Interpolação

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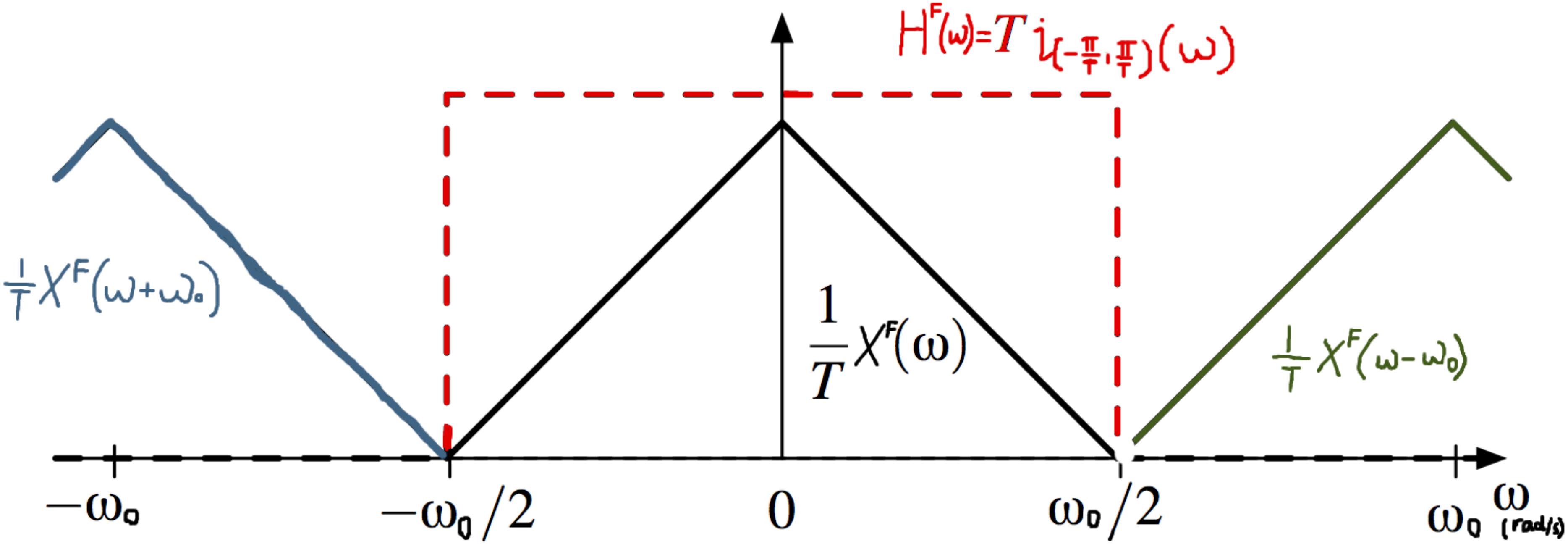
Interpolation

- In the mathematical field of numerical analysis, **interpolation** is a method of constructing new data points within the range of a discrete set of known data points.
- In engineering and science, one often has a number of data points, obtained by sampling or experimentation, which represent the values of a function for a limited number of values of the independent variable. It is often required to **interpolate** (i.e. estimate) the value of that function for an intermediate value of the independent variable. This may be achieved by curve fitting or regression analysis.

Recall



$H^F(\omega)=T\,1_{(-\frac{\pi}{T},\frac{\pi}{T})}(\omega)$
$\frac{1}{T}X^F(\omega+\omega_0)$
$\frac{1}{T}X^F(\omega)$
$\frac{1}{T}X^F(\omega-\omega_0)$
$-\omega_0$
$-\omega_0/2$
0
$\omega_0/2$
ω_0
ω
(rad/s)

Impulse response of an ideal low-pass filter

$$h(t) = T\,\frac{\pi/T}{\pi}\,\mathrm{sinc}\left(\frac{\pi/T}{\pi}\,t\right) = \mathrm{sinc}\left(\frac{t}{T}\right)$$

The reconstructed signal

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]\,h(t-nT) = \sum_{n=-\infty}^{\infty} x[n]\,\mathrm{sinc}\left(\frac{t-nT}{T}\right)$$

The reconstructed signal is a train of sinc pulses scaled by the samples x[n]

This system is difficult to implement because each sinc pulse extends over a long (theoretically infinite) time interval.

