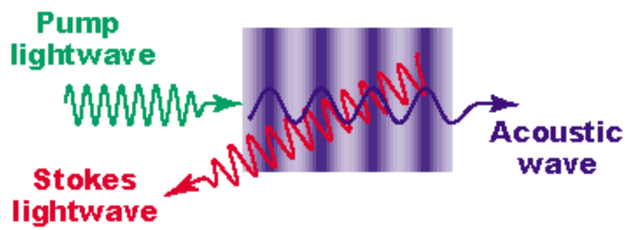


Incident radiation (λ_o=1550 nm)

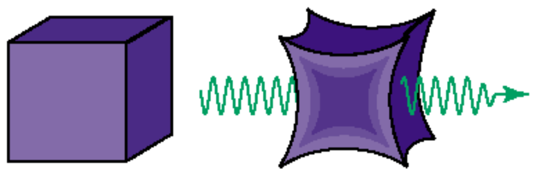
Raman radiation (λ_o ±80 nm)



Brillouin radiation (λ_o±0.08 nm)



$$n_B = \frac{2nV_s}{\lambda_o}$$



Electrostriction

Raman shift ≈ -13 THz @ 1.55 μm
Brillouin shift ≈ -11 GHz @ 1.55 μm

2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

2.14.1 Stimulated Raman Scattering (SRS)

(Spontaneous) Raman Scattering: a very small amount of light in any molecular is inelastic scattered. A lower-frequency photon is produced, and the extra energy goes into exciting a molecular vibrationsl or rotational mode.

Stimulated Raman Scattering (SRS): the lower-frequency radiation beats with the pump beam to provide a field beating at the Raman frequency. This drives the Raman oscillations directly, so that the shifted radiation experiences gain at the expense of the pump beam.

The growth of the Stokes wave along the fibre in both spontaneous and stimulated emission may be expressed in the form:

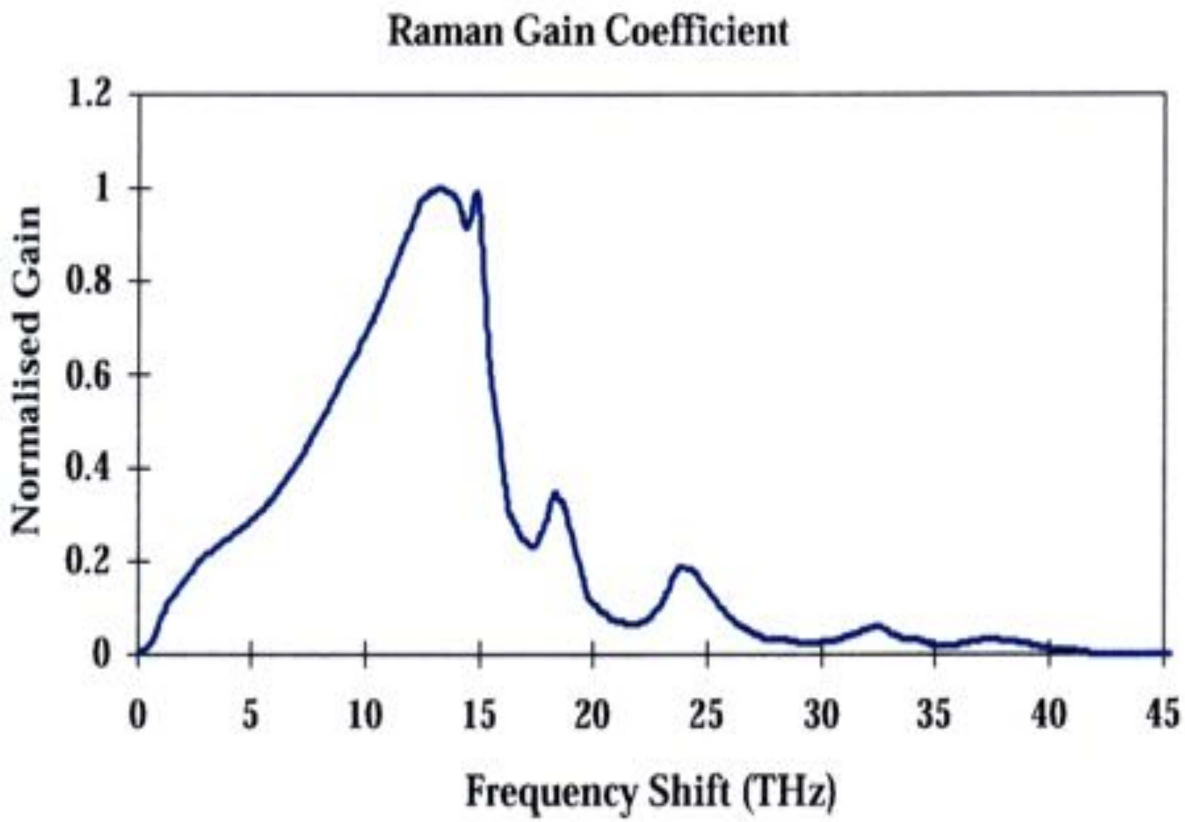
$$\frac{dI_S}{dz} = g_{SRS} I_S I_0 e^{-\alpha_P z} - \alpha_S I_S$$

