Fiber Modes

- Maxwell's equations in the Fourier domain lead to

$$\nabla^2 \tilde{\mathbf{E}} + n^2(\omega) k_0^2 \tilde{\mathbf{E}} = 0.$$

- $n = n_1$ inside the core but changes to n_2 in the cladding.
- Useful to work in cylindrical coordinates ρ, ϕ, z.
- Common to choose E_z and H_z as independent components.
- Equation for E_z in cylindrical coordinates:

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + n^2 k_0^2 E_z = 0.$$

- H_z satisfies the same equation.

Fiber Modes (cont.)

- Use the method of separation of variables:

$$E_z(\rho,\phi,z) = F(\rho)\Phi(\phi)Z(z).$$

- We then obtain three ODEs:

$$d^2Z/dz^2 + \beta^2 Z = 0,$$

$$d^2\Phi/d\phi^2 + m^2\Phi = 0,$$

$$\frac{d^2F}{d\rho^2} + \frac{1}{\rho}\frac{dF}{d\rho} + \left(n^2k_0^2 - \beta^2 - \frac{m^2}{\rho^2}\right)F = 0.$$

- β and m are two constants (m must be an integer).
- First two equations can be solved easily to obtain

$$Z(z) = \exp(i\beta z), \qquad \Phi(\phi) = \exp(im\phi).$$

- $F(\rho)$ satisfies the Bessel equation.

Fiber Modes (cont.)

- General solution for E_z and H_z:

$$E_z = \begin{cases} A J_m(p\rho)\exp(im\phi)\exp(i\beta z)\,; & \rho \le a, \\ C K_m(q\rho)\exp(im\phi)\exp(i\beta z); & \rho > a. \end{cases}$$

$$H_z = \begin{cases} B J_m(p\rho)\exp(im\phi)\exp(i\beta z)\,; & \rho \le a, \\ D K_m(q\rho)\exp(im\phi)\exp(i\beta z); & \rho > a. \end{cases}$$

$$p^2 = n_1^2 k_0^2 - \beta^2, \quad q^2 = \beta^2 - n_2^2 k_0^2.$$

- Other components can be written in terms of E_z and H_z:

$$E_\rho = \frac{i}{p^2}\left(\beta \frac{\partial E_z}{\partial \rho} + \mu_0 \frac{\omega}{\rho}\frac{\partial H_z}{\partial \phi}\right), \qquad E_\phi = \frac{i}{p^2}\left(\frac{\beta}{\rho}\frac{\partial E_z}{\partial \phi} - \mu_0 \omega \frac{\partial H_z}{\partial \rho}\right),$$

$$H_\rho = \frac{i}{p^2}\left(\beta \frac{\partial H_z}{\partial \rho} - \varepsilon_0 n^2 \frac{\omega}{\rho}\frac{\partial E_z}{\partial \phi}\right), \qquad H_\phi = \frac{i}{p^2}\left(\frac{\beta}{\rho}\frac{\partial H_z}{\partial \phi} + \varepsilon_0 n^2 \omega \frac{\partial E_z}{\partial \rho}\right).$$

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Eigenvalue Equation

- Boundary conditions: E_z, H_z, E_ϕ, and H_ϕ should be continuous across the *core–cladding interface*.

- Continuity of E_z and H_z at $\rho = a$ leads to
  $AJ_m(pa) = CK_m(qa), \quad BJ_m(pa) = DK_m(qa).$

- Continuity of E_ϕ and H_ϕ provides two more equations.

- Four equations lead to the eigenvalue equation

$$\left[\frac{J_m'(pa)}{pJ_m(pa)} + \frac{K_m'(qa)}{qK_m(qa)}\right]\left[\frac{J_m'(pa)}{pJ_m(pa)} + \frac{n_2^2}{n_1^2}\frac{K_m'(qa)}{qK_m(qa)}\right]$$

$$= \frac{m^2}{a^2}\left(\frac{1}{p^2} + \frac{1}{q^2}\right)\left(\frac{1}{p^2} + \frac{n_2^2}{n_1^2}\frac{1}{q^2}\right)$$