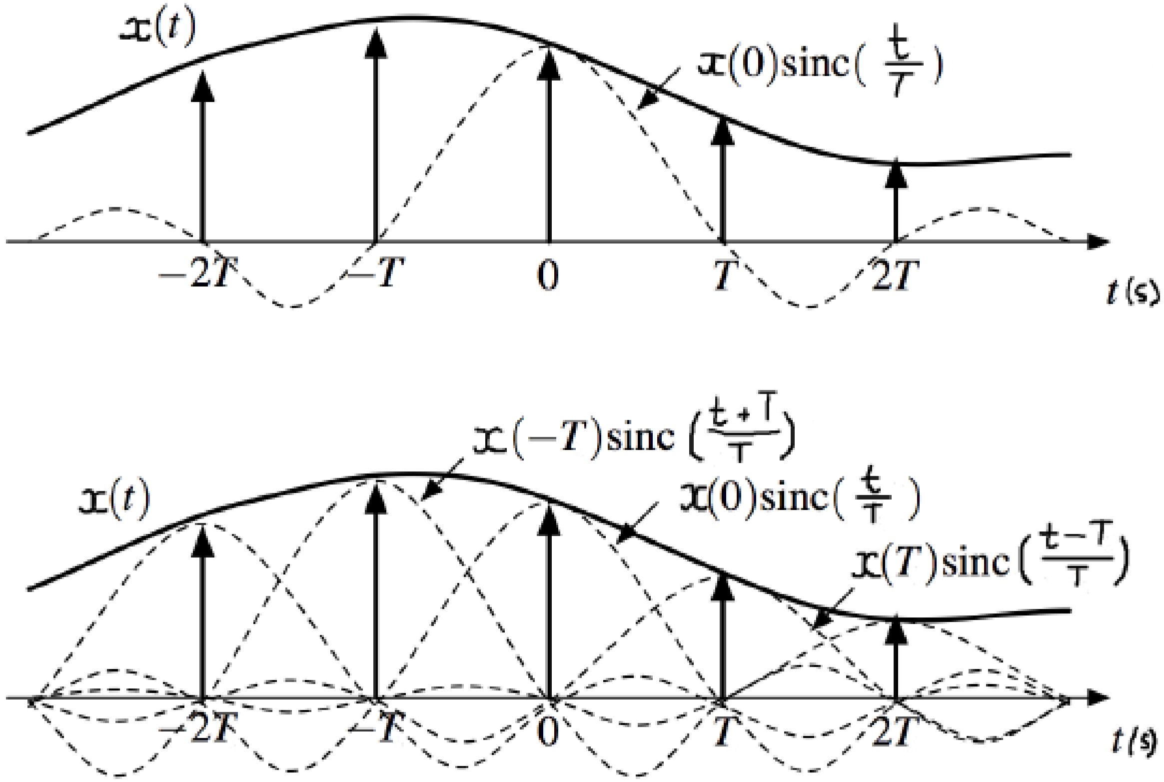


Figure 13: The interpolated signal is a sum of shifted sincs, weighted by the samples $x(nT)$. The sinc function $h(t) = \mathrm{sinc}(t/T)$ shifted to nT, i.e. $h(t-T)$, is equal to one at nT and zero at all other samples lT, $l \neq n$. The sum of the weighted shifted sincs will agree with all samples $x(nT)$, n integer.

Zero-Order hold

Many practical reconstruction systems use zero-order hold circuits for reconstruction.

Because rectangular pulses are easier to generate than sinc pulses.

- Replace the ideal sinc with a rectangular pulse[7]

$$h_{\mathrm{ZOH}}(t) = \mathrm{rect}\left(\frac{t - 0.5\,T}{T}\right)$$

yielding

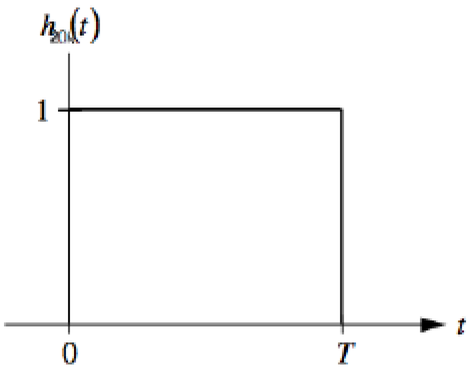
$$x_{\mathrm{ZOH}}(t) = \sum_{n=-\infty}^{+\infty} x[n]\, h_{\mathrm{ZOH}}(t - n\,T).$$