Raman gain

2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

2.14.1 Stimulated Raman Scattering (SRS)



Relative polarization factor
$(1 \leq k \leq 2)$

$$P_{th}^{SRS} \cong 16 \frac{kA_{eff}}{g_{SRS} L_{eff}}$$

$$P_{th}^{SRS} \cdot L_{eff} \approx 17 \text{ W} \cdot \text{km} \; @ \; 1.55$$

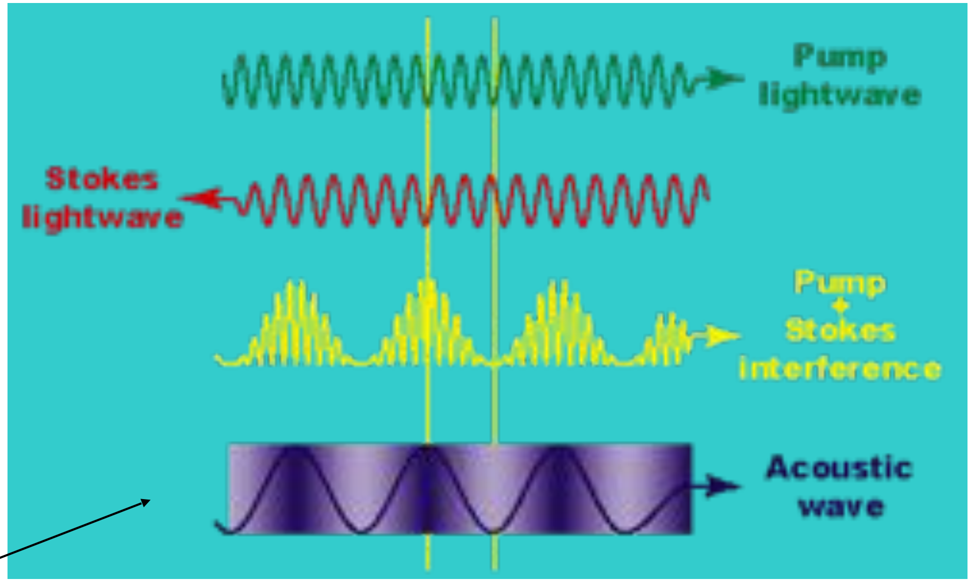
2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

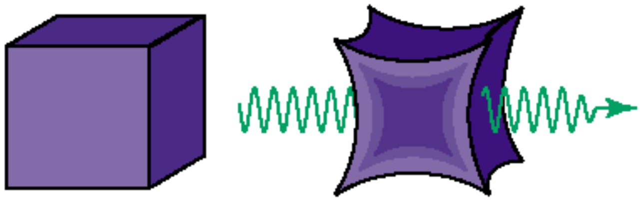
2.14.2 Stimulated Brillouin Scattering (SBS)

SBS Characteristics:

- Low power threshold
- Backward propagating Stokes wave
- Small Stokes shift (low phonon energy)
- Acoustic phonon lifetime is long (10ns), so gain bandwidth is narrow.
- Need narrow linewidth pump source for efficient excitation.