$$p^2 = n_1^2 k_0^2 - \beta^2, \quad q^2 = \beta^2 - n_2^2 k_0^2.$$

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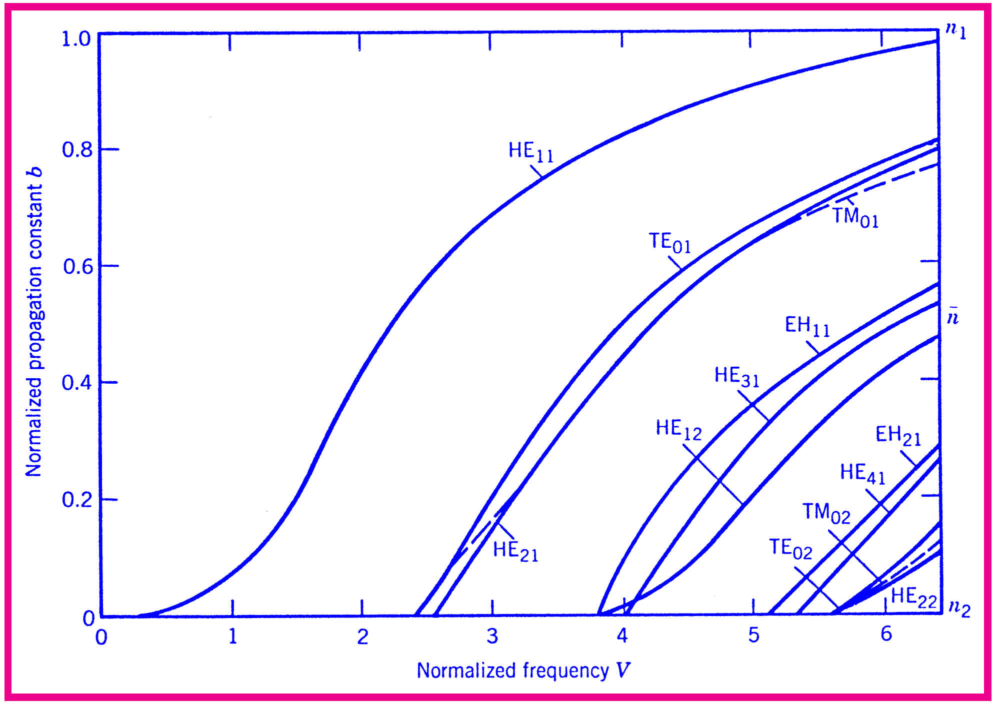
Eigenvalue Equation

- Eigenvalue equation involves Bessel functions and their derivatives. It needs to be solved numerically.
- Noting that $p^2 + q^2 = (n_1^2 - n_2^2)k_0^2$, we introduce the dimensionless V parameter as
$$V = k_0 a \sqrt{n_1^2 - n_2^2}.$$
- Multiple solutions for β for a given value of V.
- Each solution represents an optical mode.
- Number of modes increases rapidly with V parameter.
- Effective mode index $\bar{n} = \beta / k_0$ lies between n_1 and n_2 for all bound modes.

Effective Mode Index



- Useful to introduce a normalized quantity

$$b = (\bar{n} - n_2)/(n_1 - n_2), \quad (0 < b < 1).$$

- Modes quantified through $\beta(\omega)$ or $b(V)$.

Classification of Fiber Modes

- In general, both E_z and H_z are nonzero (hybrid modes).
- Multiple solutions occur for each value of m.
- Modes denoted by HE$_{mn}$ or EH$_{mn}$ $(n = 1, 2, \ldots)$ depending on whether H_z or E_z dominates.
- TE and TM modes exist for $m = 0$ (called TE$_{0n}$ and TM$_{0n}$).
- Setting $m = 0$ in the eigenvalue equation, we obtain two equations

$$\left[\frac{J'_m(pa)}{pJ_m(pa)} + \frac{K'_m(qa)}{qK_m(qa)}\right] = 0, \qquad \left[\frac{J'_m(pa)}{pJ_m(pa)} + \frac{n_2^2}{n_1^2}\frac{K'_m(qa)}{qK_m(qa)}\right] = 0$$

- These equations govern TE$_{0n}$ and TM$_{0n}$ modes of fiber.

Linearly Polarized Modes

- Eigenvalue equation simplified considerably for weakly guiding fibers $(n_1 - n_2 \ll 1)$:

$$\left[\frac{J'_m(pa)}{pJ_m(pa)} + \frac{K'_m(qa)}{qK_m(qa)}\right]^2 = \frac{m^2}{a^2}\left(\frac{1}{p^2} + \frac{1}{q^2}\right)^2.$$

- Using properties of Bessel functions, the eigenvalue equation can be written in the following compact form:

$$p\frac{J_{l-1}(pa)}{J_l(pa)} = -q\frac{K_{l-1}(qa)}{K_l(qa)},$$

where $l = 1$ for TE and TM modes, $l = m - 1$ for HE modes, and $l = m + 1$ for EH modes.

- $\mathrm{TE}_{0,n}$ and $\mathrm{TM}_{0,n}$ modes are degenerate. Also, $\mathrm{HE}_{m+1,n}$ and $\mathrm{EH}_{m-1,n}$ are degenerate in this approximation.