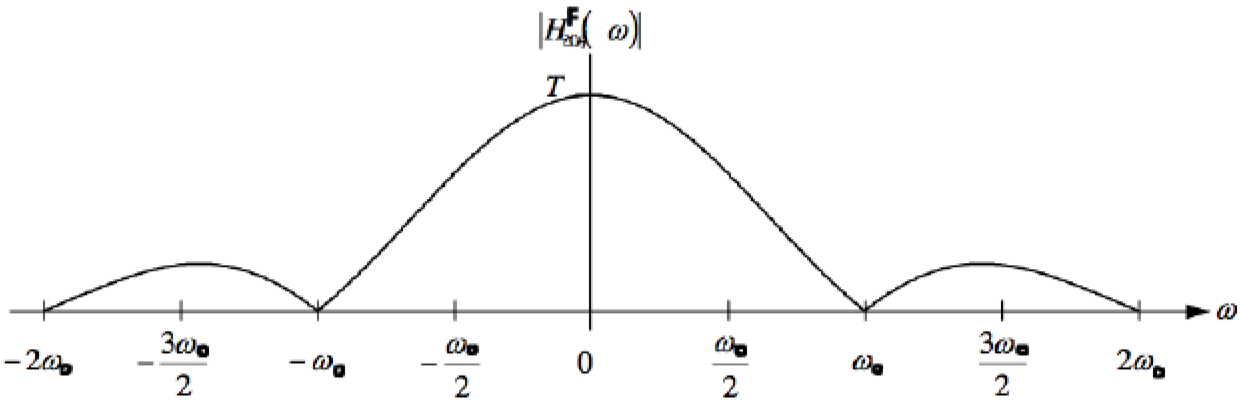
Zero-Order hold

Frequency response of the zero-order hold:

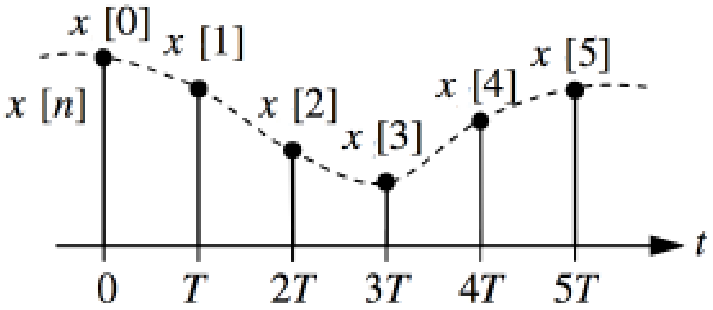
$$H^{\mathrm{F}}_{\mathrm{ZOH}}(\omega) = \int_0^T e^{-j\,\omega\,t}\,dt = \frac{1-e^{-j\,\omega\,T}}{j\,\omega} = T\,\mathrm{sinc}\left(\frac{\omega\,T}{2\,\pi}\right)e^{-j\,0.5\,\omega\,T} = T\,\mathrm{sinc}\left(\frac{\omega}{\omega_0}\right)e^{-j\,\pi\,\frac{\omega}{\omega_0}}$$

recall $\omega_0 = 2\,\pi/T$ and (1).





Zero-Order hold



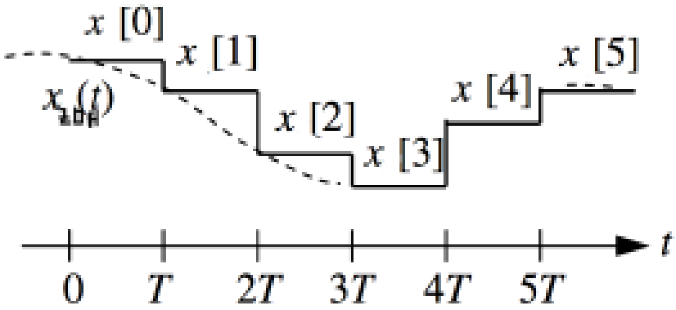
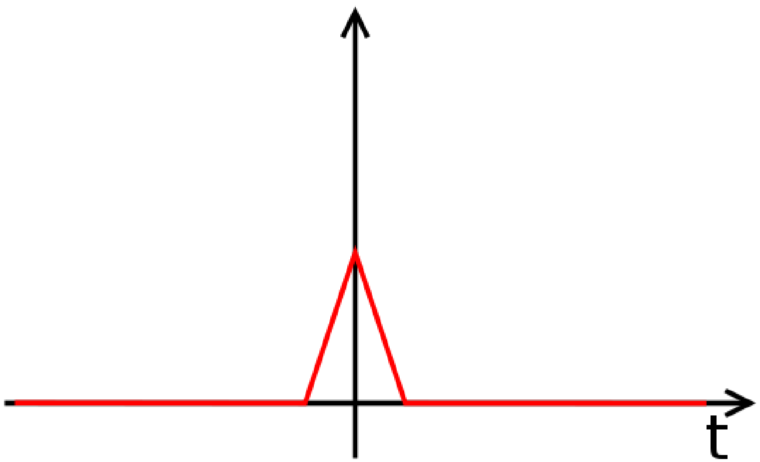
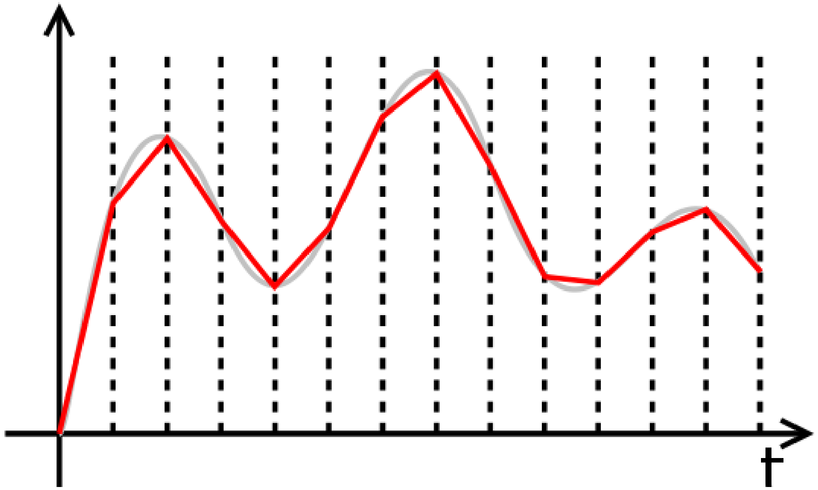