Electrostriction



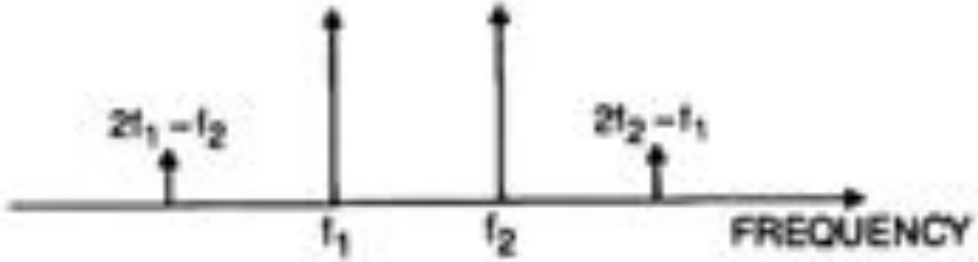
$$P_{th}^{SBS} \cong 21\left(1+\frac{\Delta\nu_{pump}}{\Delta\nu_{B}}\right)\frac{A_{eff}}{g_{SBS}L_{eff}}$$

$$P_{th}^{SBS} \cdot L_{eff} \approx 0.03\ \mathrm{W \cdot km}\ \ @\ 1.55$$

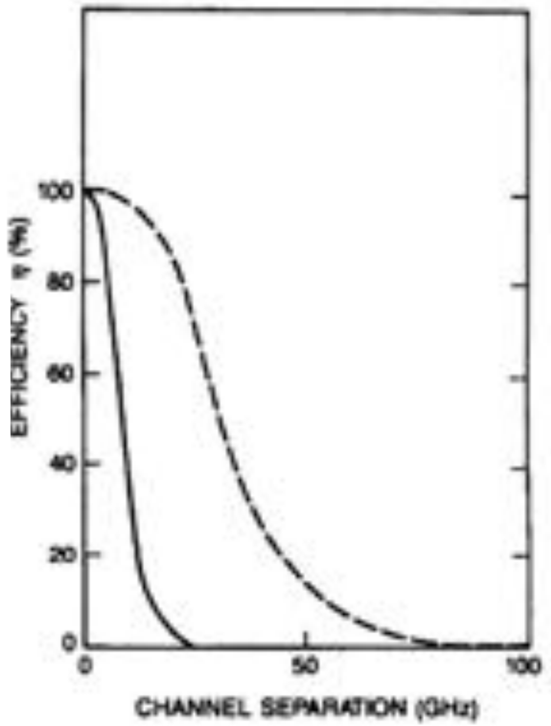
2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

2.14.3 Four-Wave Mixing (FWM)



Four-photon mixing with two injected waves at frequencies f_1 and f_2.



$$P_{ij}(L) = \eta_{FWM} \left(\frac{1024\pi^6}{n^4 \lambda^2 c^2} \right) \cdot \left(\frac{3\chi_3 L_{eff}}{A_{eff}} \right)^2 P_i P_j e^{-\alpha L}$$

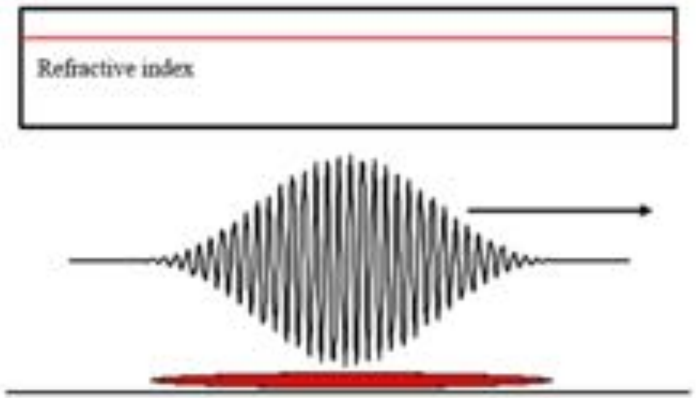
see A.R. Chraplyvy, Journal Lightwave Technology **8**, 1548-1557 (1990).