Linearly Polarized Modes

- Degenerate modes travel at the same velocity through fiber.
- Any linear combination of degenerate modes will travel without change in shape.
- Certain linearly polarized combinations produce LP_{mn} modes.
  - ⋆ LP_{0n} is composed of of HE_{1n}.
  - ⋆ LP_{1n} is composed of $\mathrm{TE}_{0n} + \mathrm{TM}_{0n} + \mathrm{HE}_{2n}$.
  - ⋆ LP_{mn} is composed of $\mathrm{HE}_{m+1,n} + \mathrm{EH}_{m-1,n}$.
- Historically, LP modes were obtained first using a simplified analysis of fiber modes.

Fundamental Fiber Mode

- A mode ceases to exist when $q = 0$ (no decay in the cladding).
- TE$_{01}$ and TM$_{01}$ reach cutoff when $J_0(V) = 0$.
- This follows from their eigenvalue equation

$$p\frac{J_0(pa)}{J_1(pa)} = -q\frac{K_0(qa)}{K_1(qa)}$$

after setting $q = 0$ and $pa = V$.

- Single-mode fibers require $V < 2.405$ (first zero of J_0).
- They transport light through the fundamental HE$_{11}$ mode.
- This mode is almost linearly polarized ($|E_z|^2 \ll |E_x|^2$)

$$E_x(\rho, \phi, z) = \begin{cases} A[J_0(p\rho)/J_0(pa)]e^{i\beta z}\,; & \rho \le a, \\ A[K_0(q\rho)/K_0(qa)]e^{i\beta z}; & \rho > a. \end{cases}$$

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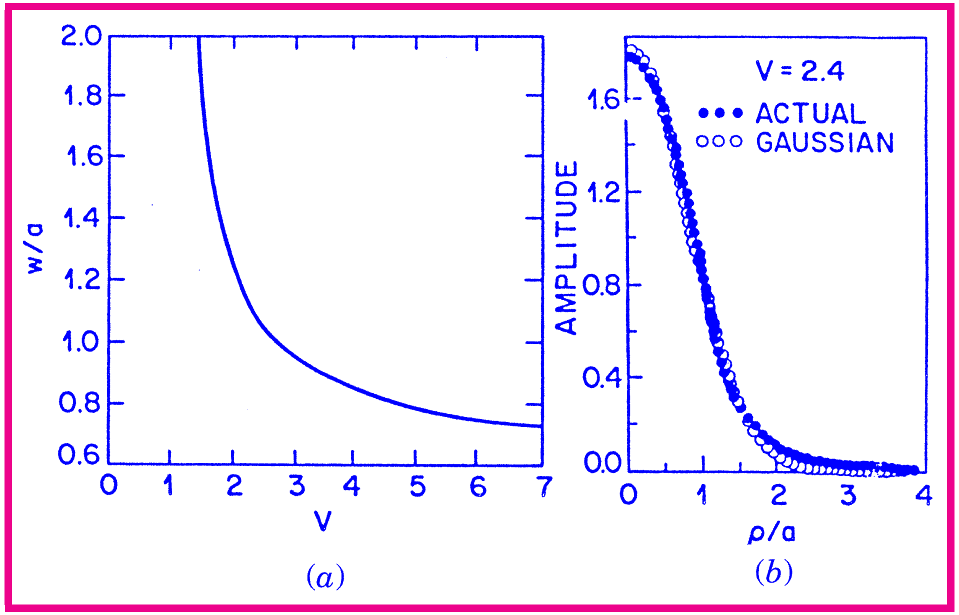
Fundamental Fiber Mode

- Use of Bessel functions is not always practical.
- It is possible to approximate spatial distribution of HE_{11} mode with a Gaussian for V in the range 1 to 2.5.
- $E_x(\rho, \phi, z) \approx A \exp(-\rho^2/w^2) e^{i\beta z}$.
- Spot size w depends on V parameter.



Single-Mode Properties

- Spot size: $w/a \approx 0.65 + 1.619V^{-3/2} + 2.879V^{-6}$.
- Mode index:

$$\bar{n} = n_2 + b(n_1 - n_2) \approx n_2(1 + b\Delta),$$

$$b(V) \approx (1.1428 - 0.9960/V)^2.$$

- Confinement factor:

$$\Gamma = \frac{P_{\rm core}}{P_{\rm total}} = \frac{\int_0^a |E_x|^2 \rho\, d\rho}{\int_0^\infty |E_x|^2 \rho\, d\rho} = 1 - \exp\left(-\frac{2a^2}{w^2}\right).$$

- $\Gamma \approx 0.8$ for $V = 2$ but drops to 0.2 for $V = 1$.
- Mode properties completely specified if V parameter is known.

Fiber Birefringence

- Real fibers exhibit some birefringence $(\bar{n}_x \neq \bar{n}_y)$.
- Modal birefringence quite small $(B_m = |\bar{n}_x - \bar{n}_y| \sim 10^{-6})$.
- Beat length: $L_B = \lambda / B_m$.
- State of polarization evolves periodically.
- Birefringence varies randomly along fiber length (PMD) because of stress and core-size variations.