Figure 16: The zero-order hold output $x_{\text{ZOH}}(t)$ is a train of rectangular pulses scaled by the samples $x[n]$ (a staircase approximation of $x(t)$), easy to generate.

First-order hold

$$x_{\mathrm{FOH}}(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\mathrm{tri}\left(\frac{t-nT}{T}\right) \qquad h_{\mathrm{FOH}}(t) = \frac{1}{T}\mathrm{tri}\left(\frac{t}{T}\right) = \begin{cases} \frac{1}{T}\left(1-\frac{|t|}{T}\right) & \text{if } |t| < T \\ 0 & \text{otherwise} \end{cases}$$





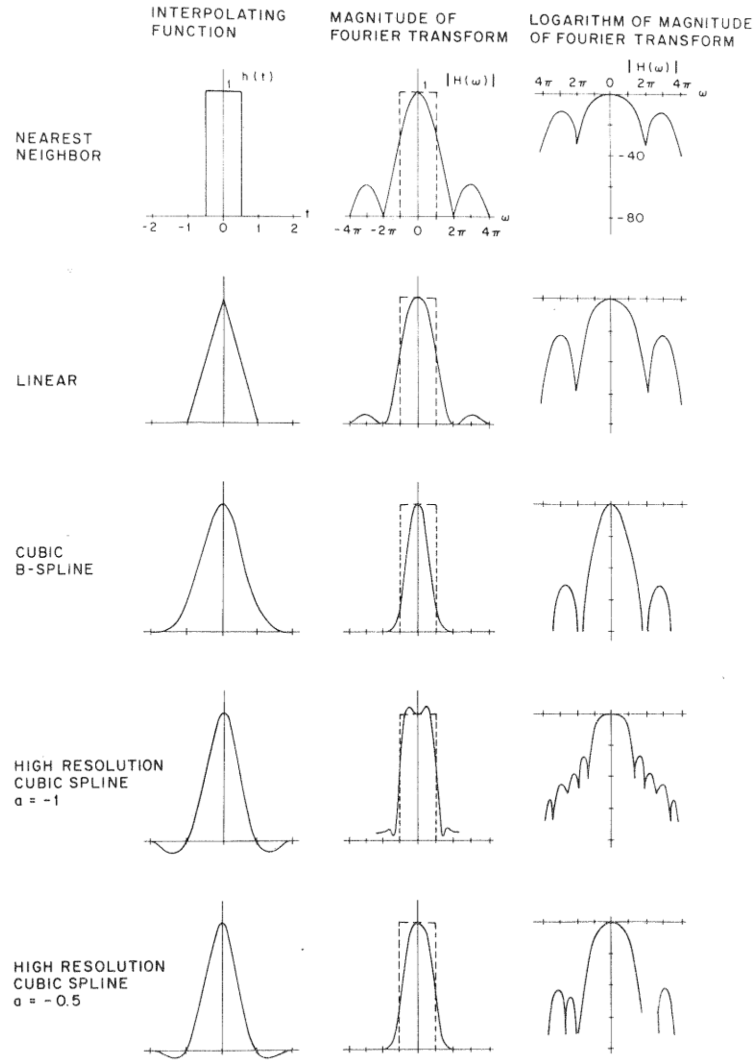


Fig. 4. Interpolating functions. This figure shows several interpolating functions (first column), their Fourier transforms (second column), and the logarithm of their Fourier transforms on an 80 dB scale (third column). The dashed box shows and ideal low-pass filter with cutoff at pi times the sampling frequency. The performance in the pass zone can be best appreciated on the linear plot of the Fourier transform; the stop zone performance can best be appreciated on the logarithm of the Fourier transform plot. The best performance (the best approximation to an ideal low-pass filter) is provided by the high-resolution cubic spline function with $a = -0.5$.

Parker et al. Comparing interpolation methods for image resampling
IEEE Transactions on Medical Imaging, 1983