2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

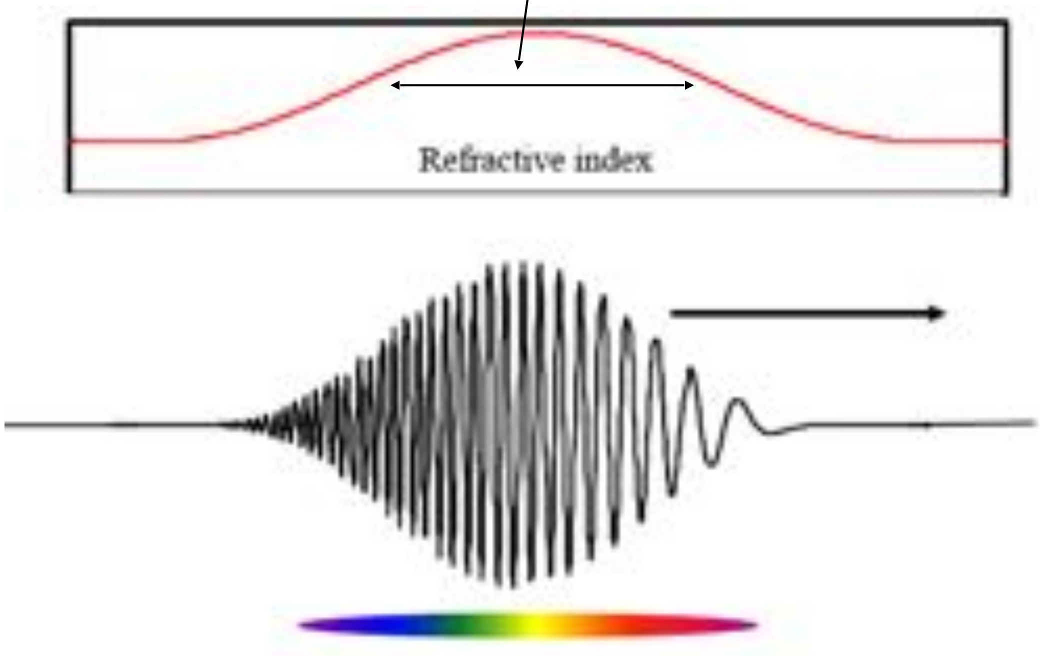
2.14.4 Self-Phase (SPM) and Cross-Phase Modulation (XPM)

Linear regime



Kerr nonlinearity

$$\Delta\nu_{SPM} \propto n_{2K} \frac{d}{dt}\left(E^2\right)$$



$$P_{th}^{SPM} \cong \frac{\lambda A_{eff}}{\pi n_{2K} L_{eff}}$$

$$P_{th}^{SPM} \cdot L_{eff} \approx 1.5 \text{ W} \cdot \text{km} \ @ \ 1.55$$

2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

2.14.4 Self-Phase (SPM) and Cross-Phase Modulation (XPM)

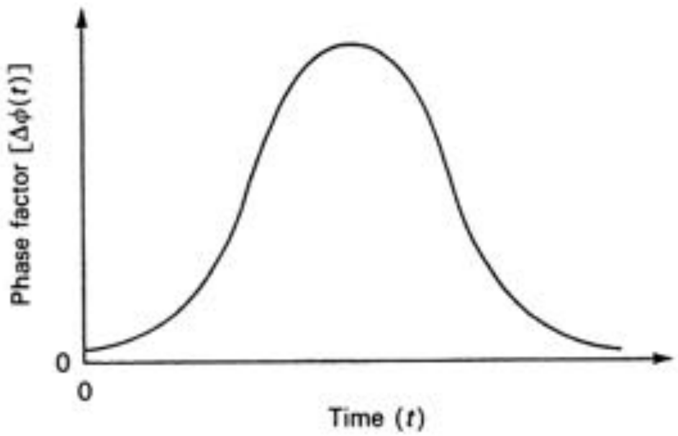
$$\left.\begin{aligned} \phi &= \frac{2\pi}{\lambda} nL \\ n &= n_0 + n_{2K} E^2 \end{aligned}\right\} \Rightarrow \phi = \frac{2\pi L}{\lambda}\left(n_0 + n_{2K} E^2\right)$$

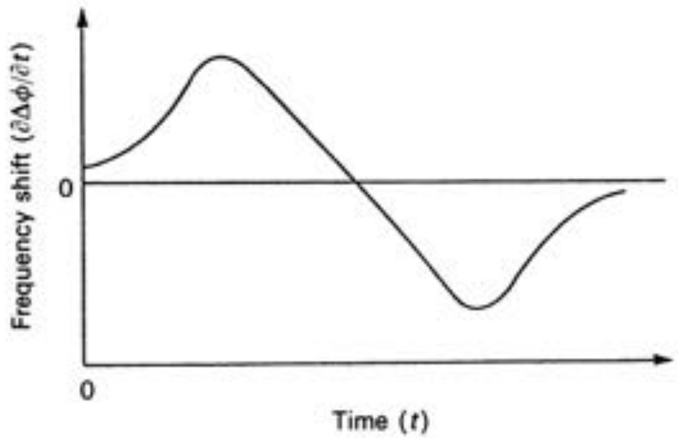
$$\omega' = \omega_0 + \omega \Leftrightarrow \omega' = \omega_0 + \frac{d\phi}{dt} \Rightarrow$$

$$\Rightarrow \omega' = \omega_0 + \frac{2\pi L}{\lambda} n_{2K} \frac{d}{dt}\left(E^2\right)$$

$$\left.\begin{aligned} \frac{d}{dt}\left(E^2\right) > 0 &\Rightarrow \omega' = \omega_0 - \omega_K(t) \\ \frac{d}{dt}\left(E^2\right) < 0 &\Rightarrow \omega' = \omega_0 + \omega_K(t) \end{aligned}\right\}$$

Pulse is CHIRPED



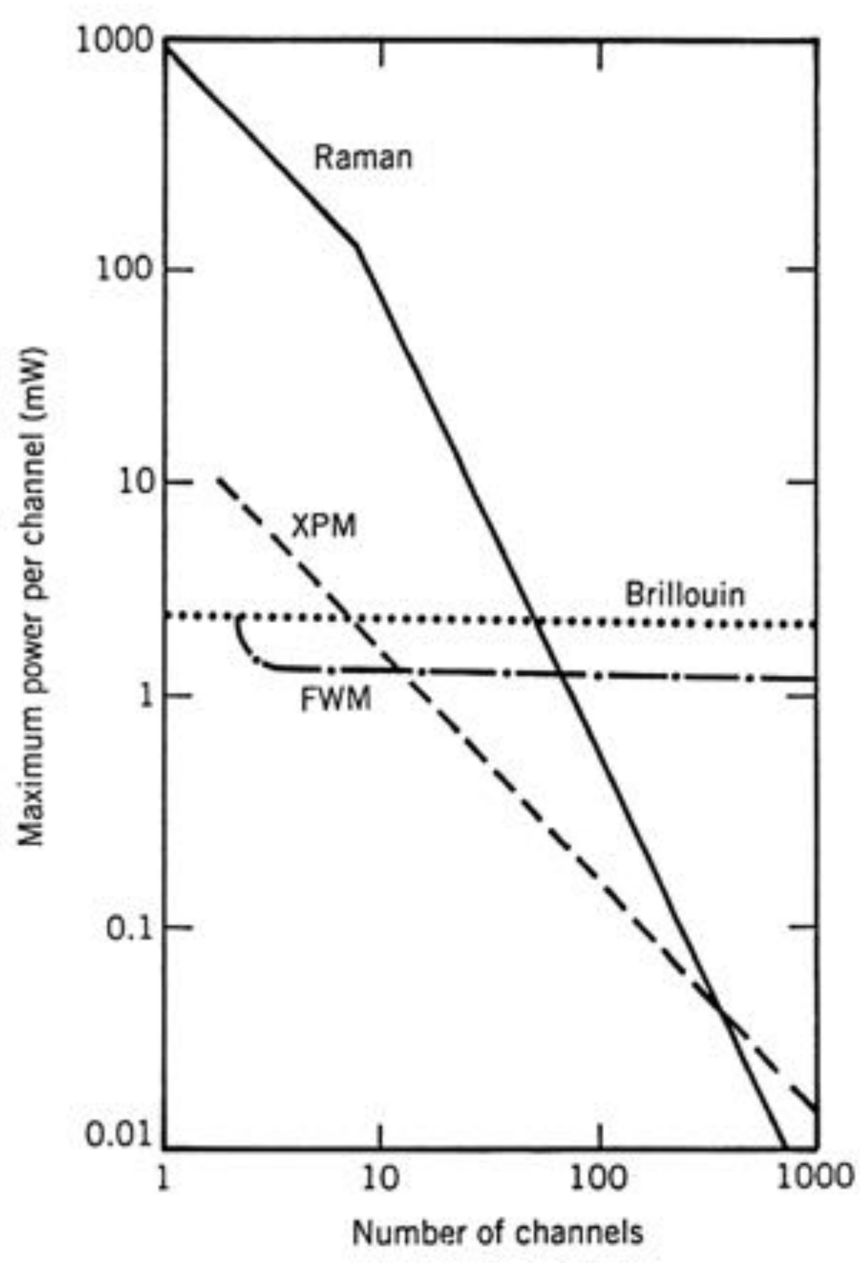


2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

Limitations over WDM

Limitations on the channel power imposed by four nonlinear effects (assumed λ=1.55 μm and fibre loss of 0.2 dB/km)



see A.R. Chraplyvy, Journal Lightwave Technology **8**, 1548-1557 (1990).

2. Light Propagation in Fibres and Related Optical Effects

2.14 Non-linear Optical Effects.

Fundamental soliton peak power

$$P_0 \simeq \frac{3.11\beta_2}{\gamma T_{FWHM}^2}$$

Soliton Propagation